Connecting time crystals to many-body protection

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Synopsis

What are time crystals?

Dissipative time crystals (DTC)

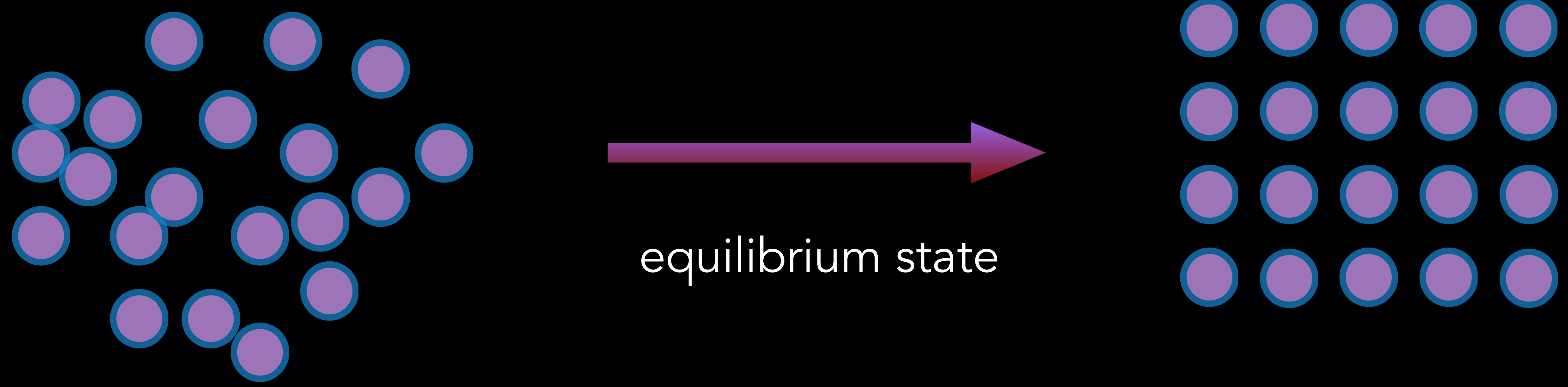
Van der Pol Oscillator: simplest DTC?

Spin models: testing our expectations

Conclusions

What are time crystals?

Space crystals



spontaneous **space**-translational symmetry breaking

robust against local fluctuations

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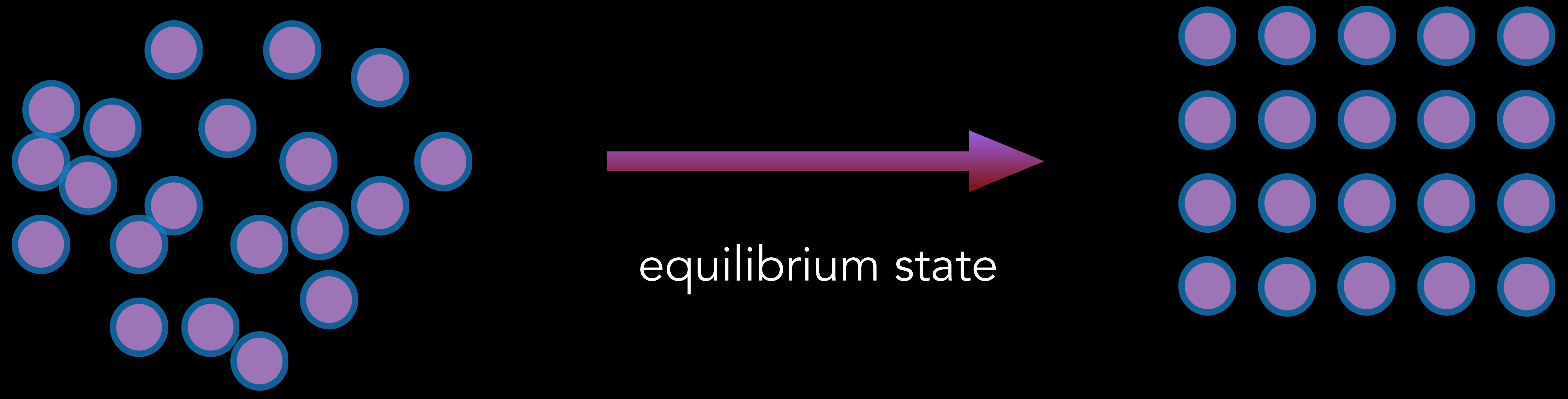

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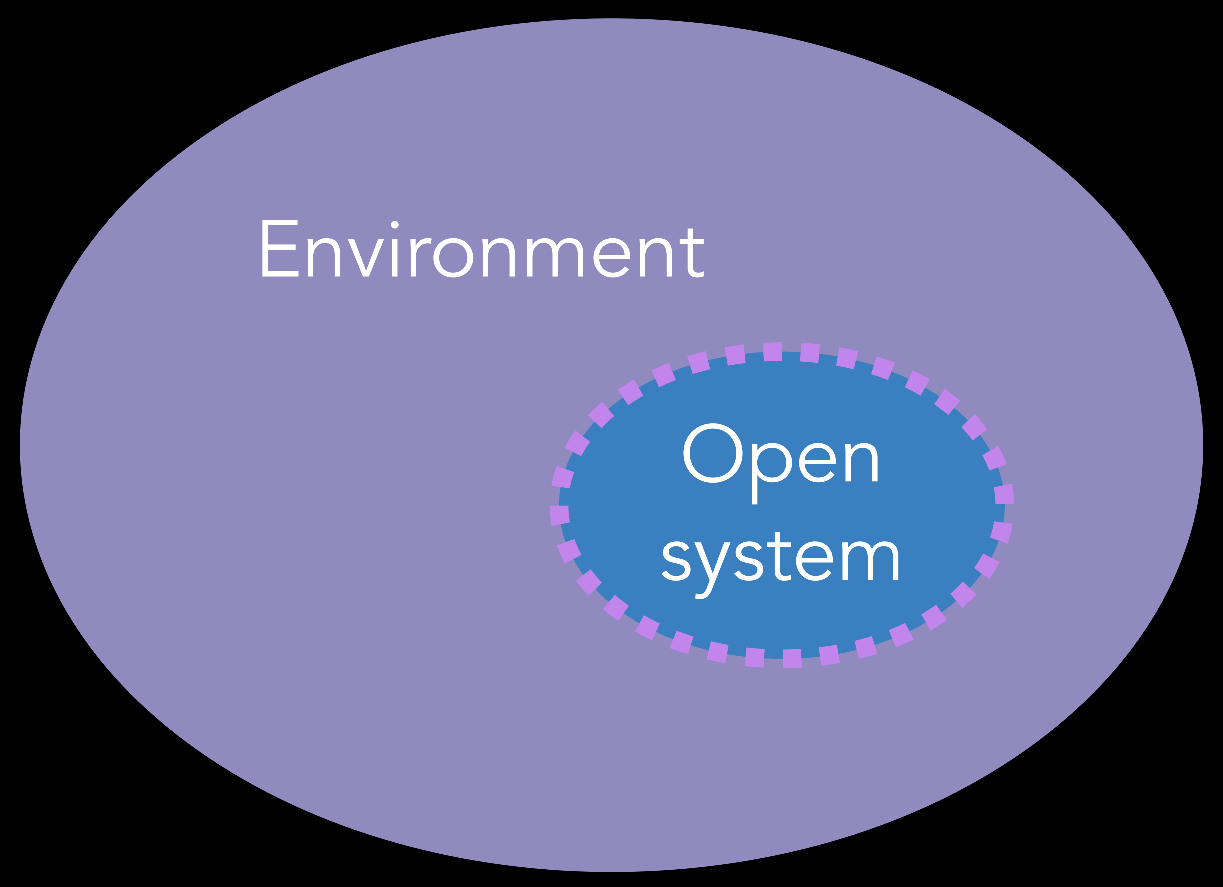
Time crystals

Introduced by Wilczek at 2012: Idea of **time**-translational breaking symmetry as temporal counterpart of spatial crystal

F. Wilczek, “Quantum time crystals,” Phys. Rev. Lett. **109**, 160401 (2012)

Dissipative time crystals (DTC)

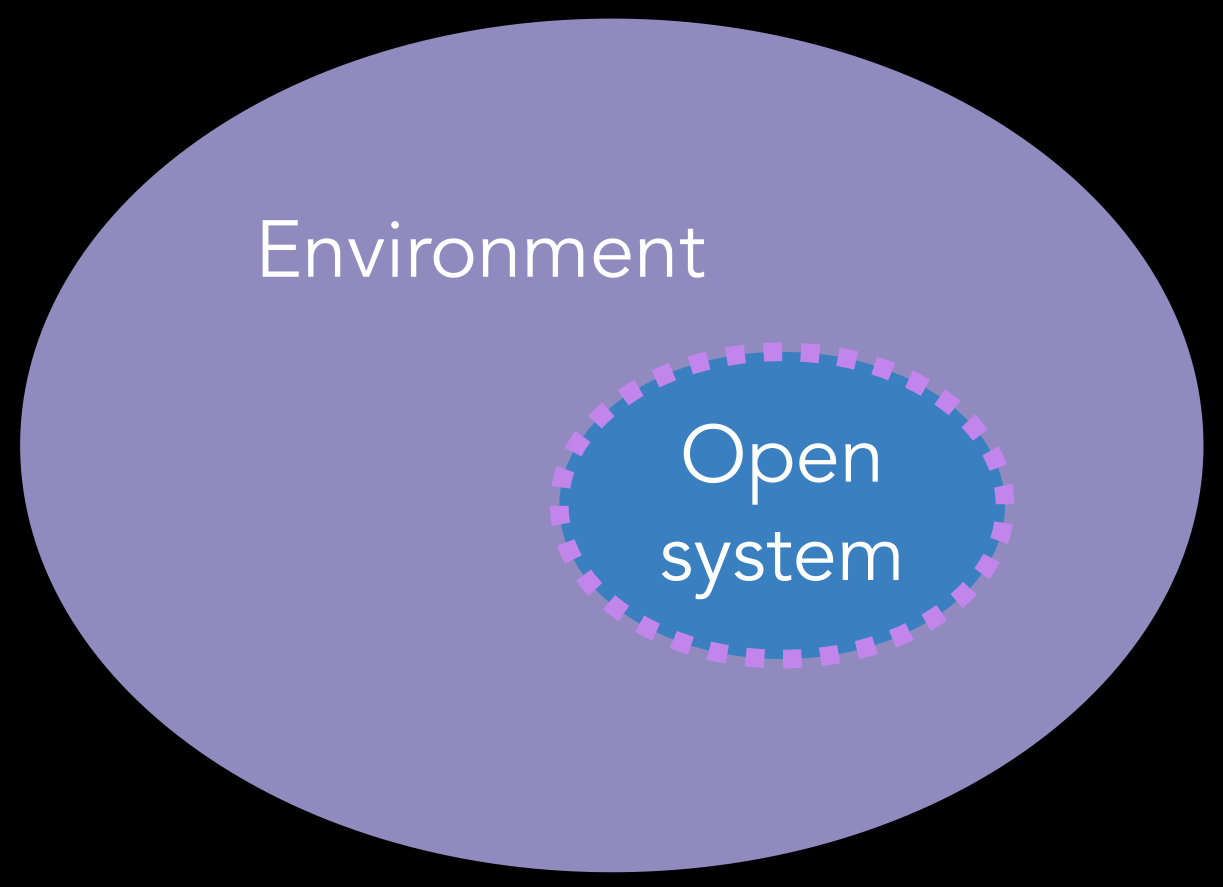


Energy and Information exchange with the surrounding environment (dissipation,…)

Imbalance between gain and loss from outside energy (no thermal equilibrium)

Open and **non** equilibrium systems

Dissipative time crystals (DTC)



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Open and **non** equilibrium systems

Time-crystalline phase:

A state where, without external periodic forcing, the system settles into long-lived, robust oscillations.

Dissipative time crystals (DTC)

A time-independent Liouvillian $\mathcal{L}$ generates the dynamics of the open system as: $\partial_t \hat{\rho} = \mathcal{L}[\hat{\rho}]$

$$\frac{d\hat{\rho}}{dt} = \left[\frac{\hat{H}}{\mathrm{i}\hbar}, \hat{\rho}\right] + \sum_m \gamma_m \mathcal{D}_{J_m}[\hat{\rho}]$$

with

$$\mathcal{D}_J[\hat{\rho}] = 2\hat{J}\hat{\rho}\hat{J}^\dagger - \hat{J}^\dagger\hat{J}\hat{\rho} - \hat{\rho}\hat{J}^\dagger\hat{J}$$

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Generally:

$$\partial_t \hat{\rho} = \mathcal{L}[\hat{\rho}]$$

$$\hat{\rho}(t) = \sum_n e^{\lambda_n t}\,\hat{C}_n$$

Liouvillian spectrum

$\mathrm{Re}\{\lambda_n\} \leq 0$

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DTC requirement:

$$\exists\ \{\lambda_k = \mathrm{i}k\Omega\}_{k\in\mathbb{Z}}$$

such that:

$$\lim_{t\to\infty} \hat{\rho}(t) = \sum_{k\in\mathbb{Z}} e^{k\mathrm{i}\Omega t}\hat{c}_k$$

$\dfrac{2\pi}{\Omega}$ — periodic observables

Van der Pol oscillator: simplest DTC?

Single cavity model with Master Equation:

$$\frac{d\hat{\rho}}{dt} = \underbrace{[-\mathrm{i}\omega\hat{a}^\dagger\hat{a},\hat{\rho}]}_{\text{free evolution}} + \underbrace{\frac{\gamma}{2}\mathcal{D}_{a^2}[\hat{\rho}]}_{\text{nonlinear losses}} + \underbrace{\mathcal{D}_{a^\dagger}[\hat{\rho}]}_{\text{pumping}} = \mathcal{L}[\hat{\rho}]$$

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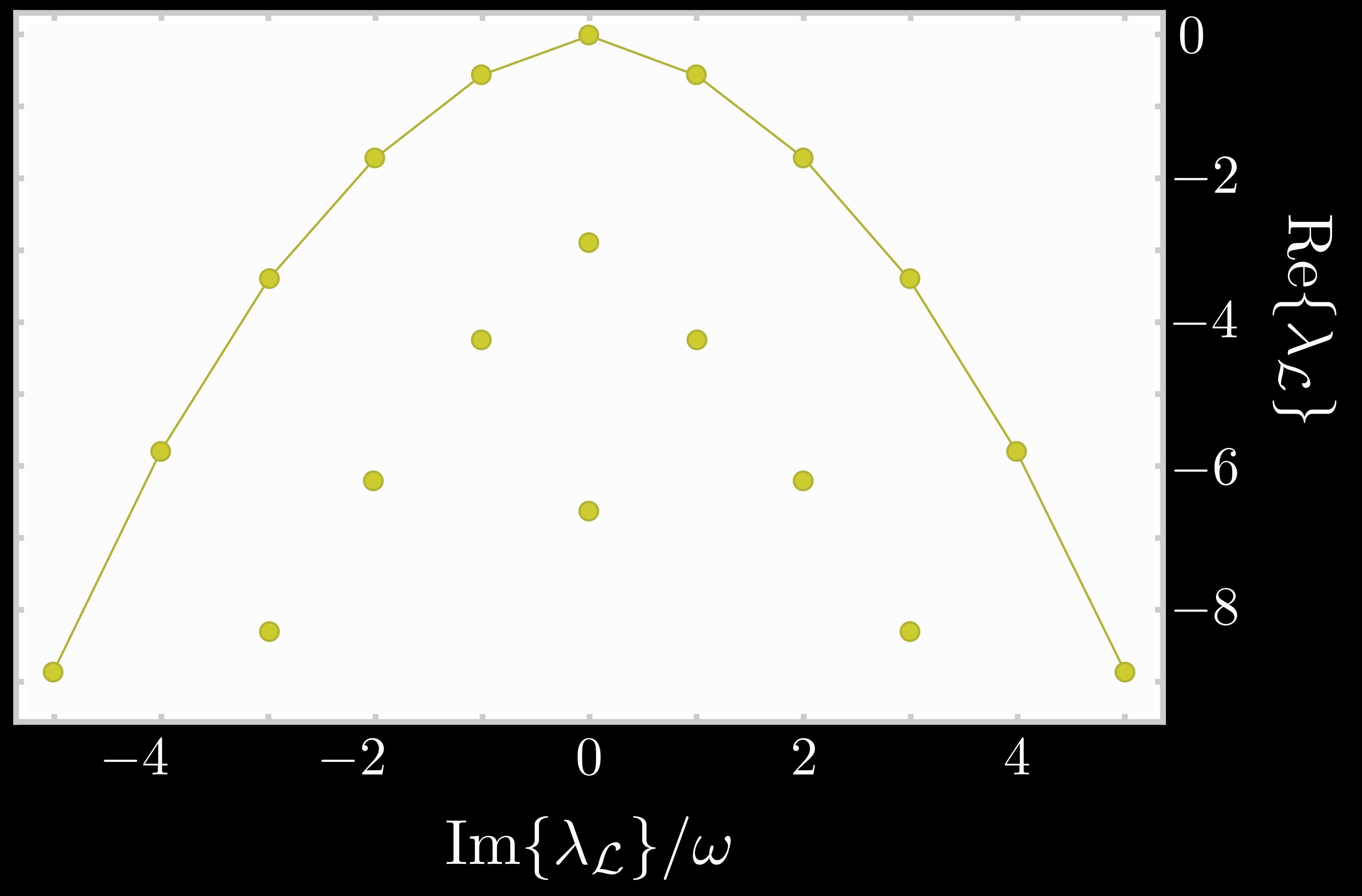
Numerical Liouvillian spectrum:

$\gamma = 1$



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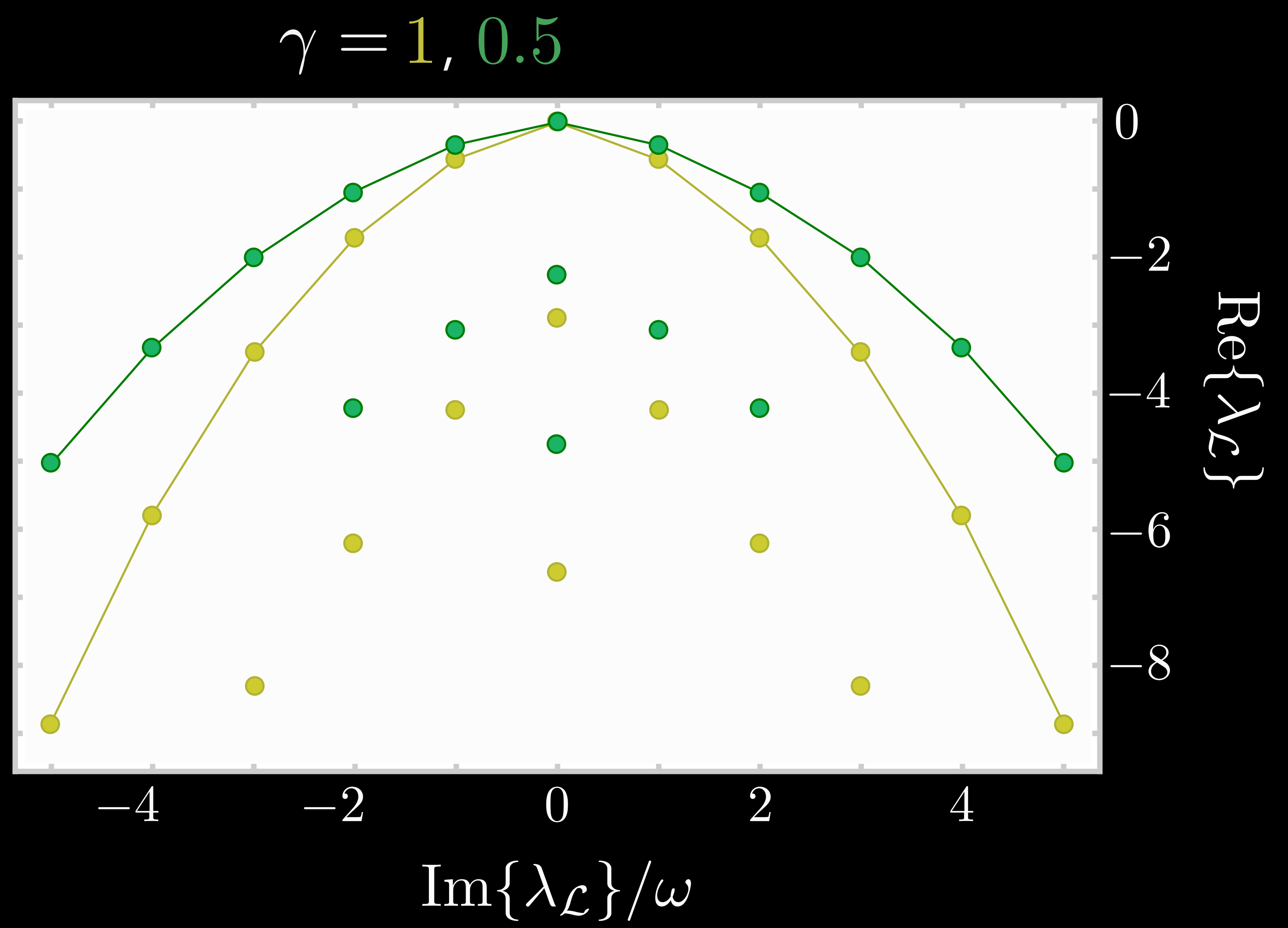

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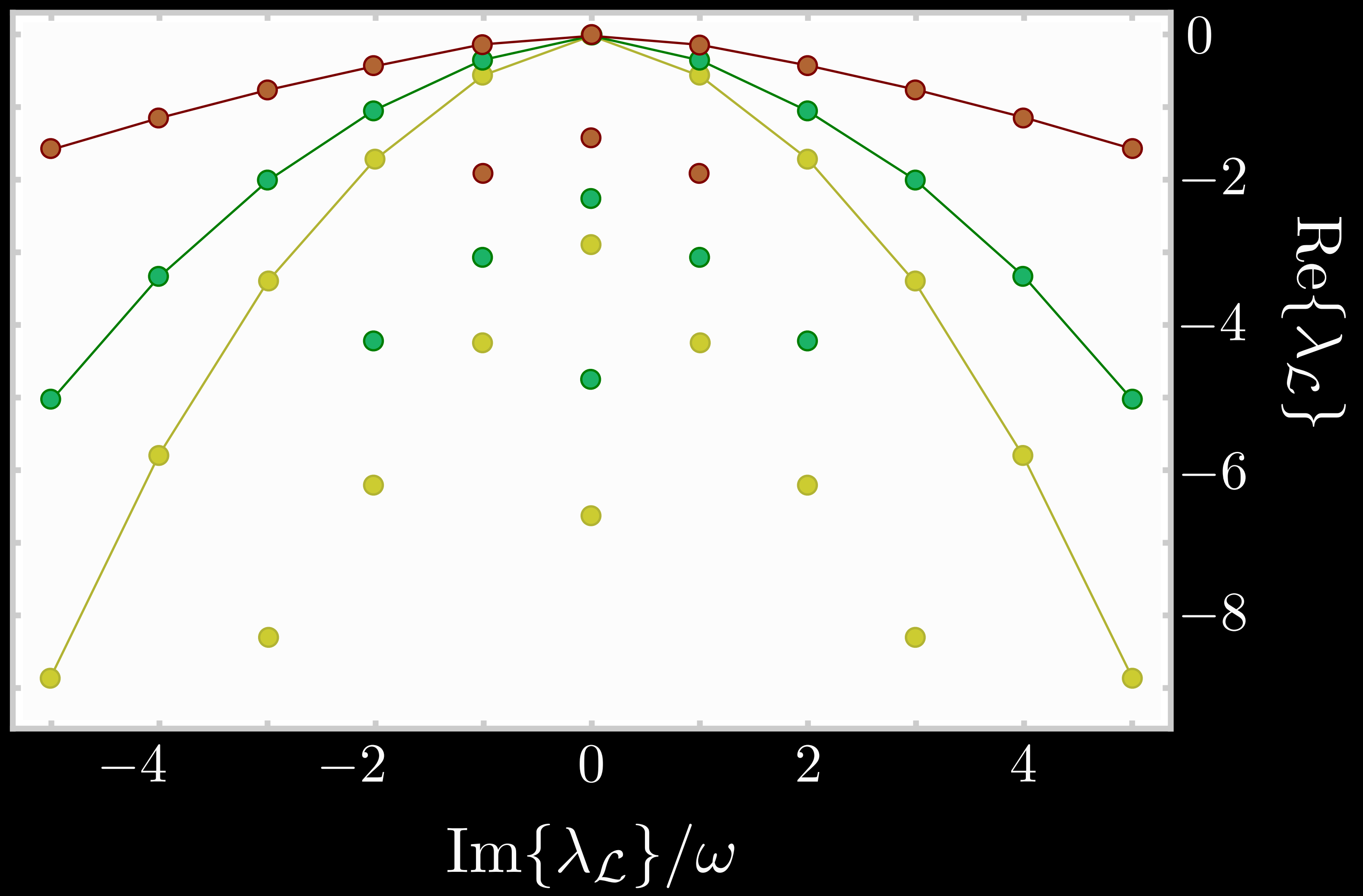
$\gamma = 1,\ 0.5,\ 0.1$



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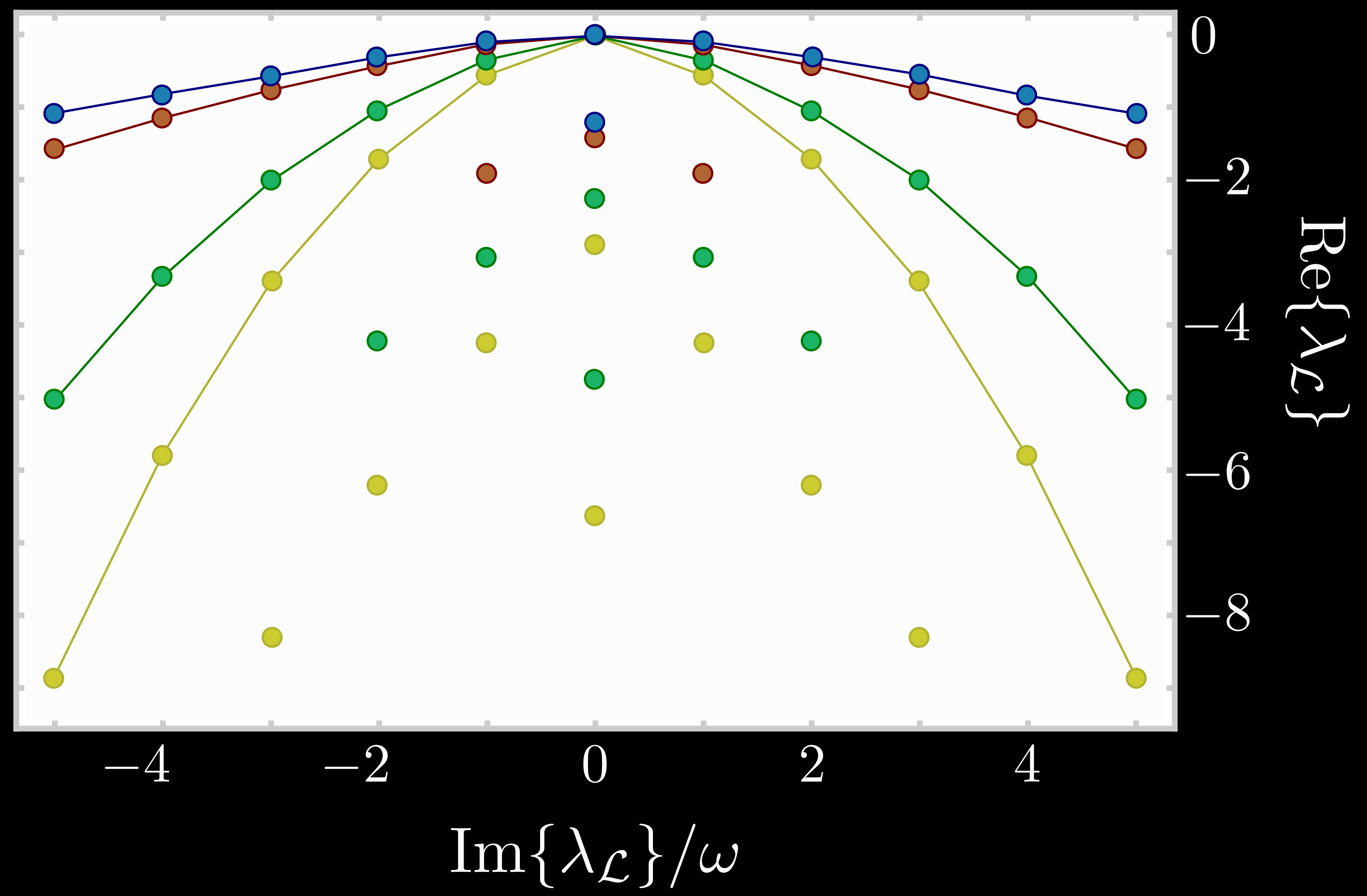

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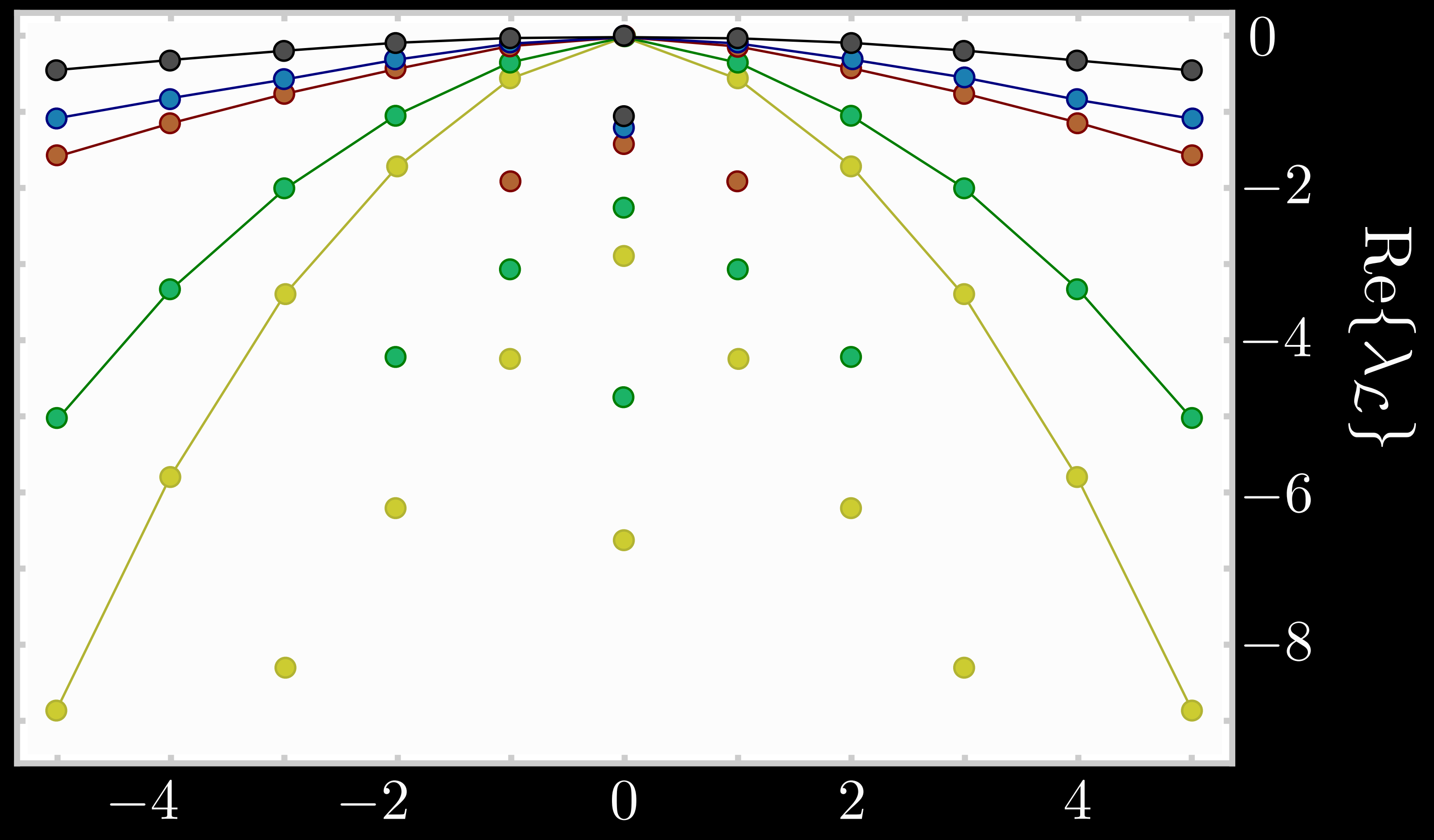

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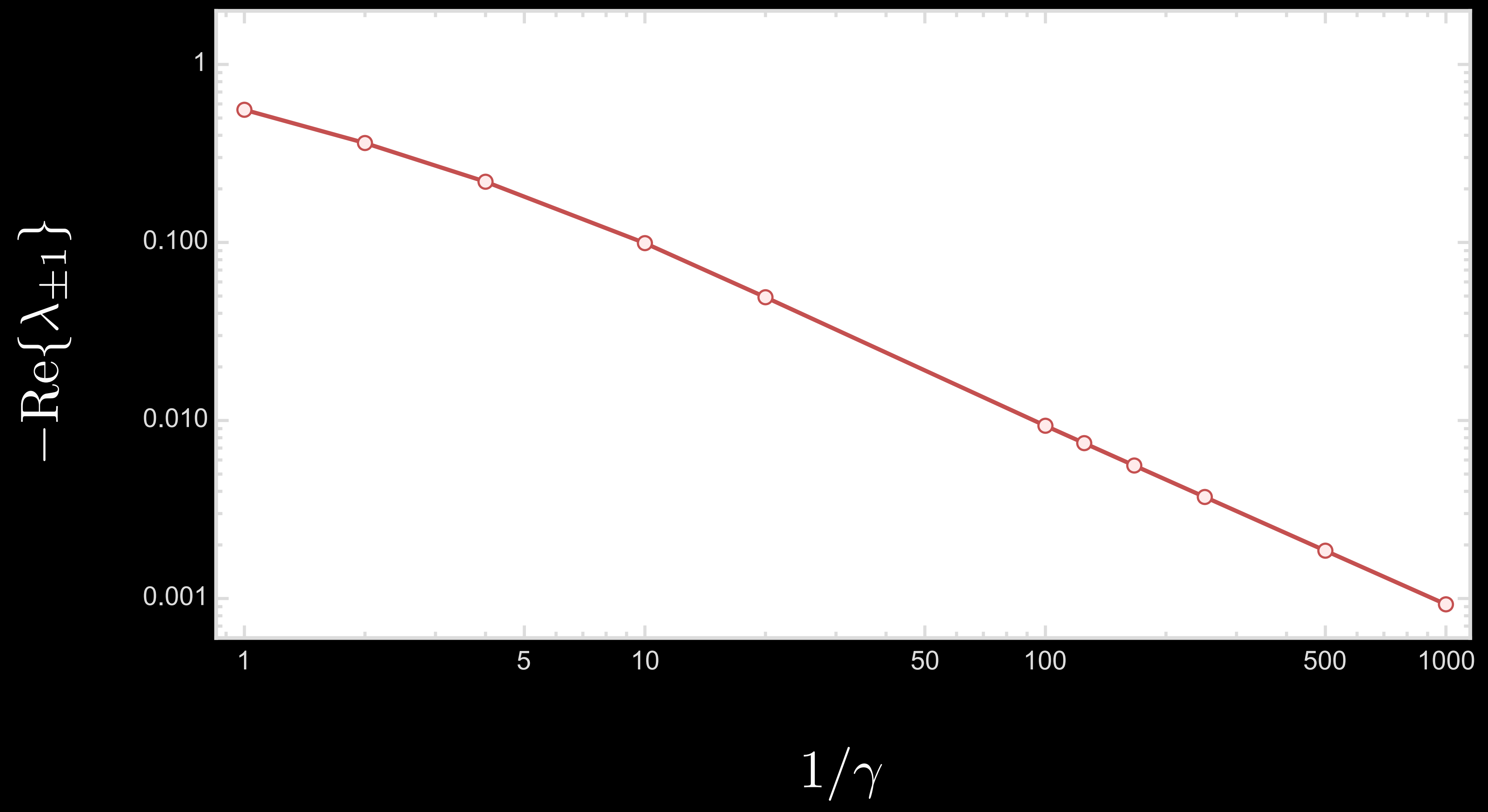

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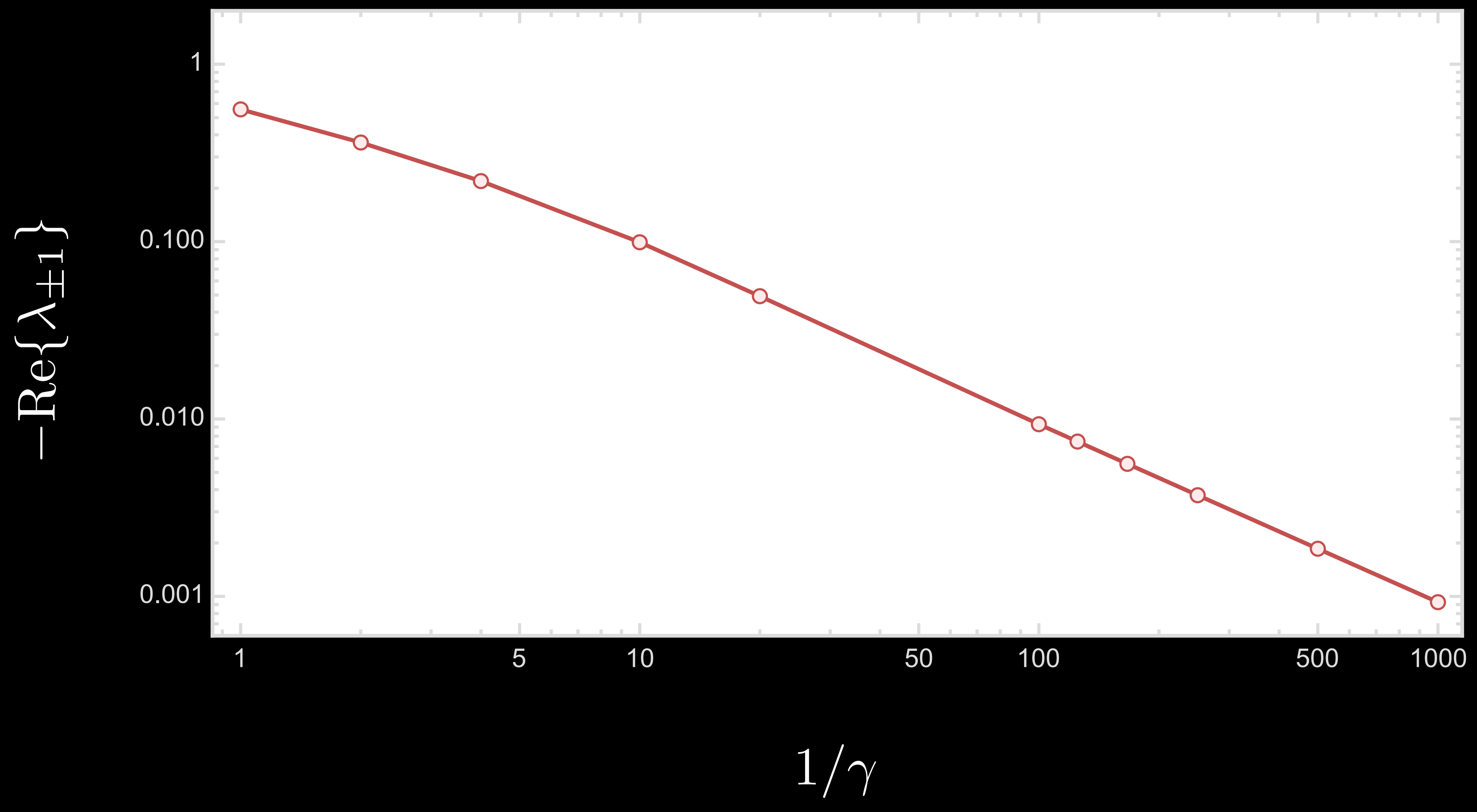

Question:

Are these oscillations robust, or do they disappear under noise?

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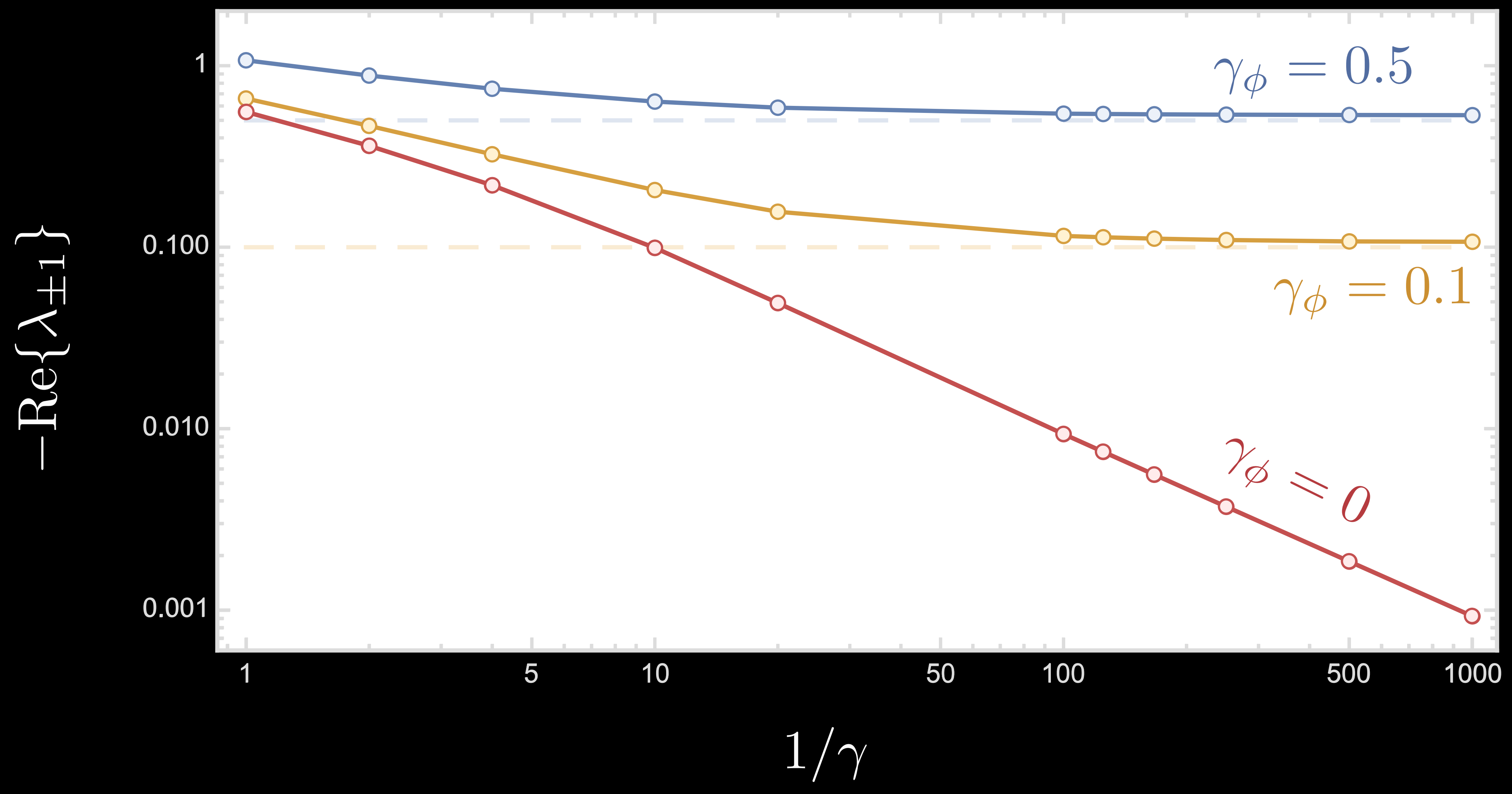

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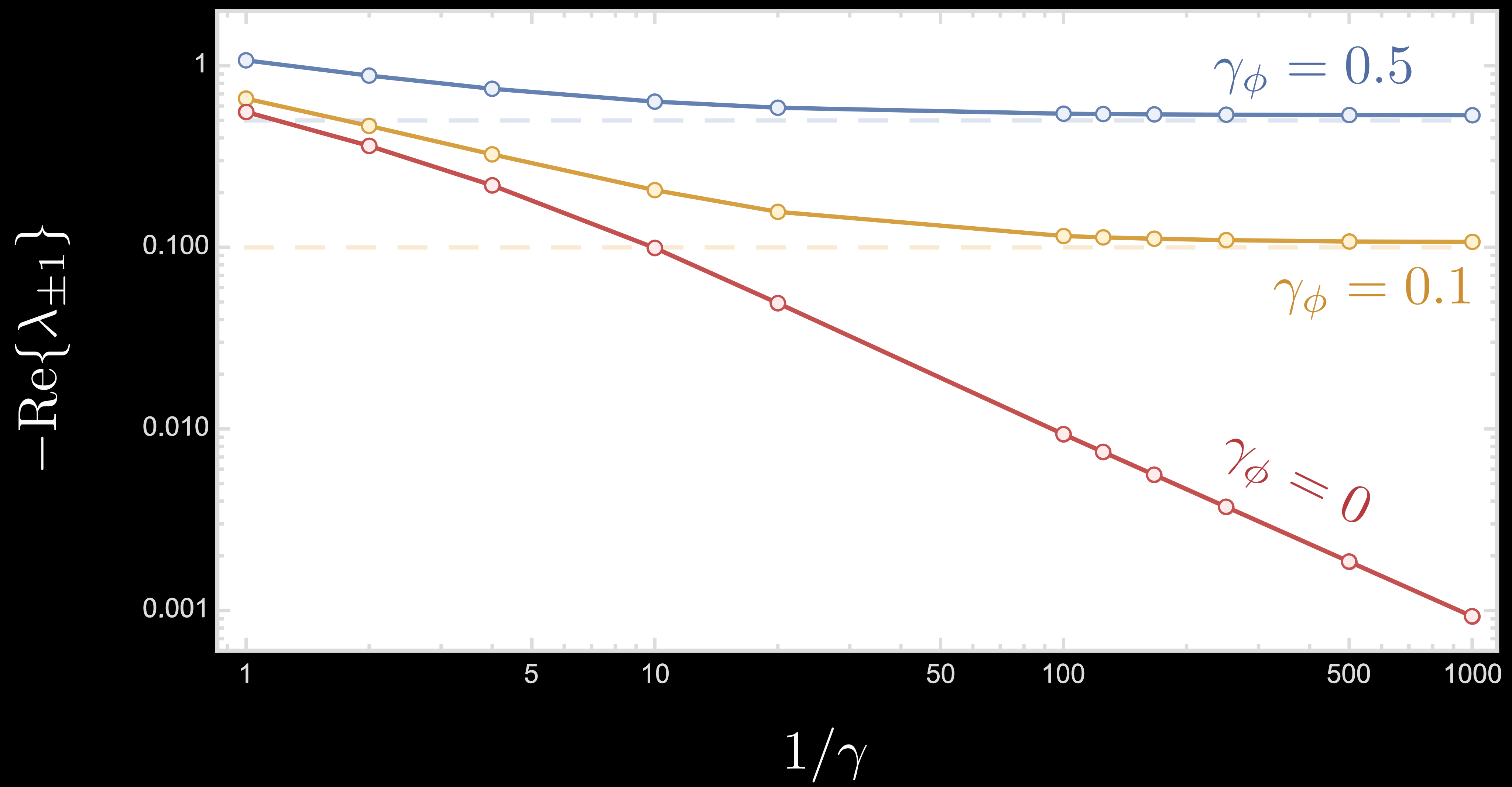

Does this mean we have a DTC?

We conjecture:

Extended systems are required to achieve true time-crystalline order, in particular because they can use many-body protection against local fluctuations

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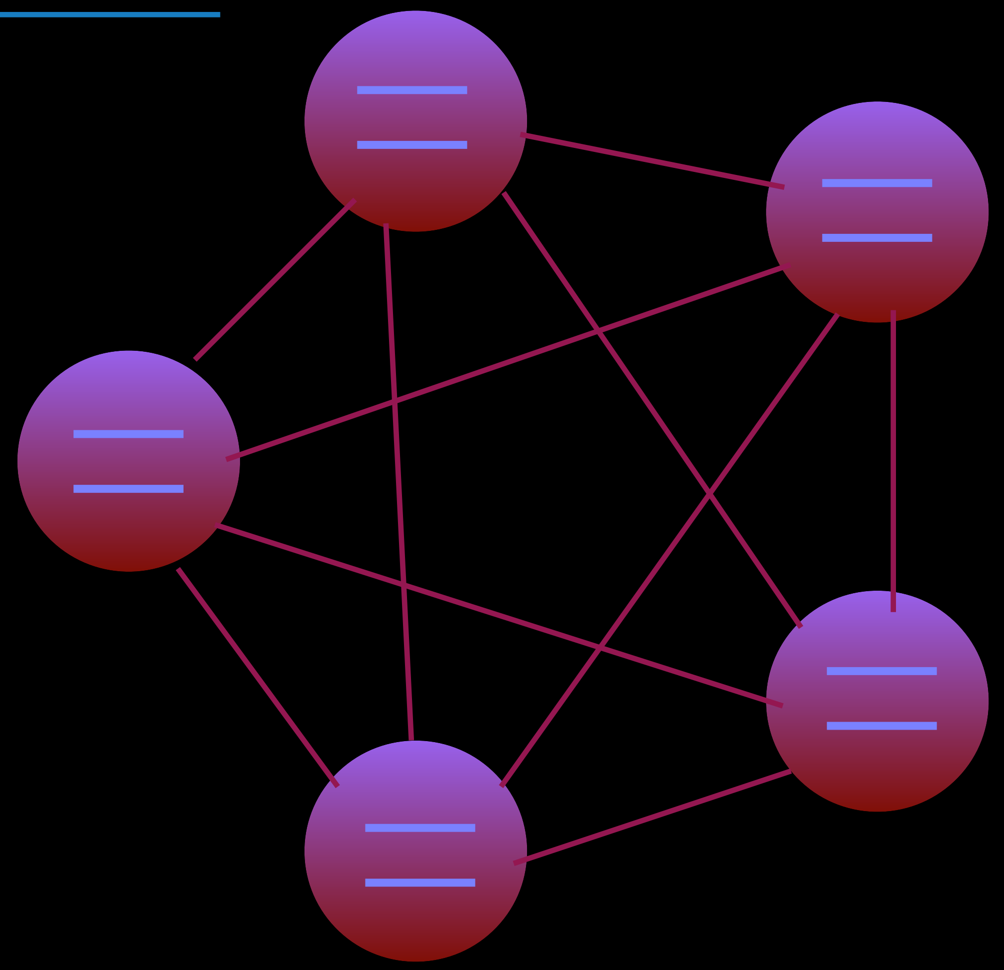
Spin models: testing our expectations



Collective spin operators:

$$\hat{J}_\alpha = \frac{1}{2}\sum_{n=1}^{N} \hat{\sigma}_\alpha^{(n)}, \quad \alpha \in \{x, y, z\}$$

$$[\hat{J}_\alpha, \hat{J}_\beta] = i \sum_{\nu=x,y,z} \varepsilon_{\alpha\beta\nu}\, \hat{J}_\nu$$

$$\hat{J}_{\pm,n} = \hat{J}_{x,n} \pm i\hat{J}_{y,n}$$

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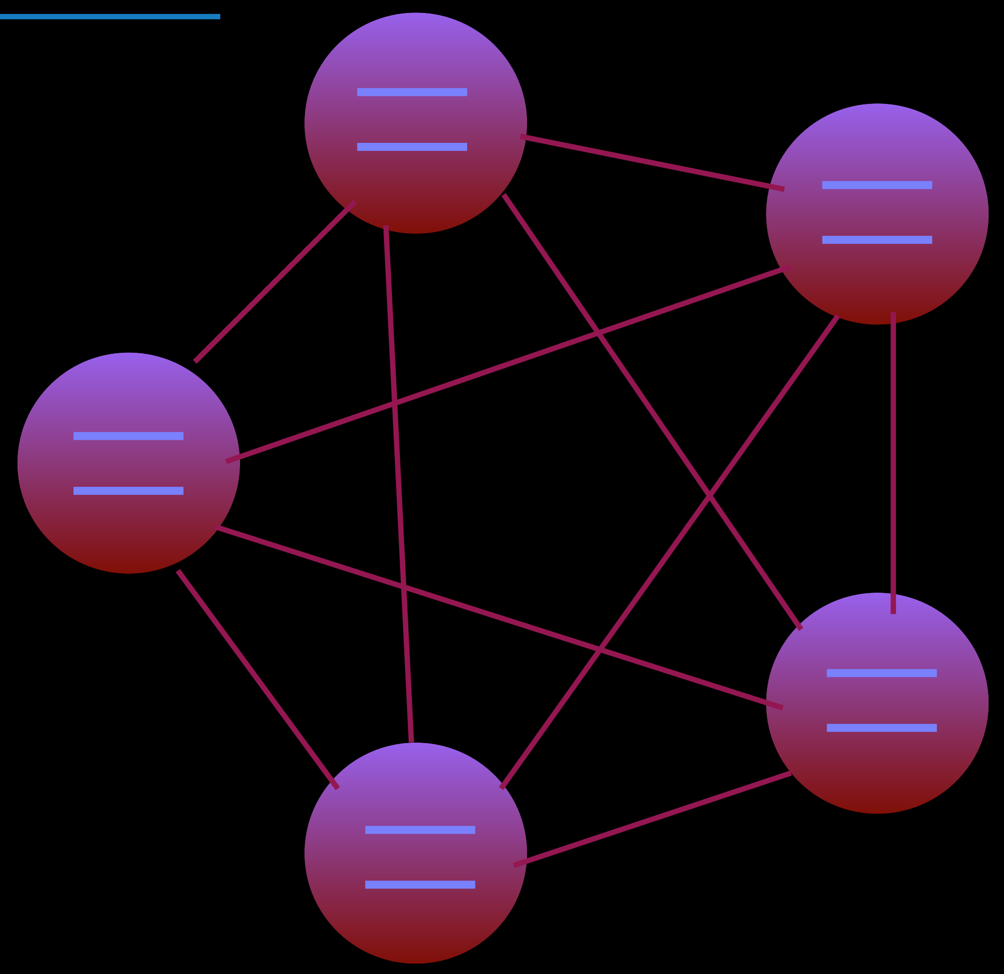

Standard DTC spin model:

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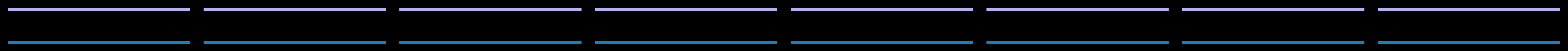

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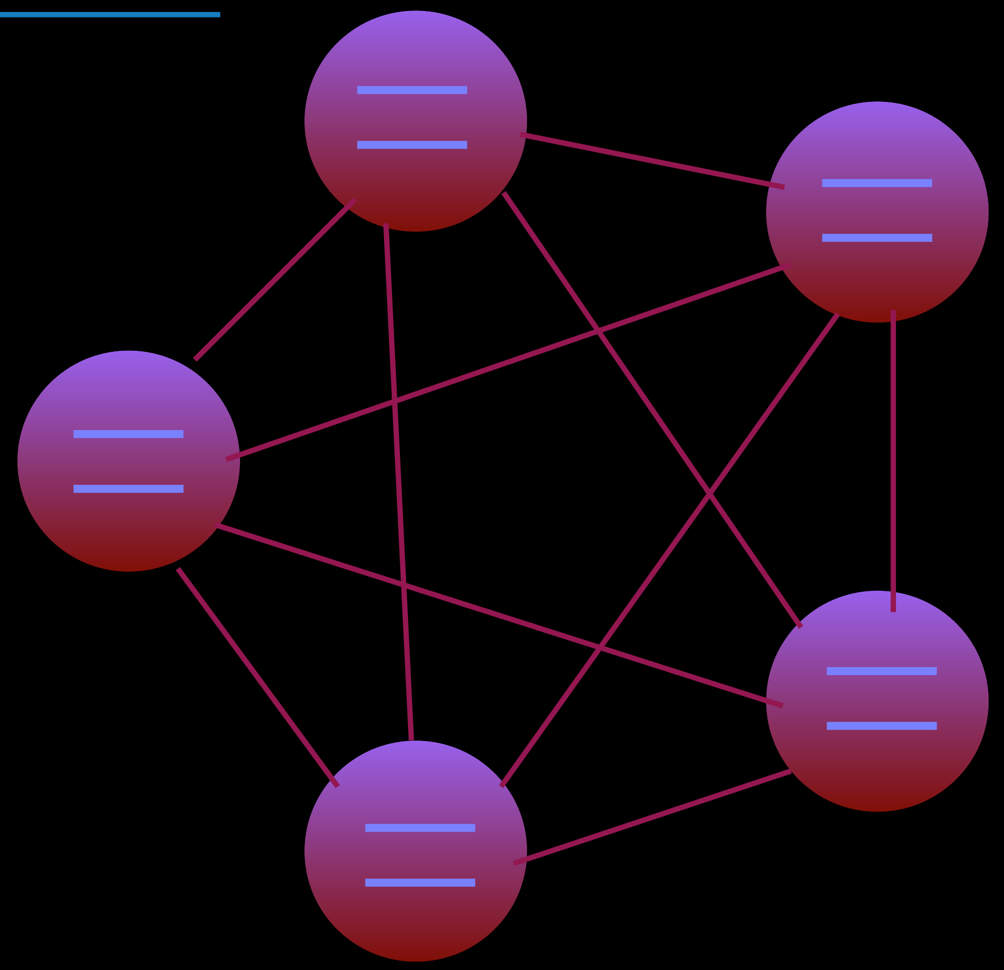

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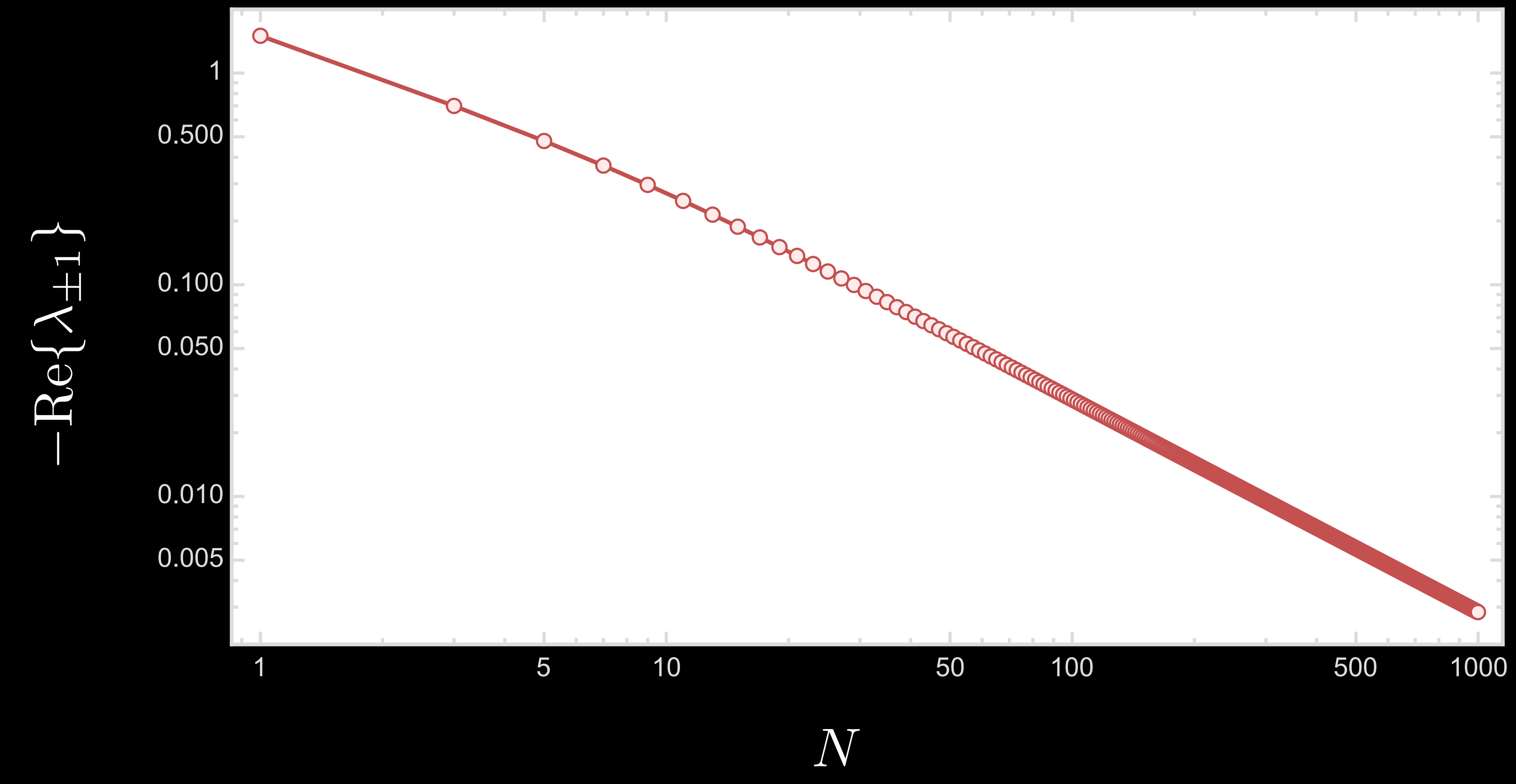

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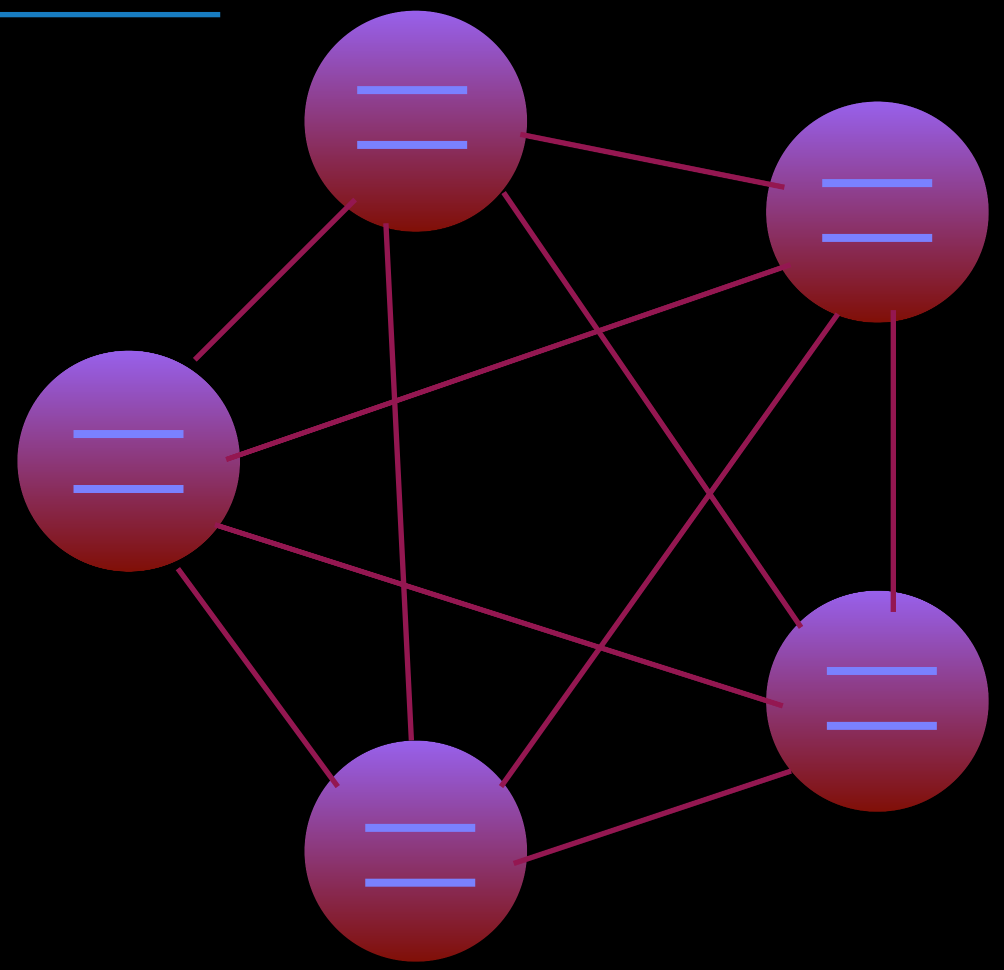

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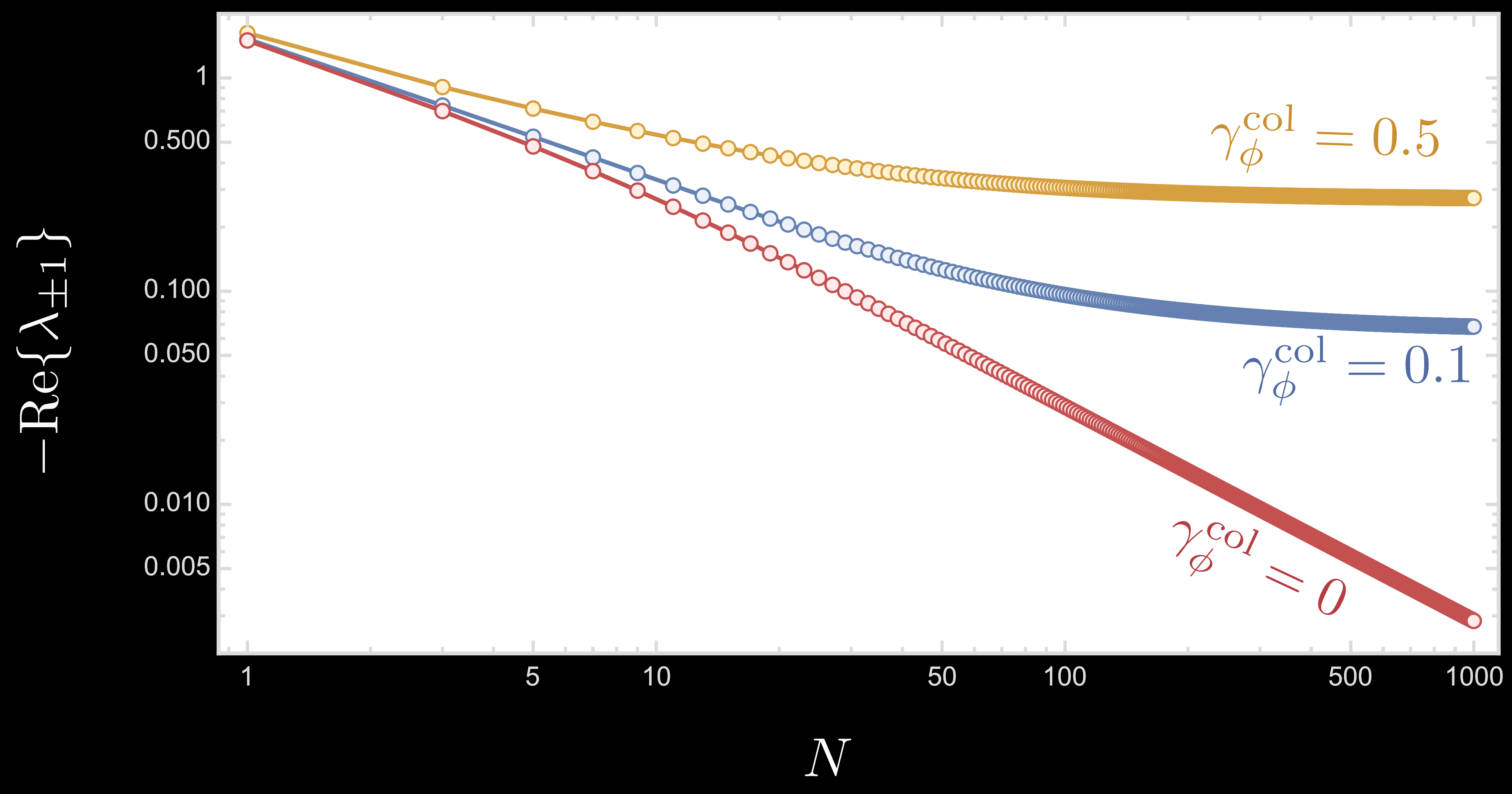

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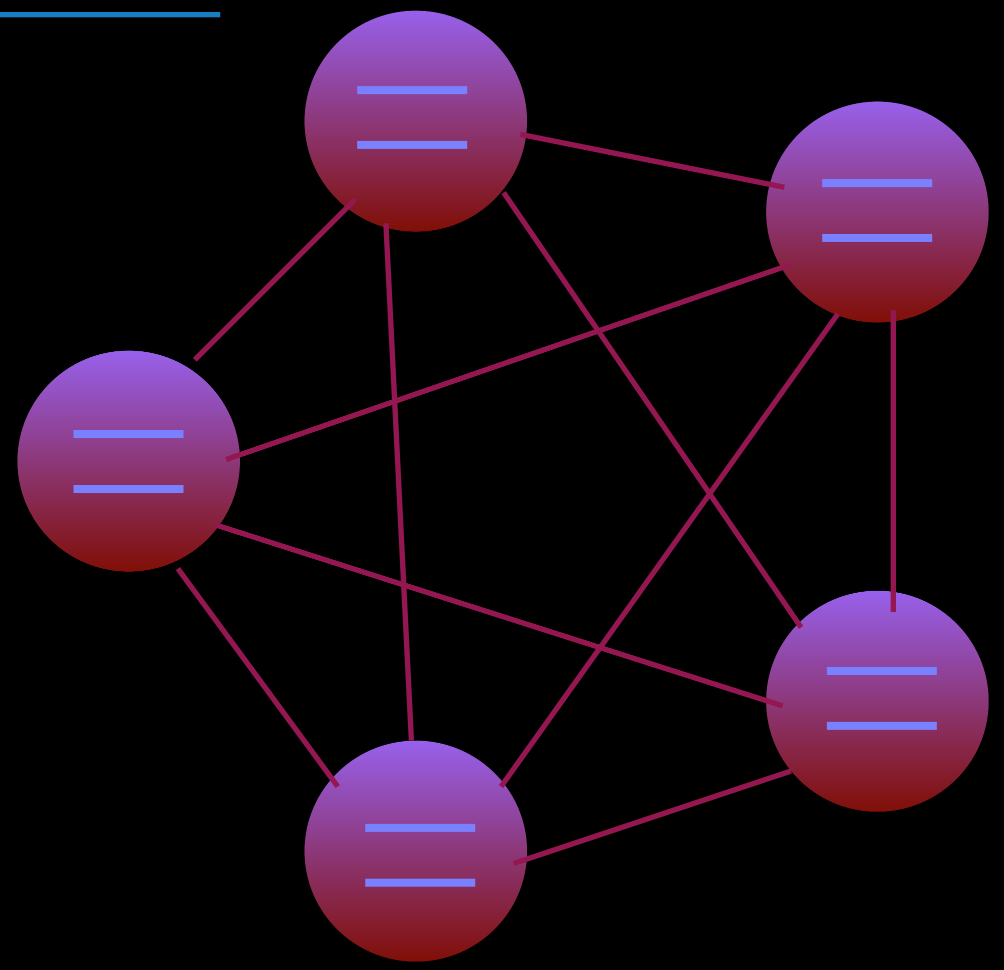

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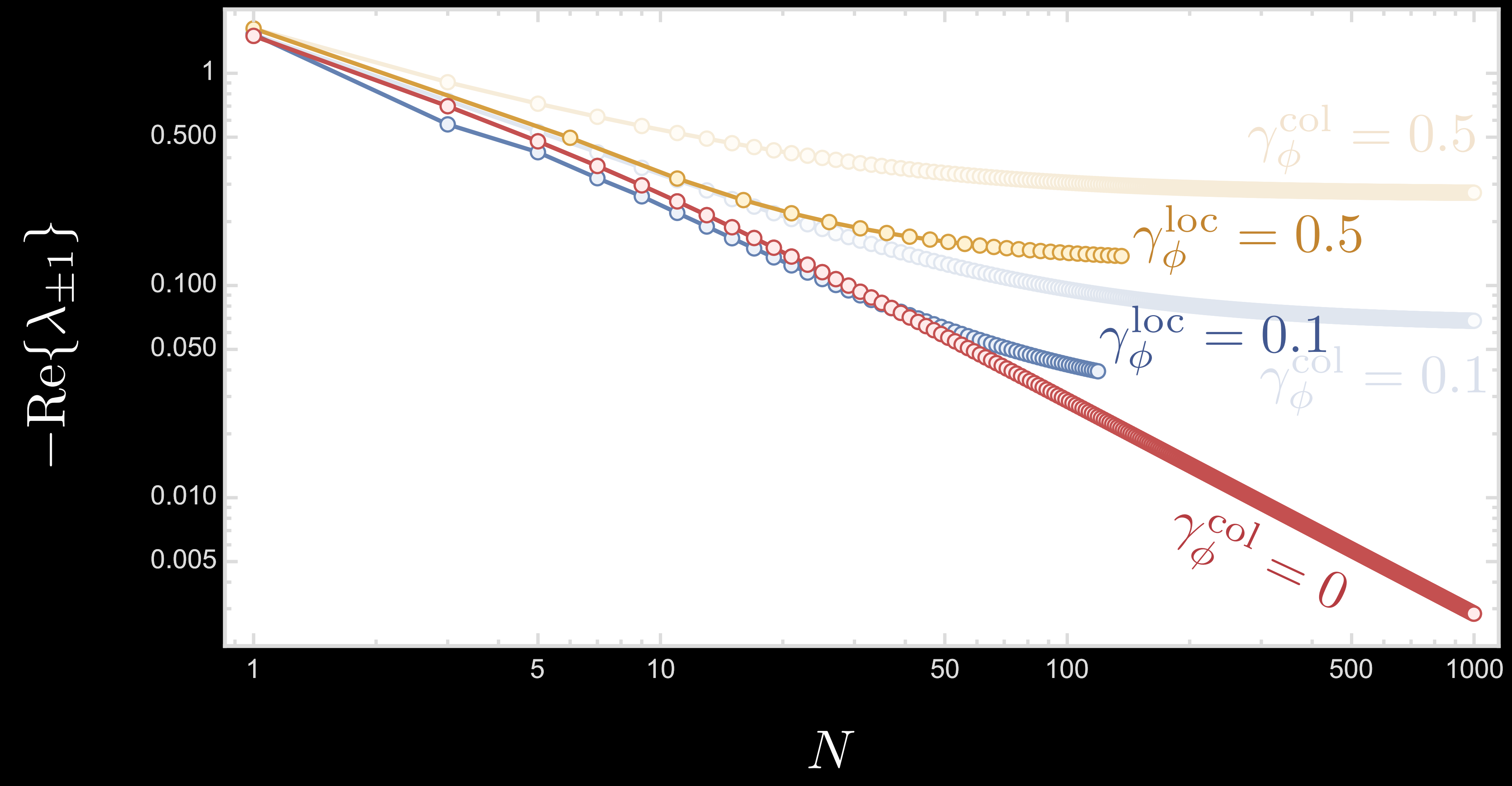

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Conclusions

Understanding Time Crystals:

Time crystals require persistent oscillations *and* robustness

Small system:

Shows oscillations, gap closes in a certain limit

But destroyed by dephasing

Many-Body System:

Gap also closes → collective oscillations emerge

Collective dephasing destroys it

Local dephasing shows partial resilience → hints of many-body protection

Thank you :)