ICE-10

Valencia, 20th October 2025

Entanglement detection with imprecise measurements

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Entanglement

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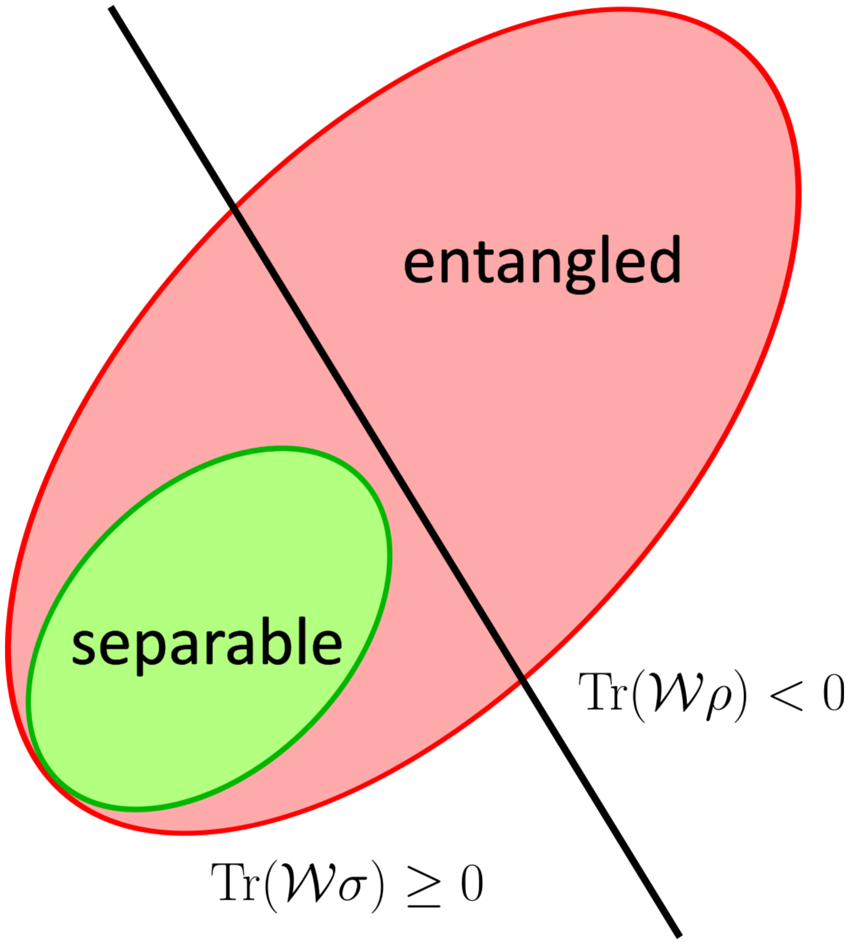

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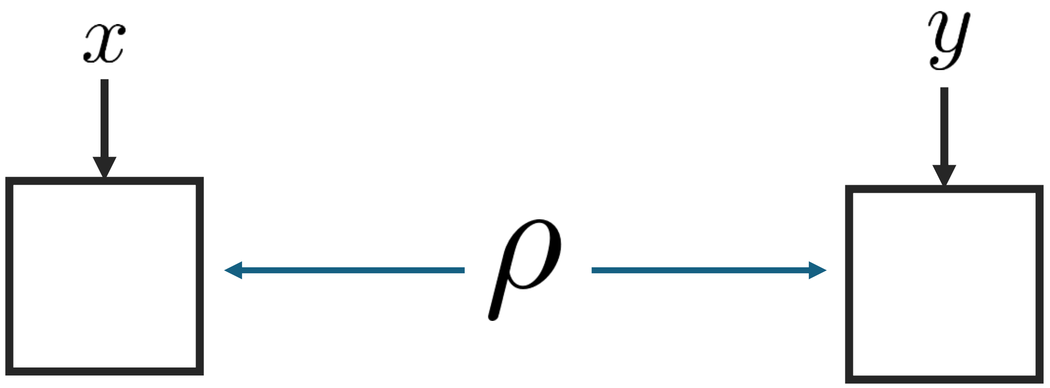

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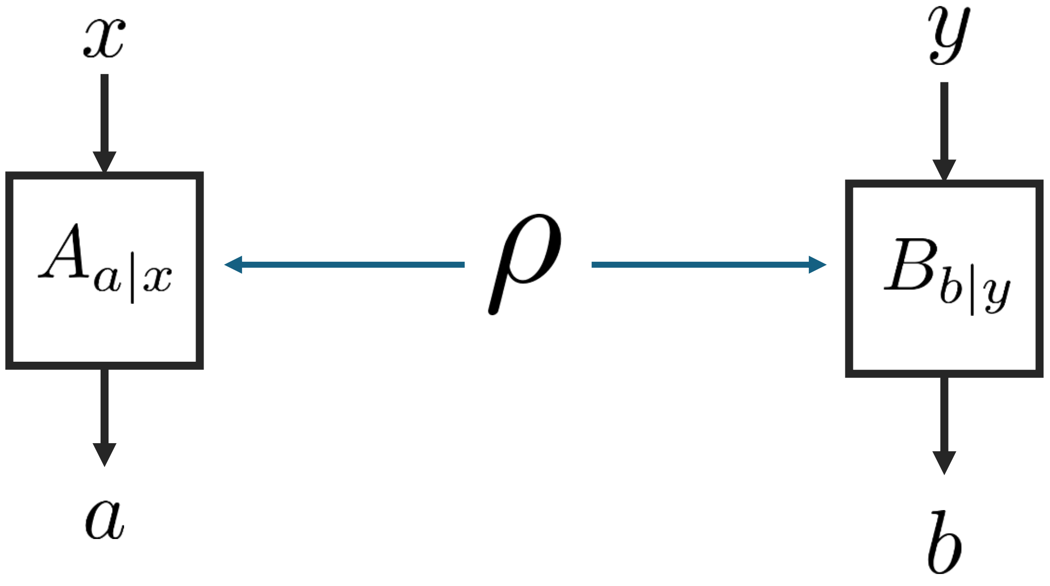

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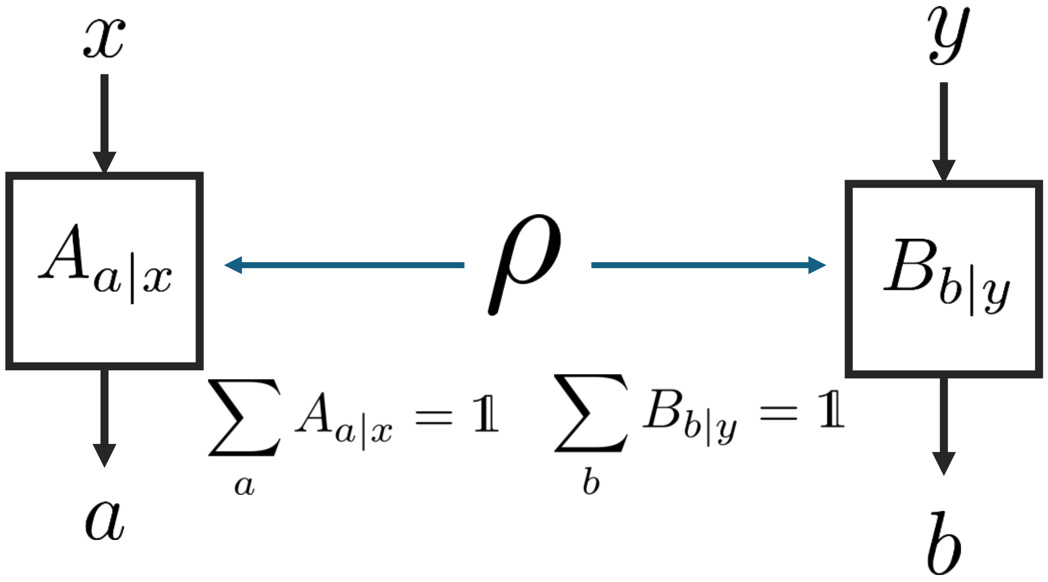

$$\sum_a A_{a|x} = \mathbb{1} \qquad \sum_b B_{b|y} = \mathbb{1}$$

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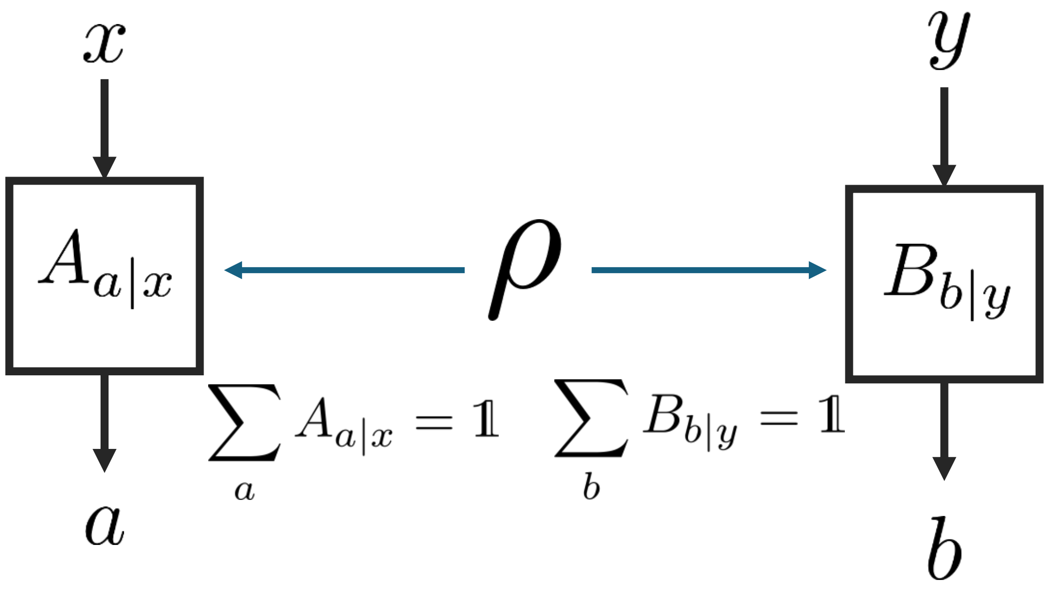
$$p(ab|xy) = \mathrm{Tr}(A_{a|x} \otimes B_{b|y}\ \rho)$$



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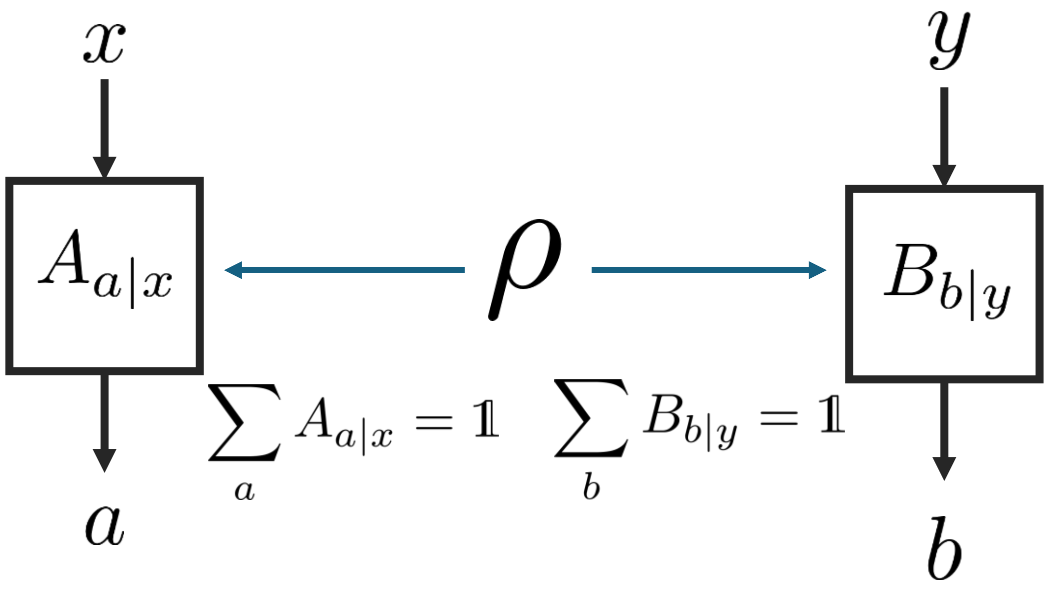

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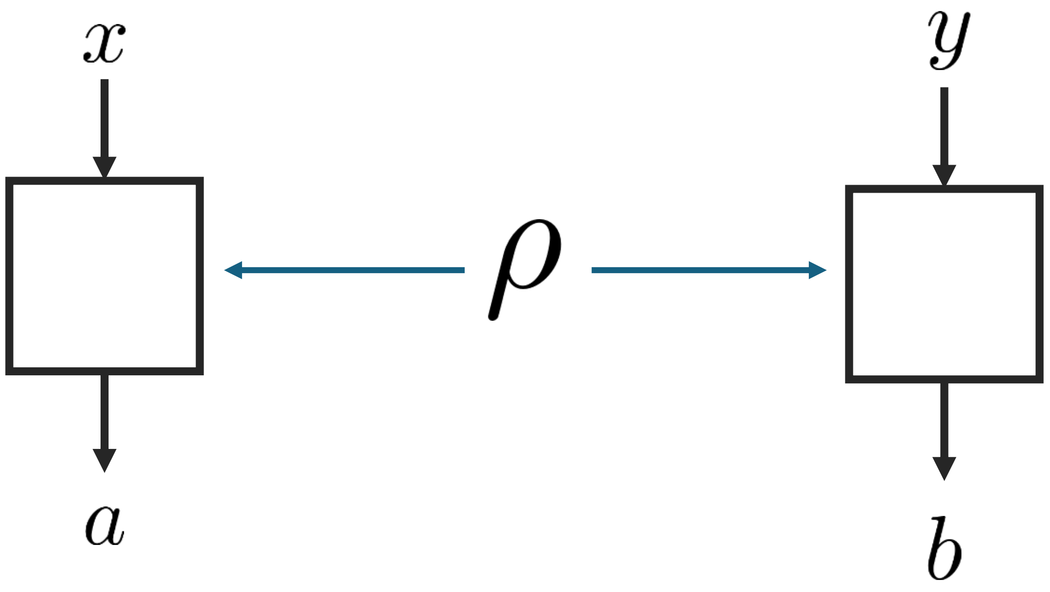

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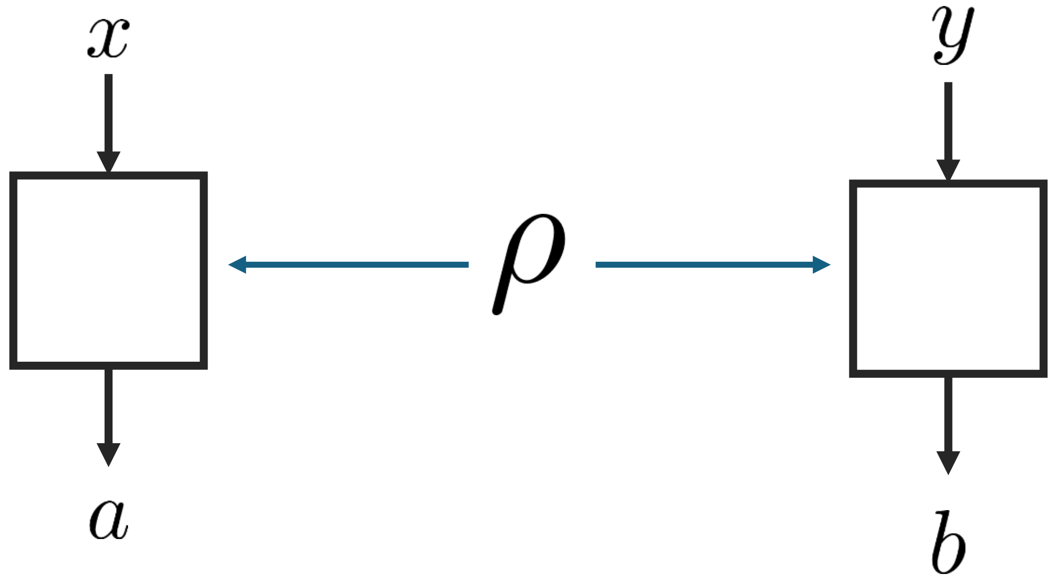

detect entanglement

if all states generating the outcome statistic for the performed measurements are entangled

observe nonlocality

if all states generating the outcome statistic with any arbitrary measurements are entangled

ED with bounded distrust

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Operational notion of
measurement inaccuracy

ED with bounded distrust

Operational notion of measurement inaccuracy

$$\mathcal{F}_x^A = \frac{1}{d}\sum_{a=1}^{d} \mathrm{Tr}(A_{a|x} A^{\varepsilon}_{a|x}) \geq 1 - \varepsilon_x^A$$

S. M.*, H. Yamasaki*, M. Huber, A. Tavakoli, PRL 128, 250501 (2022)

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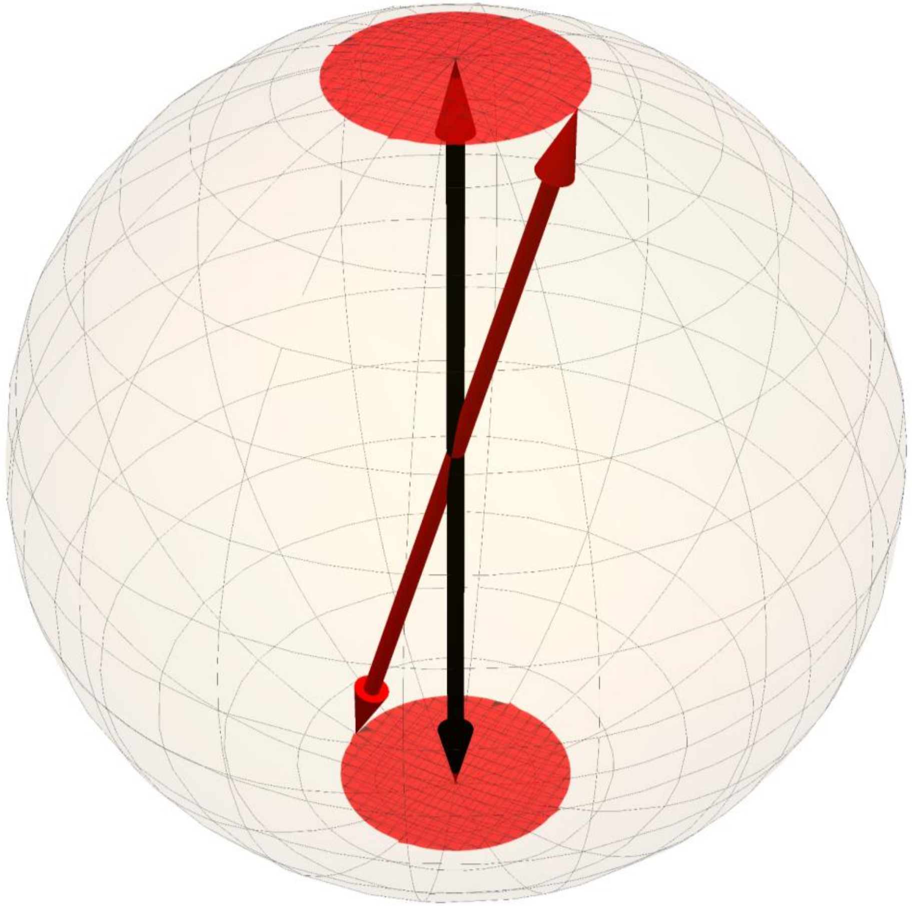

S. M.*, H. Yamasaki*, M. Huber, A. Tavakoli, PRL 128, 250501 (2022)

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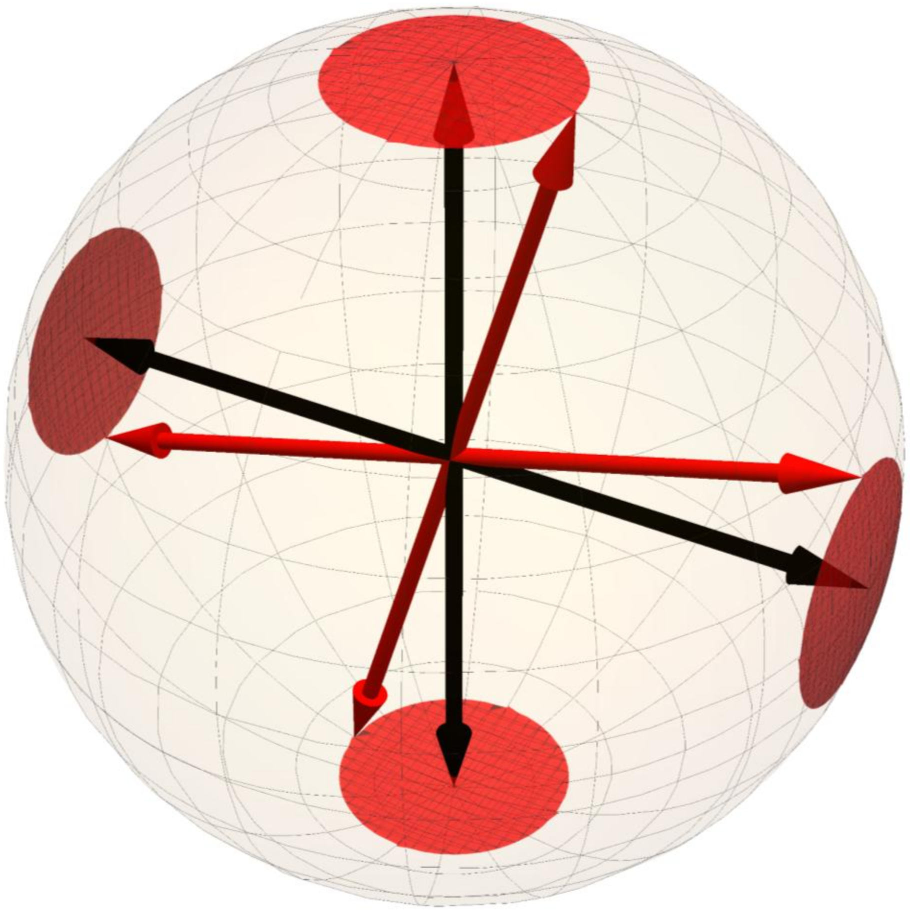

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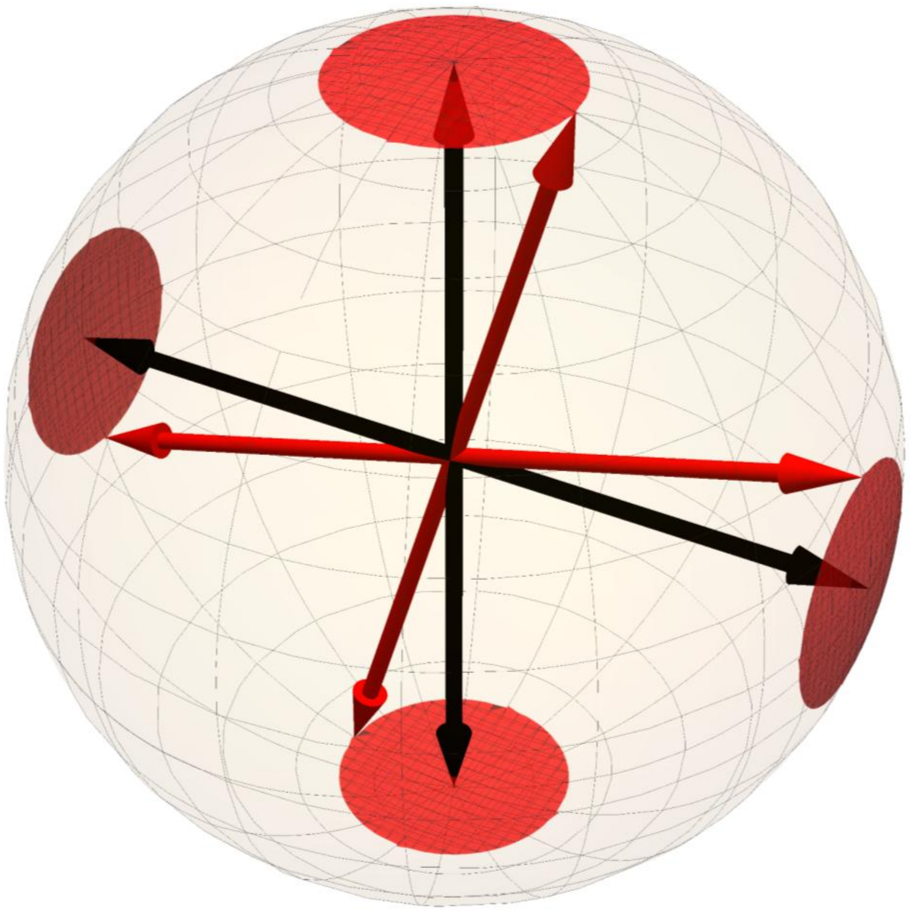
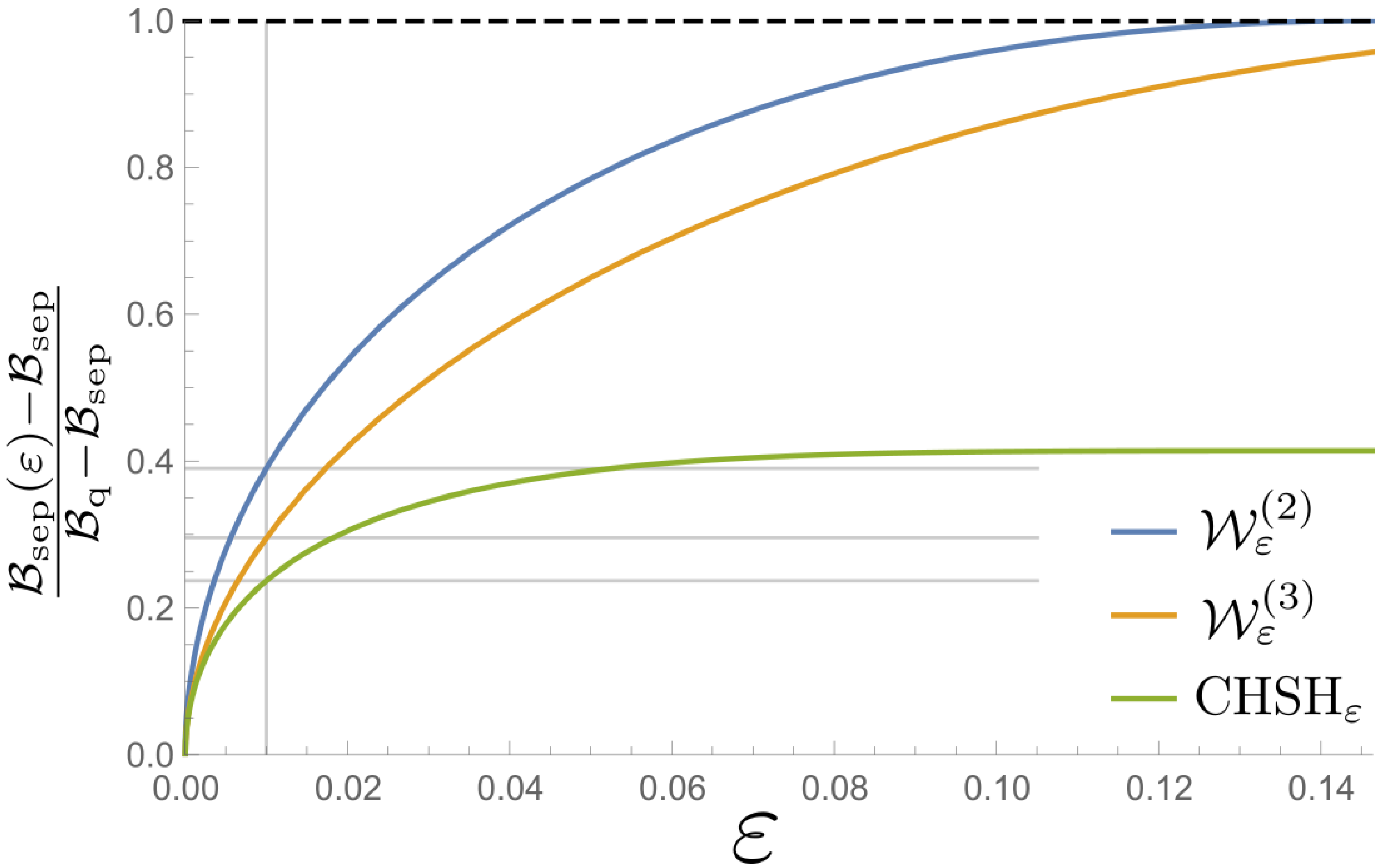
$$\langle \mathcal{W}^{(2)}_{\varepsilon} \rangle_{\rho_{\text{sep}}} \leq 1 + 4(1-2\varepsilon)\sqrt{\varepsilon(1-\varepsilon)} = \mathcal{B}_{\text{sep}}(\varepsilon)$$



Impact of inaccuracies



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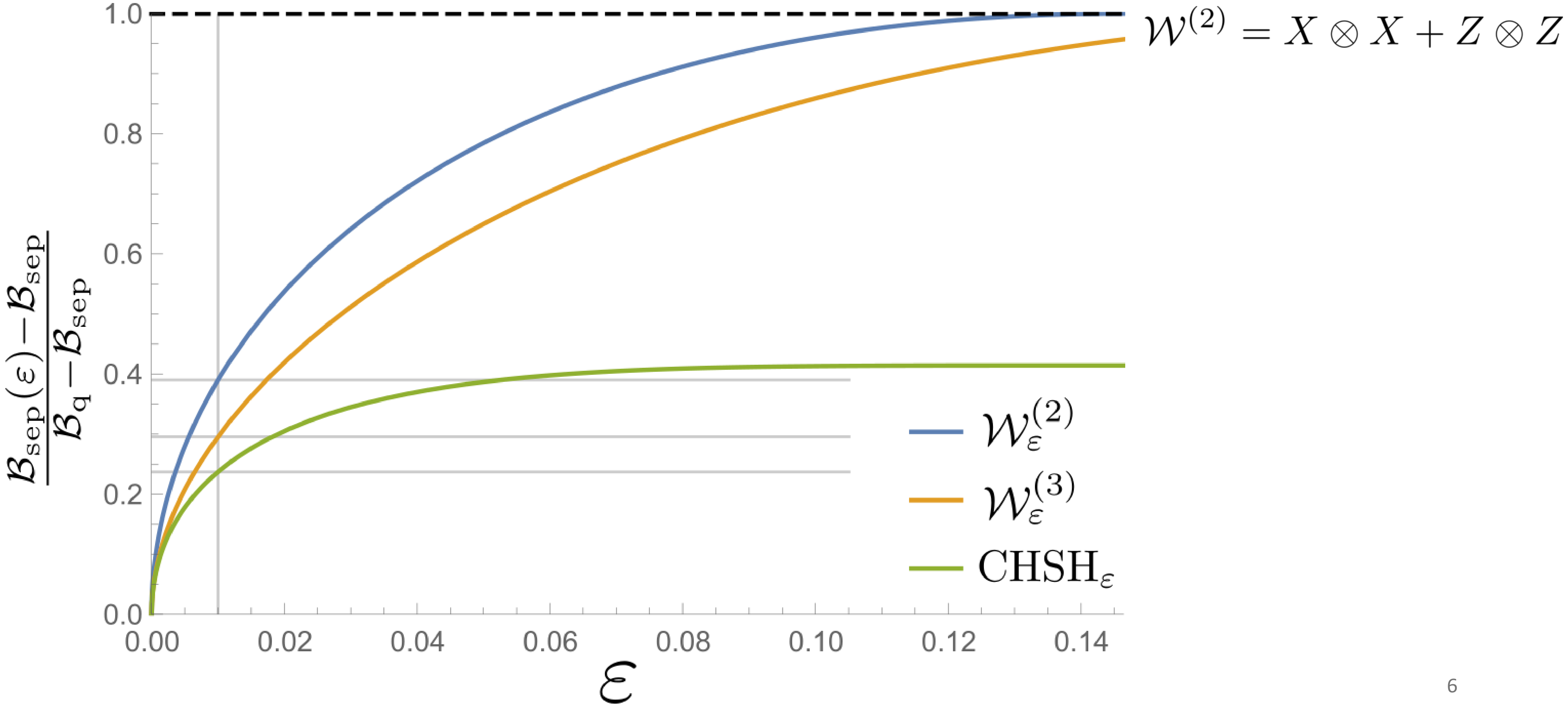

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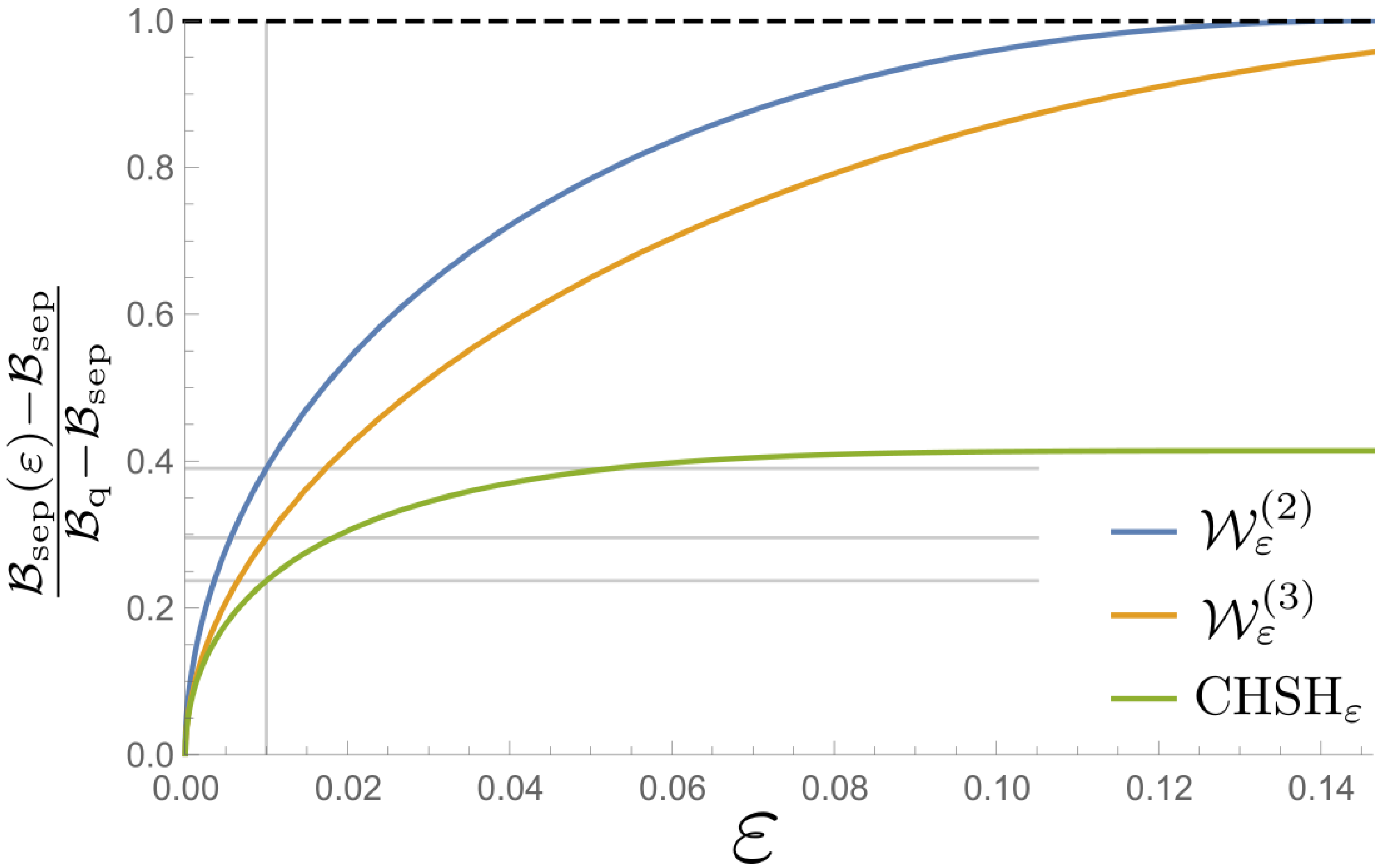

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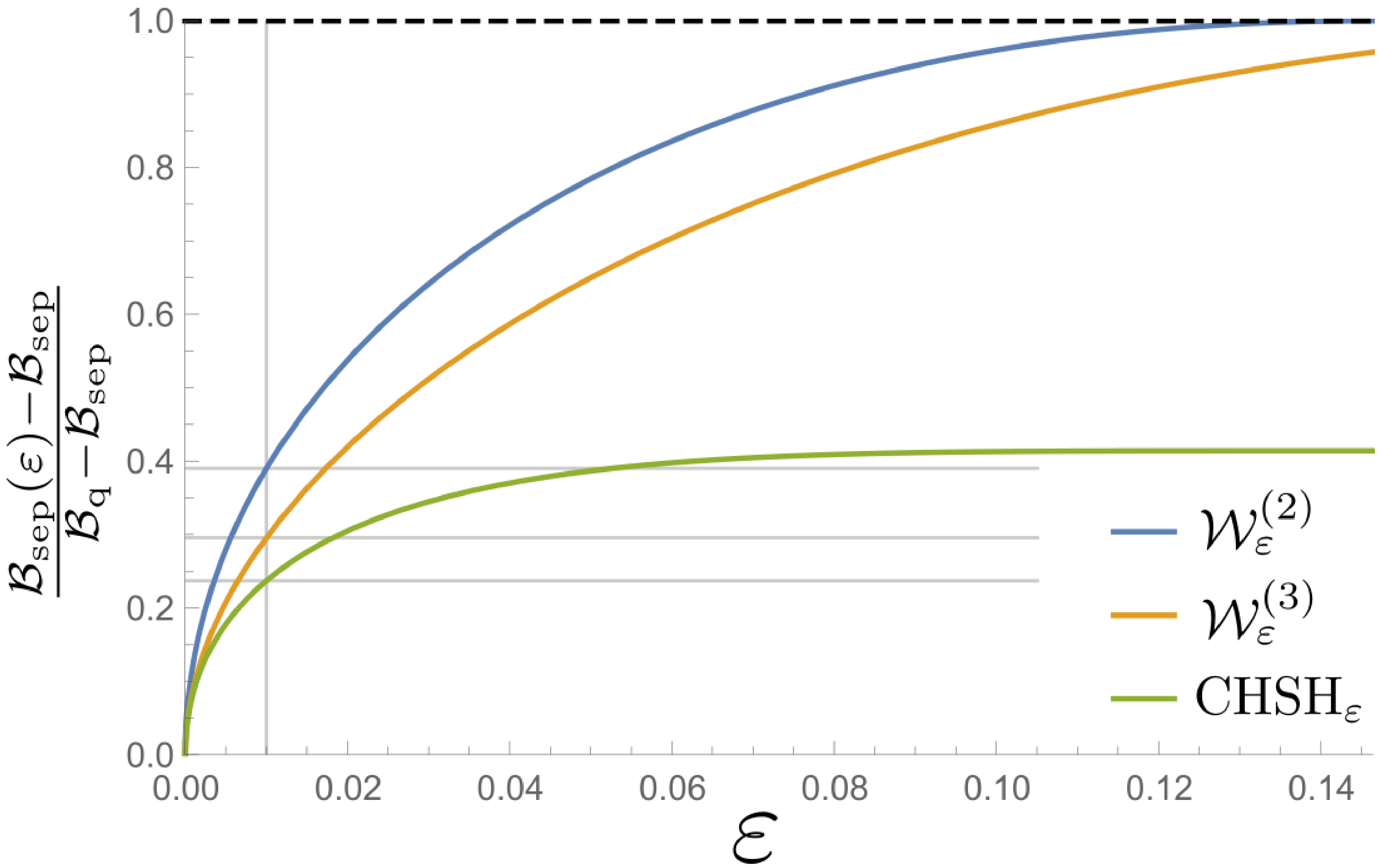

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$$\mathrm{CHSH} = X \otimes X + X \otimes Z + Z \otimes X - Z \otimes Z$$

Advantages of ED with inaccuracies

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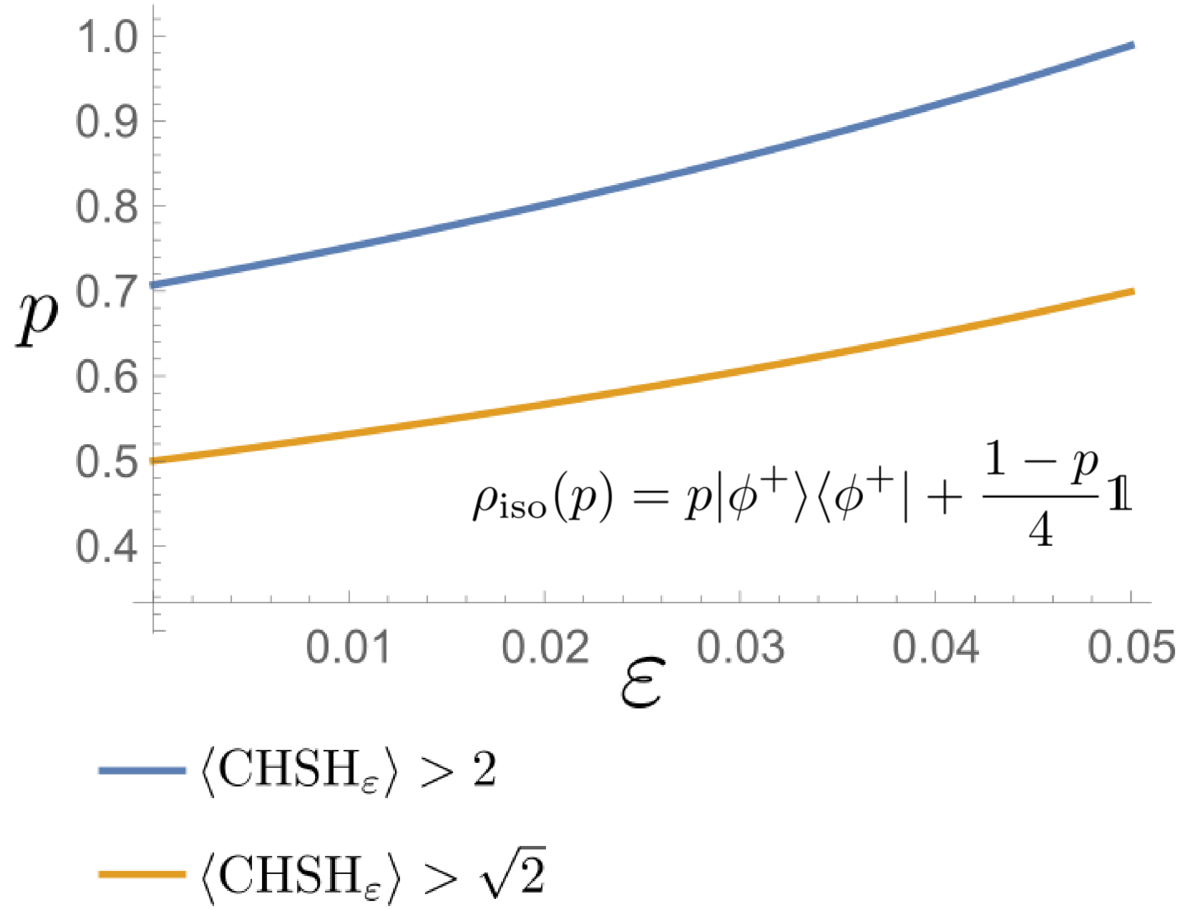
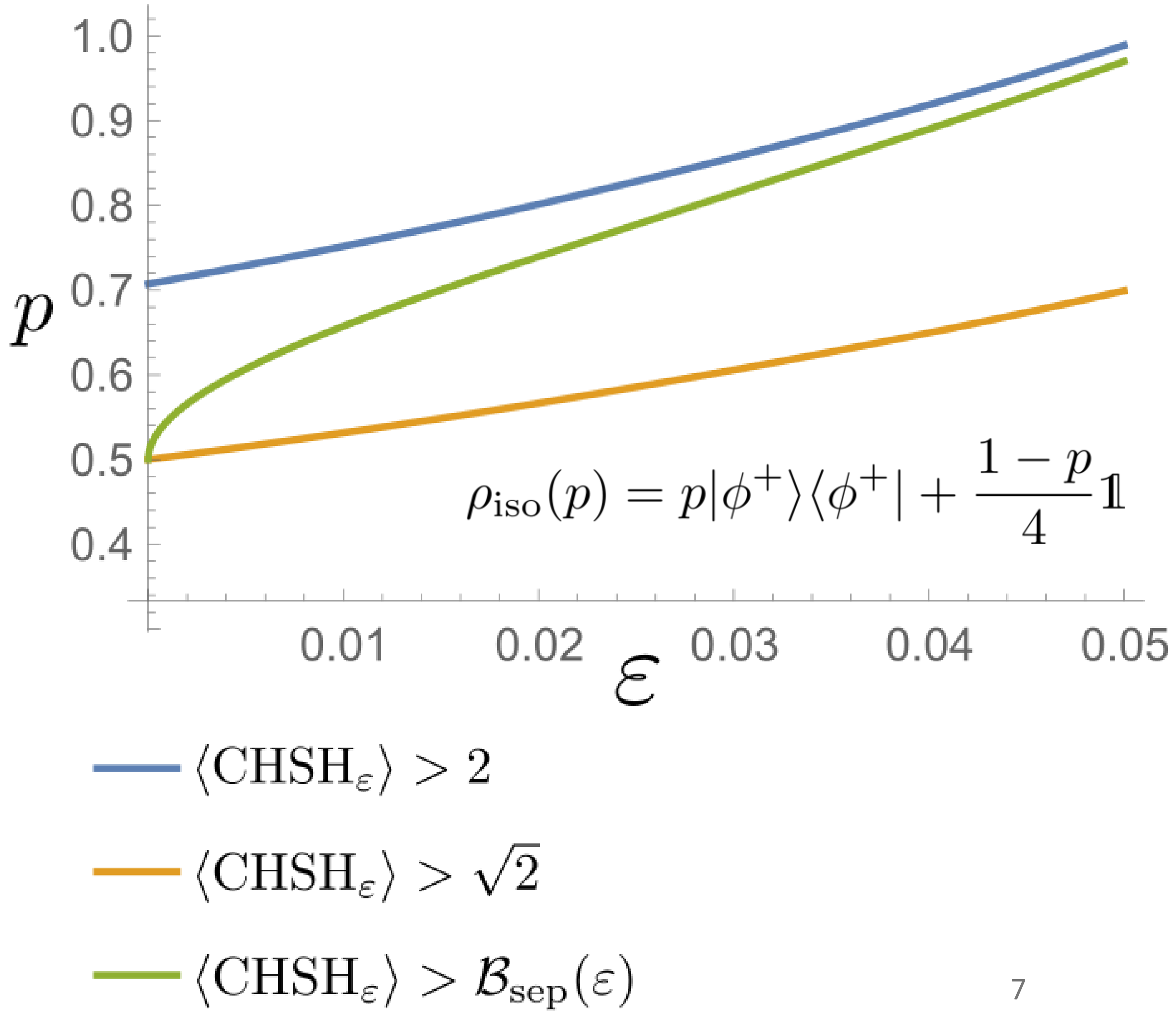


Advantages of ED with inaccuracies

- Account for inaccuracies using information on the measurements



Advantages of ED with inaccuracies

- Account for inaccuracies using information on the measurements
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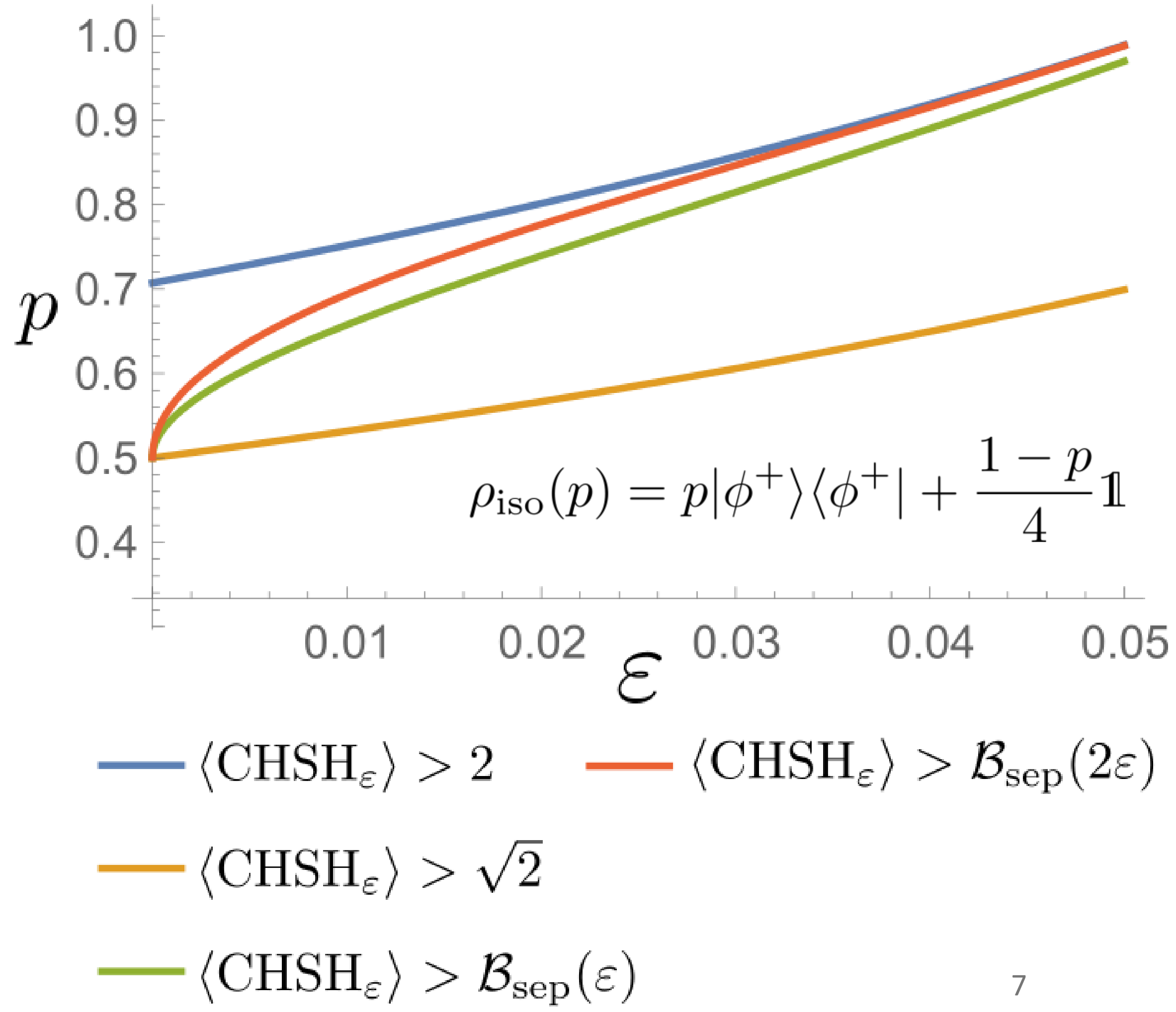

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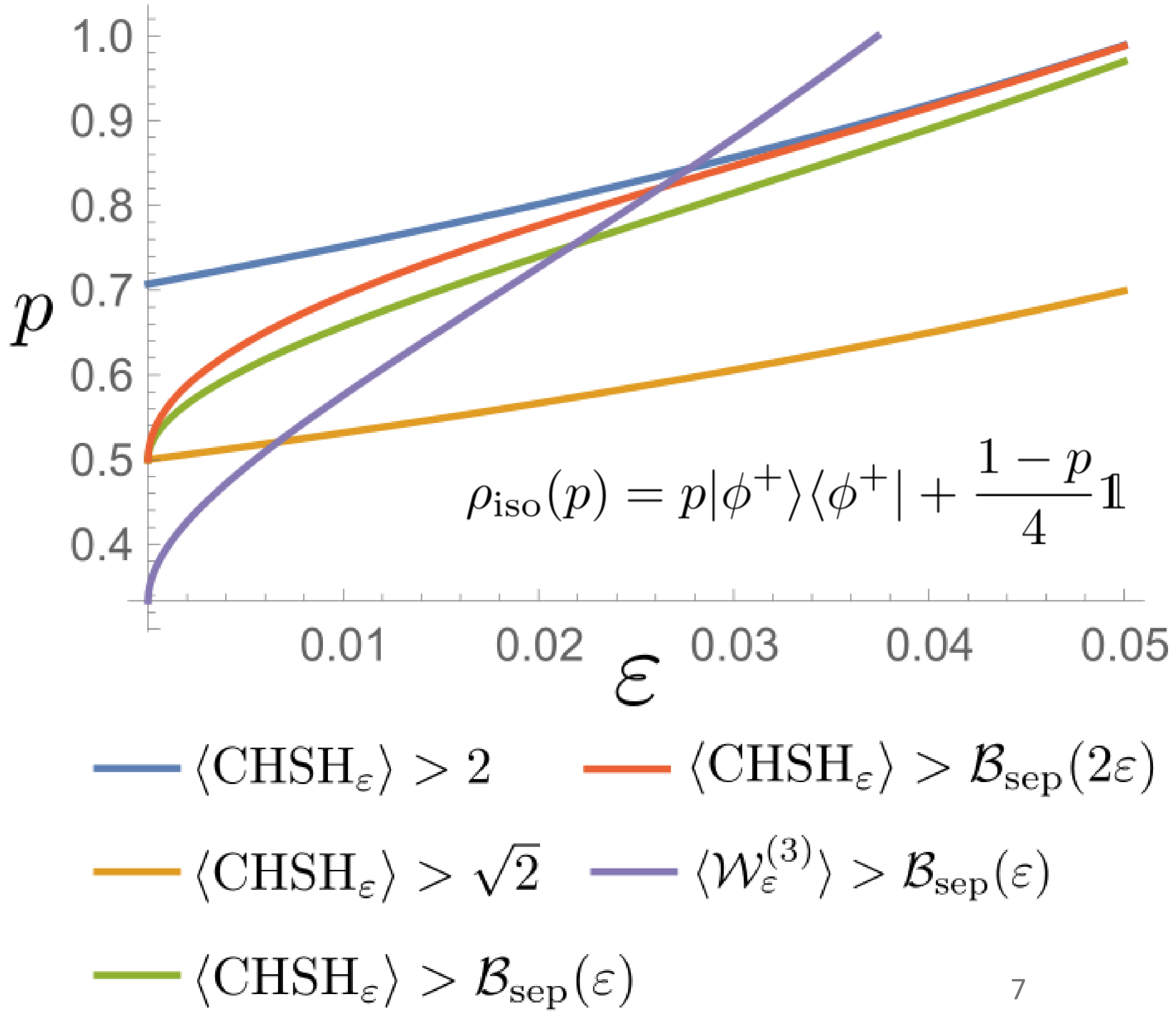

Advantages of ED with inaccuracies

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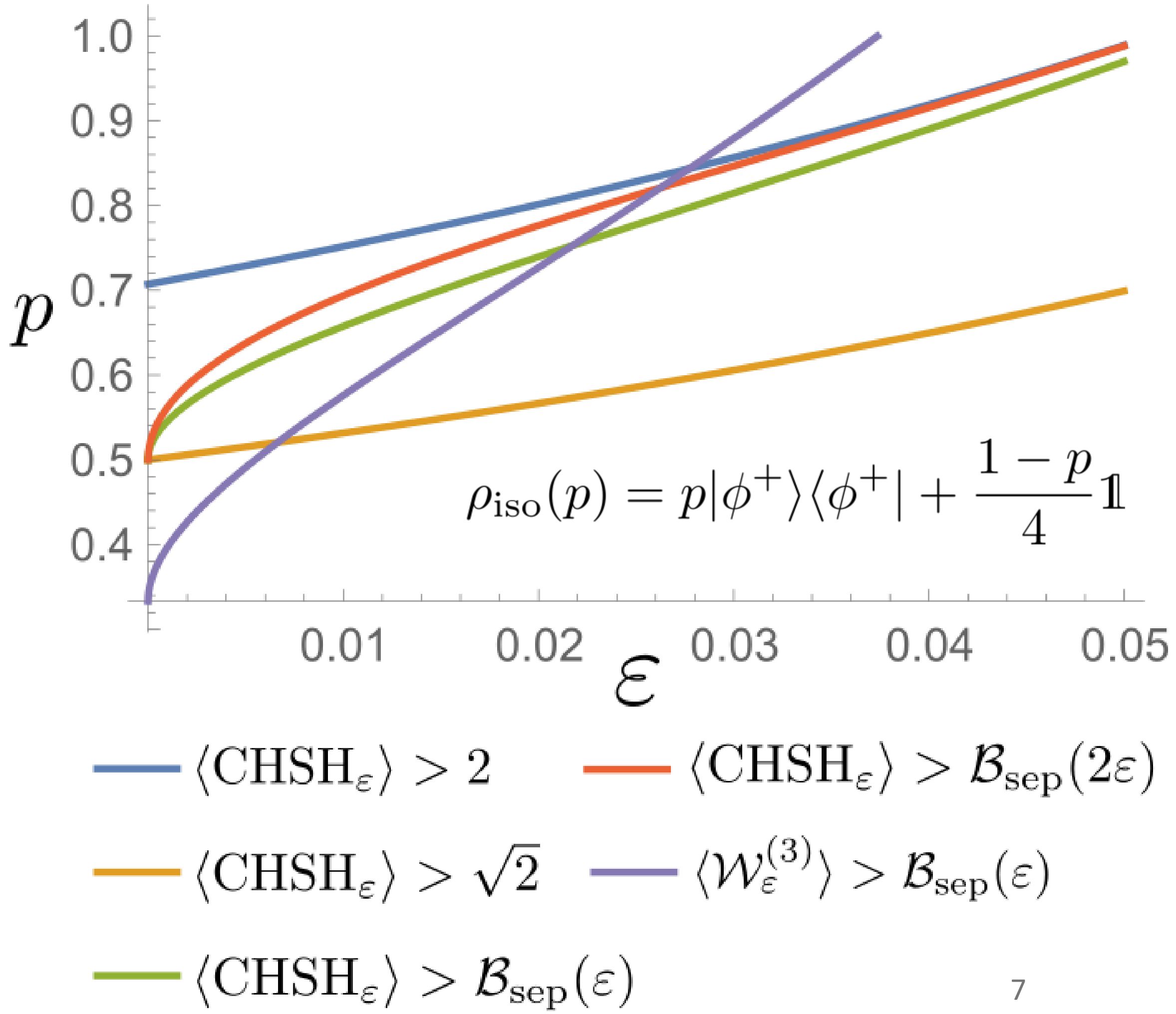
- Choose the level of trust in the set-up
- Known witnesses can be used
- Generalization to higher dimensions and more parties



Multipartite ED

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H. Cao*, **S. M.*,** L. A. Rozema, C. Zhang, A. Tavakoli, P. Walther, PRL 133, 150201 (2024)

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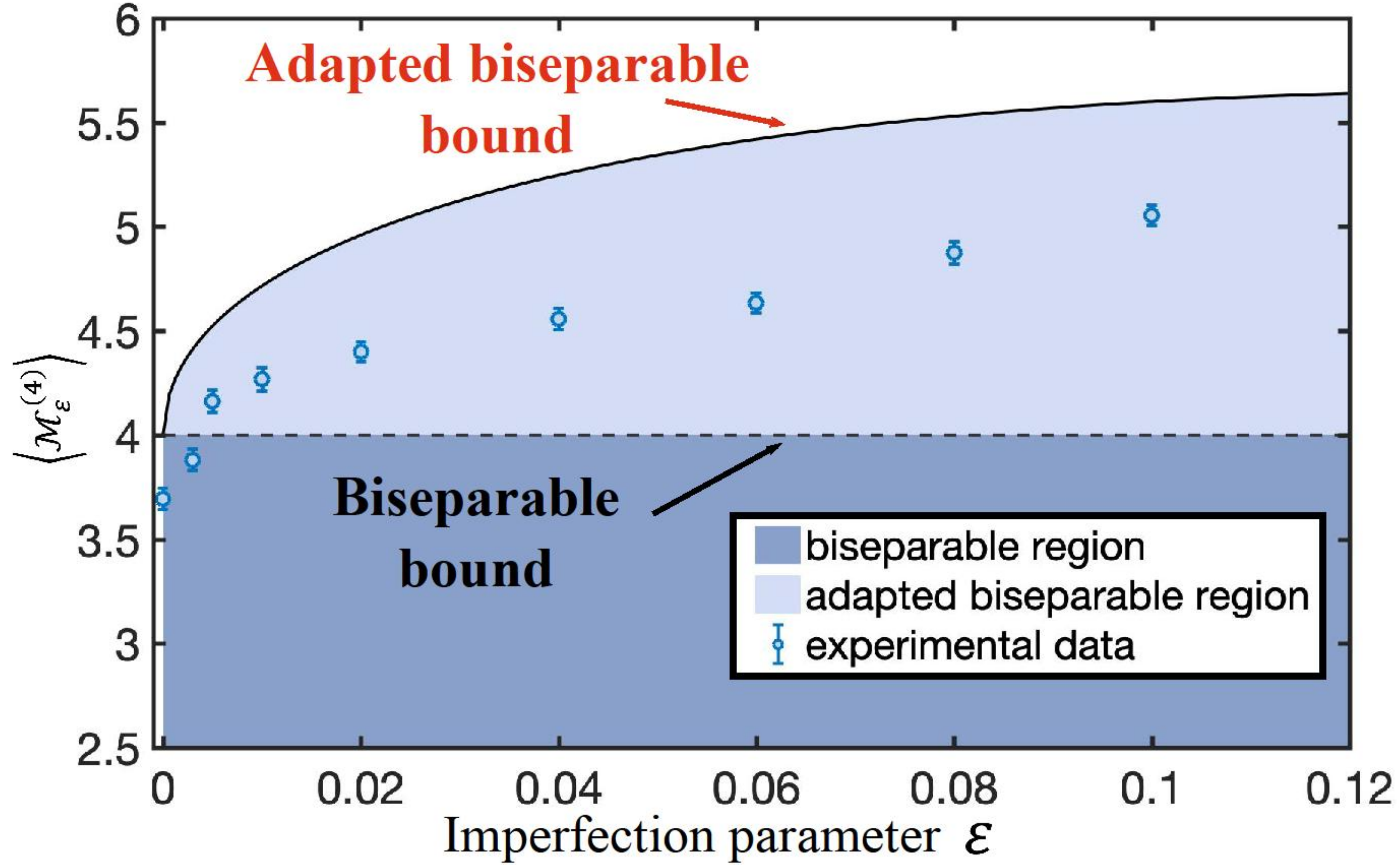

Mermin witness

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High-dimensional ED

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Measuring m mutually unbiased bases, states of Schmidt number r satisfy

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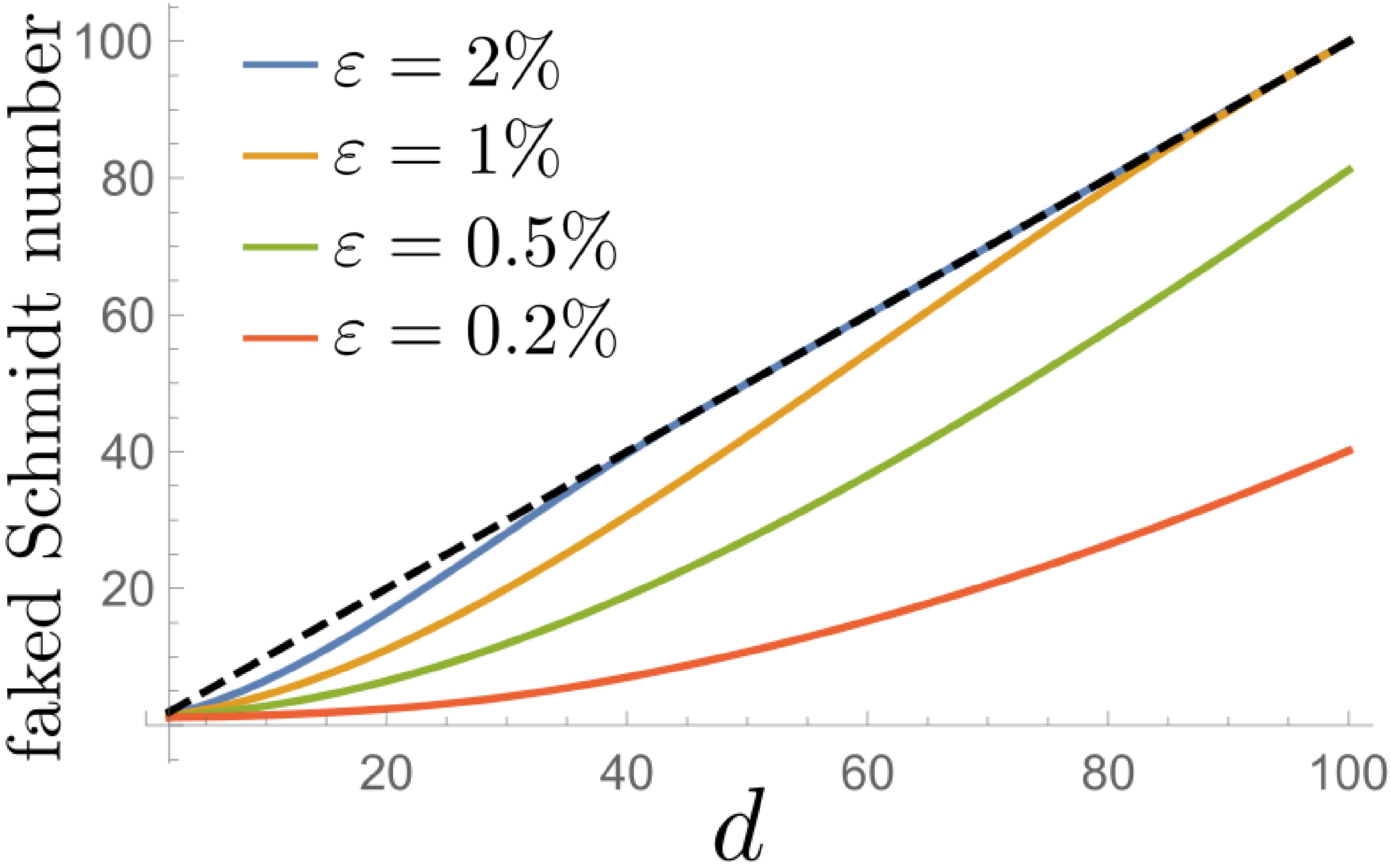
S. M., M. Huber, A. Tavakoli, PRL 131, 170201 (2023)

High-dimensional ED



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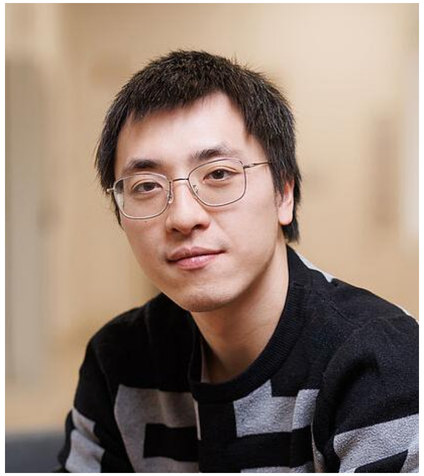
- Operational definition of measurement inaccuracy
- Imprecisions can highly impact the validity of witnesses
- Adjust separable bounds to the inaccuracy
- Extend to multiple parties and higher dimensions

Armin Tavakoli

Hayata Yamasaki

Marcus Huber



Huan Cao & Philip Walther group

Robert Fickler & team

Thank you!

(bcam) basque center for applied mathematics

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FFG Forschung wirkt.