Joint (quantum) measurements on distant physical systems

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Swiss National Science Foundation

Why study joint quantum measurements?

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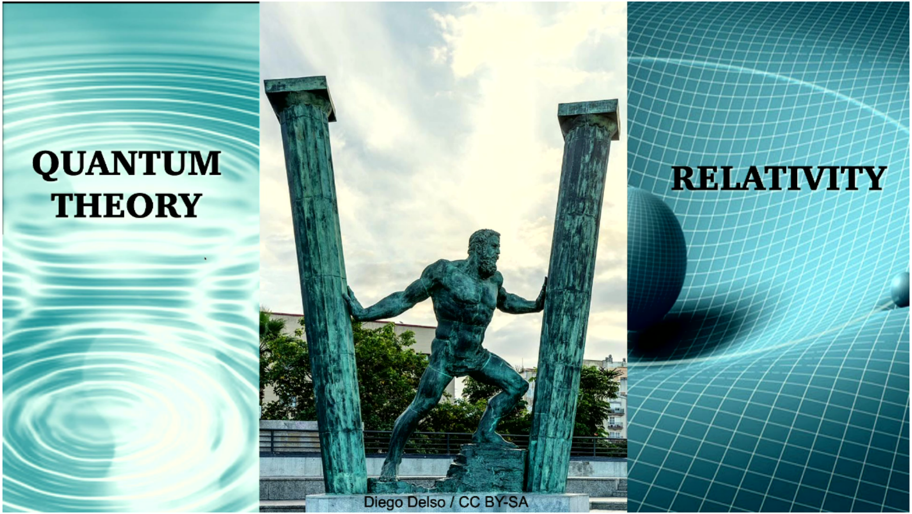
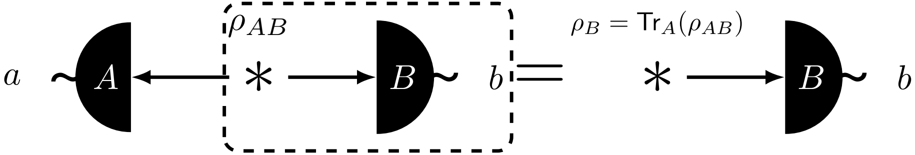
Because it is the fair thing to do

In this talk:
How do joint quantum measurements combine with relativistic causality?



Individual measurements on separated subsystems are consistent with relativistic causality



$$p(b) = \sum_a \mathrm{Tr}[\rho_{AB} \cdot (E_a \otimes F_b)] = \mathrm{Tr}\{\rho_{AB} \cdot [(\sum_a E_a) \otimes F_b]\} = \mathrm{Tr}[\rho_{AB} \cdot (\mathbb{1} \otimes F_b)] = \mathrm{Tr}(\rho_B \cdot F_b)$$

Born's rule accounts for the impossibility of instantaneous transfer of information

Bob's statistics are the same regardless of whatever (even if) Alice measures

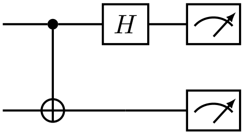
What about joint measurements?

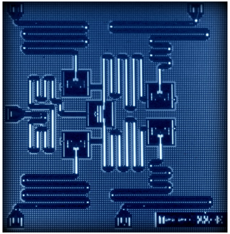
In quantum mechanics, we can perform joint measurements over several subsystems

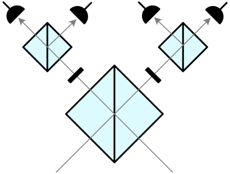
For example, the Bell state measurement

$$E_a = \{ |\phi^+\rangle\langle\phi^+|, |\phi^-\rangle\langle\phi^-|, |\psi^+\rangle\langle\psi^+|, |\psi^-\rangle\langle\psi^-| \}$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right),\ |\psi^\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$$







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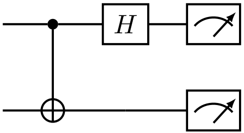

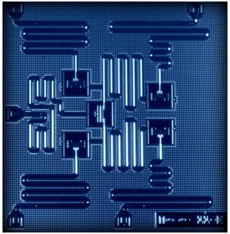

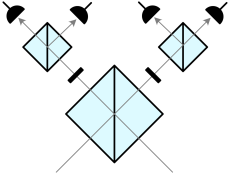

What if we cannot bring the subsystems together? Can we perform (nontrivial) joint measurements? Is there a conflict with relativity in this case?

What about joint measurements?

Erweiterung des Unbestimmtheitsprinzips für die relativistische Quantentheorie.

Von **L. Landau** und **R. Peierls** in Zürich.

(Eingegangen am 3. März 1931.)

Durch Betrachtung der möglichen Meßmethoden wird gezeigt, daß alle in der Wellenmechanik auftretenden physikalischen Größen im relativistischen Gebiet im allgemeinen nicht mehr definierbar sind. Damit hängt das bekannte Versagen der wellenmechanischen Methoden in diesem Gebiet zusammen.

Zeitschrift fur Physik 1, 56-59

What about joint measurements?

Impossible Measurements on Quantum Fields

RAFAEL D. SORKIN

Department of Physics, Syracuse University, Syracuse NY 13244-1130

Abstract

It is shown that the attempt to extend the notion of ideal measurement to quantum field theory leads to a conflict with locality, because (for most observables) the state vector reduction associated with an ideal measurement acts to transmit information faster than light. Two examples of such information-transfer are given, first in the quantum mechanics of a pair of coupled subsystems, and then for the free scalar field in flat spacetime. It is argued that this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory, removing whatever advantages it may have seemed to possess vis a vis the sum-over-histories approach, and reinforcing the view that a sum-over-histories framework is the most promising one for quantum gravity.

arXiv:gr-qc/9302018

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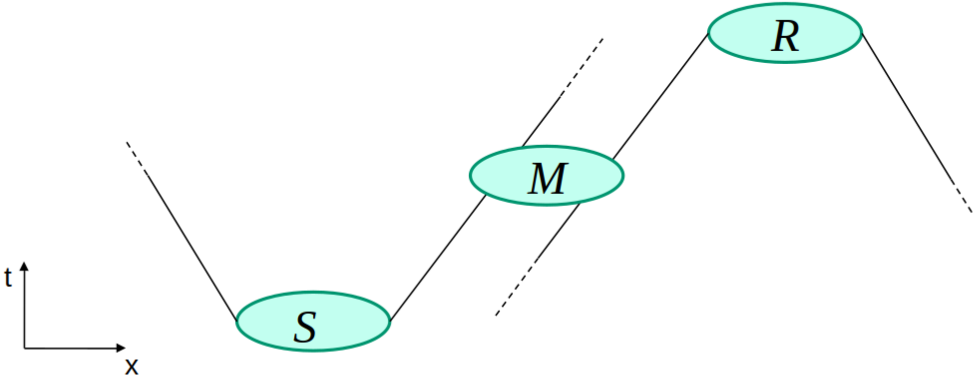
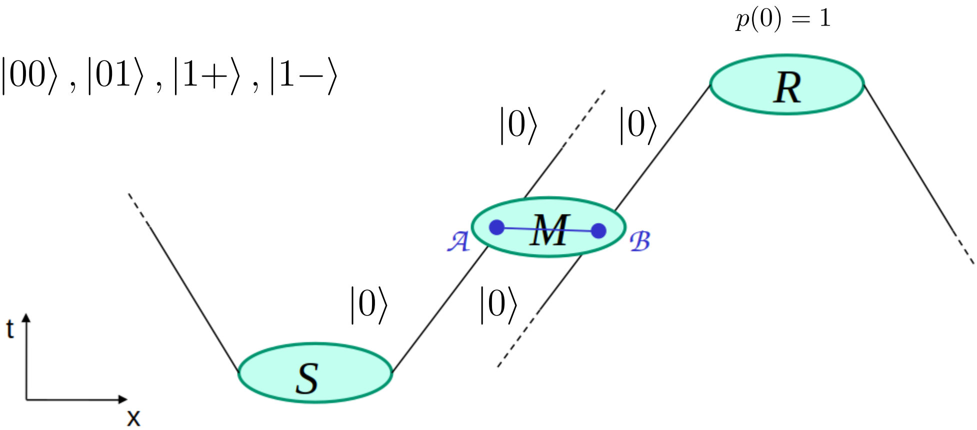
Superluminal signaling in nonrelativistic QM!



Borsten *et al.*, PRD 2019

Superluminal signaling in nonrelativistic QM!

$|00\rangle\,, |01\rangle\,, |1+\rangle\,, |1-\rangle$



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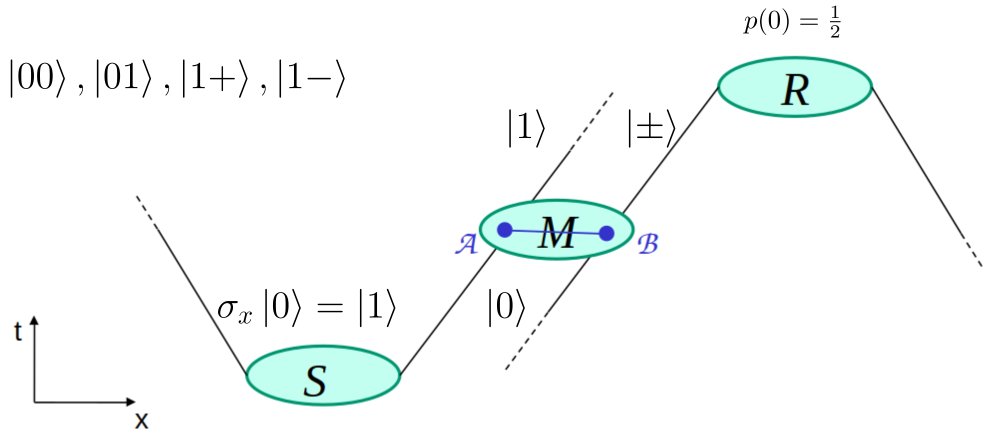

Signaling is a very big problem

There is a (real) lot of measurements that lead to signaling, even in non-relativistic QM

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PHYSICAL REVIEW A VOLUME 49, NUMBER 6 JUNE 1994

Causality constraints on nonlocal quantum measurements

Sandu Popescu
Service de Physique Théorique, Université Libre de Bruxelles, Campus Plaine CP 225, Boulevard du Triomphe, 1050 Bruxelles, Belgium

Lev Vaidman
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv, 69978 Israel
(Received 2 April 1993)

Consequences of relativistic causality for measurements of nonlocal characteristics of composite quantum systems are investigated. It is proved that verification measurements of entangled states necessarily erase local information. A complete analysis of measurability of nondegenerate spin operators of a system of two spin-$\frac{1}{2}$ particles is presented. It is shown that measurability of certain projection operators which play an important role in axiomatic quantum theory contradicts the causality principle.

Joint measurements that do not lead to signaling erase all local information
$\Rightarrow$ for two qubits, the BSM is **the only** measurement that does not lead to signaling
$\Rightarrow$ the BSM is not a typical measurement, it is exceptional

The problem may be with *ideal* measurements

$$A = \sum_i a_i P_i, \quad P_i \succeq 0 \ \forall\, i, \quad \sum_i P_i = \mathbb{1}$$

Born's rule: $p(a_i|\rho) = \mathrm{Tr}\,(\rho P_i)$

Lüders' rule: $\rho \overset{a_i}{\to} \rho_i = P_i \rho P_i / \mathrm{Tr}\,(\rho P_i)$

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PHYSICAL REVIEW A **66**, 022110 (2002)

Measurements of semilocal and nonmaximally entangled states

Berry Groisman and Benni Reznik
School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
(Received 15 November 2001; published 16 August 2002)

Consistency with relativistic causality narrows down dramatically the class of measurable observables. We argue that, by weakening the preparation role of ideal measurements, many of these observables become measurable. In particular, we show by applying entanglement assisted remote operations that all Hermitian observables of a (2×2)-dimensional bipartite system are measurable.

VOLUME 90, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 10 JANUARY 2003

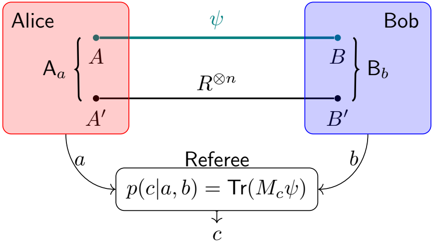
Instantaneous Measurement of Nonlocal Variables

Lev Vaidman
[1]*School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel*
(Received 14 December 2001; revised manuscript received 15 April 2002; published 2 January 2003)

It is shown, under the assumption of the possibility to perform an arbitrary local operation, that all nonlocal variables related to two or more separate sites can be measured instantaneously, except for a finite time required for bringing to one location the classical records from these sites which yield the result of the measurement. It is a verification measurement: it yields reliably the eigenvalues of the nonlocal variables, but it does not prepare the eigenstates of the system.

Performing (non-ideal) measurements in a way consistent with relativity

Definition (localized measurement): the quantum-to-classical transition occurs locally

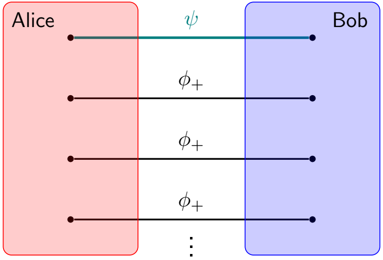


A quantum measurement $\{M_c\}_c$ is n-localizable with resource R if there exist local measurements $\{\mathsf{A}_a\}_a \subset \mathcal{H}_{AA'}$ and $\{\mathsf{B}_b\}_b \subset \mathcal{H}_{BB'}$, and distributions $p(c|a,b)$ such that

$$M_c = \sum_{a,b} p(c|a,b) \mathrm{Tr}_{A'B'} \left[(\mathsf{A}_a \otimes \mathsf{B}_b) \left(\mathbb{1}_{AB} \otimes R^{\otimes n}_{A'B'} \right) \right]$$

Localization of measurements: blind ping-pong teleportation

Rationale: “move” the full state to one party and measure it there $\rightarrow$ teleportation.



Cirac *et al.*, PRL 2001; Vaidman, PRL 2003; Clark *et al*, NJP 2010: any measurement can be localized

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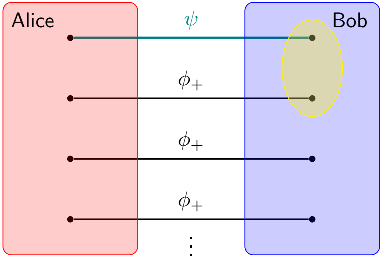

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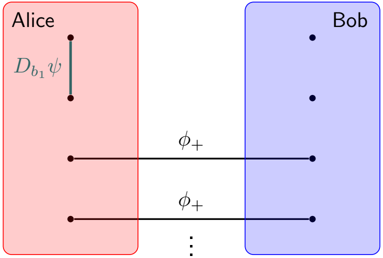

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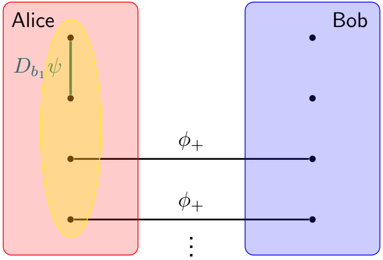

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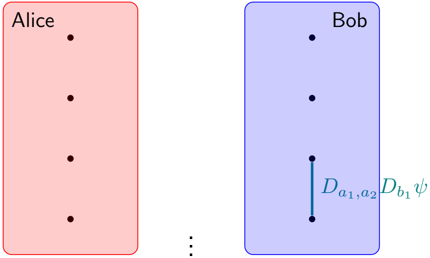

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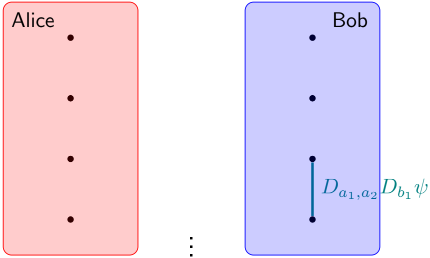


Cirac *et al.*, PRL 2001; Vaidman, PRL 2003; Clark *et al*, NJP 2010: any measurement can be localized
Need infinite entanglement either always or in the worst case.

arXiv:2408.00831 (Phys. Rev. X 15, 021013)

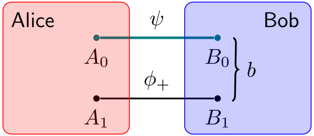
Classification of joint quantum measurements based on entanglement cost of localization

Jef Pauwels, Alejandro Pozas-Kerstjens, Flavio Del Santo, and Nicolas Gisin
Group of Applied Physics, University of Geneva, 1211 Geneva, Switzerland and Constructor University, 1211 Geneva, Switzerland

Despite their importance in quantum theory, joint quantum measurements remain poorly understood. An intriguing conceptual and practical question is whether joint quantum measurements on separated systems can be performed without bringing them together. Remarkably, by using shared entanglement, this can be achieved perfectly when disregarding the post-measurement state. However, existing localization protocols typically require unbounded entanglement. In this work, we address the fundamental question: "Which joint measurements can be localized with a finite amount of entanglement?" We develop finite-resource versions of teleportation-based schemes and analytically classify all two-qubit measurements that can be localized in the first steps of these hierarchies. These include several measurements with exceptional properties and symmetries, such as the Bell state measurement and the elegant joint measurement. This leads us to propose a systematic classification of joint measurements based on entanglement cost, which we argue directly connects to the complexity of implementing those measurements. We illustrate how to numerically explore higher levels and construct generalizations to higher dimensions and multipartite settings.

Which measurements can we localize with a **fixed** amount of entanglement?
(reproduce their statistics on any state)

The simplest case: 2-qubit state, 1 shared ebit



Step 1: Bob teleports to Alice using the ebit

$$|\psi\rangle \to \mathbb{1} \otimes \sigma_b |\psi\rangle$$

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831 (Phys. Rev. X 15, 021013)

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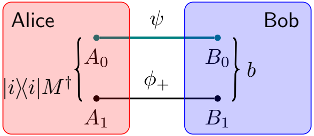

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Step 2: Alice applies $M^\dagger$ to rotate the measurement basis to the computational basis

$$M = (|v_1\rangle, |v_2\rangle, |v_3\rangle, |v_4\rangle), \quad \langle v_i|v_j\rangle = \delta_{ij}, \quad \sum_i |v_i\rangle\langle v_i| = \mathbb{1}$$

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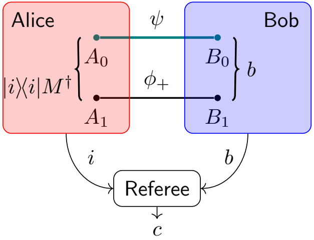

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Step 3: Alice measures in the computational basis

$$\left|\langle i, j| M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) |\psi\rangle\right|^2$$

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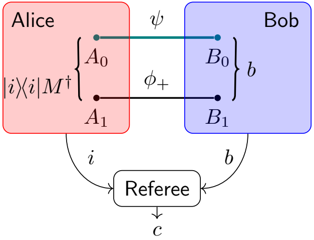

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Step 3: Alice measures in the computational basis

$$\left|\langle i,j|\, M^\dagger \cdot (\mathbb{1} \otimes \sigma_b)\, |\psi\rangle\right|^2 = \left| \langle i,j \,|\, M^\dagger \,|\psi\rangle\right|^2$$

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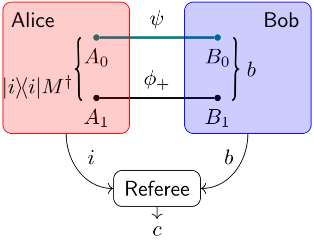

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$$\left|\langle i,j|\, M^\dagger \cdot (\mathbb{1} \otimes \sigma_b)\, |\psi\rangle\right|^2 = \left|e^{\mathrm{i}\phi_b(i,j)} \langle \pi_b(i,j)|\, M^\dagger \, |\psi\rangle\right|^2$$

$$M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M = P_b \cdot \Phi_b$$

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831 (Phys. Rev. X 15, 021013)

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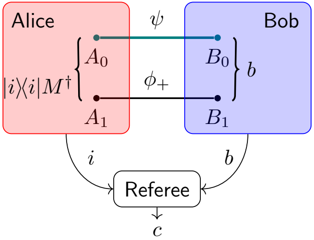


$$M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M = P_b \cdot \Phi_b$$

The solutions are intertwiners between (red.) representations of $SU(2)$

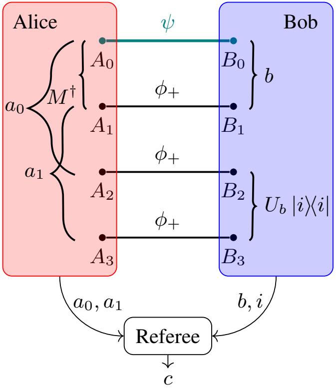
$$\{\mathbb{1} \otimes \sigma_b\}_b,\ \{P_b \cdot \Phi_b\}_b$$

Only two nontrivial solutions:

1. Bell state measurement
   - Dense coding, entanglement swapping, teleportation...
2. $\pi/2$-twisted (BB84) basis: $\{|00\rangle, |01\rangle, |1+\rangle, |1-\rangle\}$
   - Position-based cryptography

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831 (Phys. Rev. X 15, 021013)

A less simple case: 2-qubit state, 3 shared ebits



Bob teleports, Alice rotates *and teleports back*, Bob amends knowing his previous outcome

$$M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M \cdot (\sigma_{a_1} \otimes \sigma_{a_2}) \cdot M^\dagger \cdot (\mathbb{1} \otimes \sigma_b) \cdot M = P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}$$

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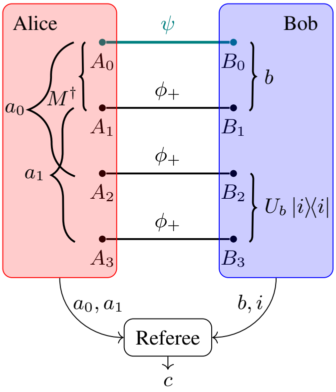

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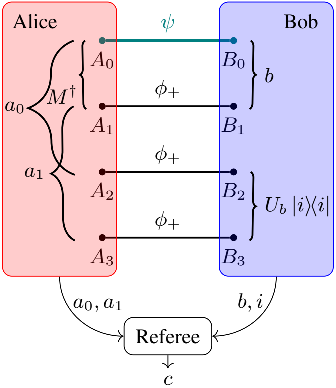

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Five new solutions:

1. Partial BSM: $\{|00\rangle, |11\rangle, |\psi^+\rangle, |\psi^-\rangle\}$
   - Linear optics
2. Elegant Joint Measurement (Gisin, Entropy 2019)
   - Genuine network nonlocality
3. $\pi/2$-twisted BSM: $\{|0+\rangle \pm |11\rangle, |0-\rangle \pm |10\rangle\}$
   - Randomness without inputs (Boreiri *et al.*, Quantum 2025)
4. Two more iso-entangled measurements

$$\{|\psi^-\rangle \pm |00\rangle, |\psi^+\rangle \pm |11\rangle\}$$
$$\{|1-\rangle \pm |01\rangle, |1+\rangle \pm \mathrm{i}\,|00\rangle\}$$

(which cannot be localized with fewer ebits)

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831 (Phys. Rev. X 15, 021013)

A criterion for measurement complexity

The set of m-e$d$it measurements localizable at the n-th level of the hierarchy is

$$\mathcal{V}_n^{d,m} \equiv \{M \in \mathcal{U}(d^m) \,|\, M^\dagger \cdot (\mathbb{1} \otimes \mathcal{P}_{m-1}) \cdot M \in \bar{\mathcal{V}}_{n-1}^{d,m}\}$$

with

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Generalization of the Clifford hierarchy $\mathcal{C}_n^{d,m} \equiv \{M \in \mathcal{U}(d^m) \,|\, M^\dagger \cdot \mathcal{P}_m \cdot M \in \mathcal{C}_{n-1}^{d,m}\}$

- BB84 basis does not belong to $\mathcal{C}_1^{2,2}$
- $\{|1-\rangle \pm |01\rangle\,, |1+\rangle \pm \mathrm{i}\,|00\rangle\}$ does not belong to (at least) $\mathcal{C}_6^{2,2}$
- Position in the Clifford hierarchy is connected to the complexity of implementing M in a quantum computer (Gottesman & Chuang, Nature 1999)

Also, other localization hierarchy (Clark *et al*, NJP 2010) relates to T-depth (Speelman, TQC 2016).

Claim:
Entanglement cost of localization is a (physically motivated) measure of measurement complexity

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831 (Phys. Rev. X 15, 021013)

Generalizations

$$\mathcal{V}_n^{d,m} \equiv \{M \in \mathcal{U}(d^m) \,|\, M^\dagger \cdot (\mathbb{1} \otimes \mathcal{P}_{m-1}) \cdot M \in \bar{\mathcal{V}}_{n-1}^{d,m}\}$$
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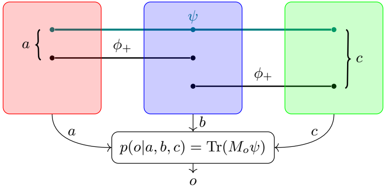
Higher levels

2 qubits, 9 ebits: (at least) 27 new bases.
In all cases, all entangled states in the basis are iso-entangled.

More parties



1st level: (at least) 8 solutions, nothing surprising
2nd level: (at least) 64 solutions, 2 generalizations of EJM

Higher dimensions

Recall 2-qubit, 2nd level equation:
$\mathcal{M}_b^\dagger \cdot (\sigma_{a_1} \otimes \sigma_{a_2}) \cdot \mathcal{M}_b = P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}$

Approach: write $\{P_{a_1,a_2,b} \cdot \Phi_{a_1,a_2,b}\}_{a_1,a_2,b}$ in dimension-free form

Goal: generalization of EJM to dimension d

Our analytical methods explode combinatorially for all cases

J. Pauwels, APK, F. del Santo, and N. Gisin, arXiv:2408.00831 (Phys. Rev. X 15, 021013)

Conclusions

- Joint quantum measurements deserve attention. We have only scratched the surface.
- If we only care about measurement outcomes, it is possible to perform any joint measurement in a way consistent with relativity, if given enough entanglement.
- Entanglement cost of localization is a sound and physically motivated measure of measurement complexity.

- Improve methods for analytical/numerical characterization.
- Applications: network nonlocality, cryptography...
- Foundations: is relativistic causality sufficient to describe joint quantm measurements? Do we need new update rules for the post-measurement states?

Thanks for your attention

Questions? Comments?

2408.00831 (Phys. Rev. X 15, 021013)

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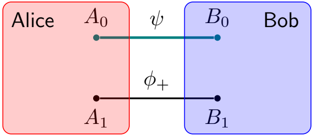
Extra: A simple example of localization

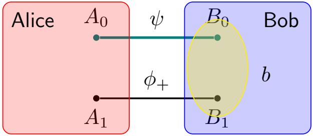
Bell state measurement



Extra: A simple example of localization

Bell state measurement

Step 1: Bob performs BSM $\rightarrow$ teleports his particle

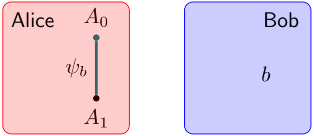


Extra: A simple example of localization

Bell state measurement

Step 1: Bob performs BSM $\rightarrow$ teleports his particle (with a distortion)

$$|\psi\rangle \rightarrow \mathbb{1} \otimes \sigma_b \, |\psi\rangle$$



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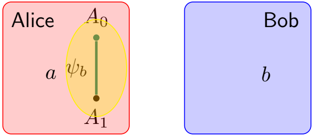

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$$|\psi\rangle \rightarrow \mathbb{1} \otimes \sigma_b |\psi\rangle$$

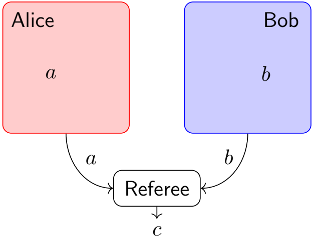
Step 2: Alice performs BSM

$$p(a) = |\langle E_a| \mathbb{1} \otimes \sigma_b |\psi\rangle|^2 \overset{*}{=} |\langle \phi^+| \mathbb{1} \otimes (\sigma_a \cdot \sigma_b) |\psi\rangle|^2 \overset{**}{=} |\langle E_{a \oplus b}|\psi\rangle|^2$$

$* \; |E_a\rangle = \mathbb{1} \otimes \sigma_a |\phi^+\rangle \,, \quad ** \; \sigma_a \cdot \sigma_b \sim \sigma_{a \oplus b}$

Extra: A simple example of localization

Bell state measurement



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Step 3: Alice and Bob put in common their results

$$c = a \oplus b$$

Alice	**Bob**	**Real outcome**
00 ($\mathbb{1}$)	b	b
a	00	a
01 (X)	01	00
01	10 (Z)	11 (Y)
	$\vdots$	

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Extra: applications

Quantum position verification

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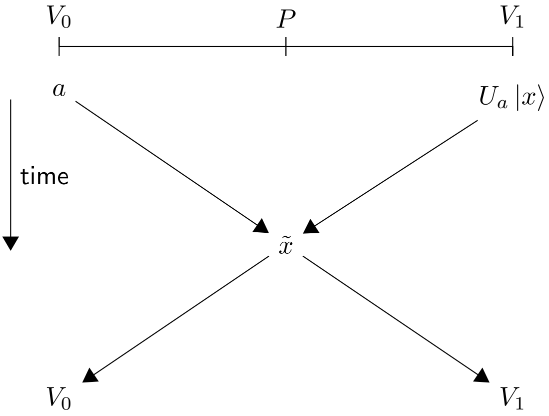
Goal: convince two verifiers (V_0 and V_1) that I'm at P



$t = 0$: V_0 sends a, V_1 sends $U_a |x\rangle$

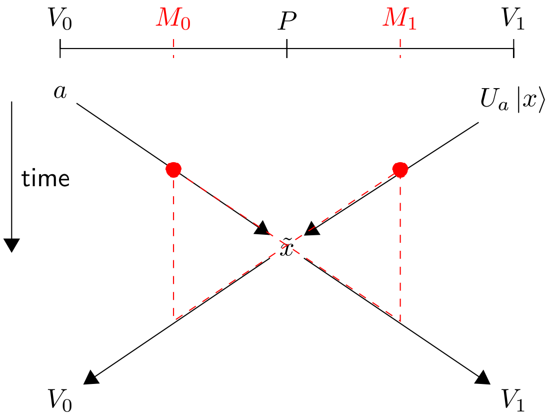
$t = 1$: P receives both pieces of information, applies $U_a^\dagger$ and measures in the computational basis, transmitting the result back to V_0 and V_1

$t = 2$: V_0 and V_1 accept if they receive x, and they receive it on time

Extra: applications

Quantum position verification

Goal: convince two verifiers (V_0 and V_1) that I'm at P



$t = 0$: V_0 sends a, V_1 sends $U_a |x\rangle$

$t < 1$: two coordinated adversaries intercept the information and run a localization protocol

$t = 1$: P receives both pieces of information, applies $U_a^\dagger$ and measures in the computational basis, transmitting the result back to V_0 and V_1

$t = 2$: V_0 and V_1 accept if they receive x, and they receive it on time

Entanglement cost of localization quantifies how secure a measurement is for QPV