

**POTENTIAL
RENORMALISATION,
LAMB SHIFT
& MEAN-FORCE
GIBBS STATE-**

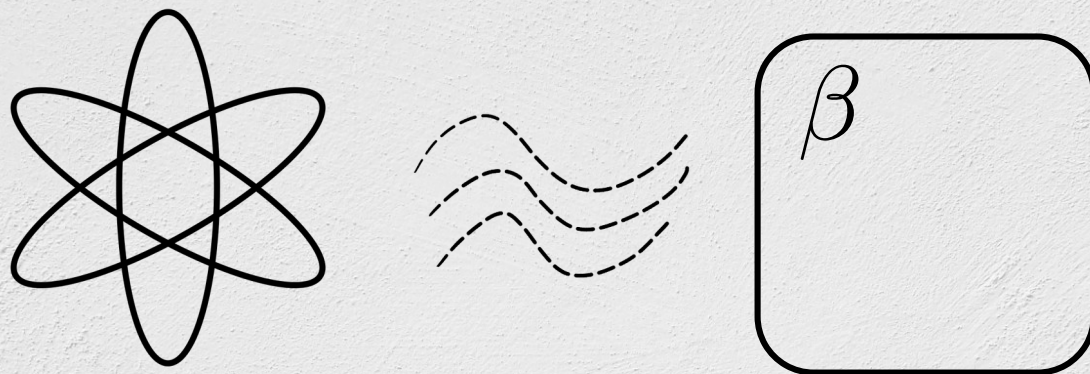


to shift or not to shift? *

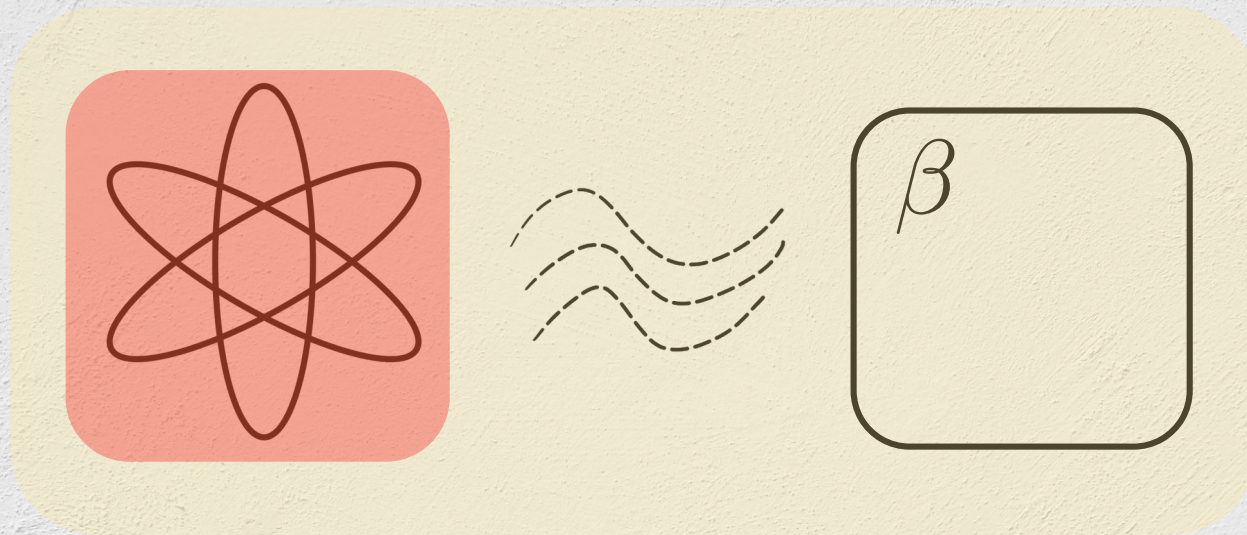
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University of Nottingham





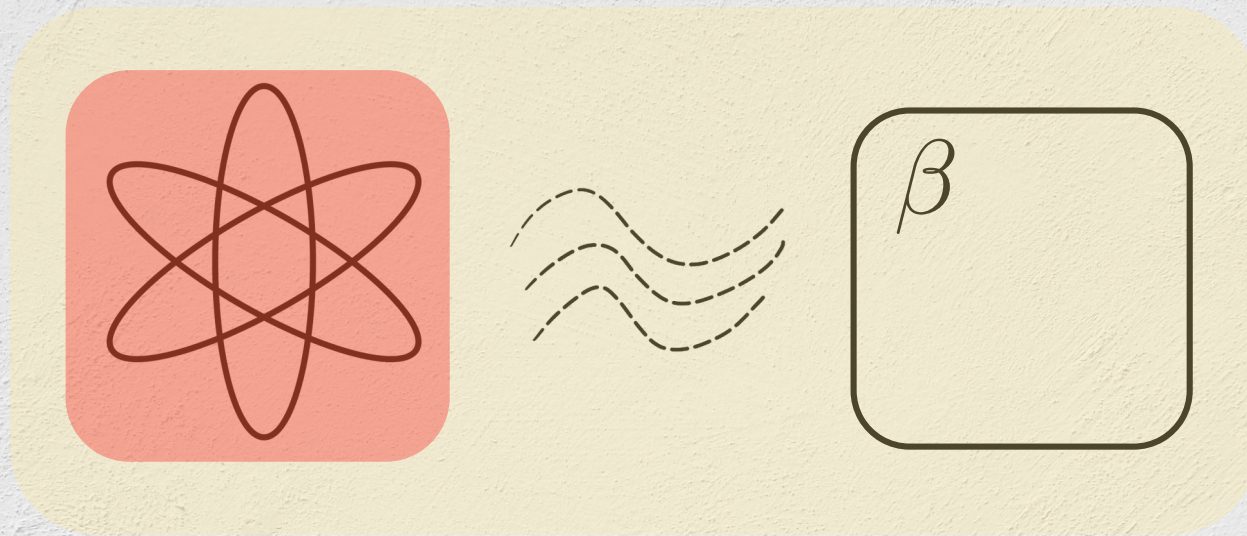
$$\boldsymbol{H}_{\text{tot}} = \boldsymbol{H}_{\text{S}} + \zeta \boldsymbol{S} \otimes \boldsymbol{B} + \boldsymbol{H}_B$$



$$\mathbf{H}_{\text{tot}} = \mathbf{H}_S + \zeta \mathbf{S} \otimes \mathbf{B} + \mathbf{H}_B$$

mean-force
Gibbs state

$$\rho_S \xrightarrow[t \rightarrow \infty]{} \text{tr}_B e^{-\beta \mathbf{H}_{\text{tot}}} \propto \tau_{MF}$$



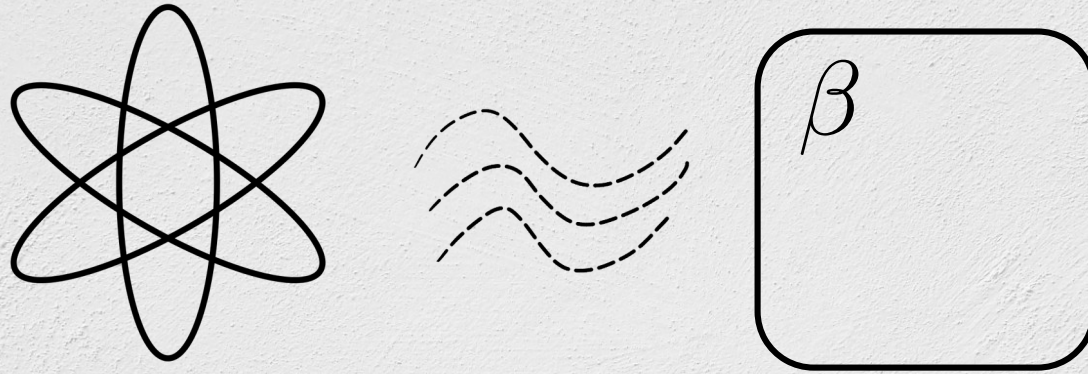
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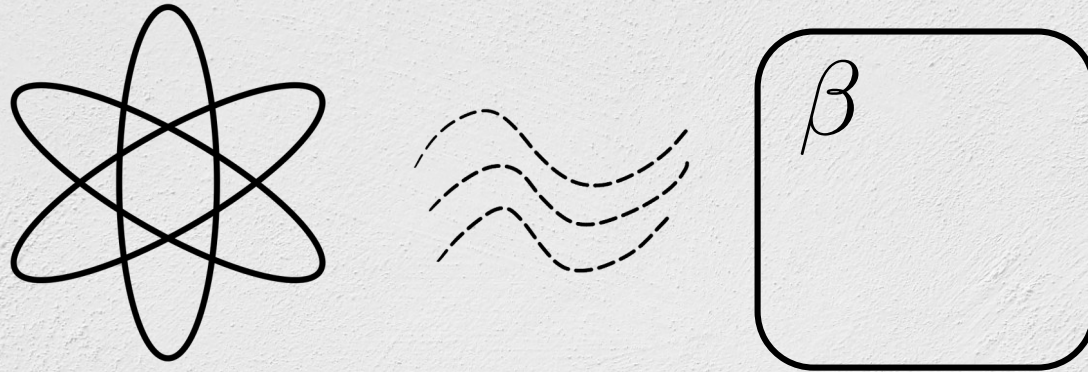
reorganisation
energy $\mathcal{O}(\zeta^2)$

$$\rho_S(t \rightarrow \infty) \underset{\beta \ll \omega_0}{\sim} e^{-\beta (\mathbf{H}_S - Q \mathbf{S}^2)}$$



renormalised Hamiltonian

$$\rho_S(t \rightarrow \infty) \underset{\beta \ll \omega_0}{\sim} e^{-\beta (H_S - Q S^2)}$$



with counter term	$H_S^{(0)} + Q S^2$	$H_S^{(0)}$
no counter term	$H_S^{(0)}$	$H_S^{(0)} - Q S^2$

microscopic Hamiltonian

renormalised Hamiltonian

$$\rho_S(t \rightarrow \infty) \underset{\beta \ll \omega_0}{\sim} e^{-\beta (H_S - Q S^2)}$$

**POTENTIAL
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to shift or not to shift? *

$$\frac{d\tilde{\boldsymbol{\varrho}}}{dt} = -i\zeta[\tilde{\mathbf{V}}(t), \tilde{\boldsymbol{\varrho}}(t)] := \zeta \tilde{\mathcal{L}}(t) \tilde{\boldsymbol{\varrho}}(t)$$



$$\tilde{\boldsymbol{\varrho}}(t) = \boldsymbol{\varrho}(0) + \zeta \int_0^t ds \tilde{\mathcal{L}}(s) \tilde{\boldsymbol{\varrho}}(s)$$

$$\boldsymbol{\varrho}(0) = \boldsymbol{\varrho}_S(0)\boldsymbol{\varrho}_B$$

$$\text{tr} \boldsymbol{\varrho}(0) \mathbf{V} = 0$$

$$\frac{d\tilde{\boldsymbol{\varrho}}_S}{dt} = \zeta^2 \int_0^t ds \text{tr}_B \tilde{\mathcal{L}}(t) \tilde{\mathcal{L}}(t-s) \tilde{\boldsymbol{\varrho}}_S(t) \boldsymbol{\varrho}_B + \mathcal{O}(\zeta^3)$$

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Markov approximation

$$\frac{d\tilde{\boldsymbol{\varrho}}_S}{dt} = \zeta^2 \int_0^\infty ds \text{tr}_B \tilde{\mathcal{L}}(t) \tilde{\mathcal{L}}(t-s) \tilde{\boldsymbol{\varrho}}_S(t) \boldsymbol{\varrho}_B + \mathcal{O}(\zeta^3)$$

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Redfield equation

$$\frac{d\tilde{\boldsymbol{\varrho}}}{dt} = -i\zeta[\tilde{\mathbf{V}}(t), \tilde{\boldsymbol{\varrho}}(t)] := \zeta \tilde{\mathcal{L}}(t) \tilde{\boldsymbol{\varrho}}(t)$$

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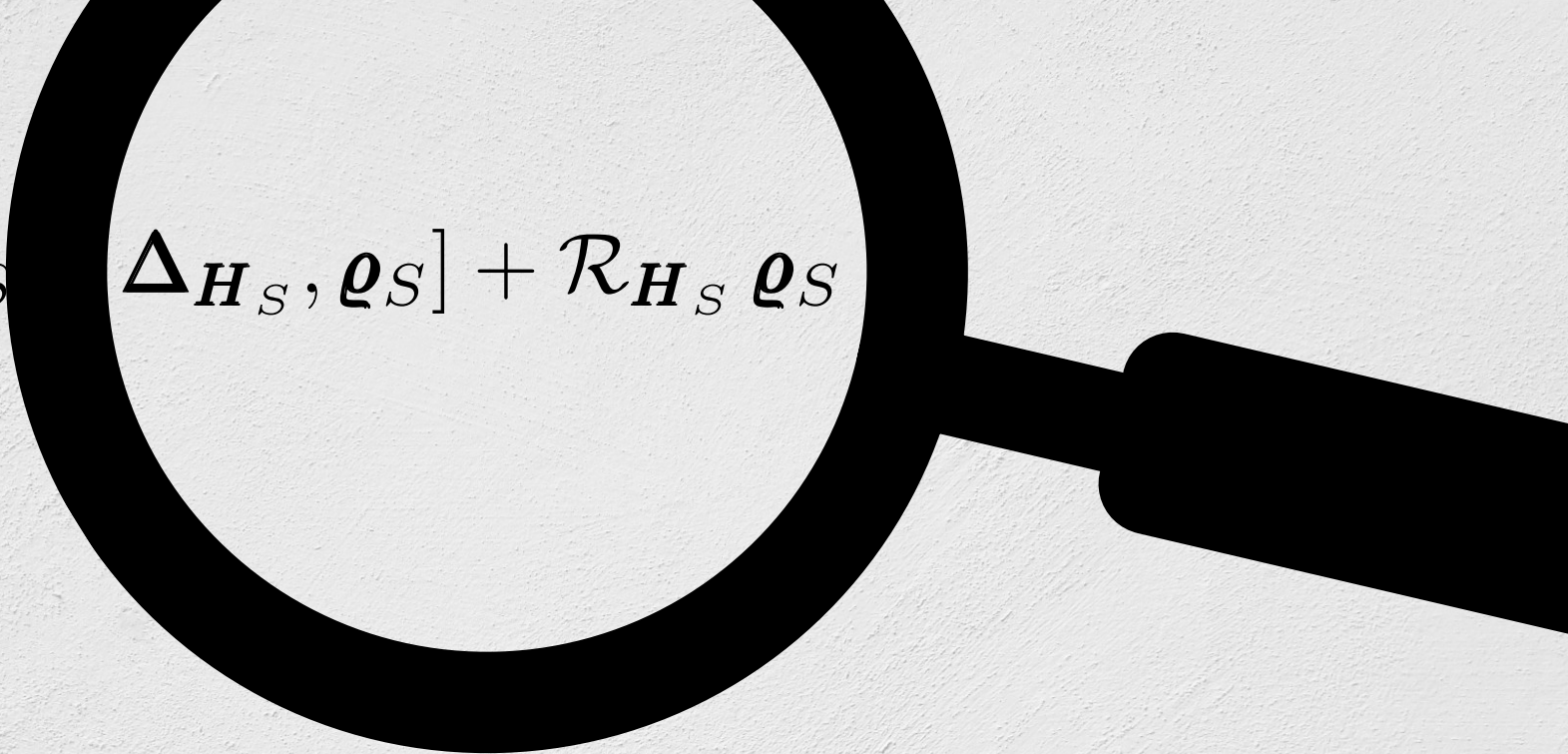
Markov approximation

$$\frac{d\tilde{\boldsymbol{\varrho}}_S}{dt} = \zeta^2 \int_0^\infty ds \text{tr}_B \tilde{\mathcal{L}}(t) \tilde{\mathcal{L}}(t-s) \tilde{\boldsymbol{\varrho}}_S(t) \boldsymbol{\varrho}_B + \mathcal{O}(\zeta^3)$$

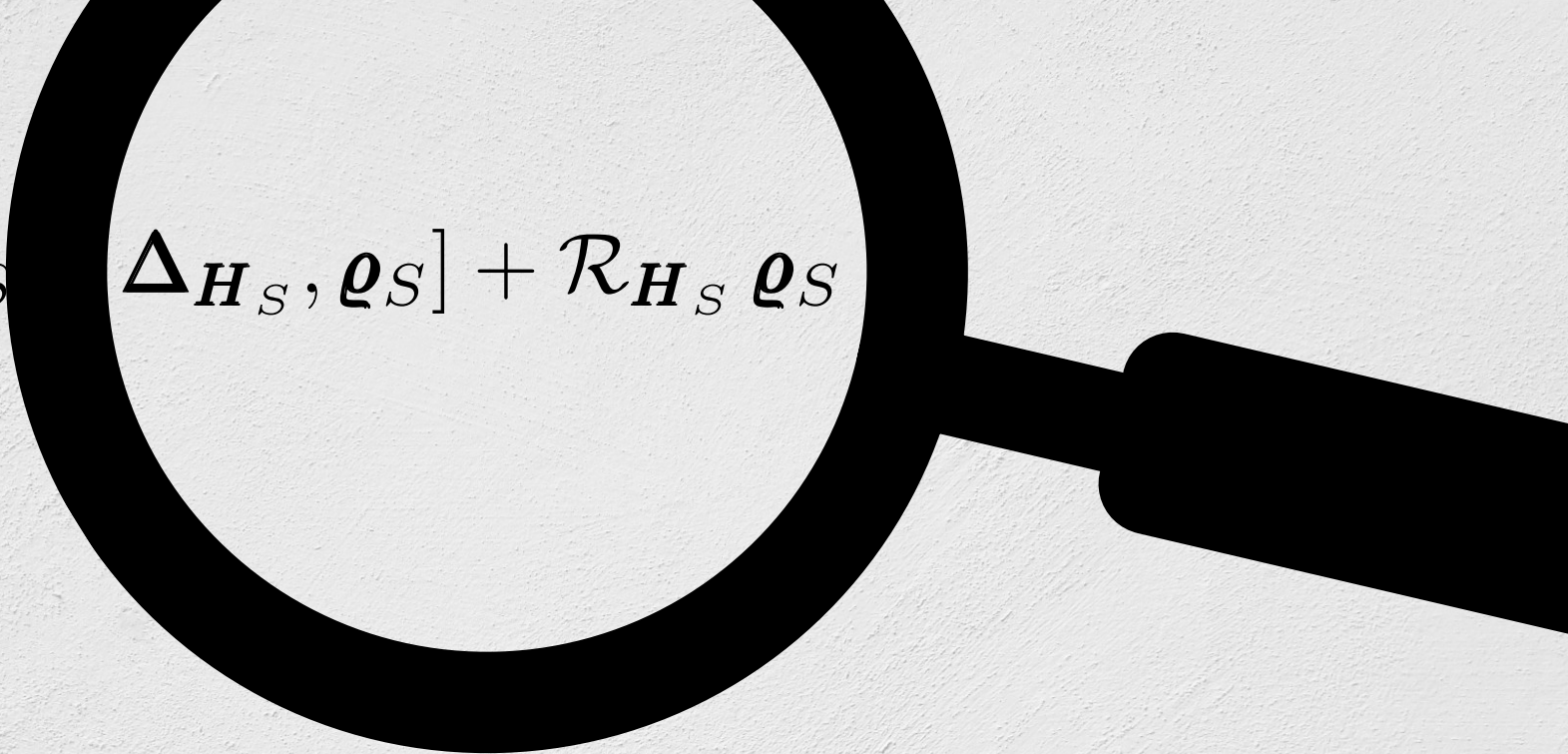
Redfield equation

$$\frac{d\boldsymbol{\varrho}_S}{dt} = -i[\mathbf{H}_S + \Delta_{\mathbf{H}_S}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\mathbf{H}_S} \boldsymbol{\varrho}_S$$

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$$\frac{d\boldsymbol{\varrho}_S}{dt} = -i[\boldsymbol{H}_S \Delta_{\boldsymbol{H}_S}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\boldsymbol{H}_S} \boldsymbol{\varrho}_S$$


$$\begin{aligned} \Gamma_\omega &= \int_0^\infty ds \, e^{i\omega s} \operatorname{tr} \tilde{\boldsymbol{B}}(t) \tilde{\boldsymbol{B}}(t-s) \boldsymbol{\varrho}_B \\ &= \frac{1}{2} \gamma(\omega) + i S(\omega) \end{aligned}$$



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$$= \underbrace{\frac{1}{2} \gamma(\omega)}_{\text{decay rate}} + i \underbrace{S(\omega)}_{\text{Lamb shift}} \quad \mathcal{O}(\zeta^2)$$

$$\begin{aligned}
\frac{d\boldsymbol{\varrho}_S}{dt} &= -i[\mathbf{H}_S + \Delta_{\mathbf{H}_S}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\mathbf{H}_S} \boldsymbol{\varrho}_S \\
&= -i[\mathbf{H}_S + \Delta_{\mathbf{H}_S}^{(\gamma)} + \Delta_{\mathbf{H}_S}^{(S)}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\mathbf{H}_S}^{(\gamma)} \boldsymbol{\varrho}_S + \mathcal{R}_{\mathbf{H}_S}^{(S)} \boldsymbol{\varrho}_S
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\end{aligned}$$

$$\frac{d\boldsymbol{\rho}_S}{dt} = -i[\mathbf{H}_S + \Delta_{\mathbf{H}_S}^{(\gamma)} + \Delta_{\mathbf{H}_S}^{(S)}, \boldsymbol{\rho}_S] + \mathcal{R}_{\mathbf{H}_S}^{(\gamma)} \boldsymbol{\rho}_S + \mathcal{R}_{\mathbf{H}_S}^{(S)} \boldsymbol{\rho}_S$$

How does one derive master equations?

$$\frac{d\varrho_S}{dt} = -i[\mathbf{H}_R + \Delta_{\mathbf{H}_R}^{(\gamma)} + \Delta_{\mathbf{H}_R}^{(S)}, \varrho_S] + \mathcal{R}_{\mathbf{H}_R}^{(\gamma)} \varrho_S + \mathcal{R}_{\mathbf{H}_R}^{(S)} \varrho_S$$

How does one derive master equations?

* **Subtract the reorganisation energy** from the microscopic Hamiltonian and derive the master equation in that basis.

with counter term	$H_S^{(0)} + Q S^2$	$H_S^{(0)}$
no counter term	$H_S^{(0)}$	$H_S^{(0)} - Q S^2$
	H_S	H_R

$$\frac{d\varrho_S}{dt} = -i[\mathbf{H}_R + \Delta_{\mathbf{H}_R}^{(\gamma)} + \cancel{\Delta_{\mathbf{H}_R}^{(S)}}, \varrho_S] + \mathcal{R}_{\mathbf{H}_R}^{(\gamma)} \varrho_S + \cancel{\mathcal{R}_{\mathbf{H}_R}^{(S)}} \varrho_S$$

How does one derive master equations?

- * **Subtract the reorganisation energy** from the microscopic Hamiltonian and derive the master equation in that basis.
- * **Vanish all the Lamb shift terms.**

$$\frac{d\boldsymbol{\rho}_S}{dt} = -i[\mathbf{H}_R + \Delta_{\mathbf{H}_R}^{(\gamma)}, \boldsymbol{\rho}_S] + \mathcal{R}_{\mathbf{H}_R}^{(\gamma)} \boldsymbol{\rho}_S$$

How does one derive master equations?

- * **Subtract the reorganisation energy** from the microscopic Hamiltonian and derive the master equation in that basis.
- * **Vanish all the Lamb shift terms.**

AND IT WORKS!

... but why?

... and when?

OUTLINE

1. Potential renormalisation and mean-force Gibbs state.
2. The Lamb shift.
3. To shift or not to shift?
4. The harmonic oscillator.

$$\mathbf{H}_S = (\mathbf{H}_S - \delta\mathbf{H}) + \delta\mathbf{H} := \mathbf{H}_R + \delta\mathbf{H} \quad \swarrow \mathcal{O}(\zeta^2)$$

$$\frac{d\boldsymbol{\varrho}_S}{dt} = -i[\mathbf{H}_S + \Delta_{\mathbf{H}_S}^{(\gamma)} + \Delta_{\mathbf{H}_S}^{(S)}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\mathbf{H}_S}^{(\gamma)} \boldsymbol{\varrho}_S + \mathcal{R}_{\mathbf{H}_S}^{(S)} \boldsymbol{\varrho}_S$$

LAC & Glatthard, Quantum 9, 1868 (2025)

Glatthard et al., PRE 112, 024109 (2025)

$\mathcal{O}(\zeta^2)$

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$$\mathbf{H}_S = (\mathbf{H}_S - Q \mathbf{S}^2) + Q \mathbf{S}^2 := \mathbf{H}_R + \underbrace{Q \mathbf{S}^2}_{\mathcal{O}(\zeta^2)}$$

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$$\mathbf{H}_S = (\mathbf{H}_S - Q \mathbf{S}^2) + Q \mathbf{S}^2 := \mathbf{H}_R + Q \mathbf{S}^2 \quad \swarrow \mathcal{O}(\zeta^2)$$

Step 1

$$\begin{aligned} \frac{d\boldsymbol{\varrho}_S}{dt} = & -i[\mathbf{H}_S + \Delta_{\mathbf{H}_R}^{(\gamma)} + \Delta_{\mathbf{H}_R}^{(S)}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\mathbf{H}_R}^{(\gamma)} \boldsymbol{\varrho}_S + \mathcal{R}_{\mathbf{H}_R}^{(S)} \boldsymbol{\varrho}_S \\ & + \mathcal{O}(\zeta^3) \end{aligned}$$

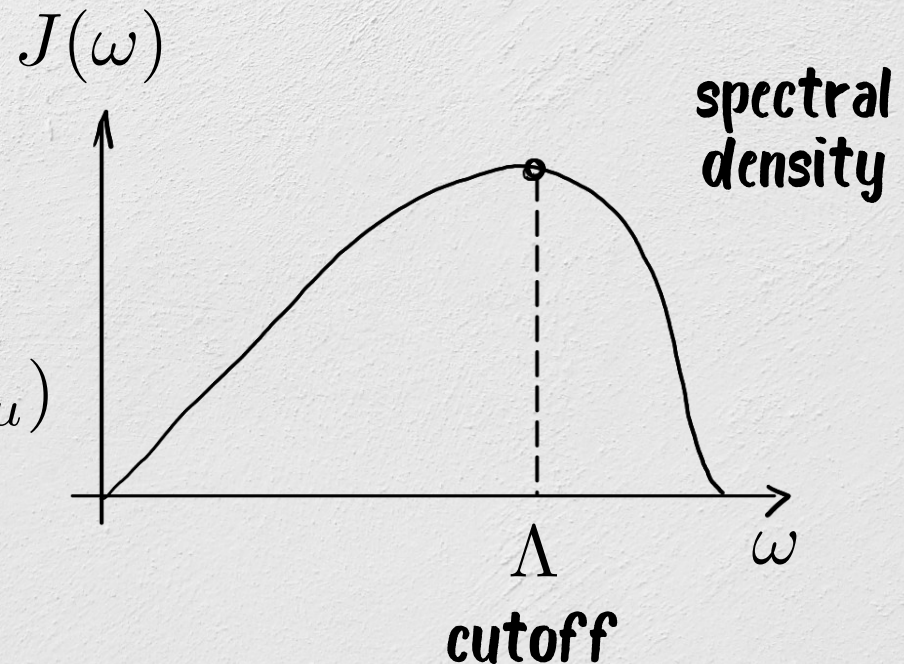
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$$\mathbf{B} = \sum_{\mu} g_{\mu} \mathbf{x}_{\mu}$$

$$J(\omega) = \sum_{\mu} \frac{g_{\mu}^2}{2 m_{\mu} \omega_{\mu}} \delta(\omega - \omega_{\mu})$$



$$\mathbf{H}_S = (\mathbf{H}_S - Q \mathbf{S}^2) + Q \mathbf{S}^2 := \mathbf{H}_R + Q \mathbf{S}^2 \quad \swarrow \mathcal{O}(\zeta^2)$$

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Step 2

$$\omega/\Lambda \ll 1 \quad -i[\Delta_{\mathbf{H}_R}^{(S)}, \boldsymbol{\varrho}_S] + \mathcal{R}_{\mathbf{H}_R}^{(S)} \boldsymbol{\varrho}_S \sim -i[-Q \mathbf{S}^2, \boldsymbol{\varrho}_S]$$

(adiabatic limit)

$$\mathbf{H}_S = (\mathbf{H}_S - Q \mathbf{S}^2) + Q \mathbf{S}^2 := \mathbf{H}_R + \overset{\mathcal{O}(\zeta^2)}{Q \mathbf{S}^2}$$

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'reorganised' master equation

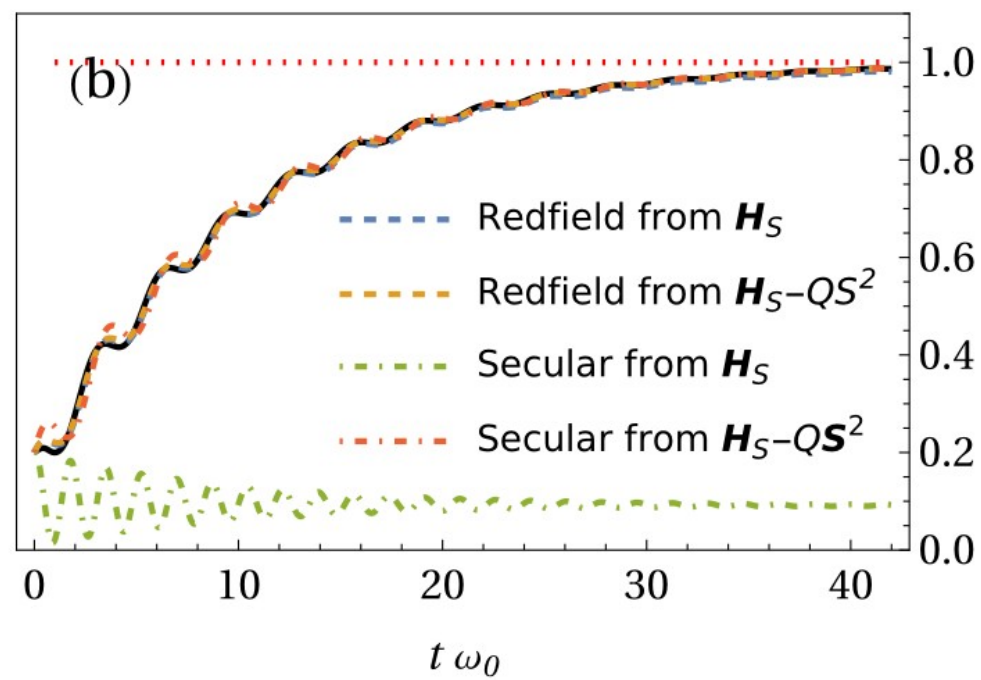
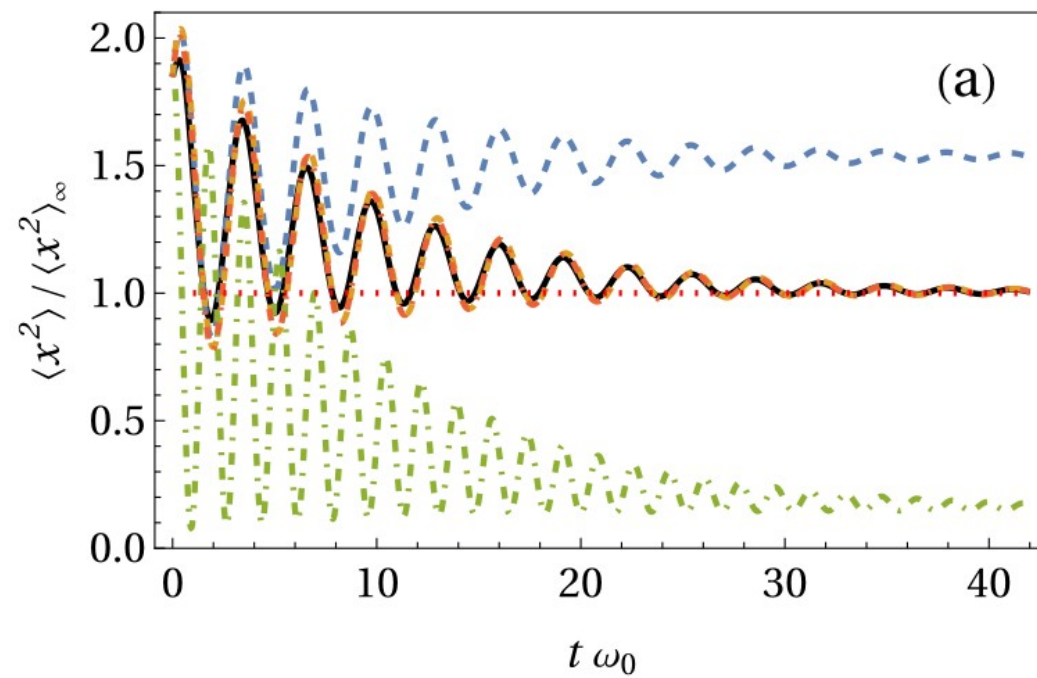
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secular
approximation

$$\frac{d\boldsymbol{\varrho}_S}{dt} = -i[\mathbf{H}_R, \boldsymbol{\varrho}_S] + \mathcal{D}_{\mathbf{H}_R} \boldsymbol{\varrho}_S$$

$$\boldsymbol{\varrho}_S(t \rightarrow \infty) \propto e^{-\beta (\mathbf{H}_S - Q \mathbf{S}^2)}$$



CONCLUSIONS

The common knowledge in open quantum systems applies only in the **adiabatic limit** and so long as the **reorganisation energy** is small.

Fixing a master equation so that it approaches the **correct steady state** tends to improve precision even during the transient dynamics.

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Plan de Recuperación,
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