Production of velocity-dependent pion SIMPs

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Fundación Carolina

Strongly Interacting Massive Particles (Carlson+92,Hochberg+14)

✔ χ is the lightest particle in a nearly secluded dark sector.

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_\chi v^2 \rangle_{3\to 2} \left[n_\chi^3 - n_\chi^2 n_{\chi,\mathrm{eq}} \right] .$$

✔ Ω_χ is set by number changing processes in the dark sector:

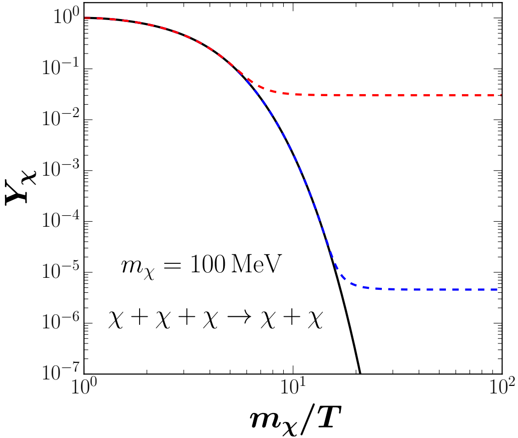
$$3 \to 2, \;\; 4 \to 2.$$

[negligible $\sigma(\chi\chi \to e^- e^+, ...)$]

✔ $m_\chi \sim \alpha_{\mathrm{eff}} (T_{\mathrm{eq}}^2 M_{\mathrm{Pl}})^{1/3}$
$\sim$ 10 MeV - 1 GeV.

☞ $\chi + \chi \to \chi + \chi$ scatterings allows for realizing SIDM.

☞ Thermal equilibrium with Standard Model particles: e.g.
$\chi + e^- \to \chi + e^-$.



QCD-like theories

✔ Based on a dark $SU(N_c)$ gauge group ($SO(N_c)$ or $Sp(N_c)$ also applies).

✔ N_f flavors of Dirac quarks in $d = N_c$ fund. rep. of $SU(N_C)$.

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{q}i\not{D}q - (\bar{q}_L M q_R + \text{h.c.})\,, \quad M = \text{diag}(m_1, \cdots, m_{N_f}).$$

☞ Massless quarks implies the global flavor sym:

$$G_F = SU(N_f)_L \times SU(N_f)_R\,; \quad q_{L(R)} \to \exp^{i\alpha^a_{L(R)}\lambda^a} q_{L(R)}.$$

✔ In analogy with ordinary QCD, strong interactions confine at some energy scale Λ.

➢ Giving rise to a fermion condensate $\langle \bar{q}q \rangle \sim \Lambda^3$

➢ For $m_q \ll \Lambda$, the condensate spontaneously breaks G_F, inducing a chiral phase transition:

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V.$$

☞ The unbroken vectorial subgroup is defined via $\alpha^a_L = \alpha^a_R$.

Dark pions

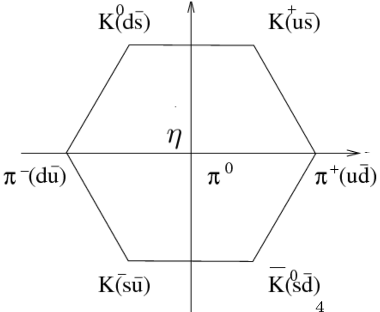
This leads to $N_f^2 - 1$ pseudo-Goldstone bosons, π^a, described by chiral perturbation theory: $U = e^{i\Pi/f_\pi}$, $\Pi = \pi^a \lambda^a$.

$$\mathcal{L}_{\rm eff} = \frac{f_\pi^2}{4}\mathrm{Tr}[|\partial_\mu U|^2] + \frac{f_\pi^2 B_0}{2}\mathrm{Tr}[MU + \mathrm{h.c.}] - \mathcal{L}_{\rm WZW}(U^5),$$

- ☞ f_π: dark meson constant such that $\Lambda \sim 4\pi f_\pi/\sqrt{N_c}$.
- ☞ For $m_q \ll \Lambda$, M softly breaks G_F (still a good symmetry).
- ☞ Degenerate quark masses $\to m_\pi^2 = 2B_0 m_q$, $\quad \langle\bar{q}q\rangle \equiv -B_0 f_\pi^2$.

For $N_c = N_f = 3$:

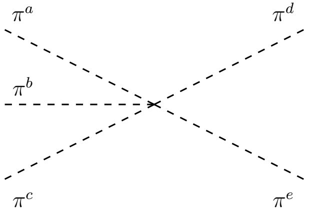
$$\frac{\Pi}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\pi_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\pi_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\pi_8 \end{pmatrix}.$$



Dark pions as SIMPs (Hochberg/Kuflik/Murayama/Volansky/Wacker 2014)

✔ DM is a thermal relic in the form of dark π.

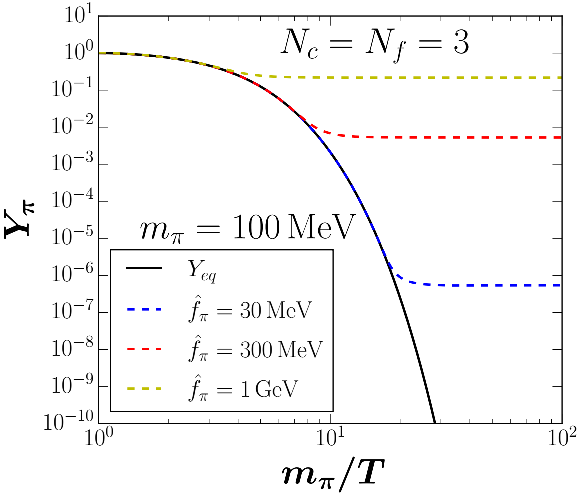
✔ Ω_π is set $3 \to 2$ proc.:



$$\langle \sigma v^2 \rangle = \frac{5\sqrt{5} m_\pi^5 N_c 3 t^2}{2\pi^5 f_\pi^{10} x^2 N_\pi^3},$$

$$\lambda = \frac{4\sqrt{10}\pi^3 M_P m_\pi^4 g_s^2}{675 g_\rho^{1/2}}.$$

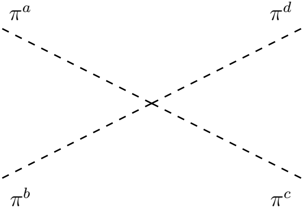
$$\frac{dY_\pi}{dx} = -\frac{\lambda \langle \sigma v^2 \rangle}{x^5} Y_\pi^2 (Y_\pi - Y_\pi^{\mathrm{eq}}).$$



Dark pions as SIMPs (Hochberg+14)

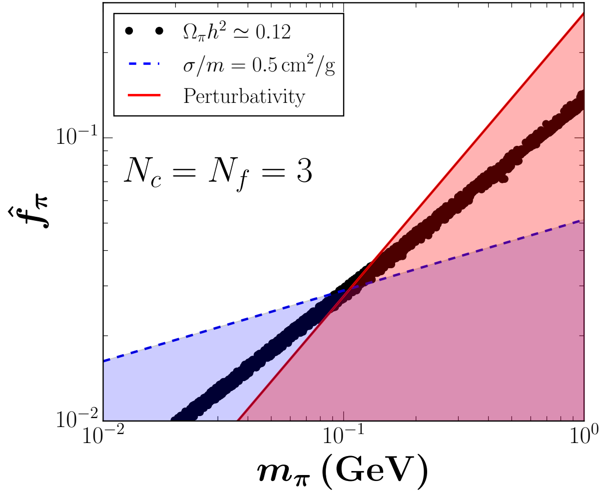
✔ π's can also scatter through a 4-point contact interaction:

$$\mathcal{L}_{2\to2} = \frac{m_\pi^2}{48 f_\pi^2}\,\mathrm{Tr}\left[\Pi^4\right] .$$



$$\sigma_{2\to2} \propto m_\pi^2/f_\pi^4 .$$

☞ v-indep: no SIDM.



Ω_π in tension with the bound from clusters of galaxies: $\sigma_0/m_{\mathrm{DM}} \gtrsim 1\ \mathrm{cm}^2/\mathrm{g}$ in the perturbativity region.

θ-vacuum in QCD-like

✔ $N_c \geq 3$, N_f flavors of light Dirac quarks in the fund. rep.:

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{g^2\theta}{32\pi^2}F\widetilde{F} + \bar{q}i\not{D}q - (\bar{q}_L M q_R + \text{h.c.})\,, \qquad \theta \ll 1.$$

$M = \text{diag}(m_1, \cdots, m_{N_f})$, with $m_i \neq 0$ for all flavors.

✔ Due to the chiral anomaly, under $q_{L,R} \to e^{\mp i\theta Q/2} q_{L,R}$:

$$\theta \to \theta(1 - \text{Tr}\, Q),$$

$$M \to M_\theta = e^{i\theta Q/2} M e^{i\theta Q/2}.$$

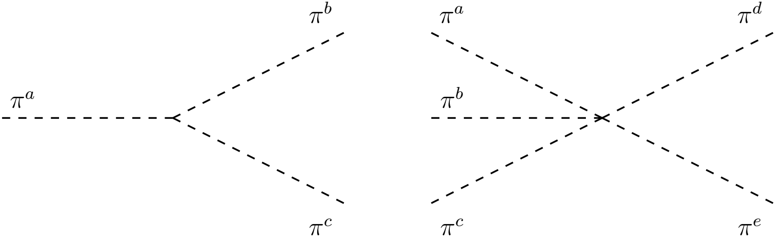
✔ Taking a transformation with $\text{Tr}\, Q = 1$, we move the θ parameter from the $F\widetilde{F}$ term to the quark mass matrix .

The Pion interactions get modified to (Kamada+17)

$$\mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4}\text{Tr}[\partial_\mu U^\dagger \partial^\mu U] + \frac{f_\pi^2}{2} B_0 \text{Tr}[M_\theta U + U^\dagger M_\theta^\dagger] + \mathcal{L}_{\text{WZW}} \,.$$

New interactions with an odd number of mesons:

$$\mathcal{L}_\theta = \frac{B_0 \theta}{3 f_\pi \text{Tr} M^{-1}} \left(d_{abc}\pi_a \pi_b \pi_c - \frac{c_{abcde}}{10 f_\pi^2} \pi_a \pi_b \pi_c \pi_d \pi_e \right),$$



Benchmark Model: $SU(3)_c$ and $m_u \leq m_d \leq m_s$

✔ Non-relativistic resonant scattering: $v_R \lesssim 0.1$.

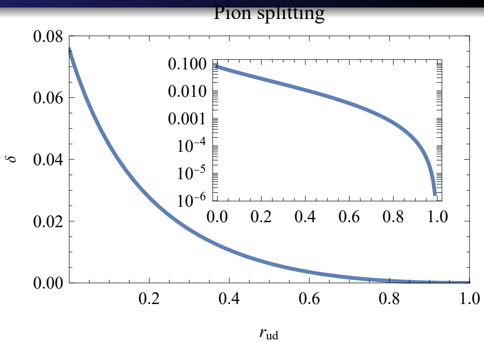
☞ m_u/m_s is a function of v_R and $r_{ud} = m_u/m_d$.

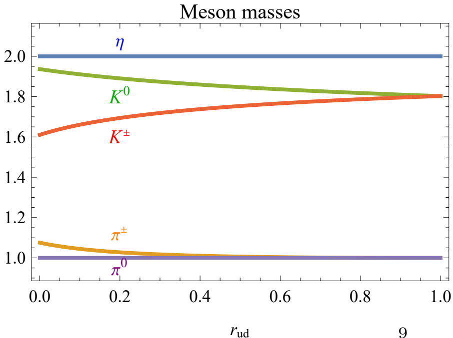
☞ Ratios of meson masses are fixed in terms of r_{ud}.

✔ η acts as unstable resonance:

$$\mathcal{L}_{\eta\pi\pi} = \frac{B_0\theta\,\cos(3\theta_{\eta\pi})}{\sqrt{3}f_\pi \mathrm{Tr}M^{-1}}\eta\pi^0\pi^0.$$

$$\begin{cases} m_{\eta^0} = \sqrt{(2+v_R^2/4)}m_{\pi^0} \\ m_K \\ m_{\pi^\pm} = (1+\delta)m_{\pi^0} \\ m_{\pi^0} \end{cases}.$$





Cosmological implications

✔ The θ angle impacts DM production due to the NCP: 3-to-2 annihilations and inverse decays DM DM $\rightarrow \eta$.

✔ All the stable mesons eventually convert to DM with a contribution to the DM relic density that depends on their mass (as usual in co-annihilating DM).

✔ DM and its coannihilating partners are referred to as π_{DM}.

☞ Kaons are sufficiently heavy so that $Y_K \sim e^{-m_K/T} << Y_\pi$.

➙ since $\delta \lesssim 0.075$ then $n_{\pi_{\mathrm{DM}}} = n_{\pi^0} + 2n_{\pi^\pm}$.

☞ Assumption: the dark sector remains in kinetic equilibrium with the SM, thereby sharing the same temperature T.
$\Rightarrow$ See Luca Marsili's talk.

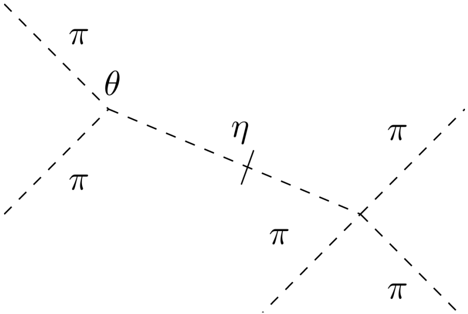
Resonant $3 \to 2$ processes

✔ η behaves as a catalyzer, inducing NCP such as

$$\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM}, \quad \eta \to \pi_{\rm DM}\pi_{\rm DM}, \quad \pi_{\rm DM}\pi_{\rm DM} \to \eta.$$

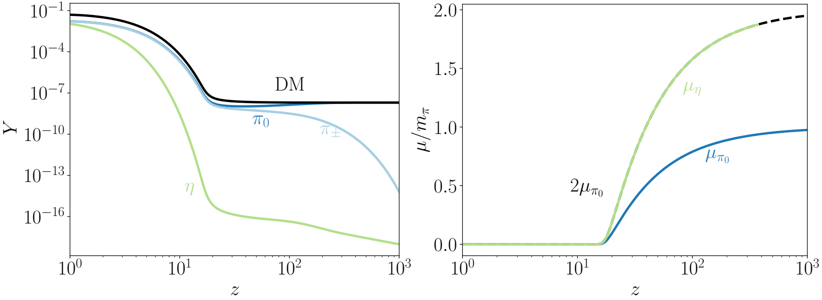
✔ For $v_R \lesssim 0.1$ and $\theta \gg 10^{-4}$: $\quad \eta \leftrightarrow \pi_{\rm DM}\pi_{\rm DM}$ are both active even after all other NCP have frozen-out.

✔ $\pi_{\rm DM}\pi_{\rm DM}\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM}$ is dominated by the exchange of an on-shell η resonance.



Resonant $3 \to 2$ processes

m_{π_0}	f_π	$m_{\pi_\pm}$	θ	r_{ud}	v_R
20.0 MeV	27.3 MeV	20.3 MeV	2.5×10^{-4}	0.33	6.7×10^{-4}



☞ As $Y_\eta \ll Y_{\pi_{\rm DM}} \Rightarrow Y_{\pi_{\rm DM}} + 2Y_\eta \simeq Y_{\pi_{\rm DM}}$.

☞ $\eta \leftrightarrow \pi_{\rm DM}\pi_{\rm DM}$ establish chemical equil.: $n_\eta/n^2_{\pi_{\rm DM}} = (n_\eta/n^2_{\pi_{\rm DM}})_{\rm eq}$.

✔ $\Omega_{\rm DM}$ is determined by s-wave 2-to-2.

Parameter space

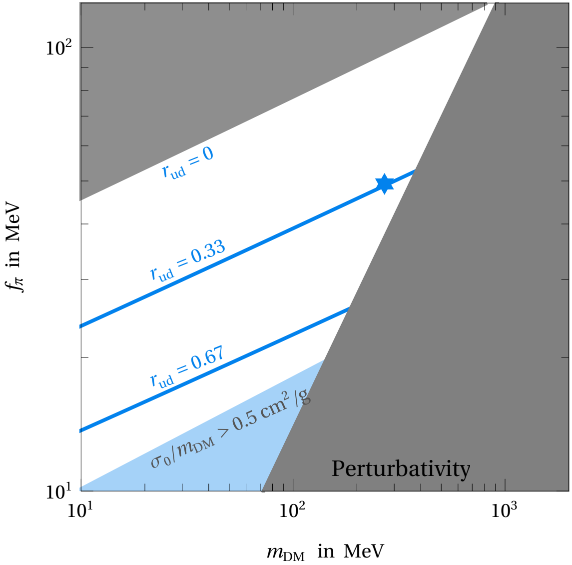
✔ $\Omega_{\rm Planck} h^2$ is reproduced along the plane for different values of r_{ud}.

☞ Upper gray region DM is overabundant for any physical value of r_{ud}.

☞ Blue region is excluded by obs. of galaxy clusters: $\sigma_0/m_{\rm DM} > 0.5$ cm^2/g.

☞ In the dark-gray region ChPT breaks down: $m_{\rm DM}/f_\pi > 4\pi/\sqrt{N_c}$.

✔ Results are independent of v_R (if $v_R \lesssim 0.1$) but depend on δ (or r_{ud}), as long as the small dependence of the masses on v_R and θ is neglected.



☞ The viable parameter regions can be compatible with cluster observations on DM self-scattering (Giacomo's talk).

Summary

✔ We presented a production mechanism of DM in QCD-like theories where $\theta \neq 0$.

☞ This requires a resonance mediating $\pi_{\rm DM}\pi_{\rm DM} \to \eta$ followed by $\eta\pi_{\rm DM} \to \pi_{\rm DM}\pi_{\rm DM}$.

☞ The θ-vacuum leads to velocity-dependent SIDM without a light mediator (Giacamo's talk)

✔ The θ-dependent effects discussed here are generic features of QCD-like theories.

Outlook

- Portal to SM: hidden dark photons (see Luca Marsili's talk); axion-like particles.
- Emission of GWs if the ChPT is first order.
- Additional source of CP violation in QCD-like sectors. Implications for the matter-antimatter asymmetry?.