Self-interacting dark matter and inelasticity

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CosmoChart
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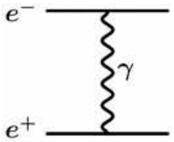
Small-scale structure of the universe and self-interacting dark matter
Valencia, 11 June 2025

Self-interacting dark matter is great, *and complicated*

- Self-interactions refer to elastic scattering between DM particles
- Light force mediators are the canonical paradigm of SIDM
  Ordinary matter analogue: $e^+ e^-$ scattering via virtual photon exchange.



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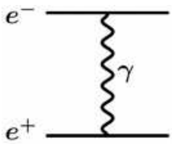

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  - @ astrophysicists: bremsstrahlung → dissipation
  - @ particle physicists: annihilation, bremsstrahlung, **bound-state formation**

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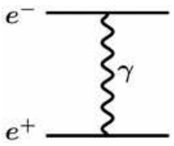

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  Bound states: ion-ion scattering very different from atom-atom scattering!

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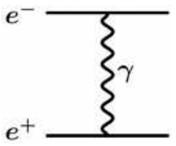

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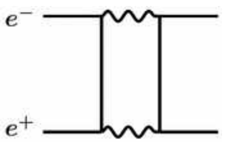
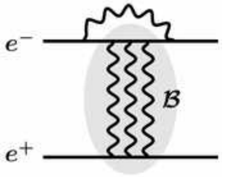
or, more importantly,

$$e^+ e^- \rightarrow \text{Positronium} + \gamma \rightarrow e^+ e^-$$





Flores, Petraki: 2405.02222;
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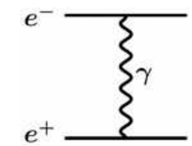

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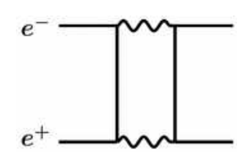

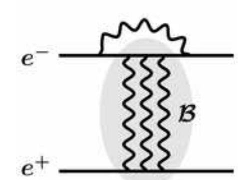

Important in many theories with long-range interactions due to BSF

(but not minimal (dark) QED)

Flores, Petraki: 2405.02222;
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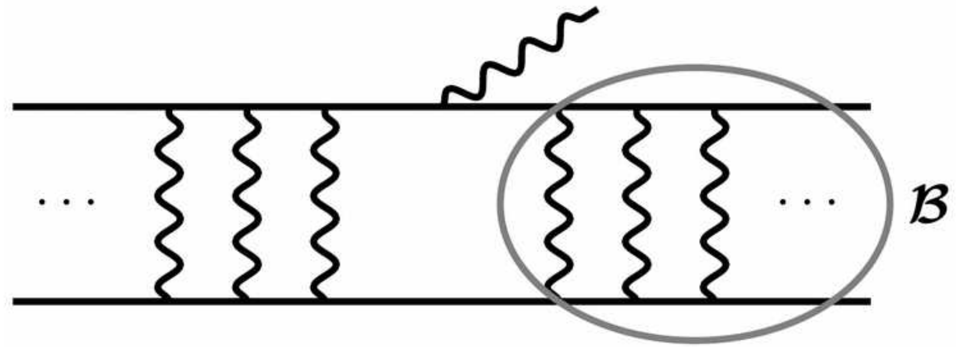
Bound-state formation:
standard calculation

Transition to a lower energy state requires dissipation of energy: emission of a force mediator

Scattering state:
two free (DM) particles interacting via a long-range force



The long-range force captures particles into a **bound state**

Cross-section depends on the overlap of scattering and bound state wavefunctions, i.e. on the potentials they experience

- If the potentials of the initial and final state are the same (e.g. minimal QED): cross-section well behaved.
- **If the potentials are different** → large wavefunction overlaps → large cross-sections that can **exceed unitarity limit, even for arbitrarily small couplings.** !

Oncala, Petraki: 1911.02605, 2101.08666, 2101.08667; Binder, Garny, Heisig, Lederer, Urban: 2308.01336; Beneke, Binder, de Ros, Garny, Lederer: 2411.08737; Petraki, Socha, Vasilaki: 2505.20443.

Bound-state formation: transitions between different potentials

- In all non-Abelian theories (e.g. QCD):

  non-Abelian gauge bosons are charged under the gauge symmetry, carry away conserved charge → different in/out potentials.

- In (slightly non-minimal) Abelian theories:

  e.g. darkly charged fermions coupled to a dark photon and a doubly charged light scalar mediator.

$$\mathcal{L} \supset \bar{X} X \gamma + X X \Phi^\dagger$$

Capture into bound states in two ways:

$$\overbrace{X + \bar{X}}^{\text{same potential}} \to \overbrace{\mathcal{B}(X\bar{X})}^{} + \gamma \qquad \textbf{well-behaved}$$

$$\underbrace{X + X}_{\text{different potentials}} \to \underbrace{\mathcal{B}(X\bar{X})}_{} + \Phi \qquad \textbf{unitarity violation!}$$

But what is Unitarity,
and what is it good for?

Cross-sections ~ (Probability for scattering) x (phase space)

Cannot be larger than 1 !

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Depend on kinematic variables (momentum), but not on any coupling

If a cross-section violates these bounds, something has gone terribly wrong!

Either the theory or the calculation need amendment.

Unitarity

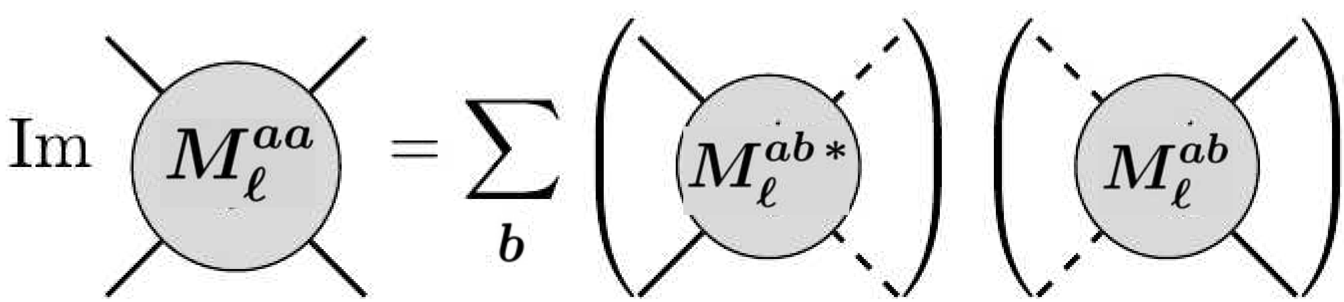
$$S^\dagger S = 1 \quad \xrightarrow{S=1+iT} \quad -i(T - T^\dagger) = T^\dagger T$$

Project on a partial wave, insert complete set of states on RHS, and rescale amplitude, $M_\ell^{ab}(p^a, p^b) = \sqrt{4p^a p^b / s} \times \mathcal{M}_\ell^{ab}(p^a, p^b)$, to obtain

$$\Downarrow$$

Generalised optical theorem: on-/off-shell and partial-wave

$$\boxed{\mathrm{Im}[M_\ell^{aa}(p, p')] = \sum_{b:\ \mathrm{on-shell}} M_\ell^{ab\,*}(p, p^b)\, M_\ell^{ab}(p', p^b)}$$



Unitarity

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$\Downarrow$ on-shell

Unitarity circle:

$$|M_\ell^{aa} - i/2| \leqslant 1/2$$

Bounds on elastic and inelastic cross-sections:

$$\sigma_\ell^{ab} = \sigma_\ell^U \times |M^{ab}|^2$$

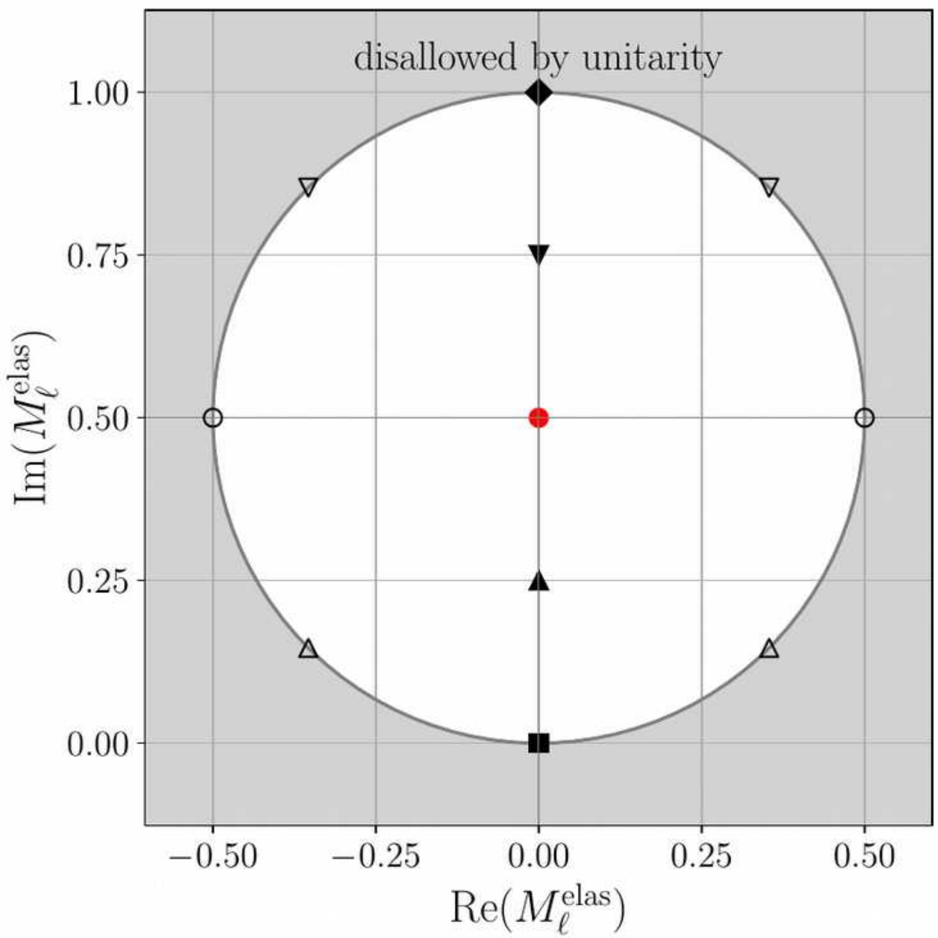
$$\sigma_\ell^U = 4\pi(2\ell+1)/p^2$$

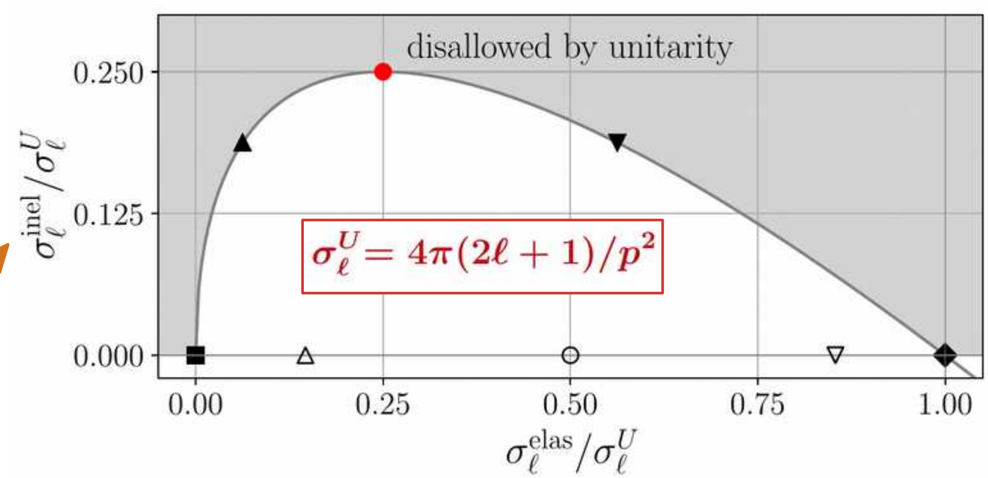
$$(\sigma_\ell^{\text{elas}} + \sigma_\ell^{\text{inel}})/\sigma_\ell^U = \sum_b |M_\ell^{ab}|^2 = \mathrm{Im}|M_\ell^{aa}| \leqslant |M_\ell^{aa}| \leqslant \sqrt{\sigma_\ell^{\text{elas}}/\sigma_\ell^U}$$

Hui: astro-ph/0102349
Flores, KP: 2405.02222

Unitarity

Bounds on partial-wave cross-sections





$$\sigma_\ell^{\mathrm{inel}}/\sigma_\ell^U \leqslant \sqrt{\sigma_\ell^{\mathrm{elas}}/\sigma_\ell^U}\left(1-\sqrt{\sigma_\ell^{\mathrm{elas}}/\sigma_\ell^U}\right)$$

$$\sigma_\ell^{\mathrm{elas}}/\sigma_\ell^U \leqslant 1$$

$$\sigma_\ell^{\mathrm{inel}}/\sigma_\ell^U \leqslant 1/4$$

Inelastic scattering generates elastic scattering!

Flores, KP: 2405.02222

Unitarity
troubles

Unitarity is a sacred principle in particle physics, however

$$\mathrm{Im}[M_\ell^{aa}(p,p')] = \sum_{b:\ \mathrm{on-shell}} M_\ell^{ab\,*}(p,p^b)\, M_\ell^{ab}(p',p^b)$$

any perturbative calculation at finite order of the couplings violates it!

Unitarity
troubles

Resolution:

Calculate at infinite order!

How?

Resummation

What is resummation?

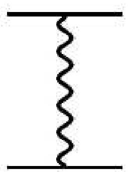
Something we learn in high-school

(but we did not know it was that)

Elastic unitarity
and resummation

Consider, for example, the interaction of two charged particles

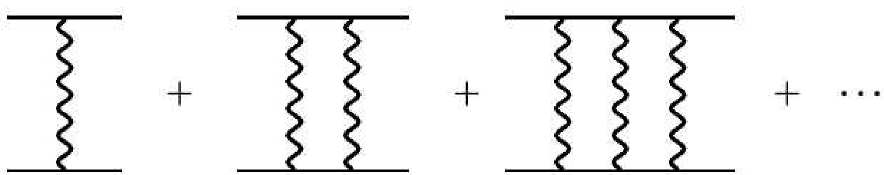
Tree-level



$$\sigma_\ell^{\text{elas}} \propto \frac{\alpha^2}{m^2 v_{\text{rel}}^4}$$

Rutherford scattering: violates unitarity at low enough velocities

Resummation



$$\sigma_\ell^{\text{elas}} = \frac{4\pi(2\ell+1)}{m^2 v_{\text{rel}}^2} \times \sin^2\left(\frac{1}{2i} \ln\left[\frac{\Gamma(1+\ell+i\alpha/v_{\text{rel}})}{\Gamma(1+\ell-i\alpha/v_{\text{rel}})}\right]\right)$$

Equivalent to solving Schroedinger's eq. for a Coulomb potential

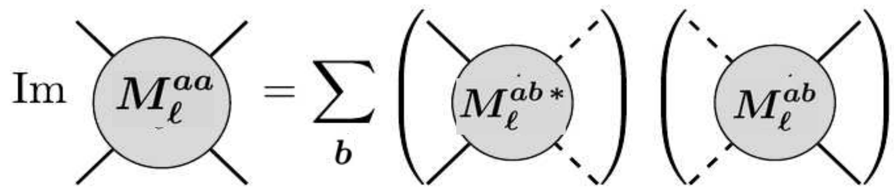
Respects *elastic* unitarity at all velocities, and spans the entire allowed range (feature of long-range interactions)

What about *inelastic* unitarity?

Unitarity

Resummation of inelastic interactions

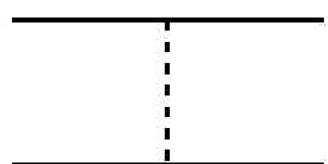
Flores, KP 2405.02222



Generalised optical theorem: on-/off-shell and partial-wave

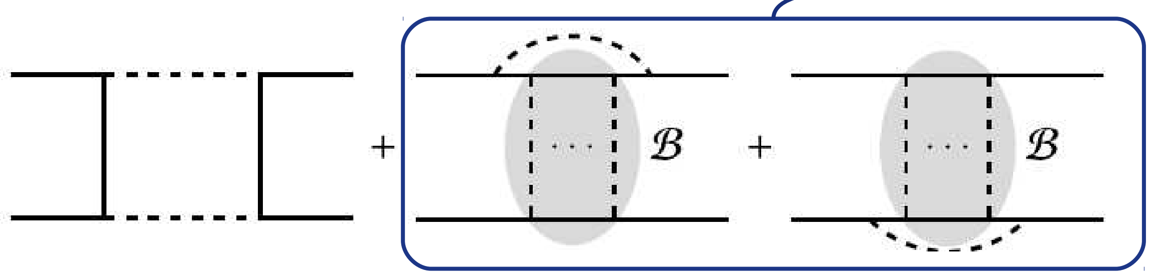
$$\mathrm{Im}[M_\ell^{aa}(p,p')] = \sum_{b:\ \text{on-shell}} M_\ell^{ab*}(p,p^b)\, M_\ell^{ab}(p',p^b)$$

Purely elastic kernel → **real potential**



Contributions exist even for non-annihilating species

Squared inelastic kernel → **imaginary potential**



Solve Schroedinger's equation with
(i) real potential only (→ unregulated cross-sections)
(ii) complex potential (→ regulated cross-sections)

Unitarity

Resumption of inelastic interactions

The resummation of squared inelastic diagrams affects both elastic and inelastic cross-sections!

Flores, KP 2405.02222

$$\frac{\sigma^{\text{inel}}_{\ell,\text{reg}}}{\sigma^U_\ell} = \frac{\sigma^{\text{inel}}_{\ell,\text{unreg}}/\sigma^U_\ell}{\left(1+\sigma^{\text{inel}}_{\ell,\text{unreg}}/\sigma^U_\ell\right)^2}$$

Corrected cross-sections

$$\frac{\sigma^{\text{elas}}_{\ell,\text{reg}}}{\sigma^U_\ell} = \frac{\sigma^{\text{elas}}_{\ell,\text{unreg}}/\sigma^U_\ell}{\left(1+\sigma^{\text{inel}}_{\ell,\text{unreg}}/\sigma^U_\ell\right)^2} + \left(1-\sigma^{\text{elas}}_{\ell,\text{unreg}}/\sigma^U_\ell\right) \times \frac{\left(\sigma^{\text{inel}}_{\ell,\text{unreg}}/\sigma^U_\ell\right)^2}{\left(1+\sigma^{\text{inel}}_{\ell,\text{unreg}}/\sigma^U_\ell\right)^2}$$

Unitarity

Resummation of inelastic interactions

The resummation of squared inelastic diagrams affects both elastic and inelastic cross-sections!

Flores, KP 2405.02222

regulating factor:
reduced flux due to inelastic scattering
(reflects continuity equation)

Important if $\sigma_{\text{unreg}} \gtrsim \sigma^U$.

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Resumming inelastic interactions

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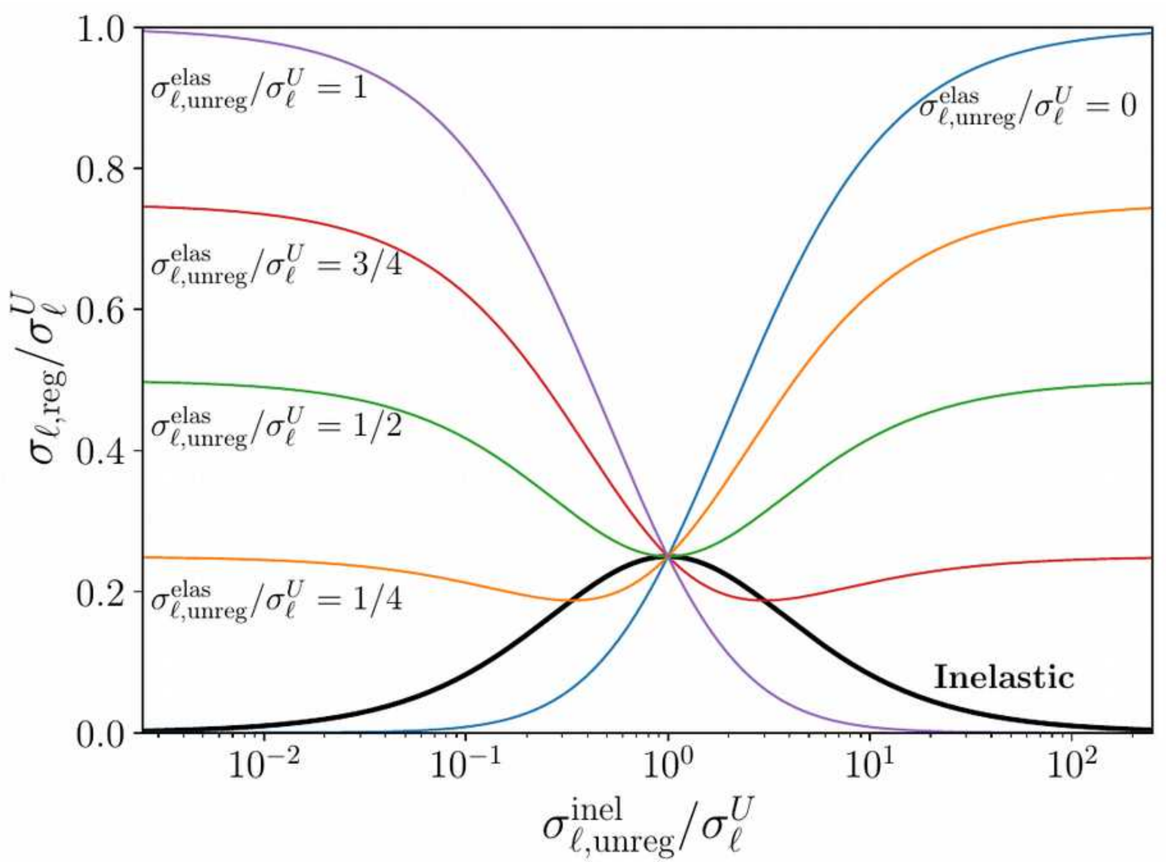
original scattering probability, suppressed by regulating factor

Additional contribution: regenerated flux

Unitarity

Resummation of inelastic interactions

corrected cross-sections



uncorrected inelastic cross-section

Flores, KP 2405.02222

- Affects both strength and velocity dependence of the cross-sections.
- Can increase or decrease elastic scattering.

- Inelastic cross-sections: affect DM production in the early universe
  → correlation of parameters (mass-coupling relation)
  → DM self-interactions
  [KP, Socha, Vasilaki 2505.20443]
- Elastic cross-section: affects DM scattering inside halos
  [Flores, KP, Pignard: in progress]

Conclusions

Inelasticity is particularly important in the presence of light force mediators and affects elastic scattering in “real” and “virtual” ways:

- “Real” ways: formation of bound states, transitions between levels, dissipation.
- “Virtual” ways: inelastic scatterings feed into elastic scatterings.

- Unitarity has always been an important guide in finding new physics (e.g. discovery of electroweak interactions, Higgs etc).

  Employ it for SIDM!

- Strong connection between unitarity limits and long-range interactions in the non-relativistic regime.