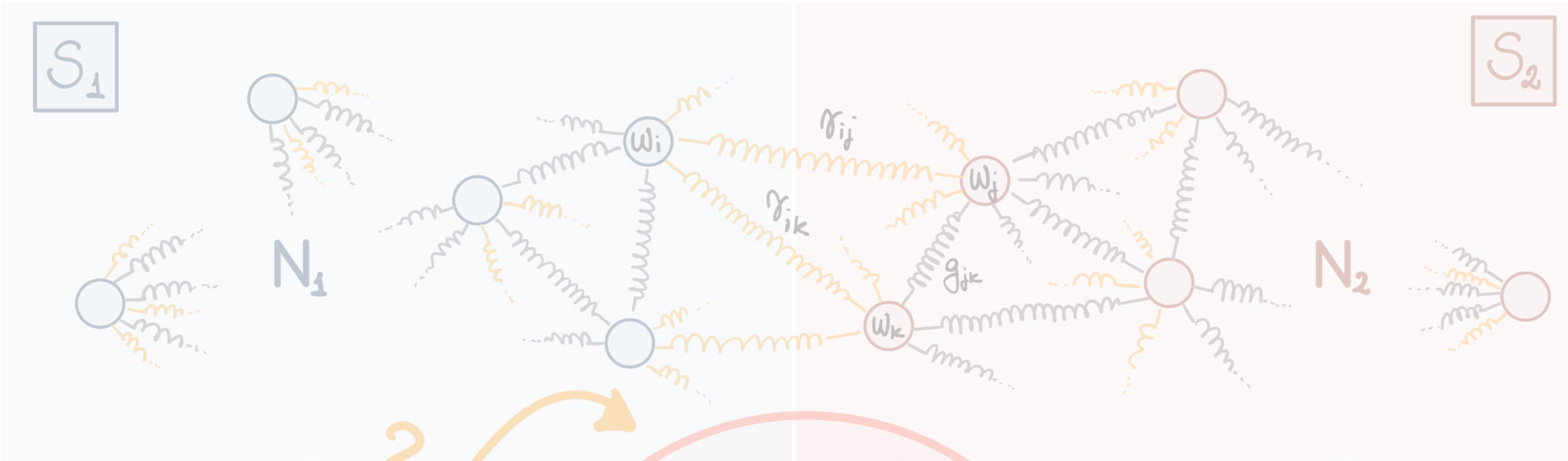

Local energy assignment for two interacting quantum thermal reservoirs

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UNIVERSITÀ DEGLI STUDI DI MILANO

Weak-coupling quantum thermodynamics

- Isolated system $\dot{\rho}_S(t) = -i[H_S, \rho_S(t)]$ $\Delta U_S(t) = 0$

- Work protocol (driving)

$$\dot{\rho}_S(t) = -i[H_S(t), \rho_S(t)]$$

$$\Delta U_S(t) = \underline{\underline{\delta W_S(t)}}$$

- Heating (bath)

$$\dot{\rho}_S(t) = -i[H_S(t), \rho_S(t)] + D_t[\rho_S(t)]$$

$$\Delta U_S(t) = \delta W_S(t) + \underline{\underline{\delta Q_S(t)}}$$

Heat and work definitions:

$$\delta W_S(t) = \int_0^t d\tau \, \mathrm{Tr}\{\dot{H}_S(\tau)\rho_S(\tau)\}$$

$$\delta Q_S(t) = \int_0^t d\tau \, \mathrm{Tr}\{H_S(\tau)\dot{\rho}_S(\tau)\}$$

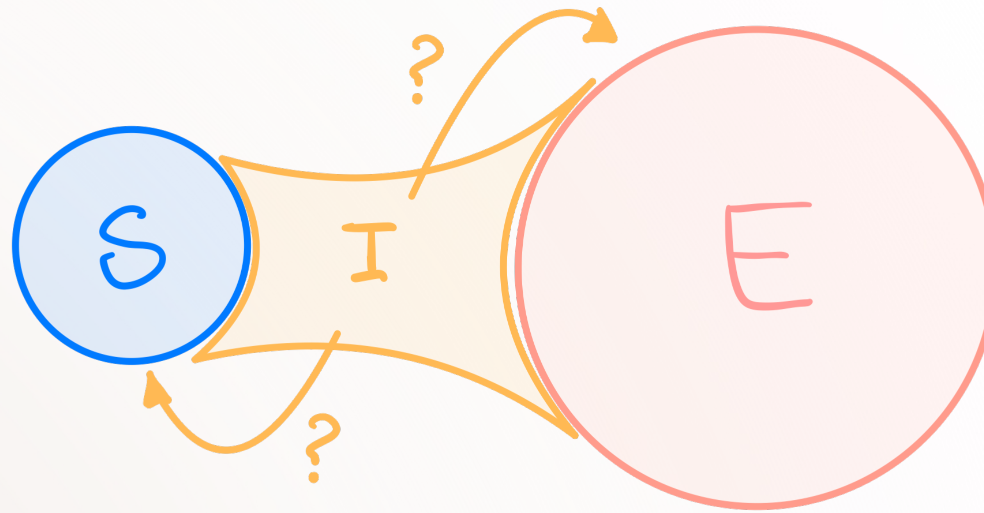
Effective modeling, relies on:
- Weak coupling
- Absence of memory
- Infinitely large thermal bath

Beyond:
- Intermediate to strong coupling
- Memory effects and finite size environments
- Coherent or non-thermal environments

Interaction energy assignment

- Microscopic modelling

$$H = H_S(t) + H_I + H_E$$



- Question of how to "assign" the interaction energy

$$X = S, E, I$$

$$\tilde{U}_X(t) = \mathrm{Tr}\{H_X(t)\varrho(t)\}$$

$$\delta\tilde{Q}_X(t) = \int_0^t d\tau\, \mathrm{Tr}\{H_X(\tau)\dot{\varrho}(\tau)\}$$

$$\delta\tilde{W}_X(t) = \int_0^t d\tau\, \mathrm{Tr}\{\dot{H}_X(\tau)\varrho(\tau)\}$$

Two popular sets of definitions

$$\begin{array}{ccccccc} \Delta\tilde{U}_S(t) & & \Delta\tilde{U}_I(t) & & \Delta\tilde{U}_E(t) & & \\ = & & = & & = & & \\ \delta\tilde{Q}_S(t) & + & \delta\tilde{Q}_I(t) & + & \delta\tilde{Q}_E(t) & = & 0 \\ + & & + & & + & & \\ \delta\tilde{W}_S(t) & & - & & - & & \end{array}$$

“Interaction” approach

$$\begin{aligned} \Delta U_S^{int}(t) &= \Delta\tilde{U}_S(t) + \underline{\Delta\tilde{U}_I(t)} \\ \delta Q_S^{int}(t) &= \delta\tilde{Q}_S(t) + \delta\tilde{Q}_I(t) \\ \delta W_S^{int}(t) &= \delta\tilde{W}_S(t) \end{aligned}$$

“Bare” approach

$$\begin{aligned} \Delta U_S^{bare}(t) &= \Delta\tilde{U}_S(t) \\ \delta Q_S^{bare}(t) &= \delta\tilde{Q}_S(t) + \delta\tilde{Q}_I(t) \\ \delta W_S^{bare}(t) &= \delta\tilde{W}_S(t) - \underline{\Delta\tilde{U}_I(t)} \end{aligned}$$

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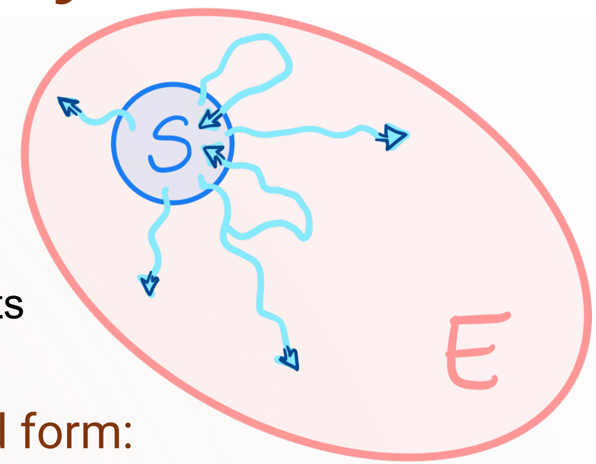
General open quantum systems

Exact generator for the open system

$$\dot{\rho}_S(t) = \dot{\Phi}_t \circ \Phi_t^{-1}[\rho_S(t)] =: \mathcal{L}_t[\rho_S(t)]$$



Describes information backflow and strong coupling effects

Time-local master equation in generalized Lindblad form:

$$\mathcal{L}_t[\rho_S(t)] = -i[\underline{\underline{K_S(t)}}, \rho_S(t)] + \mathcal{D}_t[\rho_S(t)]$$

$$\mathcal{D}_t[\rho_S(t)] = \sum_k \underbrace{\gamma_k(t)}_{\in \mathbb{R}} \left[L_k(t)\rho_S(t)L_k^\dagger(t) - \frac{1}{2}\left\{ L_k^\dagger(t)L_k(t), \rho_S(t) \right\} \right]$$

effective energy operator?

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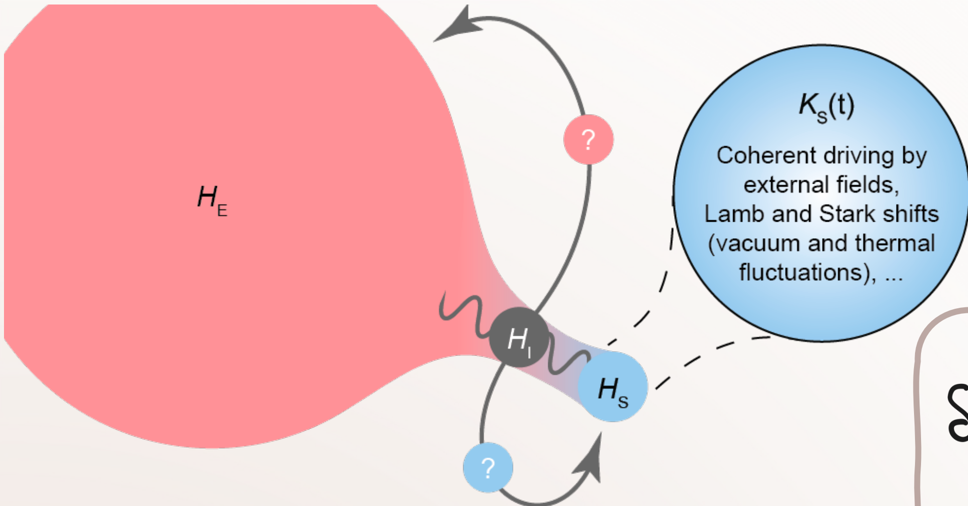
Minimal dissipation thermodynamics



emergent work protocols ←

"Minimal dissipation" approach

$$U_S^{md}(t) = \mathrm{Tr}\{K_S(t)\,\rho_S(t)\}$$

$$\delta W_S^{md}(t) = \int_0^t d\tau\, \mathrm{Tr}\{\dot{K}_S(\tau)\,\rho_S(\tau)\} \neq 0$$

even if $\underline{H_S(t) \equiv H_S}$

$$\delta Q_S^{md}(t) = \int_0^t d\tau\, \mathrm{Tr}\{K_S(\tau)\,\dot{\rho}_S(\tau)\} = \int_0^t d\tau\, \mathrm{Tr}\{K_S(\tau)\, D_t[\rho_S(t)]\}$$

- Applicable to both sides of the bipartition
- Environments can model work agents

A. Colla and H.-P. Breuer, Open-system approach to nonequilibrium quantum thermodynamics at arbitrary coupling, Phys. Rev. A **105**, 052216 (2022).

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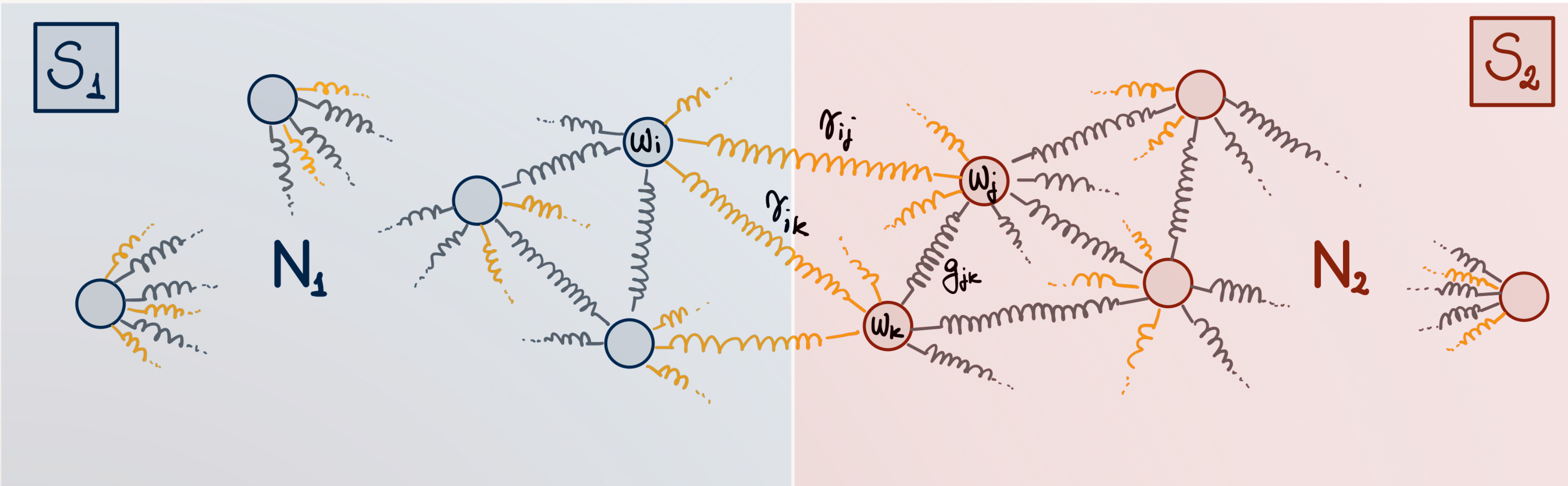
Interacting “heat baths”



$$H = \underbrace{\sum_{ij=1}^{N_1} h_{ij}^{(1)} a_i^{(1)\dagger} a_j^{(1)}}_{H_1} + \underbrace{\sum_{k\ell=1}^{N_2} h_{k\ell}^{(2)} a_k^{\dagger(2)} a_\ell^{(2)}}_{H_2} + \underbrace{\sum_{i=1}^{N_1} \sum_{k=1}^{N_2} \gamma_{ik} \left(a_i^{\dagger(1)} a_k^{(2)} + a_k^{\dagger(2)} a_i^{(1)} \right)}_{H_I}$$

A. Colla, B. Vacchini, and A. Smirne, Local energy assignment for two interacting quantum thermal reservoirs (2025), arXiv:2510.06929 [quant-ph].

Solve via exact diagonalization

Parameters and regimes

Hamiltonian parameters

- Main subsystem frequencies ω_1 , ω_2
- Intra-system coupling parameter g
- Inter-system coupling parameter γ

Leading quantities

- Leading subsystem eigenvalues $\nu_1 = \omega_1 + (N_1 - 1) g$
- Effective detuning $\Delta = \nu_1 - \nu_2$
- Effective coupling strength $\Gamma = 2\sqrt{N_1 N_2}\,\gamma$

Dispersive regime

$$\Delta \gg \Gamma$$

Ultrastrong coupling regime

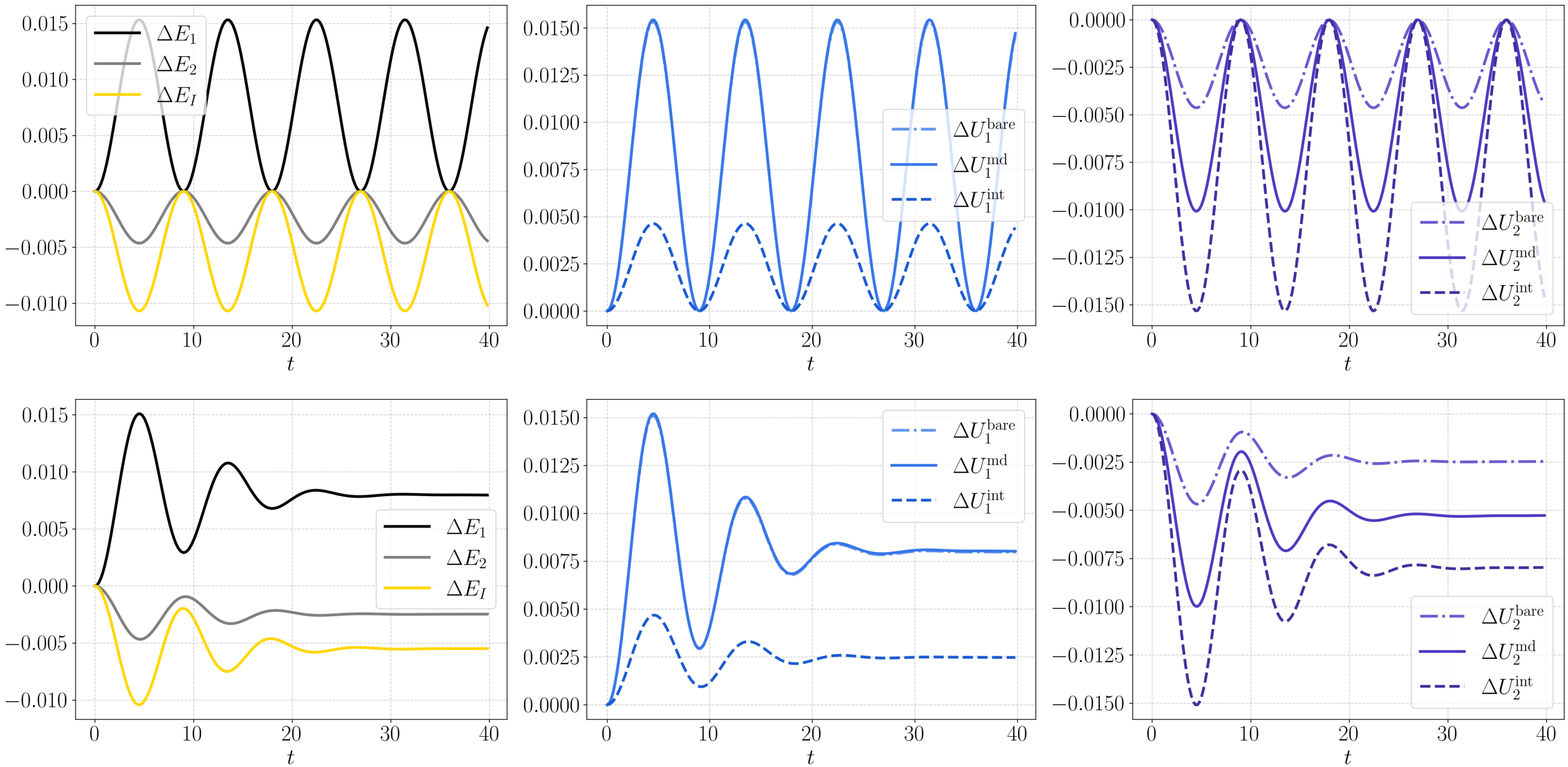
$$\Delta \ll \Gamma$$

Dispersive limit

Energies

Subsystem 1

Subsystem 2



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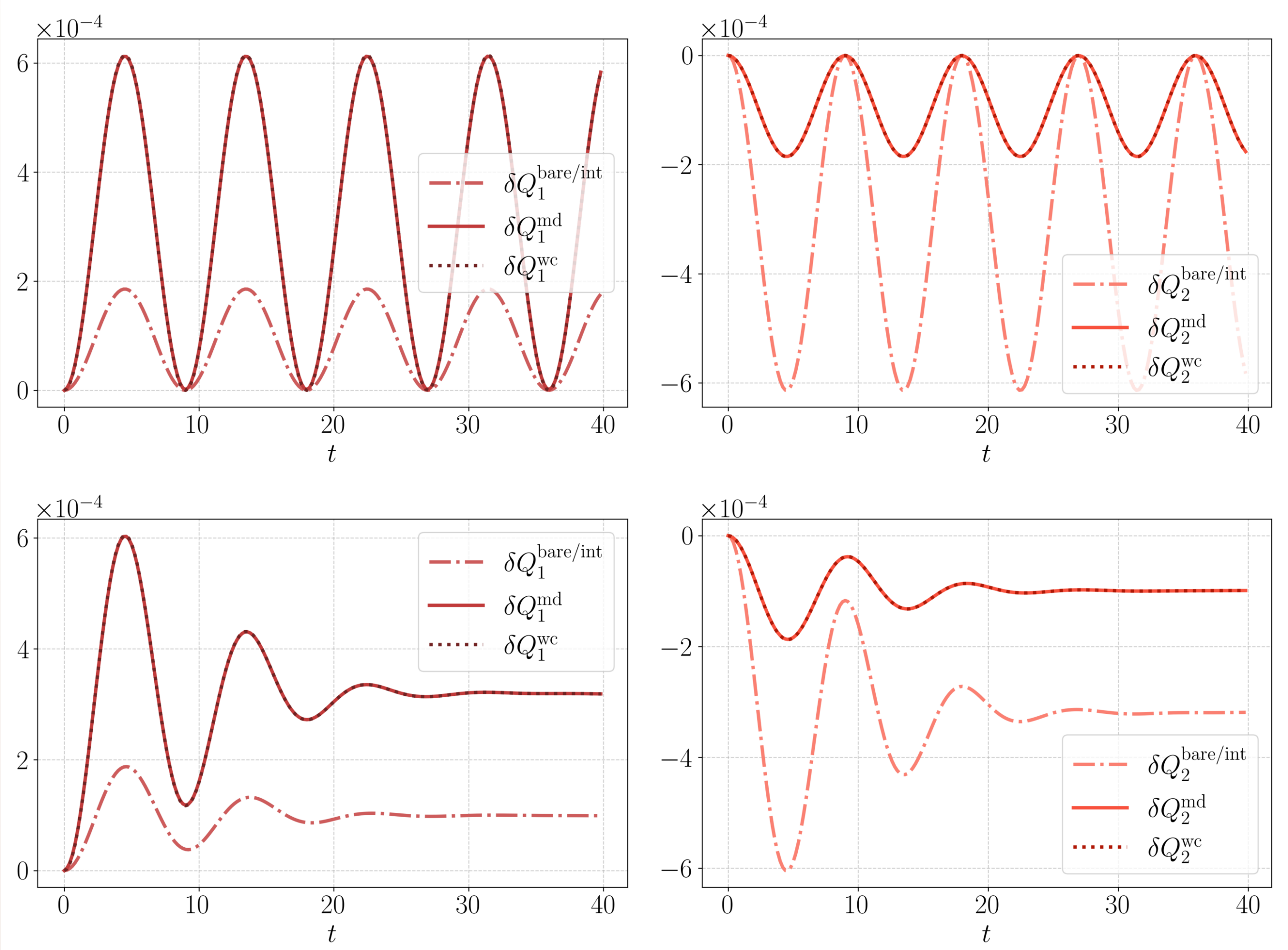
Dispersive limit

Subsystem 1 Subsystem 2



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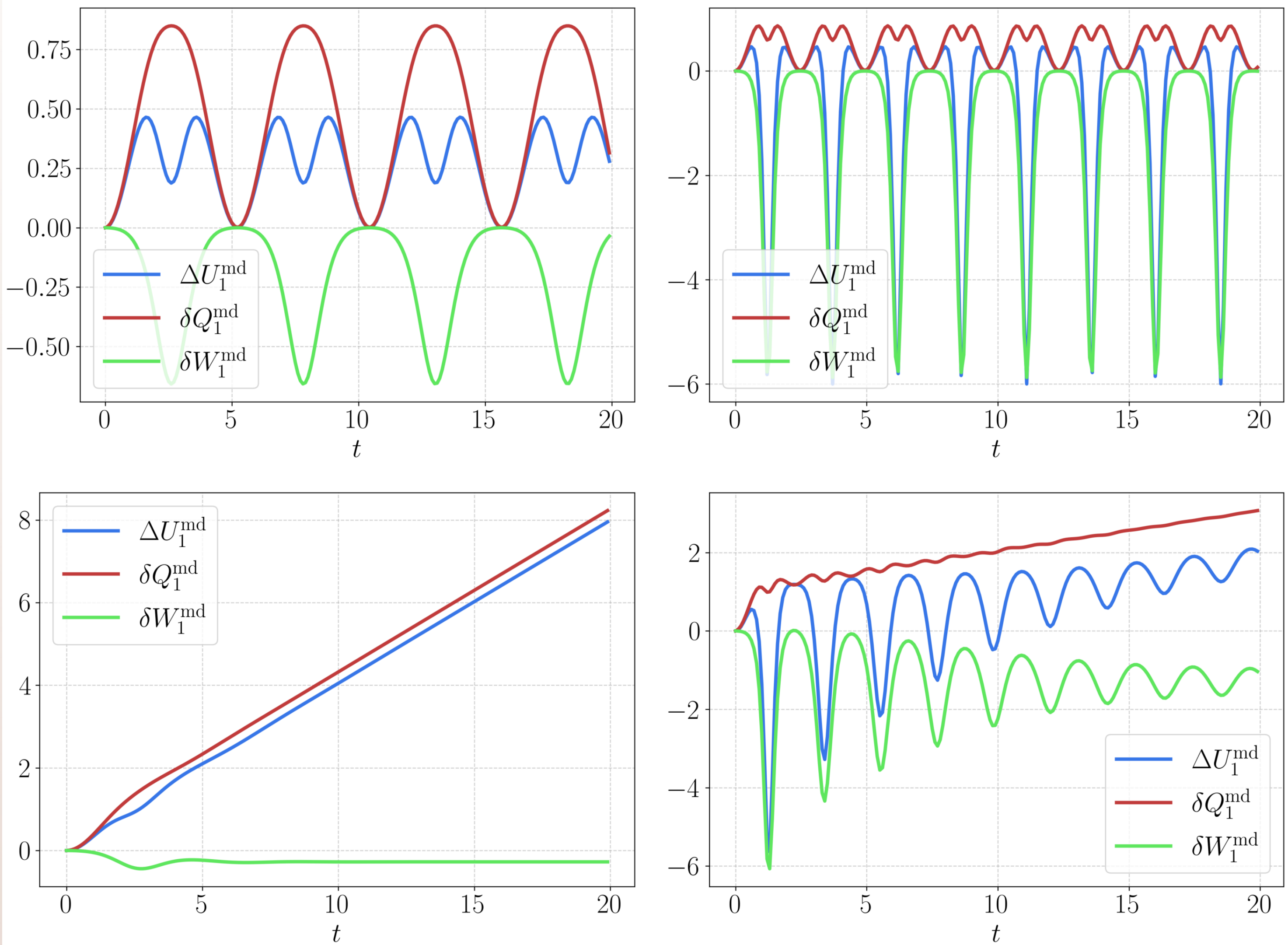
Minimal dissipation peaks

Strong coupling | Ultrastrong coupling



Recap & outlook

- Two principal set of definitions for first law quantities are incompatible → need for careful consideration
- Non-negligible role of interaction energy also in the dispersive limit
- Only minimal dissipation heat compatible with the weak-coupling limit
- Unbounded work peaks predicted in the minimal dissipation definitions → exploitable?
- Entropic aspects and the second law?
- Appropriate macroscopic limit?

thank you!