Optimal quantum transport on a ring via locally monitored chiral quantum walks

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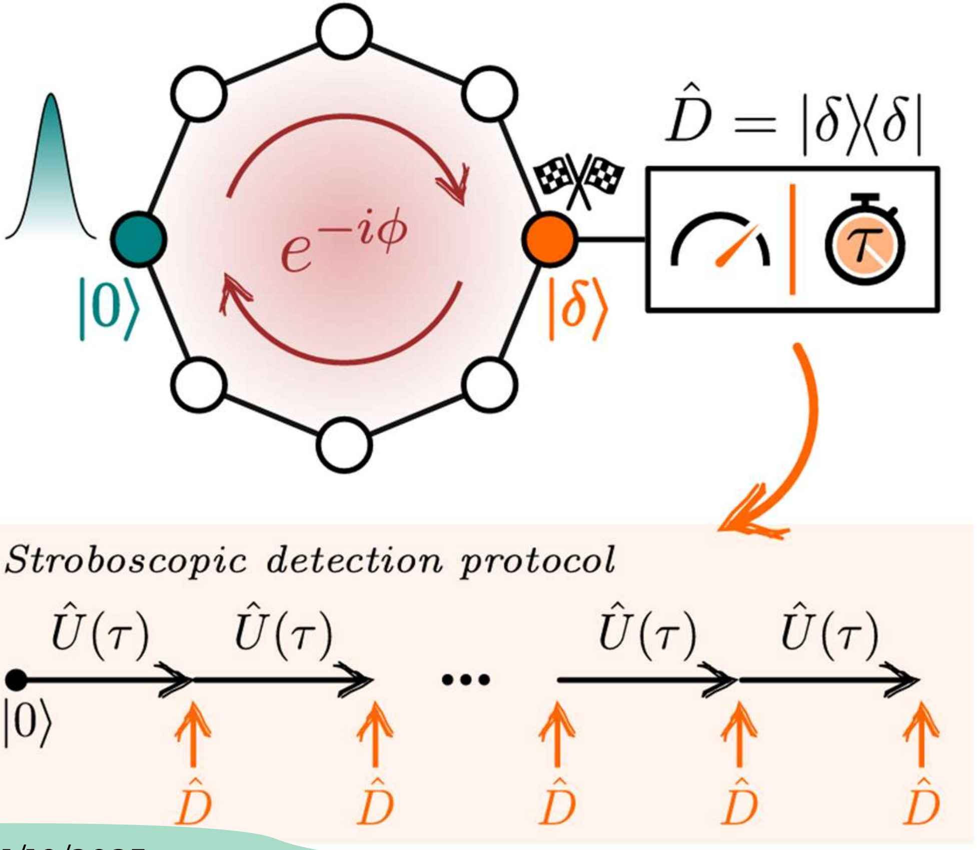
Outline

Model

We consider a continuous time chiral quantum walk on a ring, investigating the role of **chirality** and **local monitoring** in enhancing the transfer of an excitation between opposite sites.



Initial state: $|\psi_0\rangle = |0\rangle$

Target site: $|\delta\rangle = \begin{cases} |N/2\rangle & \text{even } N \\ |(N \pm 1)/2\rangle & \text{odd } N \end{cases}$

$$H = \sum_{j=0}^{N-1} e^{-i\phi} |(j+1)_N\rangle\langle j| + e^{i\phi} |(j-1)_N\rangle\langle j|$$

The walker is observed by the detector for the first time at the nth attempt → First-detected-passage time: $n\tau$

Optimization problem

- Goal: find an **optimal and robust transfer protocol** that maximizes the detection probability at the target site without requiring fine tuning of parameters.

- Detection probability up to time $n\tau$:

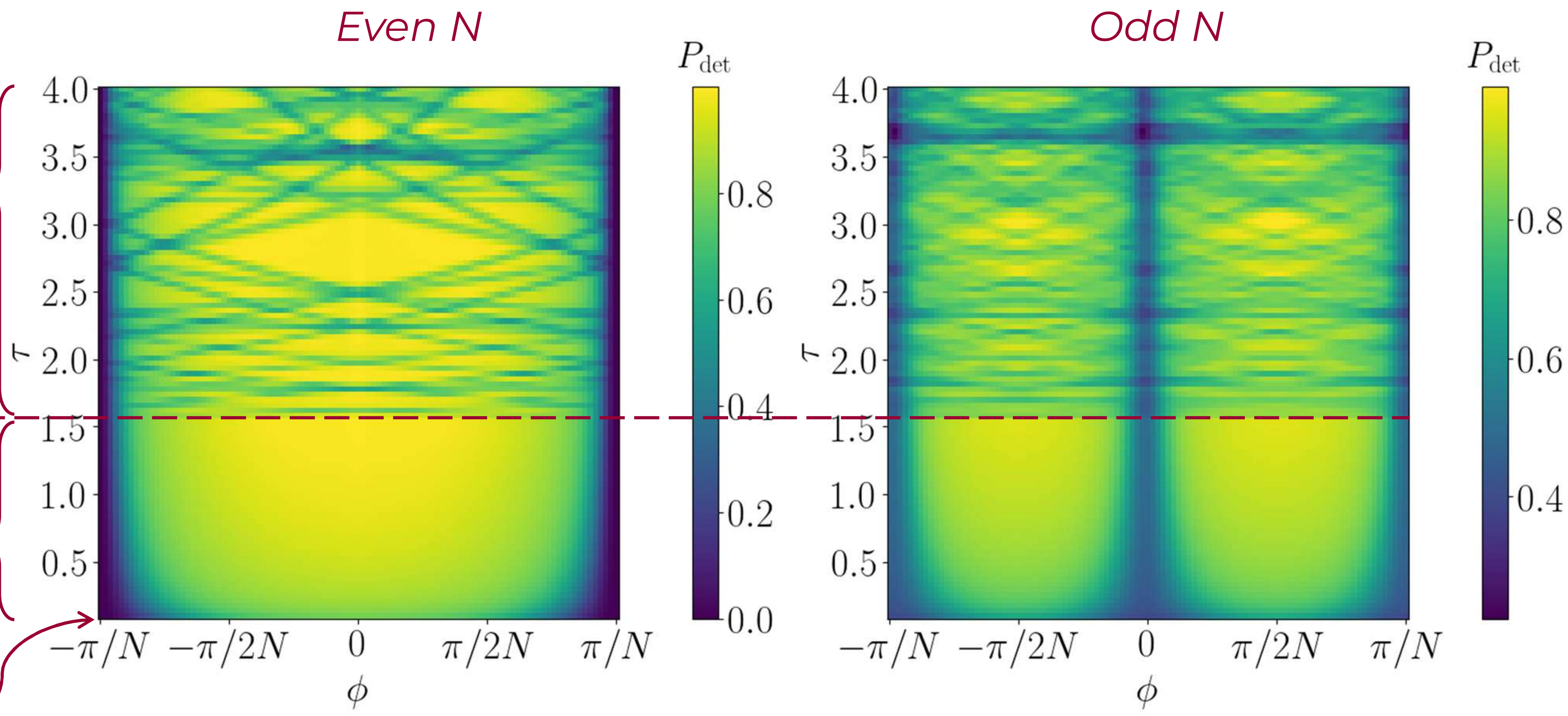
$$\mathrm{P}_{det}(n) = \sum_{m=1}^{n} F_m \text{ , with } F_n = \langle\theta_n|\hat{D}|\theta_n\rangle \text{ and } |\theta_n\rangle = \hat{U}(\tau)\left[\left(\mathbb{I} - \hat{D}\right)\hat{U}(\tau)\right]^{n-1}|\psi_0\rangle.$$

- The total time of the process is assumed to be a resource and fixed at a convenient value $T = 200$. $P_{det}(\phi,\tau)$ after $n = \lfloor T/\tau \rfloor$ detection attempt is maximized over ϕ and τ.

H. Friedman, D. A. Kessler, and E. Barkai, Phys. Rev. E 95, 032141 (2017).

Optimal detection period

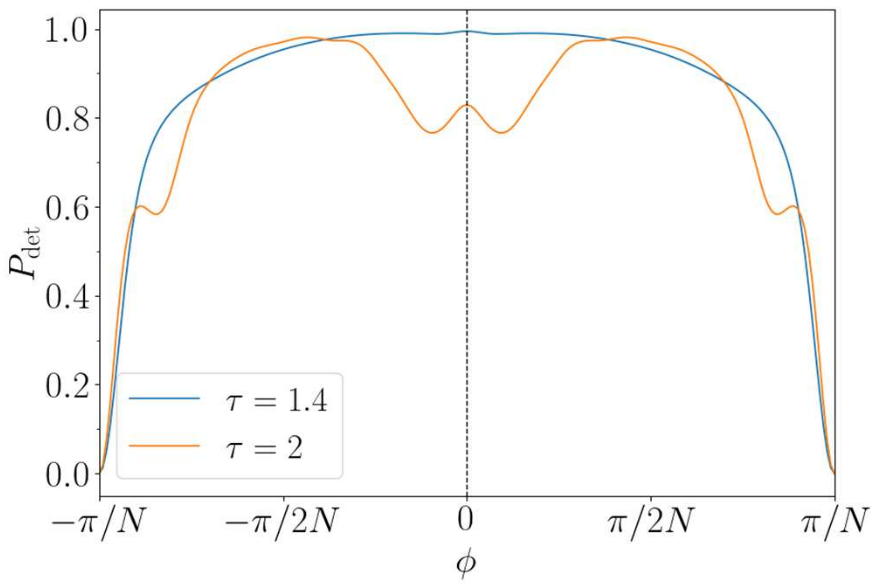
- Highly structured pattern with sharp oscillations
- **Threshold $\boldsymbol{\tau^*}$**
- Smooth behavior, generally increasing with τ
- Quantum Zeno effect: $P_{det} \to 0$ as $\tau \to 0$



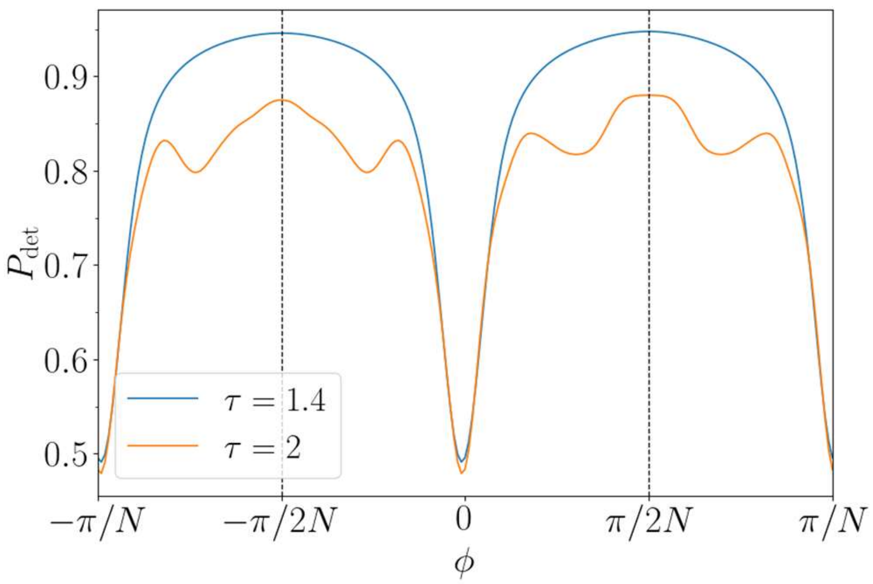
We aim for a robust protocol that does not require prior knowledge of parameters, so we restrict the optimization to the region $0 < \tau \leq \tau^*$ (We will show that for large N $\tau^* \to \pi/2^+$).

Optimal phase

Even N



Odd N



$P_{det}(\phi,\tau) = P_{det}(-\phi,\tau)$

For $\tau < \tau^*$ $\boldsymbol{\phi_{opt}} = \mathbf{0}$ and $P_{det} = 0$ at $\phi = \pm\pi/N$

↓

chirality inhibits excitation transfer between opposite sites of the symmetric cycle

$P_{det}(\phi,\tau) \neq P_{det}(-\phi,\tau)$

For $\tau < \tau^*$ $\boldsymbol{\phi_{opt}} = \boldsymbol{\pm\pi/2N}$ and P_{det} is minimized at $\phi = 0, \pm\pi/N$

↓

chirality enhances the transfer in the asymmetric cycle at target sites $\delta = (N \pm 1)/2$

Dark states

The presence of the detector divides the Hilbert space into two parts:

Bright subspace

↓

$P_{det}(\infty) = 1$

Dark subspace

↓

$P_{det}(\infty) = 0$

Dark states are orthogonal to the detection site $|\delta\rangle$ and stationary with respect to evolution and detection attempts.
They satisfy the condition

$$\lambda_m \tau = \lambda_n \tau (\mathrm{mod}\ 2\pi)$$

with $\lambda_{m,n}$ eigenvalues of the Hamiltonian. This equation can hold true in two cases:

1. Spectral degeneracies
2. Particular values of ϕ and τ

F. Thiel, I. Mualem, D. Meidan, E. Barkai, and D. A. Kessler, Phys. Rev. Res. 2, 043107 (2020).

Dark states

1. Spectral degeneracies

These arises for $\phi = 0, \pm\pi/N$:

- *Even N* → dark states at $\phi = 0$ do not affect the evolution of $|\psi_0\rangle$ yielding $P_{det}(\infty) = 1$, while for $\phi = \pm\pi/N$ the initial state is dark, yielding $P_{det}(\infty) = 0$.
- *Odd N* → the initial state has a finite overlap with dark states at $\phi = 0, \pm\pi/N$, yielding $P_{det}(\infty) = 1/2$.

2. Particular values of ϕ and τ

When the eigenvalues are nondegenerate, dark states appear for

$$\tau_{dark} = \frac{k\pi}{2\sin\left(\frac{\pi(m-n)}{N}\right)\sin\left(\phi - \frac{\pi(m+n)}{N}\right)} \geq \pi/2.$$

The solutions of this expression correspond to <u>low-detection-probability curves</u> in the parameters space. The first dark state associate with nondegenerate levels arises at $\tau_{dark} \to \pi/2$ as $N \to \infty$. This establishes an upper bound for the optimal τ consistent with the threshold τ^*.

Perron-Frobenius analysis

Perron-Frobenius operator

$$\hat{O}(\phi,\tau) = \left[\mathbb{I} - \hat{D}\right]\hat{U}(\phi,\tau)$$

describes the elementary step of the non-unitary evolution.

Survival probability

$$S_n = \left\|\hat{O}^n(\tau)|\psi_0\rangle\right\|^2 = 1 - P_{det}$$

is the probability that the walker has not been detected after n attempts.

In the long-time limit $P_{det} = 1$ when S_n vanishes.
Eigenvectors of $\hat{O}$ with $|\mu_j| < 1$ give exponentially decaying contributions to S_n, while eigenvectors with $\left|\mu_j\right| = 1$ yield non-decaying contributions that remain finite for $n \to \infty$ (dark states).
So the asymptotic dynamics is determined by the **largest-modulus eigenvalue of P-F operator,** $\mu_{\boldsymbol{PF}}$.

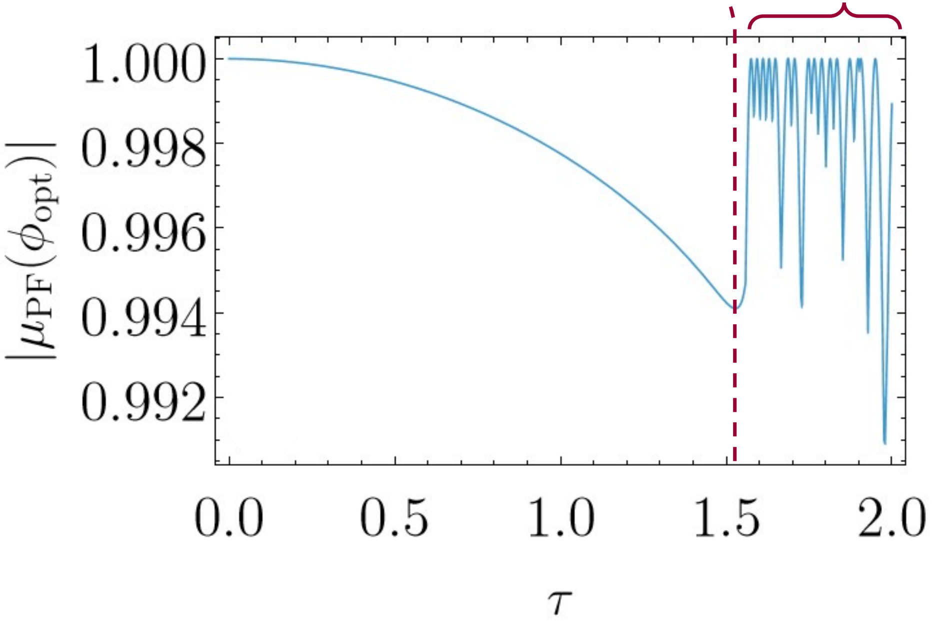
F. Thiel, I. Mualem, D. Meidan, E. Barkai, and D. A. Kessler, Phys. Rev. Res. 2, 043107 (2020).

Perron-Frobenius analysis

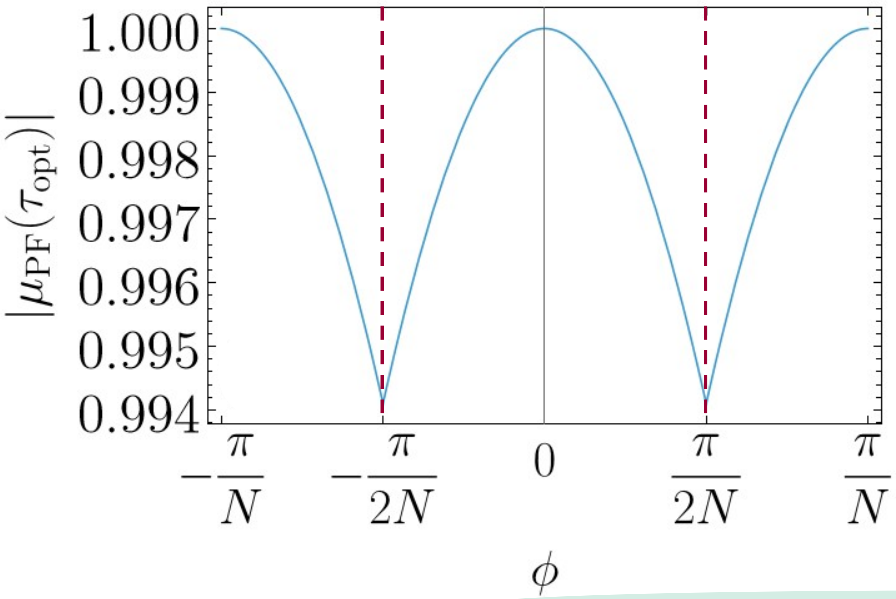
The optimal parameters, τ_{opt} and ϕ_{opt}, can be estimated by minimizing the modulus $|\mu_{PF}|$:

Minimum at $\tau = \tau_{opt}$

Oscillatory behavior with $|\mu_{PF}| = 1$ for specific τ → non-decaying modes



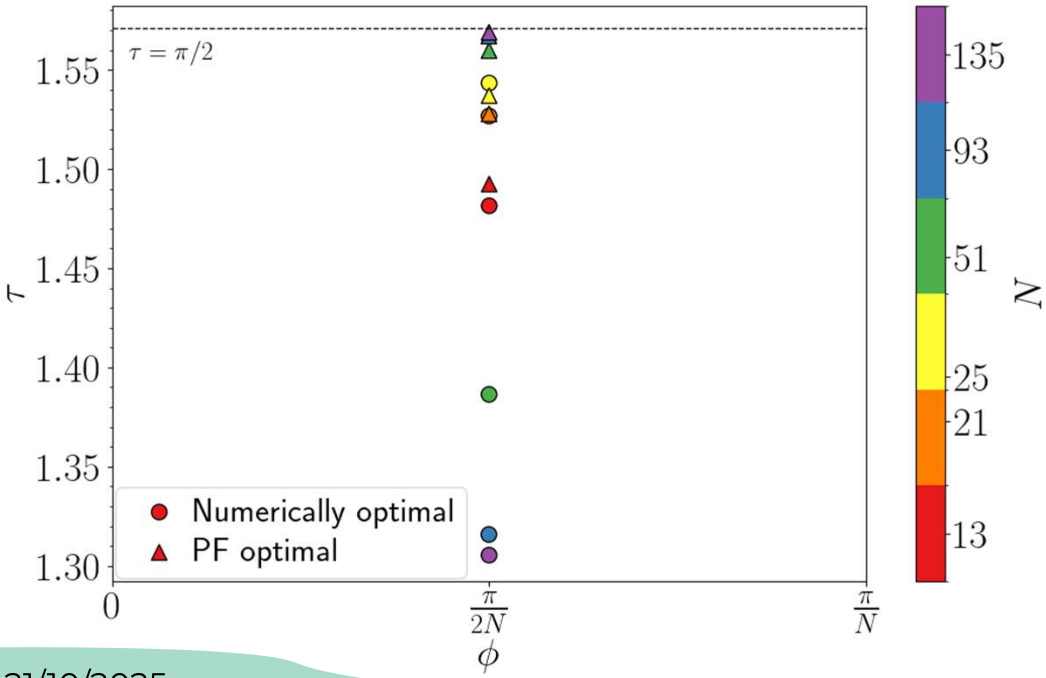
Minimum at the optimal phase ϕ_{opt}



Finite-time effects

How predictive is the Perron-Frobenius analysis for the optimal parameters at finite time? It depends on how T is close to the **asymptotic time scale t_{as}**:

- $\boldsymbol{T > t_{as}}$ → the asymptotic regime holds true: $\phi_{opt} = \phi_{PF}$ and $\tau_{opt} = \tau_{PF}$
- $\boldsymbol{T < t_{as}}$ → P_{det} does not saturate to its asymptotic value, the accuracy of the predictions is worsened



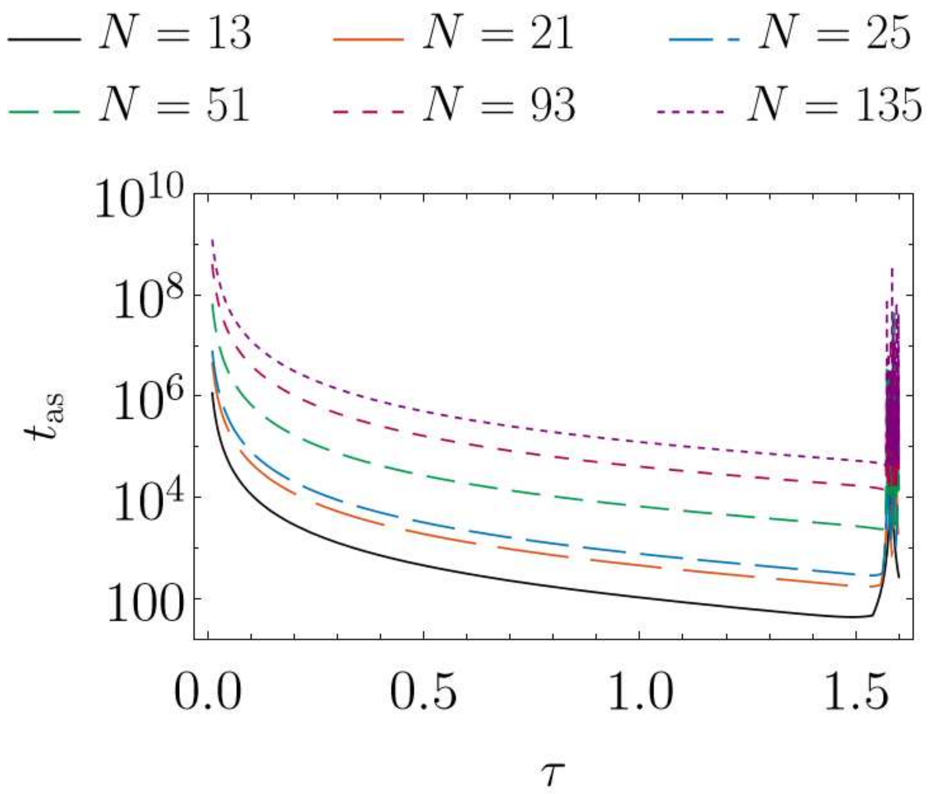
This means that for $T = 200$:

High N → $T \ll t_{as}$ → $\begin{cases} \phi_{opt} = \phi_{PF} \\ \tau_{opt} \text{ decreasing}, \tau_{PF} \to \pi/2^- \end{cases}$

Small N → $T \gtrsim t_{as}$ → $\begin{cases} \phi_{opt} = \phi_{PF} \\ \tau_{opt} \approx \tau_{PF} \end{cases}$

Asymptotic time scale

The asymptotic time scale is governed by the spectral gap of the Perron-Frobenius operator $\Delta \equiv 1 - |\mu_{PF}|$, specifically, it can be estimated as $\boldsymbol{t_{as}} \sim \mathbf{1}/\Delta$.



- $\tau \to 0$: t_{as} diverges due to the quantum Zeno effect
- $0 < \tau < \tau^*$: t_{as} decreases and reaches a **minimum at $\boldsymbol{\tau_{PF}}$**
- $\tau > \tau^*$: t_{as} reflects the system's sensitiveness to small changes in τ and ϕ.

This trend persists with increasing N, which leads to larger t_{as} (longer time for the excitation to reach the target).

Conclusions

- We developed an **optimal and robust protocol** which exploits **chirality** and **local monitoring** to enhance excitation transfer on a ring, with joint optimization of the chiral phase ϕ and detection period τ overcoming the limits of purely unitary dynamics.

- Our approach combines two key insights:
  1. The identification of dynamically relevant **dark states**,
  2. The spectral analysis of the non-unitary **Perron-Frobenius operator** to determine optimal parameters.

- The analysis is exact asymptotically and remains effective at finite times as long as the observation time T scales with the system size N. This offers a general framework for **enhancing transport in monitored quantum systems** beyond the simple model investigated here.

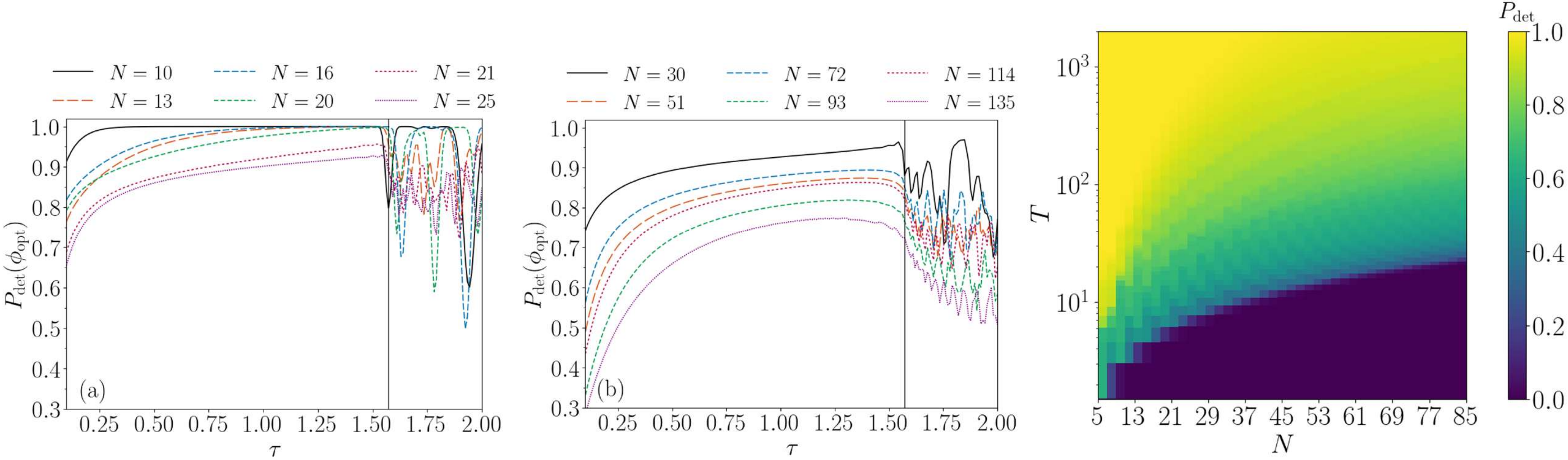
Collaborators:
Giuliano Benenti, *University of Insubria*
Luca Razzoli, *University of Pavia*
Matteo G. A. Paris, *University of Milan*
Giovanni O. Luilli, *University of Milan*

Thank you for your attention!

S. Finocchiaro, G. O. Luilli, G. Benenti, M. G. A. Paris, and L. Razzoli (2025),
Optimal quantum transport on a ring via locally monitored chiral quantum walks,
https://arxiv.org/pdf/2507.10669

Finite-time effects

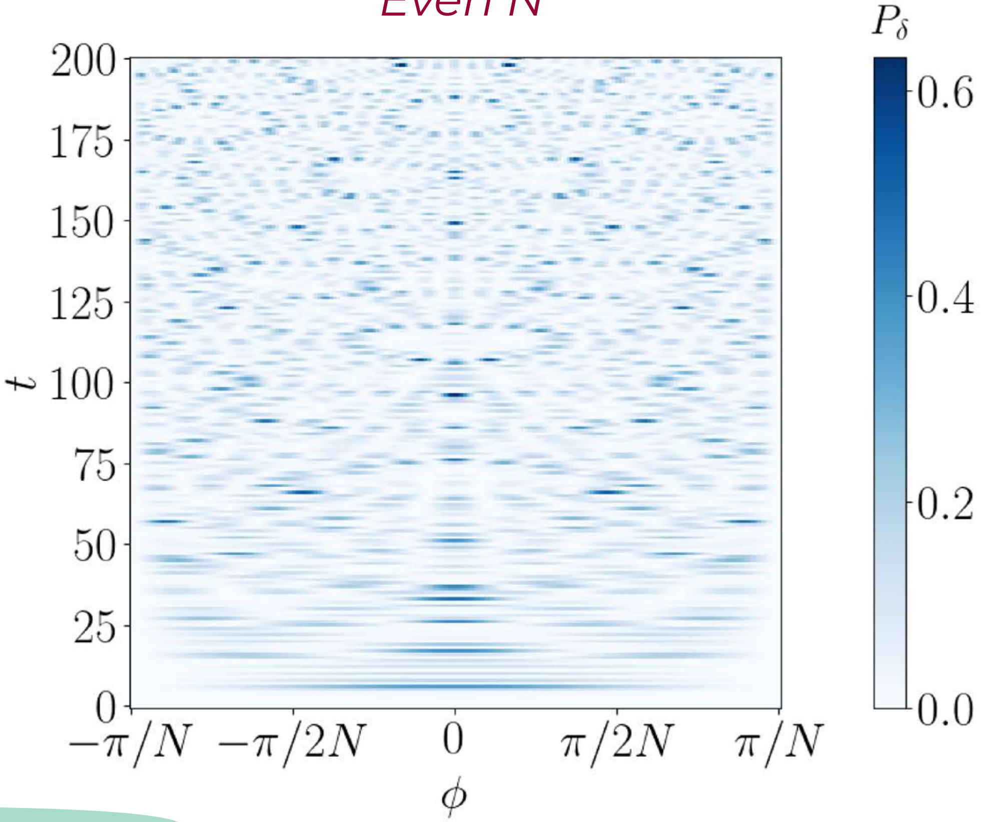
The detection probability correctly saturates to the asymptotic value for low system sizes, $N \leq 20$.
Larger system size → longer time for the excitation to reach the target → longer time scale over which the asymptotic behavior emerges and $P_{det} = 1$.



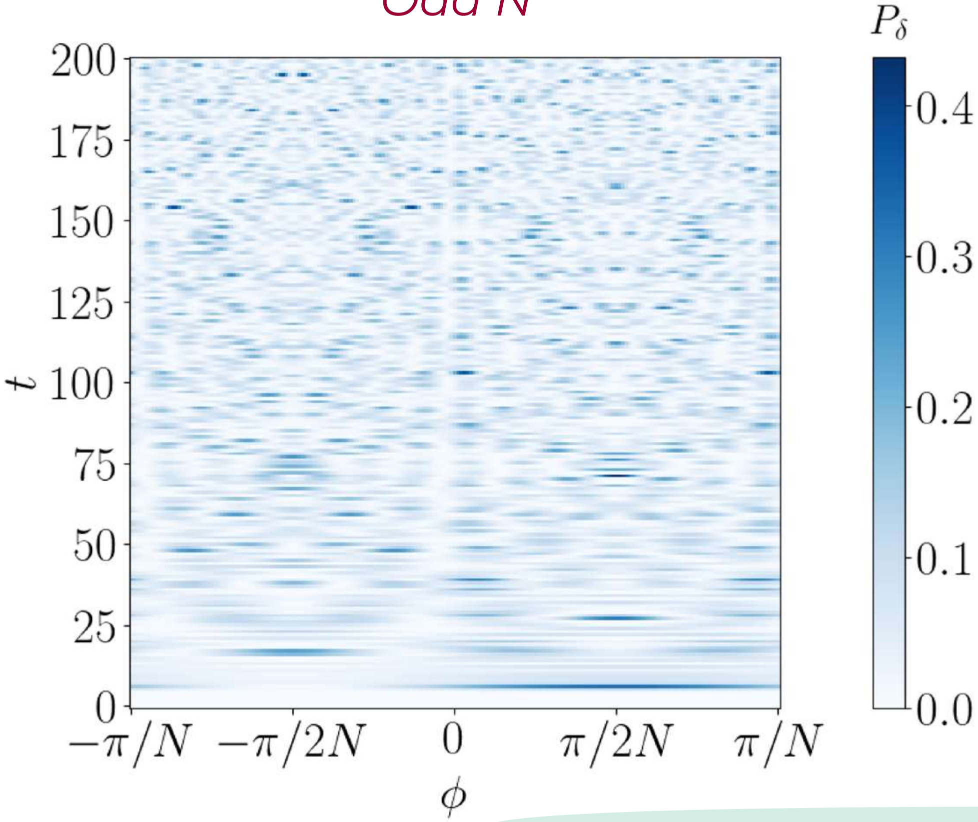
Purely-coherent transport under unitary evolution

The purely-coherent transport under unitary dynamics is not efficient. The instantaneous probability $P_\delta(\phi, t) = \left|\langle\delta|\hat{U}(\phi,t)|\psi_0\rangle\right|^2$ remains low throughout the evolution, punctuated by narrow, sharp peaks.

Even N



Odd N



Perron-Frobenius analysis for even N

$|\mu_{PF}|$ shows two symmetric minima at $\phi \neq 0$, in contrast with the numerically optimal phase $\phi_{opt} = 0$ that we expect for even N. On the other hand, dark states built using degenerate energy levels are irrelevant for excitation transfer at $\phi = 0$. Here one should consider the modulus of the subleading eigenvalue, $|\tilde{\mu}_{PF}|$, which is lower than the two local minima of $|\mu_{PF}|$ at $\phi \neq 0$.