Measuring temporal entanglement in experiments

arXiv:2409.05517

Aleix Bou-Comas

Institute for Fundamental Physics IFF-CSIC, Madrid

23rd of October 2025 | *ICE-10: Quantum Information in Spain*

INSTITUTO DE FÍSICA FUNDAMENTAL

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Plan de Recuperación, Transformación y Resiliencia

Funded by the European Union
NextGenerationEU

Measuring temporal entanglement in experiments

arXiv:2409.05517 | in review at PRX

ABC[1] Carlos Ramos Marimón[2] Jan T. Schneider[1] Stefano Carignano[3] Luca Tagliacozzo[1]

[1] Institute for Fundamental Physics IFF-CSIC, Madrid

[2] Departament de Física Quàntica i Astrofísica, Universitat de Barcelona

[3] Barcelona Supercomputing Center, Barcelona

Carlos

Jan

Stefano

Luca

Motivation

Spatial entanglement

Same time correlations between different spatial regon.

Essential role in:

- Quantum Phase Transitions
- Quantum Information Theory
- Proxy for ergodicity in out-of-equilibrium dynamics

Motivation

Spatial entanglement

Same time correlations between different spatial regon.

Essential role in:

- Quantum Phase Transitions
- Quantum Information Theory
- Proxy for ergodicity in out-of-equilibrium dynamics

Temporal Entanglement

Measures correlations of constituents across ***different temporal regions***.

Connected to:

- Equilibration and thermalization
- Geometric information (holographic theories)

Motivation

Q1: Can we measure temporal entanglement in experiments?

Motivation

Q1: Can we measure temporal entanglement in experiments?

Q2: What information about the system can we learn from it?

Spatial and Temporal Entanglement

Diagrams

Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

Diagrams

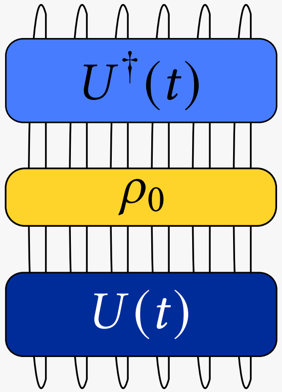
Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

$$1 = \mathrm{tr}(\rho(t)) = \mathrm{tr}\left(U(t)\rho_0 U^\dagger(t)\right) =$$

Diagrams

Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

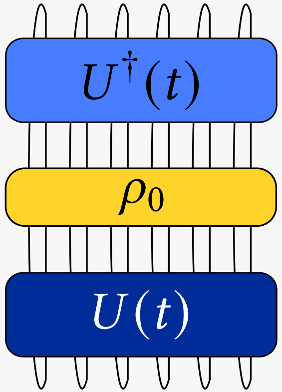
$$1 = \mathrm{tr}(\rho(t)) = \mathrm{tr}\big(U(t)\rho_0 U^\dagger(t)\big) =$$



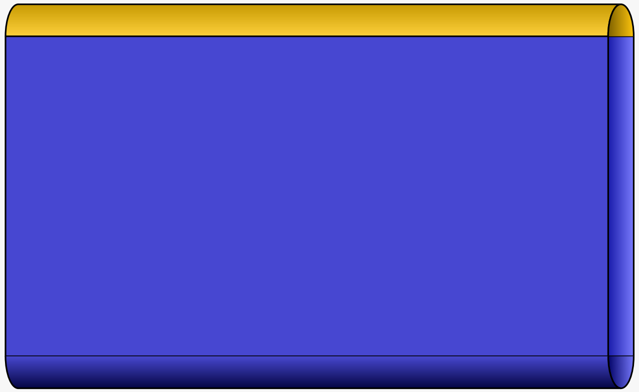
$$=$$

Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

$$1 = \mathrm{tr}(\rho(t)) = \mathrm{tr}\big(U(t)\rho_0 U^\dagger(t)\big) =$$



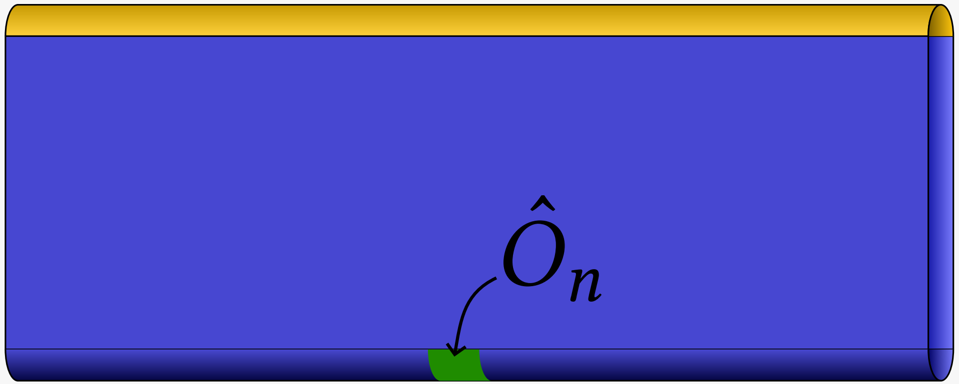
$$=$$



Diagrams

Most quantities can be found through the trace of the initial density matrix ρ_0, the unitary evolution operator $U(t)$ and some combination of operators $\{O_j\}$.

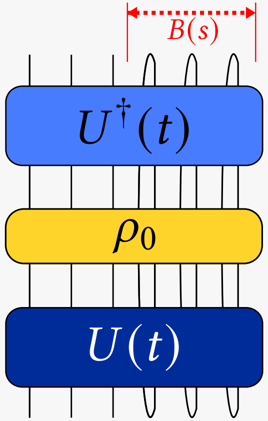
$$\left\langle \hat{O}_n(t) \right\rangle = \mathrm{tr}\left(\rho(t)\hat{O}_n\right) =$$



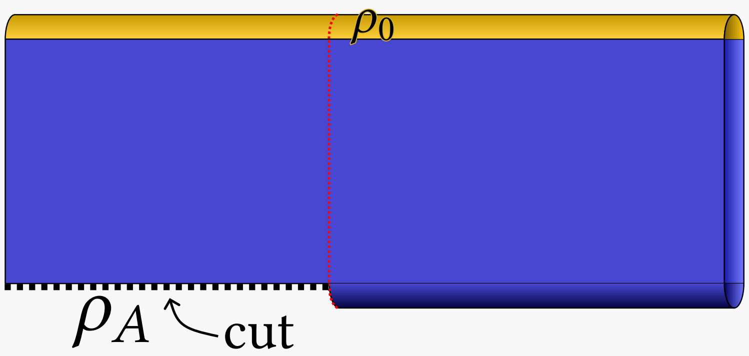
Spatial entanglement

It is characterised by *space-like cut* in the partition function dividing the space in subsystem A and B.

$$\rho_A(t) = \mathrm{tr}_{B(s)}\left(U^\dagger(t)\rho_0 U(t)\right) =$$



$=$



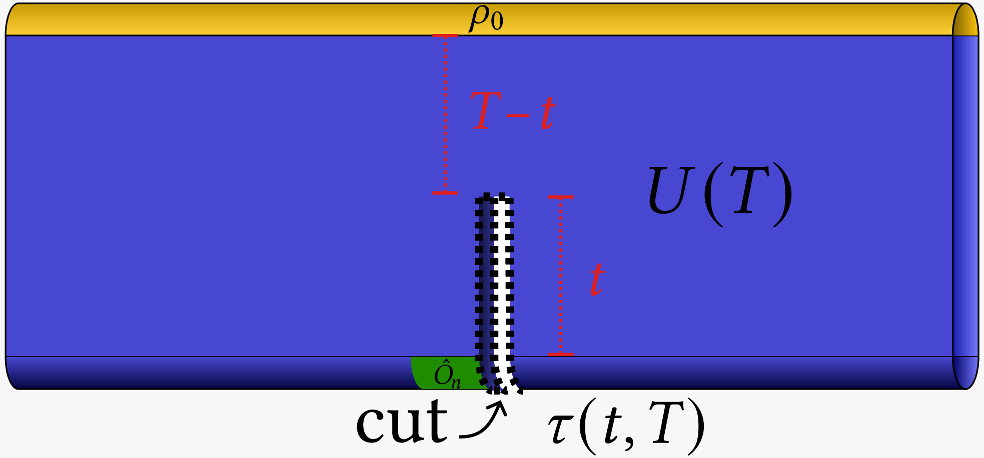
Renyi entropies quantify the entanglement of a system,

$$S_\alpha(t) = \frac{1}{1-\alpha}\log(\mathrm{Tr}(\rho_A(t)^\alpha))$$

Temporal entanglement

It is characterised by *time-like cut* in the partition function dividing the time in subsystem $(0, t)$ and (t, T).

The reduced transition matrix $\tau(t, T)$ acts similarly to the reduced desity matrix $\rho_{A(t)}$.

$\tau(t, T) =$ 

Analogous quantity with a 90 degree rotation.

Measurement protocol

$$\mathcal{T}^{\alpha}(t)_{O_j} = \mathrm{tr}\Big[\big(\tau_{\hat{O}}(t,T)\big)^{\alpha}\Big]$$ trace of reduced transition matrix

$$\mathcal{S}^{\alpha}(t)_{O_j} = \frac{1}{1-\alpha}\log\Big[\mathcal{T}^{\alpha}(t)_{O_j}\Big]$$ generalized temporal entropies

Measurement protocol

$$\mathcal{T}^{\alpha}(t)_{O_j} = \mathrm{tr}\left[\left(\tau_{\hat{O}}(t,T)\right)^{\alpha}\right]$$ trace of reduced transition matrix

$$\mathcal{S}^{\alpha}(t)_{O_j} = \frac{1}{1-\alpha}\log\left[\mathcal{T}^{\alpha}(t)_{O_j}\right]$$ generalized temporal entropies

for $\alpha \in \mathbb{N}$, $\mathcal{S}^{\alpha}$ accessible in experiment $\Rightarrow$ replica trick:

two ($\alpha = 2$) copies of the same system

Measurement protocol

$$\mathcal{T}^{\alpha}(t)_{O_j} = \mathrm{tr}\left[\left(\tau_{\hat{O}}(t,T)\right)^{\alpha}\right]$$ trace of reduced transition matrix

$$\mathcal{S}^{\alpha}(t)_{O_j} = \frac{1}{1-\alpha}\log\left[\mathcal{T}^{\alpha}(t)_{O_j}\right]$$ generalized temporal entropies

for $\alpha \in \mathbb{N}$, $\mathcal{S}^{\alpha}$ accessible in experiment $\Rightarrow$ replica trick:

two ($\alpha = 2$) copies of the same system

$$\left[\tau_{\hat{O}}(t,T)^2\right]_{a,b} = \sum_c \left[\tau_{\hat{O}}(t,T)\right]_{a,c}\left[\tau_{\hat{O}}(t,T)\right]_{c,b}$$

Measurement protocol

$$\mathcal{T}^{\alpha}(t)_{O_j} = \mathrm{tr}\left[\left(\tau_{\hat{O}}(t,T)\right)^{\alpha}\right]$$ trace of reduced transition matrix

$$\mathcal{S}^{\alpha}(t)_{O_j} = \frac{1}{1-\alpha}\log\left[\mathcal{T}^{\alpha}(t)_{O_j}\right]$$ generalized temporal entropies

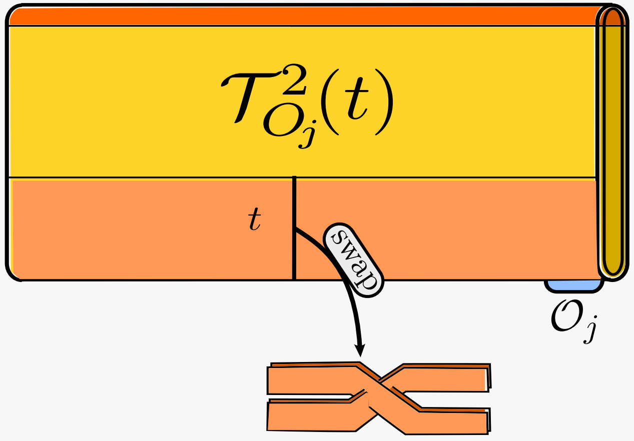
for $\alpha \in \mathbb{N}$, $\mathcal{S}^{\alpha}$ accessible in experiment $\Rightarrow$ replica trick:

two ($\alpha = 2$) copies of the same system

$$\left[\tau_{\hat{O}}(t,T)^2\right]_{a,b} = \sum_c \left[\tau_{\hat{O}}(t,T)\right]_{a,c}\left[\tau_{\hat{O}}(t,T)\right]_{c,b}$$

$$\mathrm{tr}\left[\tau_{\hat{O}}(t,T)^2\right] = \sum_{d,c} \left[\tau_{\hat{O}}(t,T)\right]_{d,c}\left[\tau_{\hat{O}}(t,T)\right]_{c,d}$$

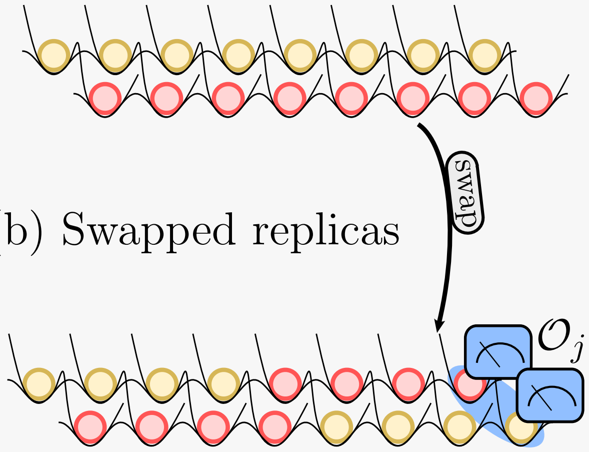
Generalized temporal entropies



Physical Experiments

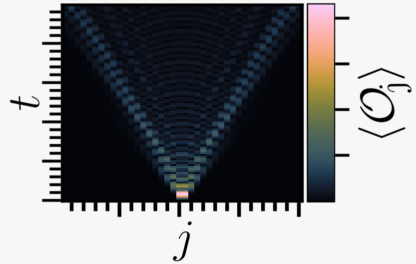
(a) Replicated system

(b) Swapped replicas

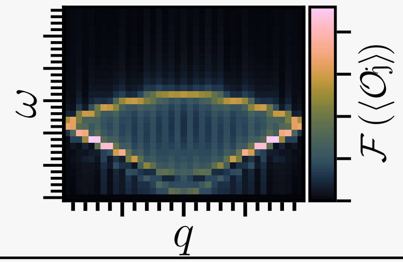


Characterization

(c) Temporal evolution: replicated operator



(d) Fourier Transform



Results

1: Generalized temporal entropies are real valued

1: Generalized temporal entropies are real valued

2: Generalized temporal entropies can be interpreted as dynamical proves of geometric quenches

1: Generalized temporal entropies are real valued

2: Generalized temporal entropies can be interpreted as dynamical proves of geometric quenches

3: Generalized temporal entropies can be measured in state-of-the-art quantum simulators.

Numerical results

Ising model

$$H = -J\sum_{n=1}^{N-1}\sigma_n^x\sigma_{n+1}^x + \sum_{n=1}^{N} g\sigma_n^z + h\sigma_n^x,$$

$$N = 40, \quad \frac{g}{J} = 0.4 \quad \text{(FM)}$$

$$\hat{O}_n = \sigma_n^x$$

$$\mathcal{T}^2(t,T)_{\hat{O}_j} = \frac{\mathrm{tr}\left[\hat{O}_j^{(1)} \otimes \hat{O}_j^{(2)} \rho^{(1,2)}(t,T)\right]}{\left\langle \hat{O}_j(T)\right\rangle^2}$$

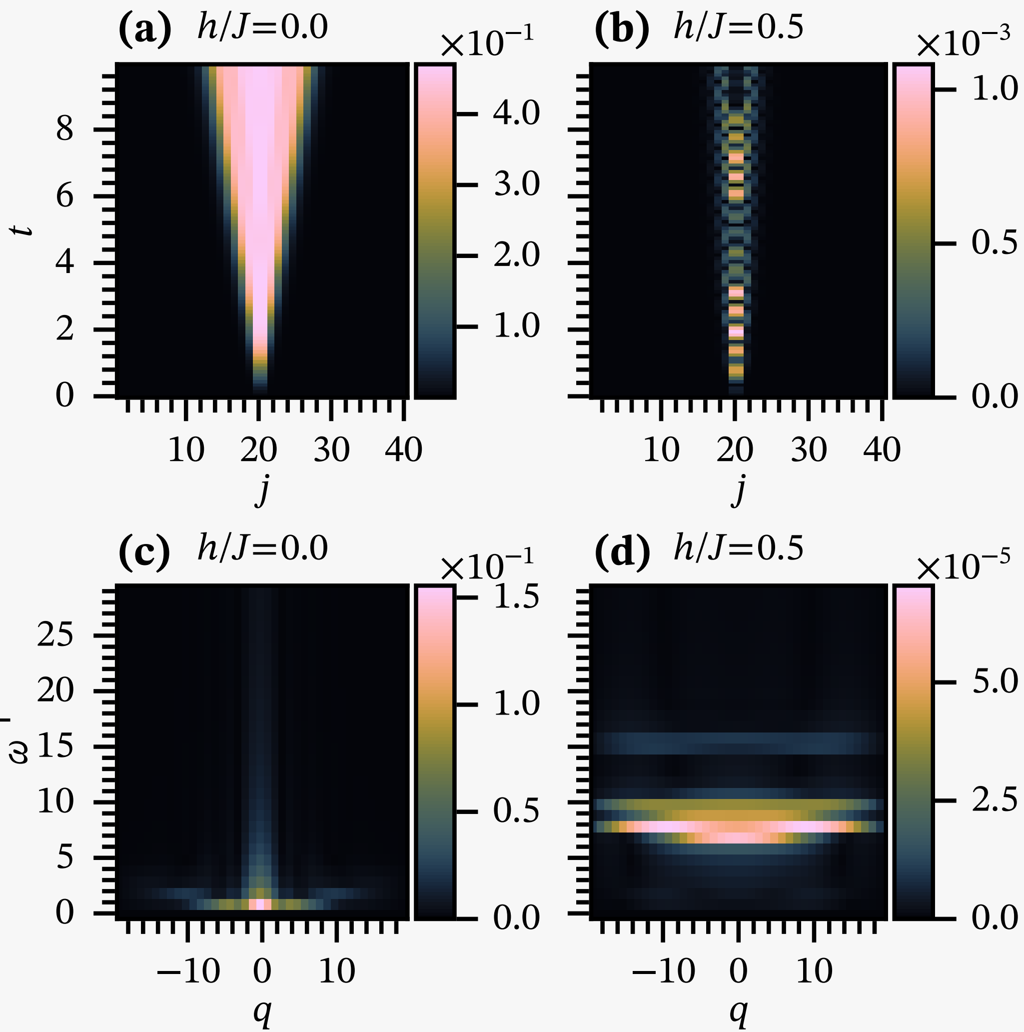
Ising model

$$H = -J \sum_{n=1}^{N-1} \sigma_n^x \sigma_{n+1}^x + \sum_{n=1}^{N} g\sigma_n^z + h\sigma_n^x,$$

$$N = 40, \quad \frac{g}{J} = 0.4 \quad \text{(FM)}$$

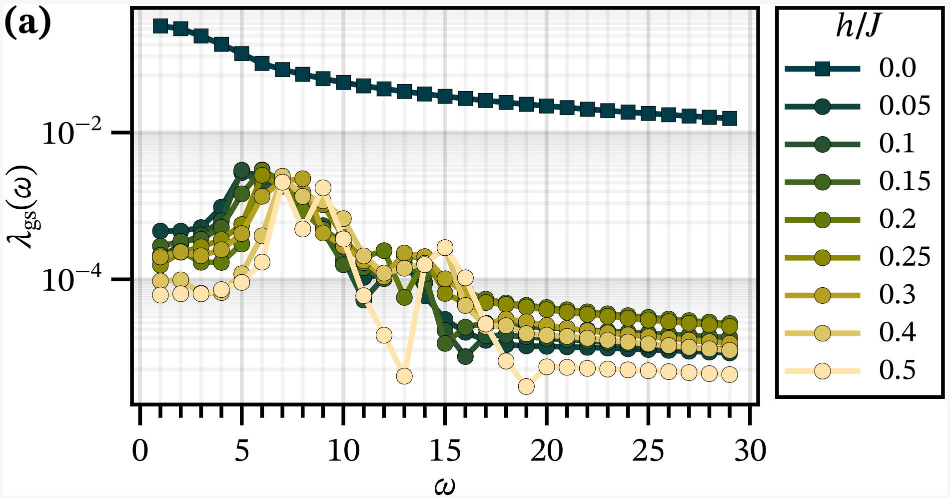
$$\hat{O}_n = \sigma_n^x$$

$$\mathcal{T}^2(t,T)_{\hat{O}_j} = \frac{\mathrm{tr}\left[\hat{O}_j^{(1)} \otimes \hat{O}_j^{(2)} \rho^{(1,2)}(t,T)\right]}{\left\langle \hat{O}_j(T) \right\rangle^2}$$

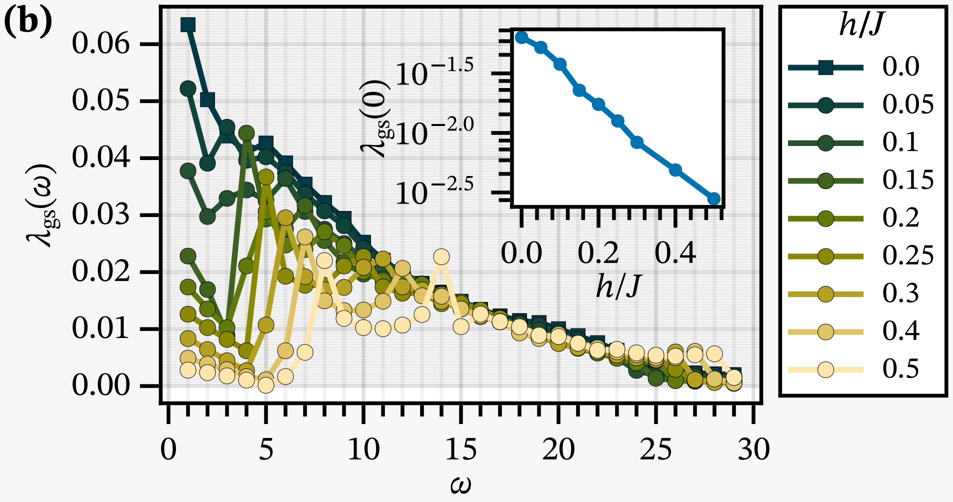


Decay of the soft mode ($q = 0$)

Ferromagnetic phase $\frac{g}{J} = 0.4$



Paramagnetic phase $\frac{g}{J} = 1.5$



Integrabilty $\Leftrightarrow$ gap

Integrabilty $\Leftrightarrow$ exponential decay of gap

$(\hat{O}_n = \sigma_n^z)$

Thanks for your attention!

Measuring temporal entanglement in experiments

arXiv:2409.05517 | aleix.bou@iff.csic.es

ABC[1] Carlos Ramos Marimón[2] Jan T. Schneider[1] Stefano Carignano[3] Luca Tagliacozzo[1]

[1] Institute for Fundamental Physics IFF-CSIC, Madrid

[2] Departament de Física Quàntica i Astrofísica, Universitat de Barcelona

[3] Barcelona Supercomputing Center, Barcelona

WE ARE HIRING! For more information visit: https://quinfog.hbar.es/

Funding from

Grant MMT24-IFF-01

GOBIERNO DE ESPAÑA

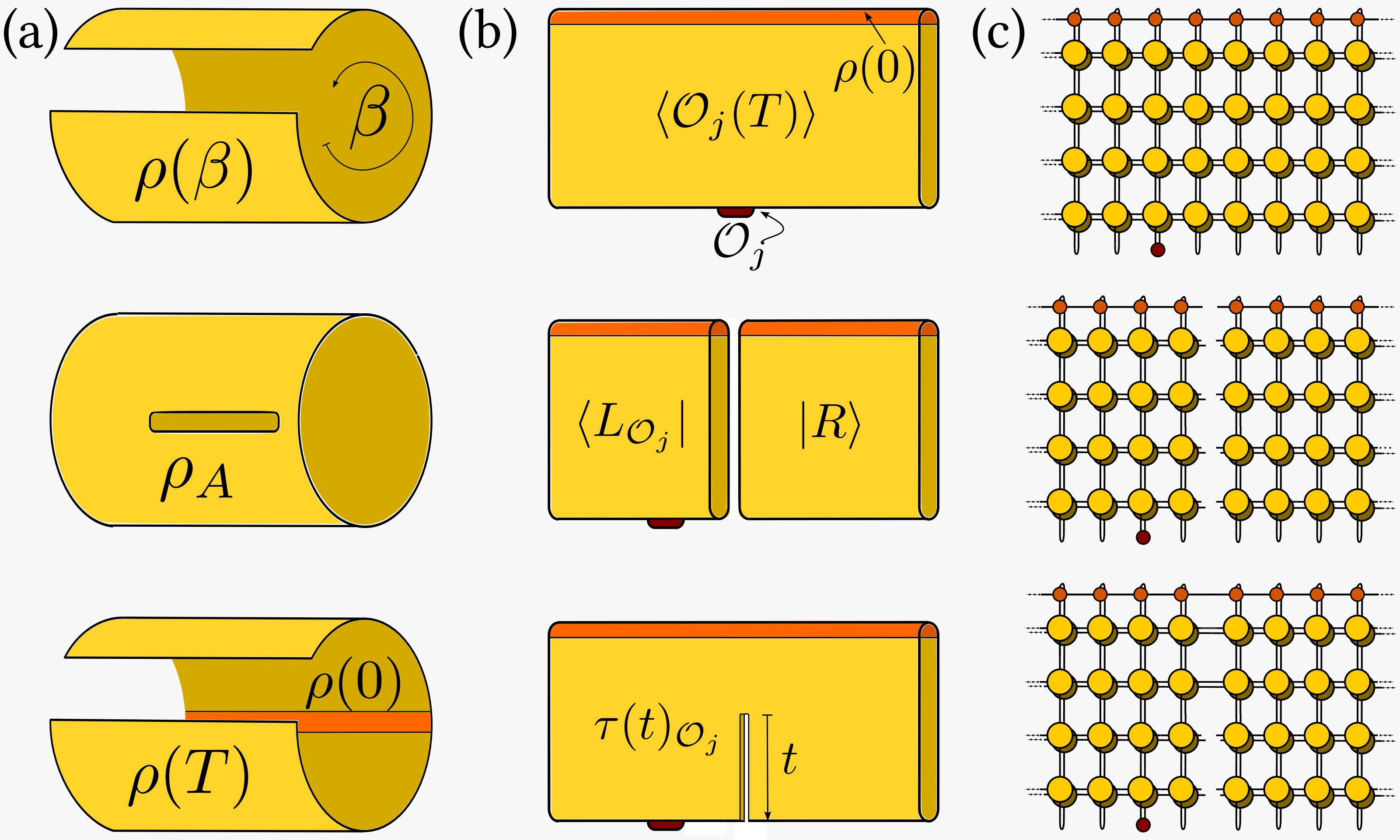
MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

AGENCIA ESTATAL DE INVESTIGACIÓN

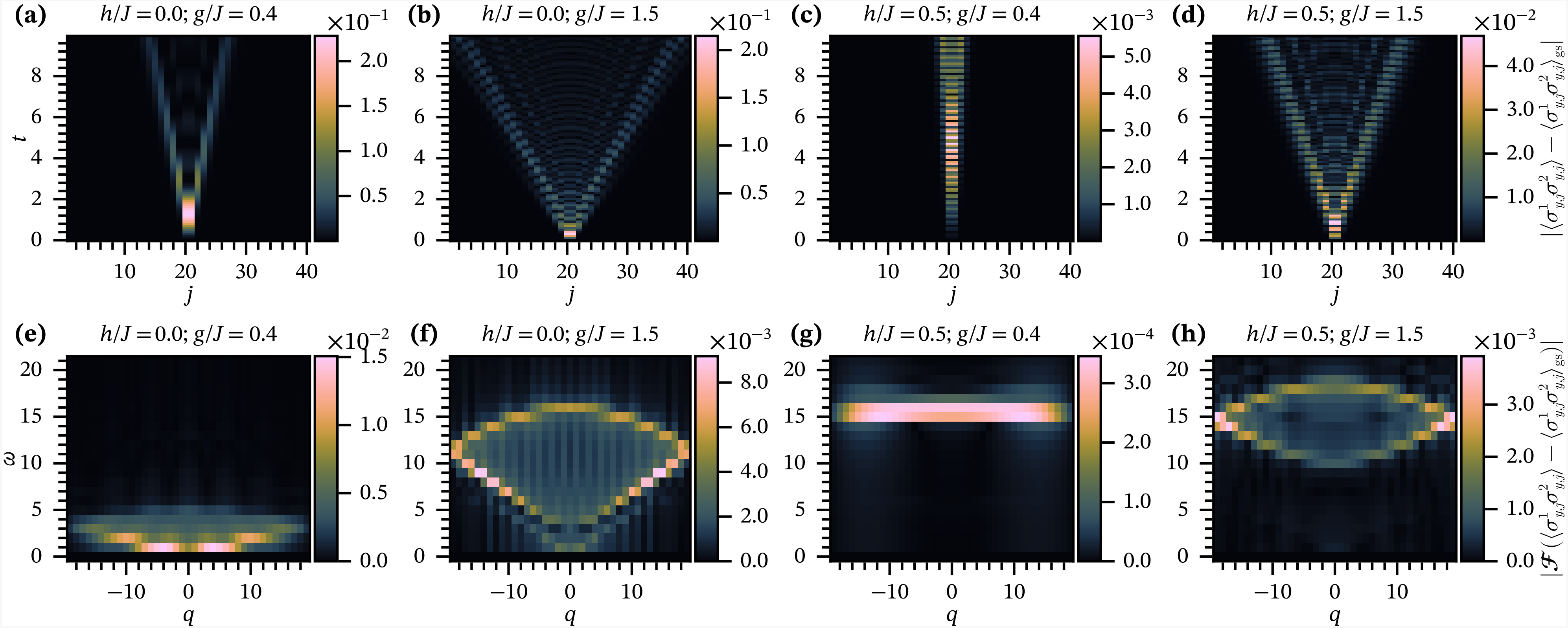
Plan de Recuperación, Transformación y Resiliencia

Funded by the European Union

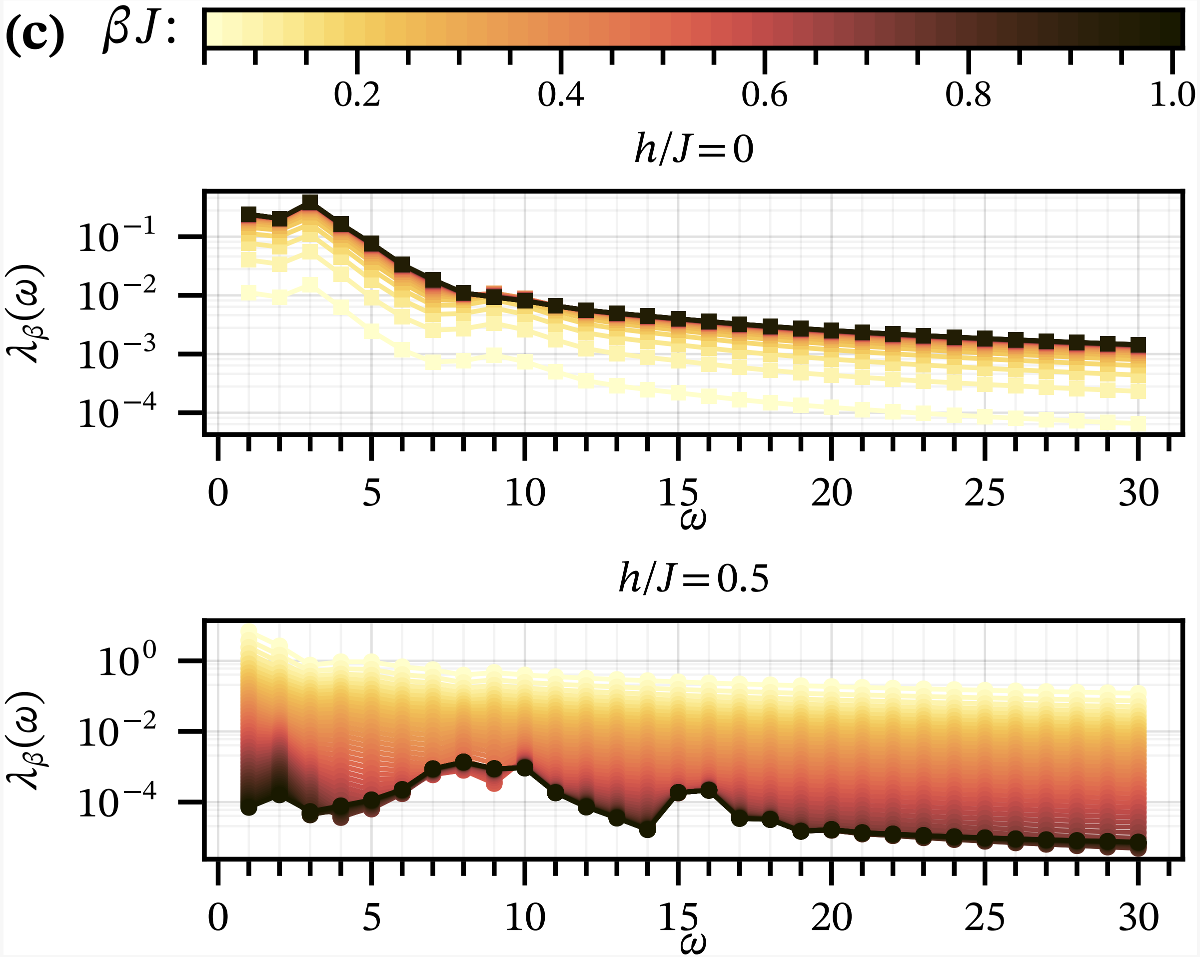
NextGenerationEU



(a)
β
$\rho(\beta)$
ρ_A
$\rho(0)$
$\rho(T)$
(b)
$\rho(0)$
$\langle \mathcal{O}_j(T) \rangle$
$\mathcal{O}_j$
$\langle L_{\mathcal{O}_j} |$
$| R \rangle$
$\tau(t)_{\mathcal{O}_j}$
t
(c)



(a) $h/J = 0.0$; $g/J = 0.4$
(b) $h/J = 0.0$; $g/J = 1.5$
(c) $h/J = 0.5$; $g/J = 0.4$
(d) $h/J = 0.5$; $g/J = 1.5$
(e) $h/J = 0.0$; $g/J = 0.4$
(f) $h/J = 0.0$; $g/J = 1.5$
(g) $h/J = 0.5$; $g/J = 0.4$
(h) $h/J = 0.5$; $g/J = 1.5$
t
j
ω
q
$\times 10^{-1}$
$\times 10^{-3}$
$\times 10^{-2}$
$\times 10^{-4}$
$|\langle \sigma^1_{y,j}\sigma^2_{y,j}\rangle - \langle \sigma^1_{y,j}\sigma^2_{y,j}\rangle_{\mathrm{gs}}|$
$|\mathcal{F}(\langle \sigma^1_{y,j}\sigma^2_{y,j}\rangle - \langle \sigma^1_{y,j}\sigma^2_{y,j}\rangle_{\mathrm{gs}})|$


(c)
βJ:
0.2
0.4
0.6
0.8
1.0
$h/J = 0$
$\lambda_\beta(\omega)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25
30
ω
$h/J = 0.5$
$\lambda_\beta(\omega)$
10^{0}
10^{-2}
10^{-4}
0
5
10
15
20
25
30
ω