Valencia Workshop on the Small-Scale Structure of the Universe and Self-Interacting Dark Matter

Particle Production from Metric Perturbations

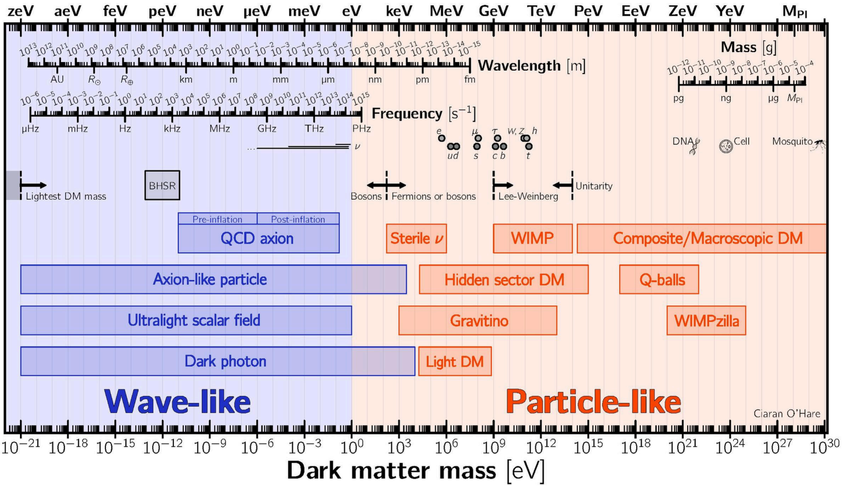
Raghuveer Garani

Based on *Phys.Rev.Lett.* 134 (2025) 10, 101005 and 2502.12249 [hep-th]

with Michele Redi and Andrea Tesi

What is dark matter?

Plentitude of candidates but mass scale unknown!



Credit: Ciaran O'Hare

What is dark matter?

How dark can dark matter be?

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How dark can dark matter be?

- All evidences $\rightarrow$ only gravitational interactions!

How to gravitationally produce particles?

All evidences for DM inferred from gravitational dynamics

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  Particle production from inhomogeneities: general metric perturbations RG, Redi, Tesi, 2502.12249 [hep-th]

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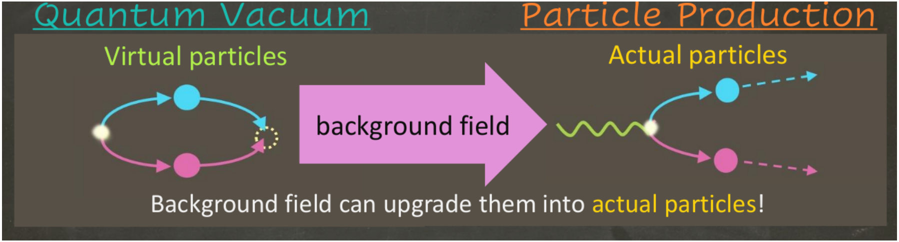
The basic idea

Schwinger



Credit: Azadeh Maleknejad

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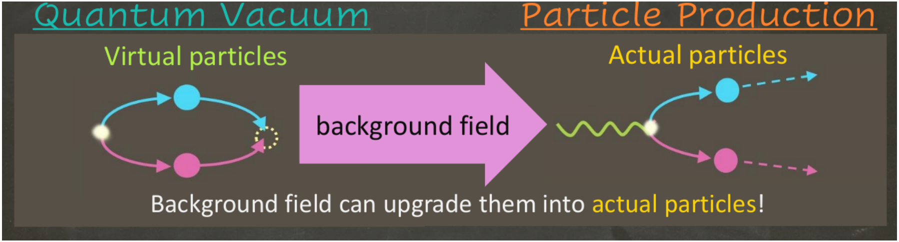

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- Rate $\propto \alpha E^2 e^{-\frac{m^2}{eE}}$

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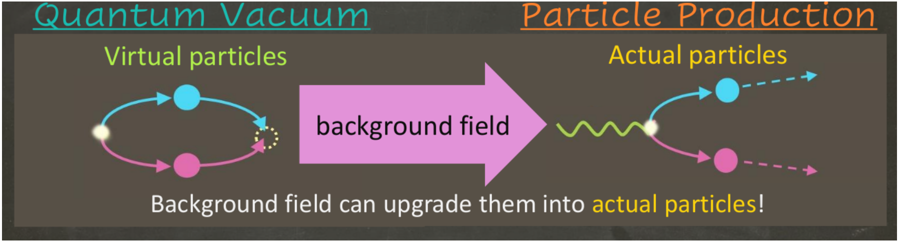

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- Rate $\propto \alpha E^2 e^{-\frac{m^2}{eE}}$
- If work done by Lorentz force: $eE\lambda_{\text{compton}} = m_e$
  $\implies$ Electron pairs created for $E \sim m_e^2 \approx 10^{18}$ V/m

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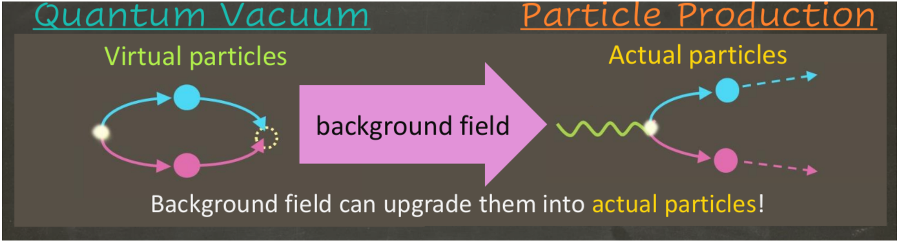

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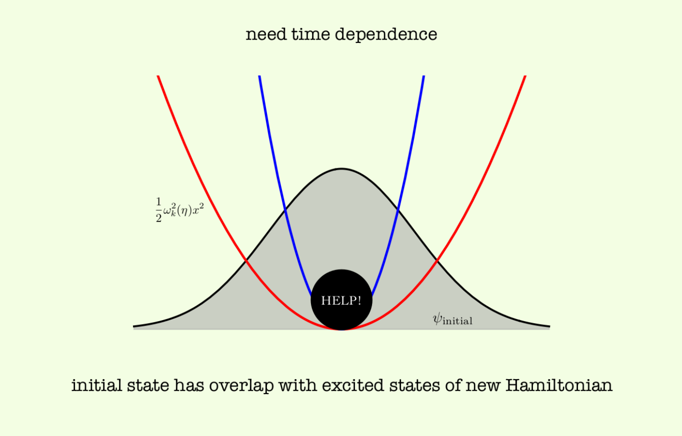
- External field $\to h_{\mu\nu}(\tau, x)$ in expanding Universe

The basic idea

Building intuition: The harmonic oscillator



Credit: Andrea Tesi

The basic idea

Building intuition: The harmonic oscillator

- $\hat{H}(\tau) = \frac{1}{2}\hat{p}^2 + \frac{1}{2}\omega^2(\tau)\hat{x}^2$ $\qquad \hat{x} = \nu(\tau)a + \nu(\tau)a^\dagger$
- At all times $\ddot{\nu} + \omega^2(\tau)\nu = 0$ $\qquad \dot{\nu}\nu^\star - \dot{\nu}^\star\nu = -i$
- Bunch-Davis $a|0\rangle = 0$ and $\qquad \nu_0 = \frac{1}{\sqrt{2\omega}}e^{-i\omega\tau}$
- $\tau \to \infty$

$$\nu = \frac{\alpha(\tau)}{\sqrt{2\omega(\tau)}}e^{-i\omega\tau} + \frac{\beta(\tau)}{\sqrt{2\omega(\tau)}}e^{i\omega\tau}$$

Parker '69; Ford '76

The basic idea

Building intuition: The harmonic oscillator

- Bogoliubov

$$\begin{pmatrix} a_{\mathrm{out},\vec{k}} \\ a^\dagger_{\mathrm{out},-\vec{k}} \end{pmatrix} = \int \frac{d^3q}{(2\pi)^3} \begin{pmatrix} \alpha_{\vec{k},\vec{\omega}} & \beta^*_{\vec{k},\vec{\omega}} \\ \beta_{-\vec{k},-\vec{\omega}} & \alpha^*_{-\vec{k},-\vec{\omega}} \end{pmatrix} \begin{pmatrix} a_{\mathrm{in},\vec{\omega}} \\ a^\dagger_{\mathrm{in},-\vec{\omega}} \end{pmatrix}$$

- Occupation number per mode

$$\langle 0 | a_k^\dagger a_k | 0 \rangle = |\beta_k(\tau)|^2$$

- Energy and number density

$$\frac{dn}{d\log k} = \frac{k^3}{2\pi^2} |\beta_k(\tau)|^2 \quad \frac{d\rho}{d\log k} = \frac{k^4}{2\pi^2} |\beta_k(\tau)|^2$$

- For QFT in curved space $\rightarrow$ gravitational particle production

Outline

- Standard gravitational particle production $(M \neq 0$ & $h_{\mu\nu} = 0)$ WIMPzillas Parker '69; Ford '76; Rubakov '84; Kolb, Riotto '98;

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New Stochastic particle production ($M = 0$ & $h_{\mu\nu} \neq 0$) RG, Redi, Tesi 2408.15987 (Phys. Rev. Lett. 134, 101005 (2025)) & 2502.12249 (hep-th)

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- Production of dark matter
- Impact of self interactions

Standard Gravitational Particle Production

Massive fermion: the algorithm

- Massive fermion conformally coupled to the metric
- $ds^2 = a(\tau)^2 \eta_{\mu\nu} dx^\mu dx^\nu$
- $g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x)$
- $\mathscr{L} = i\bar{\psi}\bar{\sigma}^\mu(\partial_\mu + \omega_\mu)\psi - \frac{M}{2}(\psi\psi + \bar{\psi}\bar{\psi})$ Wess and Bagger
- Rescale $\psi \to \chi/a^{3/2}$

Gravitational Particle Production

Massive fermion, no perturbations

- $i\bar{\sigma}^{\mu}\partial_{\mu}\chi = Ma\bar{\chi}$
- $\chi_0(\tau,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[u_{\vec{k}}(\tau)e^{+i\vec{k}\cdot\vec{x}}a_{\vec{k}} + v_{\vec{k}}(\tau)e^{-i\vec{k}\cdot\vec{x}}b^{\dagger}_{\vec{k}}\right]$
- Mode equation:

$$\partial_{\tau}^2\nu_k + k^2\nu_k + a^2(\tau)M^2\nu_k = 0$$

- Characteristic frequency $\omega^2(\tau) = k^2 + M^2a^2(\tau)$.
- Recall Energy and number density e.g. WIMPZILLA

$$\frac{dn}{d\log k} = \frac{k^3}{2\pi^2}|\beta_k(\tau)|^2 \quad \frac{d\rho}{d\log k} = \frac{k^4}{2\pi^2}|\beta_k(\tau)|^2$$

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- Can we make massless particles?

Stochastic Particle production [New]

Cosmological perturbations break Weyl invariance

- Production of conformally coupled particles $\to 0$ as $M \to 0$.
- GPP works since mass breaks Weyl invariance
- $M_{\rm pl}^2$ R breaks Weyl invariance
- Fluctuations of FLRW metric

Stochastic Particle production [New]

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  - Curvature perturbations from inflation RG, Redi, Tesi 2408.15987 & 2502.12249
  - Tensor perturbations from phase transitions Maleknejad, Kopp '24
- $g_{\mu\nu} = a^2\left(\eta_{\mu\nu} + h_{\mu\nu}\right)$

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- $g_{\mu\nu} = a^2 \left(\eta_{\mu\nu} + h_{\mu\nu}\right)$
- $$ds^2 = a^2(\tau)\left((1+2\Phi)d\tau^2 - (1-2\Psi)dx^2\right)$$

Stochastic particle production

Massless fermions with $h_{\mu\nu} \neq 0$ RG, Redi, Tesi 2408.15987 (Phys. Rev. Lett. 134, 101005 (2025)) & 2502.12249

$$\gamma^a e_a^\mu(\partial_\mu + \frac{1}{8}\omega_\mu^{bc}[\sigma^b, \bar{\sigma}^c])\chi = 0$$

$$i\partial_\tau \chi_{\vec{k}} + (\vec{k}\cdot\vec{\sigma})\chi_{\vec{k}} = J_{\vec{k}}(\tau)\,.$$

$$J_{\vec{k}}(\tau) = \int \frac{d^3q}{(2\pi)^3}\left[2\Psi_{\vec{q}}\dot{\chi}_{\vec{\omega}} - \frac{i}{2}\Psi_{\vec{q}}(\vec{q}\cdot\vec{\sigma})\chi_{\vec{\omega}} + \frac{3}{2}\dot{\Psi}_{\vec{q}}\chi_{\vec{\omega}}\right]\,.$$

- RHS: source of Weyl breaking $\to$ produces particles
- Equation linear in field $\to$ quadratic Hamiltonian
- **New: apply Bogoliubov type calculation** (translations broken)

Stochastic particle production

Bogoliubov RG, Redi, Tesi 2408.15987 (Phys. Rev. Lett. 134, 101005 (2025))

$$\chi_{\vec{k}}(\tau) = \chi_{0,\vec{k}}(\tau) + \int_{-\infty}^{\tau} d\tau' G^R_{\vec{k}}(\tau - \tau') J_{\vec{k}}(\tau')|_{\chi_{0,\vec{\omega}}}$$

$$G^R_{\vec{k}}(\tau) = i\theta(\tau)\left[P^-_{\vec{k}} e^{-ik\tau} + P^+_{\vec{k}} e^{ik\tau}\right] ,$$

$$\beta_{\vec{k}\vec{\omega}} = \int d\tau e^{-i(k+\omega)\tau} \Psi_{\vec{q}}(\tau)\, i(k-\omega)\, \xi^\dagger_{\vec{k},+} \xi_{\vec{\omega},-} \ .$$

- Notice $\beta \propto \Psi \to \langle \beta_k \rangle = 0$
- But $\langle |\beta_k|^2 \rangle \neq 0$ **!**

Stochastic particle production

Abundance of particles RG, Redi, Tesi 2408.15987 (Phys. Rev. Lett. 134, 101005 (2025))

$$\langle \Psi_{\vec{q}}(\tau)\Psi^*_{\vec{q}'}(\tau')\rangle = (2\pi)^3\delta^3(\vec{q}-\vec{q}')\frac{2\pi^2}{q^3}\Delta_\Psi(q,\tau,\tau')\ .$$

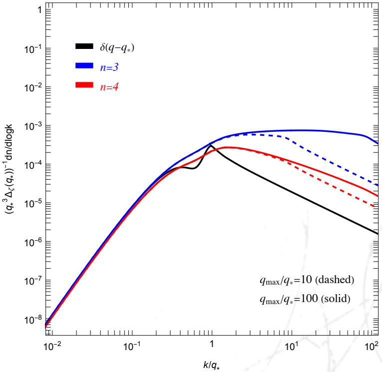
$$\langle|\beta_{\vec{k}}|^2\rangle = \frac{1}{2}\int d\tau\int d\tau'\int d(\log q)\int dx \\ \times e^{-i(k+\omega)(\tau-\tau')}\Delta_\Psi(q,\tau,\tau')\mathcal{K}[k\,,q\,,x]\,,$$

- Well behaved
- Kernel $\mathcal{K}$ depends on spin: $\frac{(k-\omega)^2\left(q^2+2k\omega-k^2-\omega^2\right)}{4k\omega}$
- $\Delta_\Psi(q,\tau,\tau') = T(q,\tau)\,T(q,\tau')\Delta_\zeta(q)$

Stochastic particle production

The spectrum RG, Redi, Tesi 2408.15987 (Phys. Rev. Lett. 134, 101005 (2025))

$$\frac{d(na^3)}{d\log k} = \frac{k^3}{4\pi^2}\int d(\log q)\, dx\, \Delta_\zeta(q)|\mathcal{I}(q, k+\omega)|^2 \mathcal{K}[k\,, q\,, x]\ \ ,$$



Stochastic Particle production [New]

Master formula for Scalars, Fermions and Vectors RG, Redi, Tesi 2502.12249 [hep-th]

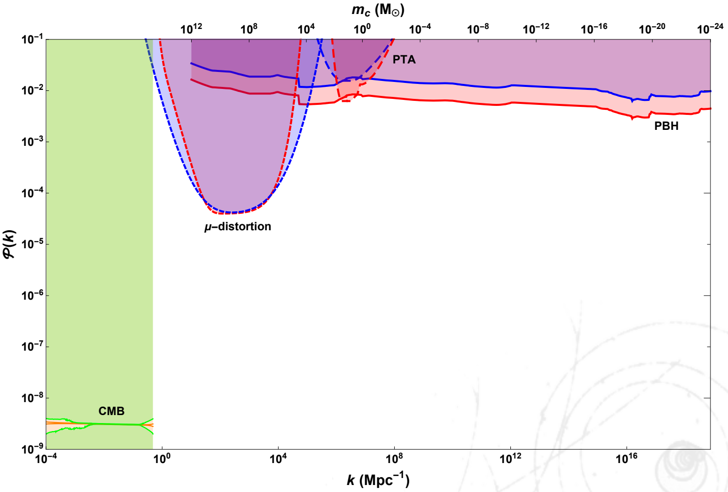
$$\frac{dN}{dkd(\cos\theta_k)d\phi_k} = \frac{k^2}{(2\pi)^3}\int \frac{d^3q}{(2\pi)^3} h_{\mu\nu}(\vec{q}, k+\omega) h^*_{\rho\sigma}(\vec{q}, k+\omega) \times \frac{A^{\mu\nu}_J A^{\rho\sigma\,*}_J}{4k\omega}$$

$$N = \frac{c_J}{2560\pi}\int \frac{d^4q}{(2\pi)^4}\theta(q^2) W^{\mu\nu\rho\sigma}(q) W_{\mu\nu\rho\sigma}(-q)$$

see also Starobinsky and Zeldovich '77

Stochastic Particle production

Application to Dark Matter: Evidences so far ONLY gravitational



2008.03289 Gow et al.

Stochastic particle production

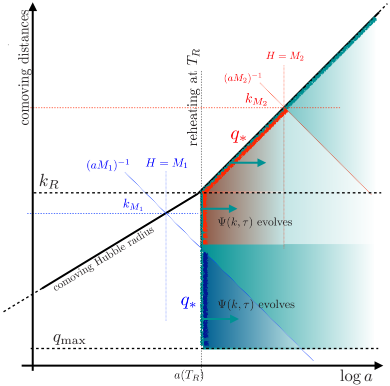
Abundance of Dark Matter from curvature perturbations RG, Redi, Tesi 2408.15987

- $$na^3 = \frac{A}{4\pi^2}\int d(\log q)\, q^3 \Delta_\zeta(q)\,, \qquad A \approx 0.015\ .$$
- $$\Omega_{\rm DM}\big|_{\rm stochastic} \approx \frac{A}{4\pi^2}\frac{M\, q_*^3}{3M_{\rm pl}^2 H_0^2}\Delta_\zeta(q_*)\,.$$
- $$q_* \approx 10^{22}\,{\rm Mpc}^{-1}\left(\frac{10^6\,{\rm GeV}}{M}\right)^{\frac{1}{3}}\left(\frac{0.01}{\Delta_\zeta(q_*)}\right)^{\frac{1}{3}}\left(\frac{0.01}{A}\right)^{\frac{1}{3}}\ .$$

Stochastic particle production

Qualitative features

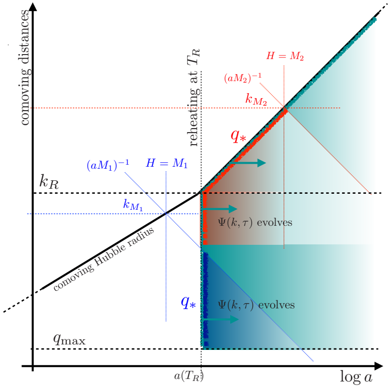
- Computation valid for $k \sim q_\star \gtrsim aM$



Stochastic particle production

Qualitative features

- Computation valid for $k \sim q_\star \gtrsim aM$
- Production begins when Ψ begins to evolve



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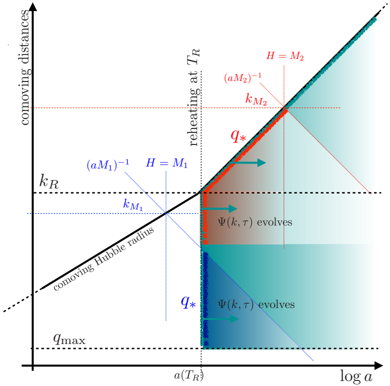

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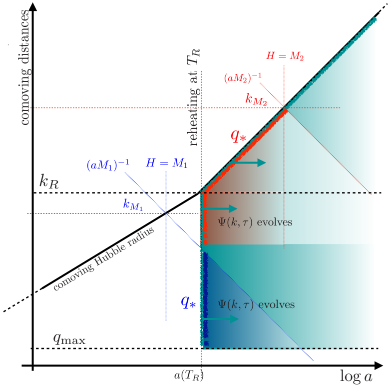

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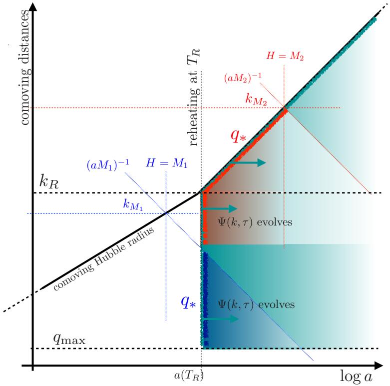
- If k_M re-enters during reheating need $q_\star \gtrsim T_0 M / T_R$



Stochastic particle production

Abundance of Dark Matter from curvature perturbations

- **Need a sizable primordial power spectrum**
- Need to happen at the last few e-foldings of inflation
- Related to ultra-slow roll scenarios
- Phase transition after production required to make them massive

Stochastic particle production

Summary: comparison to other mechanisms

- Gravitational freeze-in

$$\Omega_{\rm DM} \approx 4 \times 10^{-5} \frac{M k_R^3}{3M_{\rm Pl}^2 H_0^2}\ .$$

- Gravitational particle production

$$\Omega_{\rm DM} \approx 10^{-2} \frac{M\, k_M^3}{3M_{\rm pl}^2 H_0^2} \times \min\left[1\,, \frac{T_R}{\sqrt{M\, M_{\rm pl}}}\right],$$

- Stochastic particle production

$$\Omega_{\rm DM} \approx \frac{A}{4\pi^2} \frac{M\, q_*^3}{3M_{\rm pl}^2 H_0^2} \Delta_\zeta(q_*)\,.$$

Conclusions and Outlook

- Massless particles can be gravitationally produced from perturbations!
- Generalizations $\rightarrow$ self-interactions $+$ small mass term
- Example: $\lambda\phi^4$ and non-abelian vectors $\rightarrow$ larger cored proto-halos [work in progress]
- A new mechanism to produce primordial magnetic fields

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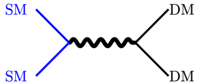
Thank you for your attention!

Gravitational Freeze-in



$$\mathcal{A} = \frac{1}{M_p^2 s}\left(T^{\mathrm{SM}}_{\mu\nu} T^{\mathrm{DM}}_{\alpha\beta}\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2}T^{\mathrm{SM}}T^{\mathrm{DM}}\right)$$

- SM particles annihilate to DM: $\frac{1}{M_{\mathrm{pl}}} h_{\mu\nu} T^{\mu\nu}$
- Yield is controlled by reheating temperature:

$$Y_{\mathrm{D}} = 6\times 10^{-6}\, c_D \left(\frac{T_R}{M_{\mathrm{pl}}}\right)^3 .$$

- $\Omega_{\mathrm{DM}} \approx 10^{-5} \frac{M}{3H_0^2 M_{\mathrm{pl}}^2} \left(\frac{T_R T_0}{M_{\mathrm{pl}}}\right)^3$

Gravitational Freeze-in

- Universal, depends only on central charge

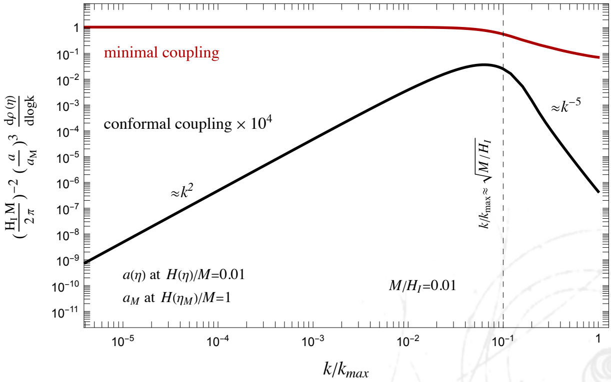
$$M_{\mathrm{DM}} \sim \frac{1}{c_{\mathrm{D}}} 10^6 \mathrm{GeV} \left(\frac{10^{15}\mathrm{GeV}}{T_R} \right)^3$$

- Typical candidate Glueball Dark Matter Faraggi and Pospelov '02

Gravitational Particle Production

Massive fermion

- Production peaked $k/a_M \sim H \sim M \implies$ suppressed by mass
- $\Omega_{\rm DM} = 10^{-2}\frac{M k_M^3}{3M_{\rm pl}^2 H_0^2}$



Redi, Tesi '23; Long, Kolb '23

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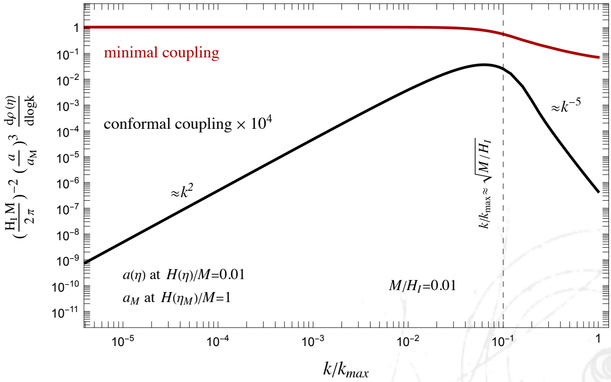

Redi, Tesi '23; Long, Kolb '23

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