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Quantum Batteries and Quantum Phase Transitions

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ICE-10: Quantum Information in Spain

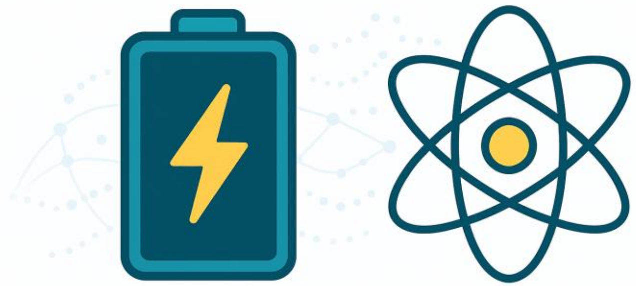
21/10/2025

Why study quantum batteries?

- The rise of quantum technologies strongly enhanced the interest in studying energetics at the quantum level: quantum thermodynamics now guides how we envision and develop novel devices based on quantum effects.

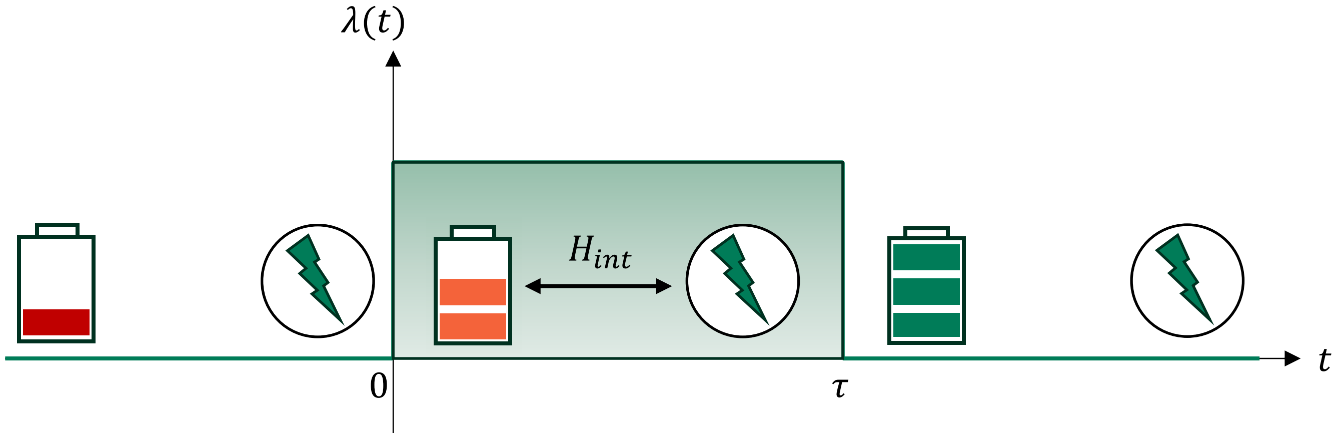
- The pursuit of efficient energy storage and transfer at the quantum scale has led to the development of **quantum batteries**: devices able to store and release energy on demand exploiting purely quantum features. *[R. Alicki, M. Fannes, Phys. Rev. E 87.4, 042123 (2013)]*

- The interest in quantum batteries is **threefold**:
  - they address fundamental questions about quantum energy exchange;
  - they can outperform classical batteries in charging power;
  - they may offer advantages in energy and time efficiency within complex quantum devices.



Double quench protocol

- The Hamiltonian of the full system is given by $\boldsymbol{H} = \boldsymbol{H_b} + \boldsymbol{H_c} + \boldsymbol{\lambda(t)} \boldsymbol{H_{int}}$



- Typical figures of merit to characterize these systems are:
  - **Energy stored** in the battery at time τ:

$$\Delta E(\tau) = Tr[\rho(\tau) H_b] - Tr[\rho(0) H_b]$$

  - **Ergotropy**, which is the maximum amount of work that can be extracted from a quantum state by means of unitary transformations:

$$\epsilon(\rho) = Tr[\rho H_b] - \min_U Tr[U \rho U^\dagger H_b]$$

Quantum phase transitions

- A **quantum phase transition** (QPT) is an abrupt change, driven by quantum fluctuations, of a many-body system's ground state induced by tuning a physical parameter at absolute zero temperature.

- Their effects on quantum systems are difficult to study numerically, as they only become apparent in the thermodynamic limit, which is challenging to explore using exact diagonalization.

- Studying **integrable models** enables the consideration of potentially infinite system sizes, leading to a deeper understanding of the underlying physics.

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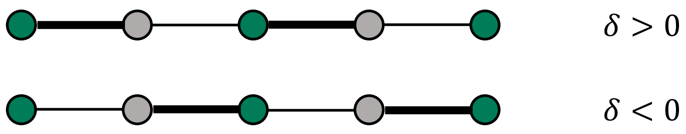
Controlling Energy Storage Crossing Quantum Phase Transitions in an Integrable Spin Quantum Battery

R.Grazi, D.Sacco Shaikh, M.Sassetti, N.Traverso Ziani and D.Ferraro

*Physical Review Letters **133**, 197001 (2024)*

Dimerized XY spin chain

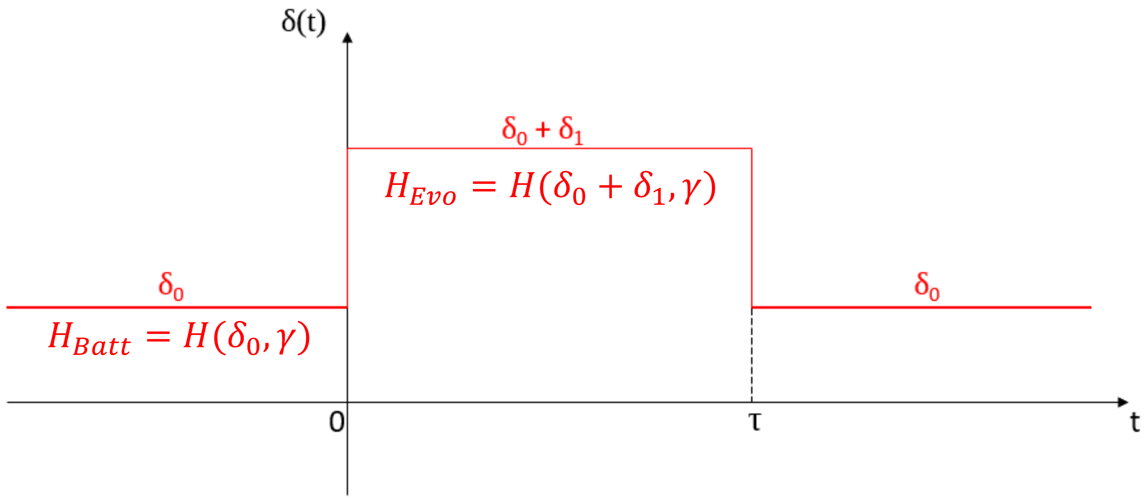
$$H(\delta,\gamma) = -J\sum_{j=1}^{N}\left\{\left[1-(-1)^j\delta\right]\left[\left(\frac{1+\gamma}{2}\right)\sigma_j^x\sigma_{j+1}^x+\left(\frac{1-\gamma}{2}\right)\sigma_j^y\sigma_{j+1}^y\right]\right\}$$

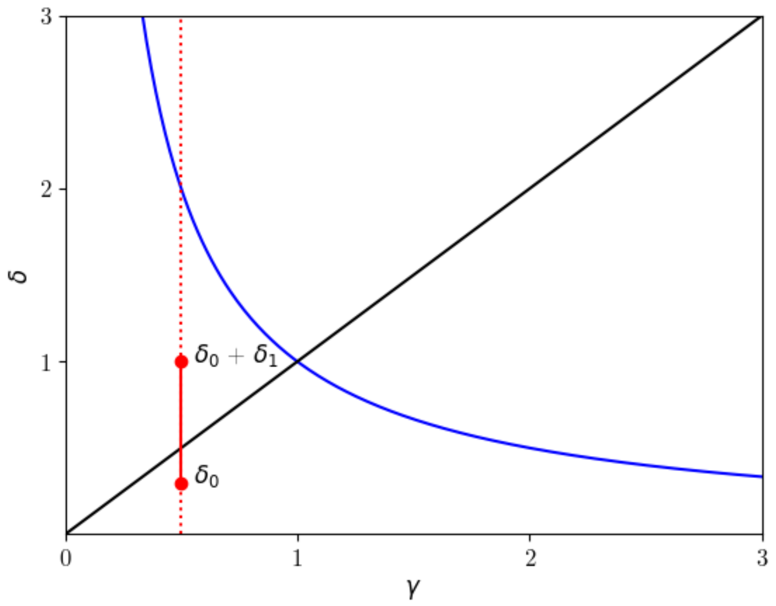


- After diagonalization we can derive the **quantum phase diagram** of the model

- We perform a double quench of the **dimerization parameter** from an initial value δ_0 to a final value $\delta_0+\delta_1$, then coming back to δ_0. From now on we define $H_{Batt} = H(\delta_0,\gamma)$ and $H_{Evo} = H(\delta_0+\delta_1,\gamma)$

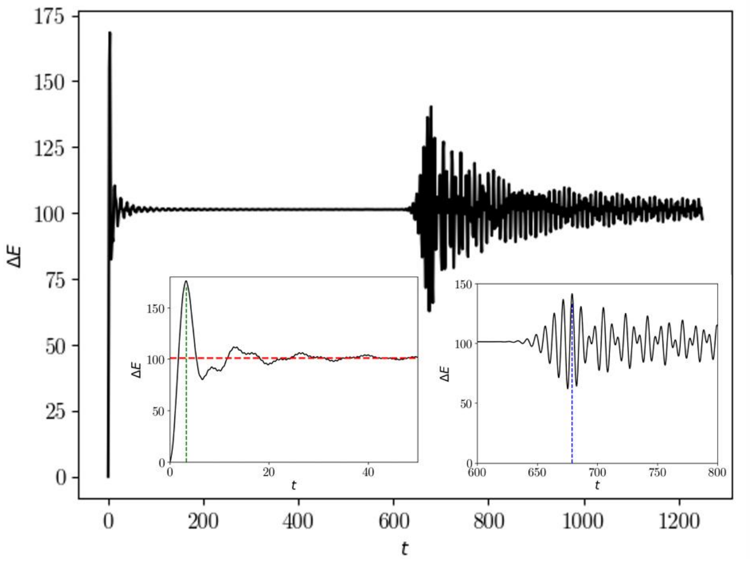
- The goal is to study the maximum energy stored in the battery as a function of δ_0 with γ and δ_1 kept fixed.





Results

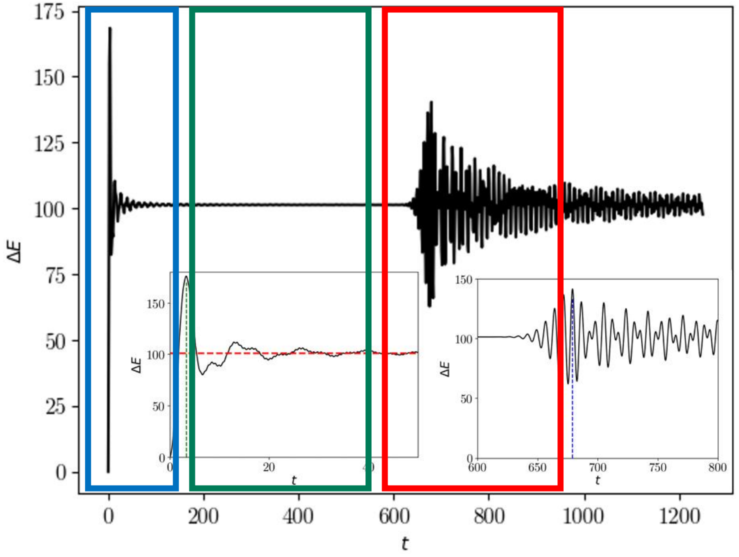
- The study of the stored energy as a function of the charging time reveals the presence of **three different time regimes**

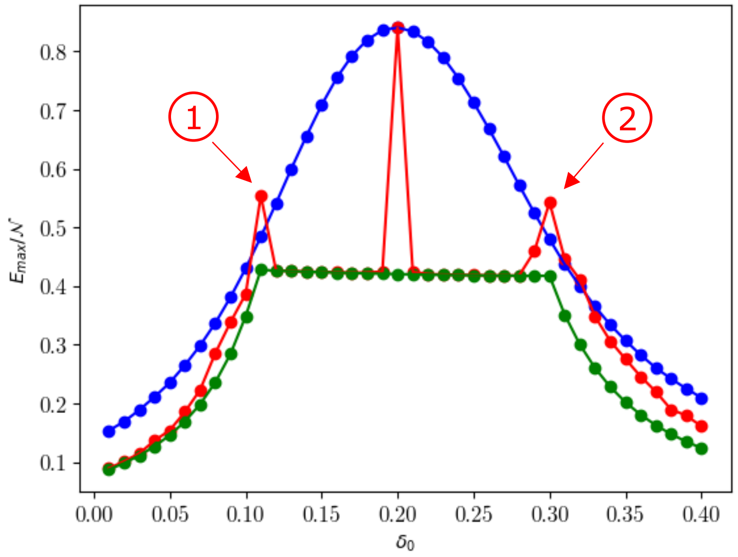


Results

- The study of the stored energy as a function of the charging time reveals the presence of **three different time regimes**

$\gamma = 1.1, \quad \delta_1 = 0.8$

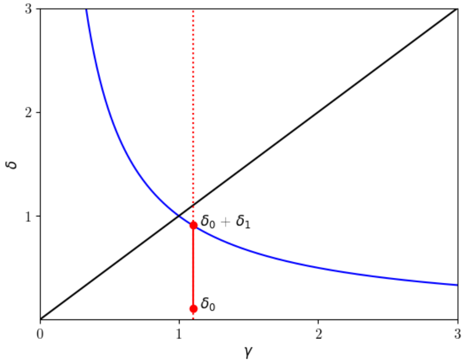




- **Short-time regime:** peak at $\delta_0 = 0.2$, so when $\delta_0 + \delta_1 = 1$ ⟹ H_{Evo} is fully dimerized

- **Long-time regime:** peaks at $\delta_0 = 0.11, \delta_0 = 0.2$ and $\delta_0 = 0.3$ ⟹ H_{Evo} is fully dimerized and critical
- **Thermodynamic regime:** kinks when H_{Evo} is critical and plateau between the two QPTs

1



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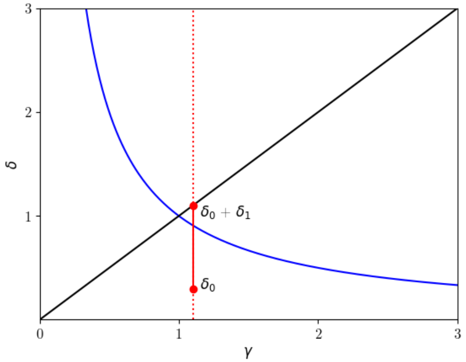

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Charging Free Fermion Quantum Batteries

R.Grazi, F.Cavaliere, M.Sassetti, D.Ferraro and N.Traverso Ziani

Chaos, Solitons and Fractals ***196****, 116383 (2025)*

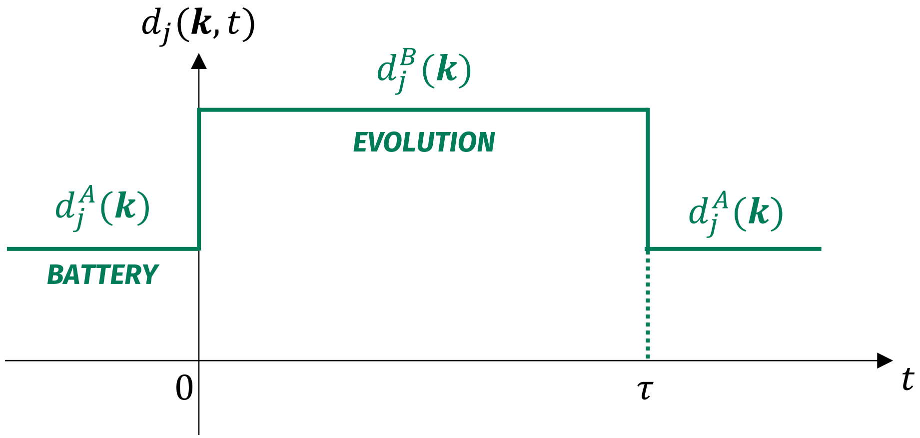
Our framework

- **Model:** quantum systems whose Hamiltonians can be reduced to 2 x 2 free fermion problems.

$$H(t) = \sum_{\boldsymbol{k}\in BZ} \begin{pmatrix} c_{a,\boldsymbol{k}}^{\dagger} & c_{b,\boldsymbol{k}}^{\dagger} \end{pmatrix} \left(d_0(\boldsymbol{k},t)\mathrm{I}_{2x2} + \boldsymbol{d}(\boldsymbol{k},t)\cdot\boldsymbol{\sigma}\right) \begin{pmatrix} c_{a,\boldsymbol{k}} \\ c_{b,\boldsymbol{k}} \end{pmatrix}$$

$$H(t) = \frac{1}{2}\sum_{\boldsymbol{k}\in BZ} \begin{pmatrix} c_{\boldsymbol{k}}^{\dagger} & c_{-\boldsymbol{k}} \end{pmatrix} \left(Z(\boldsymbol{k},t)\sigma_z + X(\boldsymbol{k},t)\sigma_x\right) \begin{pmatrix} c_{\boldsymbol{k}} \\ c_{-\boldsymbol{k}}^{\dagger} \end{pmatrix}$$

- **Charging process:** sudden quench of one internal parameter of the system.



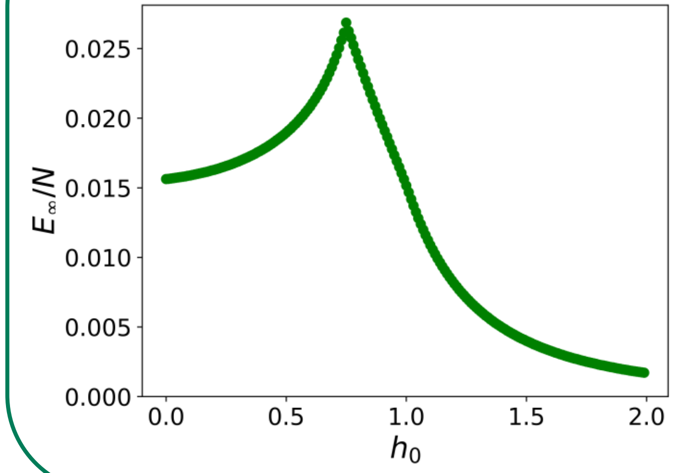
- Energy stored after a sudden quench, assuming the initial state of the quantum battery is a **thermal state** at temperature T

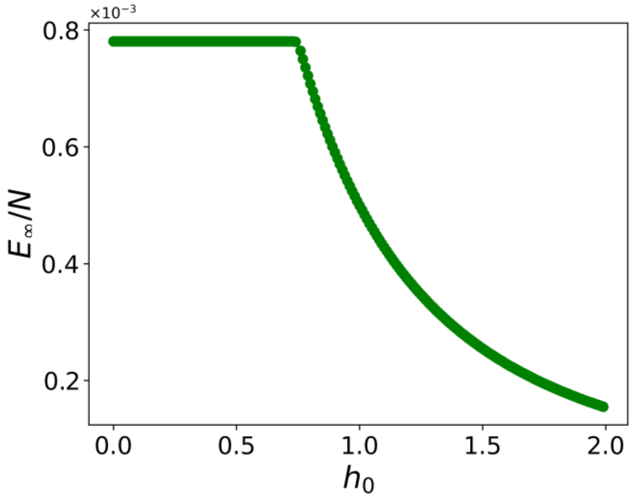
$$\Delta E(\tau) = \sum_{\boldsymbol{k}\in BZ} \frac{1-\cos(2\omega_{\boldsymbol{k}}\tau)}{\epsilon_{\boldsymbol{k}}\omega_{\boldsymbol{k}}^2} F_0(\boldsymbol{k}) F_T(\boldsymbol{k},T,\mu)$$

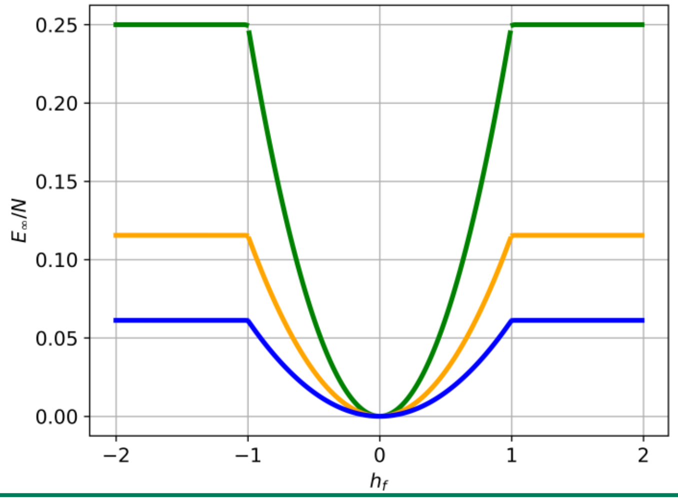
with $\epsilon_{\boldsymbol{k}} = d_0^A(\boldsymbol{k}) + |\boldsymbol{d}^A(\boldsymbol{k})|$ and $\omega_{\boldsymbol{k}} = d_0^B(\boldsymbol{k}) + |\boldsymbol{d}^B(\boldsymbol{k})|$

Applications

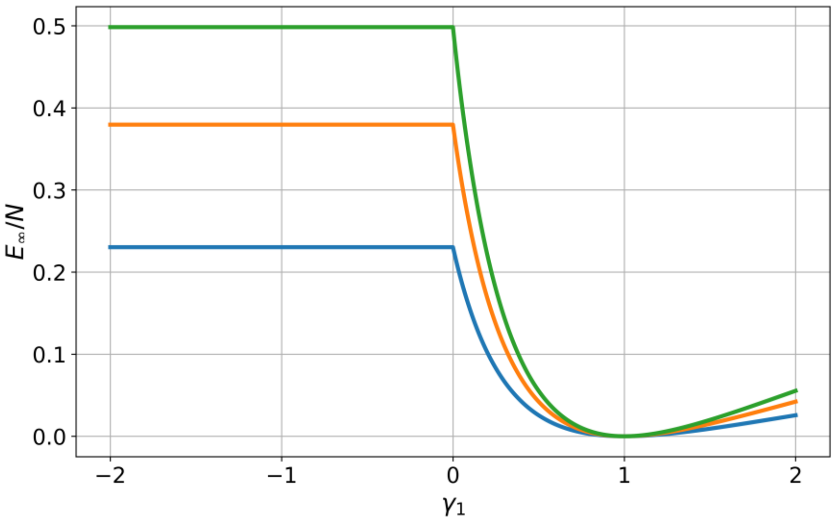
ISING CHAIN



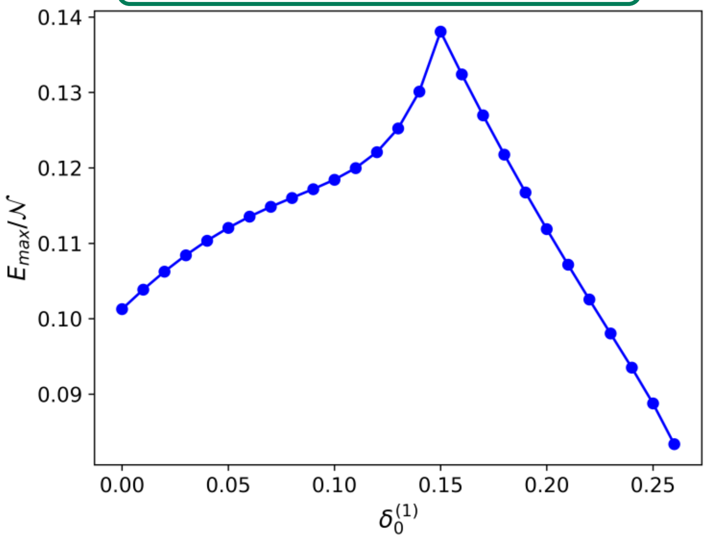




XY CHAIN



SSH CHAIN



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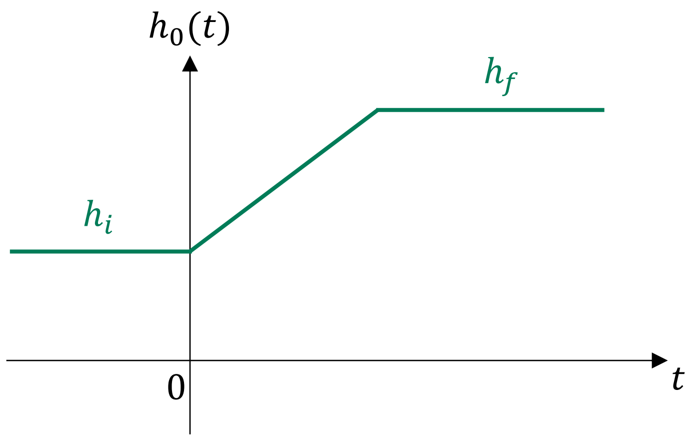
Energy and ergotropy robustness under noisy finite-time charging in an Ising quantum battery

R.Grazi, H.Johannesson, D.Ferraro and N.Traverso Ziani

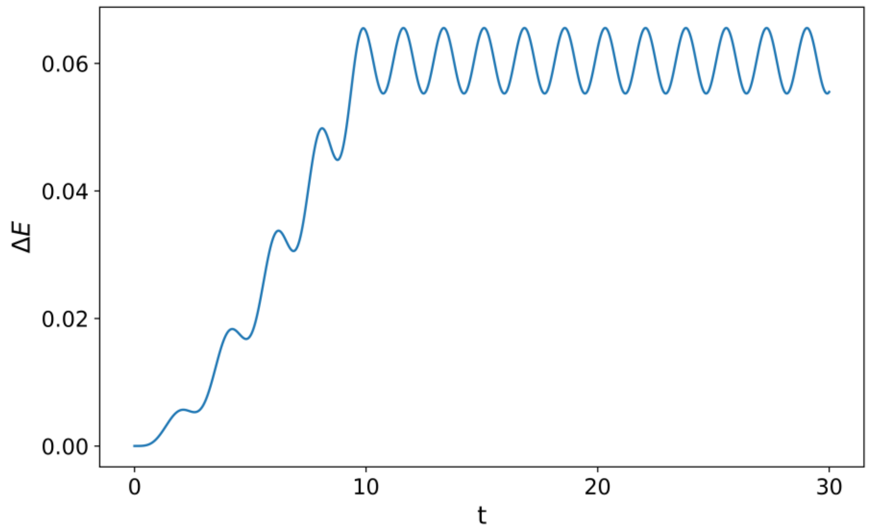
Work in progress...

Robustness against early-time oscillations

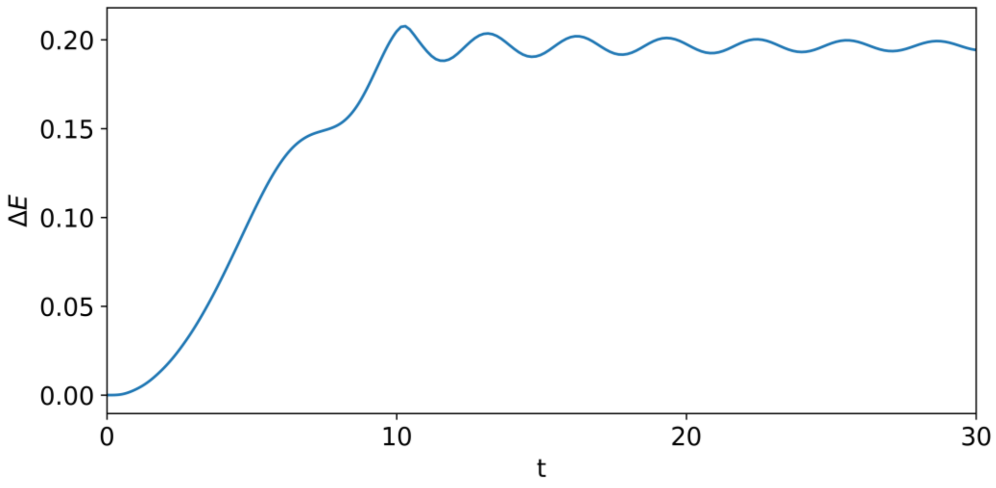
- Is it possible to achieve a more controlled charging process at short times, avoiding temporal oscillations, by moving from a sudden to a **finite-time ramp quench**?



Single-qubit system



Ising chain



Robustness against noise

- We introduce **noise** in the ramp

$$h(t) \rightarrow h(t) + \eta(t),$$

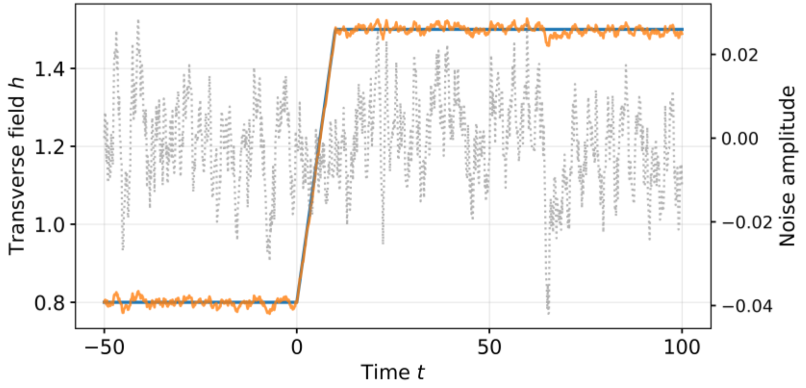
where $\eta(t)$ is a Gaussian stochastic process with $\langle \eta(t) \rangle = 0$ and **Ornstein-Uhlenbeck correlations**

$$\langle \eta(t)\eta(t') \rangle = \frac{\xi^2}{2\tau} e^{-\frac{t-t'}{\tau}}$$

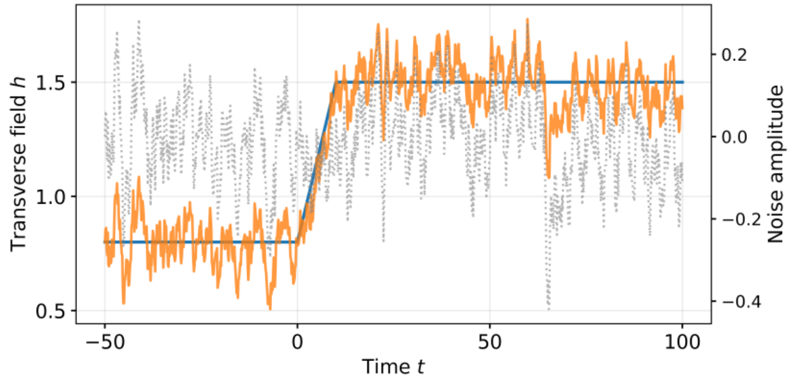
- The ensemble-averaged density matrix $\rho_k(t)$ satisfies an exact **noise master equation**

[R. Jafari, A. Langari, S. Eggert, H. Johannesson Physical Review B 109 (18), L180303 (2024)]

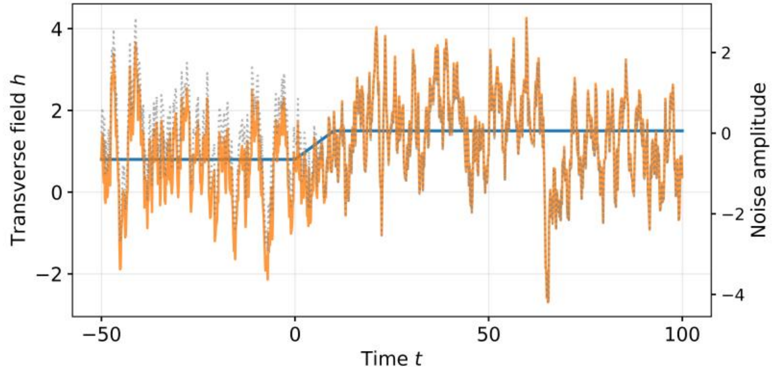
$\xi = 0.01$



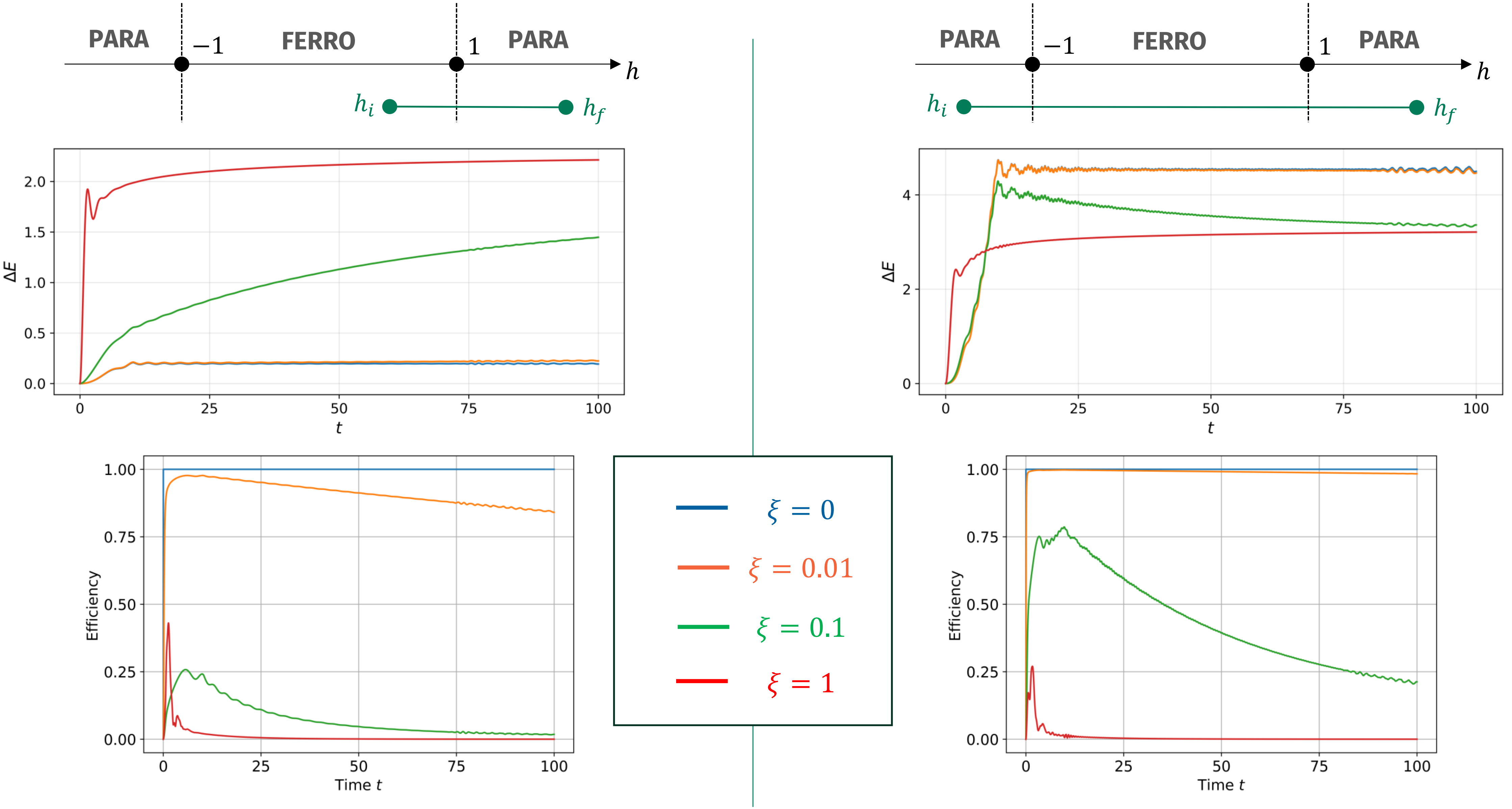
$\xi = 0.1$



$\xi = 1$



Robustness against noise



Conclusions

- We studied the implementation of systems with a large number of quantum degrees of freedom as quantum batteries by using quench charging protocols.

- The stored energy strongly depends, in a non-analytical fashion in the thermodynamic limit, on the presence of quantum phase transitions.

- Studying such type of quantum batteries can lead to promising new features such as:
    - peaks in the stored energy when the evolution Hamiltonian is critical, both considering as initial state of the system its ground state or a thermal state;
    - formation of plateau regions where the stored energy is not affected by the specific charging parameters, offering more design control;
    - enhanced robustness to external noise considered into the charging process.

R.G. *et al, «Controlling Energy Storage Crossing Quantum Phase Transitions in an Integrable Spin Quantum Battery», Phys. Rev. Lett.* 133, 197001 (2024)

R.G. *et al, «Charging free fermions quantum batteries», Chaos, Solitons & Fractals* 196, 116383 (2025)

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