Italian National Research Council

SAPIENZA UNIVERSITÀ DI ROMA

NQSTI National Quantum Science and Technology Institute

Fondazione Compagnia di San Paolo

Critical Quantum Sensing

(Theory and Experiments)

Simone Felicetti

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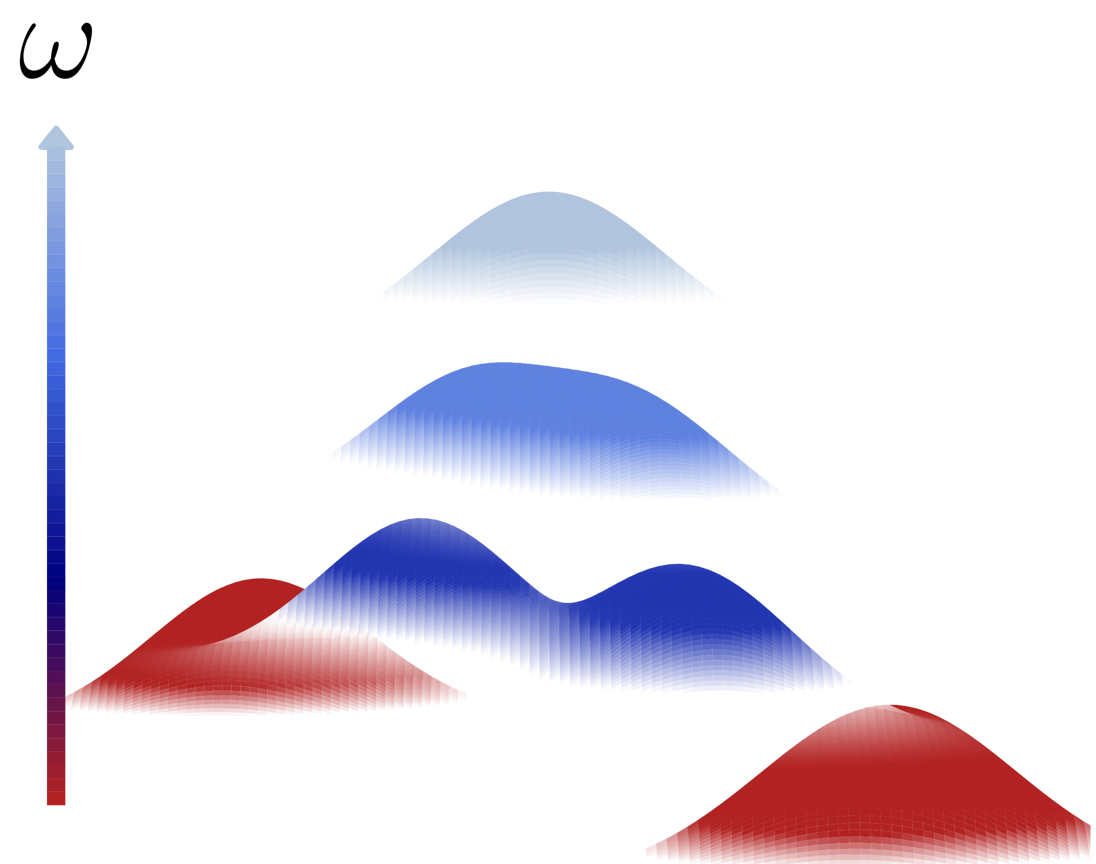
Sapienza University of Rome

ICE-10 Valencia

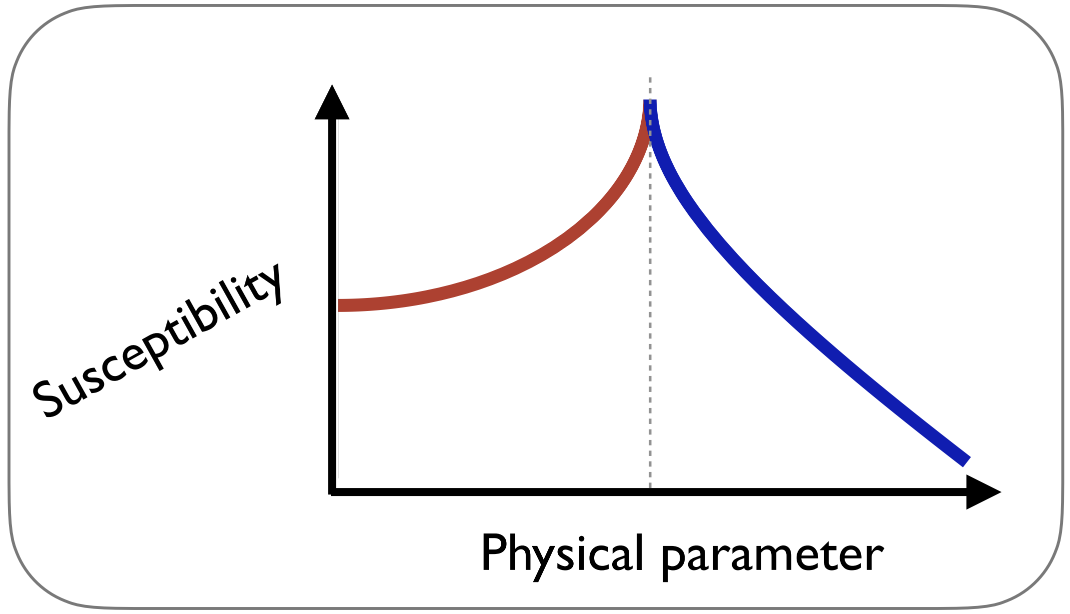
Outline

Driven-dissipative phase transitions ↔ **Quantum sensing**



Critical Quantum Sensing

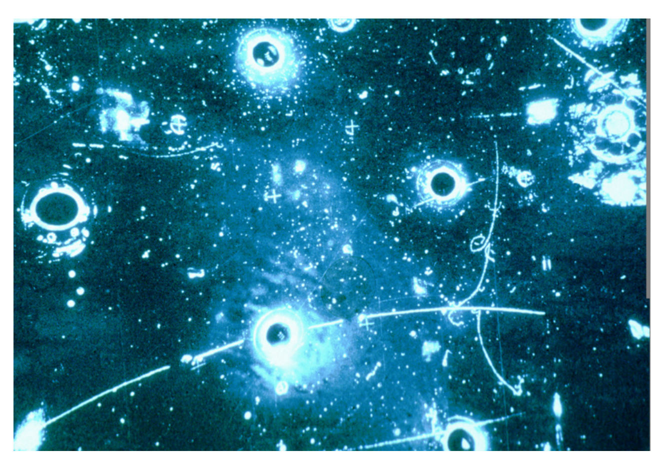
Critical phase transition



Critical sensors

Bubble chamber

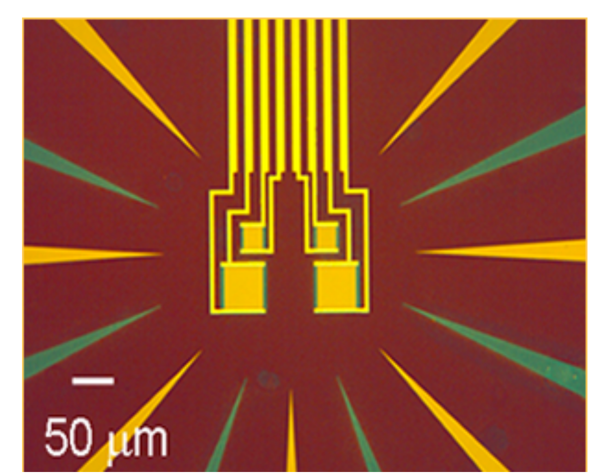
(Liquid-gas)



(CERN image archives)

Transition-edge sensors

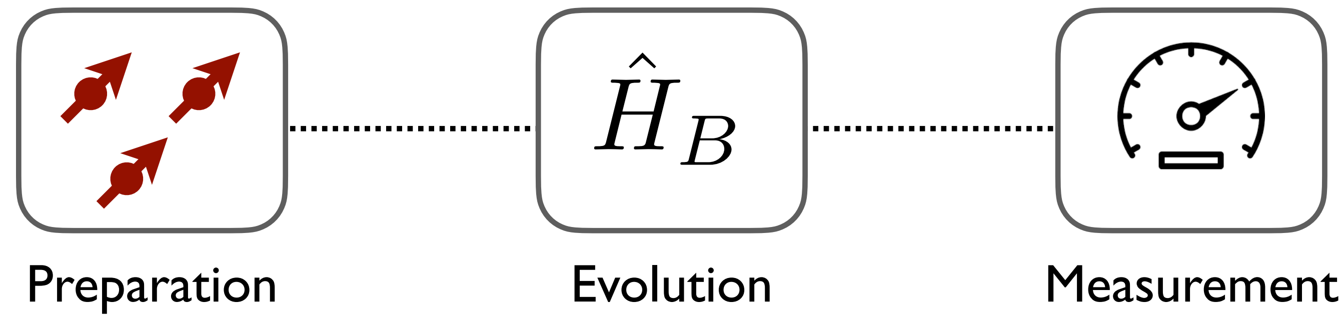
(Superconductor-normal)



(NIST image archives)

Quantum sensing

$$\hat{H}_B = \hat{H}_{sys} + B\,\hat{H}_I$$



Critical Quantum Sensing

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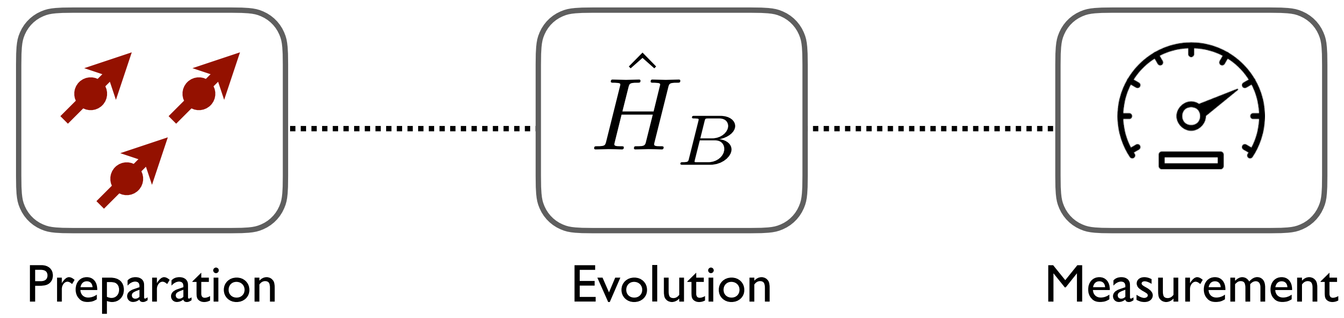


Estimation error: $$\delta B = \frac{1}{\sqrt{I_B}}$$

↓

Quantum Fisher Information

Classical probes

$$I_B \sim tN$$

Quantum probes

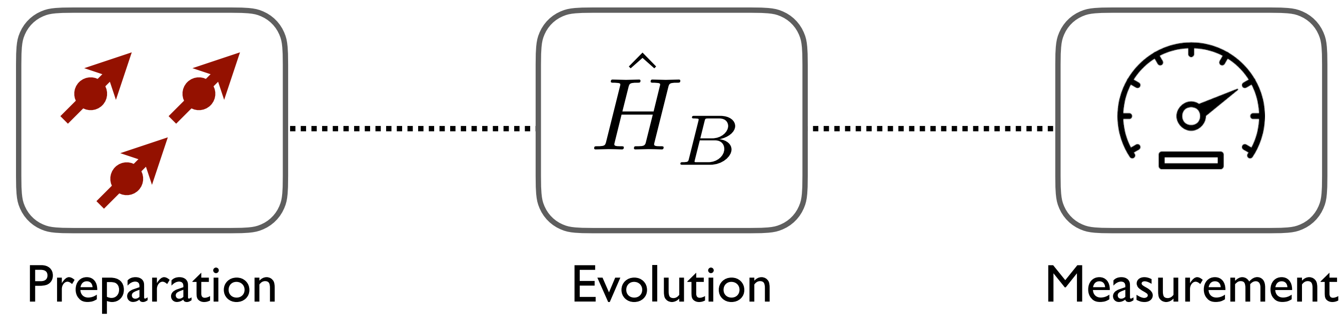
$$I_B \sim t^2 N^2$$

Heisenberg limit

Critical Quantum Sensing

Quantum sensing

$$\hat{H}_B = \hat{H}_{sys} + B\,\hat{H}_I$$



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Quantum Fisher Information

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Heisenberg limit

Critical quantum sensing: $\hat{H}_{sys}$ → **Quantum phase transition**

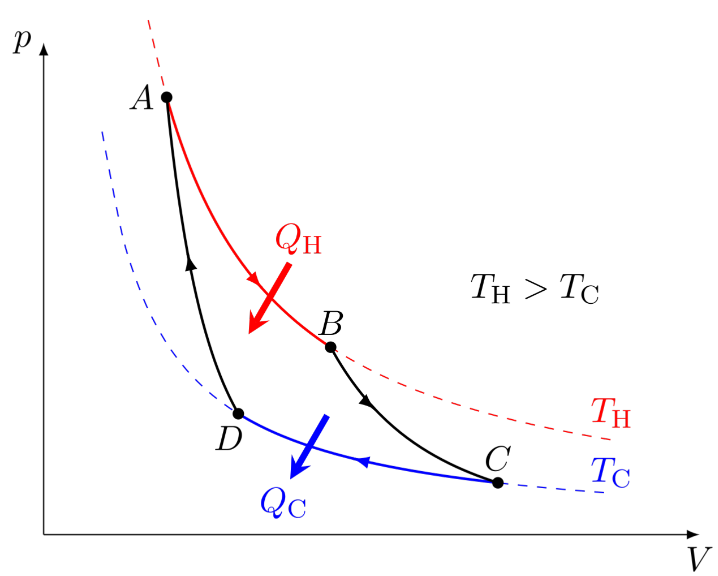
Review: Quantum Metrology and Sensing with Many-Body Systems
V. Montenegro et. al. Phys. Rep. 1134, 30 (2025)

Applied and fundamental interest

Applied and fundamental interest

Carnot cycle

- Efficiency of heat engines
- Fundamental laws of thermodynamics



Critical Quantum Sensing

Applied and fundamental interest

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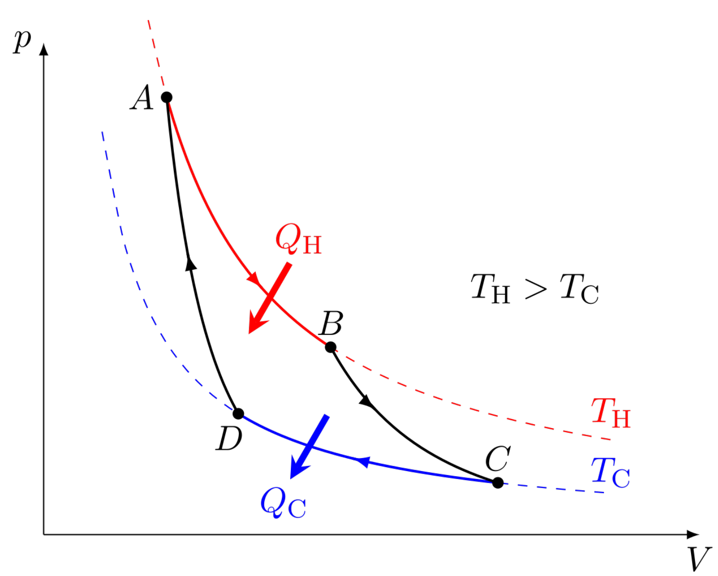
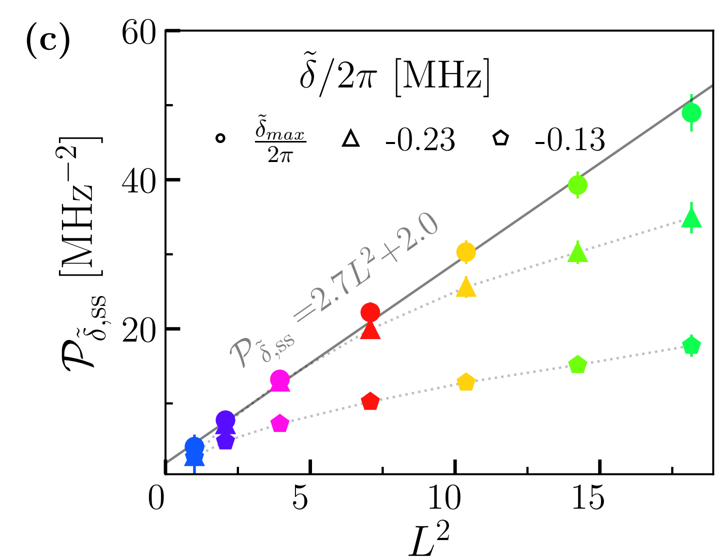


Quantum sensing protocols

- Performance of sensors
- Fundamental measurement precision



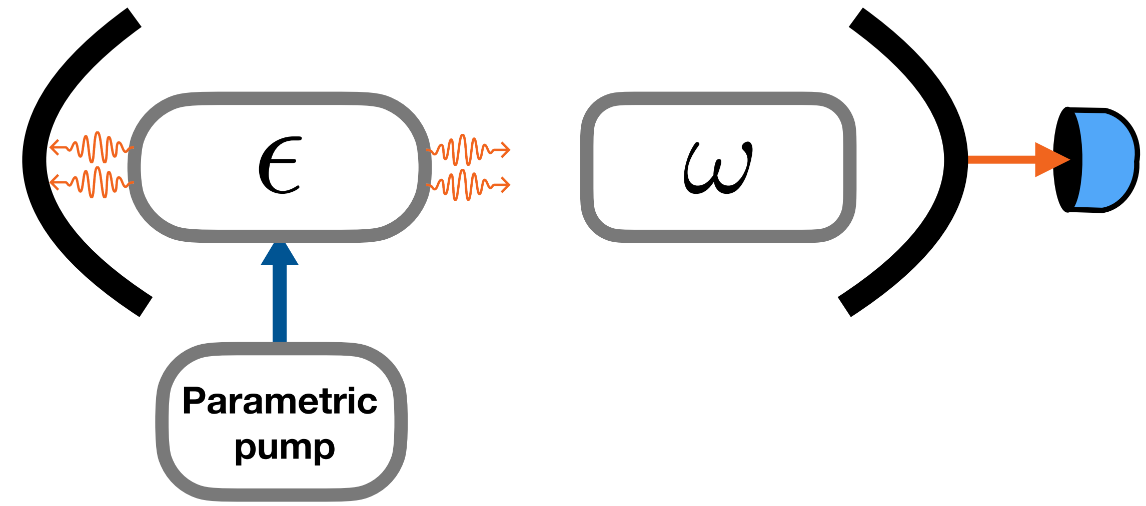
Parametric quantum sensor

Master equation

$$\dot{\rho} = -i[H, \rho] + \quad \Gamma\left(a^{\dagger}\rho a - 1/2\left\{aa^{\dagger}, \rho\right\}\right)$$

Hamiltonian

$$\hat{H}_{\mathrm{Kerr}}/\hbar = \omega \hat{a}^{\dagger}\hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^{2}) + \chi \hat{a}^{\dagger 2}\hat{a}^{2}$$



- R. Di Candia*, F. Minganti*, K.V. Petrovnin, G. S. Paraoanu, and S. Felicetti, npj Quantum Information 9,23 (2023).

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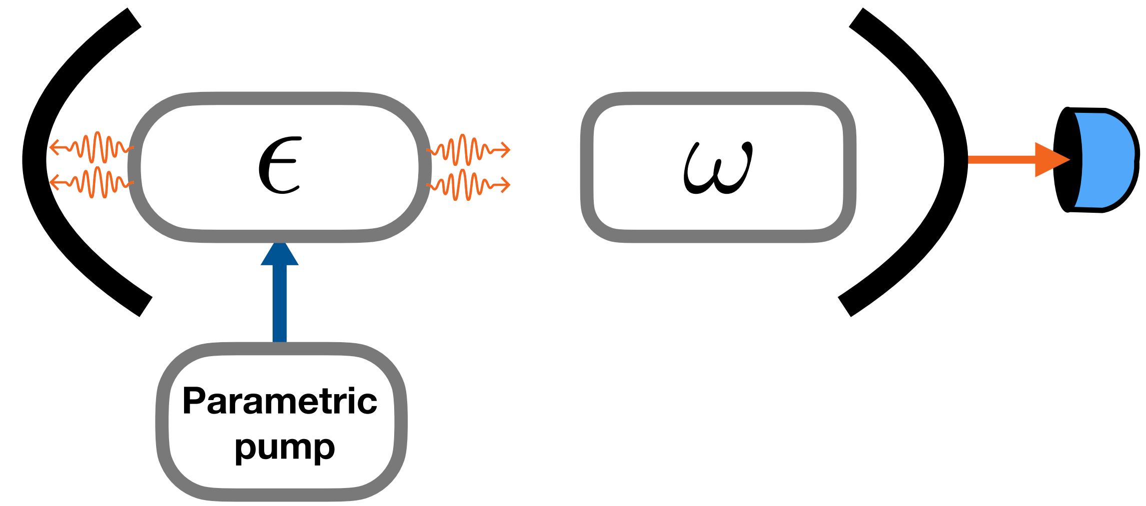

Weak nonlinearity → **Critical phase transition**

$\chi \ll 1$

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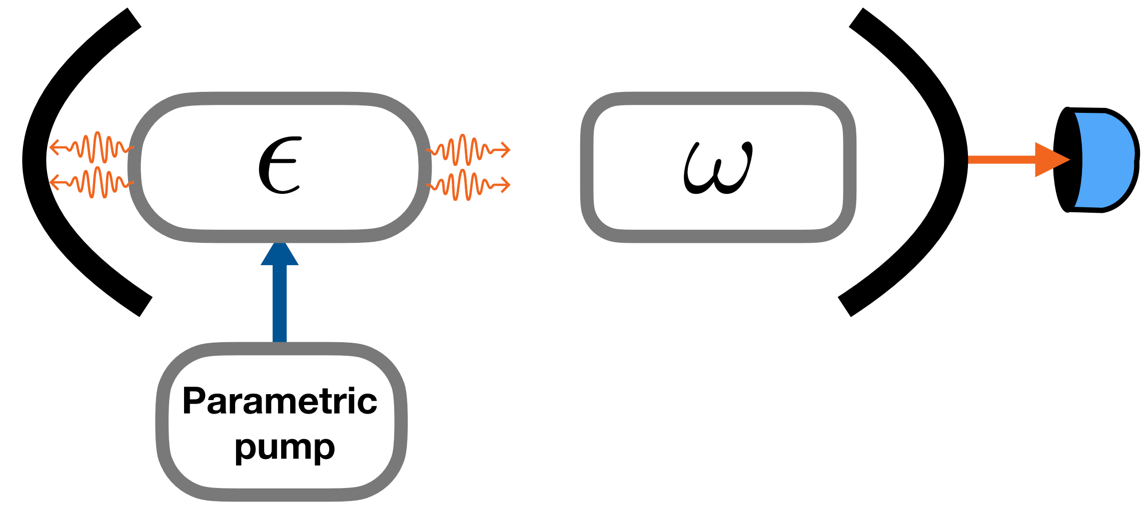

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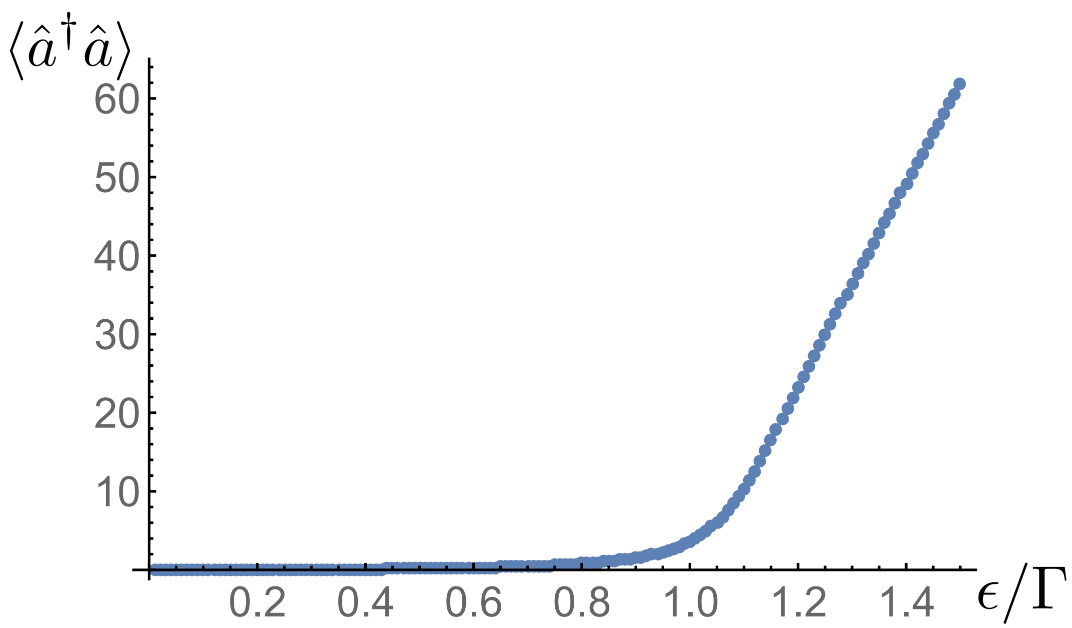

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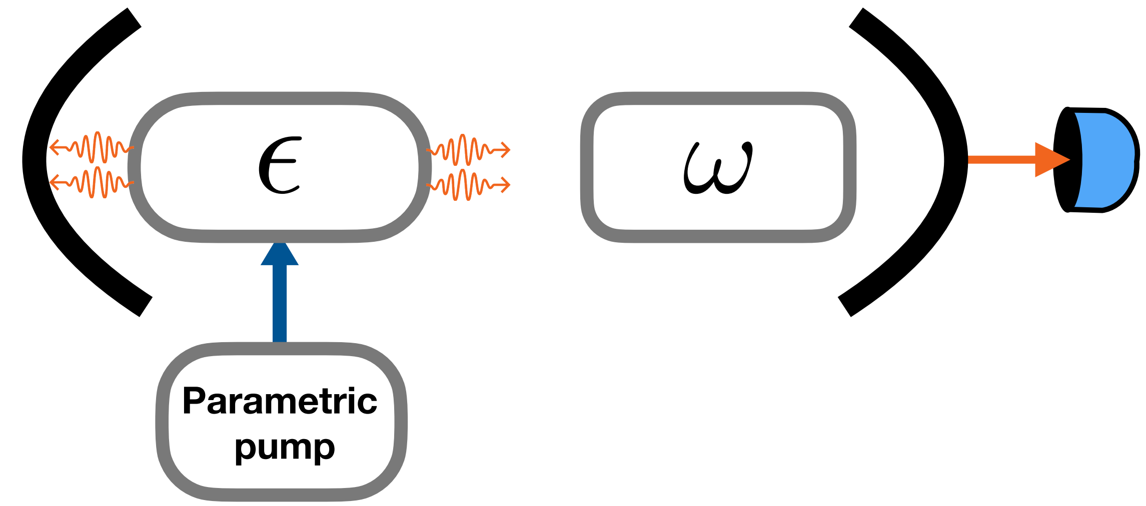

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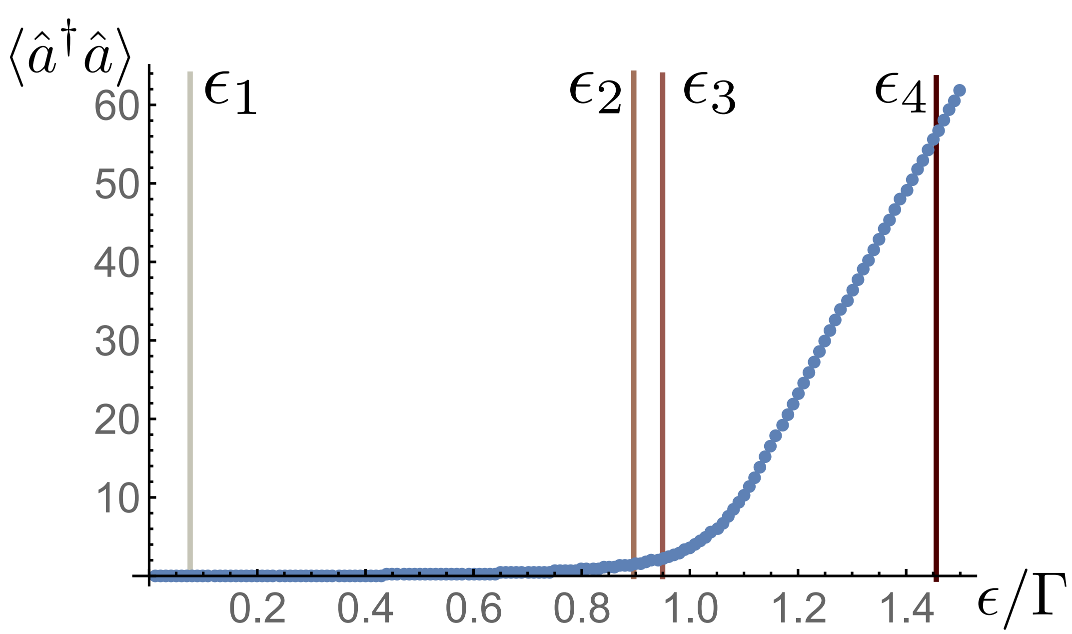

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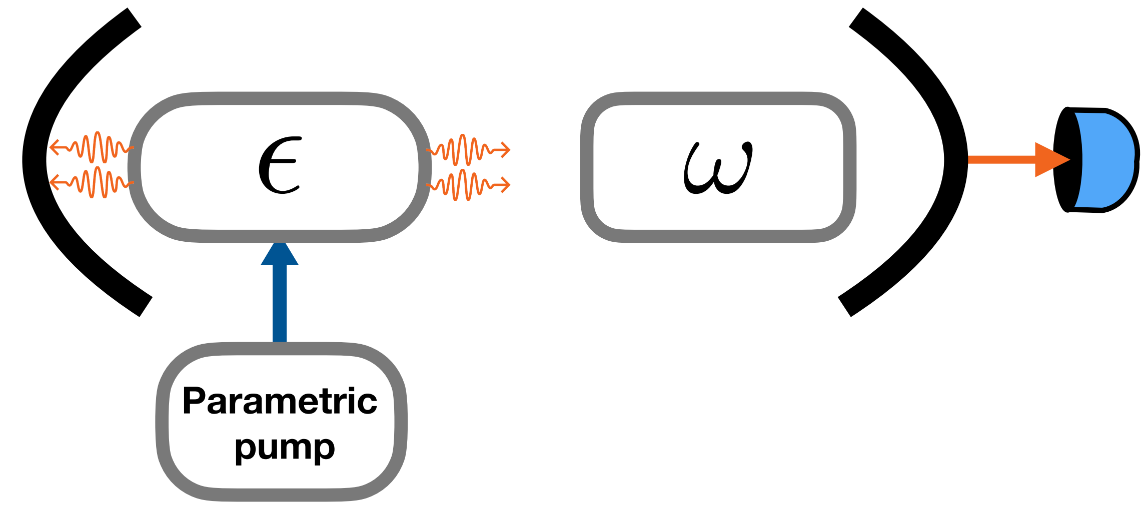

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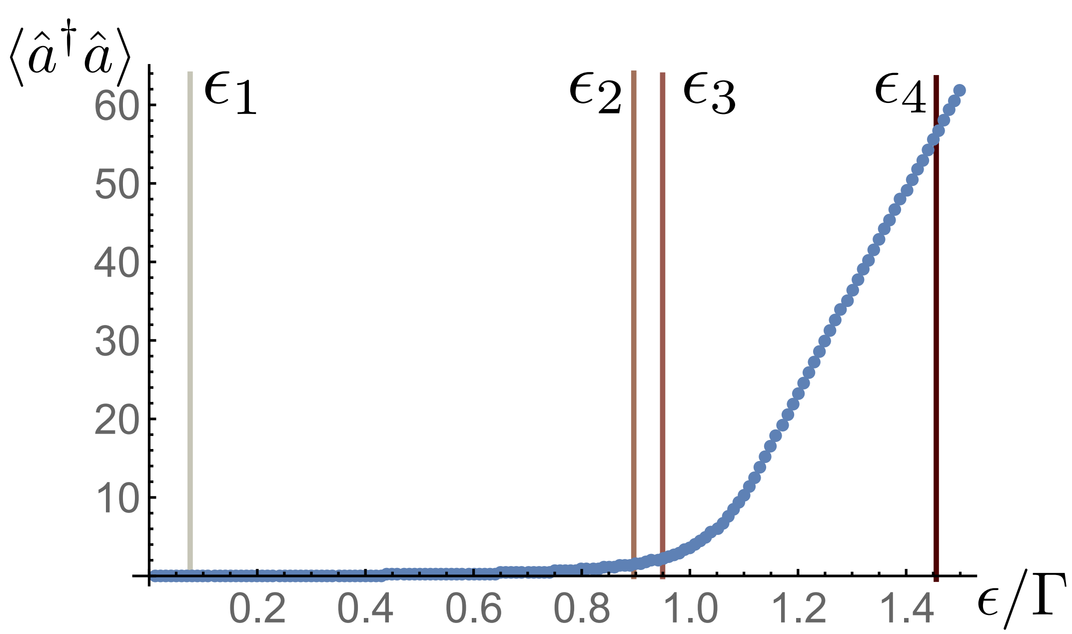

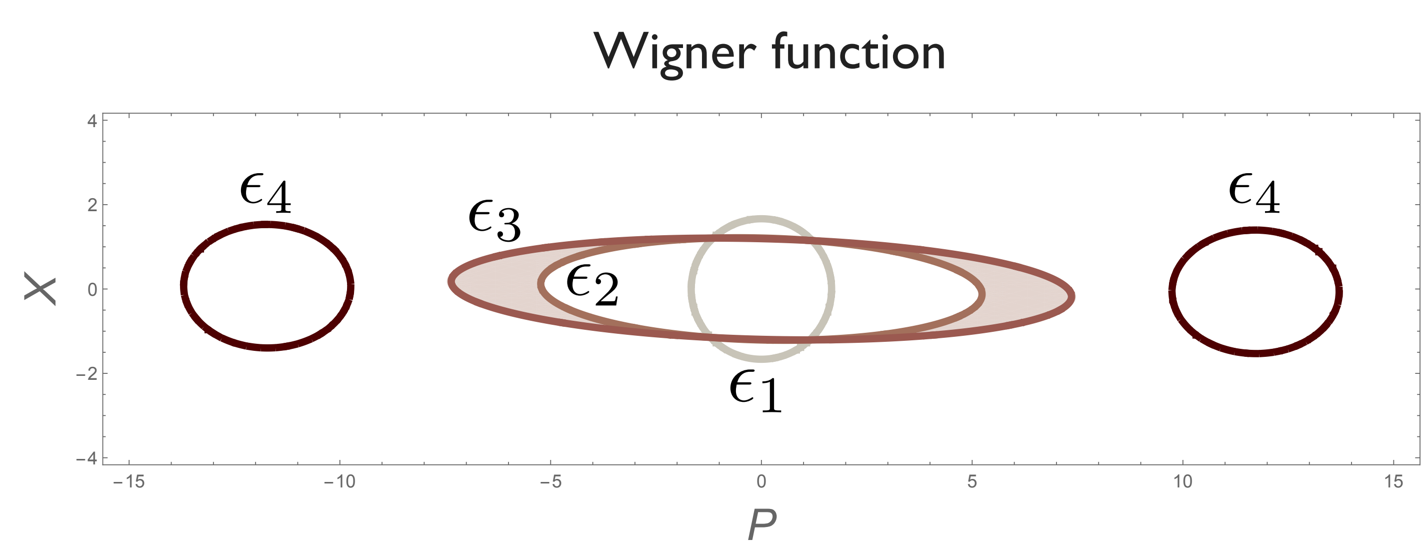

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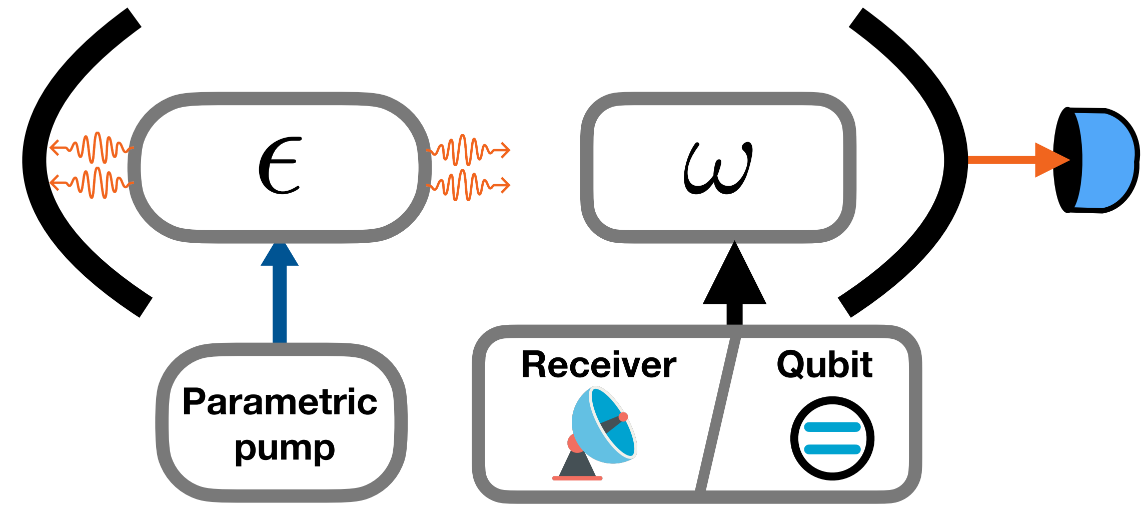
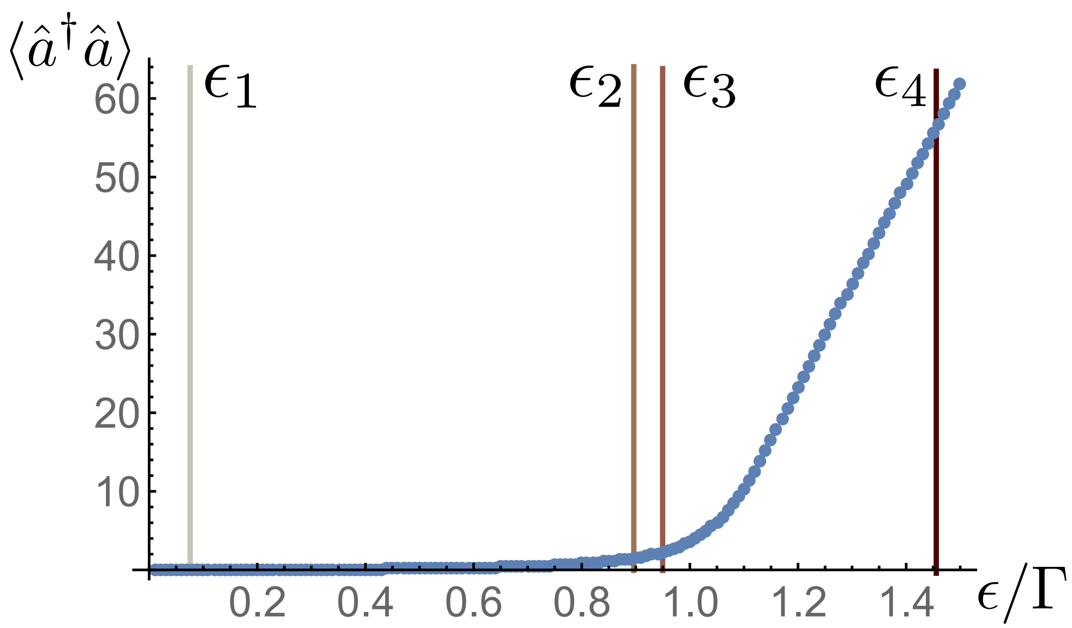


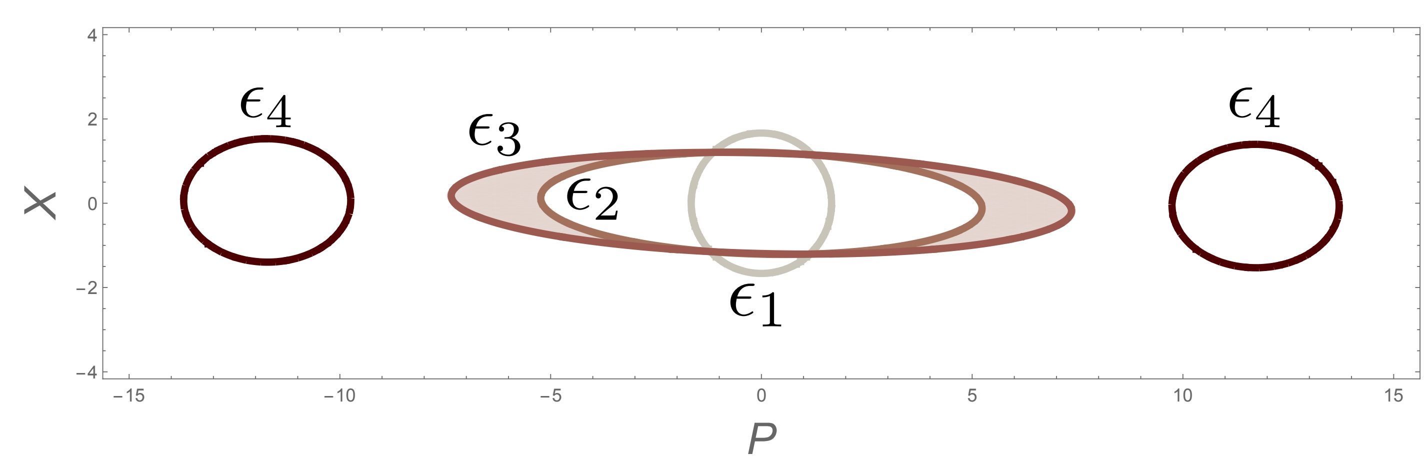
Weak nonlinearity

$\chi \ll 1$

→ **Critical phase transition**



Wigner function



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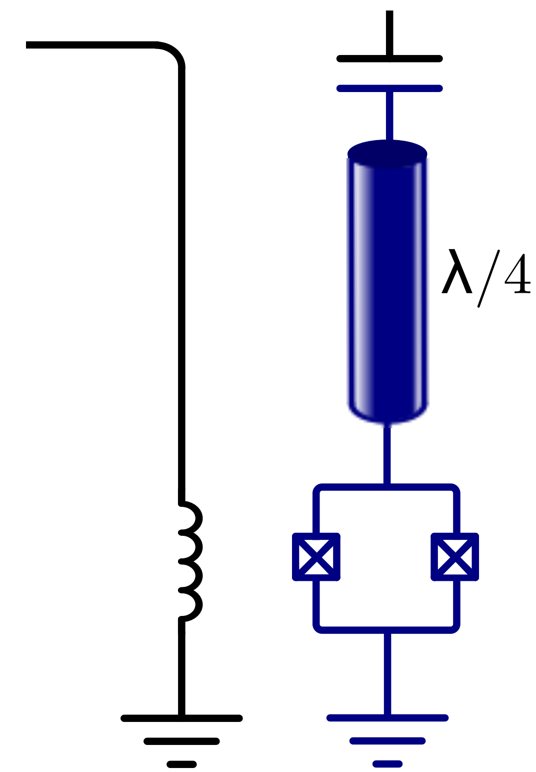
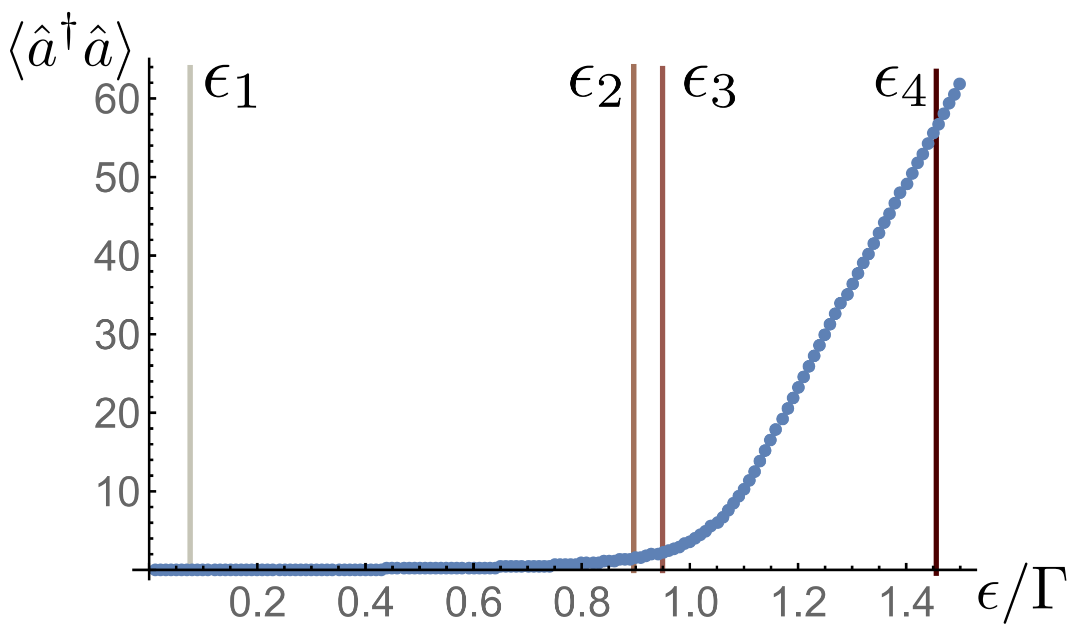
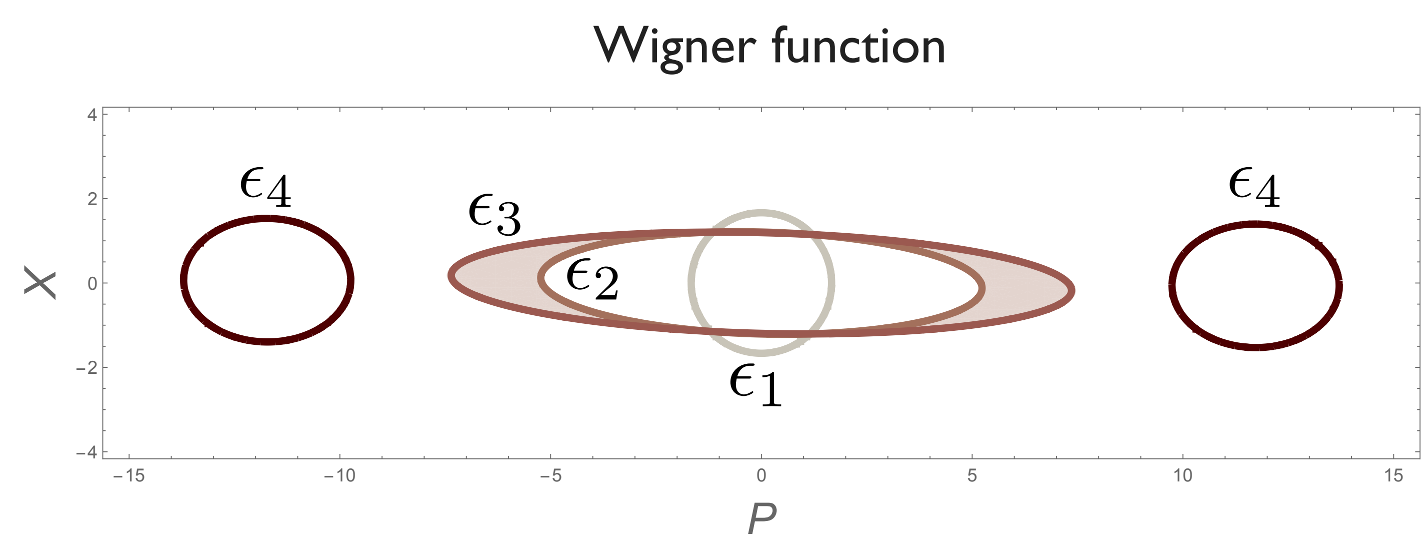
Magnetic field estimation → $\omega = \omega(B)$







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Parametric quantum sensor

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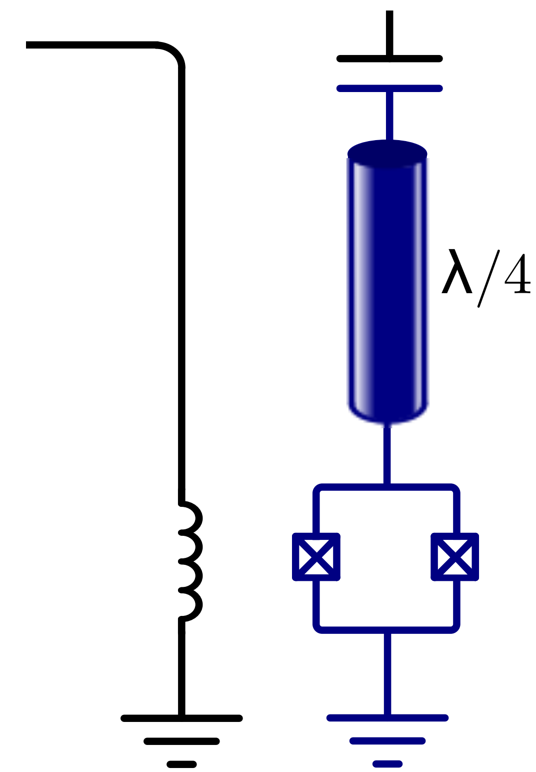
Hamiltonian

$$\hat{H}_{\rm Kerr}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2}\hat{a}^2$$



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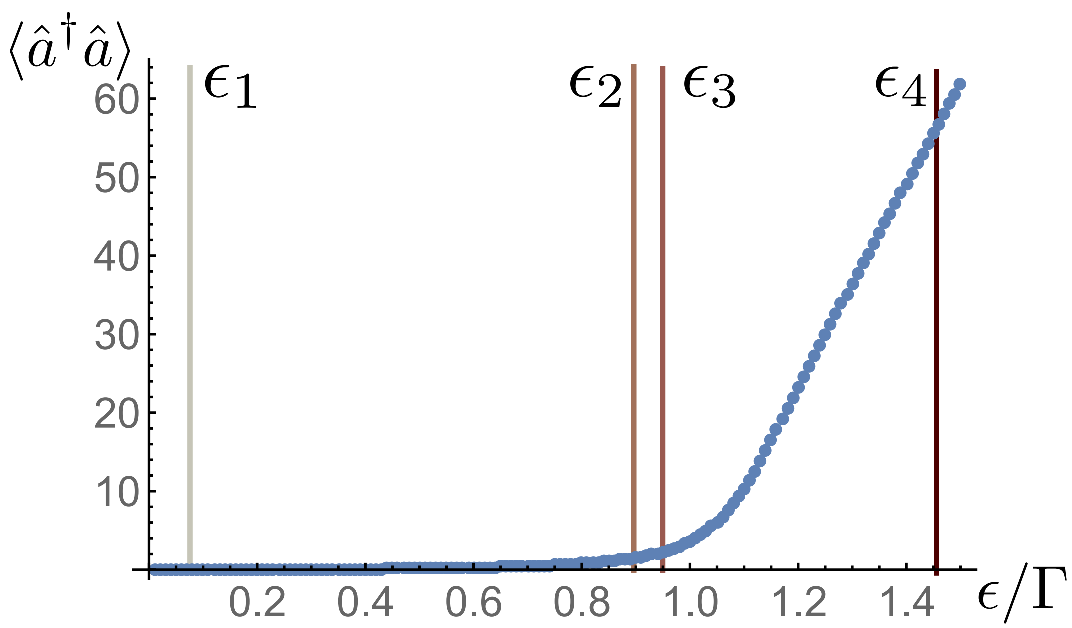

(Simplest)
Critical sensing protocol:

1) Switch on critical pump
2) Wait for steady state
3) Measure and estimate ω

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Parametric quantum sensor

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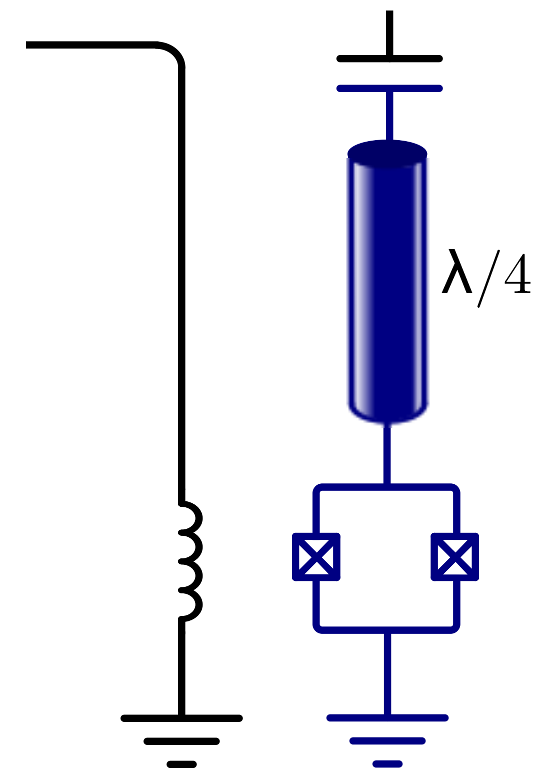
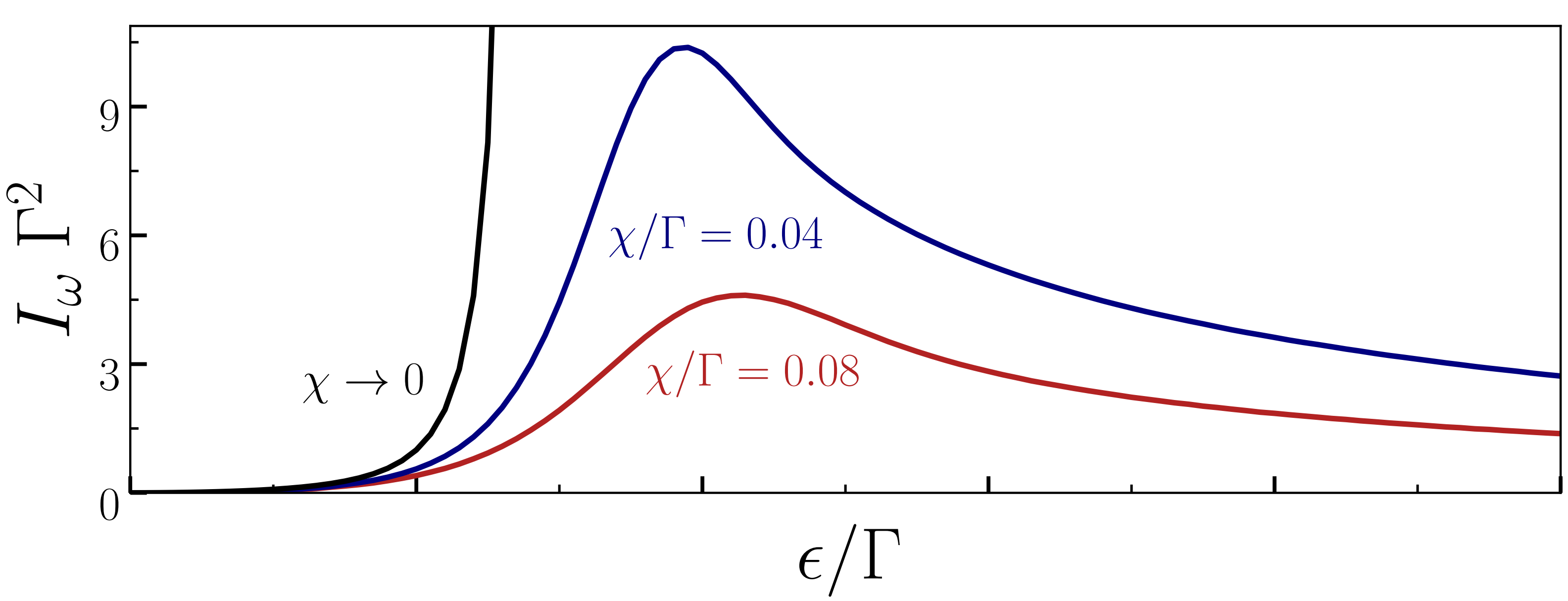


Magnetic field estimation → $\omega = \omega(B)$

Quantum Fisher information

Upper bound
to estimation precision



Experimental observation

nature communications

Article https://doi.org/10.1038/s41467-025-56830-w

Observation of first- and second-order dissipative phase transitions in a two-photon driven Kerr resonator

Received: 12 March 2024

Accepted: 3 February 2025

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Guillaume Beaulieu[1,2,9], Fabrizio Minganti[2,3,8,9], Simone Frasca[1,2], Vincenzo Savona[2,3], Simone Felicetti[4,5], Roberto Di Candia[6,7] & Pasquale Scarlino[1,2]

PRX QUANTUM **6,** 020301 (2025)

Criticality-Enhanced Quantum Sensing with a Parametric Superconducting Resonator

Guillaume Beaulieu,[1,2,*] Fabrizio Minganti,[2,3,‡] Simone Frasca,[1,2] Marco Scigliuzzo,[2,4] Simone Felicetti,[5,6] Roberto Di Candia,[7,8] and Pasquale Scarlino[1,2,†]

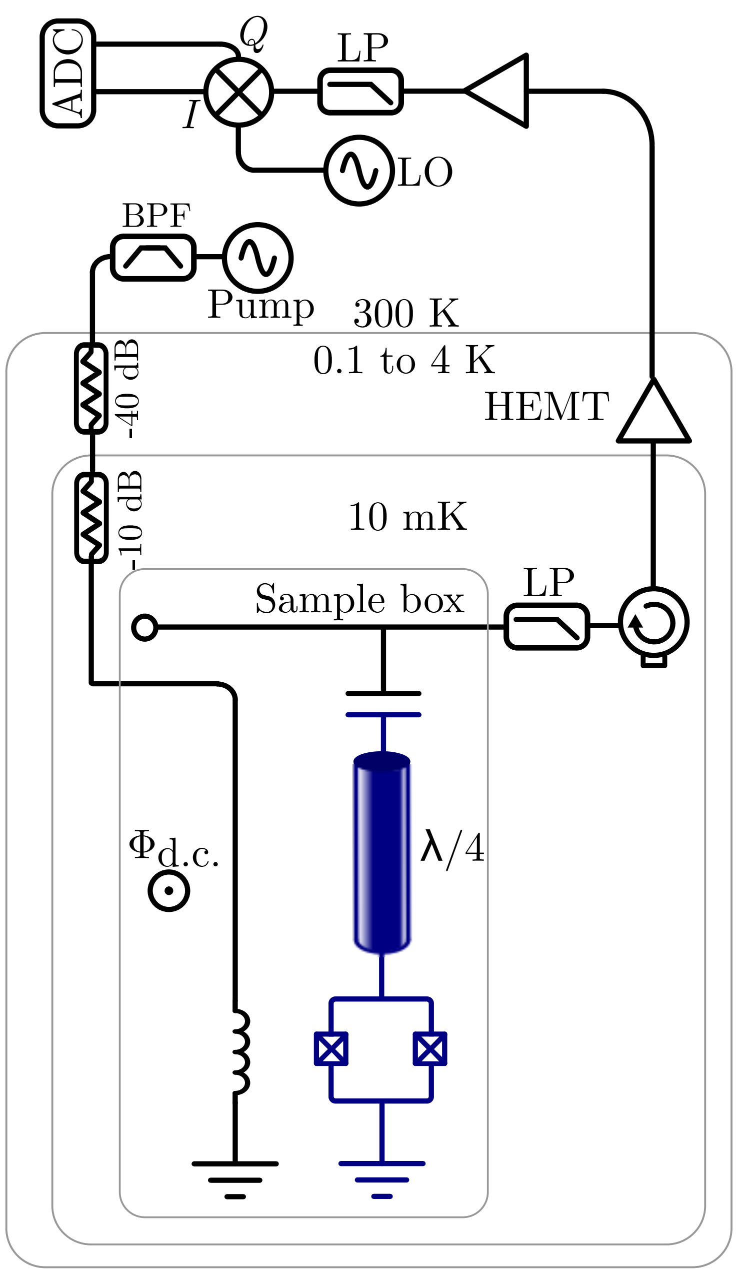
Experimental observation

Master equation

$$\frac{\partial \rho}{\partial t} = -\mathcal{L}\rho = -\frac{i}{\hbar}[\hat{H}, \rho] + \kappa(n_{\text{th}} + 1)\mathcal{D}[\hat{a}]\rho + \kappa n_{\text{th}}\mathcal{D}[\hat{a}^\dagger]\rho + \kappa_\phi \mathcal{D}[\hat{a}^\dagger \hat{a}]\rho + \kappa_2 \mathcal{D}[\hat{a}^2]\rho$$

Hamiltonian

$$\hat{H}/\hbar = \Delta \hat{a}^\dagger \hat{a} + \frac{U}{2}\hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a} + \frac{G}{2}\left(\hat{a}^\dagger \hat{a}^\dagger + \hat{a}\hat{a}\right)$$



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Hamiltonian

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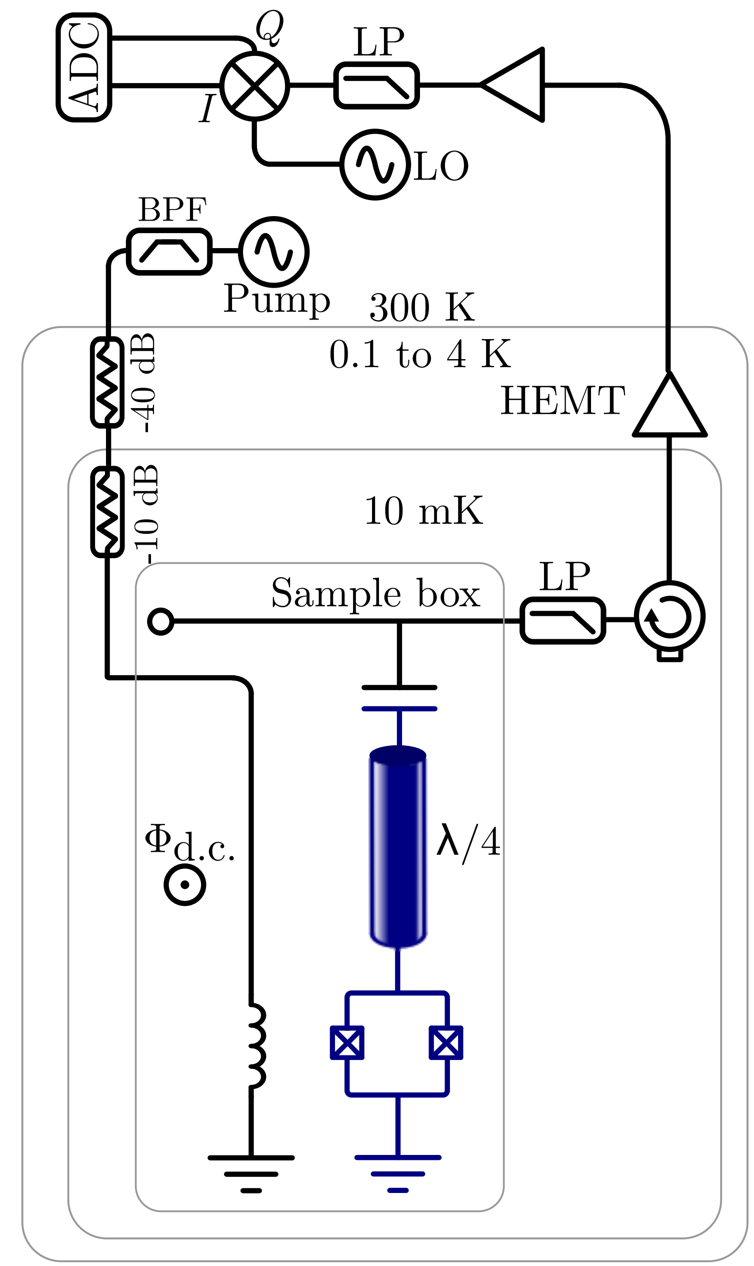
Effective system size $L \to \infty$

Pump intensity $G = \tilde{G}L$

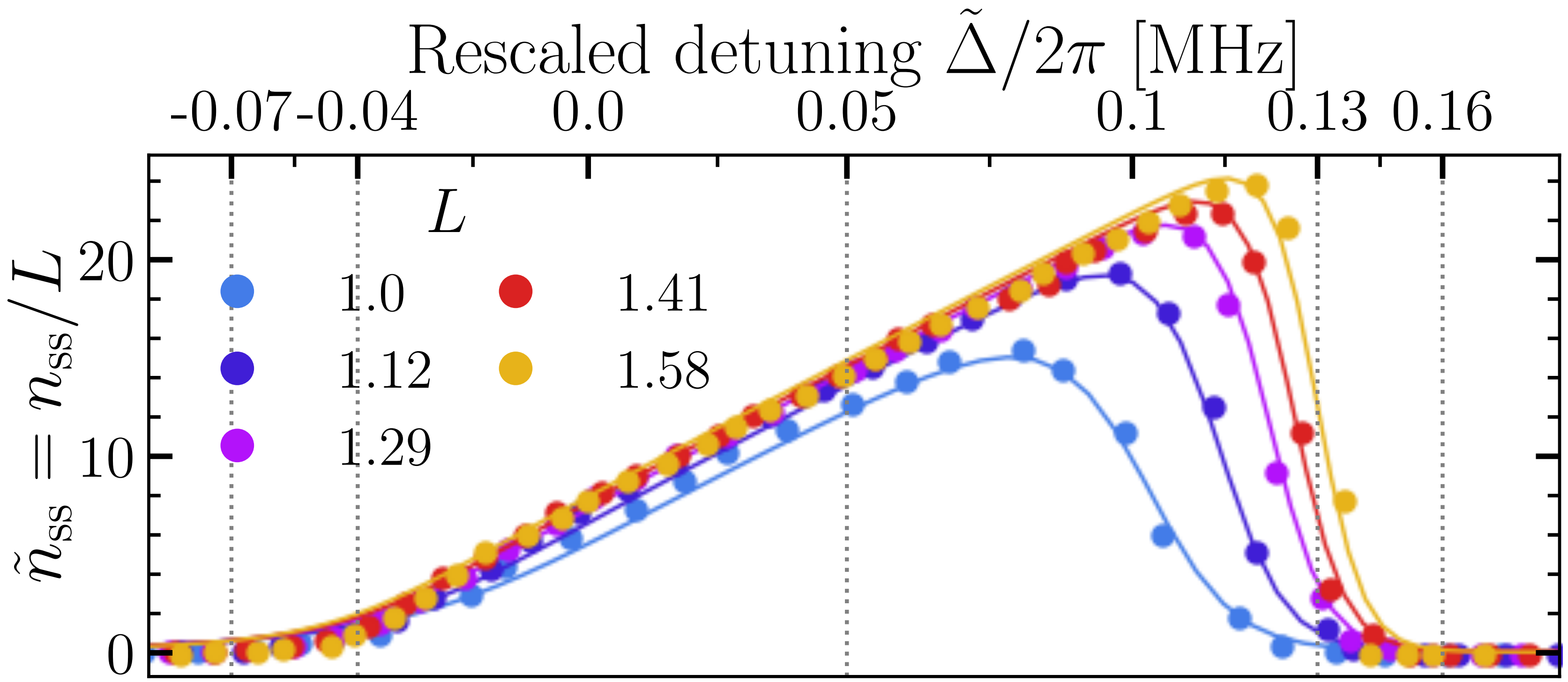
Pump-resonator detuning $\Delta = \tilde{\Delta}L$



Experimental observation

Observation of critical steady-state properties

Output photon flux (rescaled)

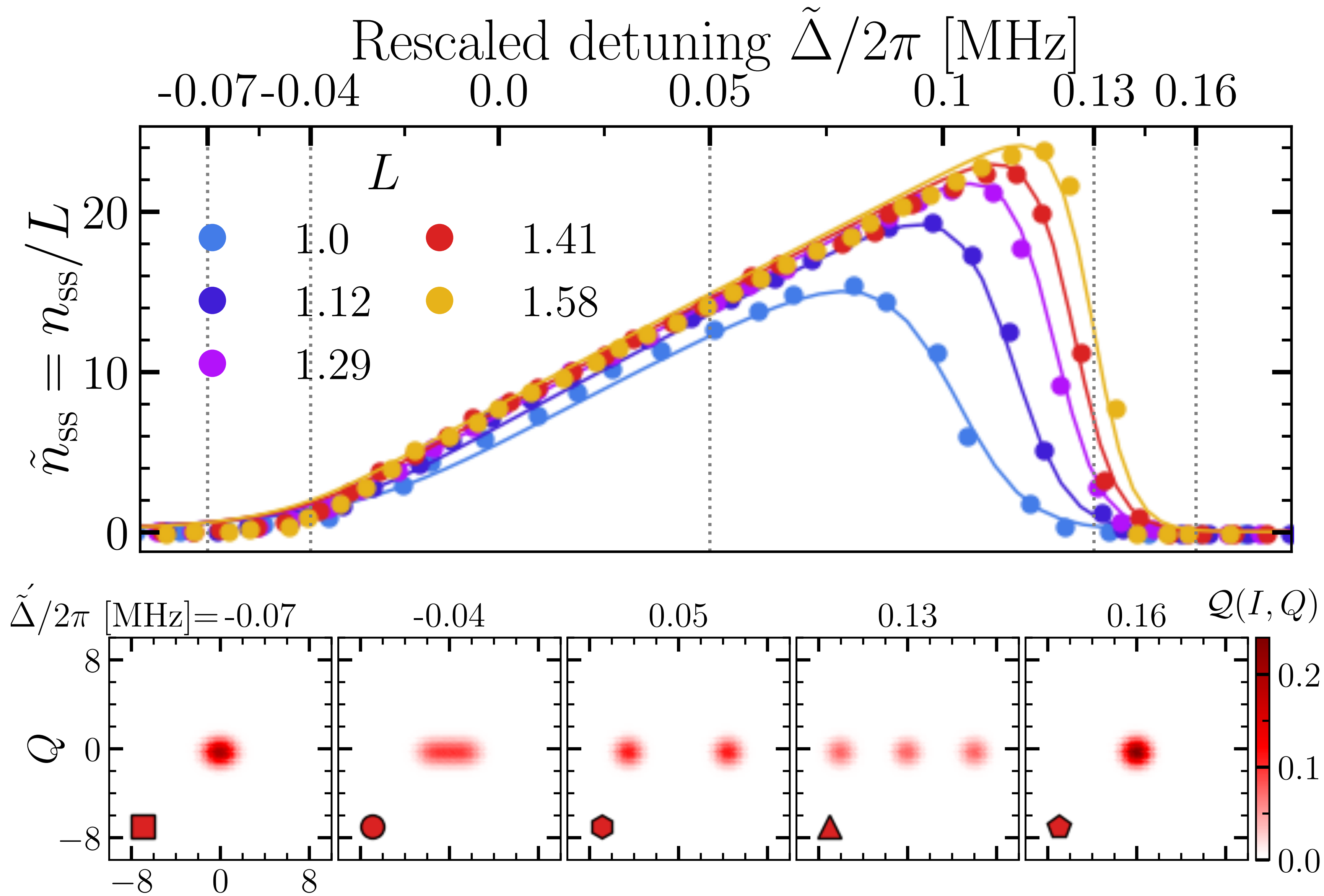


Experimental observation

Observation of critical steady-state properties

Output photon flux (rescaled)

Husimi Q-function (heterodyne)



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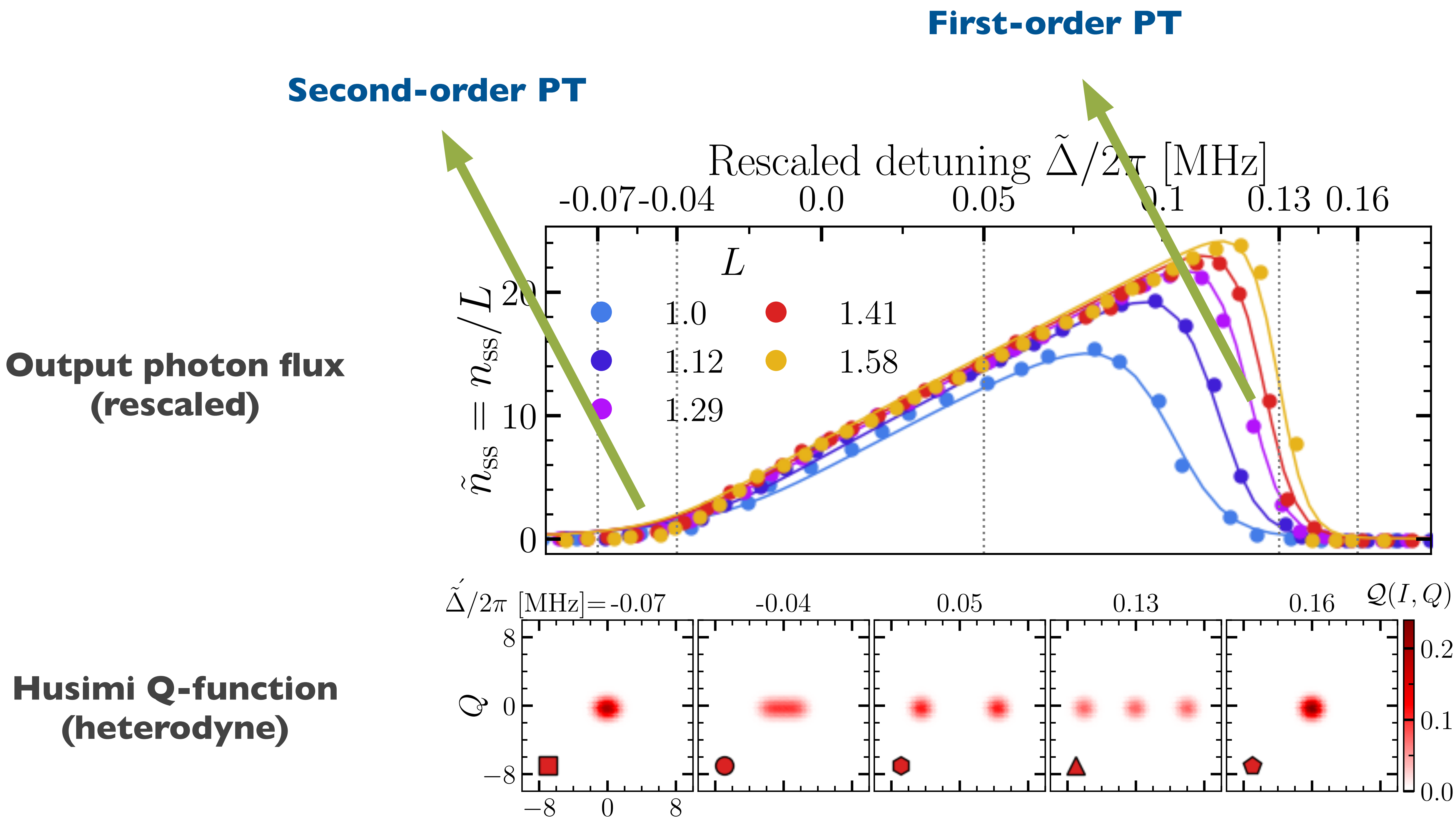
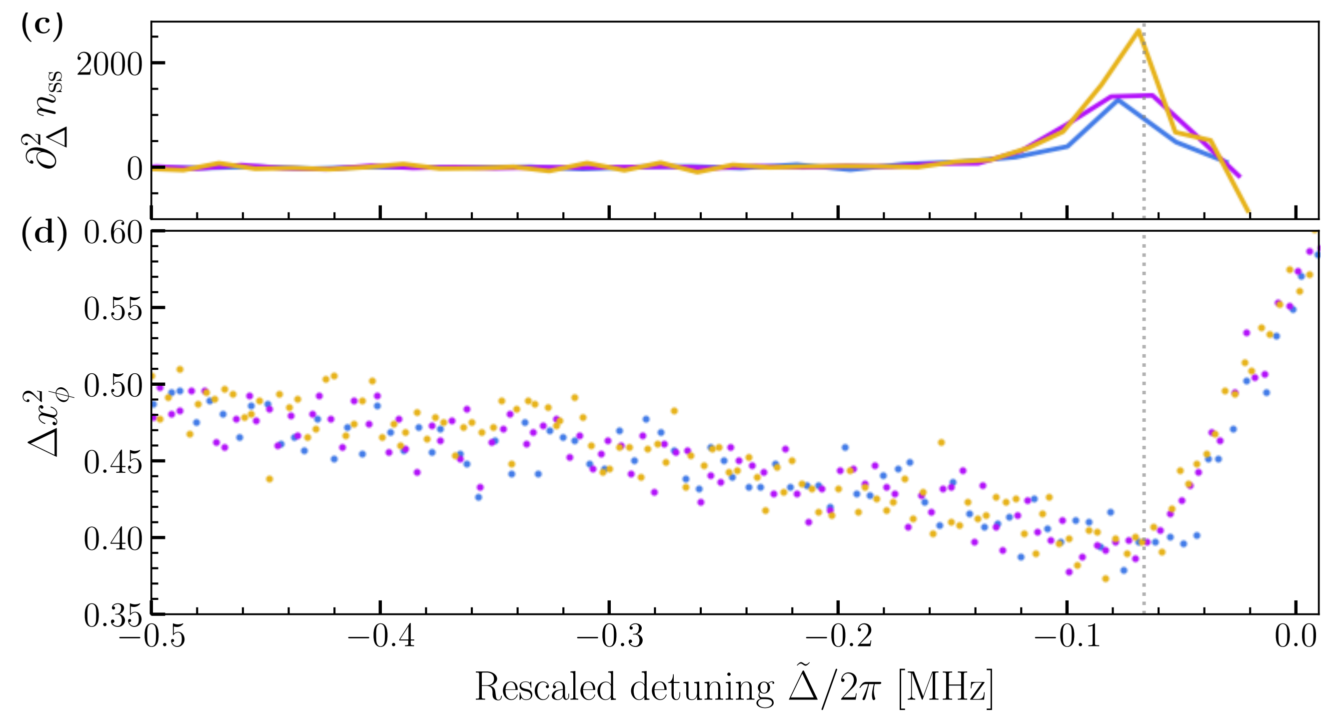


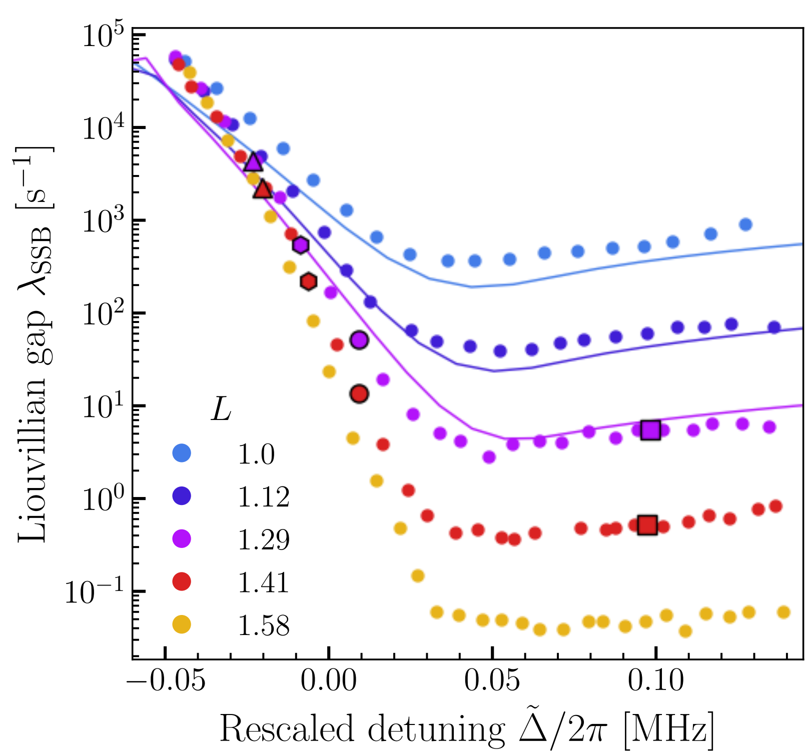
Experimental observation

Squeezing at the 2nd-order PT

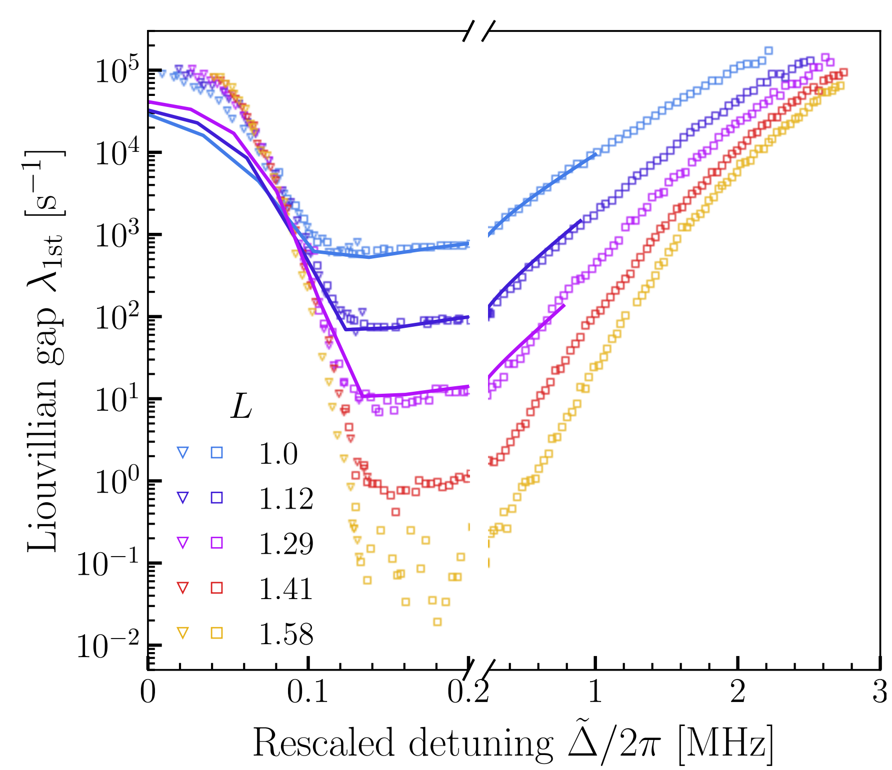


Dynamical properties

2nd-order PT



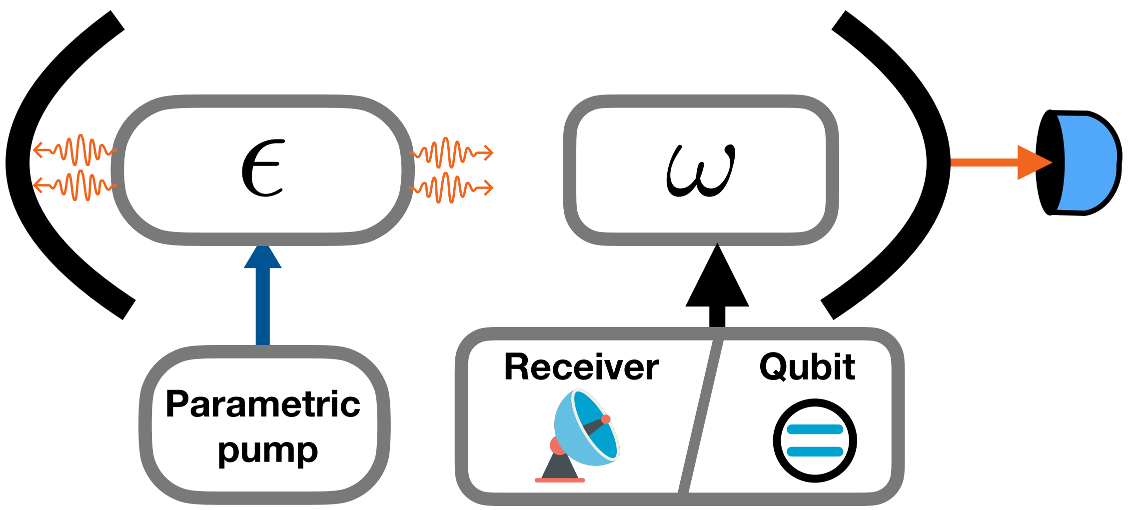
1st-order PT



Experimental observation

Observation of critically enhanced sensing precision

$$\mathcal{P}_\omega = \frac{\left|\partial_\omega \langle \hat{n} \rangle\right|^2}{\Delta n^2}$$

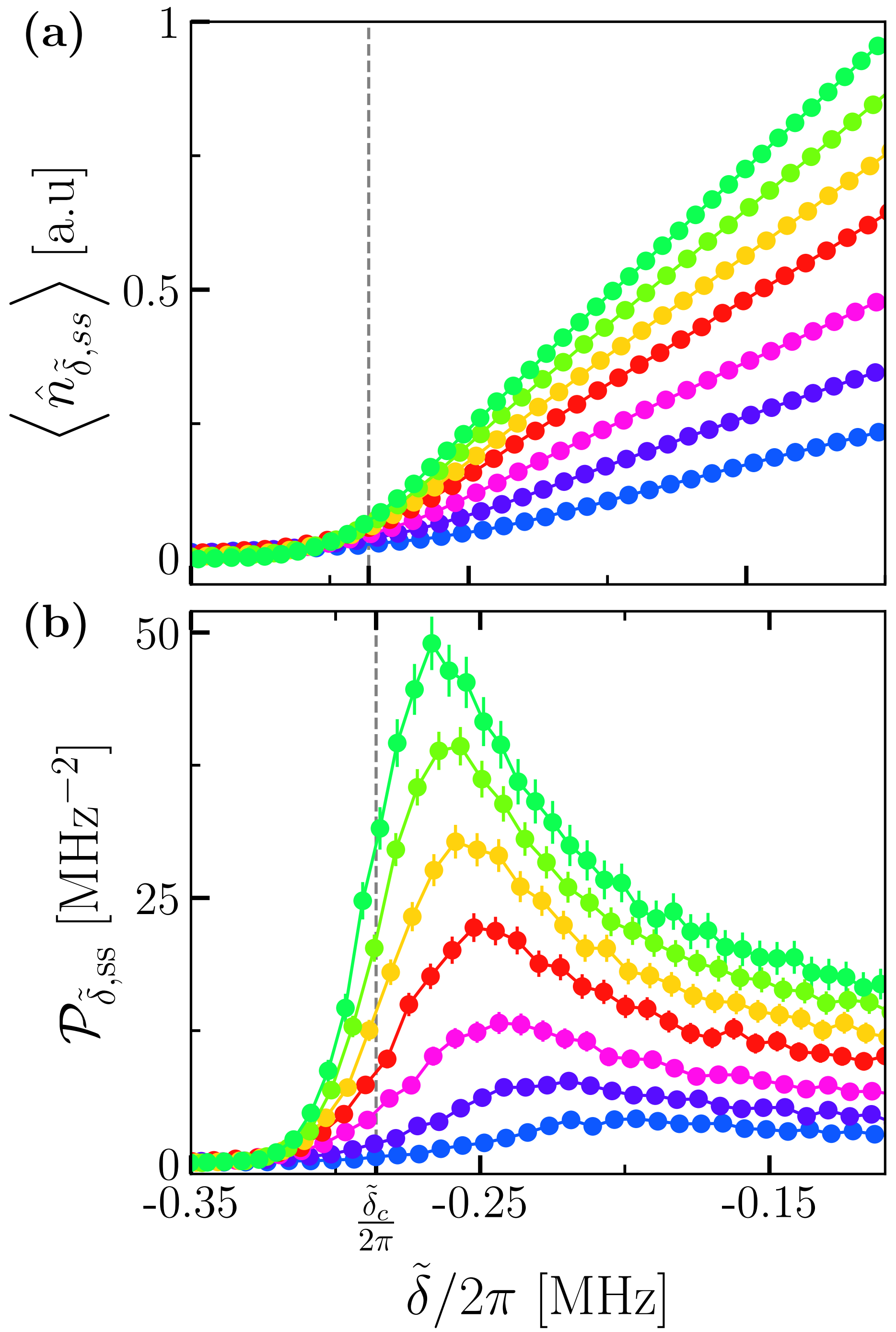


Experimental observation

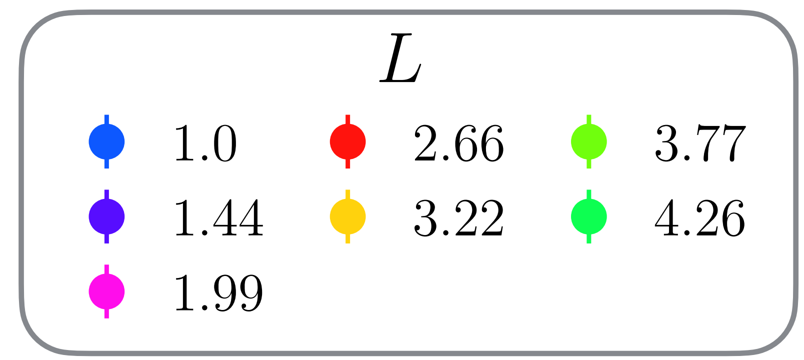
Observation of critically enhanced sensing precision

Output photon flux

Estimation precision



Detuning



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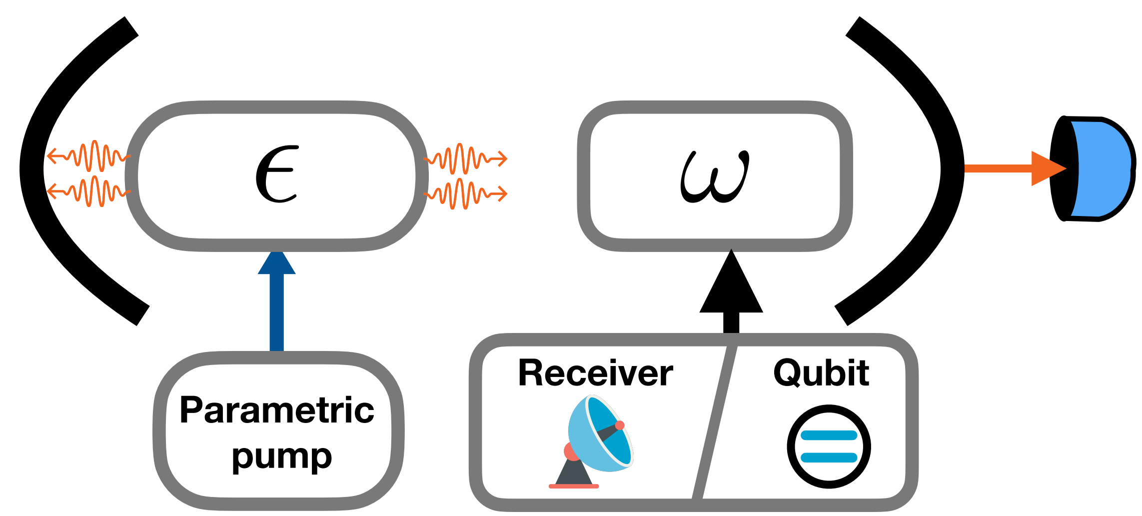
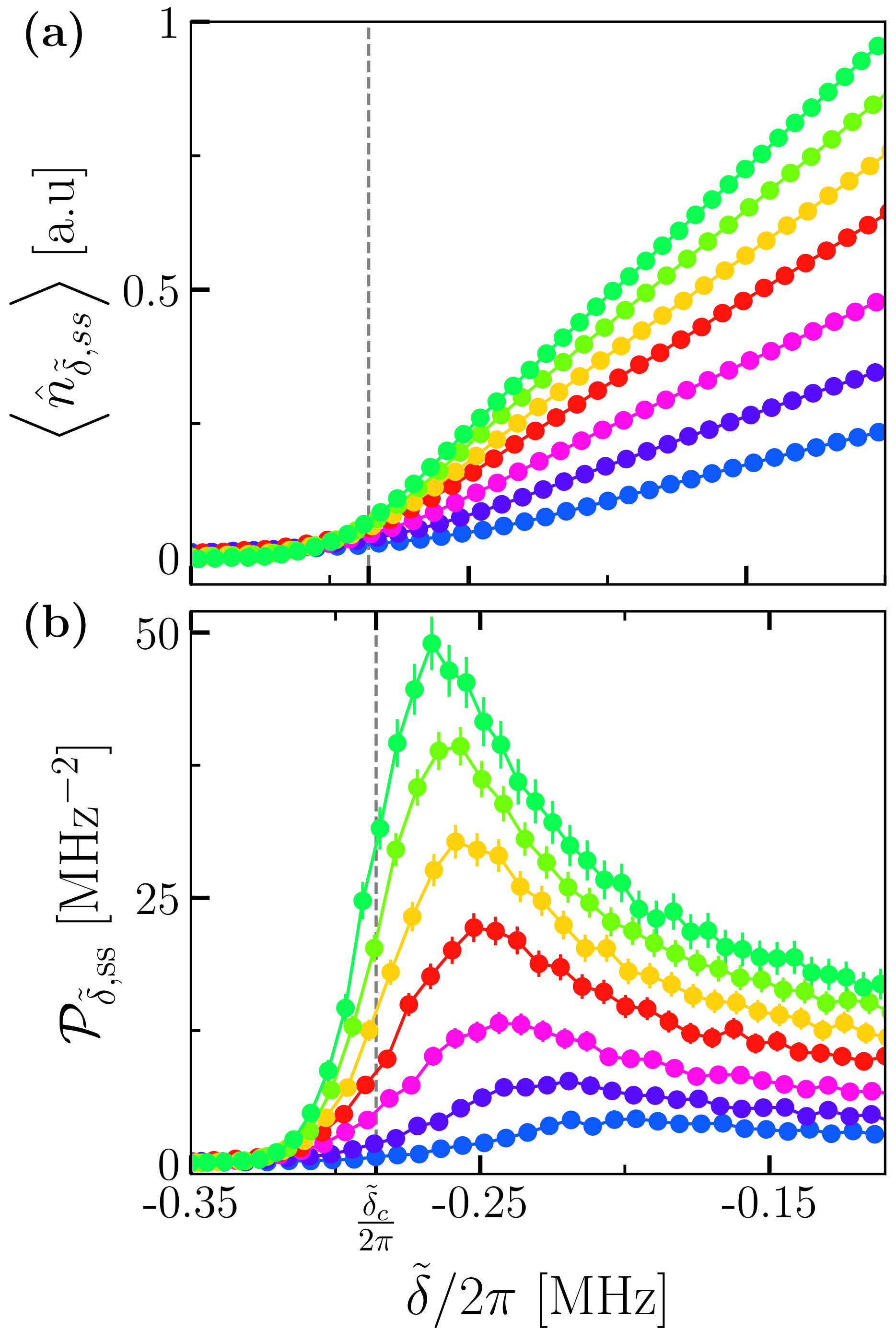


Experimental observation

Observation of critically enhanced sensing precision

Output photon flux

Estimation precision



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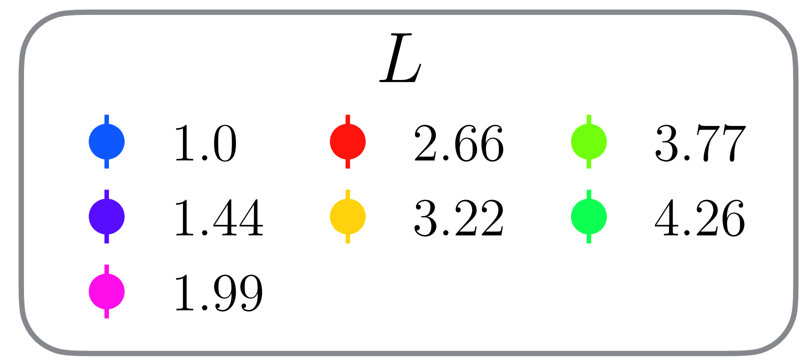

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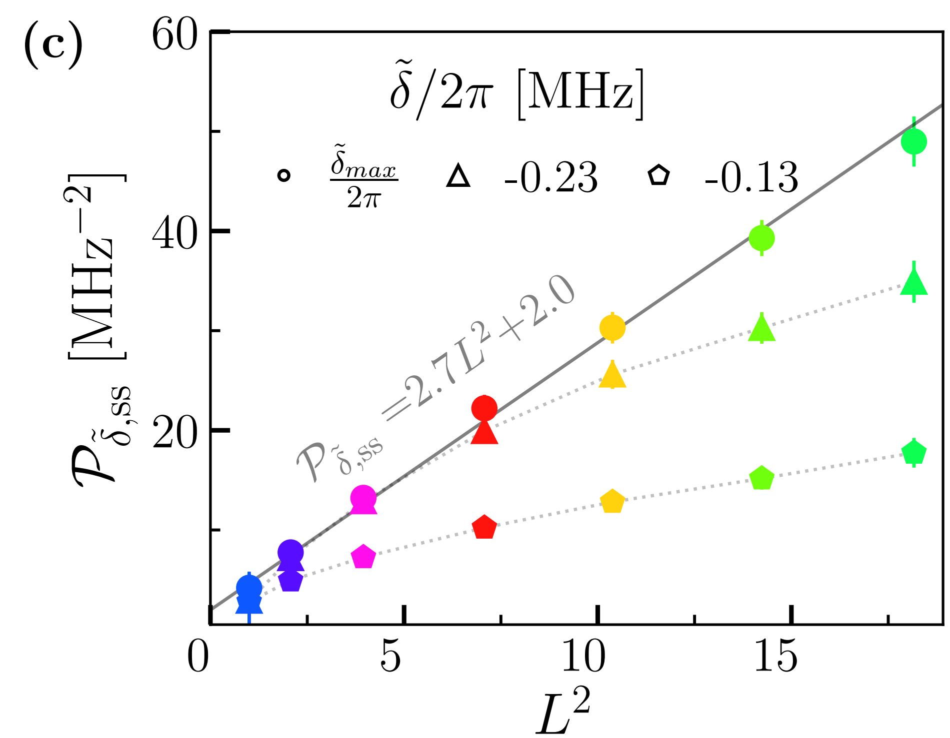

Scaling parameter

Experimental observation

PRX QUANTUM **6**, 020301 (2025)

Criticality-Enhanced Quantum Sensing with a Parametric Superconducting Resonator

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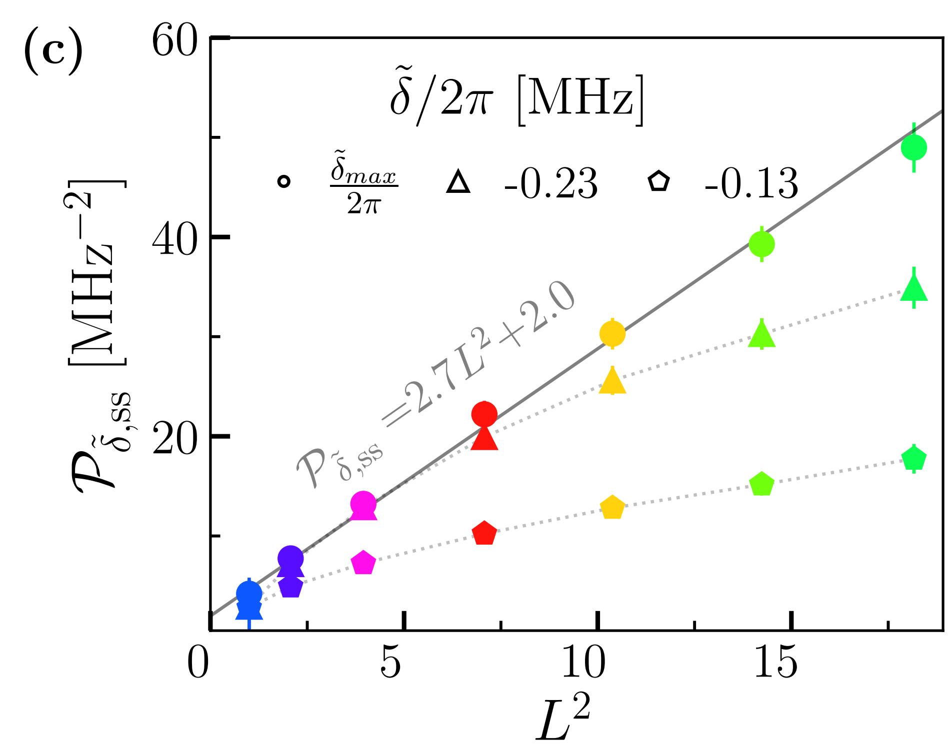
Take-home message:

Experimental demonstration of quadratic scaling

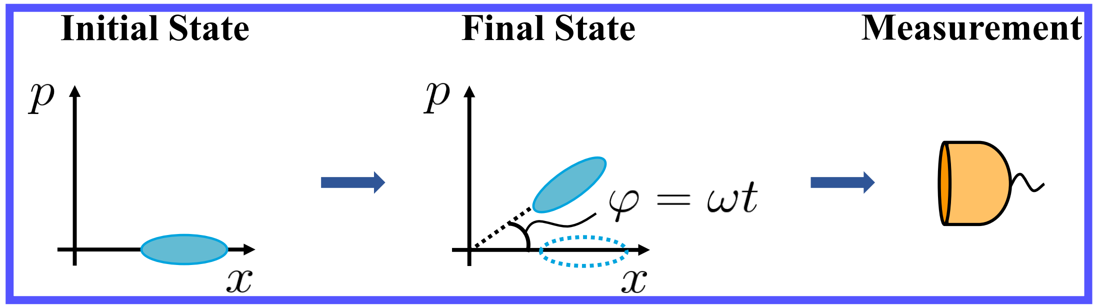


Scaling parameter

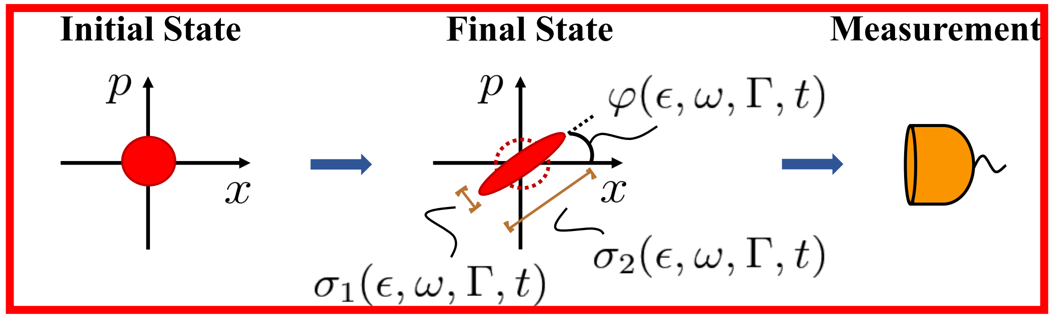
Fundamental resources

Optimal scaling with time and energy

Passive strategies



Critical quantum sensing

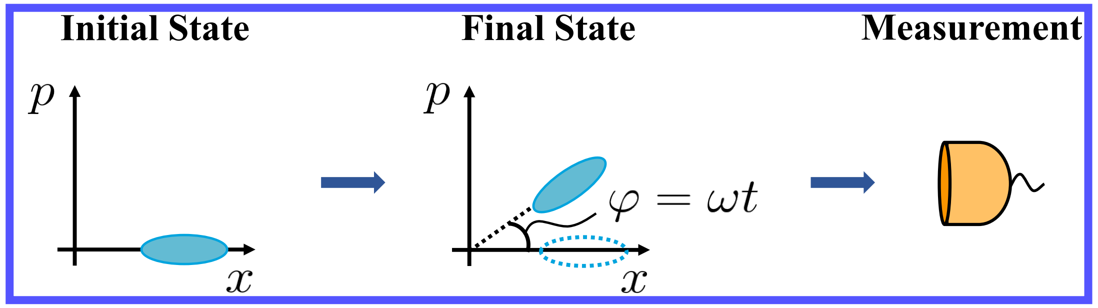


- U Alushi, W Górecki, S Felicetti, R Di Candia, Physical Review Letters 133 (4), 040801(2024)

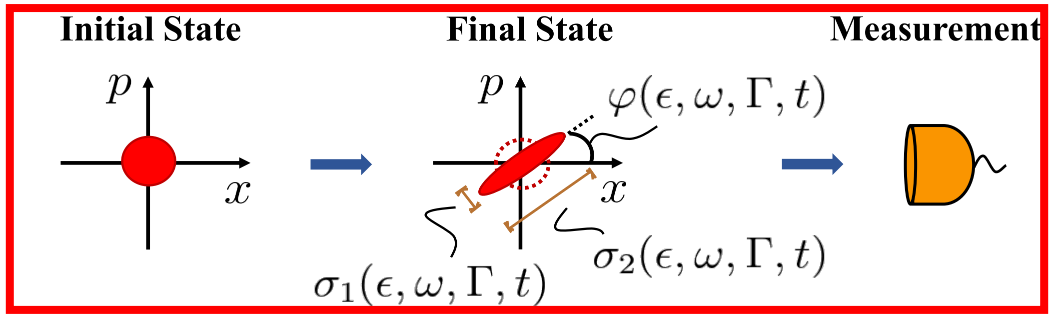
Fundamental resources

Optimal scaling with time and energy

Passive strategies

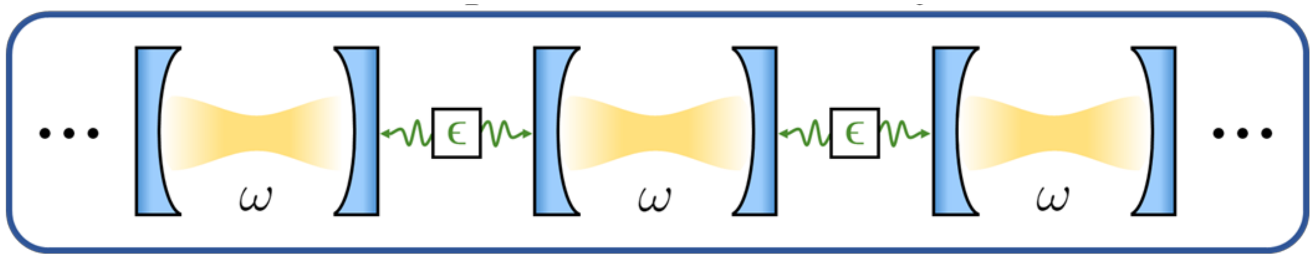


Critical quantum sensing



- U Alushi, W Górecki, S Felicetti, R Di Candia, Physical Review Letters 133 (4), 040801(2024)

Optimal scaling with number of resonators

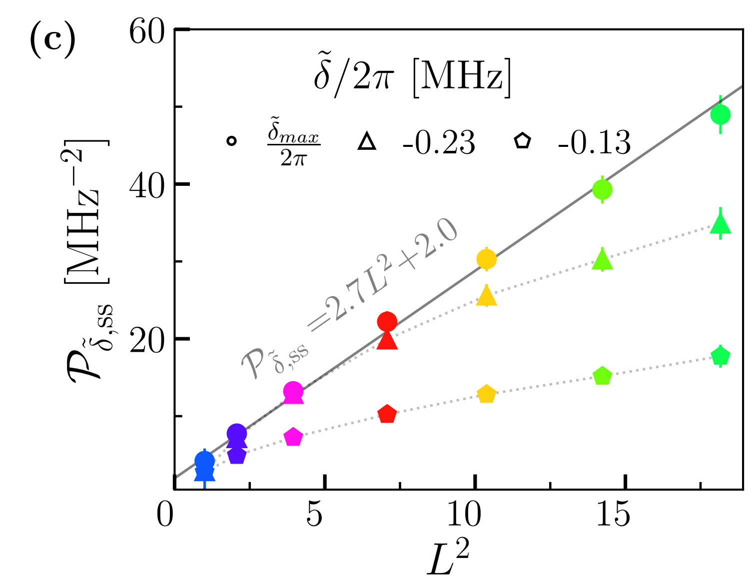


Collective enhancement

- U Alushi, A. Coppo, V. Brosco, R. Di Candia, S Felicetti, Comm. Phys. 8 (1), 74 (2025)

Conclusions

- **Critical parametric quantum sensor (Theory & Experiment)**



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- **Optimal scalings with time, photons and resonators (Theory)**

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- U Alushi, A. Coppo, V. Brosco, R. Di Candia, S Felicetti, Comm. Phys. 8 (1), 74 (2025)