Dark subhaloes in dwarf spheroidal galaxies

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Valencia
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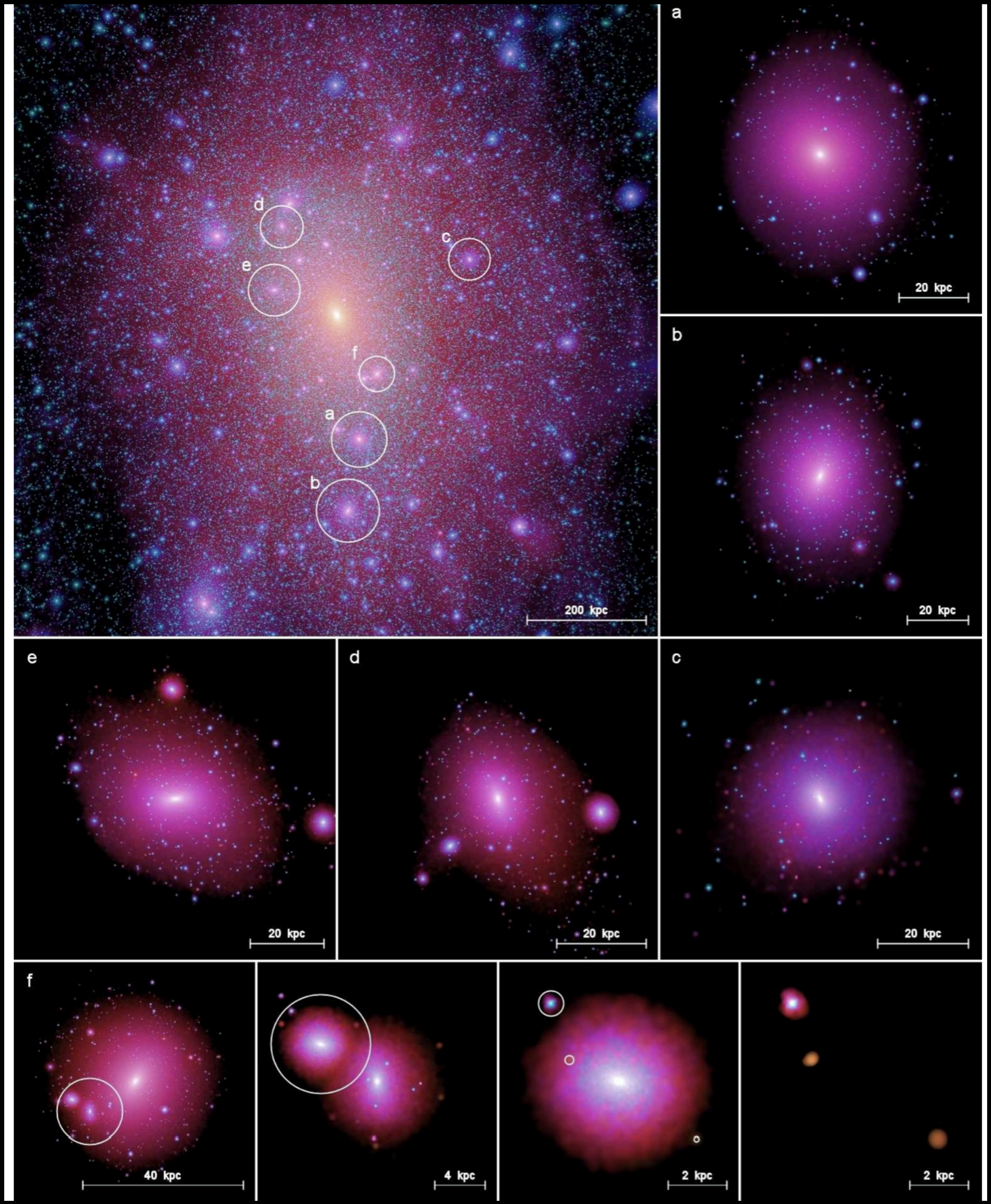
CDM SUB- SUB - (..?..) - HALOES



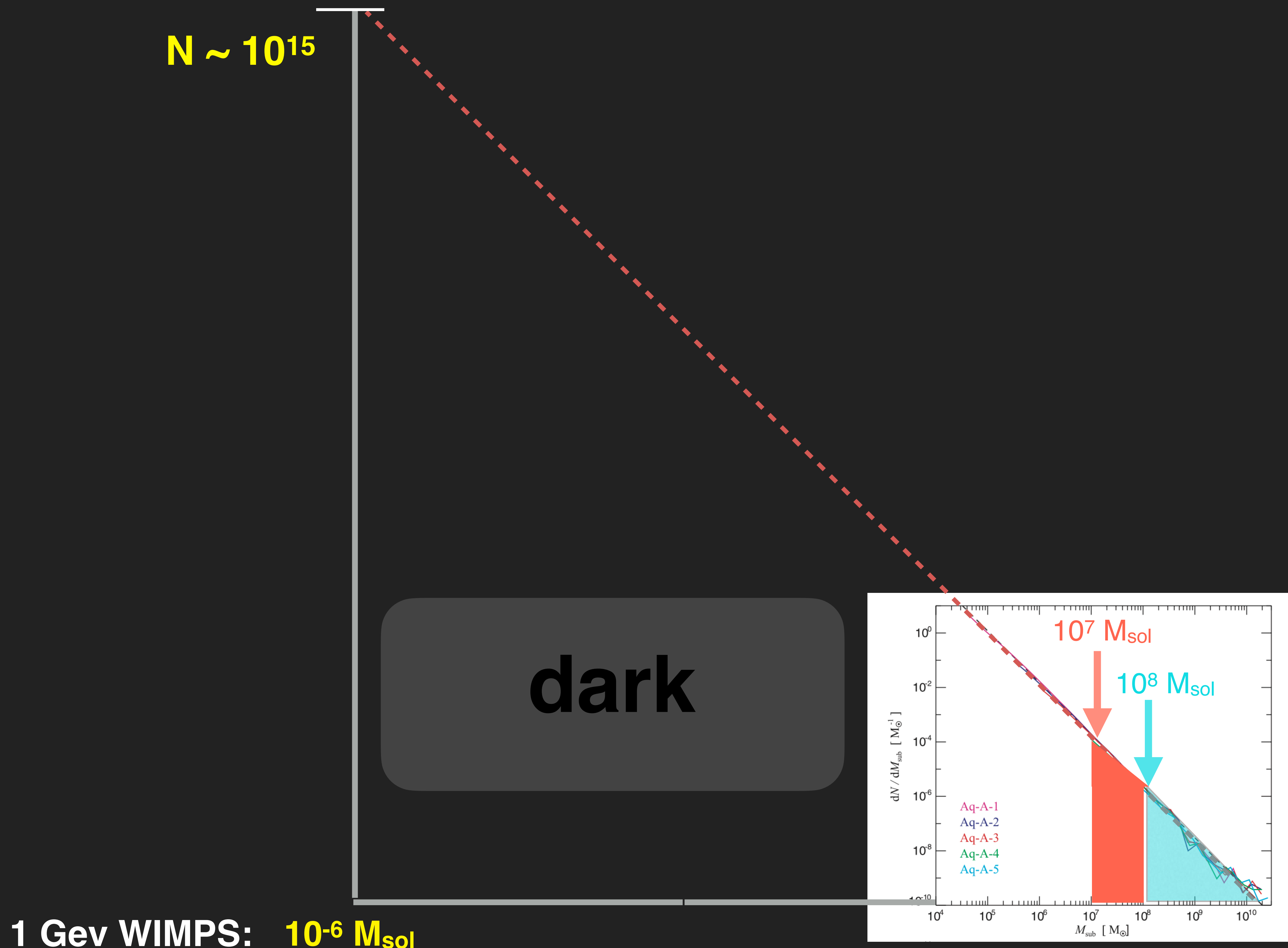
Hierarchical structure formation

Substructures within substructures within substructures… all the way down to **free-streaming length** scales !

Smallest substructures in CDM M<1E-6 Msol (planet size)

Springel+08

EXTRAPOLATION DOWN TO FREE-STREAMING LENGTH

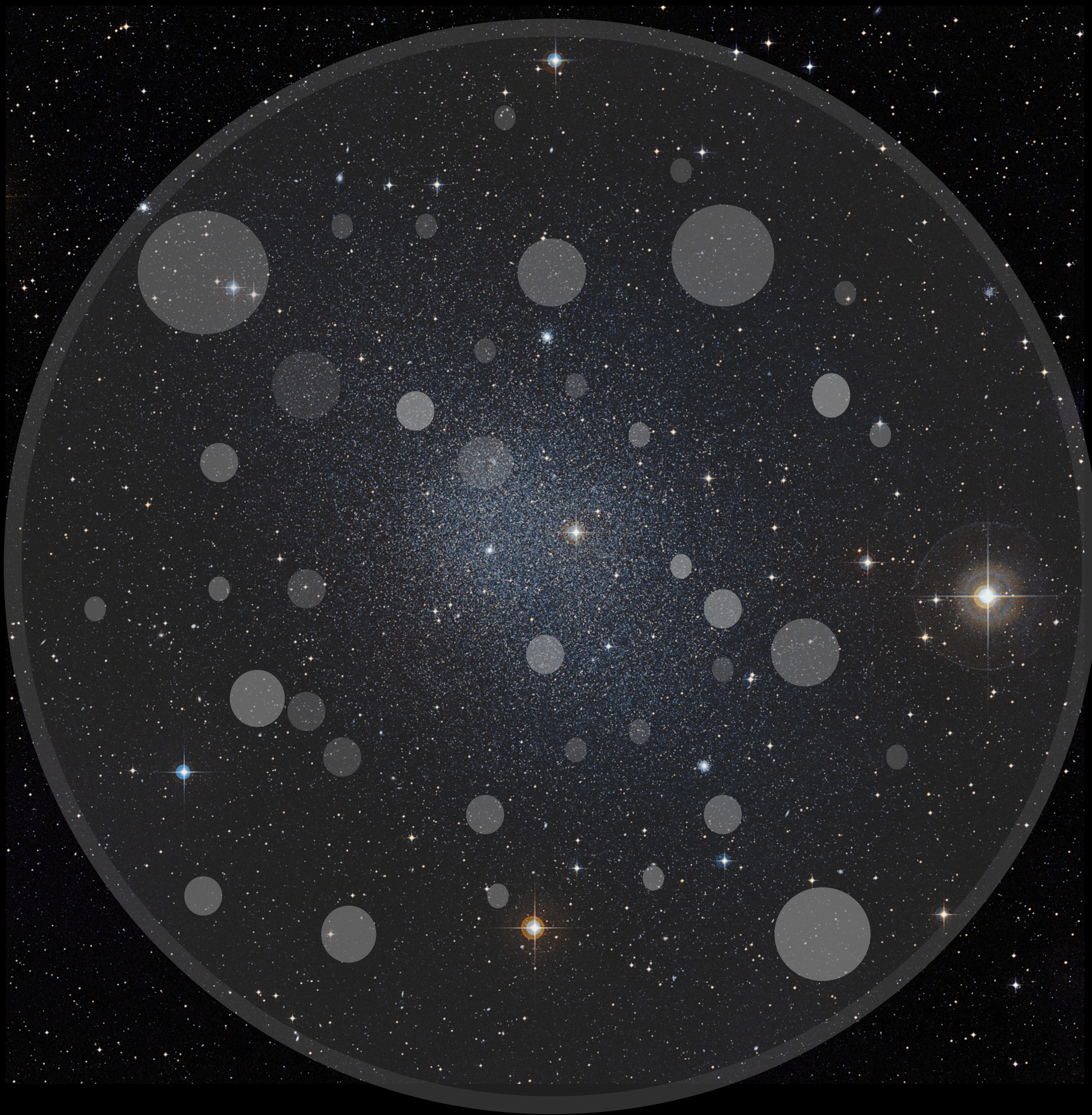


CDM SUB-SUB-HALOES IN DWARF SPHEROIDALS

Dwarf Spheroidals are the most DM-dominated galaxies of the known Universe

In CDM, gravitational potential is NOT smooth… but **clumpy**

CDM SUB-SUB-HALOES IN DWARF SPHEROIDALS



Dwarf Spheroidals are the most DM-dominated galaxies of the known Universe

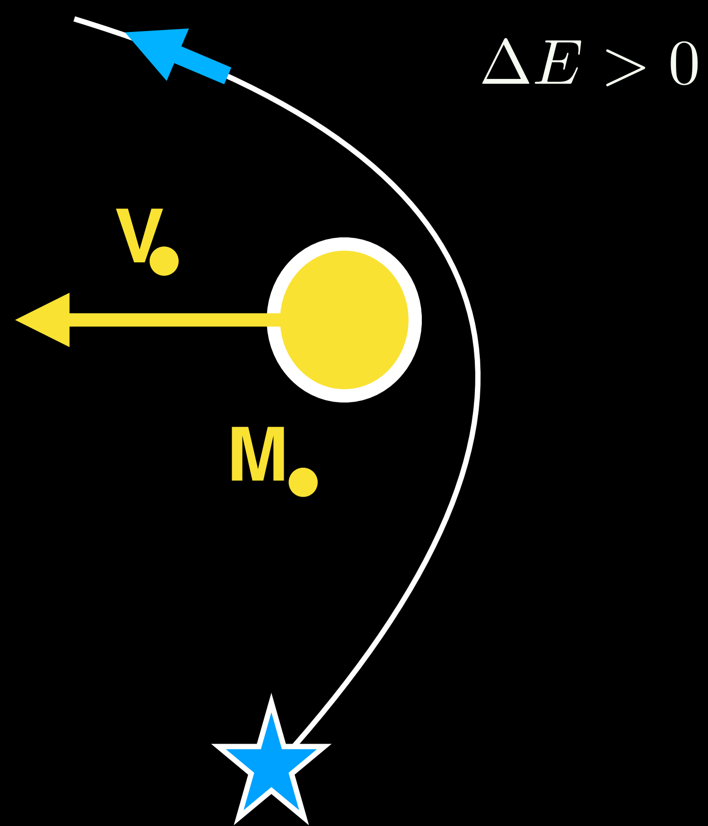
In CDM, gravitational potential is NOT smooth… but **clumpy**

What are the effect of sub-subhaloes self-gravity on the motion of stars?

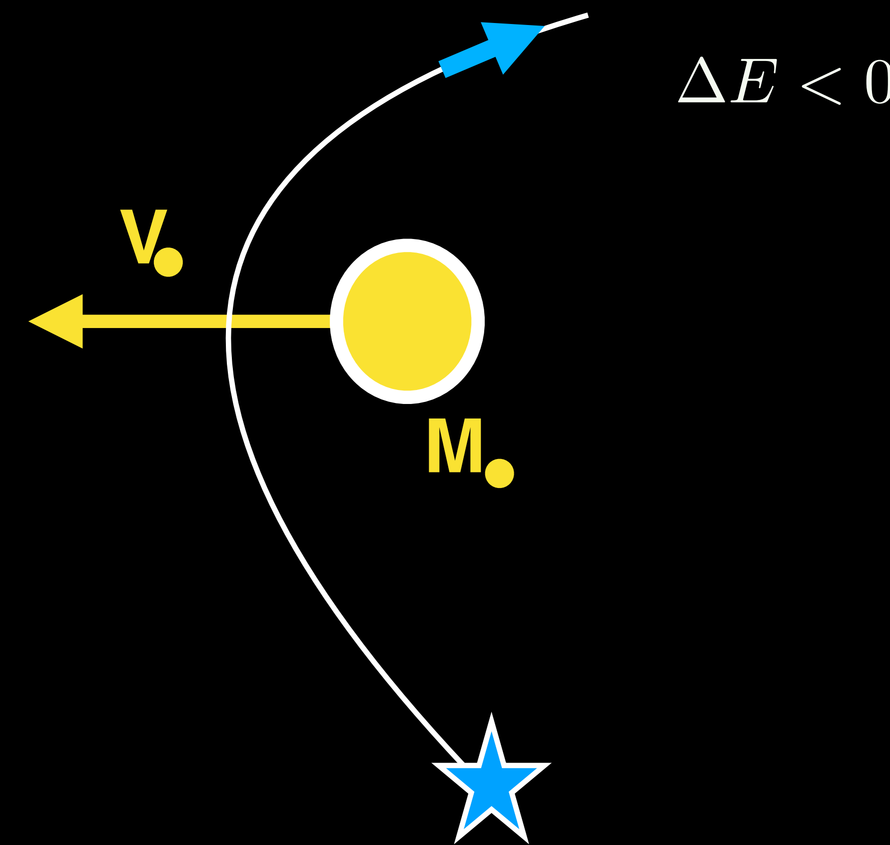
1- Gravitational capture of field stars by single subhaloes

2- Dynamical heating by a large population of dark subhaloes

GRAVITATIONAL CAPTURE ~ SLINGSHOT-MANOEUVRES(*)



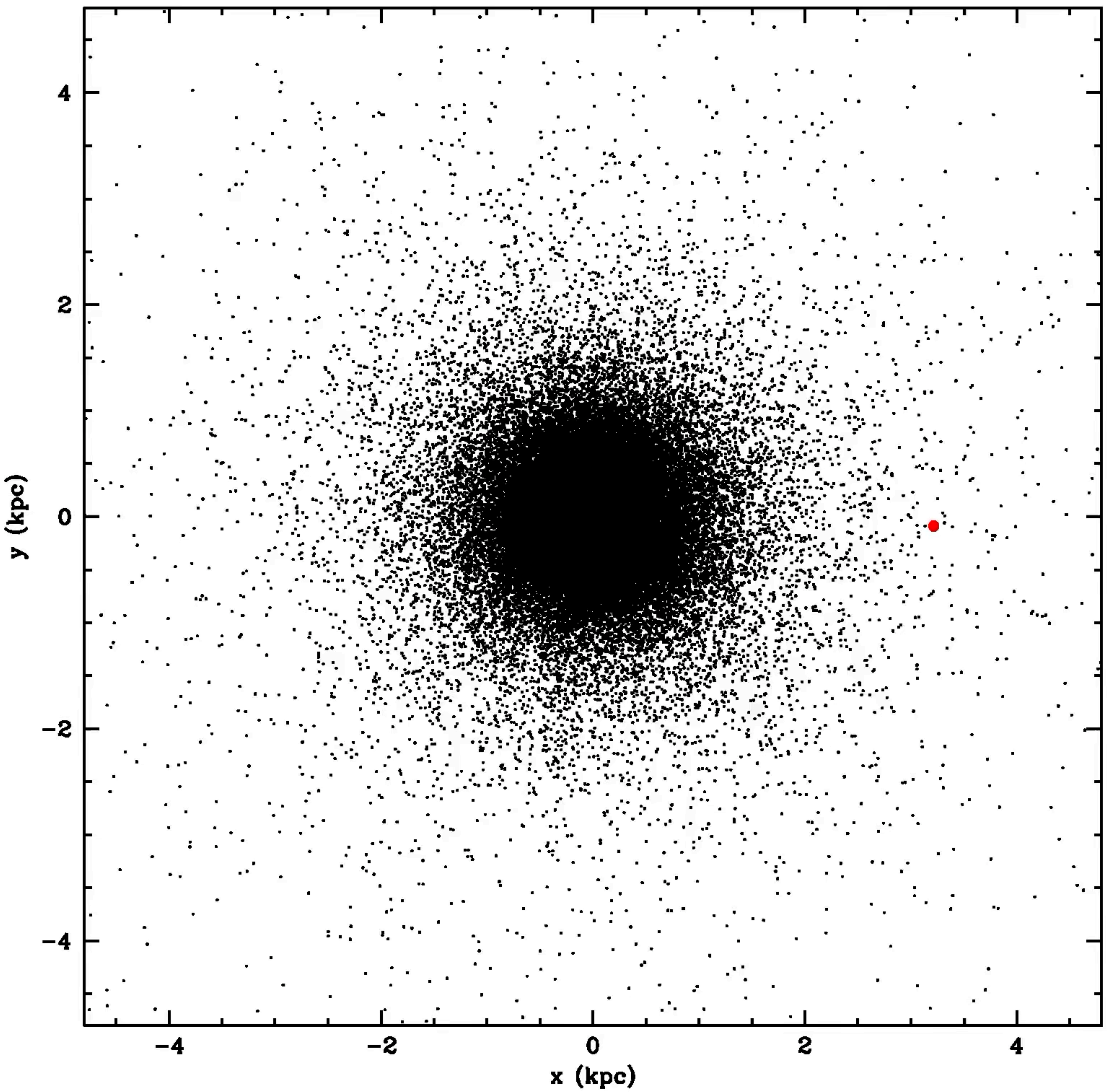
Star gains kinetic energy in the subhalo frame
Acceleration
it can not result in capture



Star loses kinetic energy in the subhalo frame
Deceleration
it can result in capture

(*) This is a local 2-body approximation of a a 3-body system. In reality, trajectories of captured objects are **chaotic** and show extreme complexity (e.g.Petit & Henon 1984)

Capture from the galactic field



M^{sub} = 5e7 Msol, r_s=130pc (**truncated cusp**. See Errani + Navarro 21)
M^{dSph} = 3e9 Msol (Dehnen prof.)
N^{dSph} = 40000

Gravitational Capture: *"process by which objects orbiting around the galactic potential begin to orbit around the subhalo potential"*

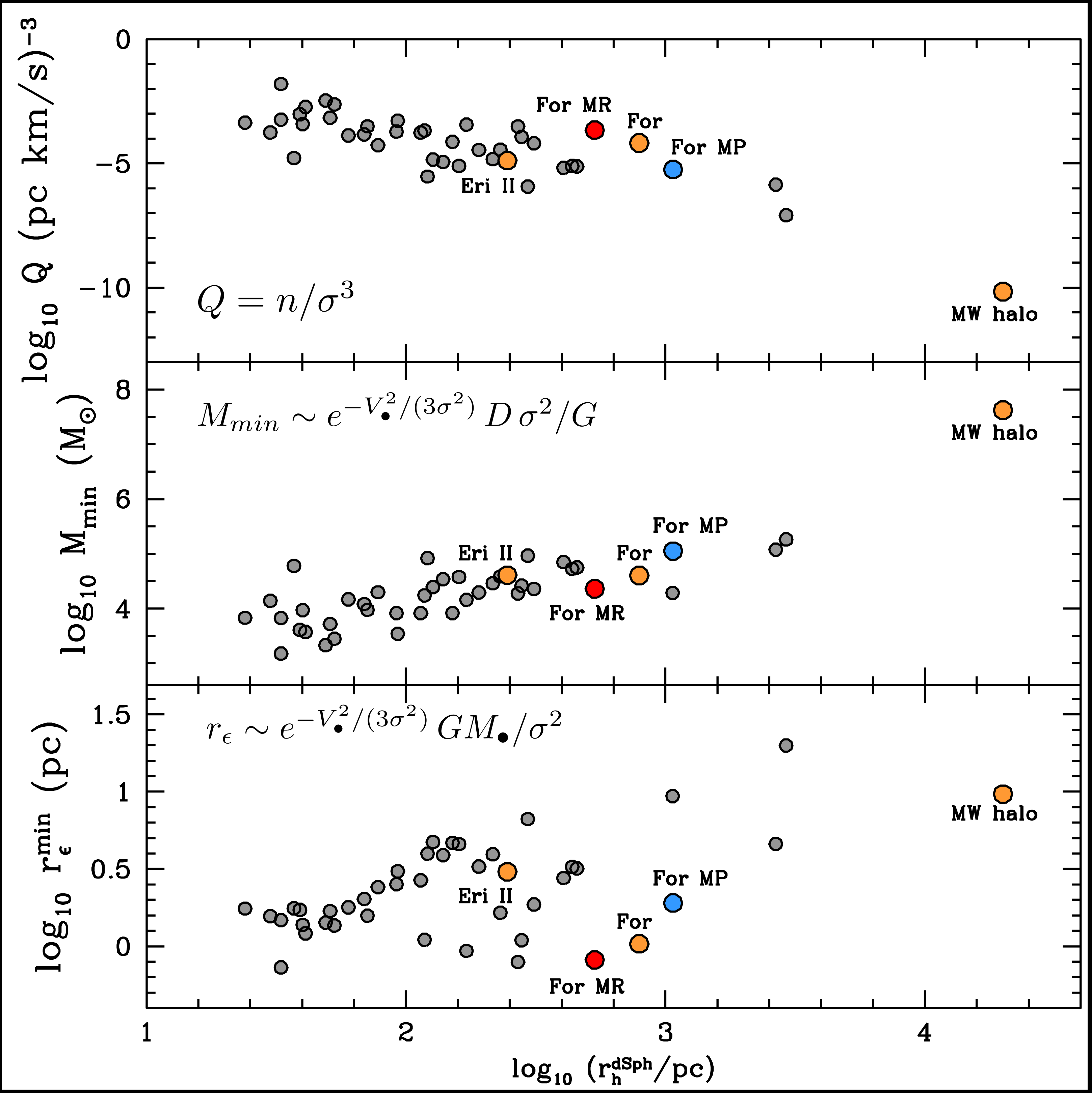
Numerical Experiments:

* (mass-less) stellar tracers in dynamical equilibrium at t=0
* static galactic (DM) potential
* Restricted 3-body eqs solved for each individual star independently

Temporary capture leads to **over-densities of field stars co-moving with the subhalo**

STATISTICAL THEORY : WHAT SUBHALOES CAPTURE FIELD STARS?

ArXiv.2404.19069



$N_\star \sim Q$

Gravitational capture most efficient in **dSphs**

Subhaloes must be *massive* enough to capture field stars

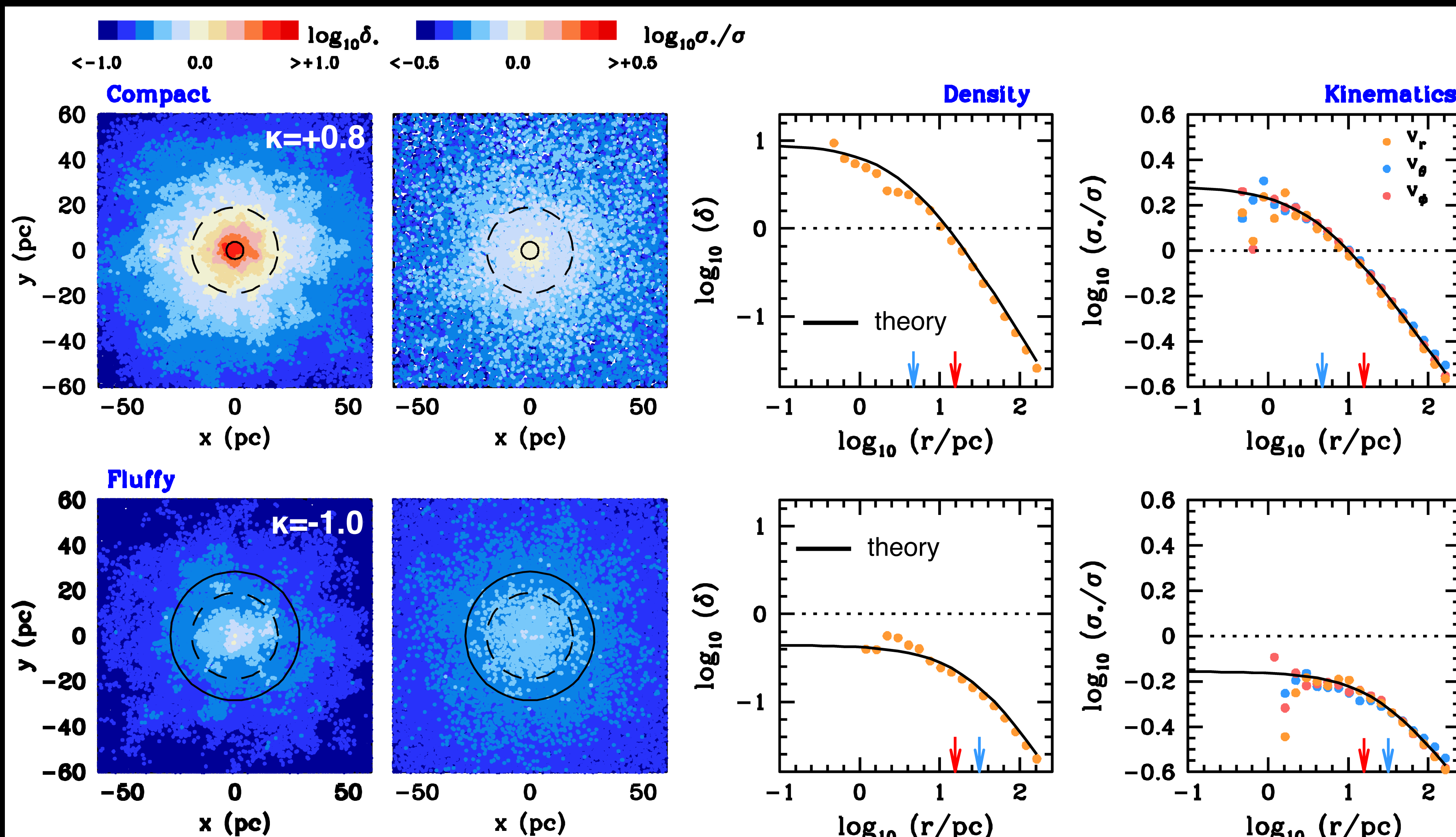
$N_\star(< r_\epsilon) > 1$

M_{min} : minimum **subhalo mass** that captures more than one field star

Stellar over-densities:

- **r_ϵ ~** size of overdensity
- size of large stellar clusters
- w/ same chemical composition as the host galaxy
- DM dominated

COMPACT VS FLUFFY SUB-SUBHALOES



$$\kappa \equiv 1 - c_\bullet / r_\epsilon > 0$$

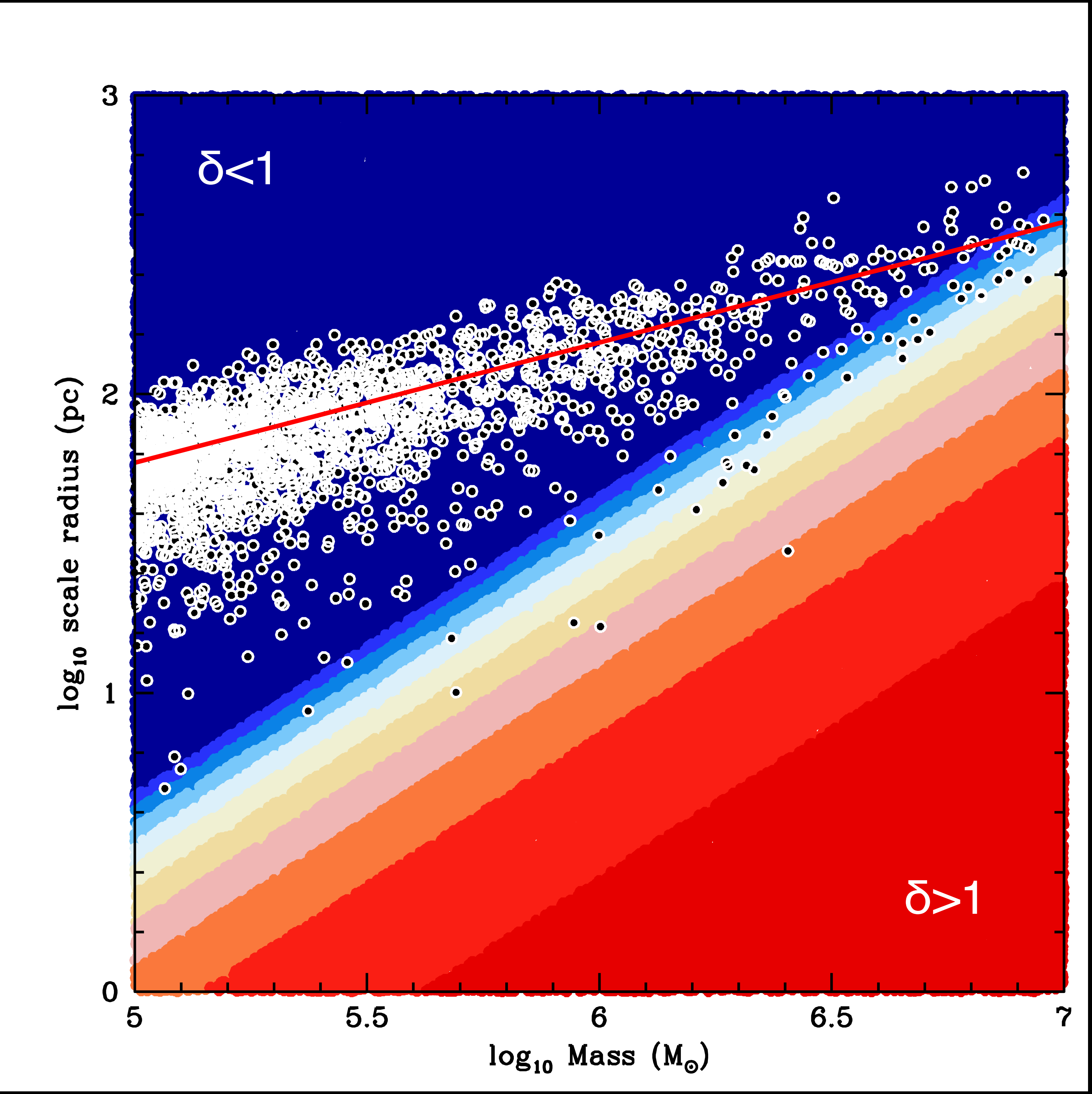
Subhaloes must be *compact* enough to generate overdensities

- To generate localized over-densities of field stars, the scale radius must be smaller than the thermal critical radius
- velocity dispersion is comparable to that of the field
- Theory works well for low-mass subhaloes on circular orbits, but accuracy decreases for very eccentric orbits and/or massive satellites (N-body models needed)

Field : MR stars in Fornax dSph. In equilibrium at t=0 within a cored halo (note: similar results in cuspy halo)

Sub-subhalo placed on circular orbit at R=0.3kpc with M=1E6 Msol

DISCUSSION: MASS-SCALE RADIUS OF SUB-SUBHALOES ??



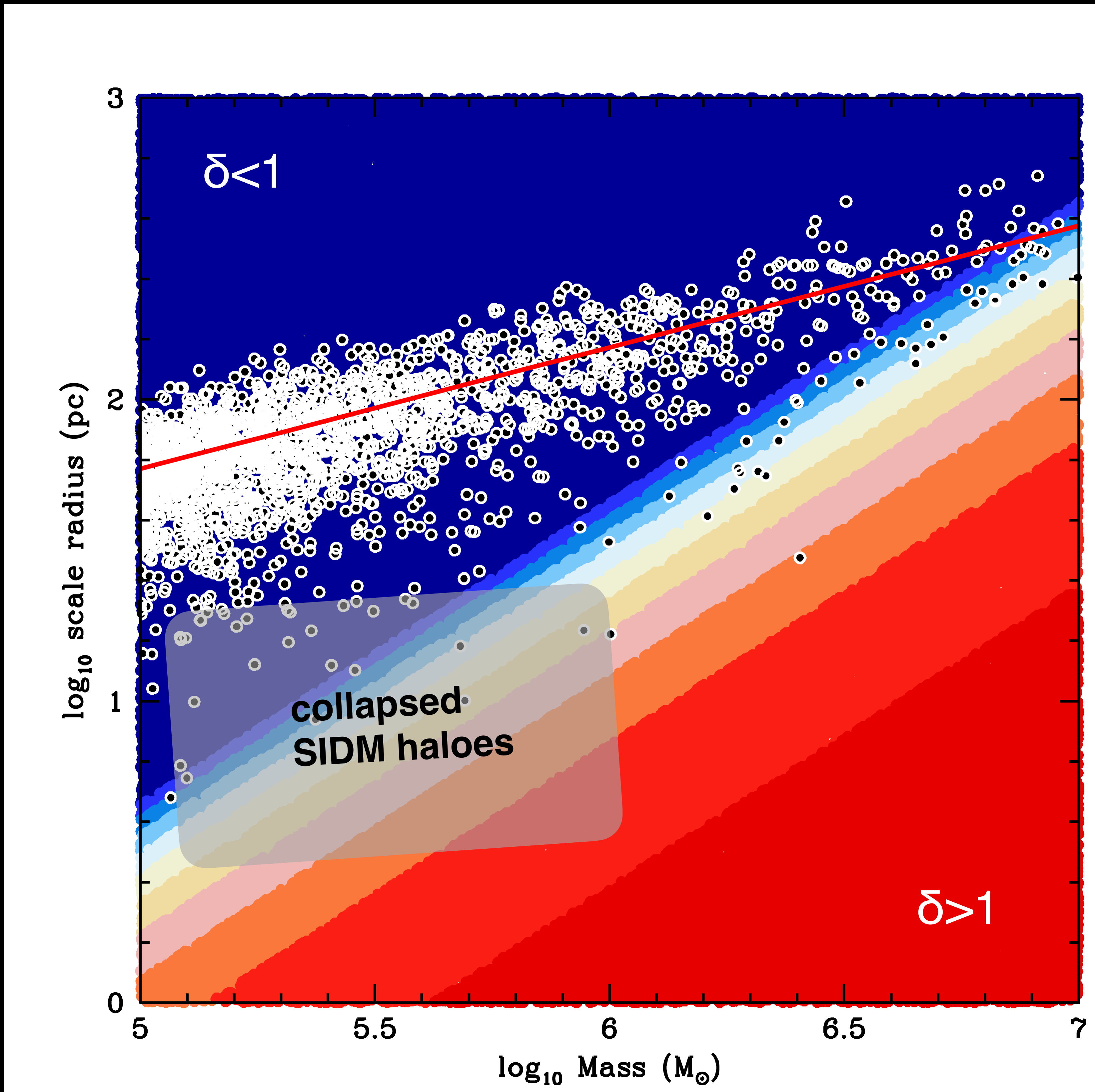
- Aquarius extrapolation of mean relation (Springel+08)

  +

- Gaussian scatter **σ = 0.13 dex** (Nadler +21)

- The majority of CDM sub-subhaloes **not compact enough** to generate visible over-densities
- Number very sensitive to mass-scale radius relation of sub-subhaloes in dSphs (cosmological simulations needed!)

DISCUSSION: MASS-SCALE RADIUS OF SUB-SUBHALOES ??



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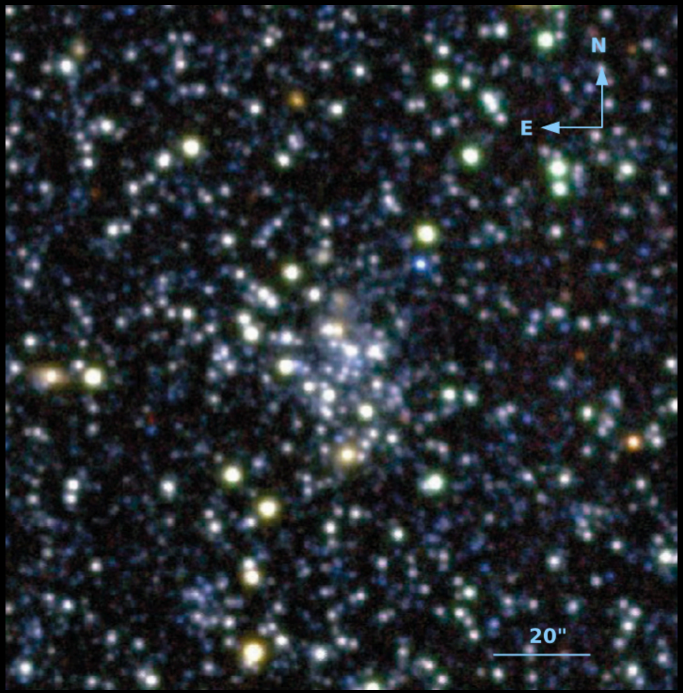
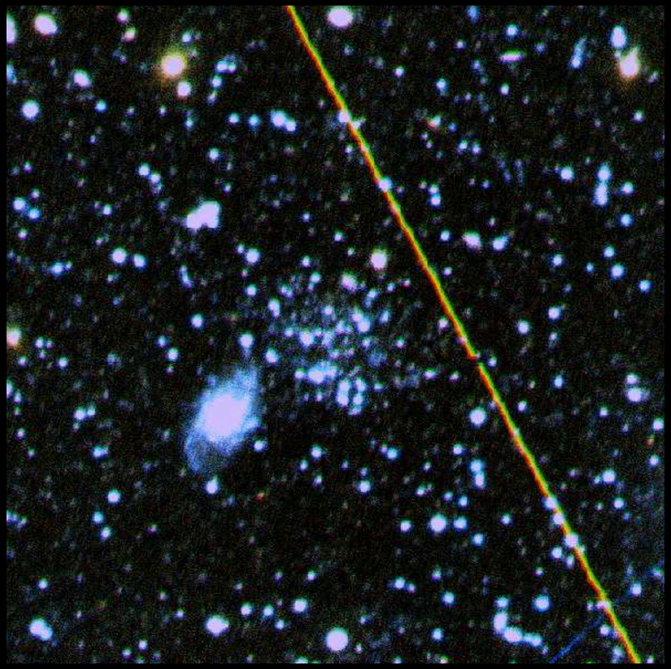
- The majority of CDM sub-subhaloes **not compact enough** to generate visible over-densities
- Number very sensitive to mass-scale radius relation of sub-subhaloes in dSphs (cosmological simulations needed!)
- Very sensitive to the presence **collapsed** subhaloes

DISCUSSION: HAVE WE ALREADY DETECTED DM SUB-SUBHALOES?



Fornax 6

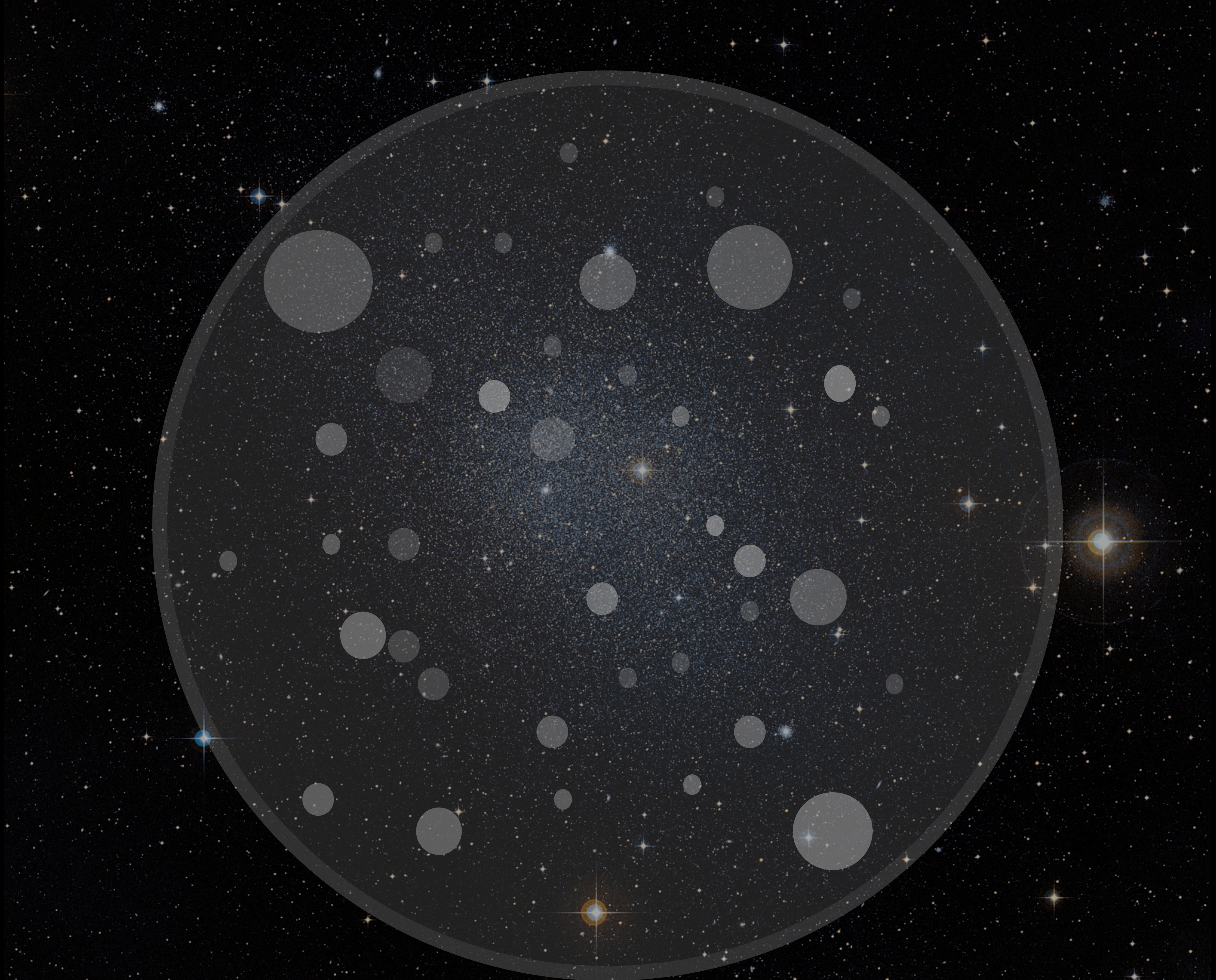
- L ~ 3E3 Msol, rc~23 pc (large size for luminosity)
- Metallicity/age undistinguishable from metal-rich stars in the Fornax dSph (Wang +19)
- Dark-Matter dominated (M/L~200 ; Pace +21)



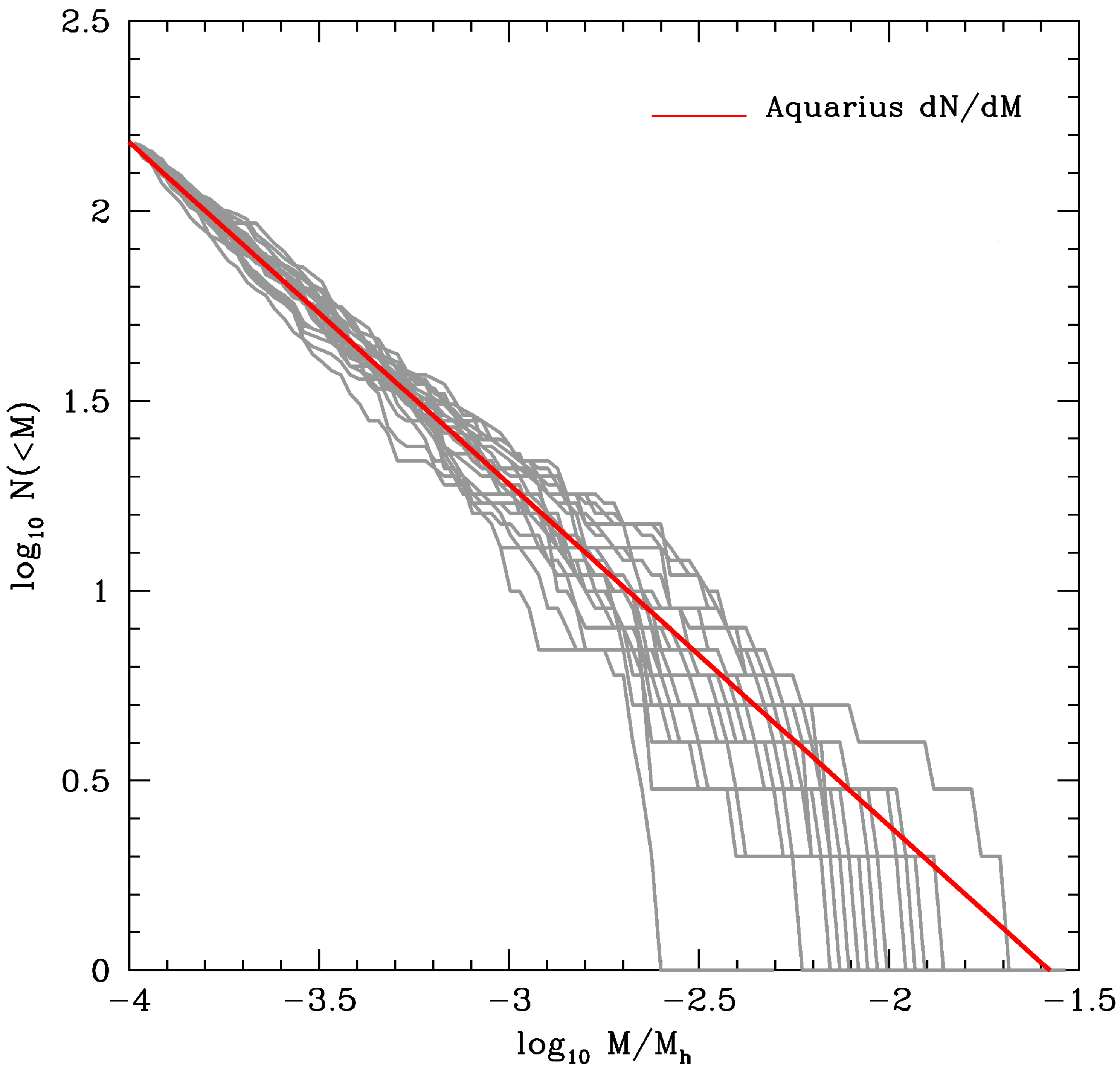
Eridanus II lone cluster

- M~ 1E3 Msol, rc~13 pc (large size for luminosity)
- Metallicity/age undistinguishable from stars in Eri II dSph (Crnojevic+18; Weisz+23; Simon+21)
- Velocity dispersion unknown

2- Dynamical heating by large population of dark subhaloes



IDEALIZED N-BODY MODELS



- **Subhalo mass function** re-scaled from Aquarius simulations
- Host & subhaloes source **static** DM potentials
- In dynamical **equilibrium**
- Opsikov-Merritt (1985) distribution function, which is isotropic at small radii and becomes **radially anisotropic velocities** at large radii.
- Number density follows the dark matter distribution
- Individual subhaloes follow **exponentially-truncated NFW profiles.**
- **Mean density** = 16 mean host density at pericentre
- Subhaloes are `**dark**' (i.e. they do not form stars in-situ).
- Stars = massless tracers in equilibrium at t=0

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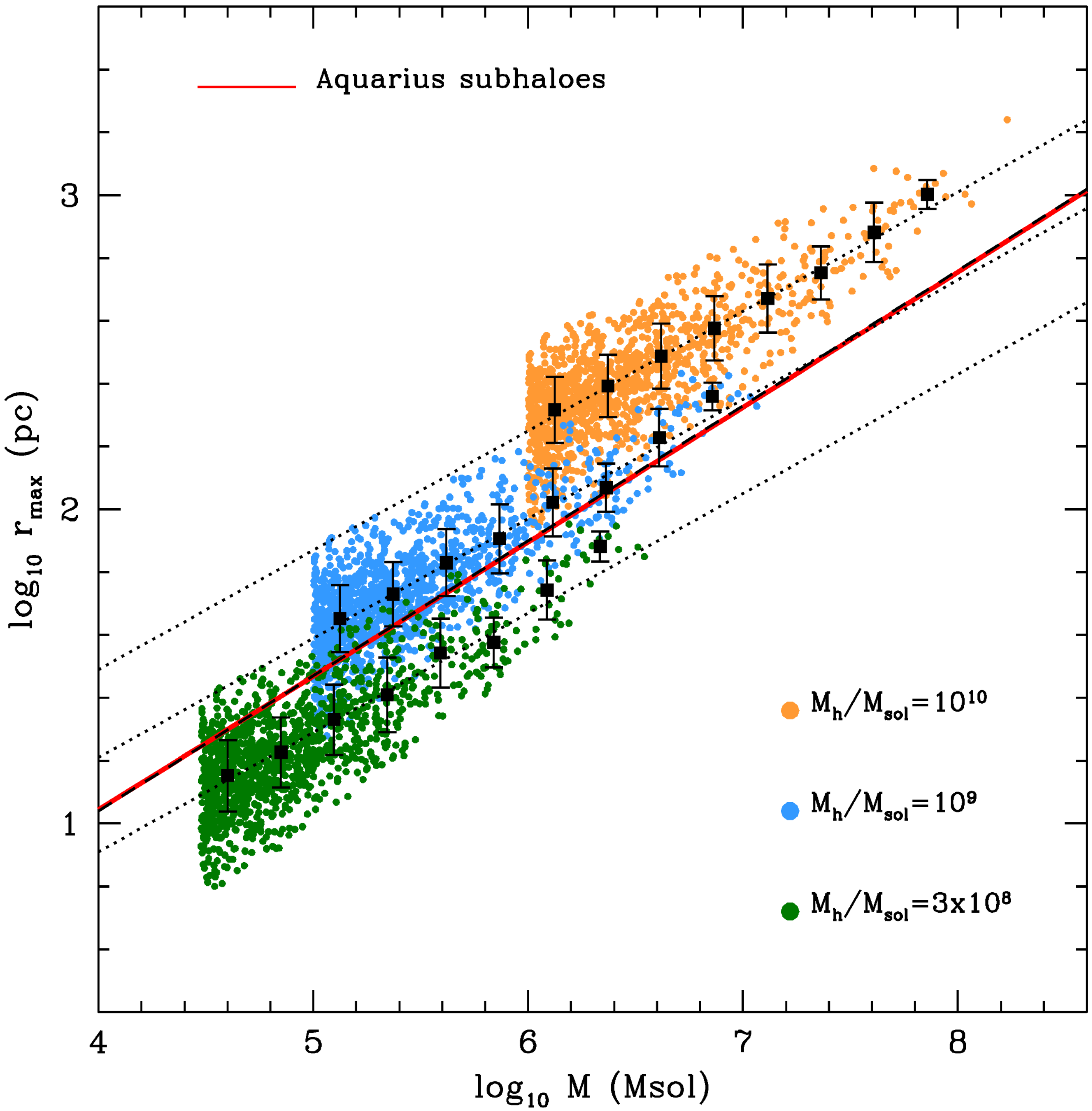

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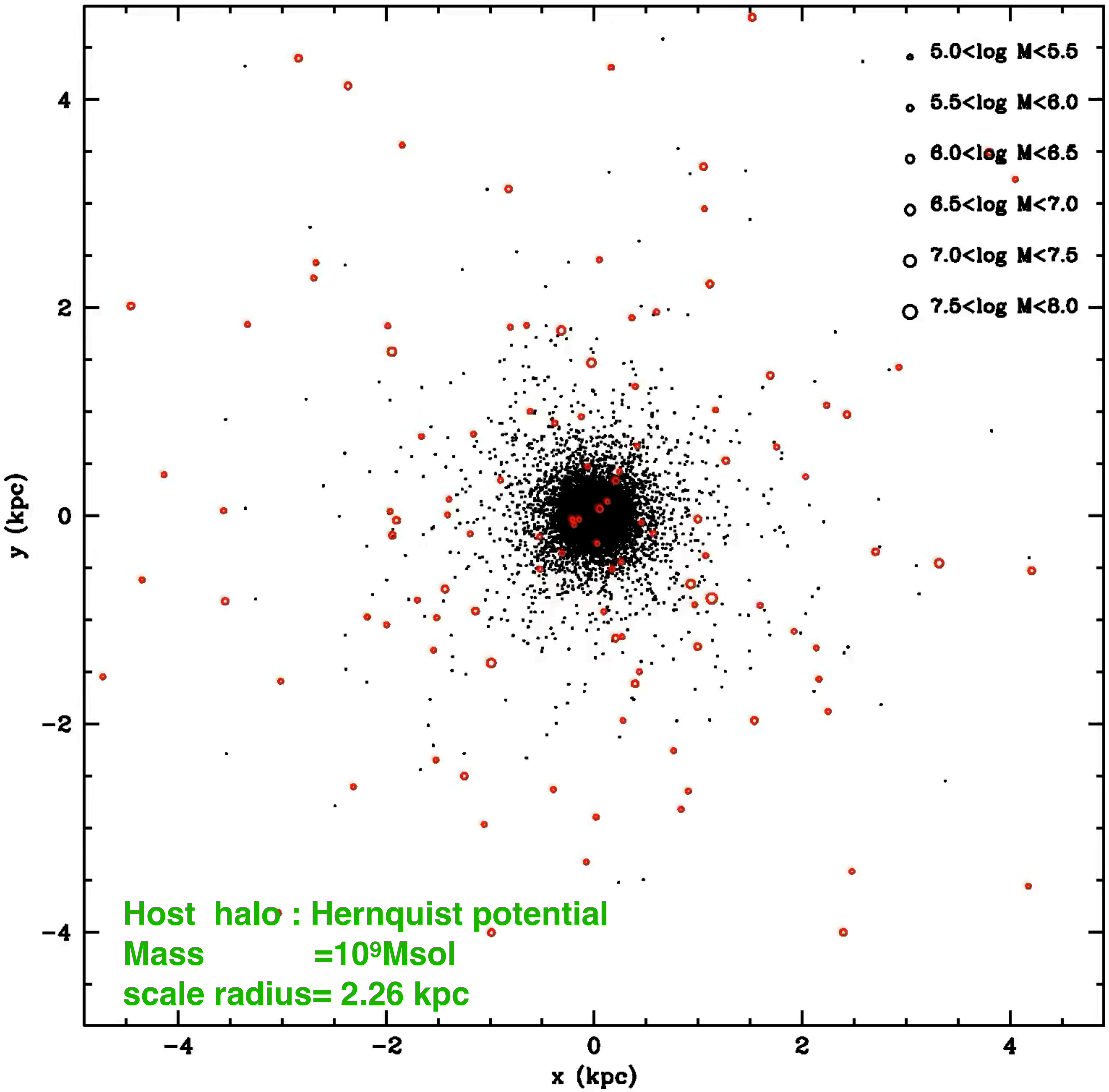

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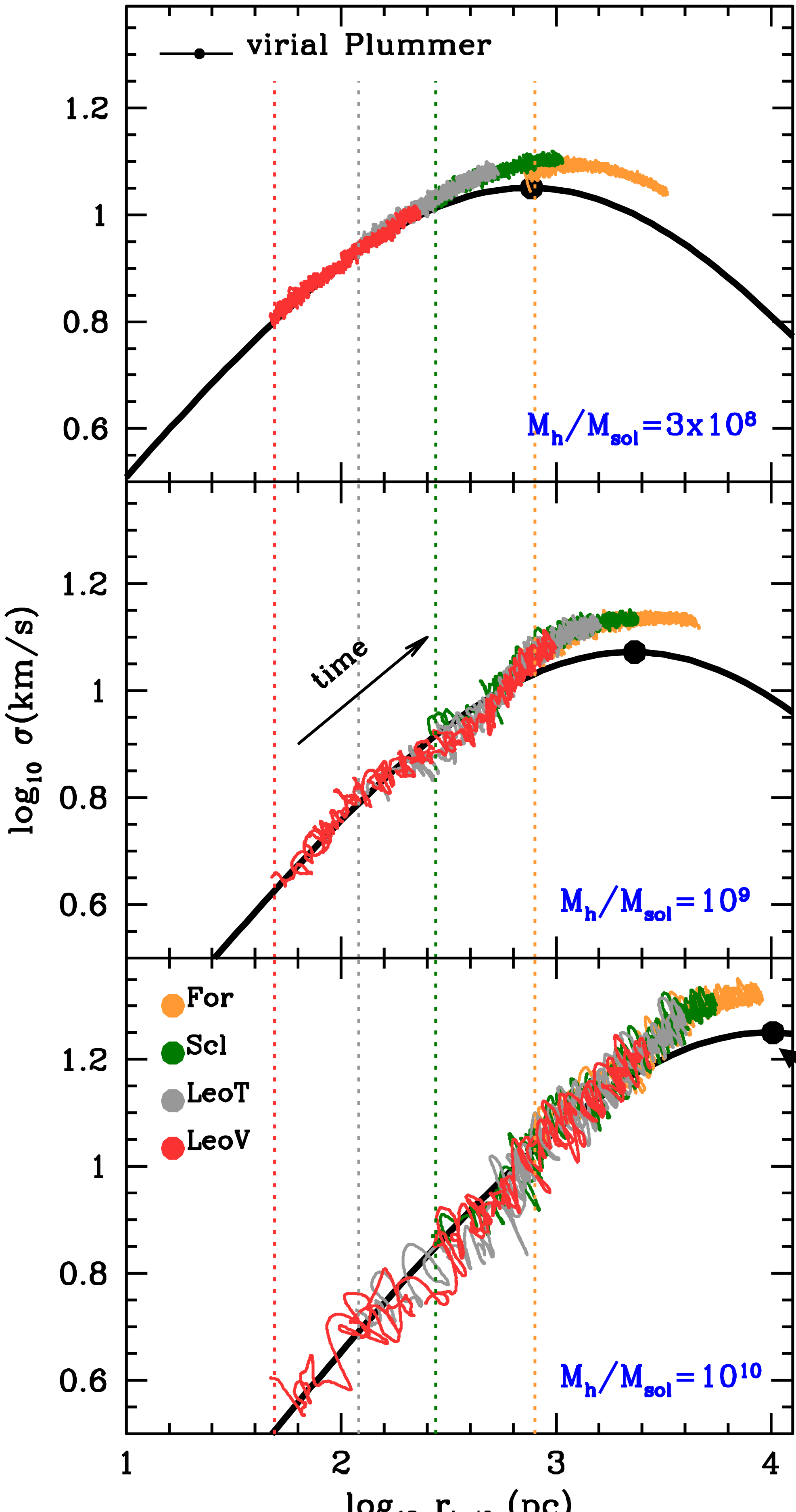
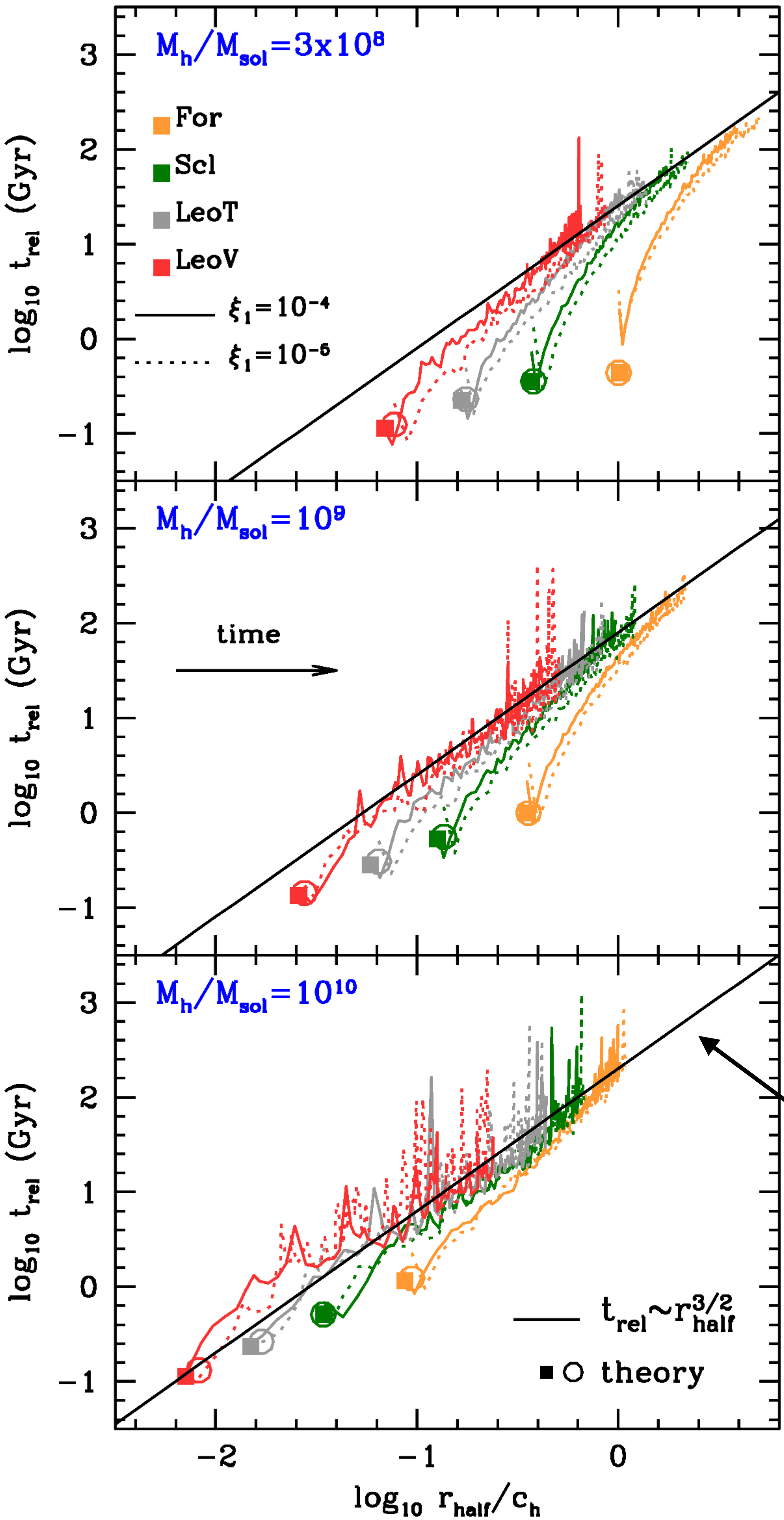
EXPANSION IN QUASI-VIRIAL EQUILIBRIUM



- stars remain close to **virial equilibrium**
- **sigma** cannot exceed maximum set by virial theorem $\boldsymbol{\sigma_{max} = \sigma(r_{half}=r_{max})}$

peak velocity radius of host halo

SELF-LIMITED EXPANSION

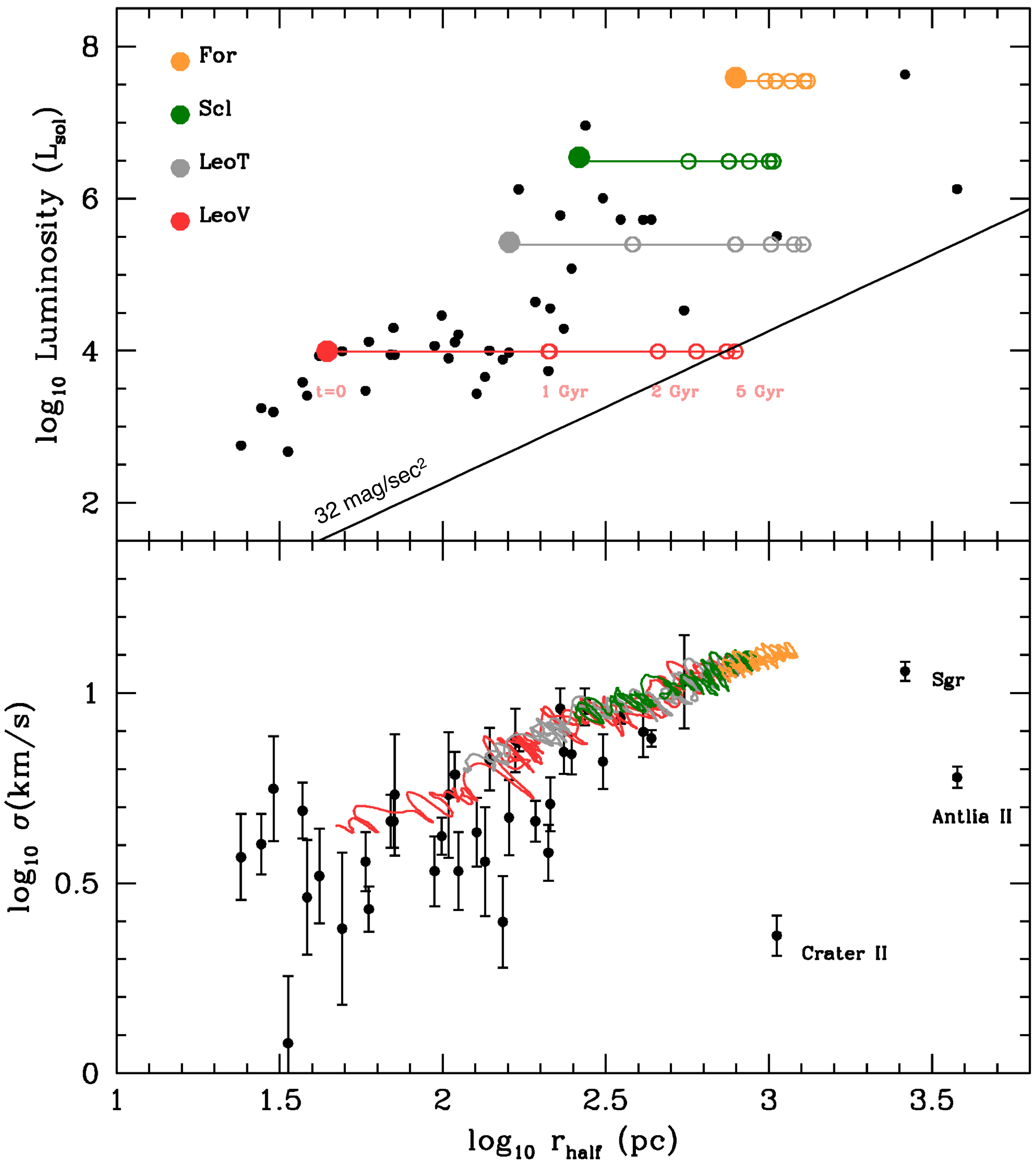


- **relaxation time** increases as galaxy expands
- expansion becomes inefficient and eventually stalls

power-law behaviour $t_{\rm rel} \sim r_{\rm half}^{3/2}$

WHY ARE ULTRA-FAINT SO SMALL?

Halo Mass =10^9Msol
scale radius= 2.26 kpc



- **ultra-faints:** relaxation times << age
- expand beyond detection within ~1–3 Gyr
- becoming UDGs (r_{half}>1kpc)

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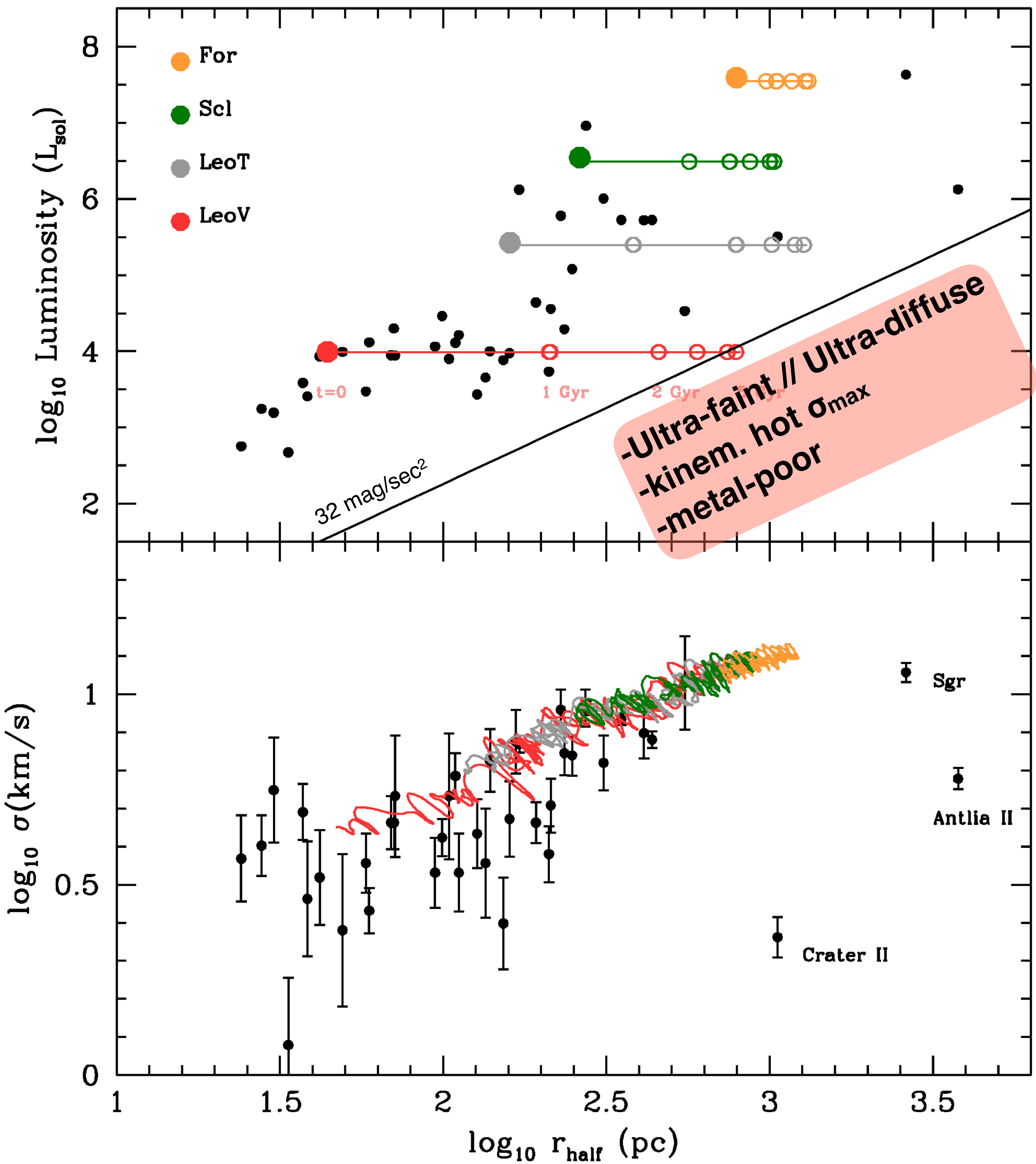

- **ultra-faints:** relaxation times << age
- expand beyond detection within ~1–3 Gyr
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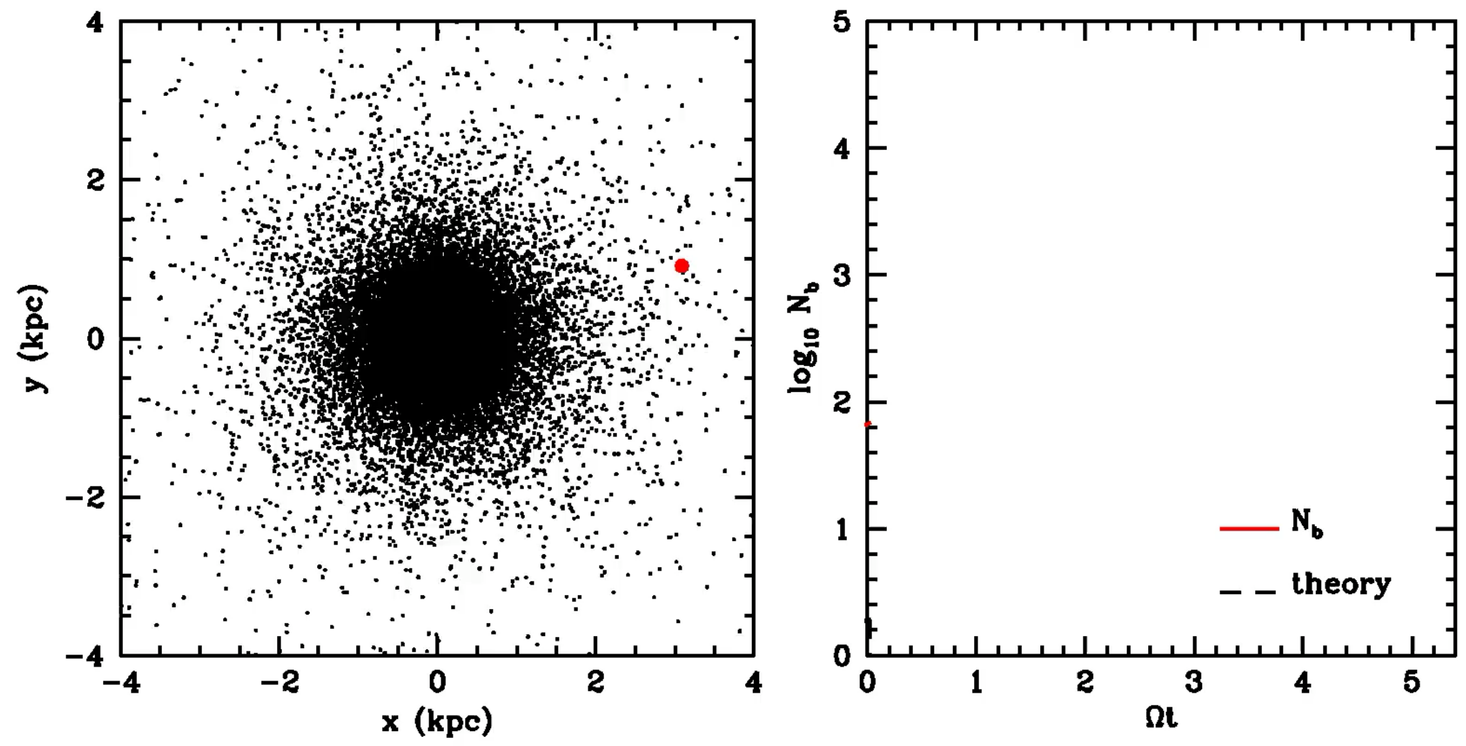
SUMMARY

- Subhaloes perturb the orbits of stars in DM-dominated dSphs
- Subhaloes **massive** enough can capture stars from the galactic field **($M>M_{min}$)**
- Subhaloes **compact** enough generate localized stellar over-densities (**$\kappa>0$** ⟷ **$\delta>1$**)
- Implication: **dark** sub-subhaloes w/ no in-situ SF may not be invisible *If they contain gravitationally-bound baryonic matter, they must emit and absorb radiation*

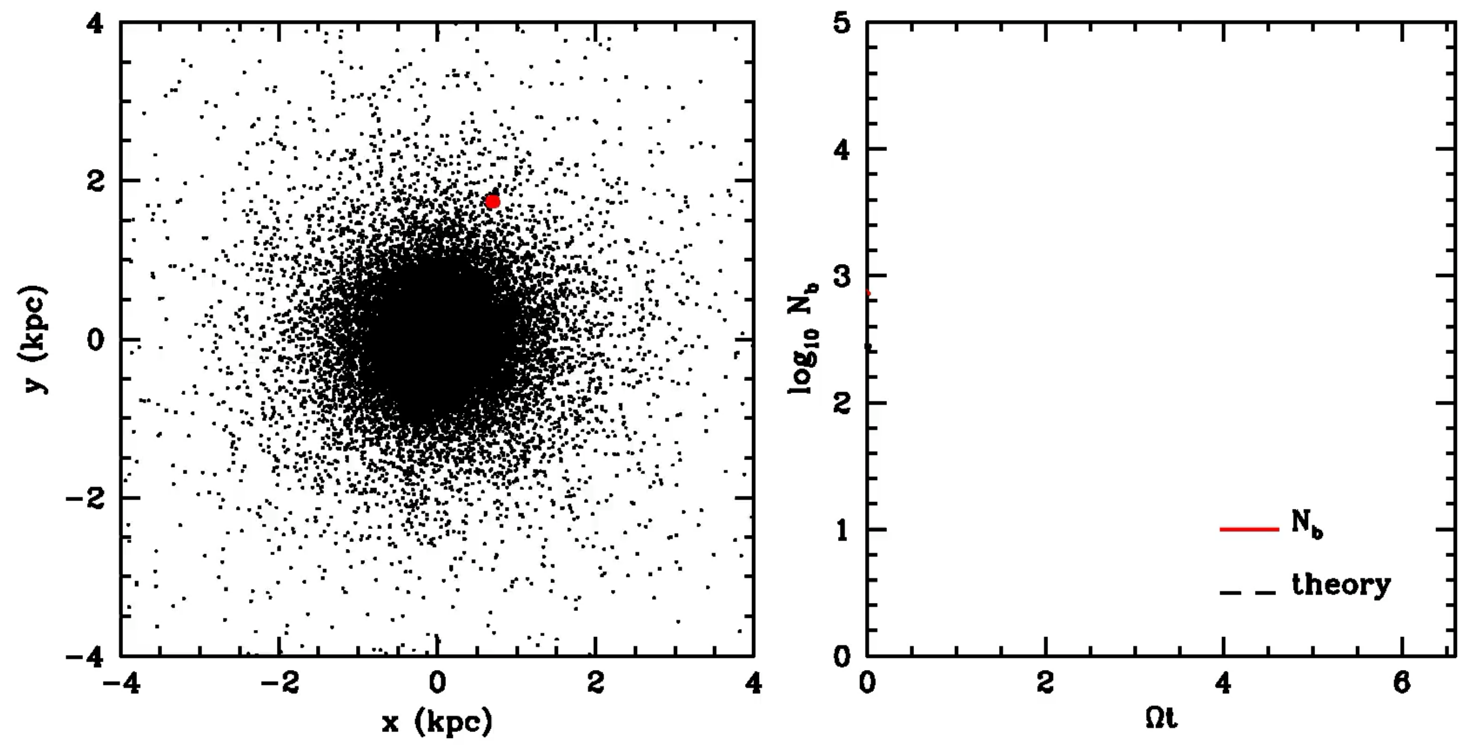
- Given analytical limitations, follow-up **N-body** modelling of sub-subhalo populations needed
- **Predictions** on Number & Masses & scale radii of dSph sub-subhaloes are **very uncertain**
- Differences between **CDM, WDM** and **SIDM** to be expected
- **Observations** of objects like F6 and Eri II clusters are still **poor** (current photometric data covers ~ 1% members. Only 16 stars of F6 with measured velocities. No kinematic information for Eri II cluster

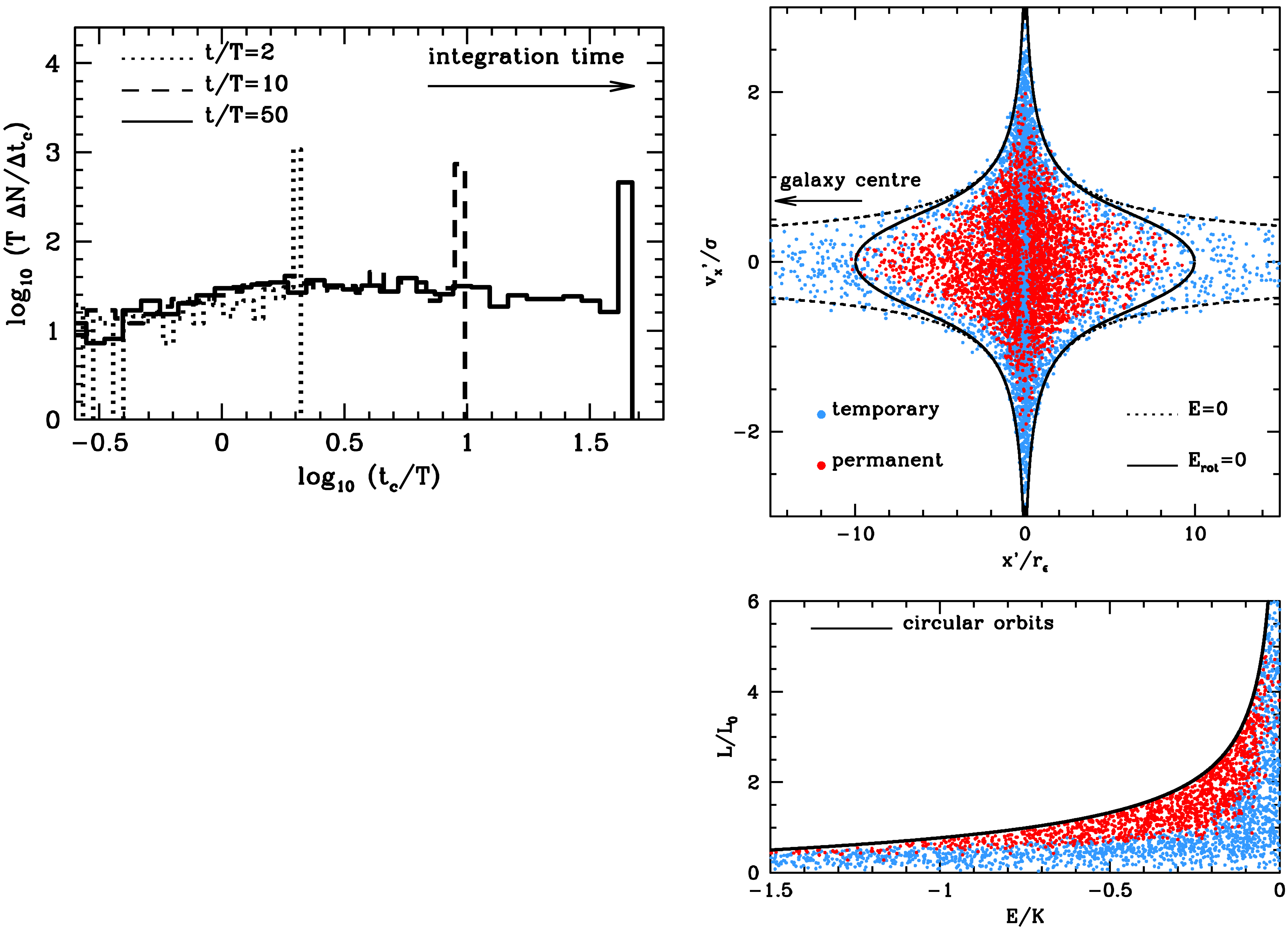
- dSphs **expand** due to subhalo perturbations
- Self-similar gravothermal expansion **saturates** as **$r_{half} \sim r_{max}$**
- Small sizes of ultra-faint dSphs are puzzling

HIGH-ECC ORBIT

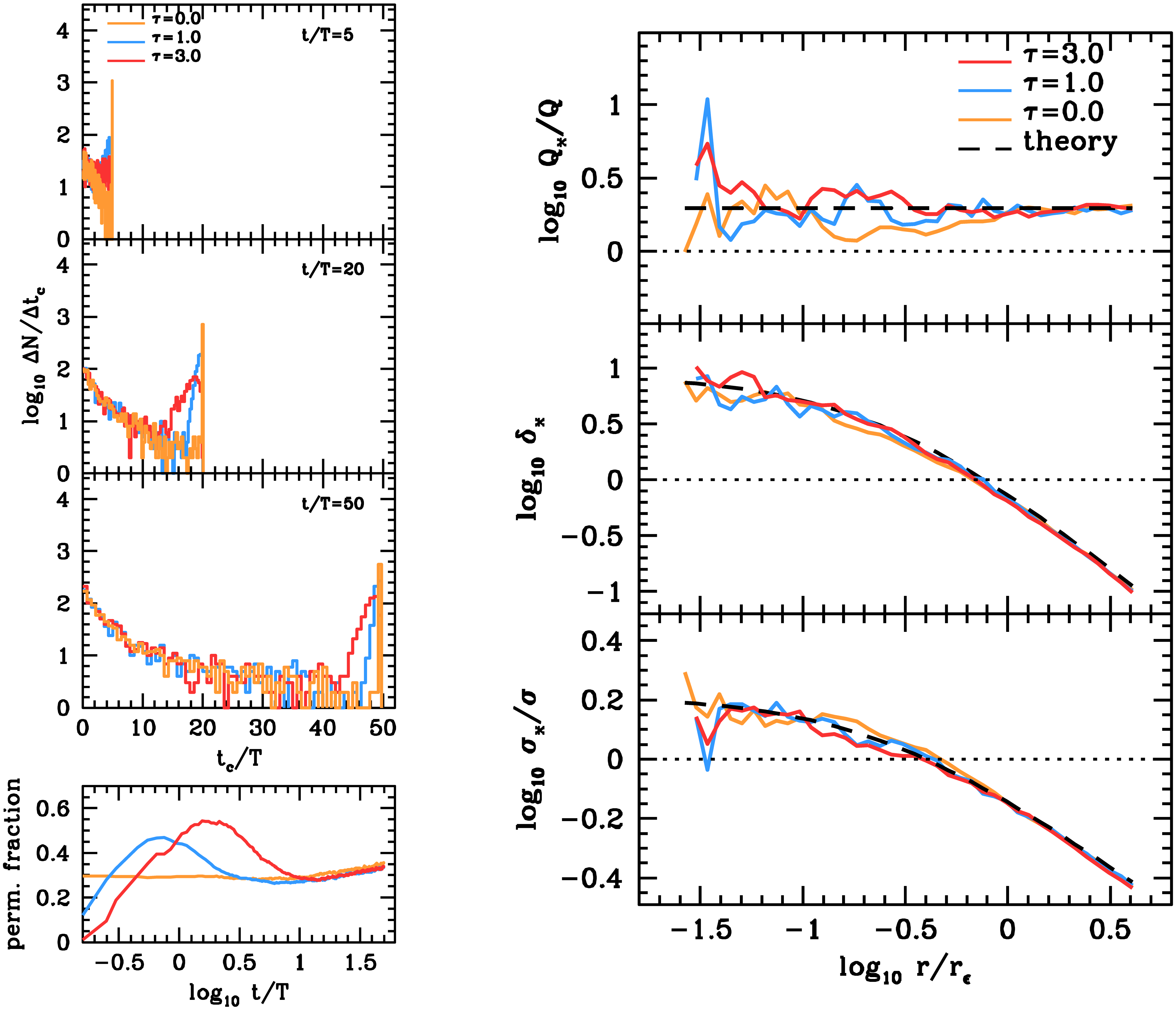


LOW-ECC ORBIT

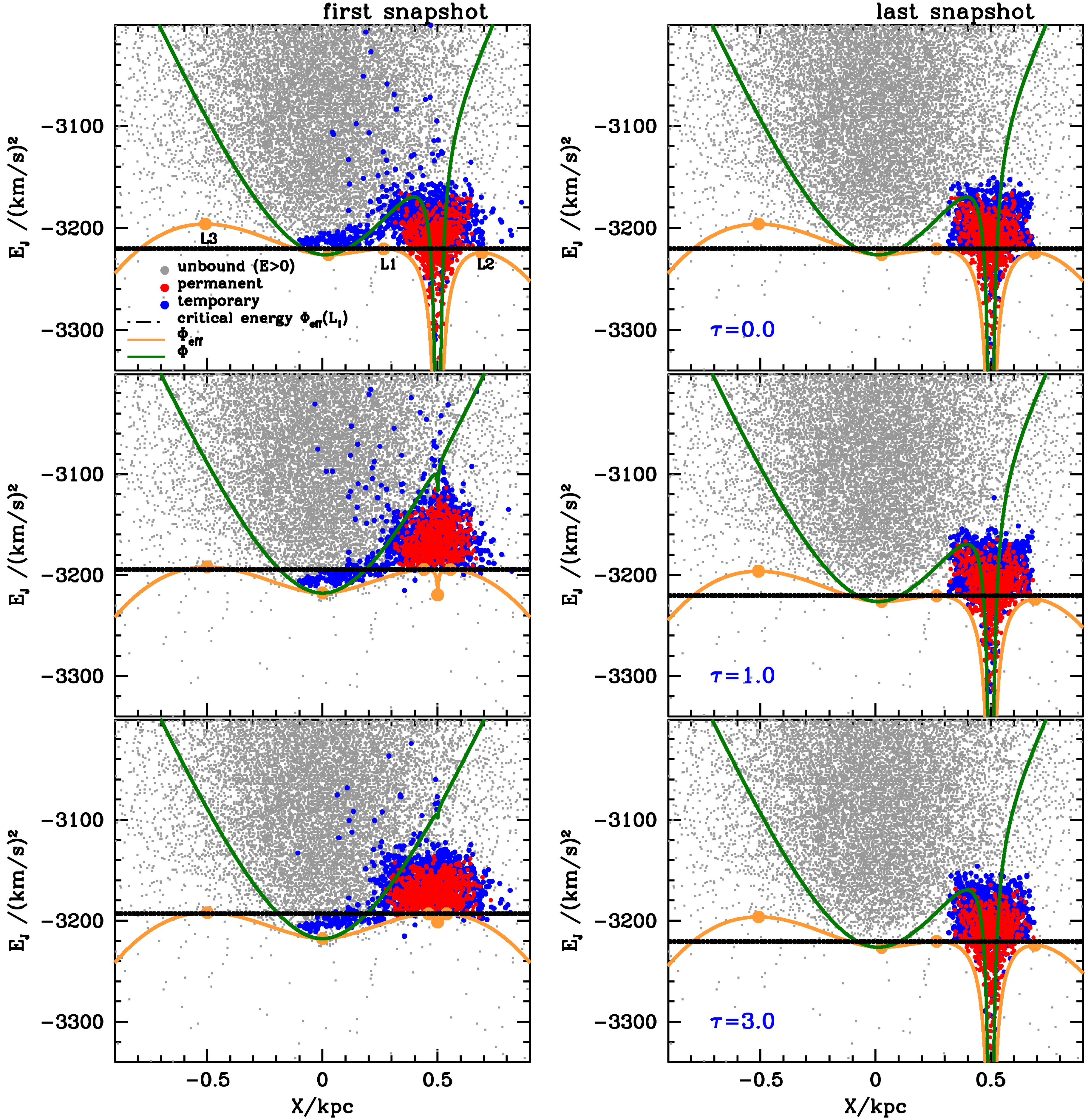




t/T=2
t/T=10
t/T=50
integration time
$\log_{10}$ (T ΔN/Δt_c)
$\log_{10}$ (t_c/T)
galaxy centre
v_x'/σ
x'/r_ϵ
temporary
permanent
E=0
E_{rot}=0
circular orbits
L/L_0
E/K

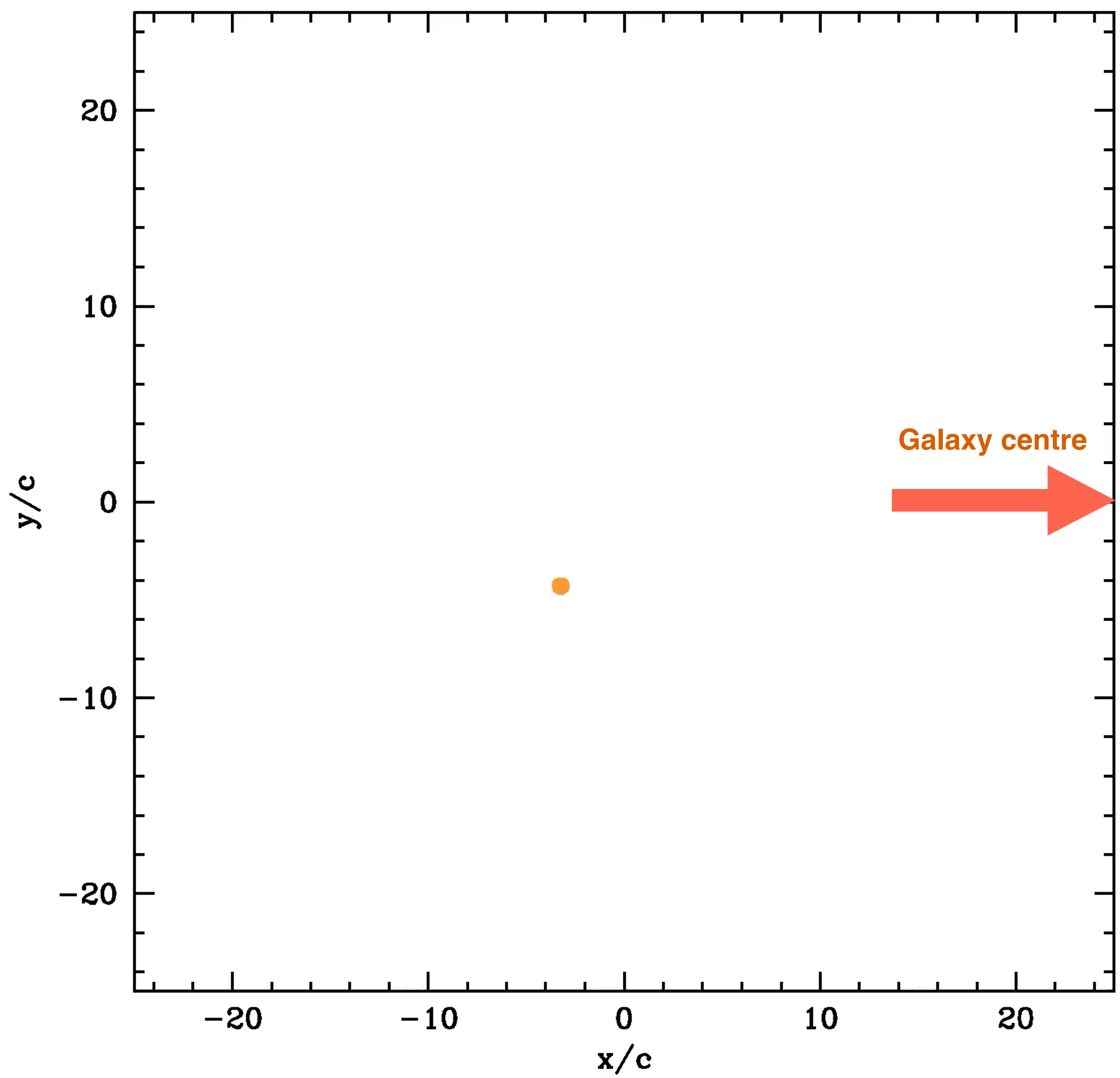


τ=0.0
τ=1.0
τ=3.0
t/T=5
t/T=20
t/T=50
$\log_{10} \Delta N/\Delta t_c$
t_c/T
perm. fraction
$\log_{10} t/T$
τ=3.0
τ=1.0
τ=0.0
theory
$\log_{10} Q_*/Q$
$\log_{10} \delta_*$
$\log_{10} \sigma_*/\sigma$
$\log_{10} r/r_\epsilon$



first snapshot
last snapshot
E_J /(km/s)²
X/kpc
L1
L2
L3
unbound (E>0)
permanent
temporary
critical energy Φ_eff(L_1)
Φ_eff
Φ
τ=0.0
τ=1.0
τ=3.0

I mean… really complicated!

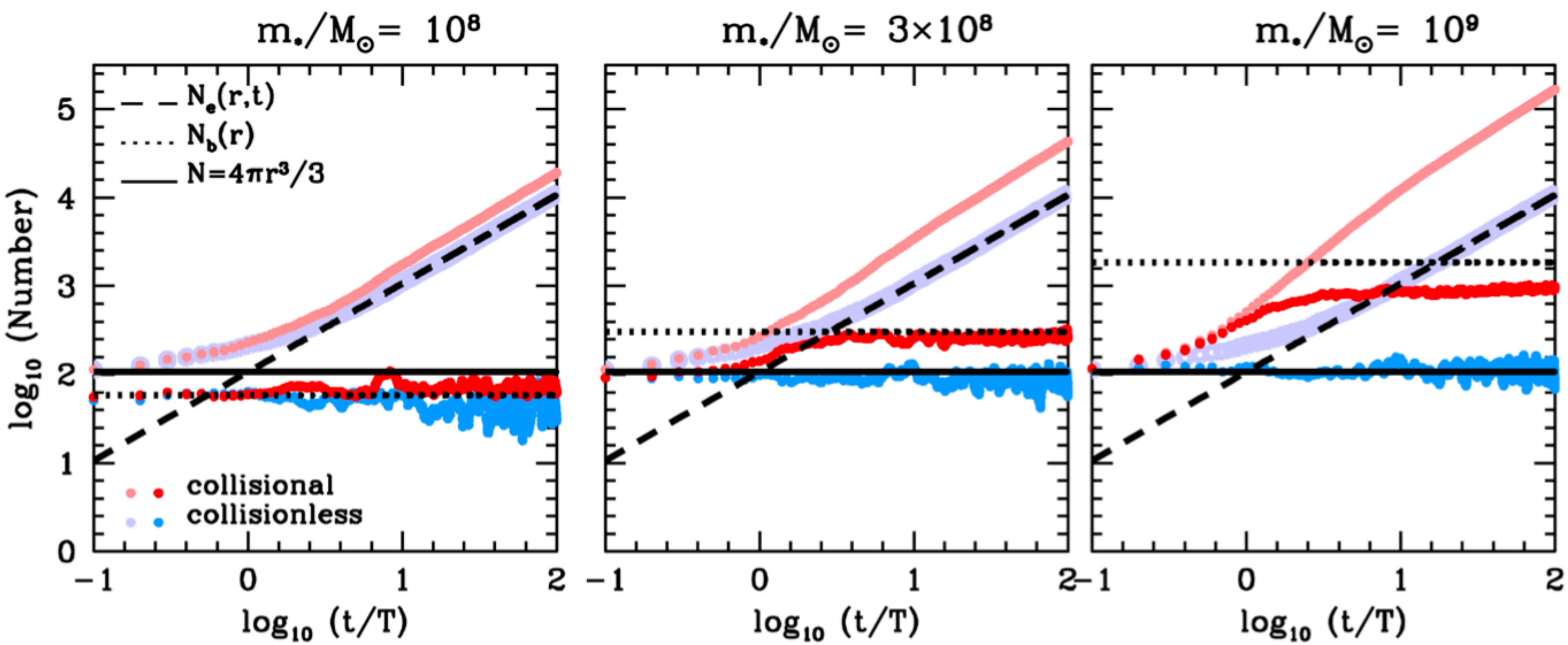


- Point-mass perturber moving on a circular orbit around a MW-like potential
- Trajectory of a field particle during time interval with E<0
- co-rotating frame centred at point-mass

Captured particles follow **Irregular** orbits

No integrals of motion are conserved

Steady-state population of trapped particles



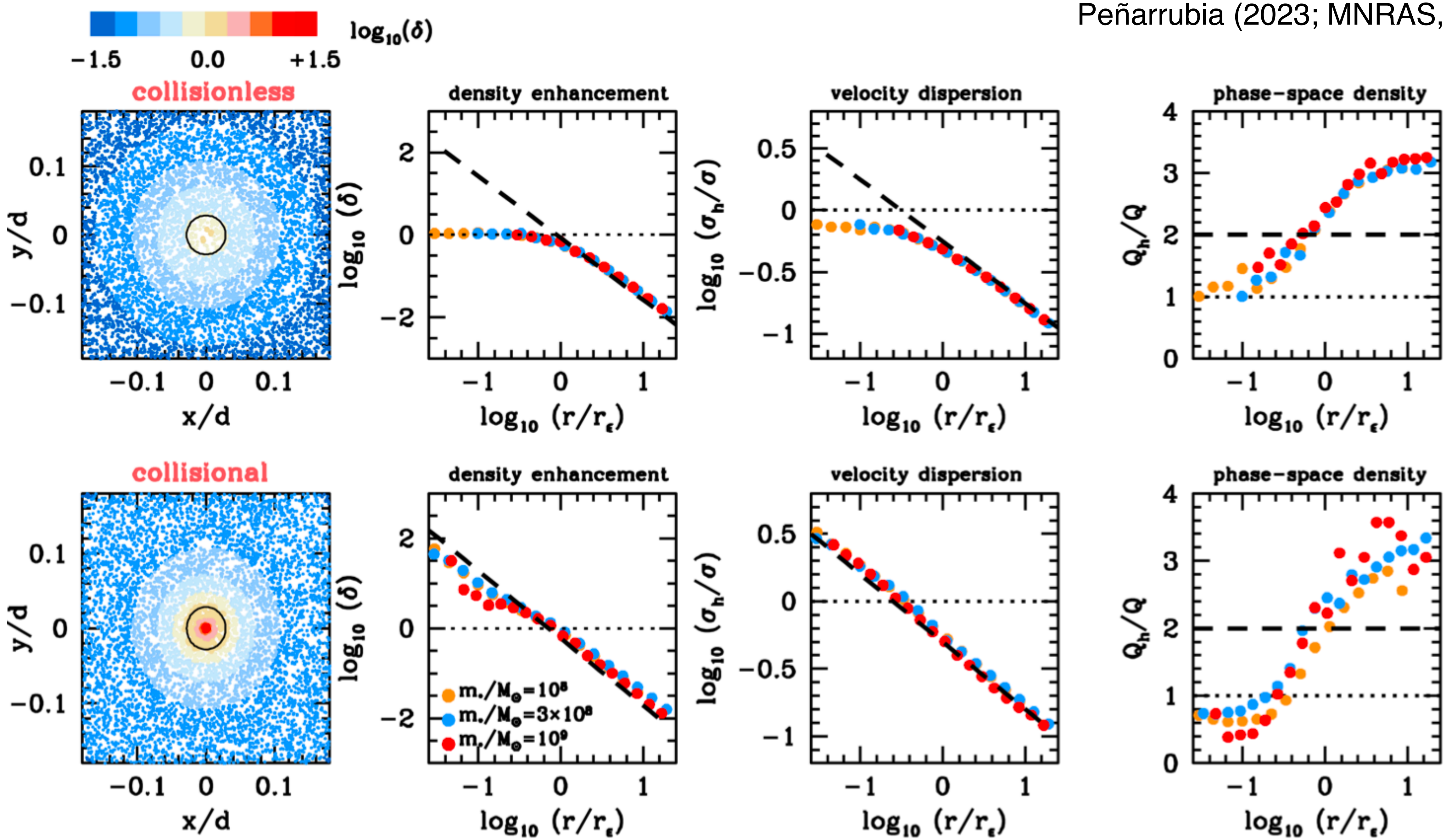
A **steady-state** is reached as the number of particles being captured equals that being unbound.

This happens on a time-scale of the order of the crossing time $\mathbf{T \sim r / \langle v^2 \rangle^{1/2}}$

The gravitational attraction from the subhalo increases the number of bound particles (**N_b**) with respect to the number expected in the unperturbed field density (**N**)

Tests with numerical models

Peñarrubia (2023; MNRAS, 519, 1955)

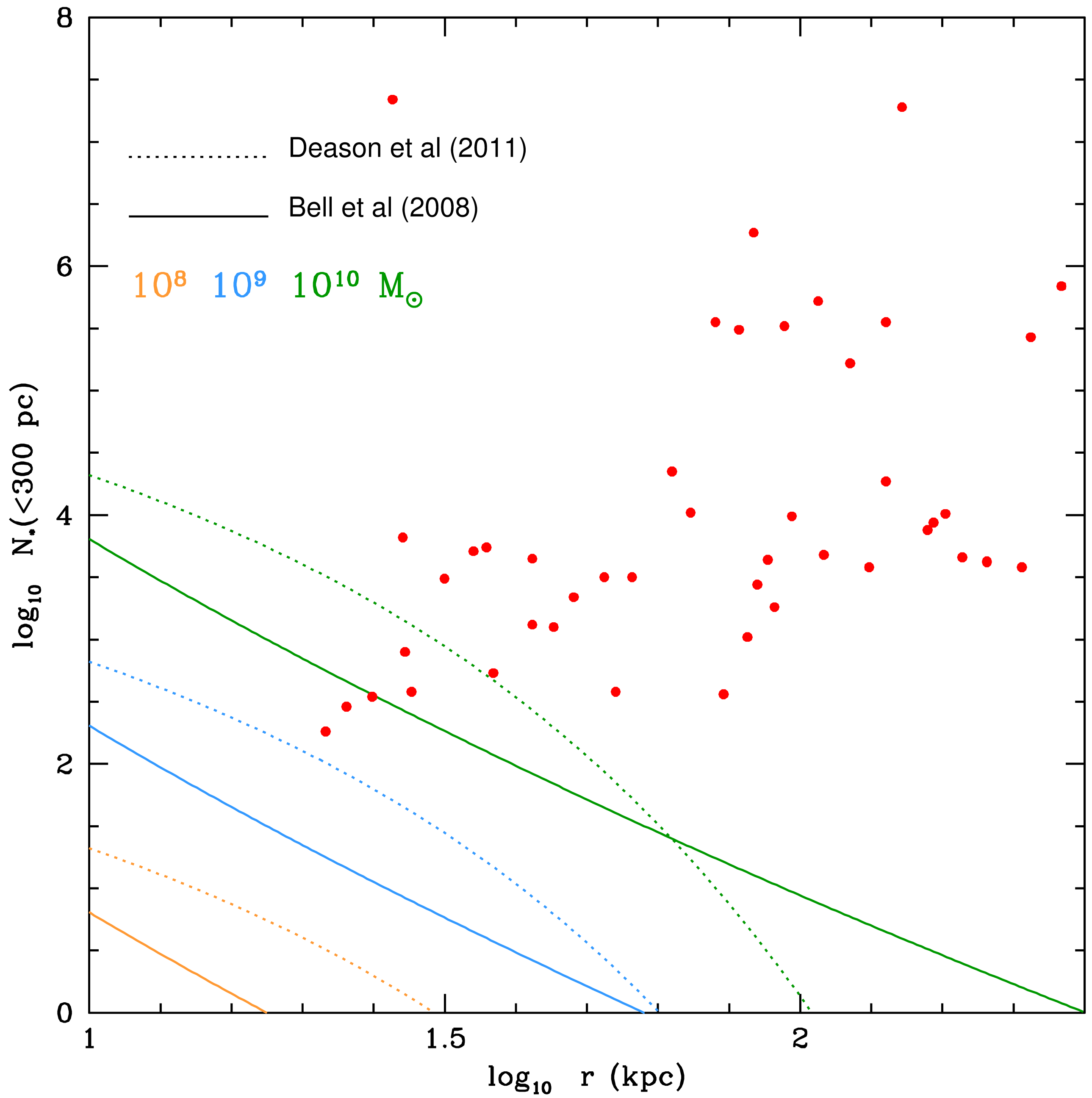


The effect of the subhalo attraction arises below a “**thermal critical radius**”

$$r_\epsilon = \left(\frac{16}{9\pi}\right)^{1/3} e^{-V_\bullet^2/(3\sigma^2)} \frac{GM_\bullet}{\sigma^2}$$

at $r=r_\varepsilon$ potential energy **$W=-GM_\bullet/r_\varepsilon$** approx. equal to mean kinetic energy of field stars **$K=3\sigma^2/2$**

(preliminary) Milky Way subhalo estimates



This estimate:

- subhalo = point-mass
- circular orbit $V = V_c$ (r)
- MW potential (McMillan 07)
- 2 stellar halo models in equilibrium
- Compute N_* = Number of MW halo stars within < 300 pc with E<0

Results:

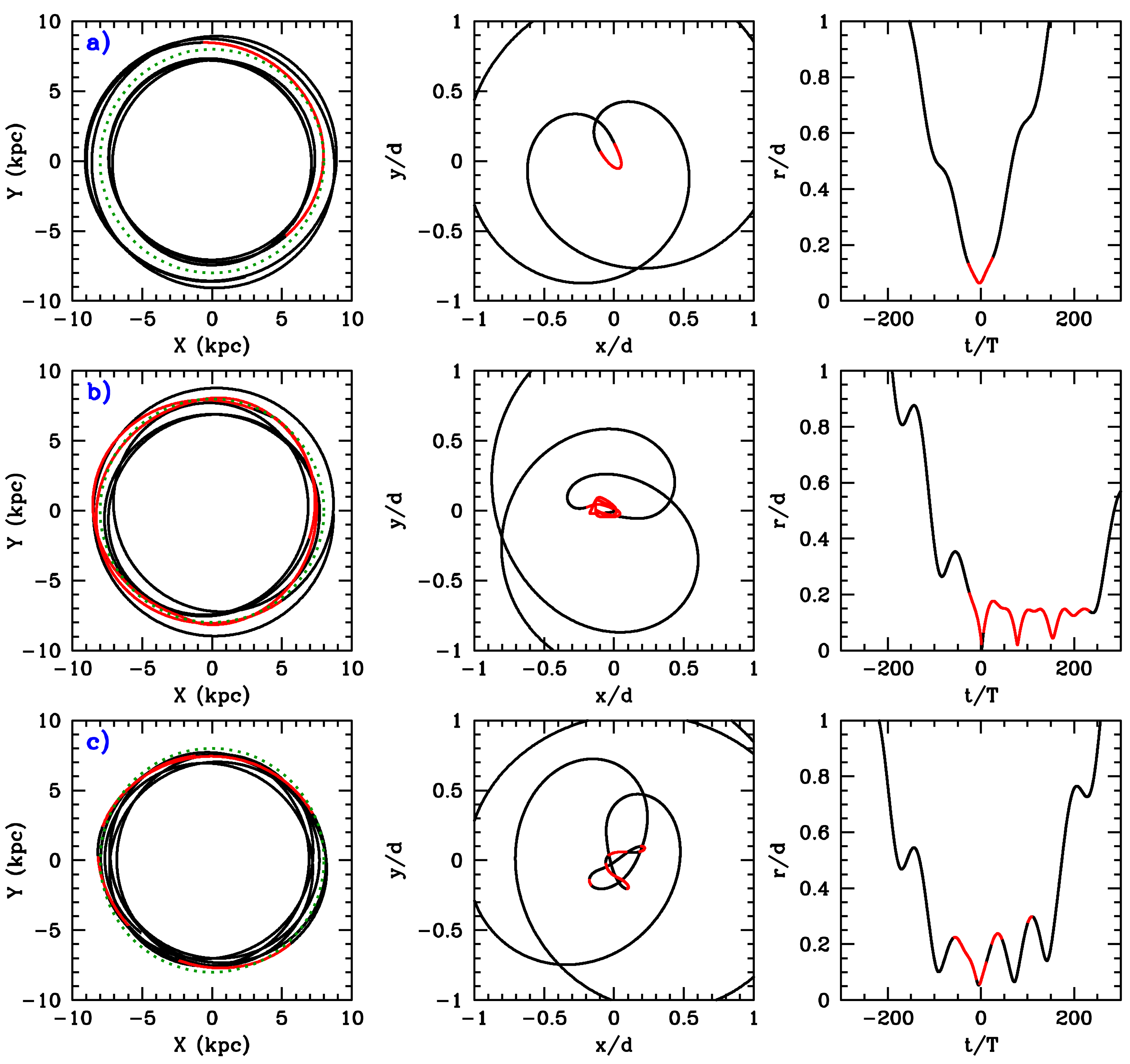
ve

- Estimates very sensitive to the properties of the outer halo (largely unknown)
- r >100 kpc too few stars to capture
- r < 50 kpc number of captured field stars comparable to dSph pop. formed in-situ

Conclusions:

- (nearby) dSphs may be surrounded by a halo of captured field (MW halo) stars.
- Captured stars likely in steady-state, with kinematics tracing the subhalo potential.
- dark subhaloes <10^8 M_{sol} not completely dark they can capture interstellar (baryonic) particles. Are they visible/detectable?
- Models for individual MW dSphs running as we speak … TBC

CAPTURE OF INTERSTELLAR OBJECTS



1) Bound particles show extremely intricate trajectories

2) Tidal trapping leads to transient capture events

STATISTICAL THEORY

Step 0 : approximations

* local approximation. $$n(\mathbf{R}_\star + \mathbf{r}) \approx n(\mathbf{R}_\star) \equiv n$$

at small distances from the point-mass $r \ll d(\mathbf{R}_\star) \equiv |\nabla\rho/\rho|^{-1}_{\mathbf{R}_\star}$

* Maxwellian approximation. $$p(\mathbf{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\mathbf{v}+\mathbf{V}_\star)^2}{2\sigma^2}\right],$$

STATISTICAL THEORY

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Step 1: energetically-bound particles within volume V=4 π r³ /3.

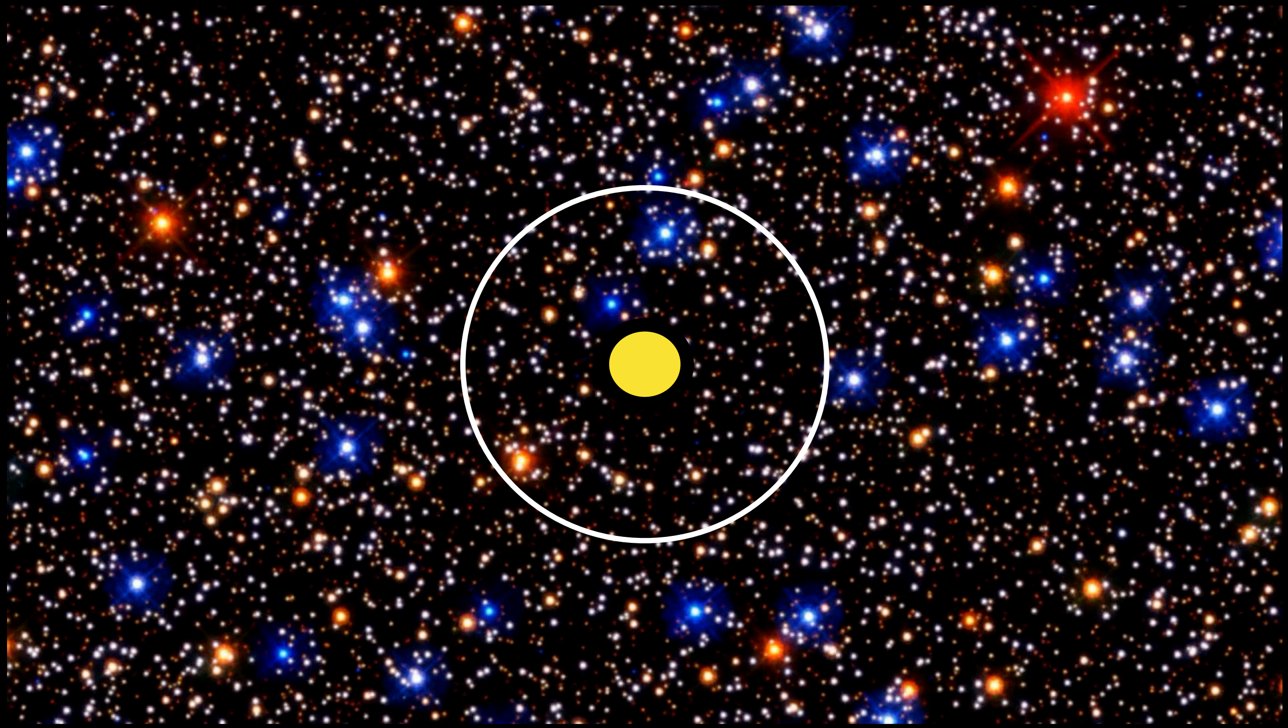
Galaxy= thermal bath (perturbations by point-mass neglected)

$$N_b(r) = \int_V d^3r\, n(\mathbf{r}) \int_{\mathbf{E}<0} \mathbf{d^3v\, p(v)} \simeq \frac{32\sqrt{\pi}}{9}(Gm_\star)^{3/2}\frac{n}{\sigma^3}e^{-V_\star^2/(2\sigma^2)}r^{3/2}$$

$$v_e = \left(\frac{2Gm_\star}{r}\right)^{1/2} \ll \sigma \quad \Rightarrow \quad r \gg r_0 = \frac{2Gm_\star}{r}$$

`critical radius'

STATISTICAL THEORY



Step 2 : accretion rate

$$C_{\rm acc} \equiv \lim_{\Delta t \to 0} \frac{\Delta N_b}{\Delta t}$$

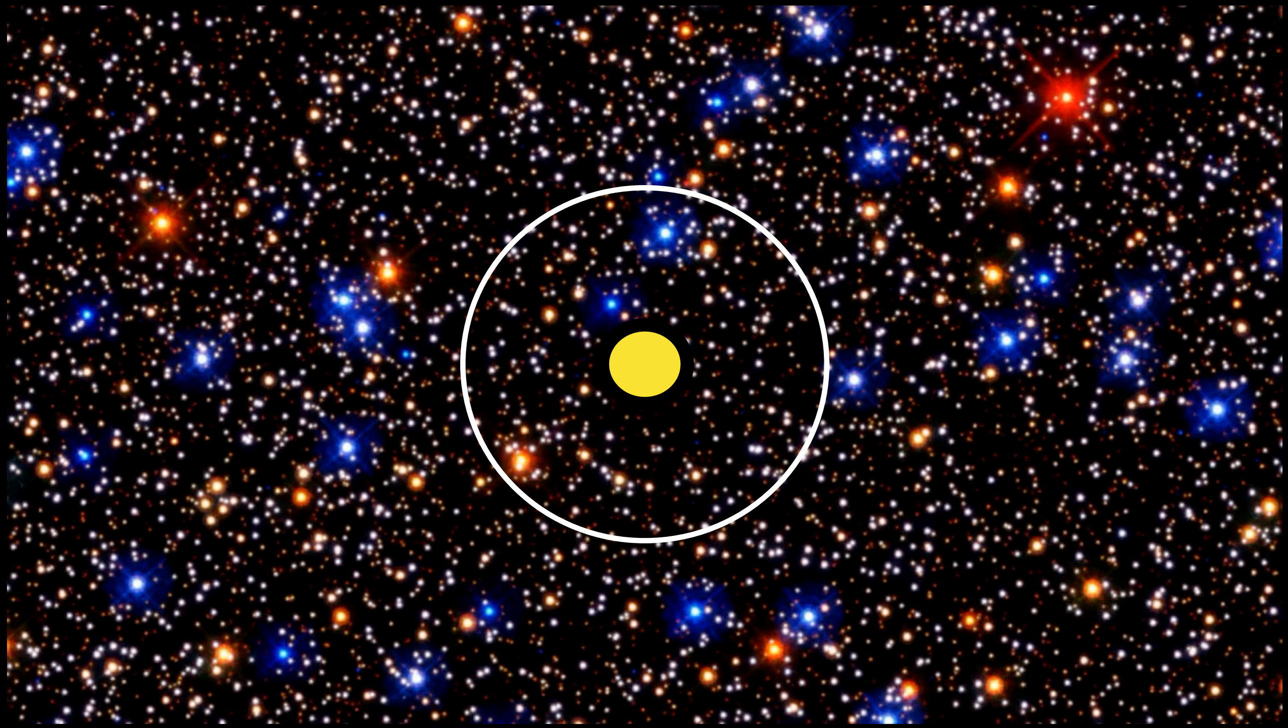
Complication: position of particles correlated as the time interval Δt -> 0

E.g. If time interval sufficiently small, no particles have time to enter/leave the volume

Correlations vanish when time interval long enough

STATISTICAL THEORY

Step 2 : accretion rate



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$$P_N(t)dt = e^{-t/T} \frac{dt}{T}$$

probability **N** particles inside **V** follows a law of decay that is analogous to the law of decay of radioactive substances

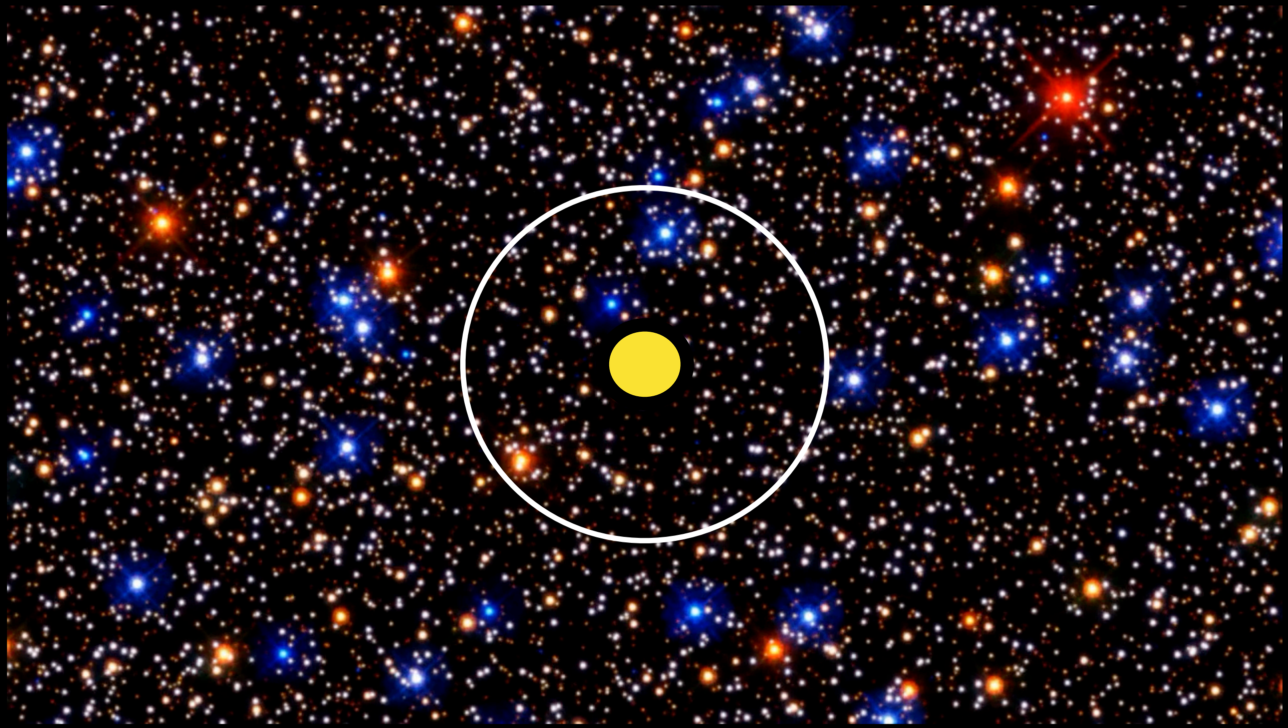
Smoluchowski (1916)

$$T(r) = \sqrt{\frac{2\pi}{3}} \frac{r}{\langle v^2 \rangle^{1/2}}$$

time-scale ~ crossing time

STATISTICAL THEORY

Step 2 : accretion rate



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time-scale ~ crossing time

$$W(N_e) = \frac{e^{-NP}(NP)^{N_e}}{N_e!}$$

Poisson probability $\mathbf{N_e}$ particles enter volume **V**

$$N_e = NP$$

Average number of particles *entering* the volume **V** == number *leaving* it (equilibrium)

$$P = \frac{\Delta t}{T}$$

Whahrscheinlichtkeitsnachwirkung = probability after-effect factor

STATISTICAL THEORY

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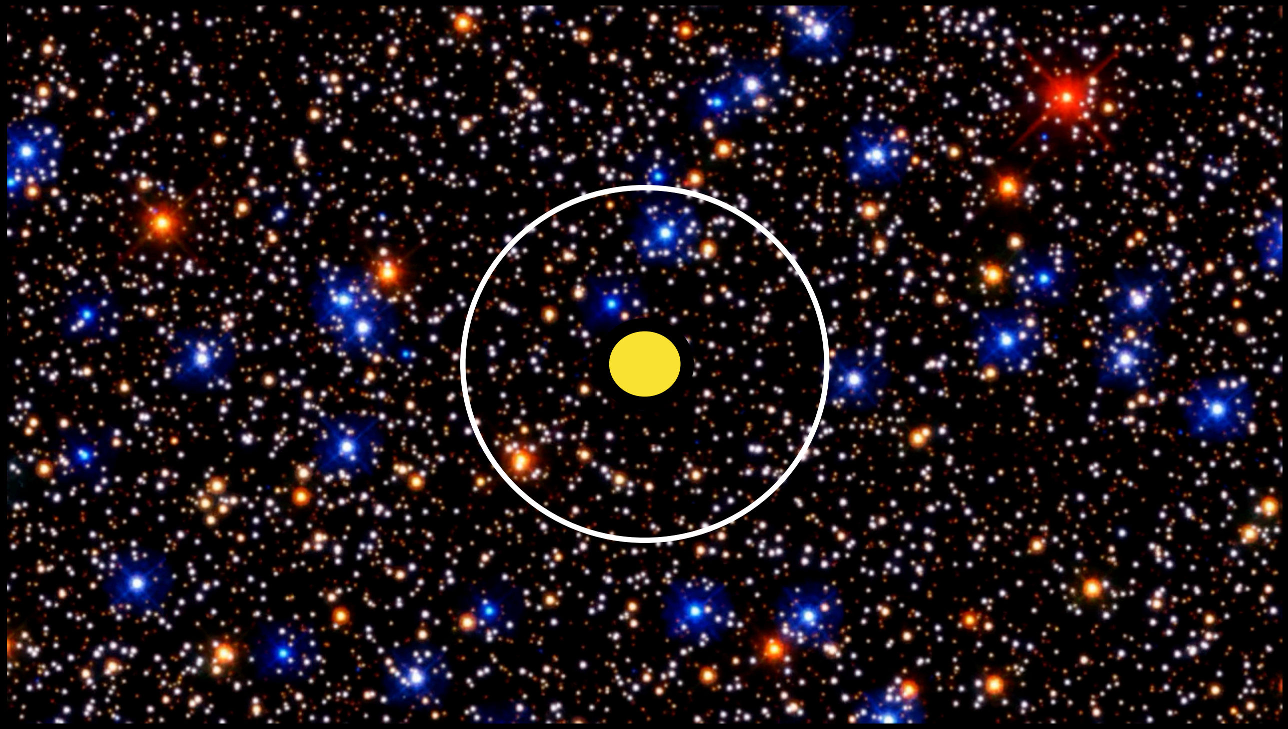


Replace average number of particles *entering* the volume **V** $\quad N_e = N P$

with average number of particles *entering* the volume **V** **with E<0** (*) $\quad \Delta N_b = N_b\, P$

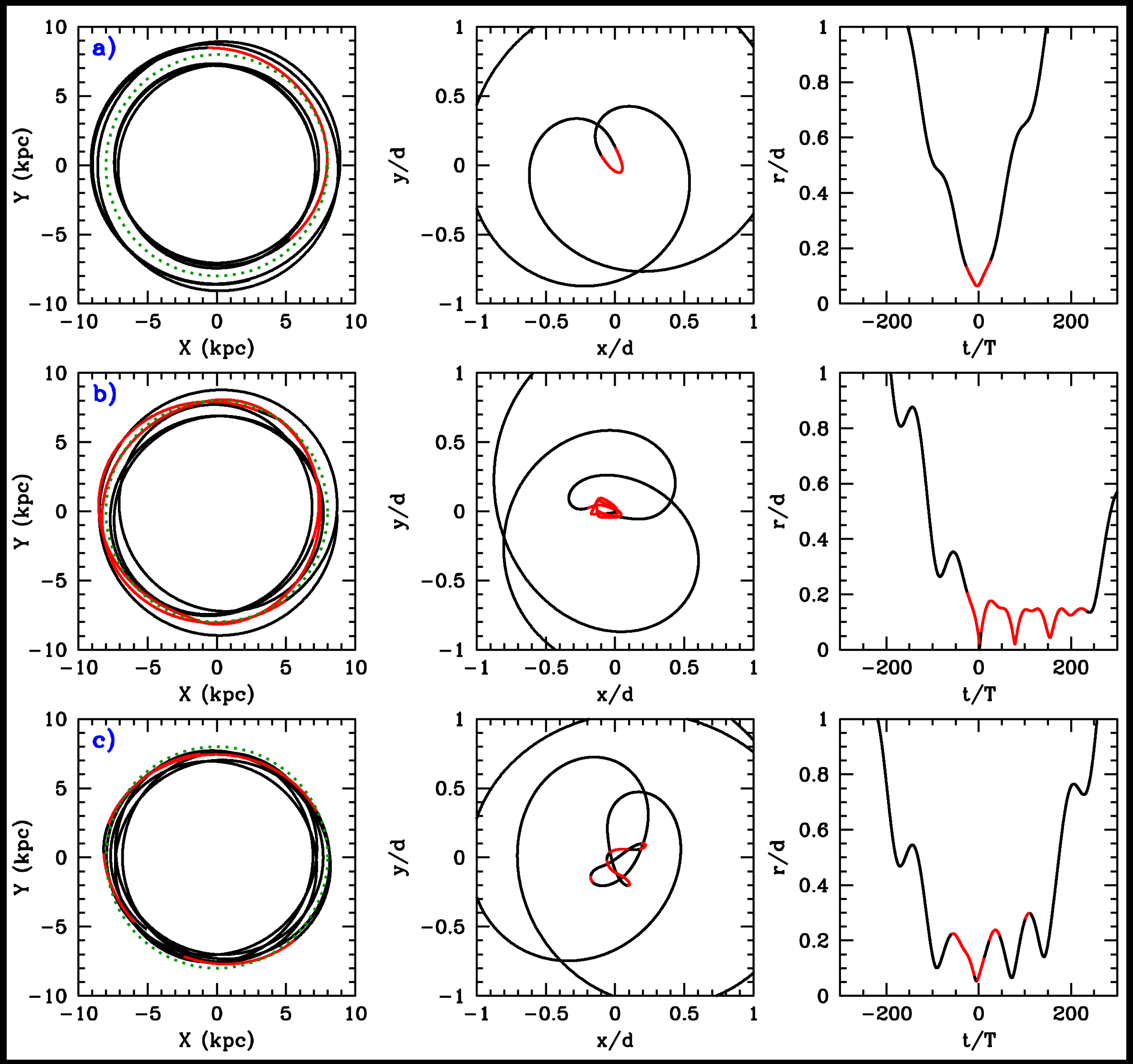
$$C_{\rm acc}(r) = \lim_{\Delta t \to 0} N_b \frac{P}{\Delta t} = \frac{N_b}{T}$$

$$N_{\rm acc}(r,t) = \int_0^t dt\, C_{\rm acc} = N_b\, \frac{t}{T}$$

*(*) the statistical assumption here is that particles within the volume **V** are statistically uncorrelated — regardless of energy **E***
Physically, this assumption is implicit to the thermal bath approach

STATISTICAL THEORY

Step 3 : survival



Complication: theory cannot predict how long particles remain bound

Run N-body models of particles moving in a Dehnen (1993) and compute distribution of survival times (**t_{surv}**)

$$\ddot{\mathbf{R}}_\star = -\nabla\Phi_g(\mathbf{R}_\star).$$

$$\ddot{\mathbf{R}} = -\frac{Gm_\star}{|\mathbf{R}-\mathbf{R}_\star|^3}(\mathbf{R}-\mathbf{R}_\star) - \nabla\Phi_g(\mathbf{R})$$

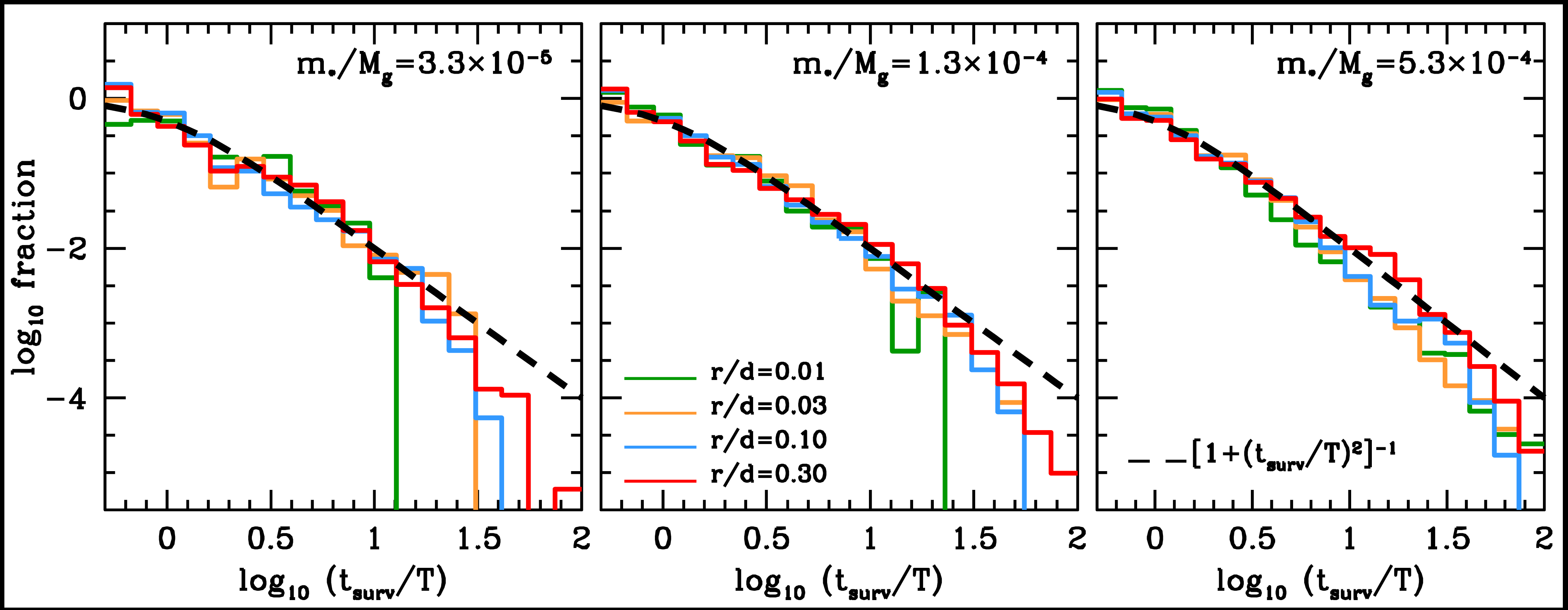
$N_g = 10^{10}$
M_g= 1.84 10^{12} M_{sol}
r_g = 15.3 kpc

3 point-masses:
m^*/M_g = 3.3×10^{-5}, 1.3×10^{-4}, 5.3×10^{-4}
m^*= 6×10^7 M_{sol}, 2.4×10^8 M_{sol}, 9.8×10^8 M_{sol}

very far from resolution required to model accretion onto Solar System
$N_g \sim 10^{20}$ (!!)

STATISTICAL THEORY

Step 3 : survival



Define: ***Dynamical lifetime function*** **$f_{surv}(t)$:= fraction of objects that remain bound as a function of time since accretion**

Empirical fit:

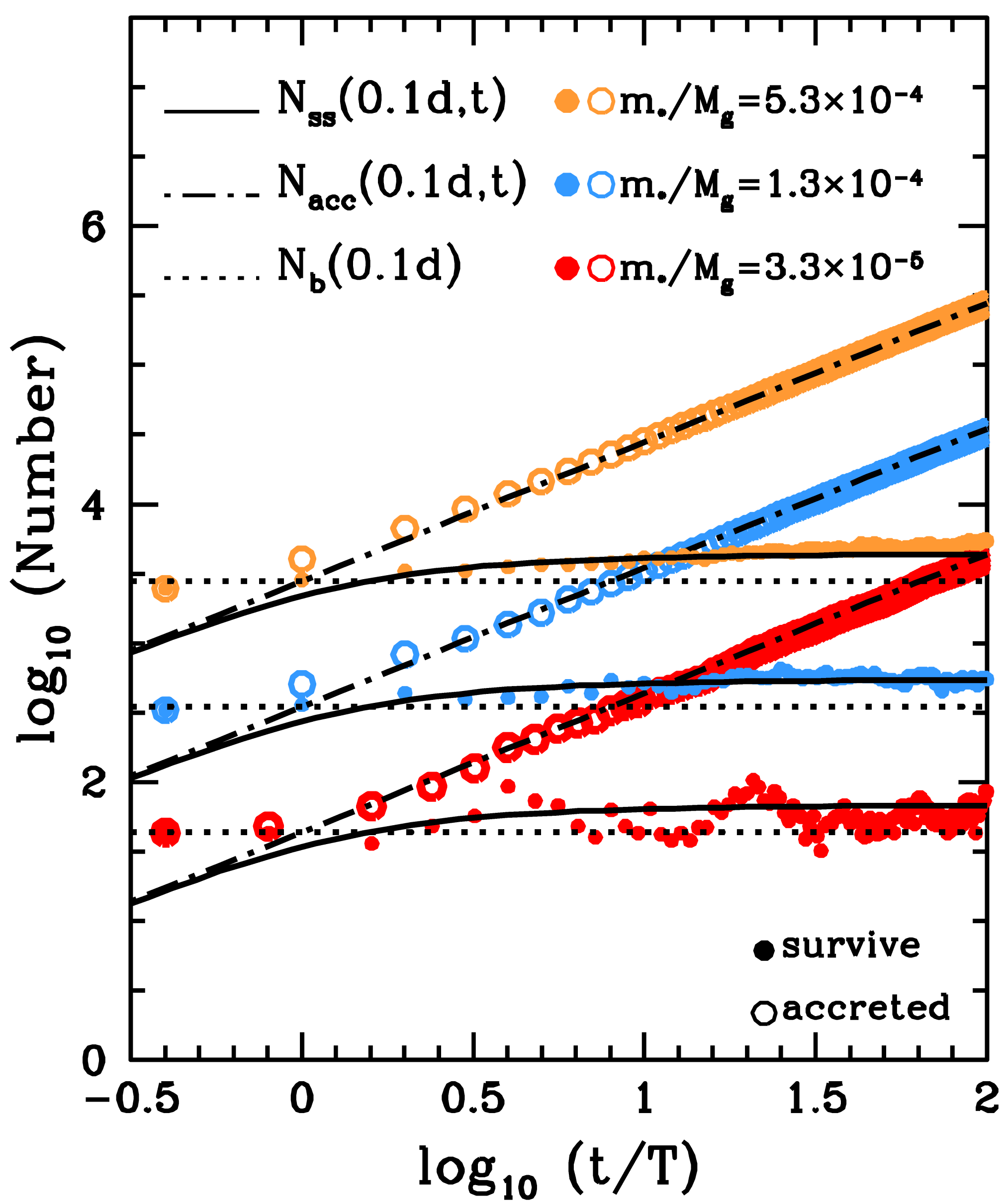
$$f_{\rm surv}(t) = \frac{1}{1+(t/T)^2}, \quad \Rightarrow \quad \alpha(t) = \frac{1}{T}\int_0^t dt\, f_{\rm surv}(t) = \arctan(t/T) \to \frac{\pi}{2}$$

Steady-state number of bound particles

$$N_{\rm surv}(t) = \int_0^t dt\, f_{\rm surv}(t)\, C_{\rm acc} \to N_{\rm ss} = N_b\, \alpha \qquad \text{for} \quad t \gg T.$$

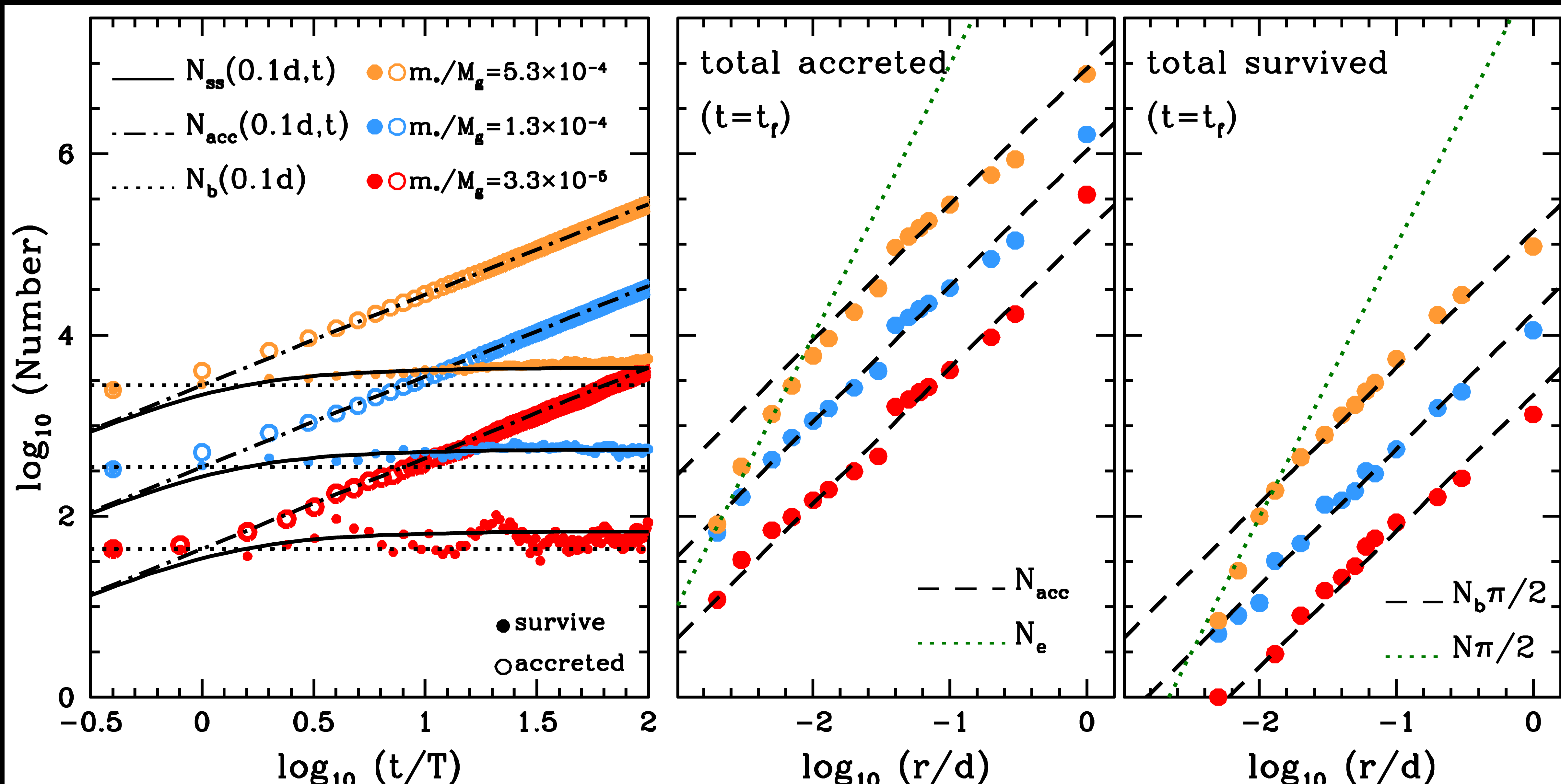
STATISTICAL THEORY

Step 4: N-body tests



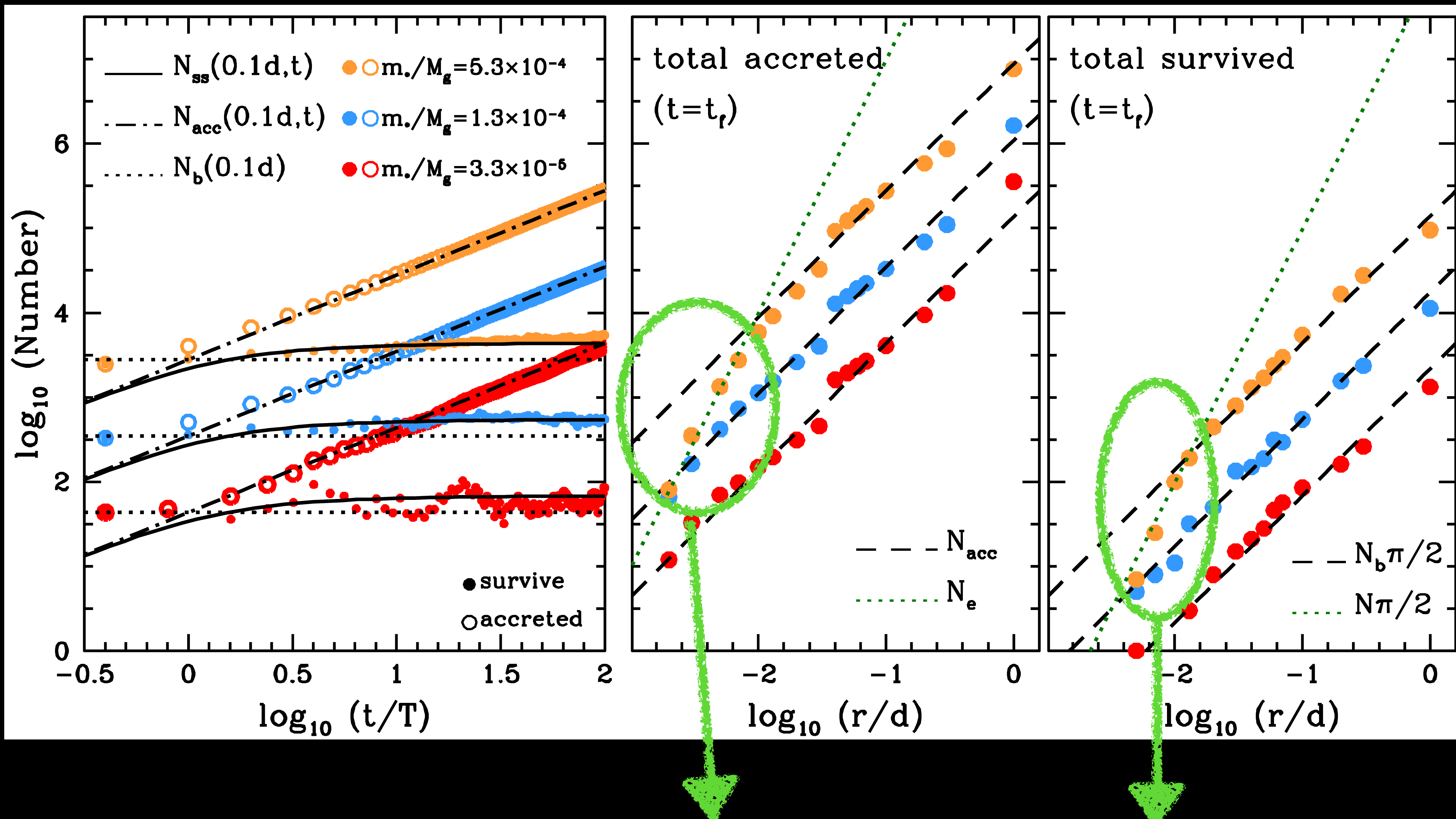
STATISTICAL THEORY

Step 4: N-body tests



STATISTICAL THEORY

Step 4: N-body tests



$N_{acc} > N_e$ at $r < r_0$!!

$N_{ss} > N$ at $r < r_0$!!

STATISTICAL THEORY

Step 5 : "halo" of temporarily-bound particles

Density enhancement: $$\delta(r) \equiv \frac{1}{4\pi\, n\, r^2}\frac{dN_{\rm ss}}{dr} \quad = \frac{2\sqrt{\pi}}{3}\frac{(Gm_\star)^{3/2}}{\sigma^3}e^{-V_\star^2/(2\sigma^2)}\frac{1}{r^{3/2}}$$

STATISTICAL THEORY

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Velocity dispersion: $$\sigma_h^2(r) = \frac{1}{\delta(r)}\int_r^\infty dr'\delta(r')\left|\frac{d\Phi}{dr}\right| \quad = \frac{2}{5}\frac{Gm_\star}{r}$$

STATISTICAL THEORY

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Mean phase-space density:

$$Q_h \equiv \frac{n\, \delta(r)}{\sigma_h^3(r)} \quad = \frac{5}{3} \left(\frac{5\pi}{2} \right)^{1/2} e^{-V_\star^2/(2\sigma^2)}\, Q$$

$$Q = \frac{n}{\sigma^3}$$ (field)

set by velocity of the point-mass w.r.t background

STATISTICAL THEORY

Step 6 : orbits

- **Phase-space density = constant**
- **Distribution function = f_0 = constant**
- **Distribution of integrals of motion**

$$p(a,e) = \omega(a,e)\, f(a,e) = 8\pi^3 (Gm_\star)^{3/2}\, e\, a^{1/2}\, f_0$$

$$p(e)de = 2e\,de$$

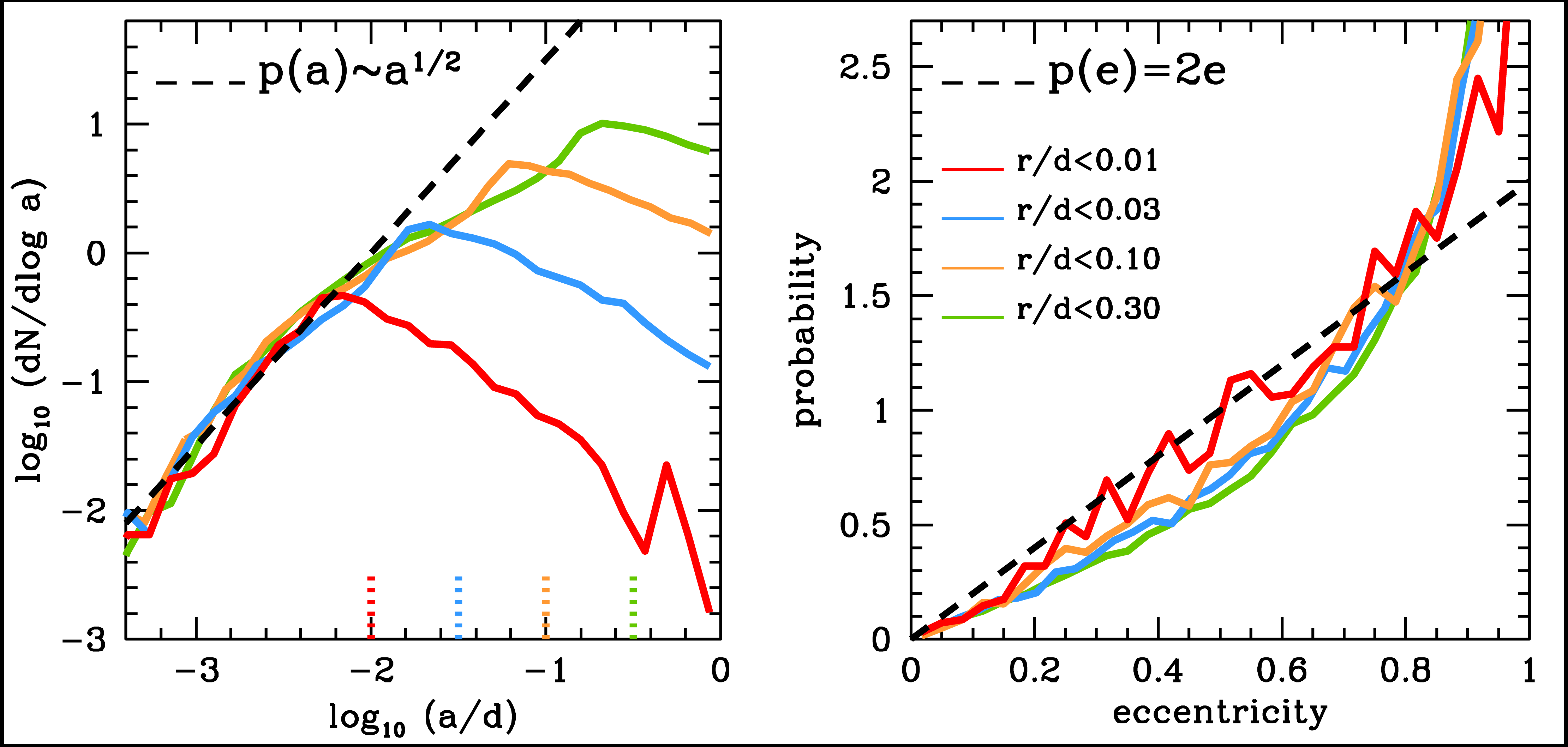
"thermal"

$$p(a)da \sim a^{1/2}da.$$

Jeans (1928) P(E) ~ $1/(-E)^{5/2}$

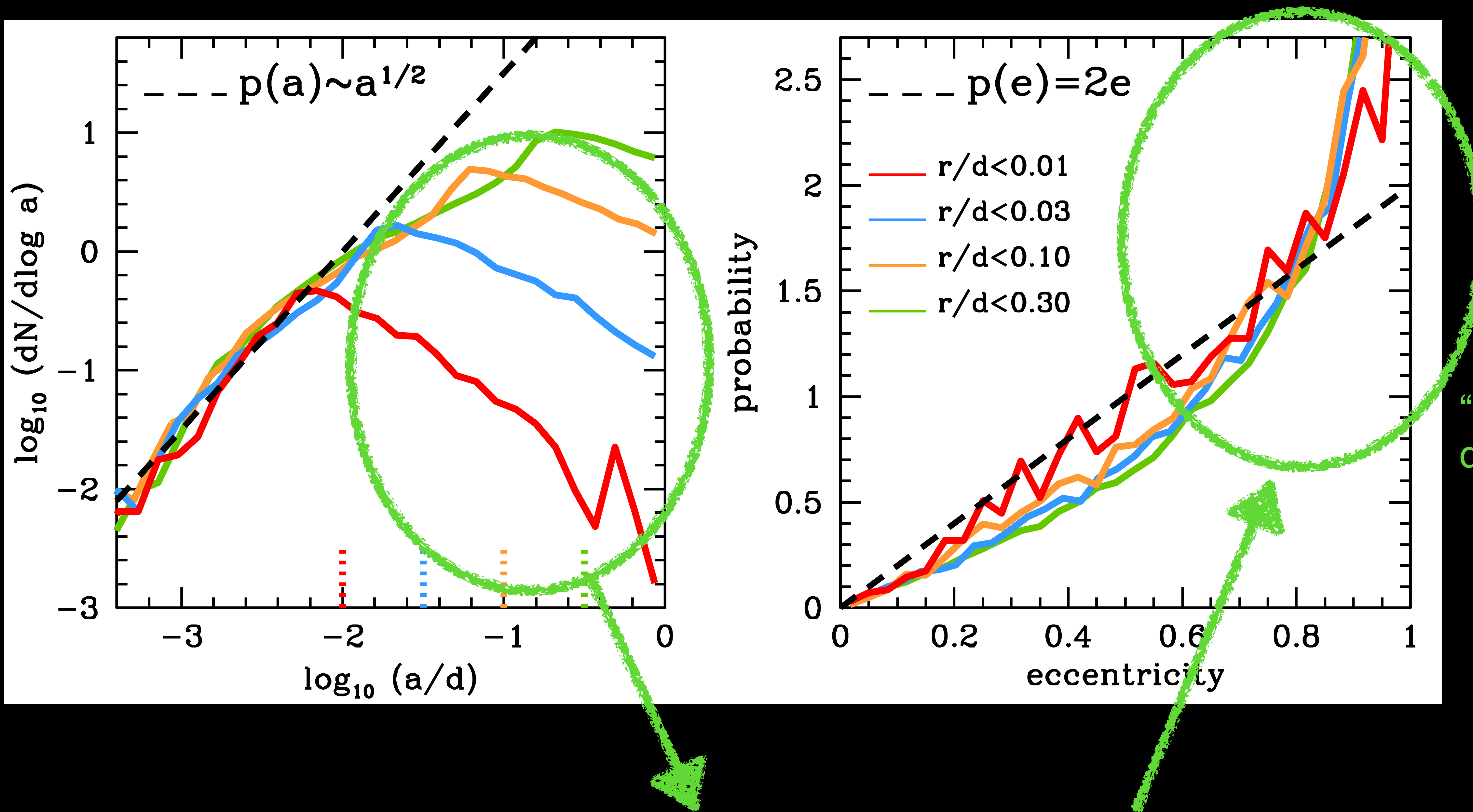
STATISTICAL THEORY

Step 6 : orbits



STATISTICAL THEORY

Step 6 : orbits



"super-thermal" distrib.

particles with apocentres that reach beyond the volume size
(very eccentric orbits, perturbed by tides)

STATISTICAL THEORY

Step 7 : solar system estimates

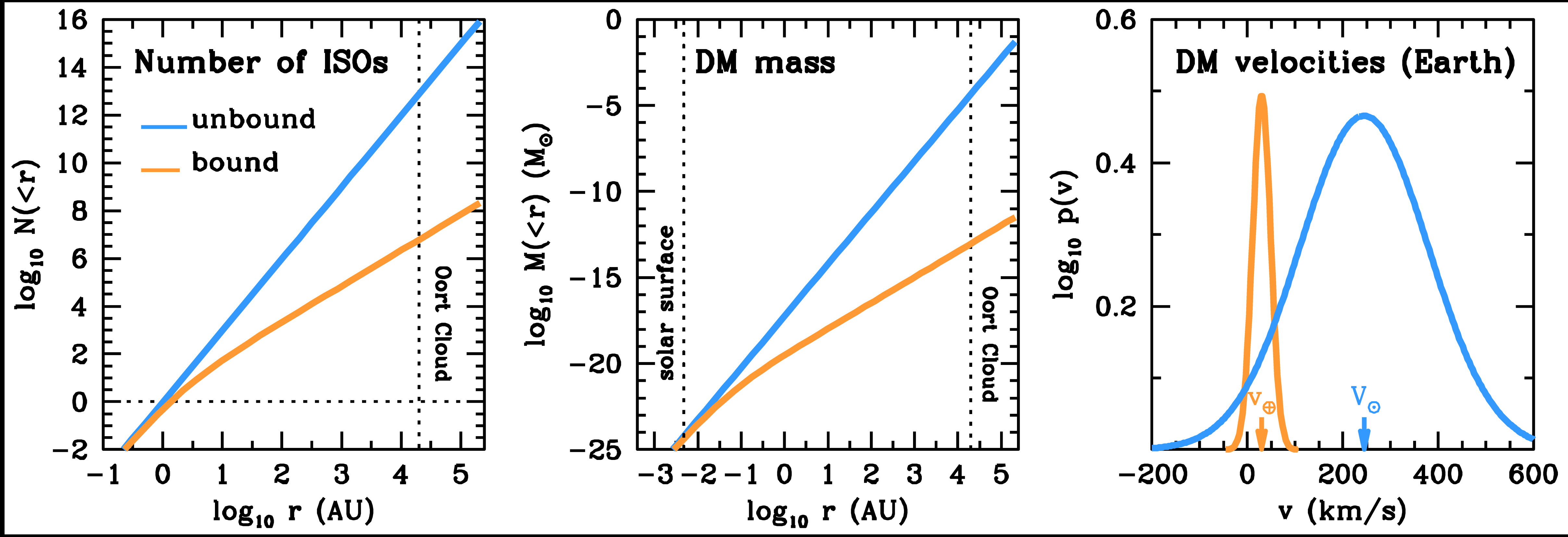
$m_* = 1\ M_{sol}$
$V_* = 237$ km/s
$R_* = 8.3$ kpc

$n^{ISO} = 2\text{x}\ 10^{15}\ pc^{-3}$ 'Oumuamua-like objects (Do et al 2018)
$\sigma^{ISO} = 28$ km/s (thin disc, Anguiano et al. 2020 from Gaia DR2 and APOGEE data)

$\rho^{DM} = 0.012\ M_{sol}\ pc^{-3}$ (Read et al 2018)
$\sigma^{DM} = 137$ km/s (NFW halo)

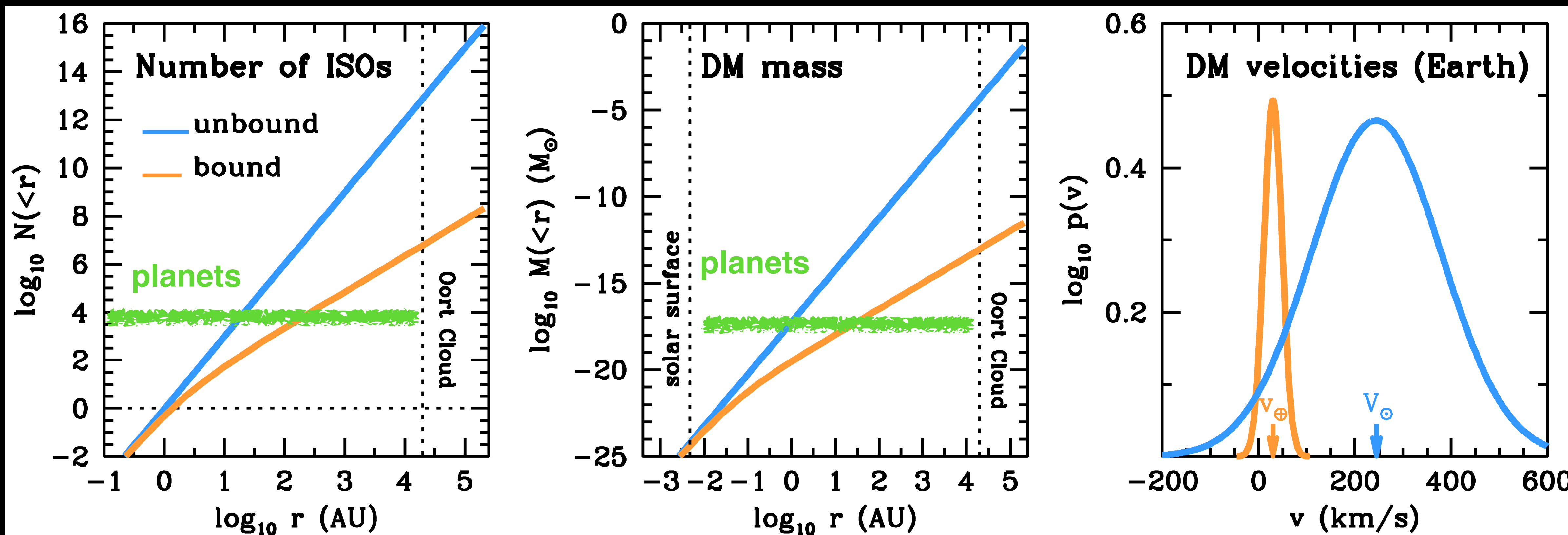
STATISTICAL THEORY

Step 7 : solar system estimates



STATISTICAL THEORY

Step 7 : solar system estimates



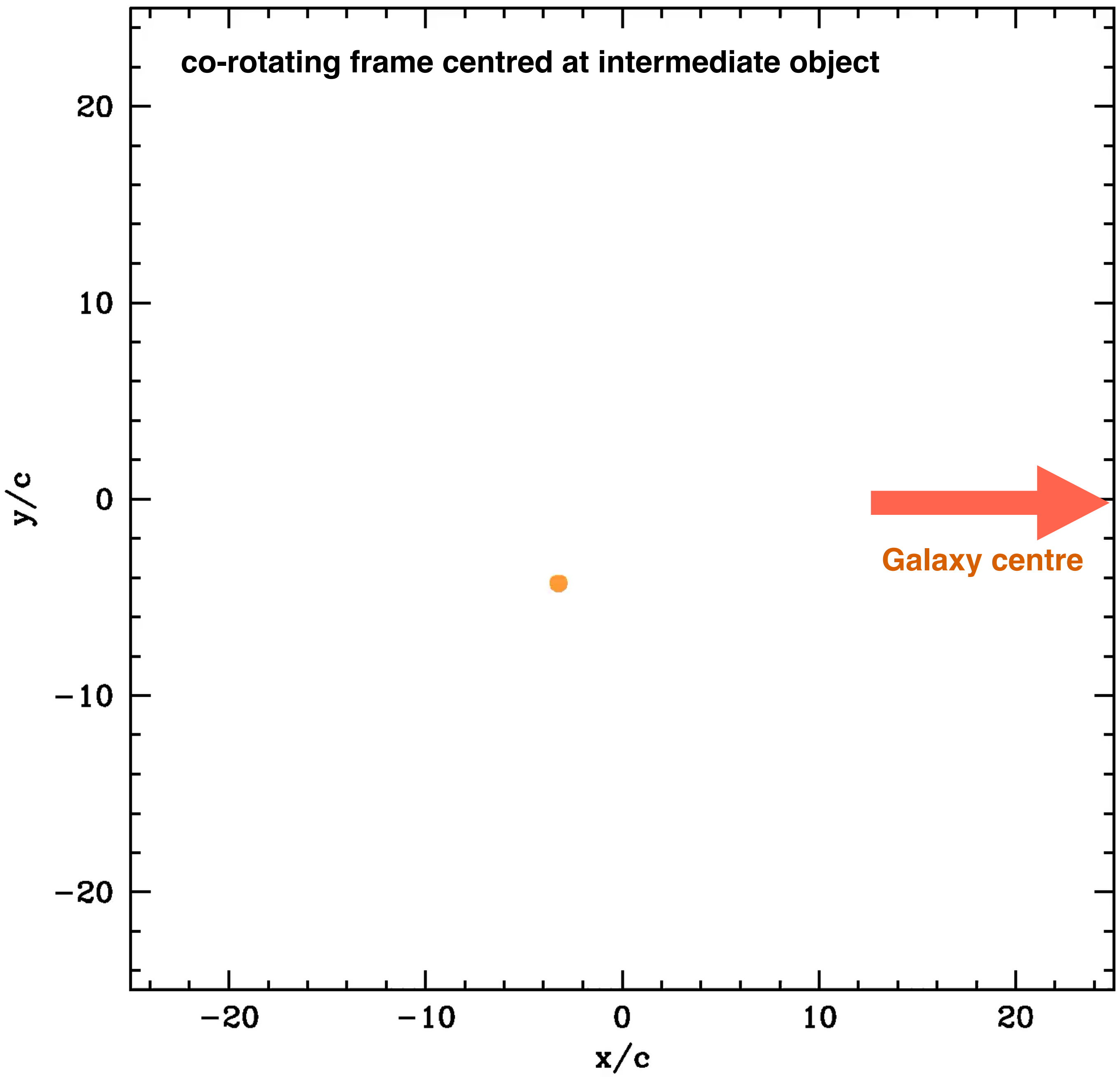
At face value tidal trapping more efficient in capturing interstellar particles than planets

But intertwined dynamical processes

- tides affect the trajectories of particles interacting with planets
- planets can both capture new interstellar particles / eject tidally-trapped ones

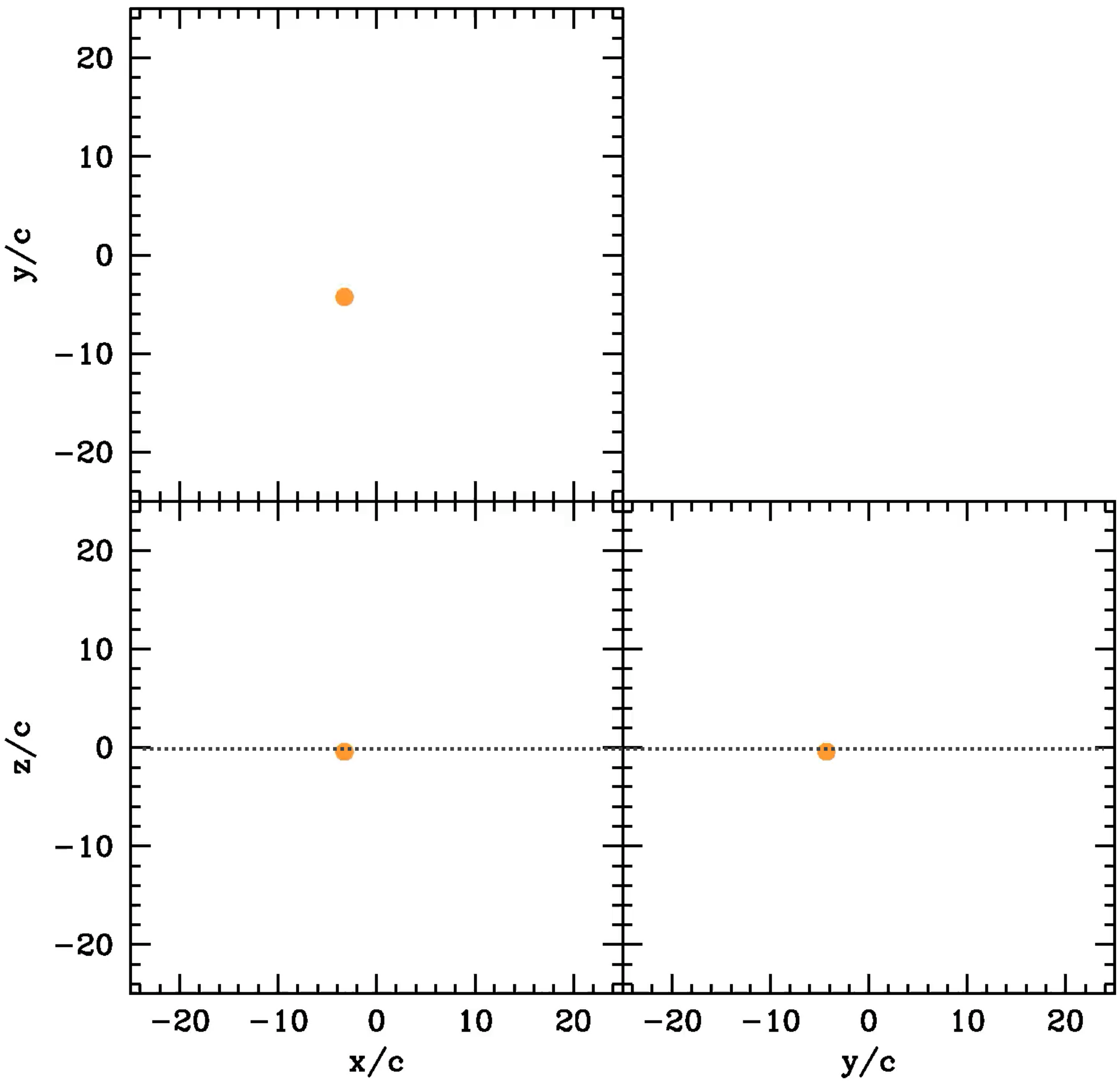
Need to run models w/ planets + tides

CHAOS IN 3-BODY SYSTEM



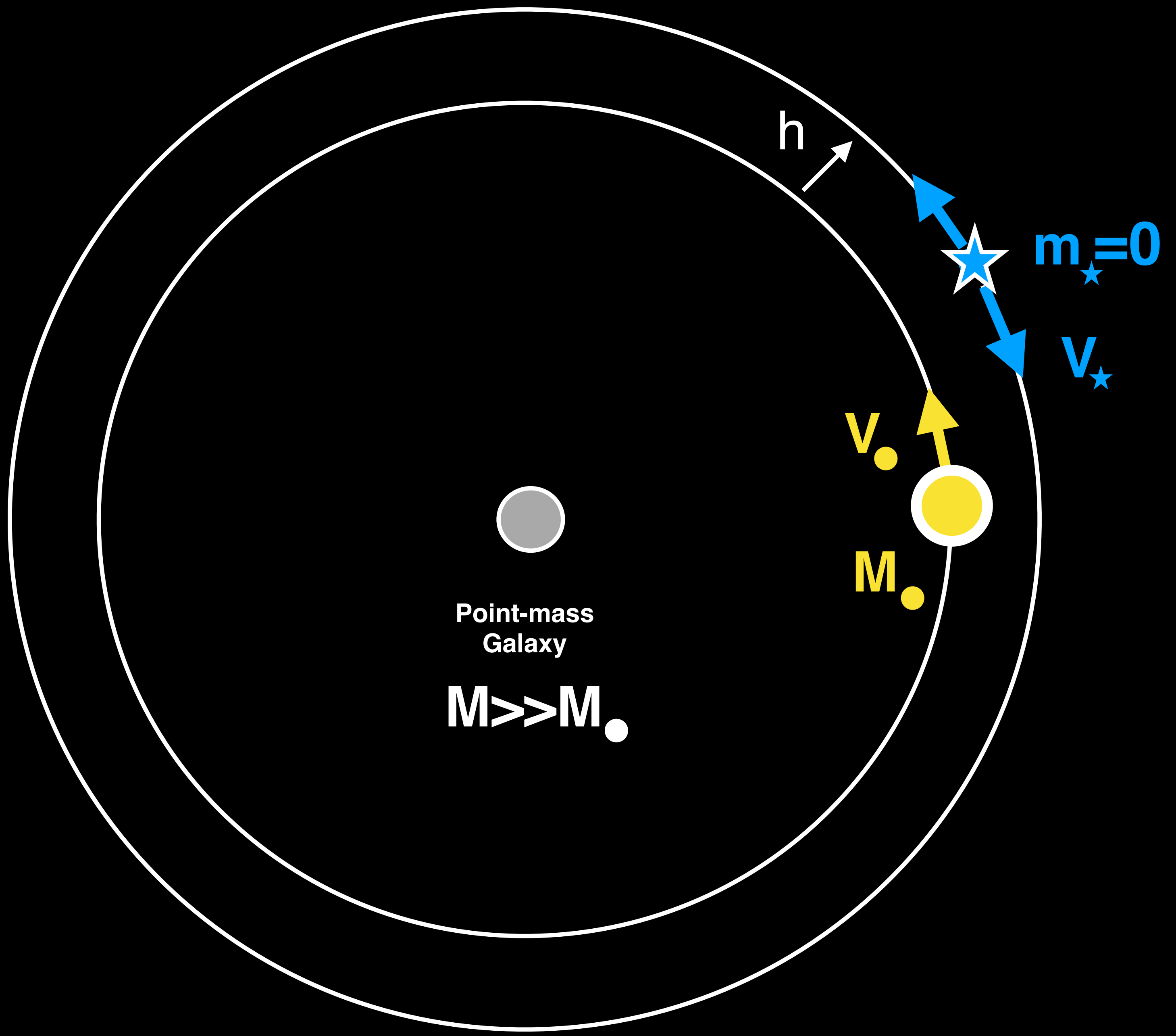
- Integrals of motion (E,L) not conserved. **Irregular orbits**
- Orbital plane & direction of motion varies in a random fashion
- In the substructure frame, orbits tend to be very eccentric
- Captured particles spend significant amount of time in the inner regions of the substructure potential
- Ultimately, they become tidally unbound

Chaos in motion



subhalo orbital plane z=0

....3-BODY TRAJECTORIES ARE MUCH MORE COMPLICATED: CHAOS



Petit & Henon (1986)
experimental set up

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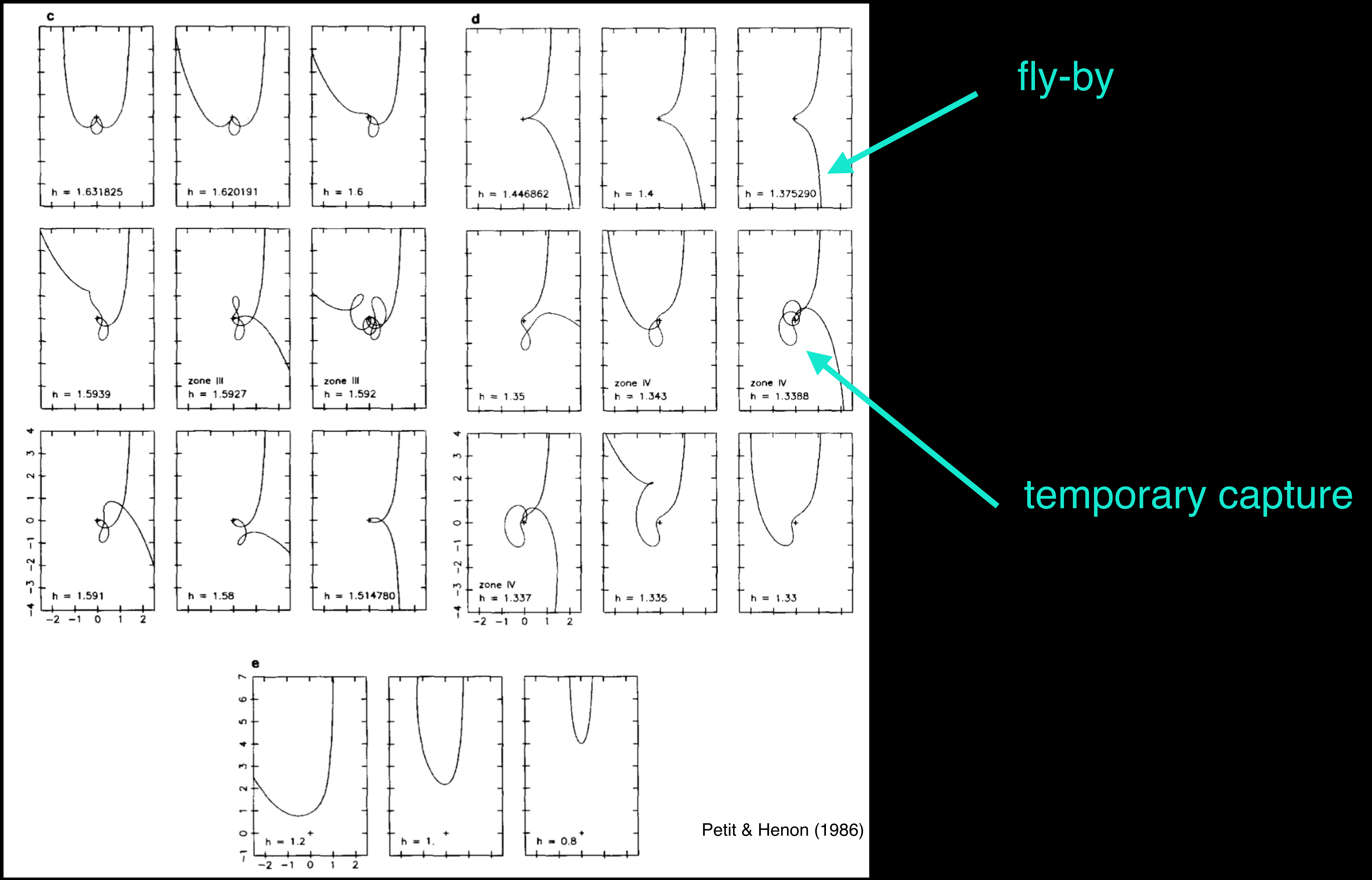

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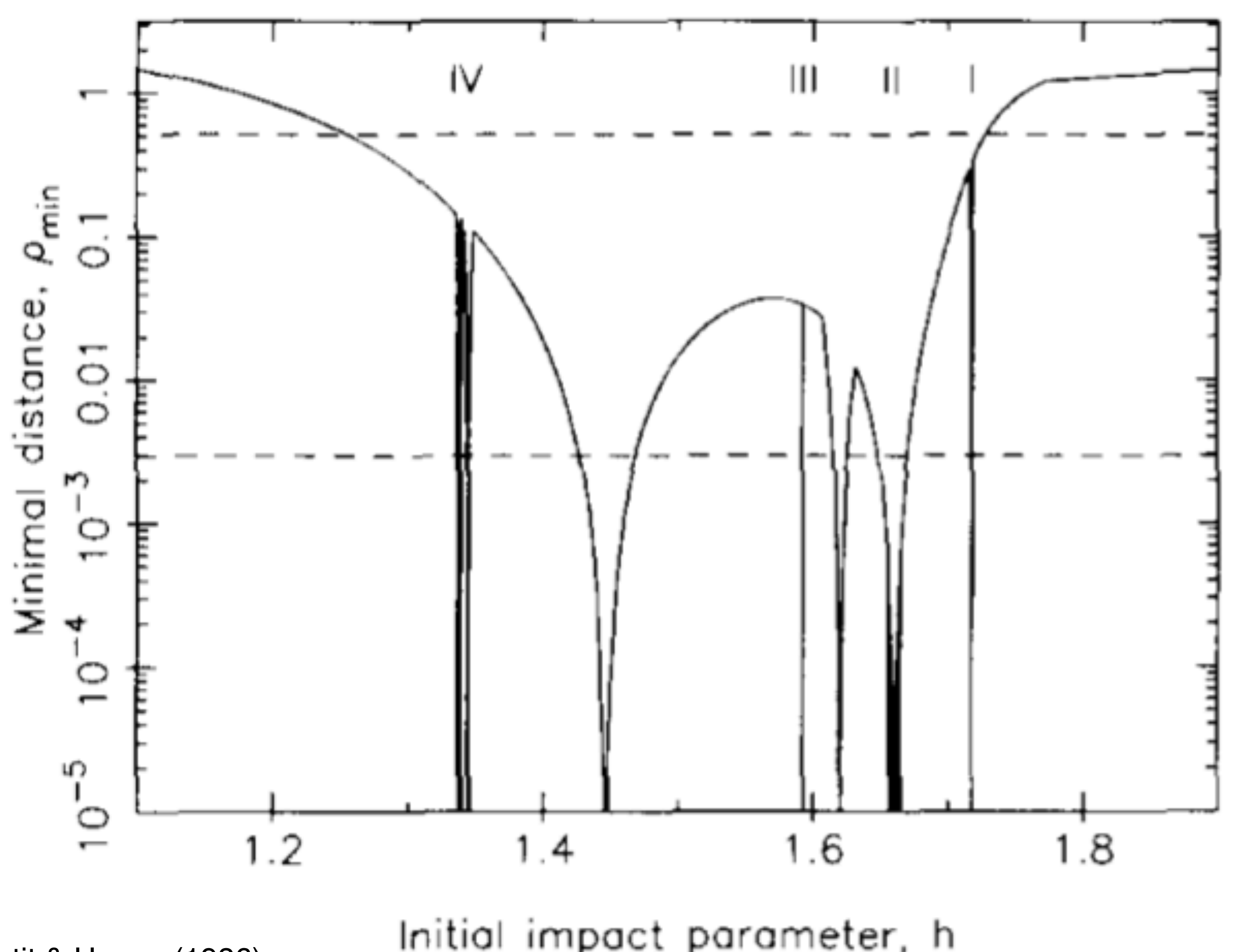


Petit & Henon (1986)

3-body eqs. have solutions where the lightest particle becomes captured by the intermediate particle

- trajectories extremely sensitive to initial phase-space location w/ **fractal structure**
- captured stars move on **chaotic orbits**
- capture always **temporary**

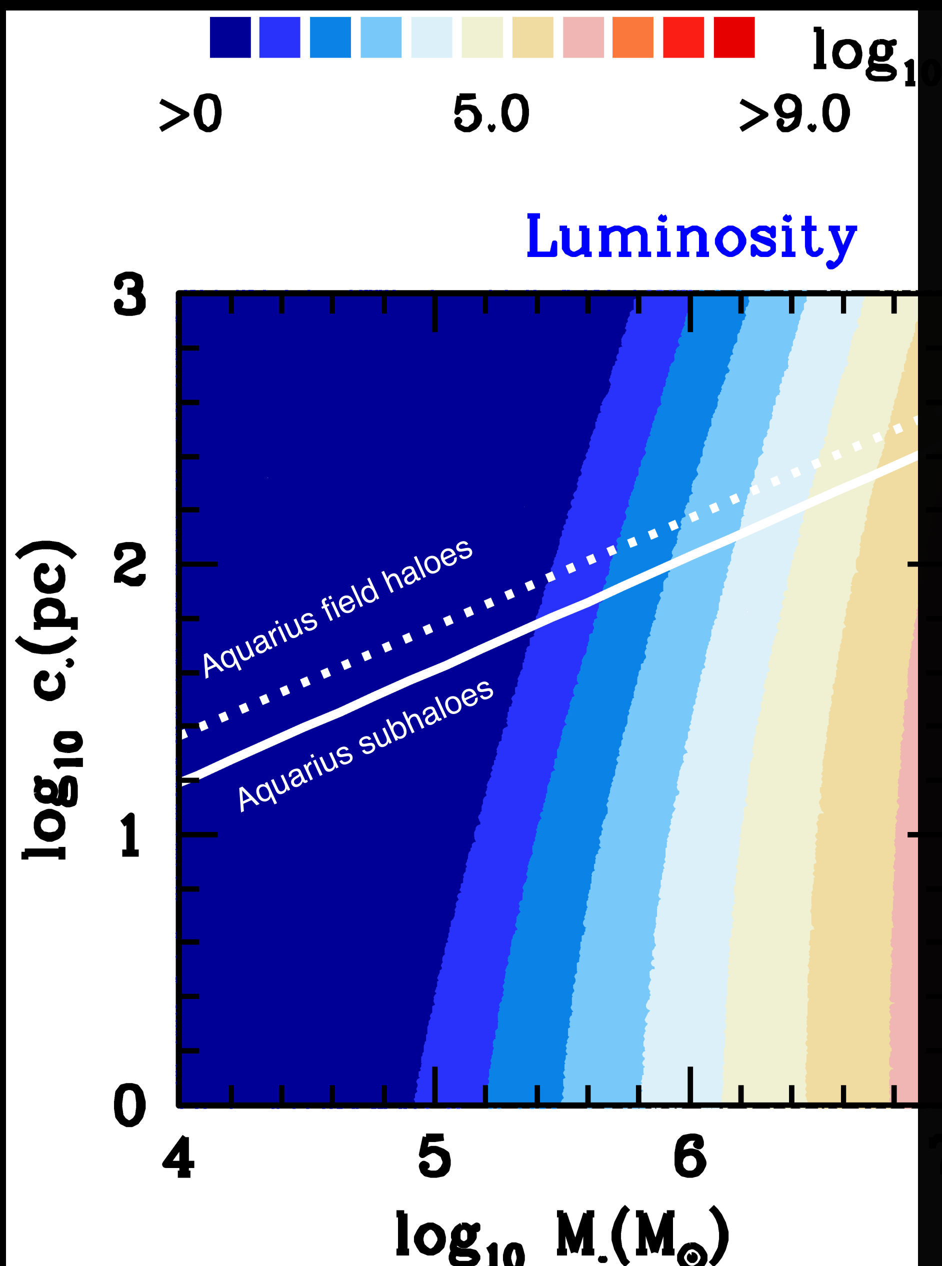
see Thibaut's talk!

ANALYTICAL ESTIMATES

JP et al. (2024, https://arxiv.org/abs/2404.19069)

- Capture Metal-rich stars in **Fornax dSph**
- Subhaloes placed with V=0 at $r \ll R_{half}$

Field: Plummer profile; N=3E7, R_{half} = 600 pc; σ =10 km/s (see Walker & JP 2011)



Luminosity = number bound stars (E<0) within the thermal critical radius

Weakly sensitive to size at fixed mass

STATISTICAL THEORY

* JP (2023; MNRAS, 519, 1955)
* JP et al. (2024, https://arxiv.org/abs/2404.19069)

N-body simulations indicate that captured particles w/ E<0 in **steady state** distribute homogeneously in phase-space

$$f(\mathbf{r},\mathbf{v}) = f_0$$

Adopting the **local approximation** @ r<< |(dn/dr)/n| [-1] and the **Maxwellian approximation**

$$f_0 = \alpha \frac{n}{(2\pi\sigma^2)^{3/2}}\, e^{-V_\bullet^2/(2\sigma^2)}.$$

$\alpha \approx 1$ (smooth subhaloes)

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Density profile of captured particles (E<0)

$$n_\star[\Phi_\bullet(r)] = \int_{E<0} d^3v \, f_\star(\mathbf{r}, \mathbf{v}) \; = \frac{8\sqrt{2}\pi}{3} f_0 |\Phi_\bullet|^{3/2}.$$

Velocity dispersion (1D) of captured particles (E<0)

$$\sigma_\star^2[\Phi_\bullet(r)] = \frac{1}{3\, n_\star(r)} \int_{E<0} d^3v \, v^2 f_\star(\mathbf{r}, \mathbf{v}) \; = \frac{2}{5} |\Phi_\bullet|$$

STATISTICAL THEORY

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Choose a **point-mass potential** $\Phi_\bullet = GM_\bullet/r$

over-density size

Define a **“thermal critical radius”** $\delta(r_\epsilon) = n_\star(r_\epsilon)/n = 1$ → $r_\epsilon = \left(\frac{16}{9\pi}\right)^{1/3} e^{-V_\bullet^2/(3\sigma^2)} \frac{GM_\bullet}{\sigma^2}$,

STATISTICAL THEORY

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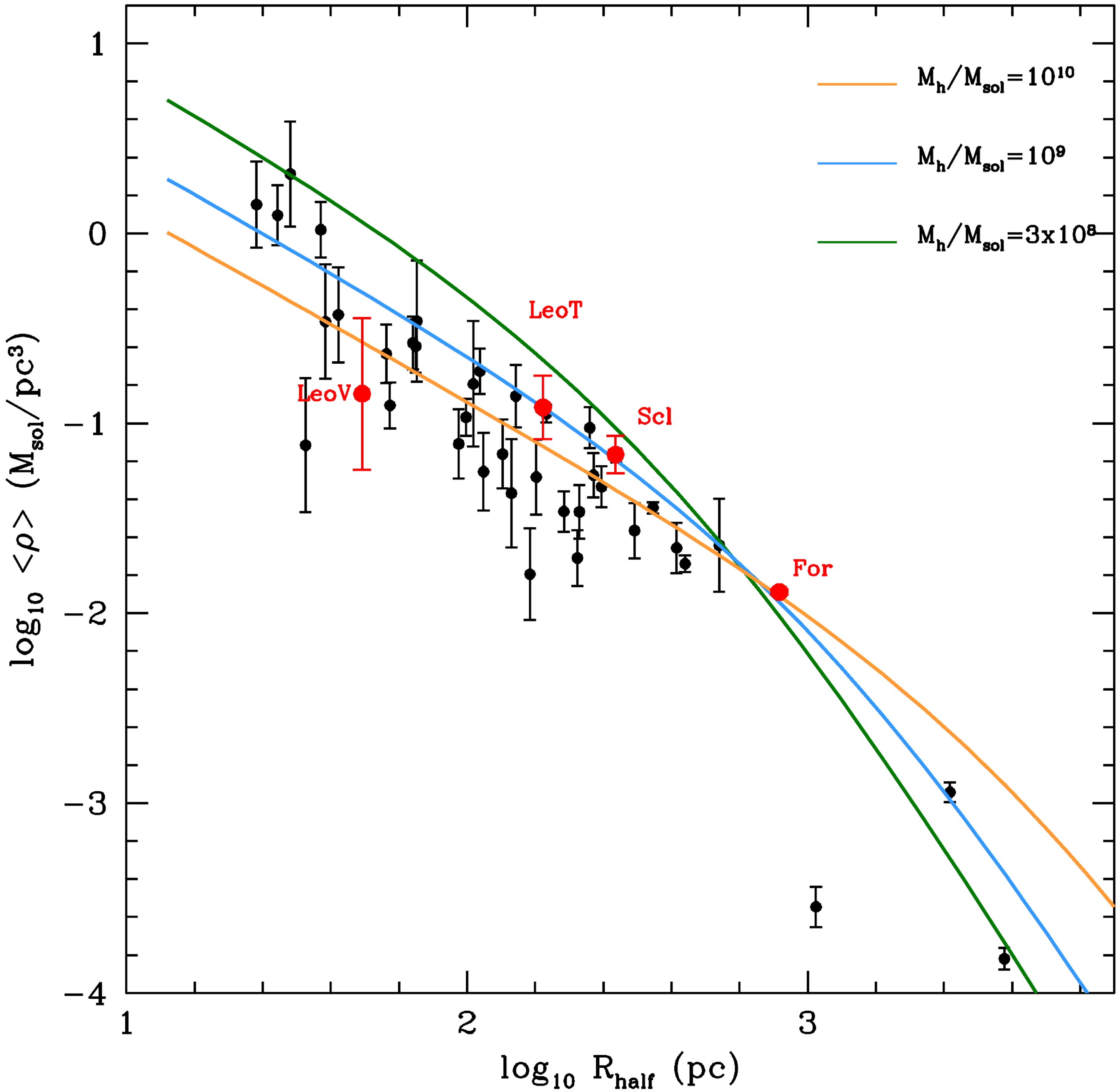
→ ***Subhaloes** sourcing a Hernquist potential generate overdensities **δ>1** if and only if*

$\Phi_\bullet = GM_\bullet/(r + c_\bullet)$

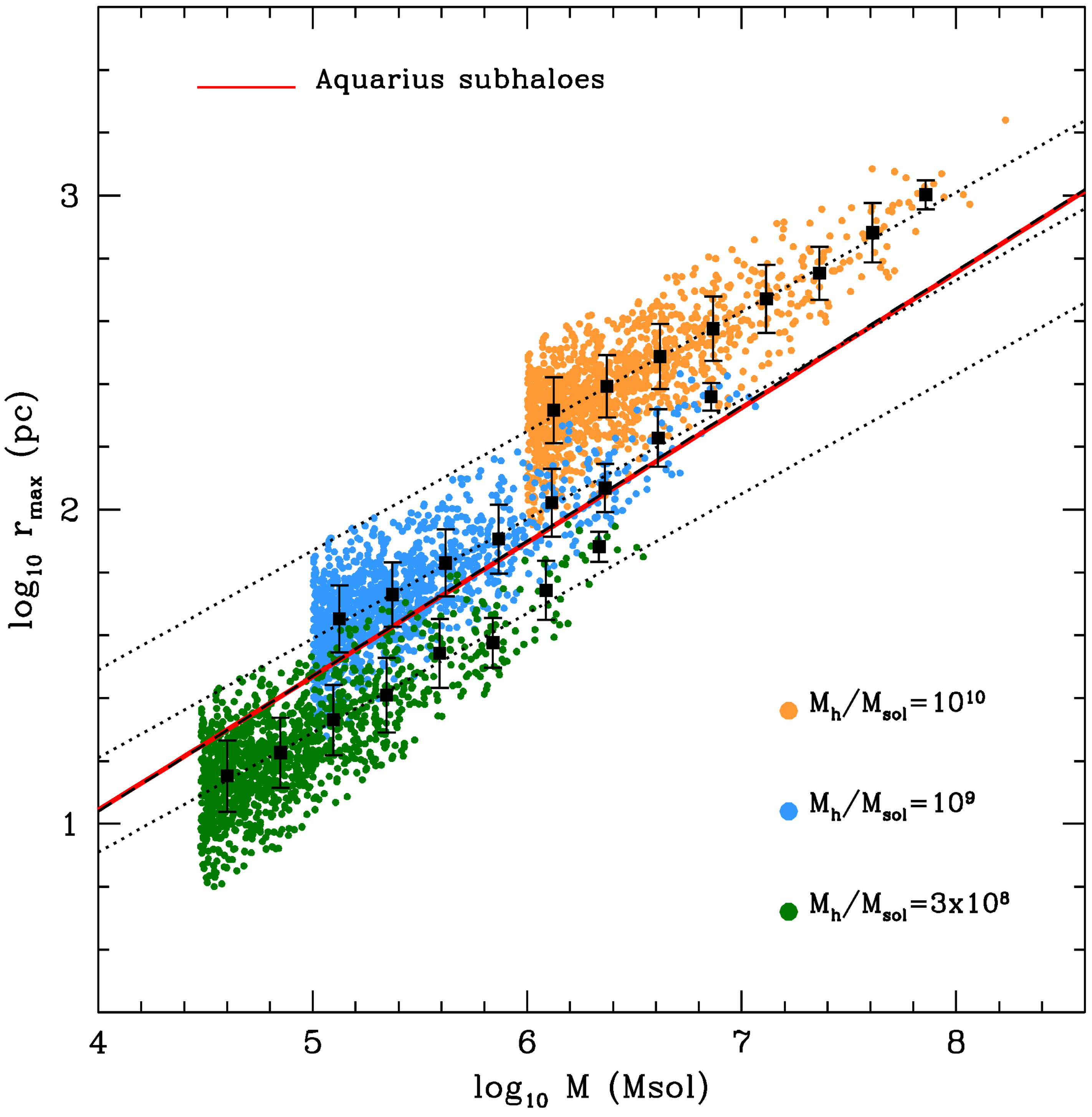
$$r_\epsilon^{Hern} = r_\epsilon - c_\bullet = \kappa\, r_\epsilon > 0$$

$$\kappa \equiv 1 - c_\bullet/r_\epsilon > 0$$ **compactness**

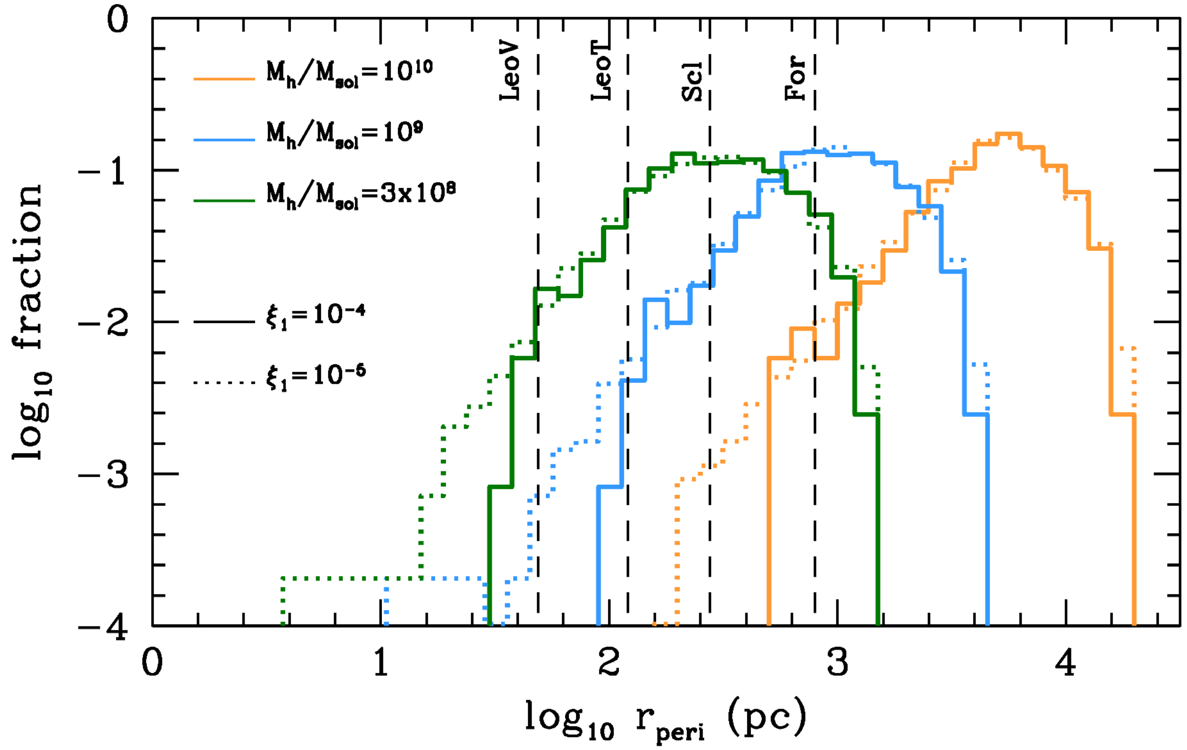
HOST POTENTIAL

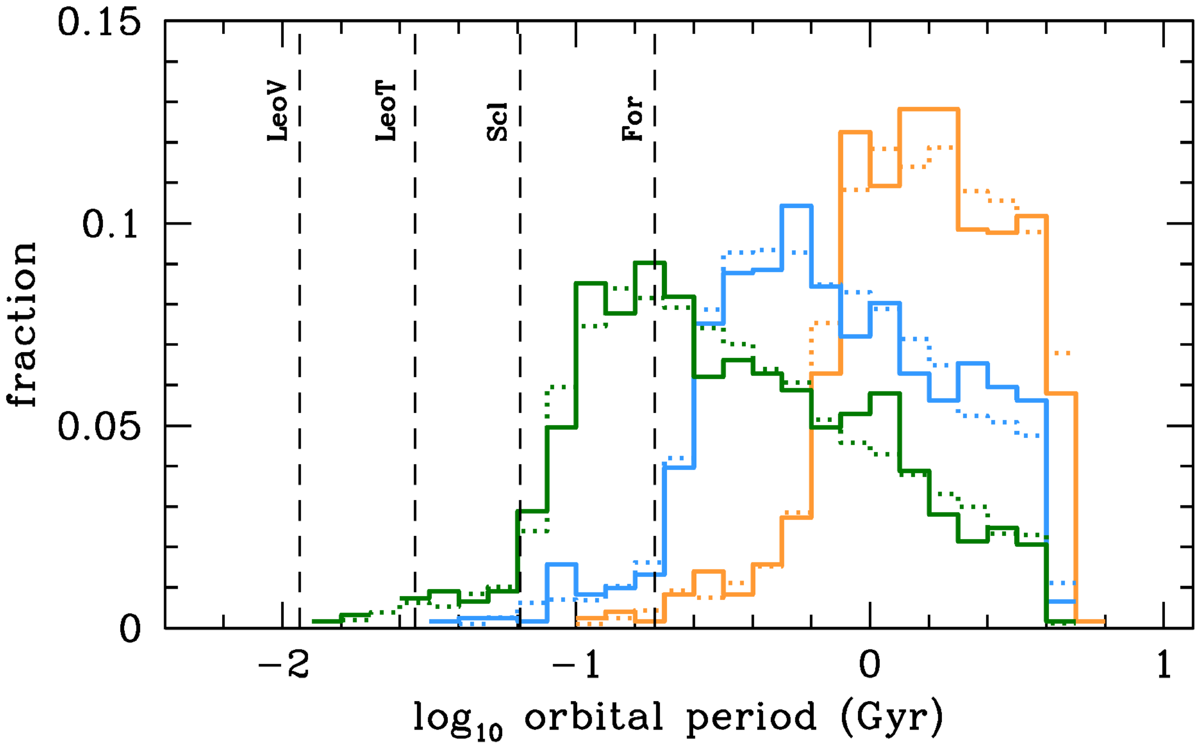


MASS - SIZE RELATION OF TRUNCATED CUSPS

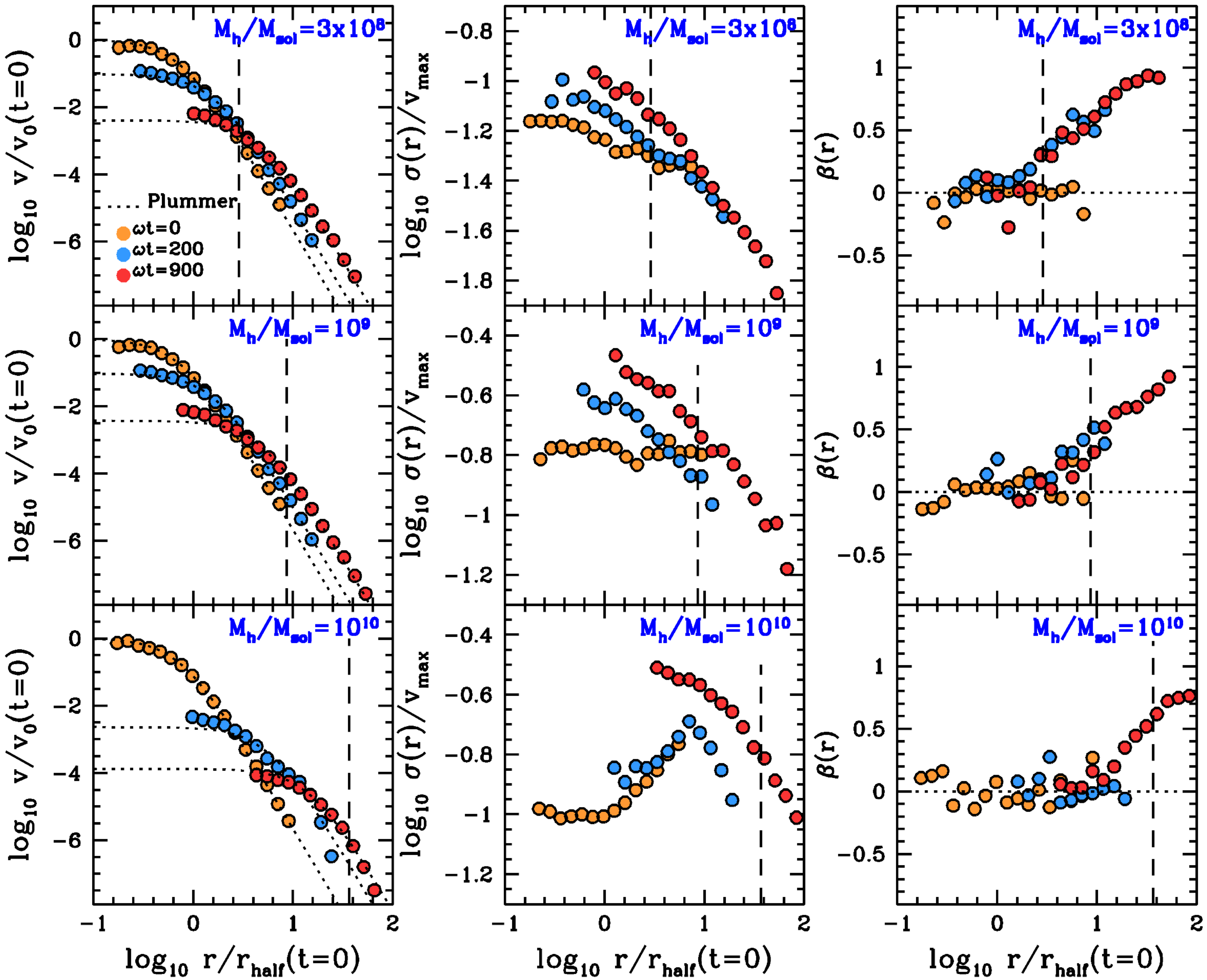


ORBITS OF SUBHALOES



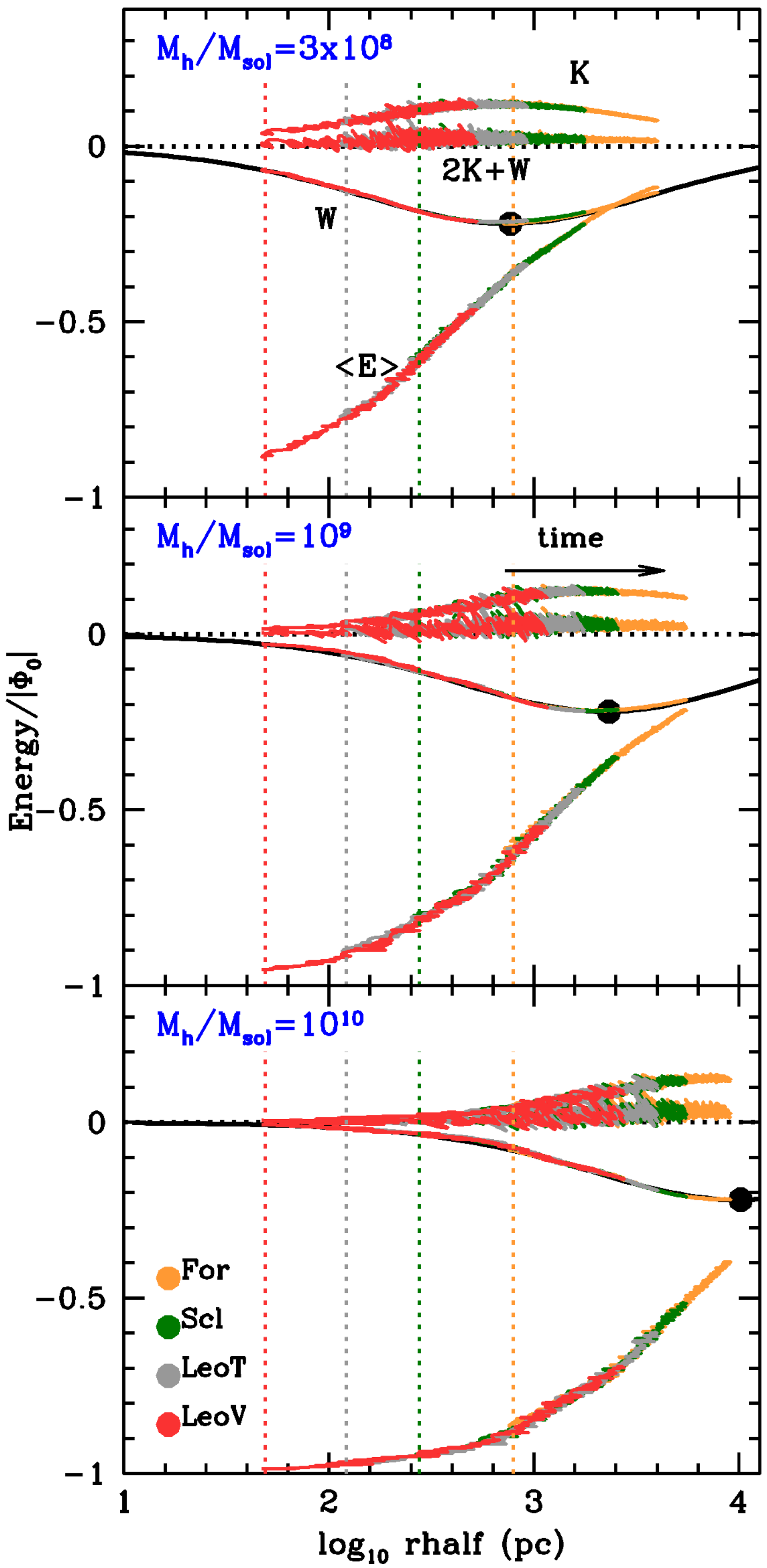


GRAVOTHERMAL EXPANSION



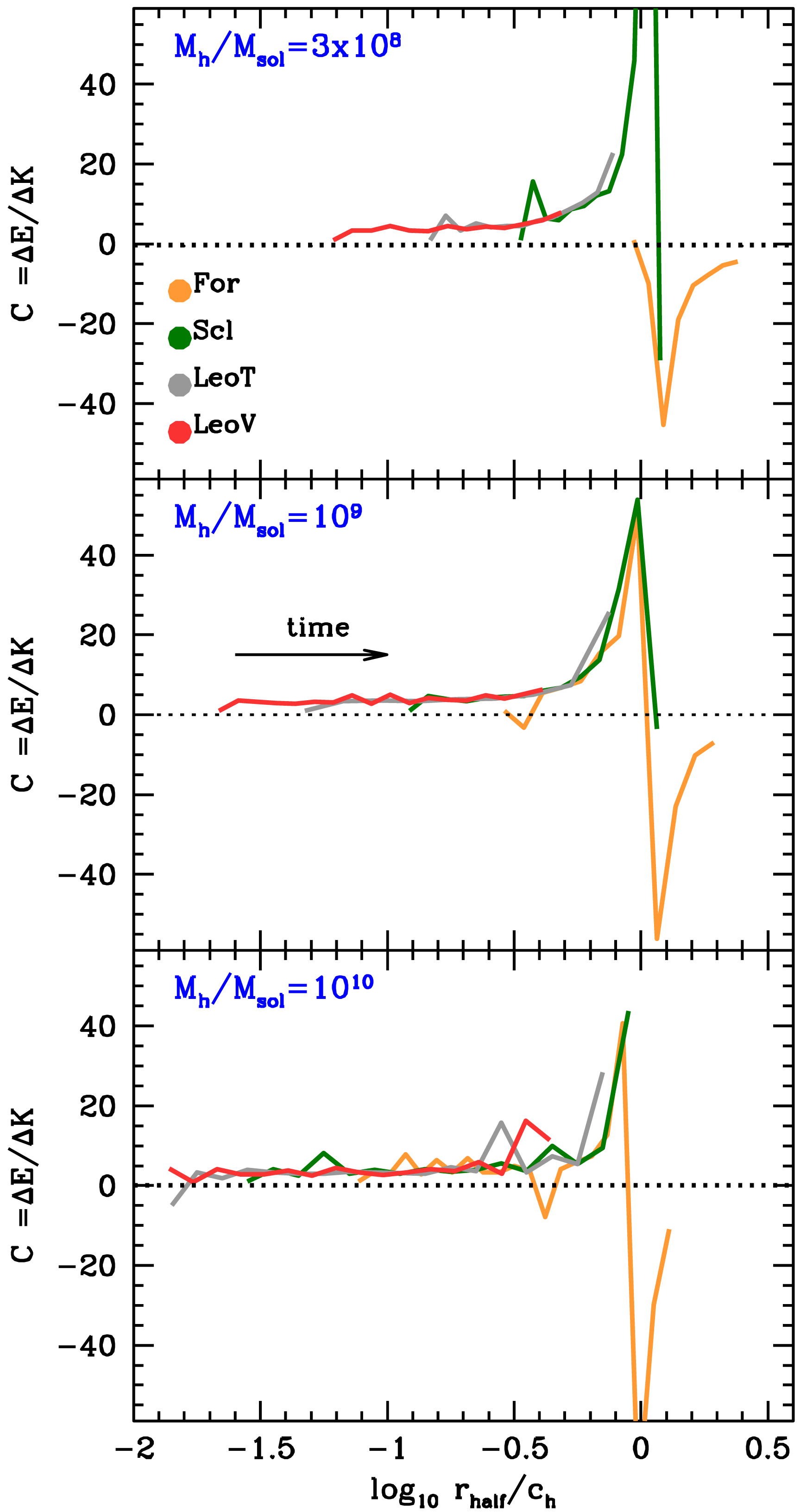
- **Self-similar evolution** of stellar profile (remains close to Plummer)
- **Gravothermal expansion** (inner regions het up, outer regions cool down)
- **radially-anisotropic** orbits

VIRIAL QUANTITIES



DIVERGENT HEAT CAPACITY

$$c_v = \frac{\partial E}{\partial K}$$



- Energy injection leads to increase of temperature $\mathbf{r_{half} < r_{max}}$
- Temperature cools down as $\mathbf{r_{half} > r_{max}}$
- This means **heat capacity** diverges at $\mathbf{r_{half} = r_{max}}$

SUBHALOES = HEATING SOURCE