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Valencia, 20-24 Oct 2025

# Harnessing strong system-bath coupling in quantum thermal machines:

## From synchronization to enhanced performance and precision

UNIVERSITÀ DI PAVIA

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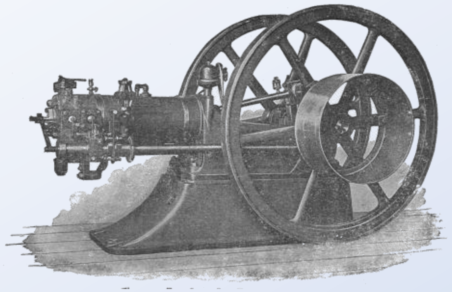
MACRO



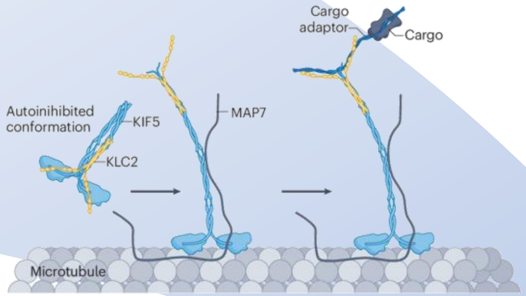
**Otto horizontal gas engine**
From: New Catechism of the Steam Engine, 1904

## Classical thermodynamics

S.J. Blundell and K.M. Blundell, *Concepts in Thermal Physics*, 2nd edn (Oxford, 2009)

## Stochastic thermodynamics

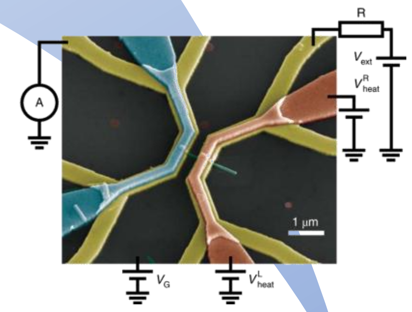
U. Seifert, Annu. Rev. Condens. Matter Phys. 10, 171 (2019)




**Kinesin 1, a microtubule-based motor**
Nat. Rev. Mol. Cell. Biol. 26, 86 (2025)

## Quantum thermodynamics

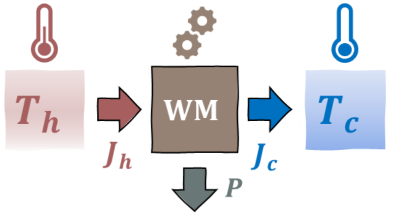
F. Binder, L.A. Correa, C. Gogolin, J. Anders, and G. Adesso, *Thermodynamics in the Quantum Regime* (Springer International Publishing, 2019)
N. M. Myers, O. Abah, and S. Deffner, AVS Quantum Sci. 4, 027101 (2022)



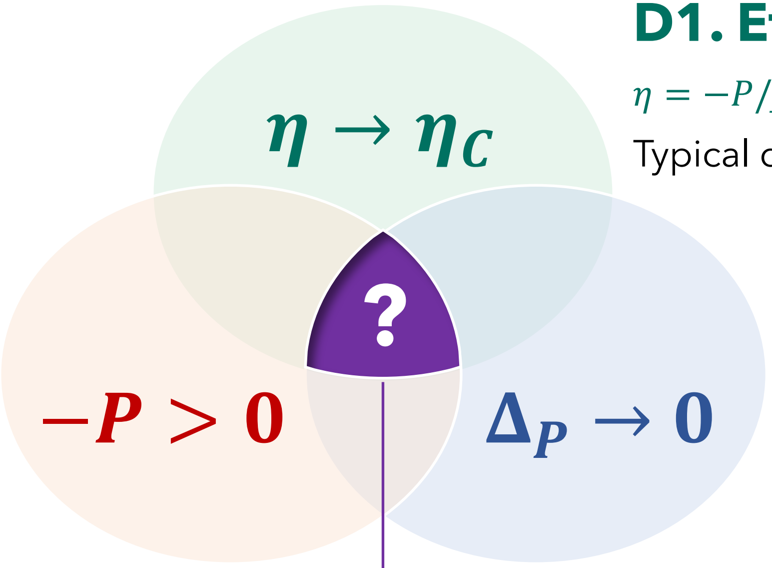

**A quantum-dot heat engine**
Nature Nanotech. 13, 920–924 (2018)

NANO

*For systems operating at the mesoscale and below, fluctuations of thermodynamic quantities can be of the same magnitude as the mean value. Plus, quantum effects enter the chat.*




# Heat engine performance
# Three *Desiderata*




**D1. Efficiency** close to Carnot,

$\eta = -P/J_h \approx \eta_C = 1 - T_c/T_h$

Typical drawback: $P \to 0$

**D2.** Finite **power output**

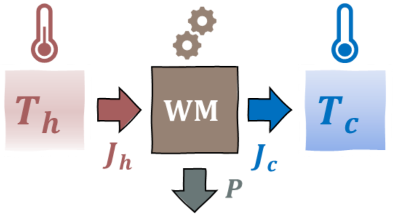
Typical drawback: $\eta \ll \eta_C$

**D3. Constancy,** vanishing fluctuations

Typical drawback: $\Delta_P \to \infty$ at finite $P$ and $\eta \approx \eta_C$

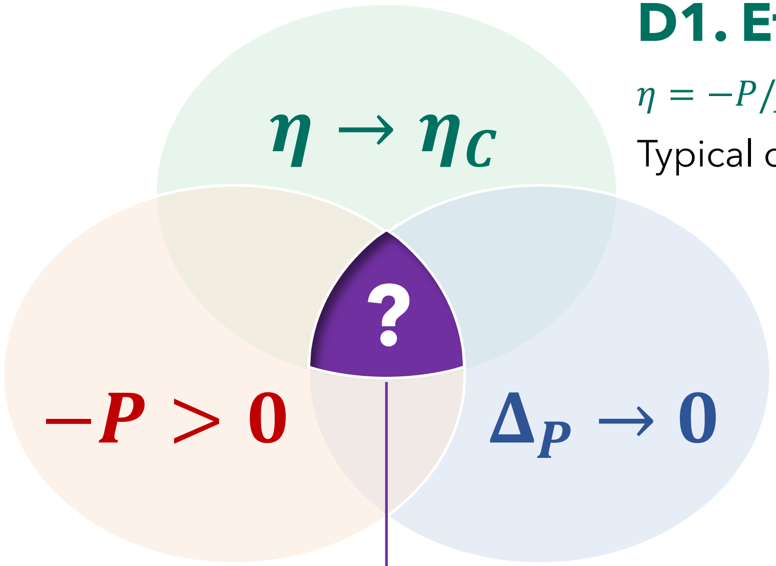
See, e.g., critical heat engine
[M. Campisi, R. Fazio, Nat. Commun. 7, 11895 (2016)]

Can a heat en***genie*** grant all the three wishes?




## Heat engine performance

# Three *Desiderata*

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Can a heat en***genie*** grant all the three wishes?

**NO**

**TRADEOFF** for steady-state heat engines

$$-P \frac{\eta}{\eta_C - \eta} \frac{k_B T_c}{\Delta_P} \leq \frac{1}{2}$$ (D1-D2-D3 are NOT compatible)

P. Pietzonka and U. Seifert, Phys. Rev. Lett. 120, 190602 (2018)

## *There is no free lunch:* 

# Precision comes at a price

## **T**hermodynamic **U**ncertainty **R**elations

*Thermodynamic bounds on fluctuations of currents in non-equilibrium steady states.*

***Higher precision requires an increase in the dissipation rate.***

$$\dot{S}\frac{\Delta_J}{J^2} \geq 2(k_B = 1)$$

- Classical
- Markovian
- Drives: time-indep. and TRS

**Precision:** The relative fluctuations (noise-to-signal ratio) of a current, $J$.

**Price:** The entropy production rate $\dot{S}$ (*dissipation*) constrains precision.

J.M. Horowitz, T.R. Gingrich, Nat. Phys. 16, 15 (2020)

**TURs...**

From exchange fluctuations theorems
A.M. Timpanaro et al., Phys. Rev. Lett. 123, 090604 (2019)

For time-dependent driving
T. Koyuk, U. Seifert, Phys. Rev. Lett. 125, 260604 (2020)

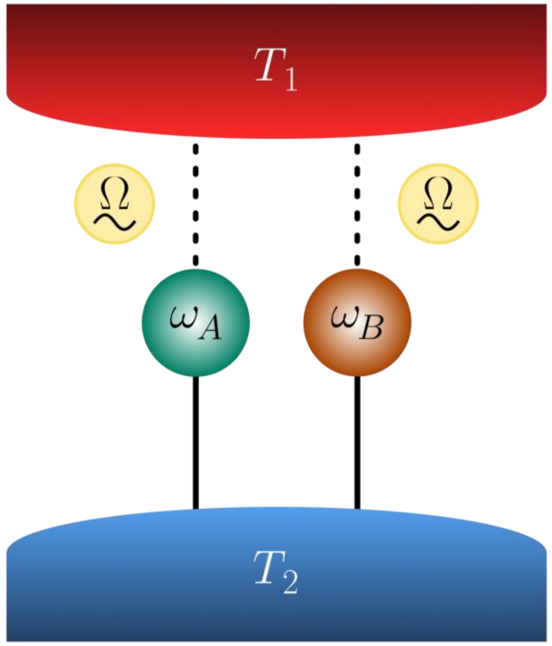
For Markovian open quantum dynamics
T. Van Vu, K. Saito, Phys. Rev. Lett. 128, 140602 (2022)

For general open quantum systems
T. Van Vu, R. Honma, K. Saito, arXiv:2508.21567 (2025)

*To be continued.*

# QTM

**with dynamical couplings**




$$H_l = \frac{p_l^2}{2m} + \frac{1}{2} m\omega_l^2 x_l^2$$

**Working medium**

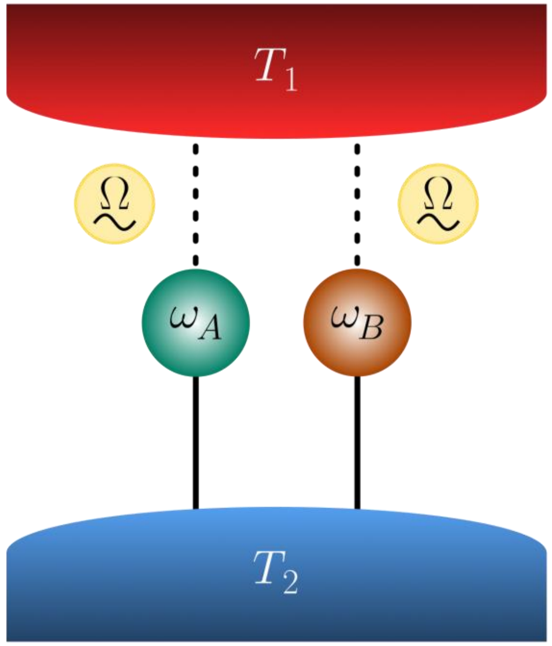
Two **uncoupled** QHOs $l = A, B$

$(\omega_A \neq \omega_B)$


M. Carrega et al., PRX Quantum 3, 010323 (2022)
F. Cavaliere et al., Phys. Rev. Research 4, 033233 (2022)
M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)
L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

# QTM

## with dynamical couplings




$$H_\nu = \sum_{k=1}^{+\infty} \left[ \frac{P_{k,\nu}^2}{2m_{k,\nu}} + \frac{1}{2} m_{k,\nu} \omega_{k,\nu}^2 X_{k,\nu}^2 \right]$$

**Bath/Dissipation** (Caldeira-Leggett)

Collection of independent harmonic oscillators

A. O. Caldeira, A. J. Leggett, Ann. Phys. 149, 374 (1983)
U. Weiss, *Quantum Dissipative Systems* (World Scientific, 2012)

**Reservoir $\nu = 1$ | Structured**

Spectral density:

$$\mathcal{J}_1(\omega) = \frac{d_1 m \gamma_1 \omega}{\left(\omega^2 - \omega_1^2\right)^2 + \gamma_1^2 \omega_1^2} \text{ or } \mathcal{J}_1(\omega) = \frac{m\gamma_1\omega}{1+\omega^2/\omega_c^2}$$

**Reservoir $\nu = 2$ | Ohmic**

Spectral density: $\mathcal{J}_2(\omega) = m\omega\gamma_2$


M. Carrega et al., PRX Quantum 3, 010323 (2022)
F. Cavaliere et al., Phys. Rev. Research 4, 033233 (2022)
M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)
L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

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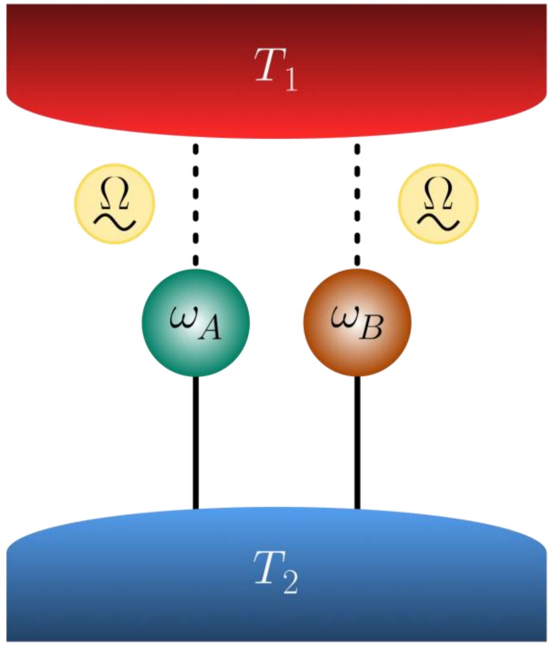



$$H^{(t)}_{\mathrm{int},\nu} = \sum_{l=A,B} \sum_{k=1}^{+\infty} \left[ -g_\nu^{(l)}(t) c_{k,\nu}^{(l)} x_l X_{k,\nu} + \mathrm{CT}_{k,\nu}^{(l)} \right]$$

**Interaction** (periodic drive)

Bilinear system-bath couplings (+counterterm*),
where $g_\nu(t) = g_\nu(t + \mathcal{T})$ with $\mathcal{T} = 2\pi/\Omega$

**Reservoir $\nu = 1$ | Dynamical coupling $g_1^{(l)}(t)$**

**Reservoir $\nu = 2$ | Static coupling $g_2^{(l)}(t) = 1$**

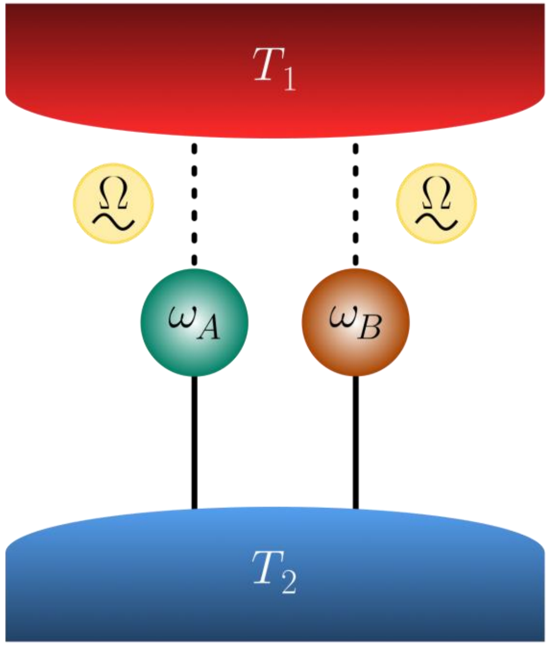
*The counterterm $\mathrm{CT}_{k,\nu}^{(l)} = \frac{\left(g_\nu^{(l)}(t) c_{k,\nu}^{(l)}\right)^2}{2 m_{k,\nu} \omega_{k,\nu}^2} x_l^2 + \frac{g_\nu^{(l)}(t) g_\nu^{(\bar{l})}(t) c_{k,\nu}^{(l)} c_{k,\nu}^{(\bar{l})}}{2 m_{k,\nu} \omega_{k,\nu}^2} x_l x_{\bar{l}}$

(i) avoids renormalizations of the characteristic frequencies of the QHOs;
(ii) cancels the direct coupling among them.


M. Carrega et al., PRX Quantum 3, 010323 (2022)
F. Cavaliere et al., Phys. Rev. Research 4, 033233 (2022)
M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)
L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

# QTM

**with dynamical couplings**




***Out-of-equilibrium quantum dissipative system:***
The total system reaches a periodic steady state sustained by the drives (period $\mathcal{T}$ of the drive)

*Heat current $J_\nu$ and Power $P$:*

$$J_\nu \equiv -\frac{1}{\mathcal{T}}\int_0^{\mathcal{T}} \mathrm{d}t\, \mathrm{Tr}[H_\nu\, \dot{\rho}(t)]$$

$$P \equiv \sum_{l=A,B} \frac{1}{\mathcal{T}}\int_0^{\mathcal{T}} \mathrm{d}t\ \mathrm{Tr}\left[\frac{\partial H^{(t)}_{\mathrm{int},1,l}}{\partial t}\rho(t)\right] = P_A + P_B$$

## Average thermodynamic quantities

Temporal (over the period $\mathcal{T}$) and quantum averages. Since the periodic drives are independent, power can be independently *injected into/extracted from* subsystems $l = A, B$.

- Energy balance relation: $J_1 + J_2 + P = 0$
- Entropy production rate: $\dot{S} = -\sum_{\nu=1,2} J_\nu/T_\nu$
- Heat engine if $P = P_A + P_B < 0$.


M. Carrega et al., PRX Quantum 3, 010323 (2022)
F. Cavaliere et al., Phys. Rev. Research 4, 033233 (2022)
M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)
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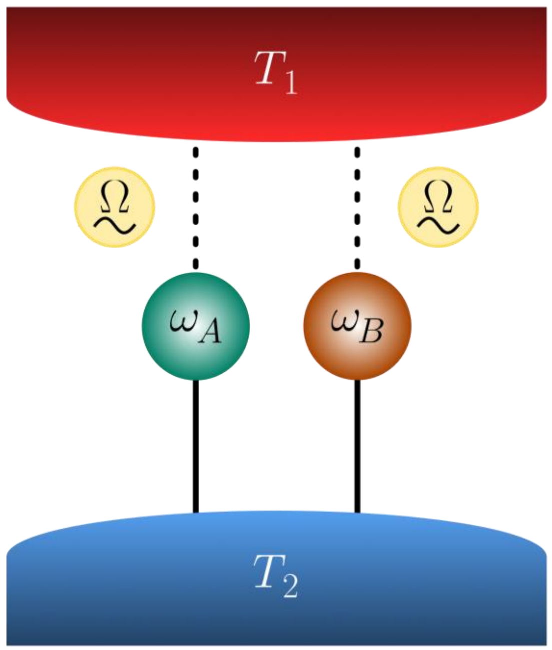
# QTM

**with dynamical couplings**




## Advantages of the model

- Allows **exact analytical solutions at arbitrary** system-bath **coupling.**
- Enables the **breaking of time-reversal symmetry** (TRS) via relative phase in periodic drives.
- Incorporates **non-Markovian effects** by engineering the bath spectral density (structured environment).
- Can address **all driving regimes**, from quasistatic to antiadiabatic.


M. Carrega et al., PRX Quantum 3, 010323 (2022)
F. Cavaliere et al., Phys. Rev. Research 4, 033233 (2022)
M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)
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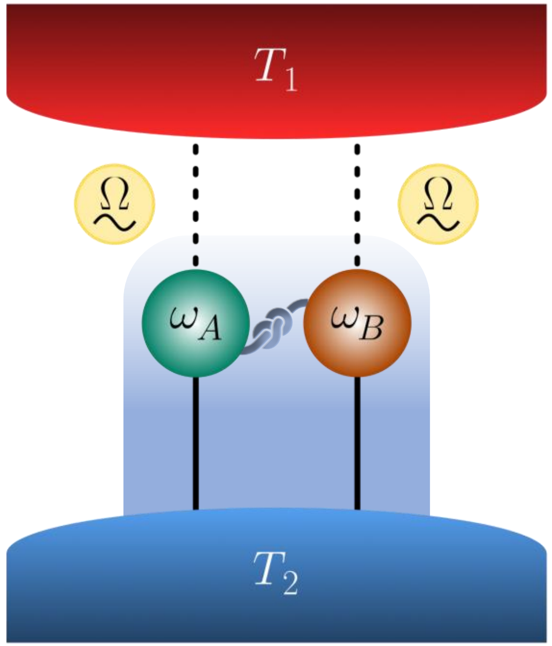
# QTM

**with dynamical couplings**

**Reservoir $\nu = 1$ | Structured**

**Weak** dynamical couplings




**Reservoir $\nu = 2$ | Ohmic**

**(Strong)** static couplings

$$\mathcal{J}_2(\omega) = m\omega\gamma_2$$

Reservoir $\nu = 2$ **mediates correlations** between the QHOs, which become pronounced at **strong coupling $\gamma_2$**

**Strong coupling typically implies:**

(i) overdamped dynamics;

(ii) poor performance of the engine.

Can **bath-mediated correlations** at strong coupling be **a resource?**

M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)

L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

# **Synchronization:** Frequency-locking in antiphase at SC

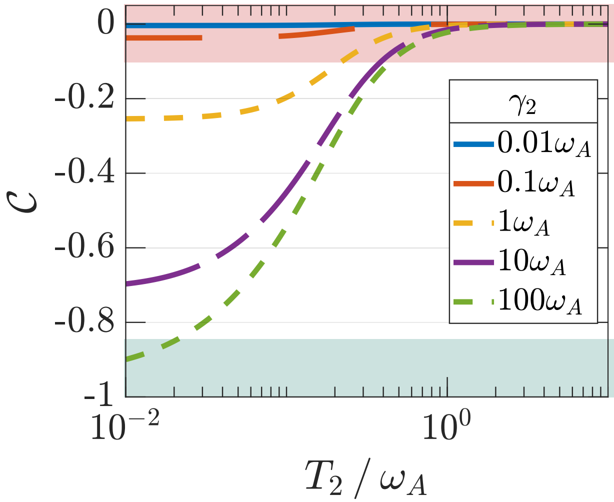
*Two otherwise independent QHOs synchronize through coupling to the common bath $\nu = 2$.*
*(no contribution from the weakly-coupled bath $\nu = 1$)*

The **Pearson (correlation) coefficient**

$$\mathcal{C} = \frac{\langle \delta x_A \delta x_B \rangle}{\sqrt{\langle \delta x_A \delta x_A \rangle \langle \delta x_B \delta x_B \rangle}}$$

with $\langle \delta x_l \delta x_{l'} \rangle \equiv \langle x_l(t) x_{l'}(t) \rangle - \langle x_l(t) \rangle \langle x_{l'}(t) \rangle$, measures the synchronization between local observables.

F. Galve, G.L. Giorgi, and R. Zambrini, *Quantum correlations and synchronization measures*, in *Lectures on general quantum correlations and their applications* (Spinger, Berlin, 2017)




**$\mathcal{C} \to 0$: No synchronization**

Anti-synchronization at low temperature and SC.

**$\mathcal{C} \to -1$: Anti-synchronization**

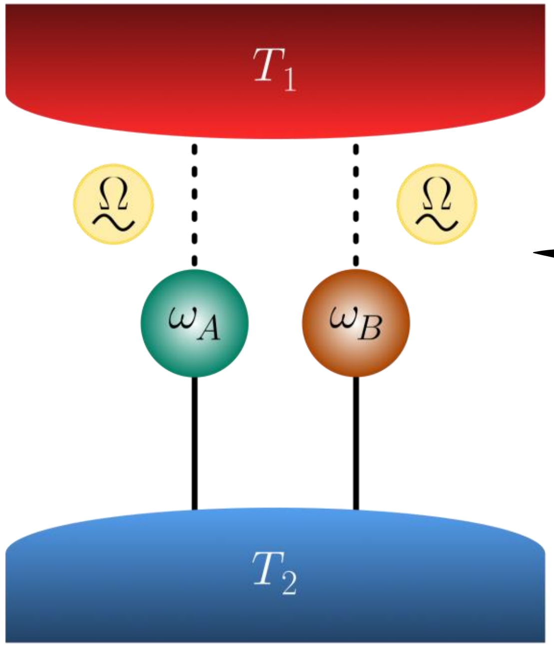
> At low temperature and strong coupling:
>
> Emergence of a weakly-damped mode with the two QHOs oscillating at frequency $\bar{\omega} = \sqrt{(\omega_A^2 + \omega_B^2)/2}$ in antiphase.

M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)
L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

**Further reading on synchronization in quantum systems:**
G.L. Giorgi et al., Phys. Rev. A 85, 052101 (2012)
G. Manzano et al., Sci. Rep. 3, 1439 (2013)
A. Mari et al., Phys. Rev. Lett. 111, 103605 (2013)
A. Roulet and C. Bruder, Phys. Rev. Lett. 121, 053601 (2018)
G. Karpat et al., Phys. Rev. A 103, 062217 (2021)

Are bath-mediated correlations a resource?

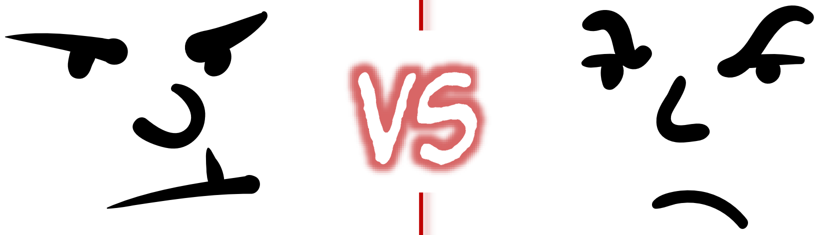
# Which QTM performs better?

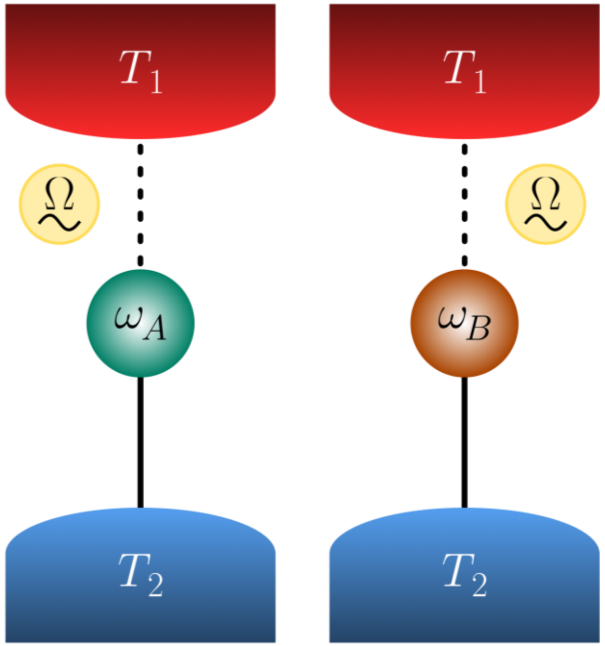



**Joint configuration**

QHOs are coupled
to common baths.

***Bath($\nu$ = 2)-induced correlations.***




**Independent configuration**

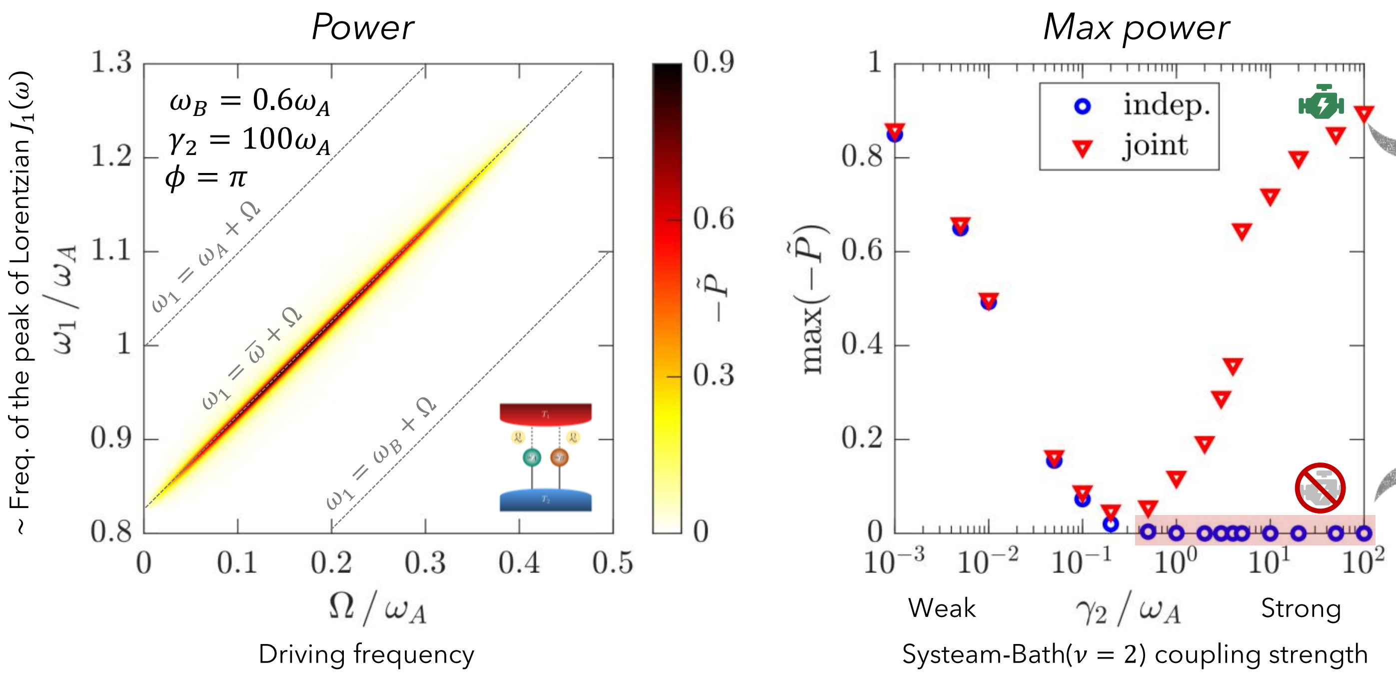
QHOs work in parallel,
with separate baths.

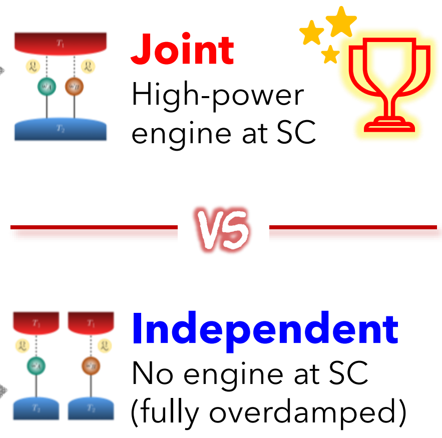
***No correlations.***

# Heat engine: **Advantage at *strong* coupling** [1/2]

**Assumptions:** (i) Monochromatic drives: $g_1^{(A)}(t) = \cos(\Omega t)$ and $g_1^{(B)}(t) = \cos(\Omega t + \phi)$ ; (ii) Lorentzian spectral density $J_1(\omega)$

*Despite the strong system-bath coupling, the joint configuration still operates as a heat engine ($P < 0$)*




*The **joint configuration** exhibits **enhanced performance** over the independent one **due to bath-mediated correlations**. The collective advantage is most pronounced **at strong coupling**, $\gamma_2$.*

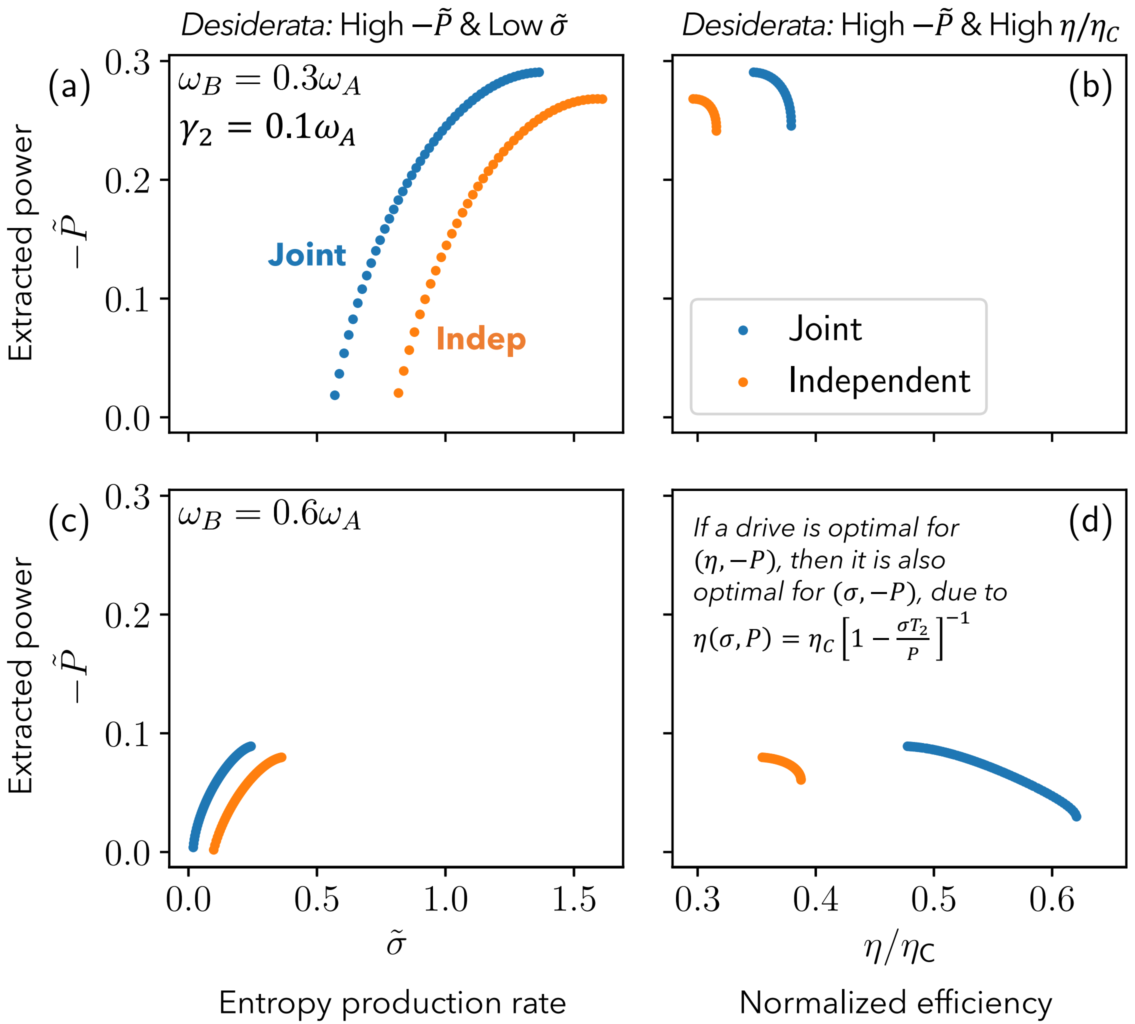
M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)

# Heat engine: **Advantage at *moderate* coupling** [2/2]

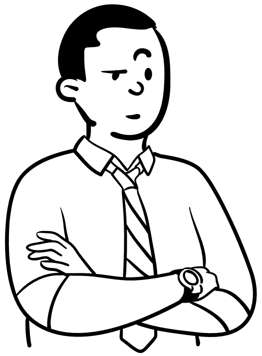
High-power engines usually exhibit low efficiency, and *vice versa*.

The **Pareto front** is the **set of optimal trade-offs** between power and efficiency, where improving one necessarily worsens the other.

*The claim of collective advantage is corroborated by* ***optimizing over generic periodic drives*** $g_1^{(l)}(t)$*, constructing the full* ***Pareto front*** $(\eta, -P)$.

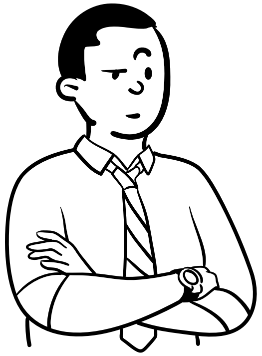



M. Carrega et al., AVS Quantum Sci. 6, 025001 (2024)



Can we harness the **synchronization** arising at strong-coupling to obtain **precise work current** (power)?

L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)



Can we harness the **synchronization** arising at strong-coupling to obtain **precise work current** (power)?

**NO**, the *total* work current cannot violate the TUR

**YES**, the *local* work currents can violate the TUR

L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

# Isothermal work-to-work converter

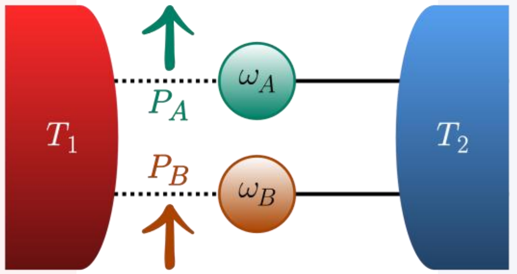
L. Razzoli et al.,
Quantum Sci. Technol. 9, 045032 (2024)

**Ohmic**
(Drude-Lorentz cut-off)

$$\mathcal{J}_1(\omega) = \frac{m\gamma_1\omega}{1+\omega^2/\omega_c^2}$$

***Driven (weak):*** ⋯⋯

$$g_1^{(A)}(t) = \cos(\Omega t)$$
$$g_1^{(B)}(t) = \cos(\Omega t + \boldsymbol{\phi})$$




*Bath-QHO couplings*

**Ohmic**

$$\mathcal{J}_2(\omega) = m\gamma_2\omega$$

***Static:*** ——

$$g_2^{(A)}(t) = g_2^{(B)}(t) = 1$$

TRS breaking: $\phi = \pi/2$

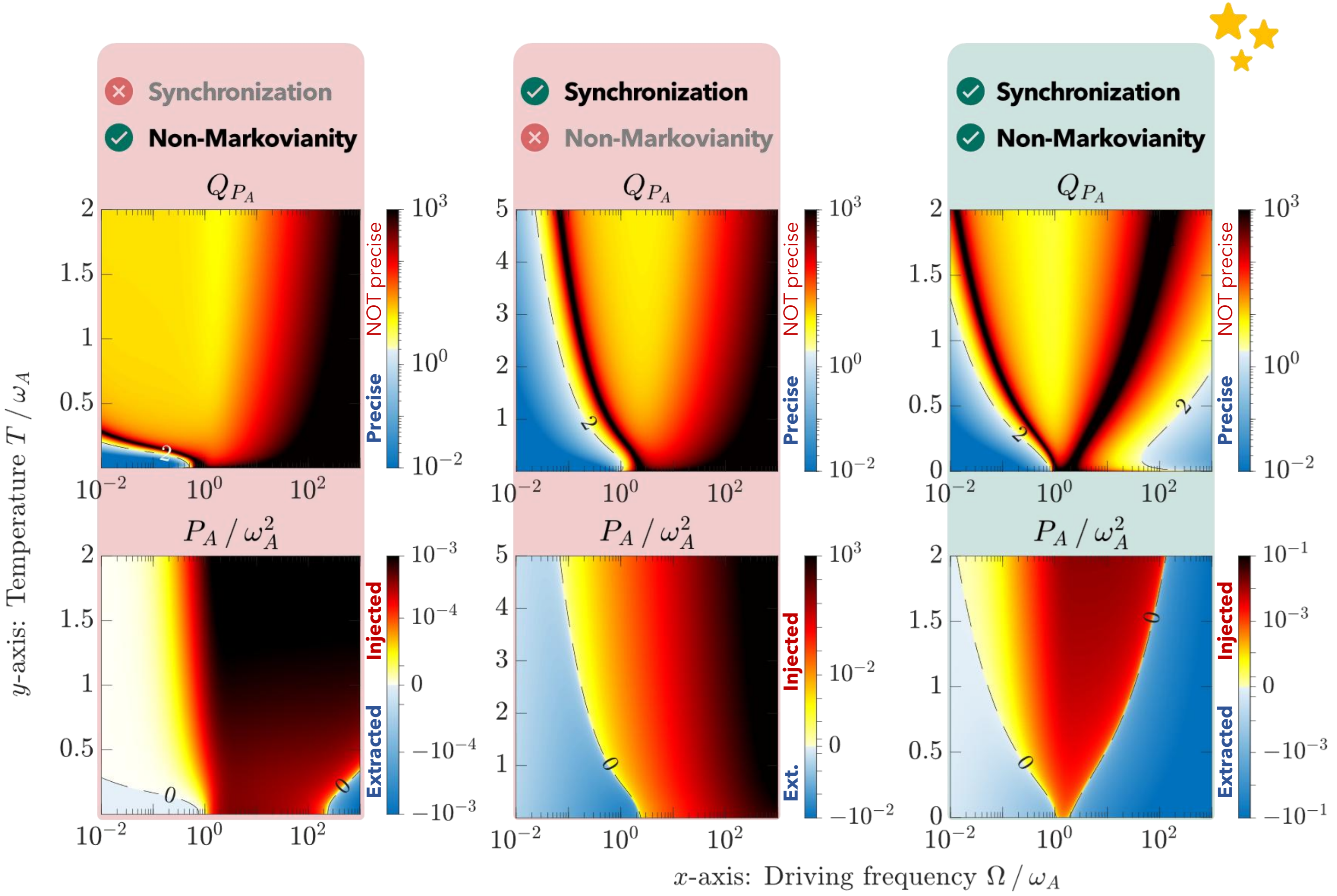
Although total power $P = \boldsymbol{P_A} + \boldsymbol{P_B} \geq 0$ is supplied to the system
(as $\dot{S} = -J_1/T - J_2/T \geq 0$ in the periodic steady-state and $P + J_1 + J_2 = 0$),
power can be locally extracted, e.g., $\boldsymbol{P_A < 0}$.

We say the work current $P_A$ is «precise» if it violates the classical TUR, i.e., if $Q_{P_A} \equiv \dot{S}\frac{\Delta_{P_A}}{P_A^2} < 2$.

# **Desiderata:** (i) $Q_{P_A} < 2$ (precise) & (ii) Finite $P_A < 0$ (extracted)

**3-INGREDIENT RECIPE**

| | |
|---|---|
| **I.** | **TRS breaking** (relative phase shift in periodic drives) |
| ✗ | $\phi = 0$ |
| ✓ | $\phi = \pi/2$ |
| **II.** | **Non-Markovianity** (finite cut-off frequency) |
| ✗ | $\omega_c = 1000\omega_A$ |
| ✓ | $\omega_c = 1.2\omega_A$ |
| **III.** | **Synchronization** (strong coupling regime) |
| ✗ | $\gamma_2 = 0.01\omega_A$ |
| ✓ | $\gamma_2 = 100\omega_A$ |

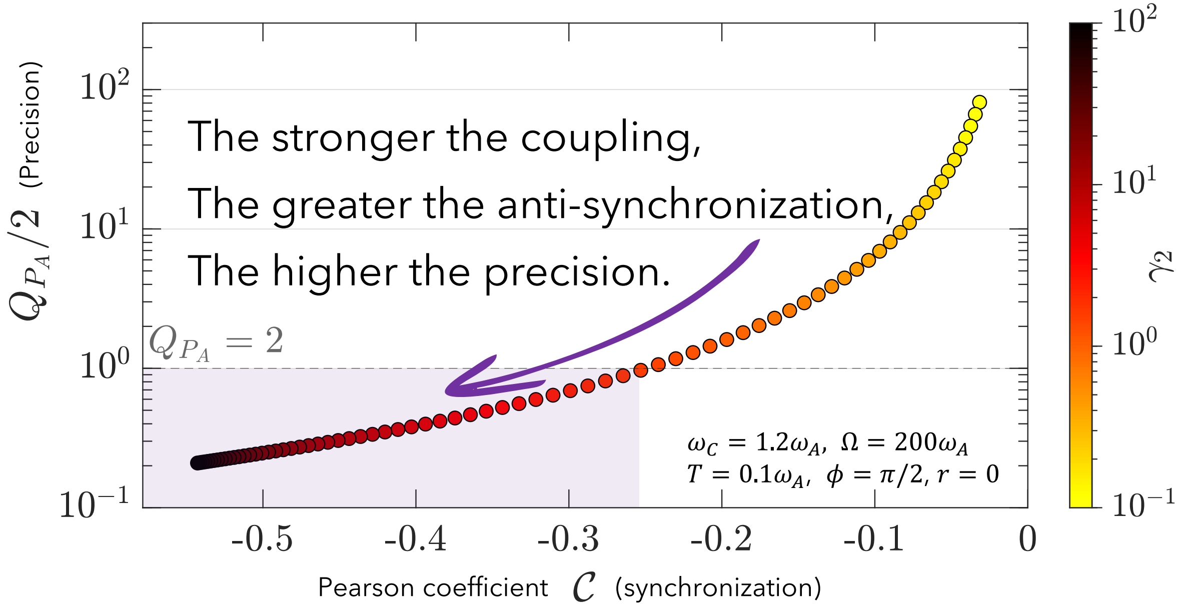



Analogous results with the TURs for periodic drives [PRL 122, 230601 (2019)] assuming weighted drives, $g_1^{(A)}(t) = \cos(\Omega t)$ , $g_1^{(B)}(t) = (\Omega/\omega_c)^r \cos(\Omega t + \phi)$.

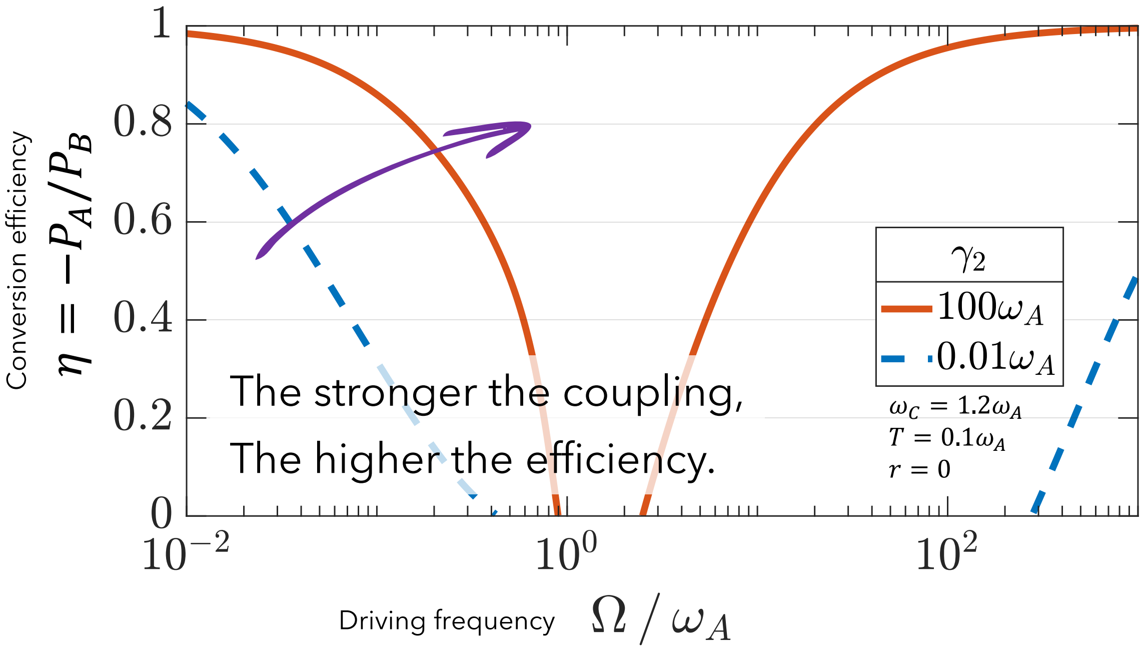
L. Razzoli et al.,
Quantum Sci. Technol. 9, 045032 (2024)

# SYNCHRONIZATION
# vs

## Precision
(Violation of classical TUR)




## Work-to-work conversion




L. Razzoli et al., Quantum Sci. Technol. 9, 045032 (2024)

# Take-home messages

- Two otherwise independent **QHOs synchronize** through coupling with a **common thermal reservoir**, at **strong coupling** and low temperature.

- When the QHOs are also dynamically coupled to a second thermal reservoir, this regime can be leveraged for thermodynamic tasks:
  - **Enhanced** heat engine **performance**[1]
  - Strong **TUR violation** for local powers[1,2]
  - **Efficient work-to-work conversion**[2]

*[1]Finite temperature gradient; [2]Isothermal regime.*

- **Outlook:** Further explore the connection between synchronization, strong coupling, and thermodynamics of precision within more complex (nonlinear) architectures for the WM.

*In collaboration with:*

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**Paolo A. Erdman**
Freie Universität Berlin, Germany

M. Carrega et al., PRX Quantum 3, 010323 (2022)

F. Cavaliere et al., Phys. Rev. Research 4, 033233 (2022)

F. Cavaliere et al., iScience 26, 106235 (2023)

L. Razzoli et al., Eur. Phys. J. Spec. Top. 233, 1263 (2024)

M. Carrega et al., **AVS Quantum Sci. 6, 025001 (2024)**

L. Razzoli et al., **Quantum Sci. Technol. 9, 045032 (2024)**

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