

Core Dynamics

**a testbed for the nature
of Dark Matter**

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Collaborators: Shashank Dattathri, Uddipan Banik, Martin Weinberg

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Key Takeaways

Cores of **collisionless** systems reveal intriguing dynamics

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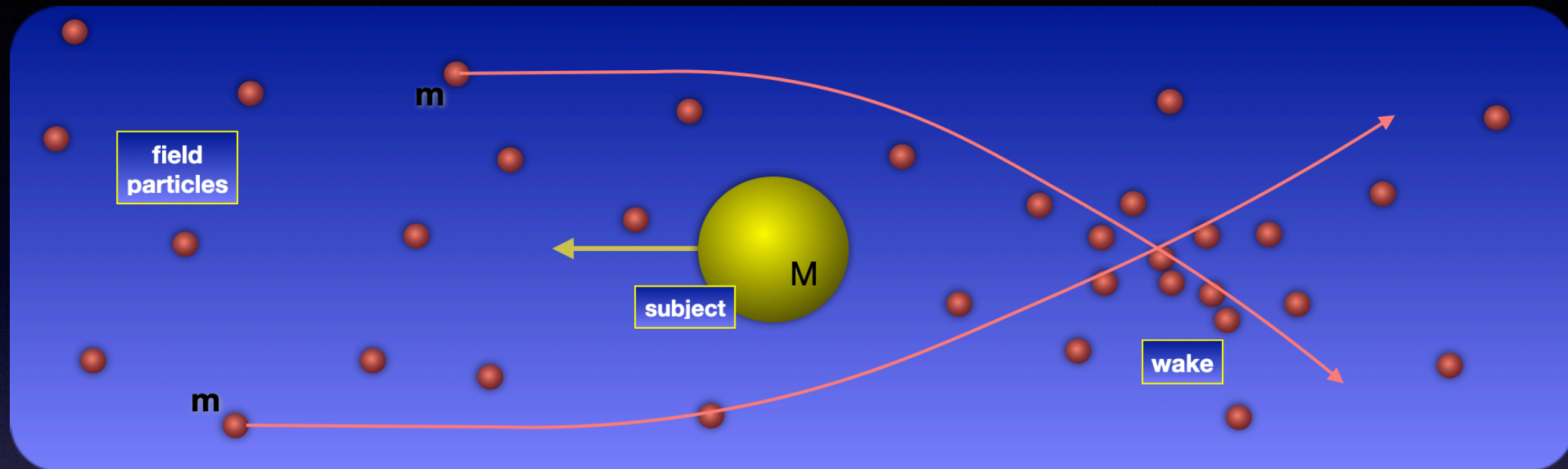
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Cores are a potential proving ground for nature of dark matter

Chandrasekhar Dynamical Friction

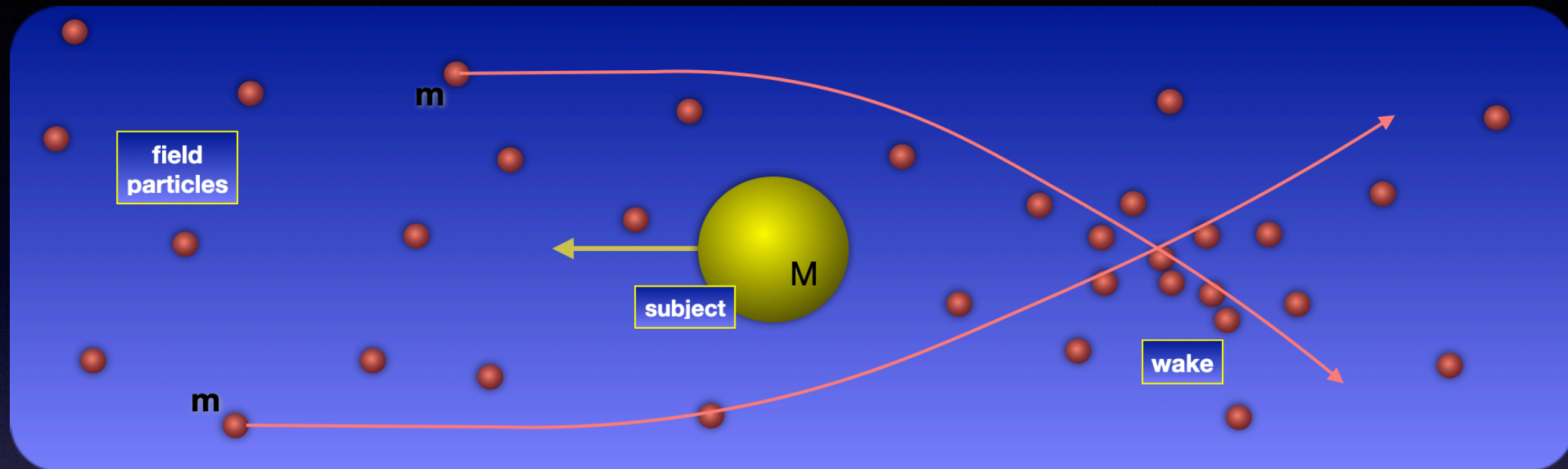
Dynamical Friction: friction due to gravity



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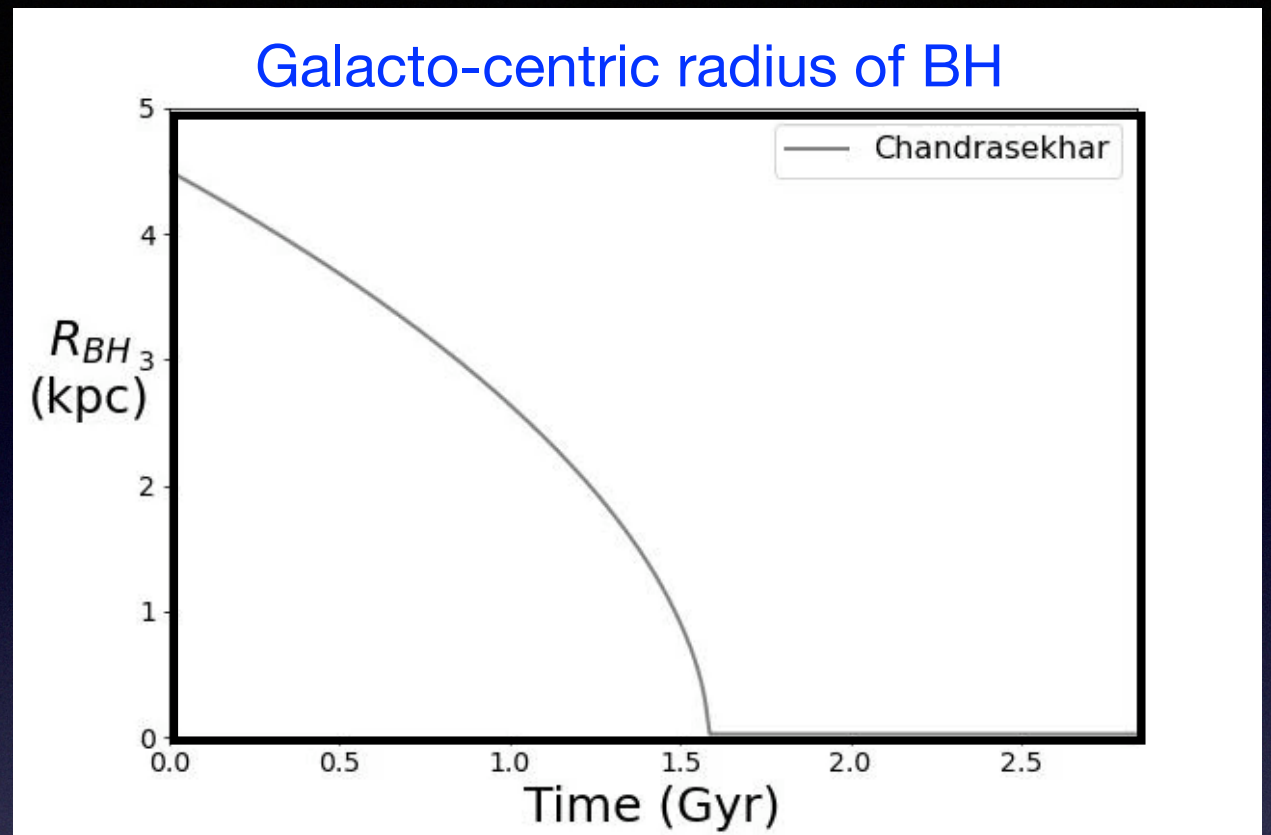
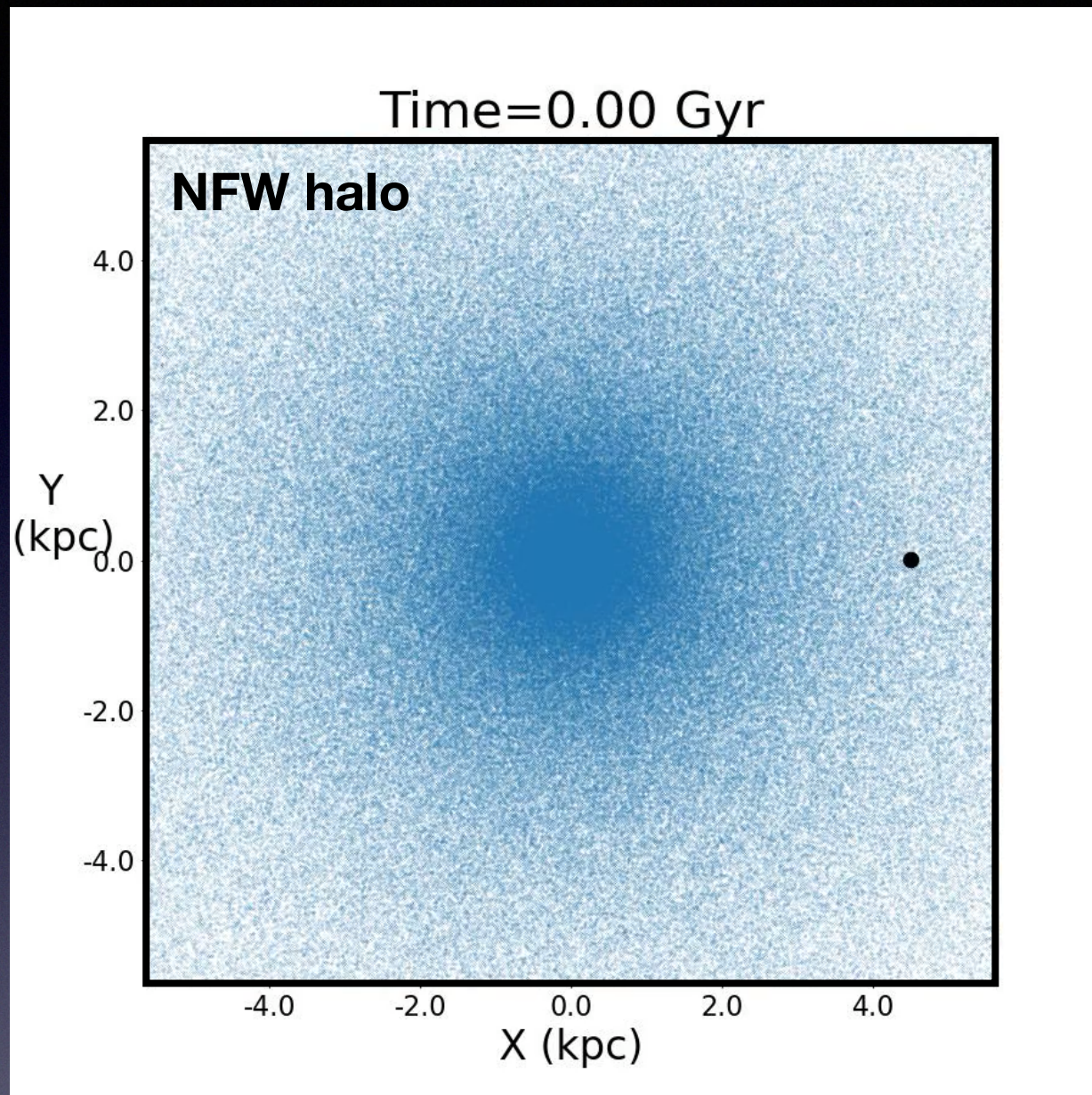
$$\mathbf{F}_{\text{df}} = -4\pi \left(\frac{GM_{\text{P}}}{v_{\text{P}}} \right)^2 \ln \Lambda \rho(< v_{\text{P}}) \frac{\mathbf{v}_{\text{P}}}{v_{\text{P}}}$$

Chandrasekhar 1943

Only valid
under following
circumstances

- The distribution of field stars is **infinite** and **homogeneous**
- The orbit of the perturber is a **straight line**

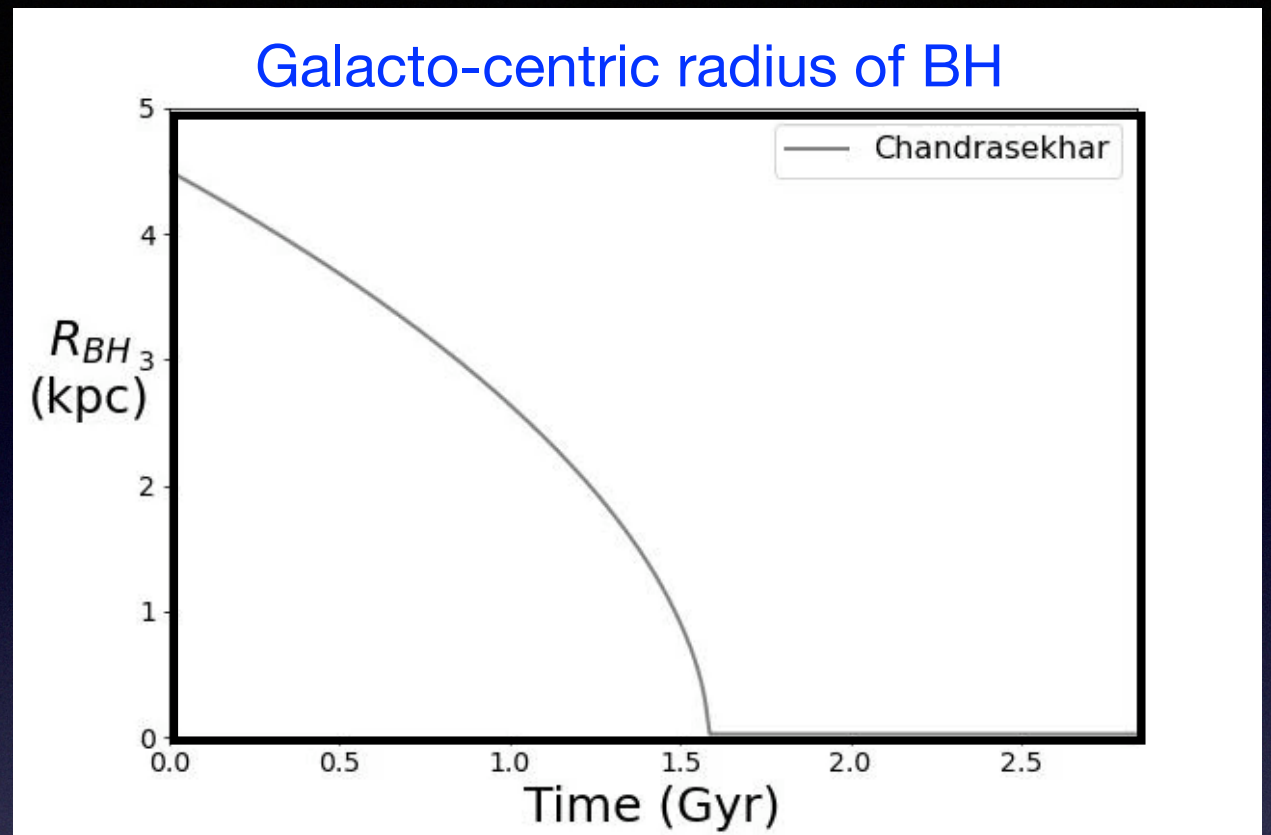
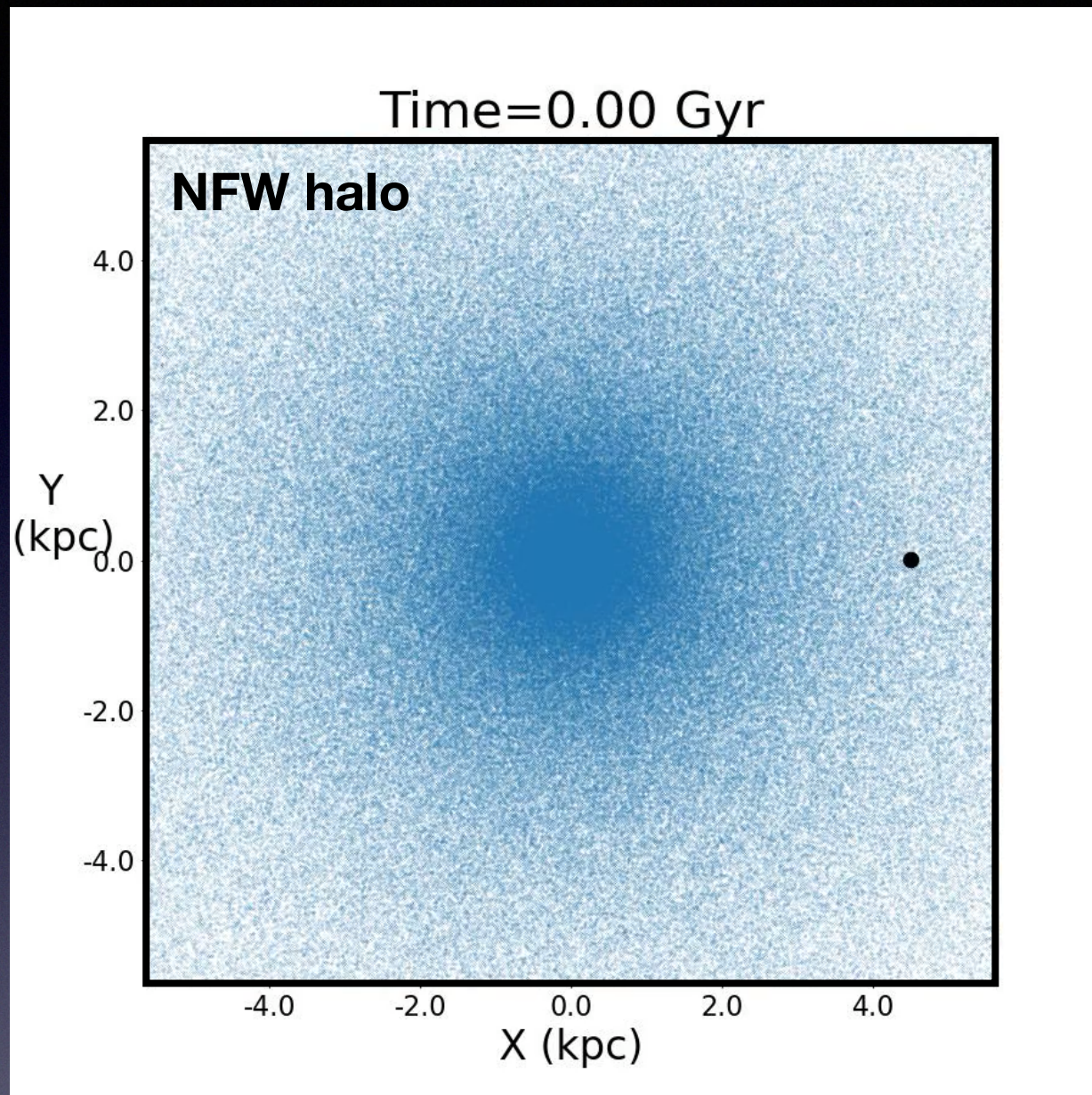
Dynamical Friction in cuspy NFW halo



Orbital decay consistent with predictions based on **Chandrasekhar dynamical friction**

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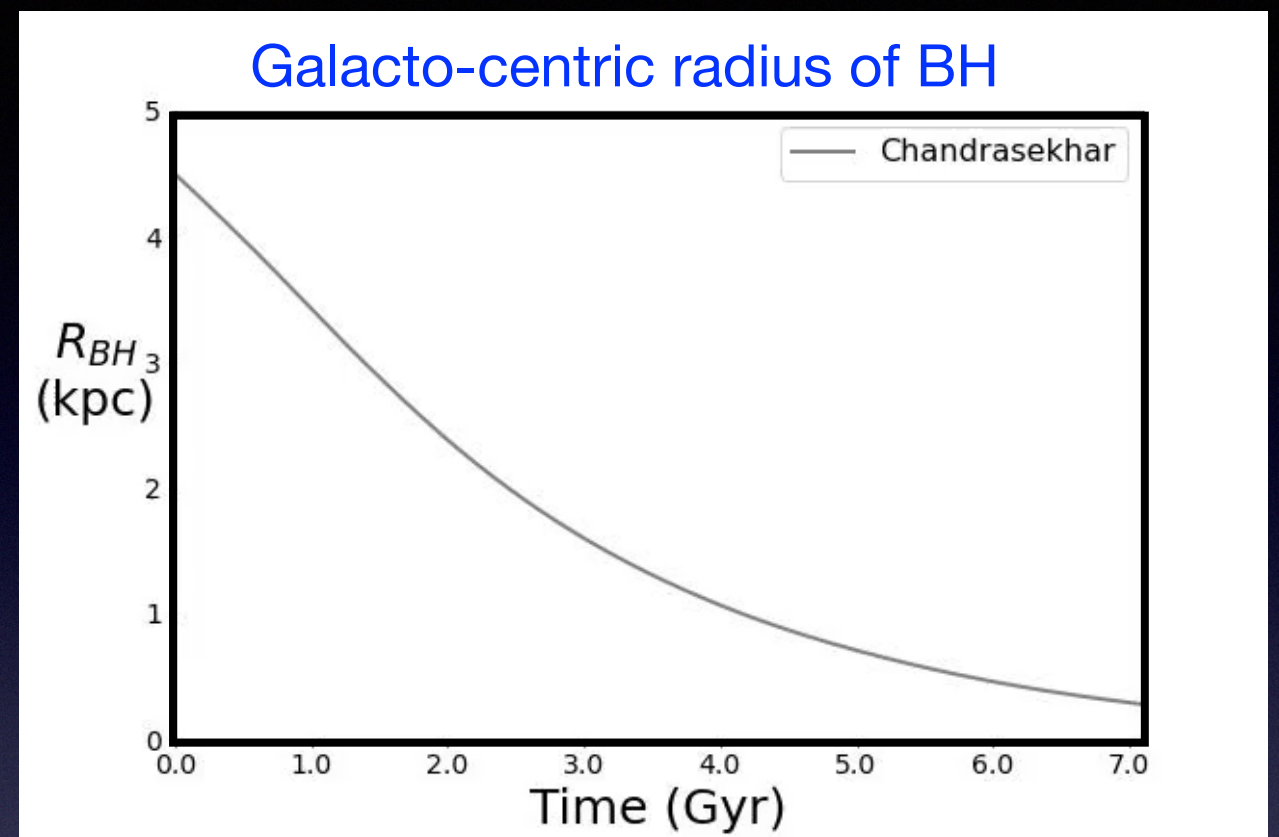
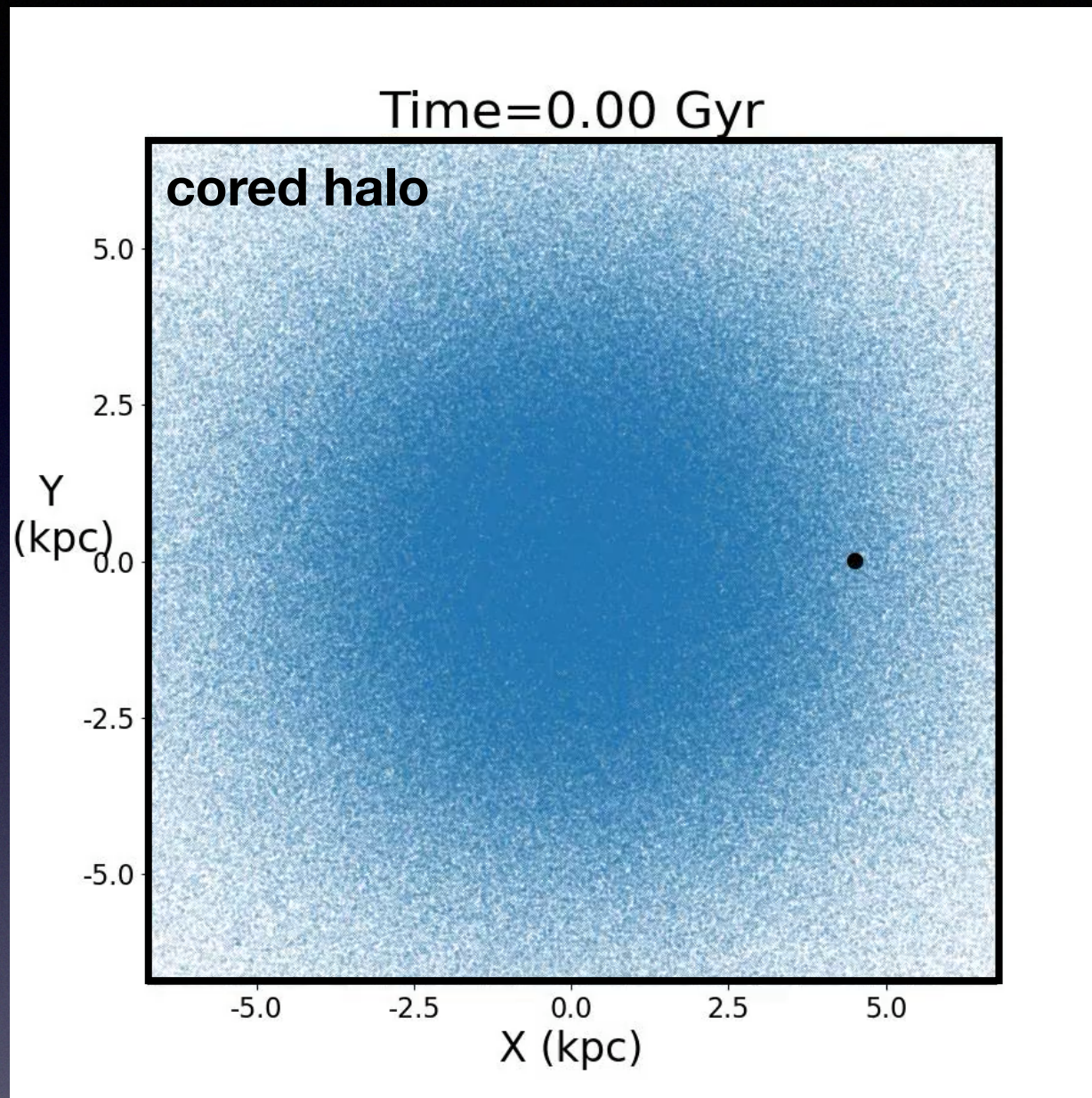
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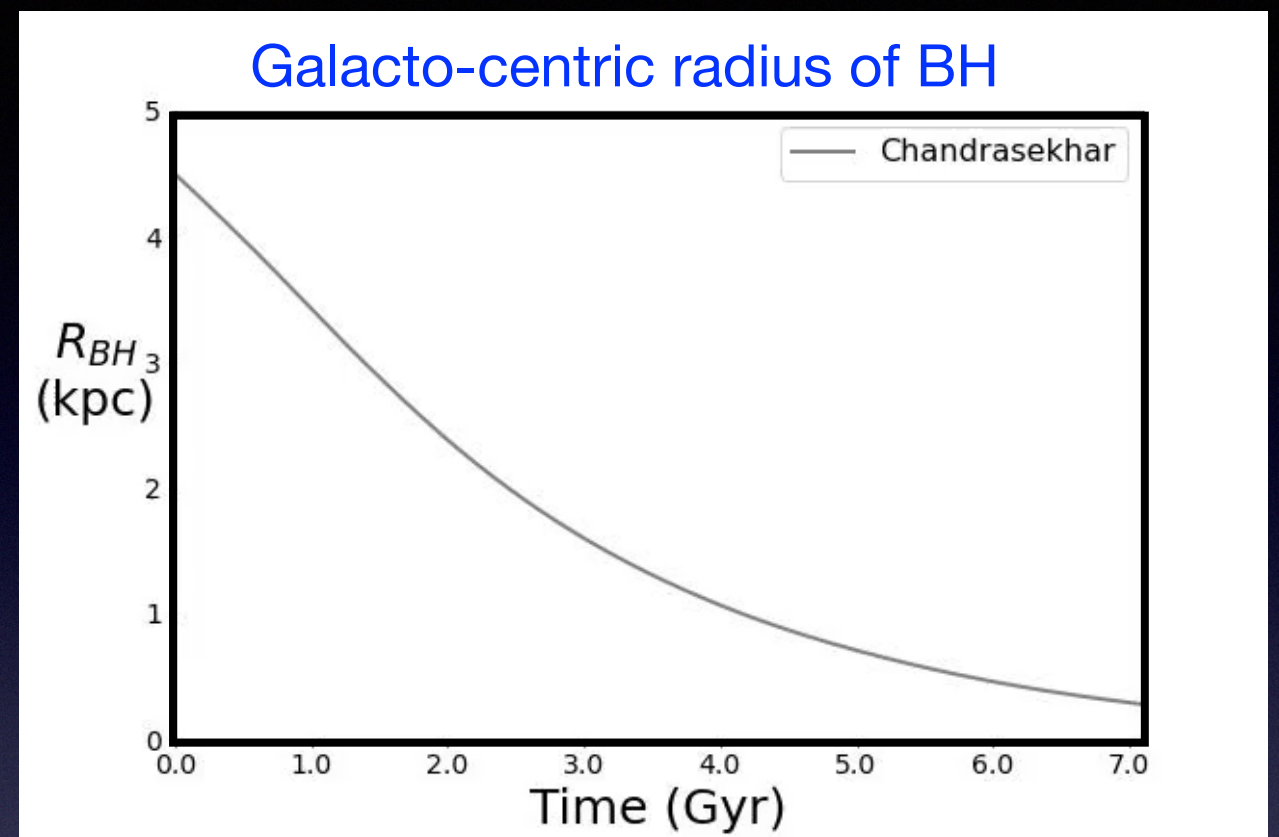
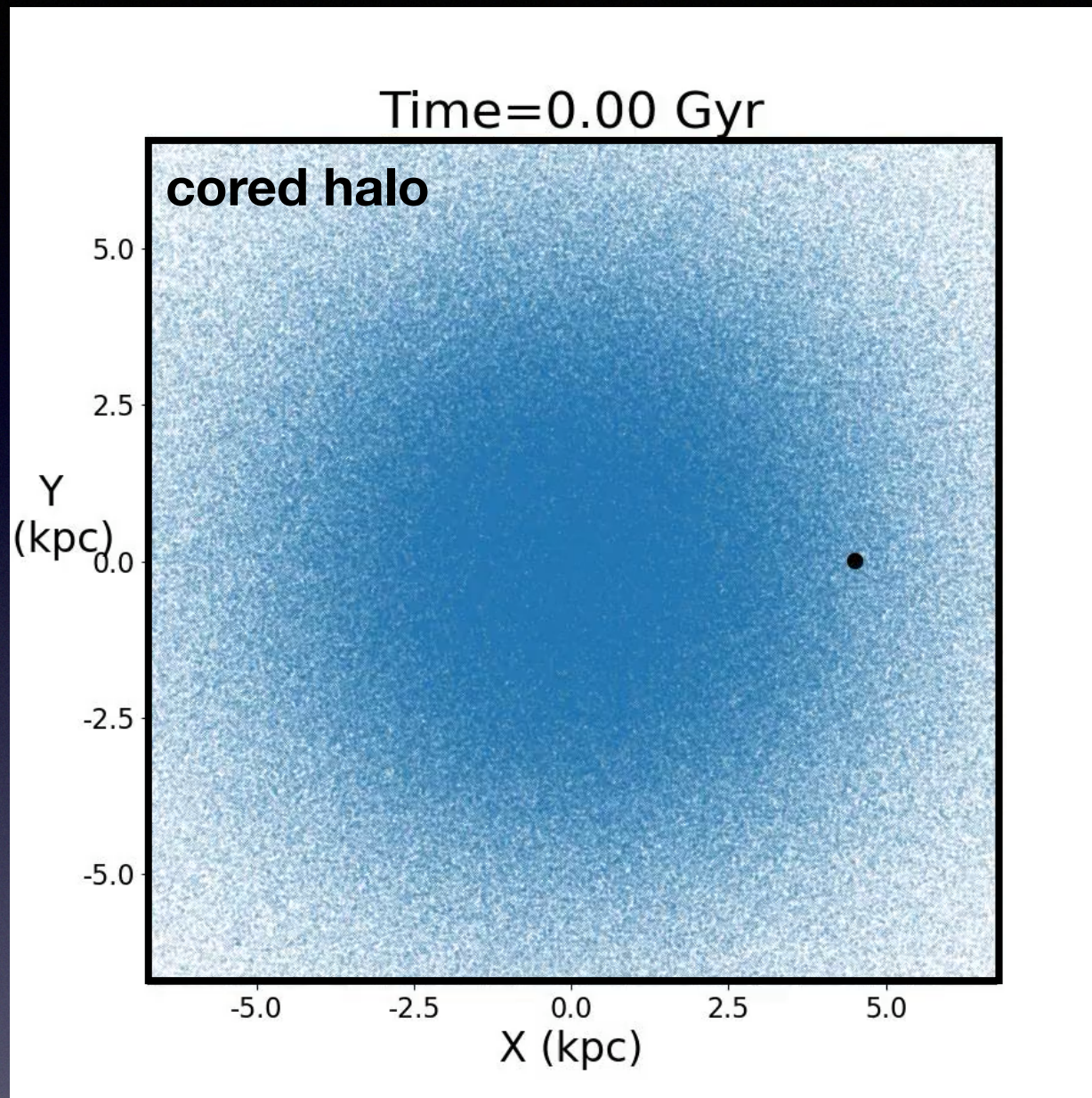
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Core Stalling



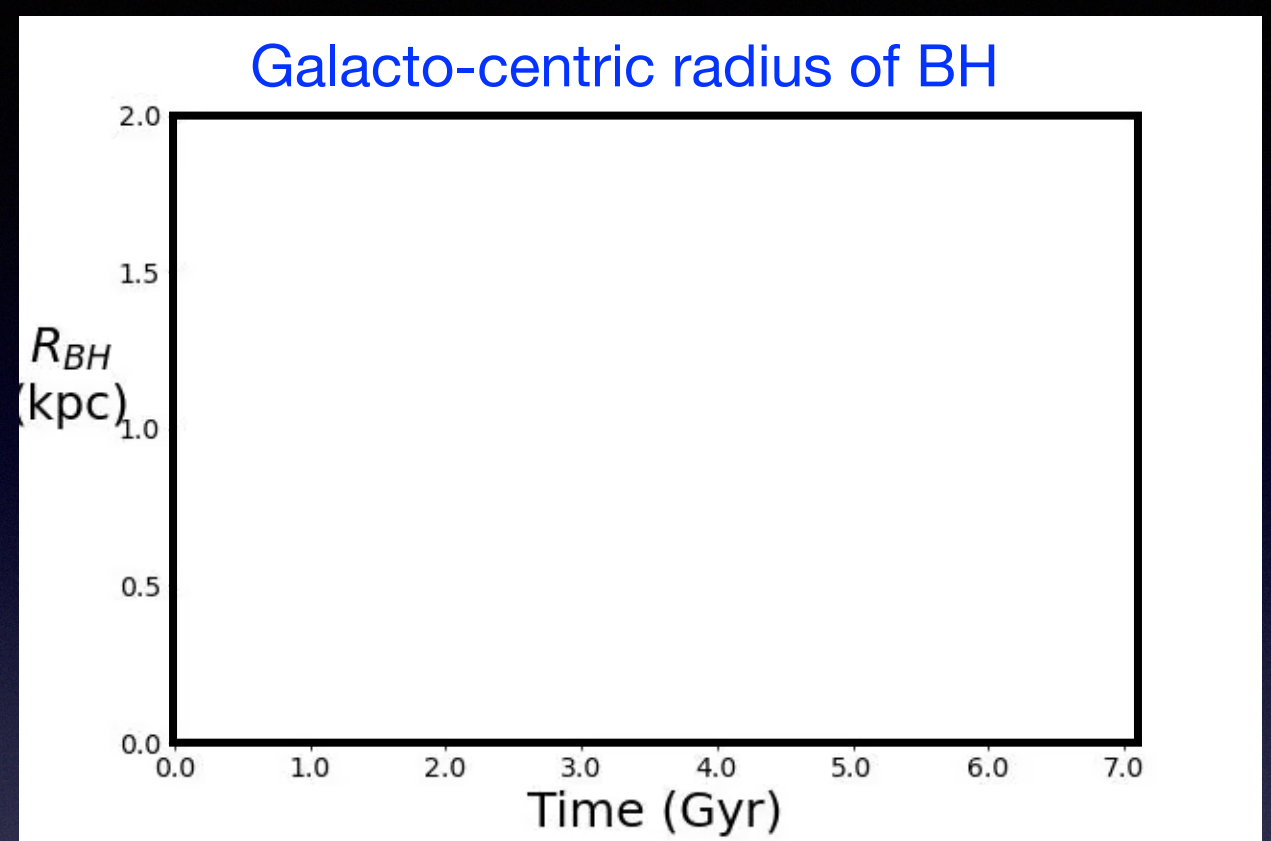
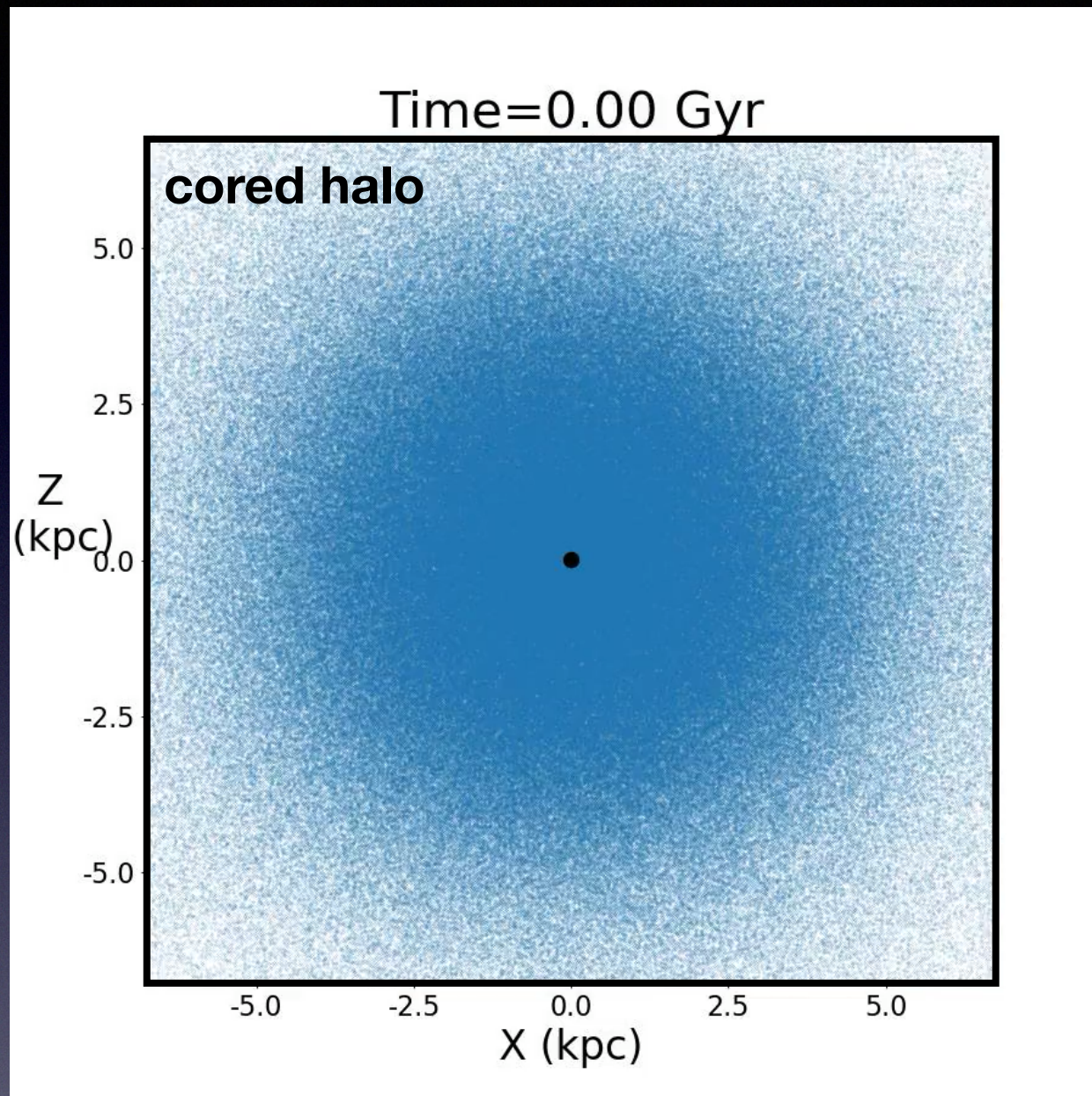
see also Read+06; Goerdt+10; Inoue 2011; Petts+16; Dutta-Chowdhury+19, Banik & vdB 2021

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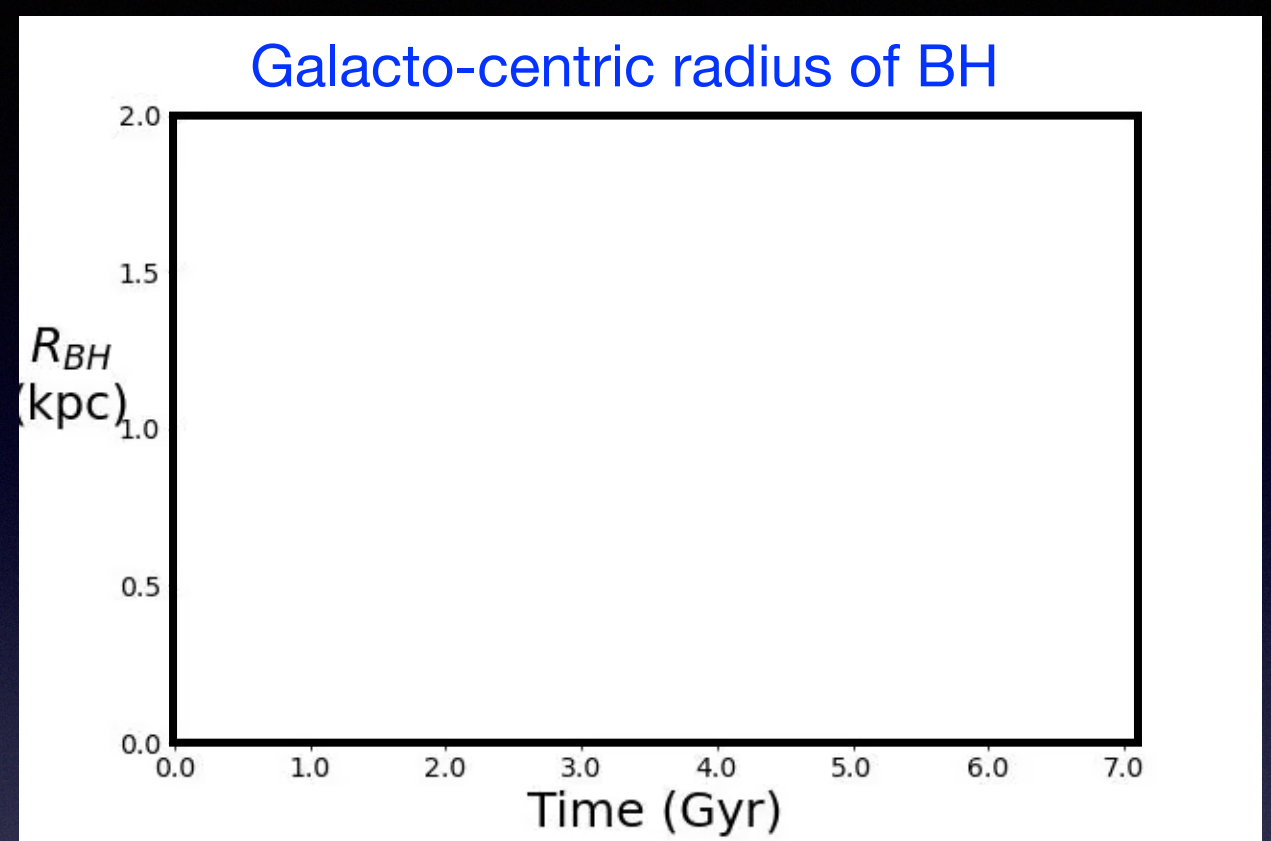
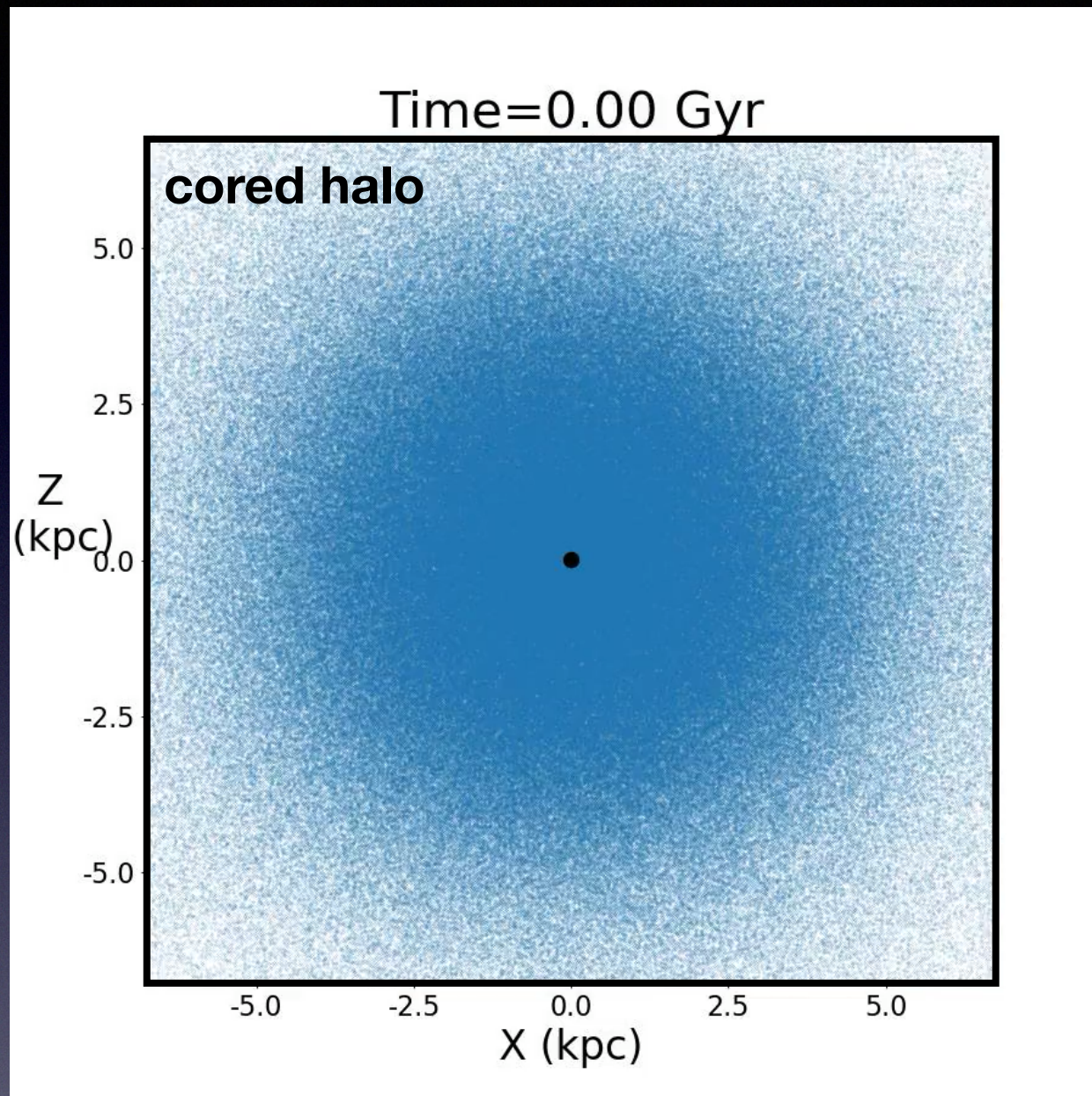
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Dynamical Buoyancy



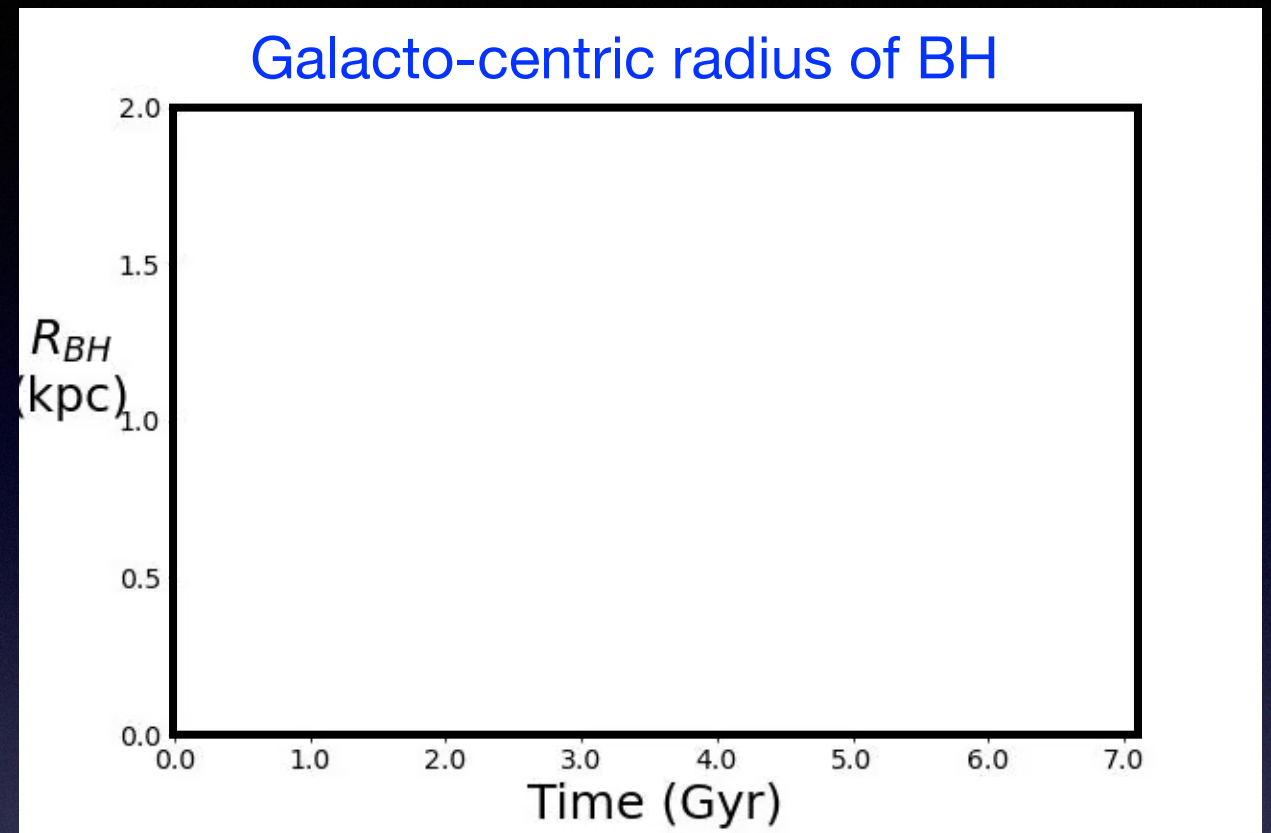
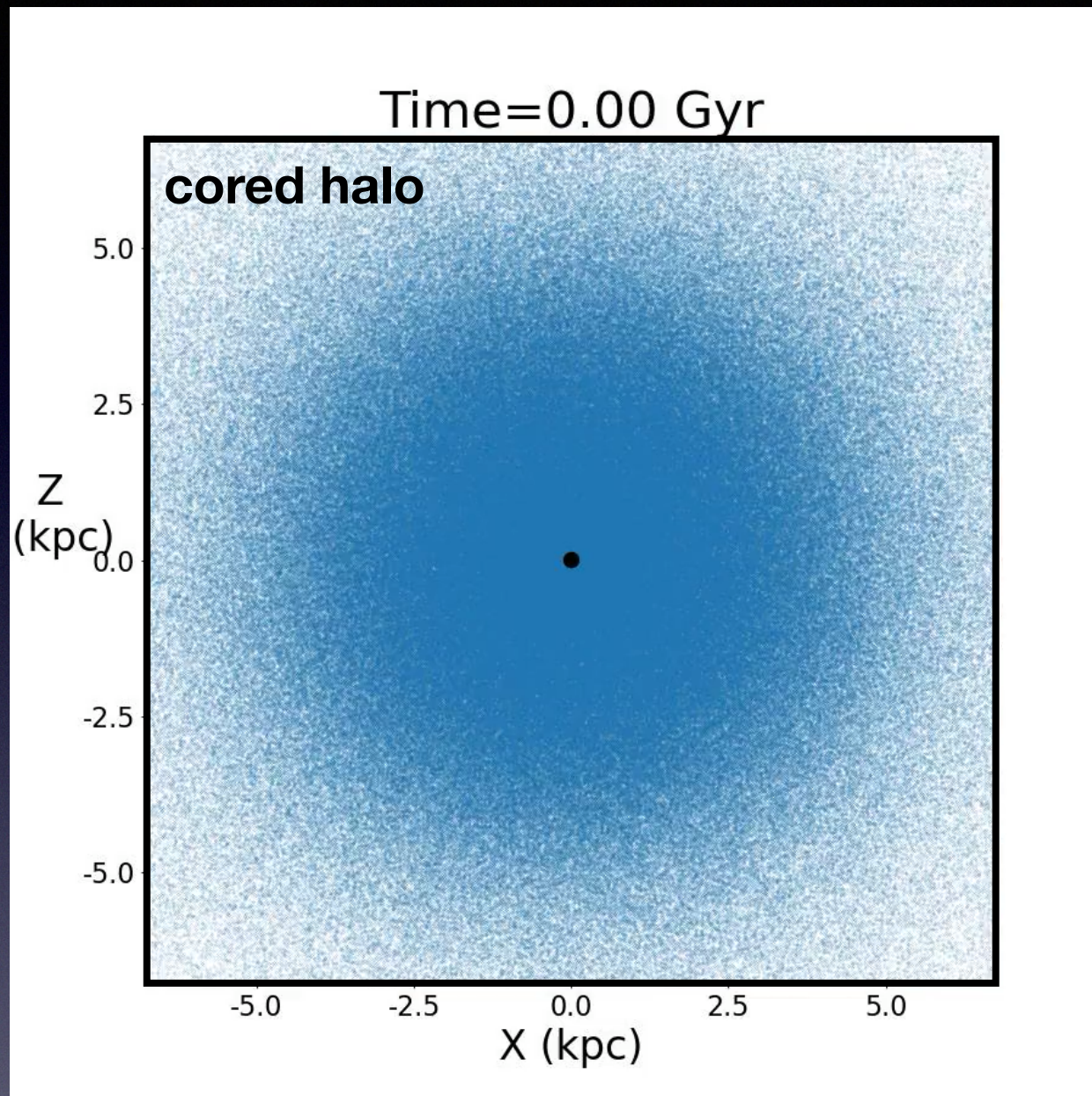
see also Cole+12; Banik & vdB 21

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Dynamical Buoyancy



**Core stalling and Dynamical buoyancy
are inconsistent with
Chandrasekhar dynamical friction**

NOTE: this is NOT due to Brownian motion,
but rather to collective effects

see also Cole+12; Banik & vdB 21

Double Power-Law Density Profiles

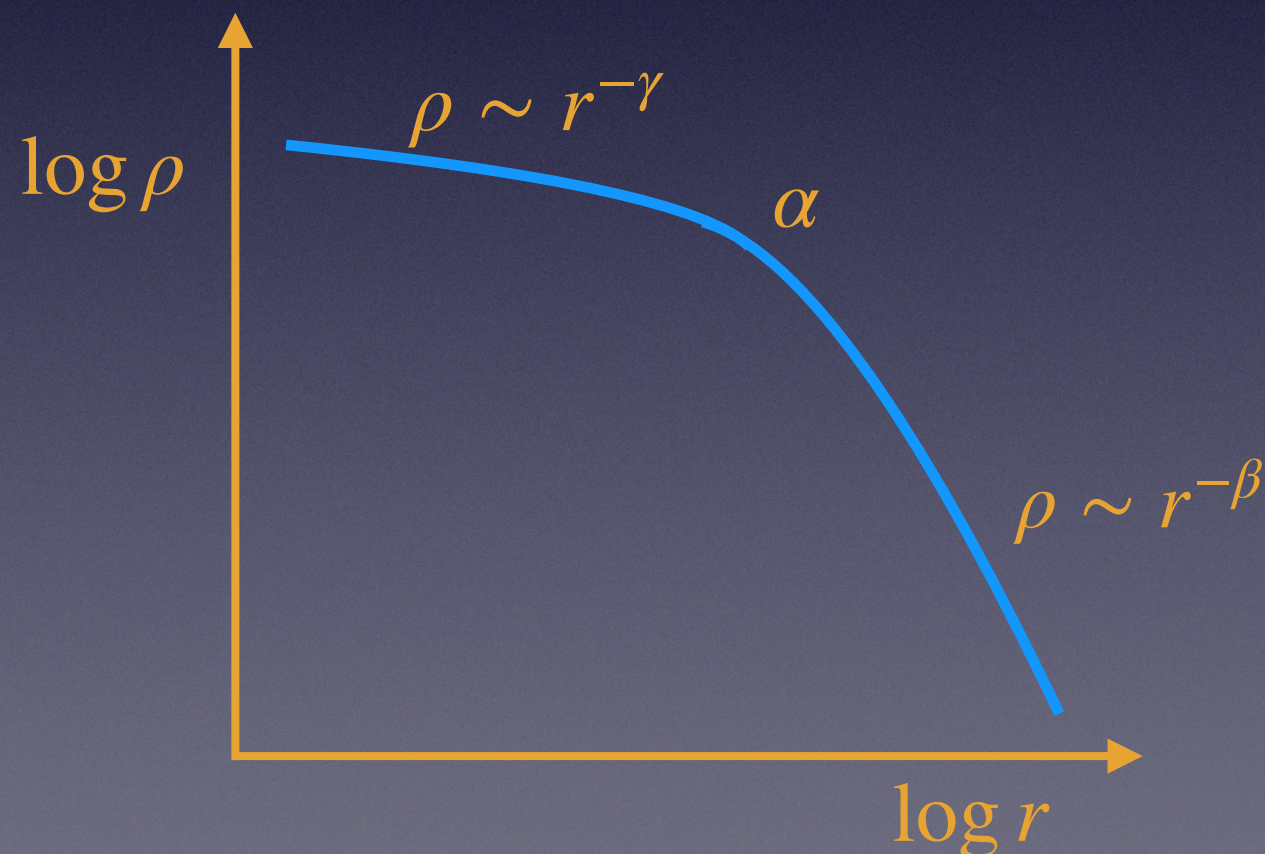
Consider the generic class of (α, β, γ) density profiles, commonly used to model/describe astrophysical systems

$$\rho(r) = \rho_0 \left(\frac{r}{R_0} \right)^{-\gamma} \left[1 + \left(\frac{r}{R_0} \right)^\alpha \right]^{(\gamma-\beta)/\alpha}$$

α : Steepness of transition

β : Outer slope

γ : Inner slope



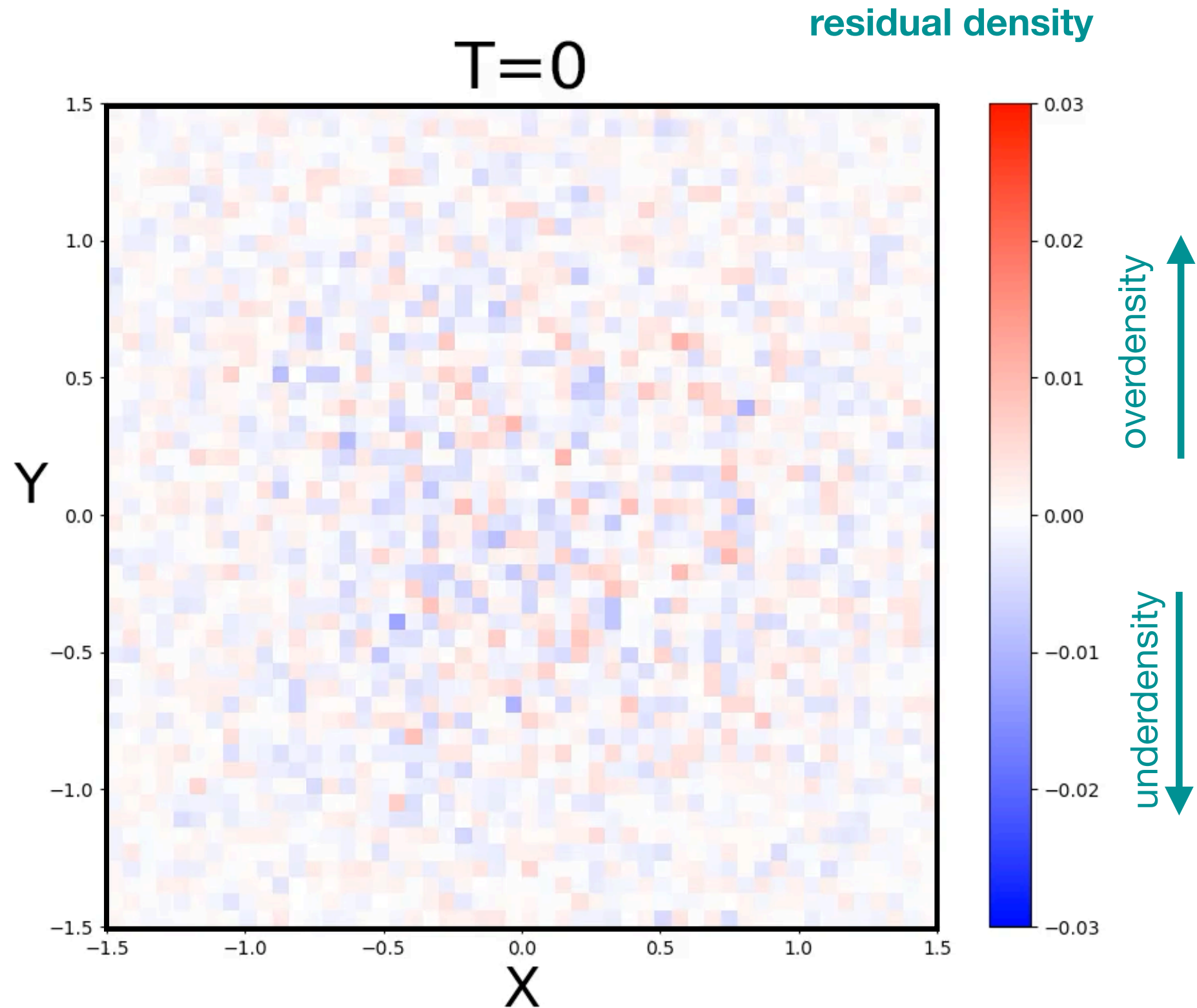
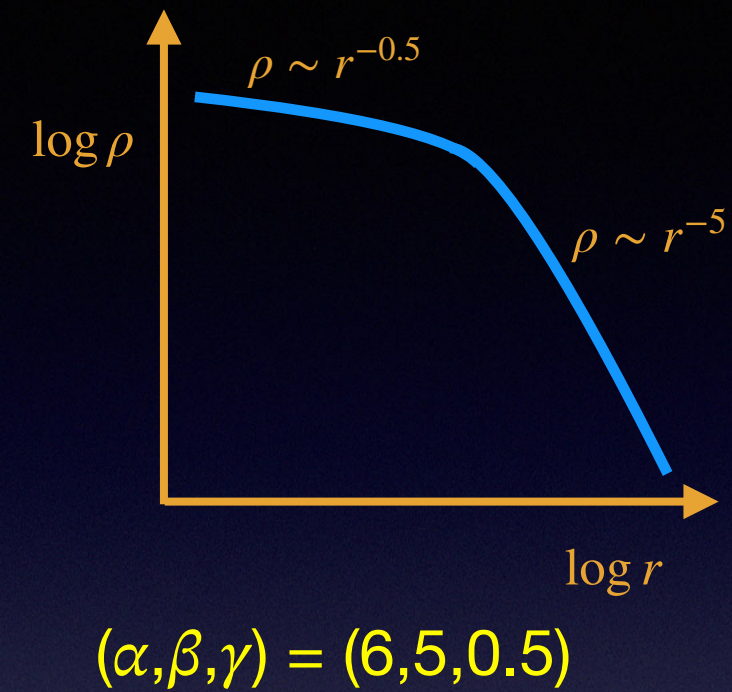
Well known examples:

NFW	$(\alpha, \beta, \gamma) = (1, 3, 1)$
Hernquist	$(\alpha, \beta, \gamma) = (1, 4, 1)$
Plummer	$(\alpha, \beta, \gamma) = (2, 5, 0)$

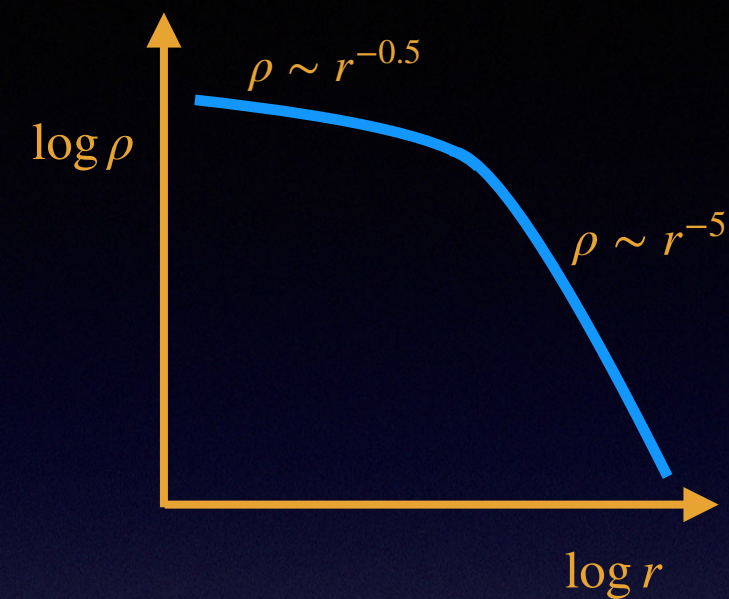
Outstanding Question

For what values of α, β, γ does core stalling/buoyancy occur?

Core Instability

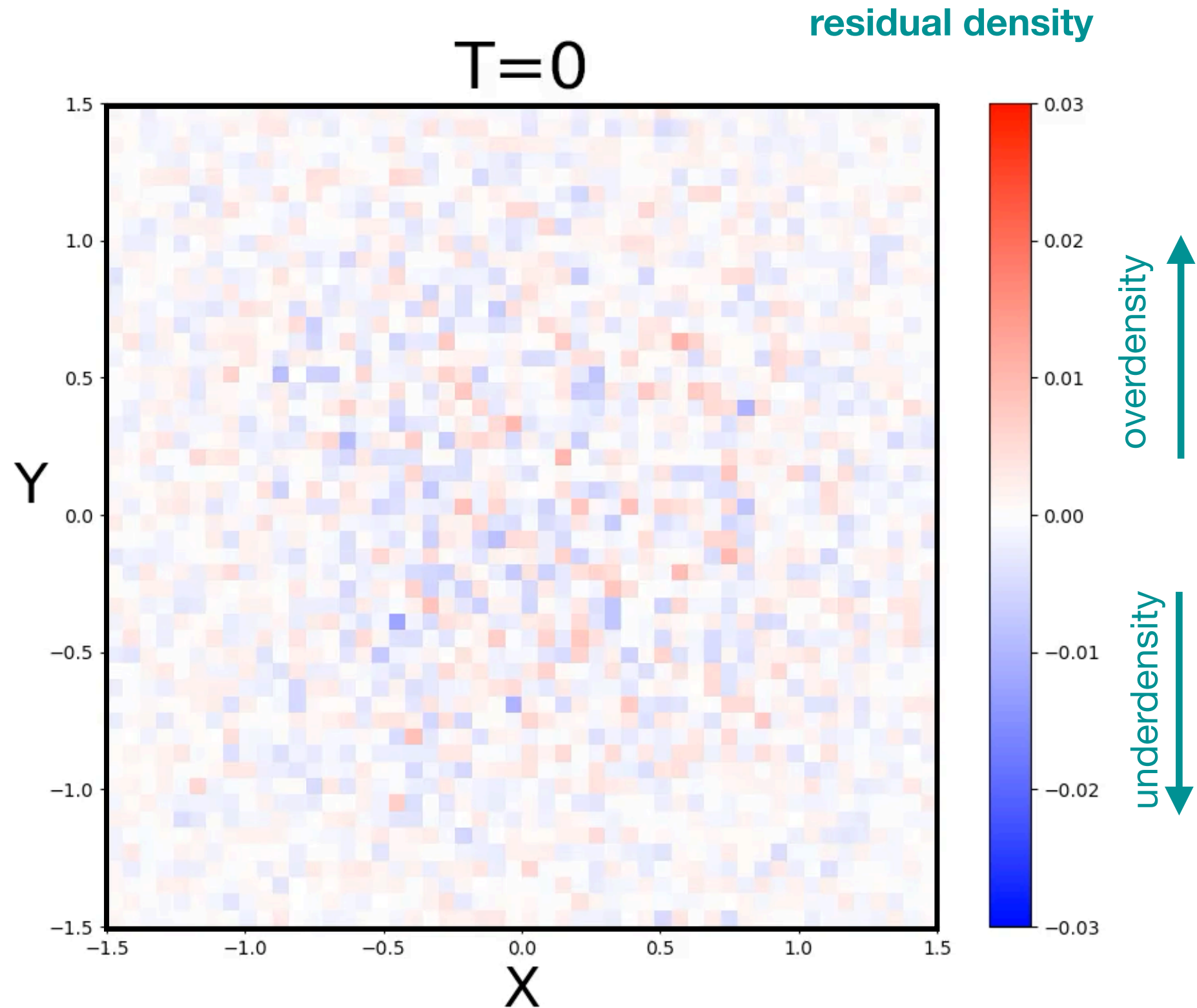
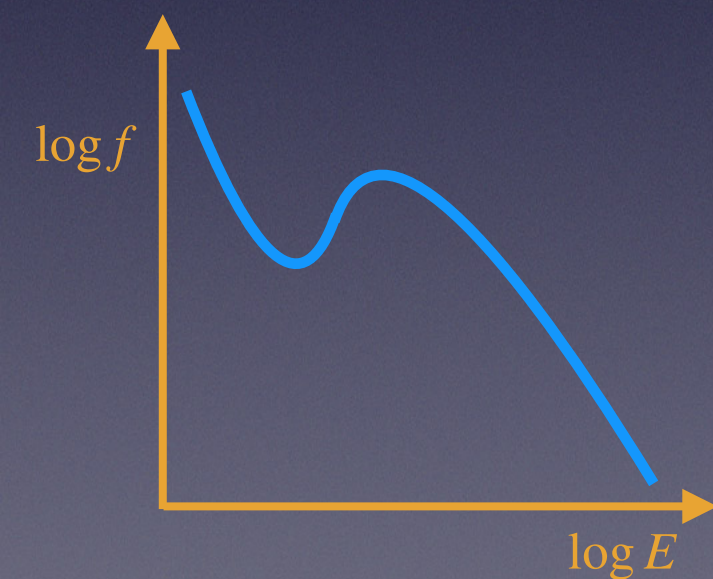


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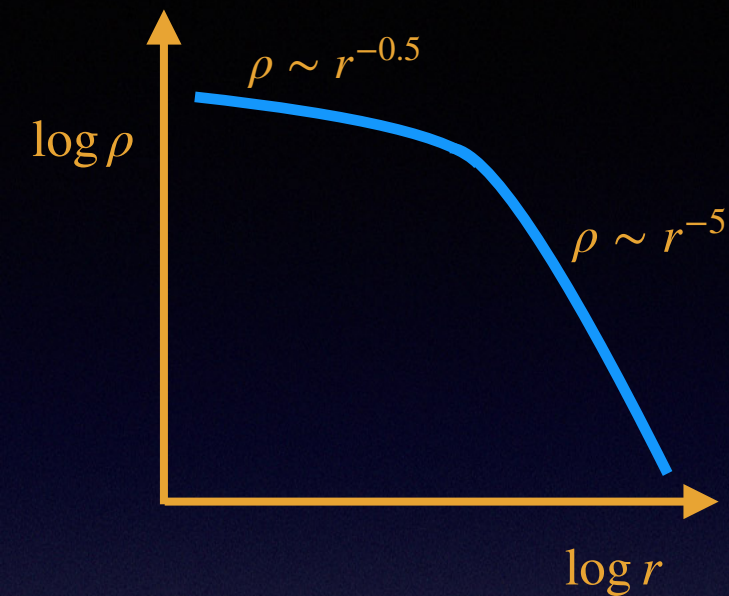


$$(\alpha, \beta, \gamma) = (6, 5, 0.5)$$

isotropic $f(x, v) = f(E)$

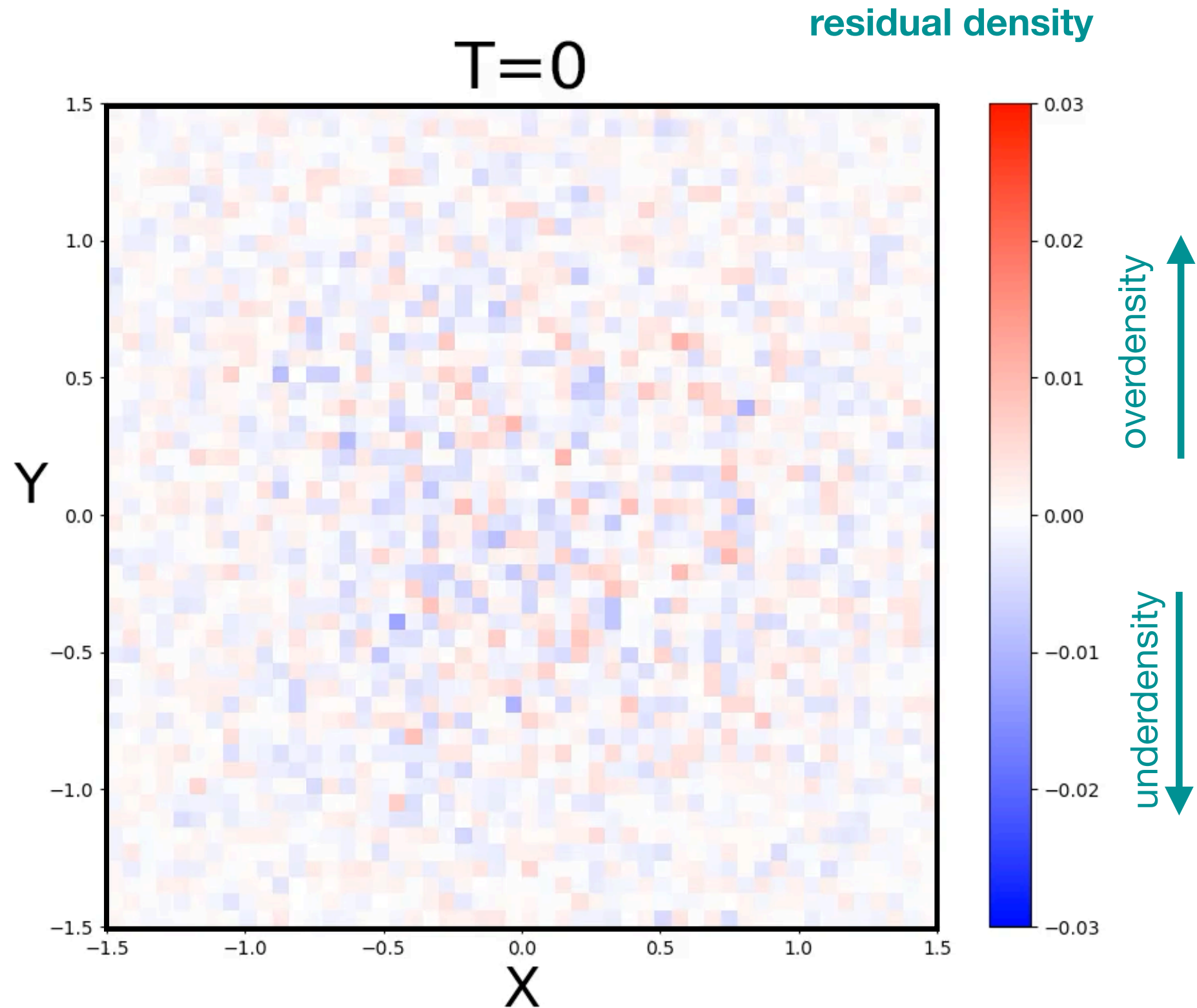
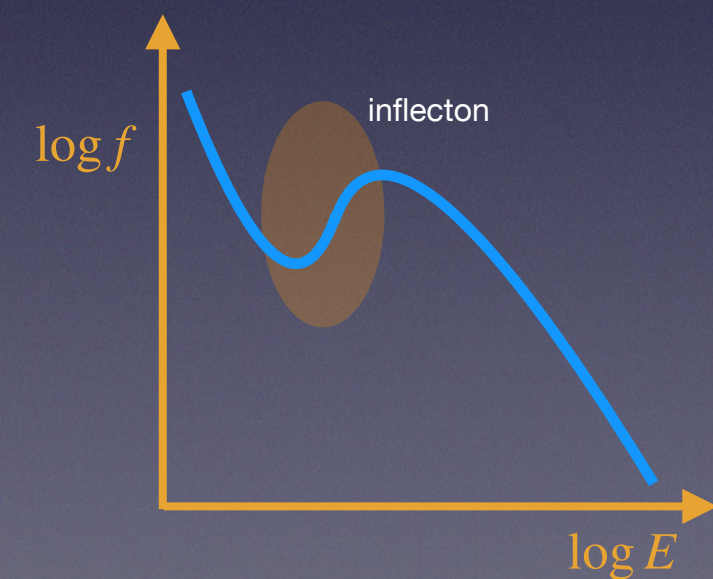


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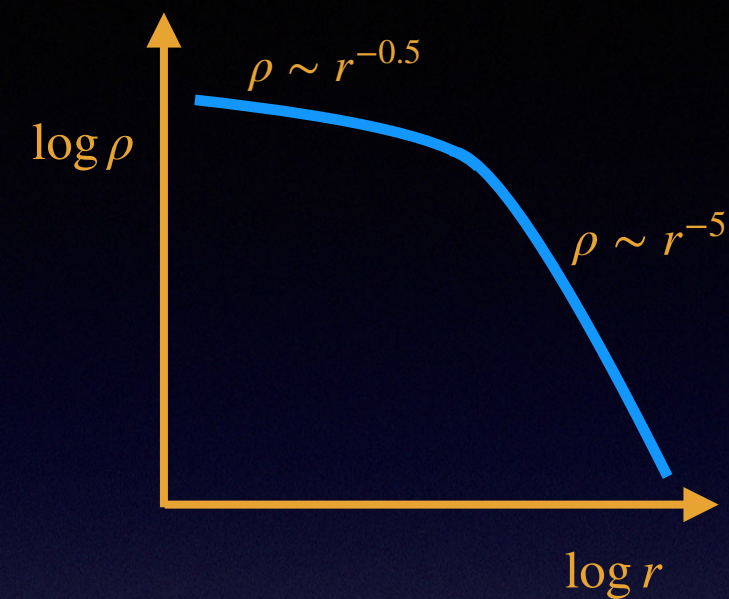


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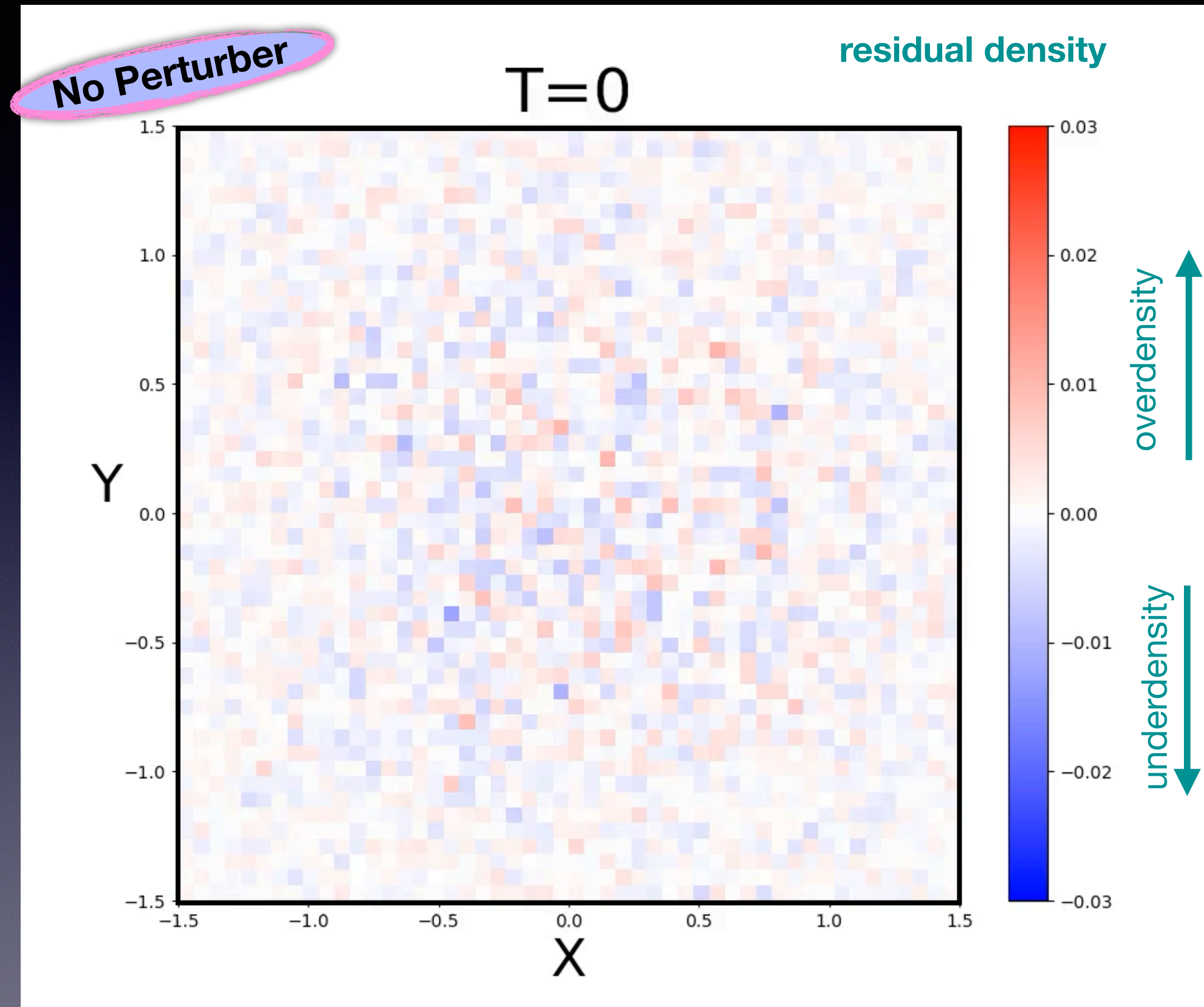
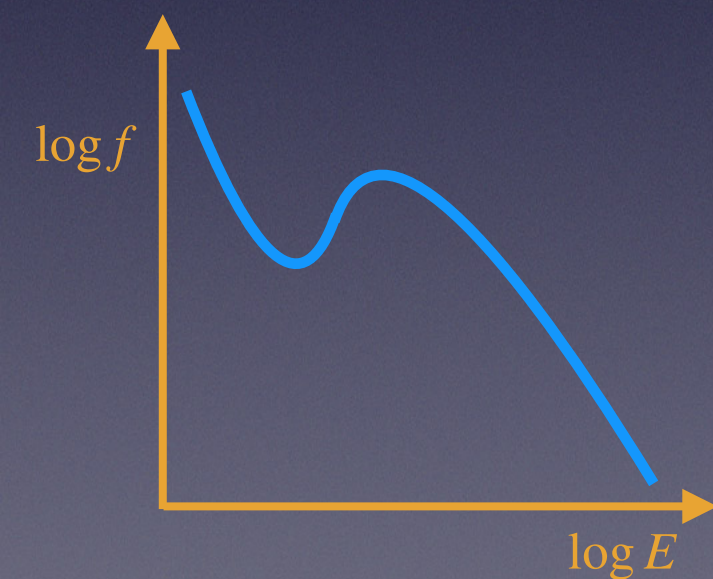


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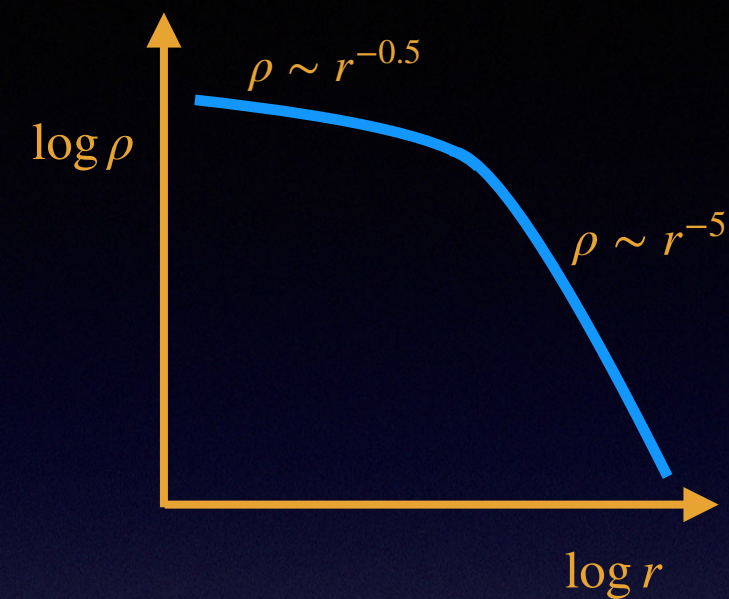


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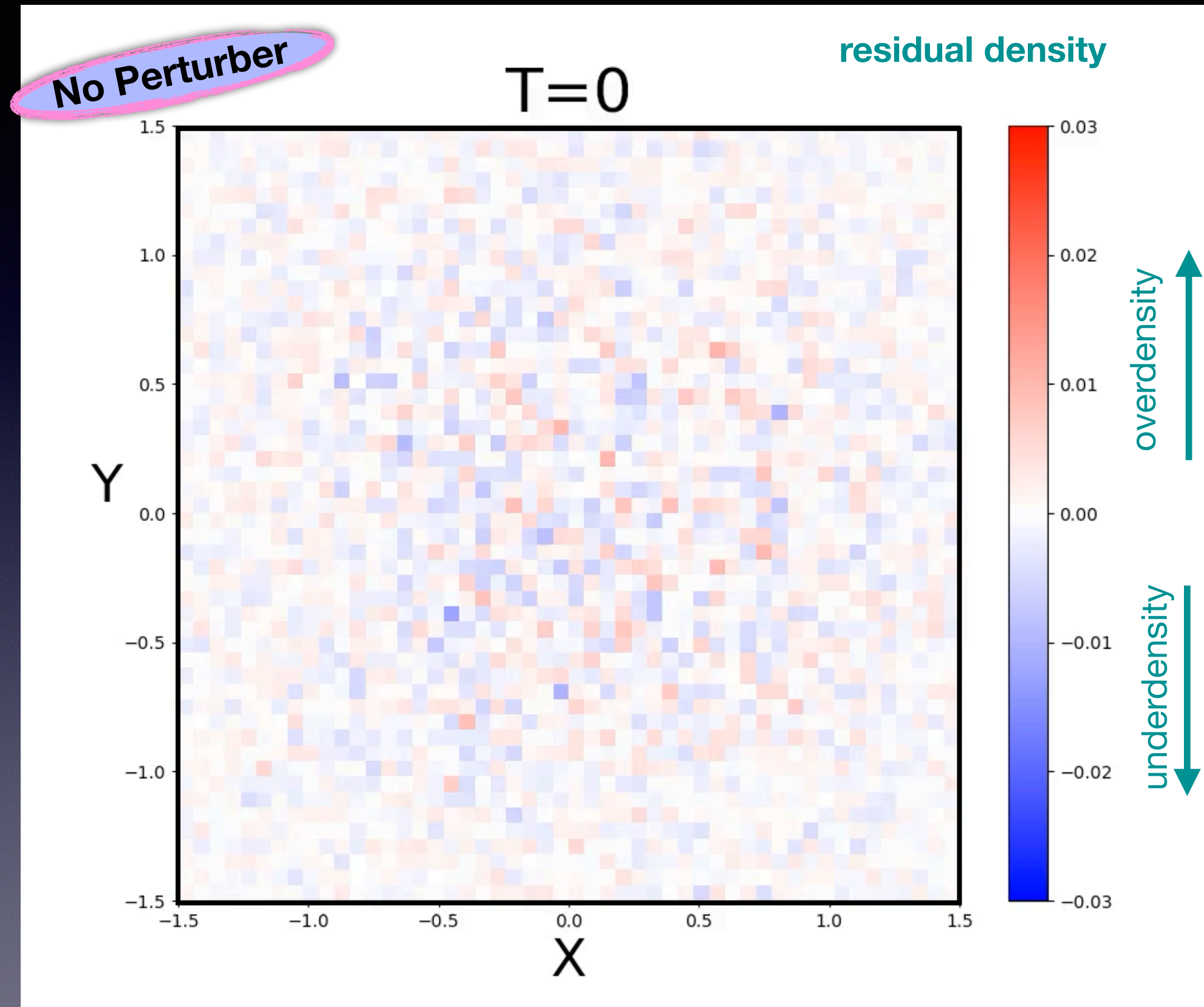
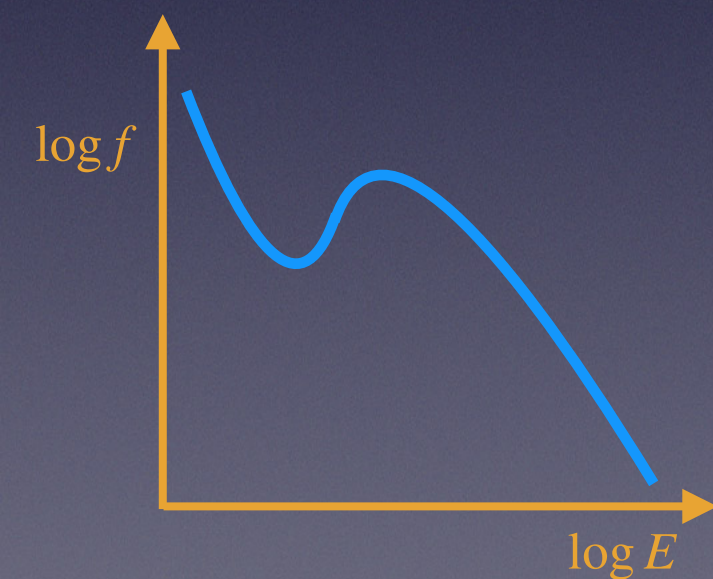


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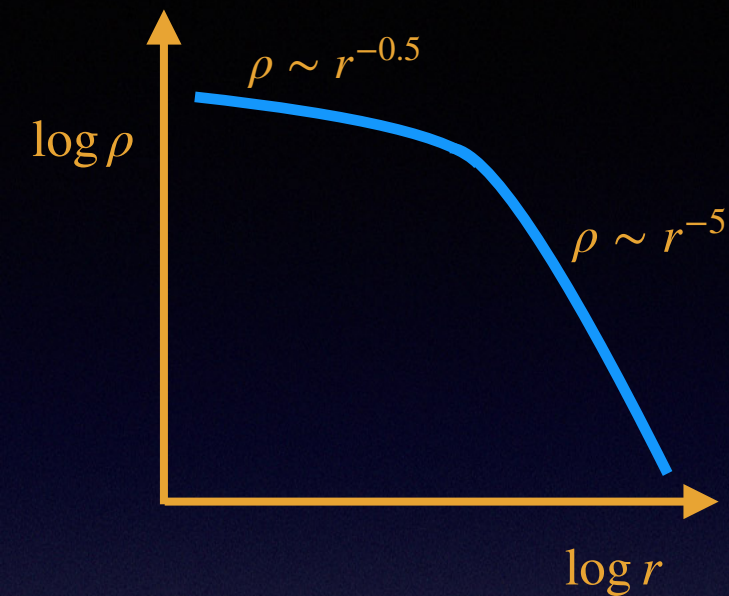


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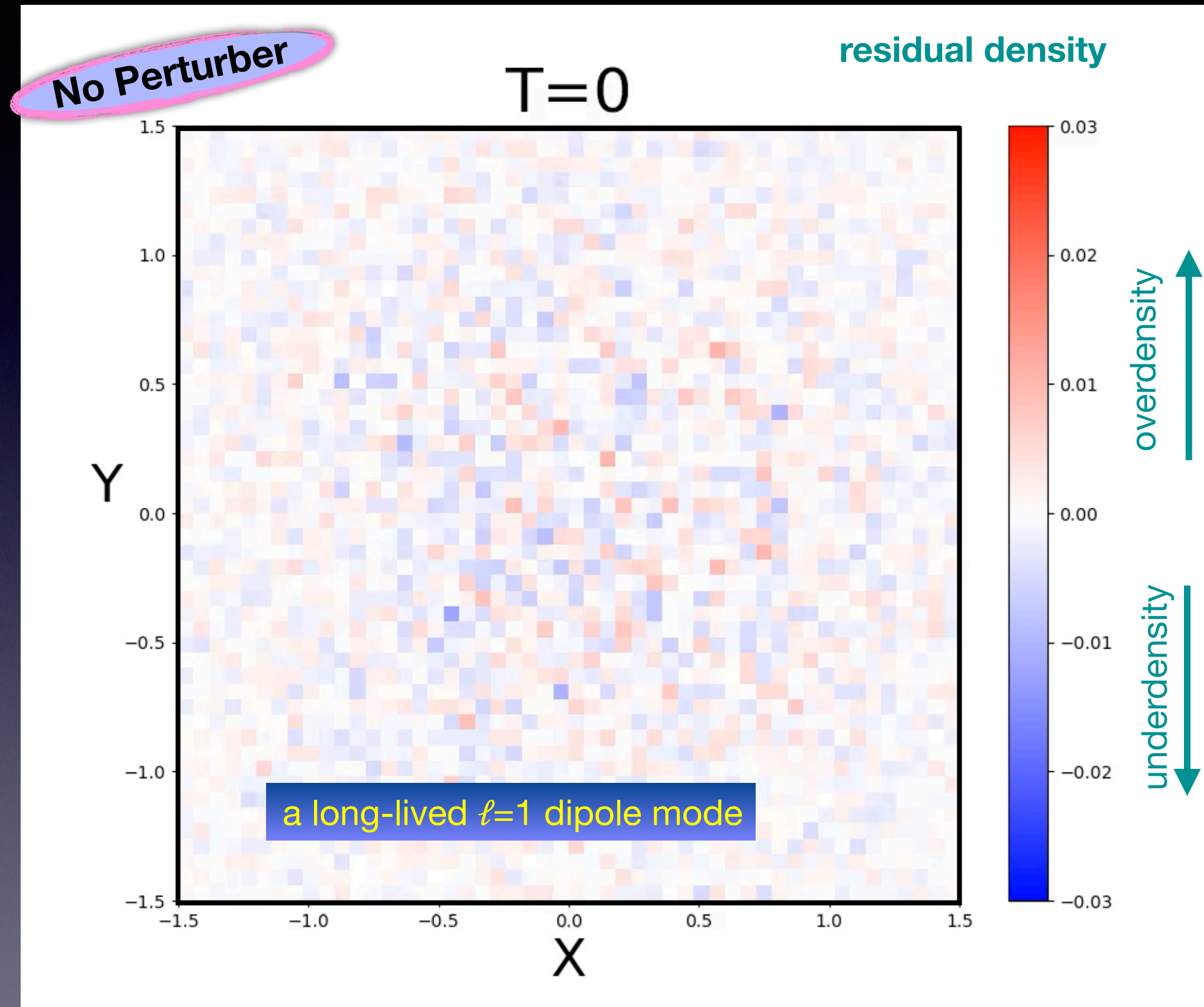
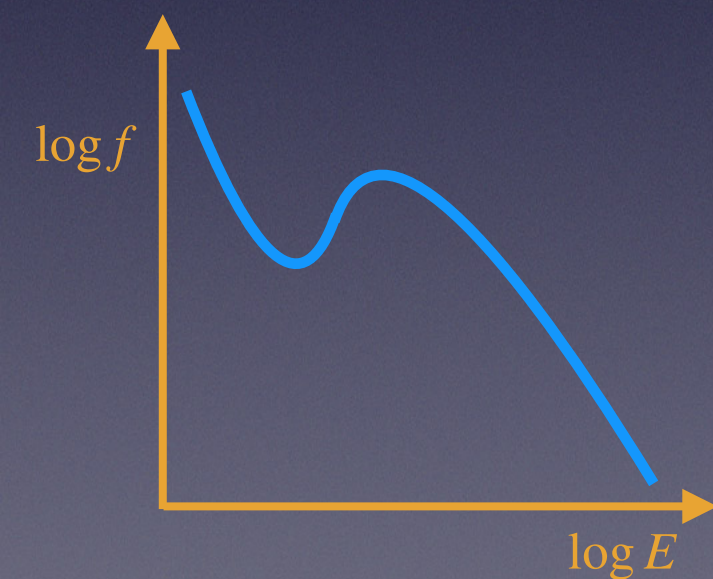


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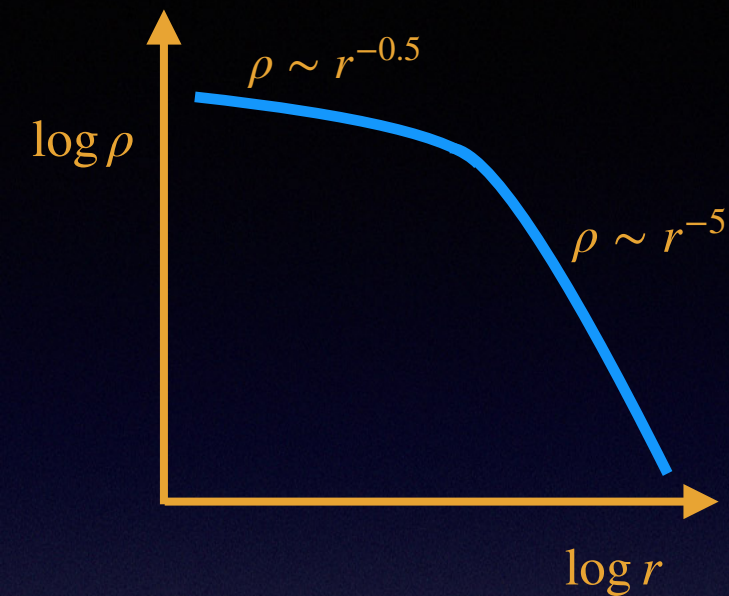


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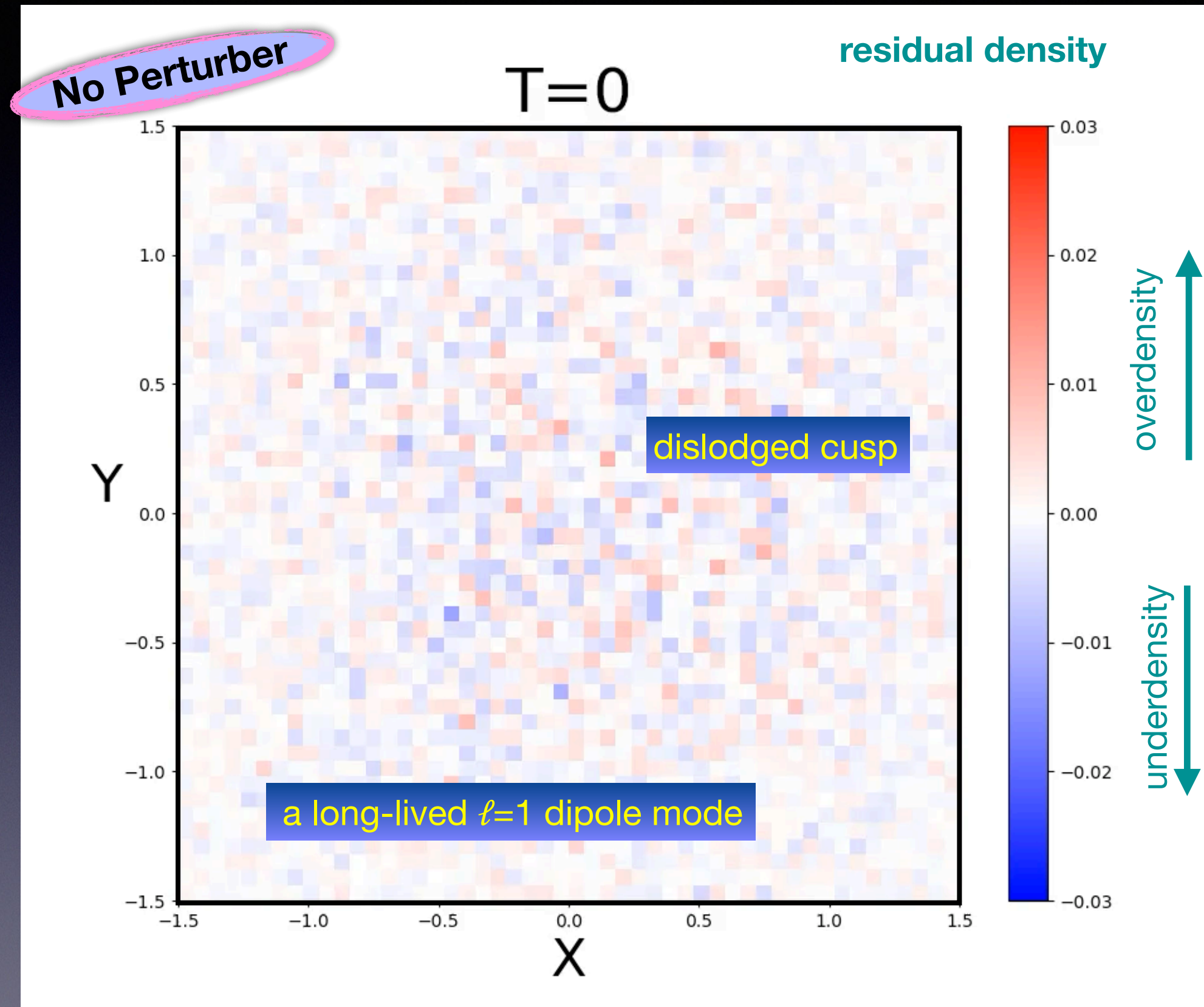
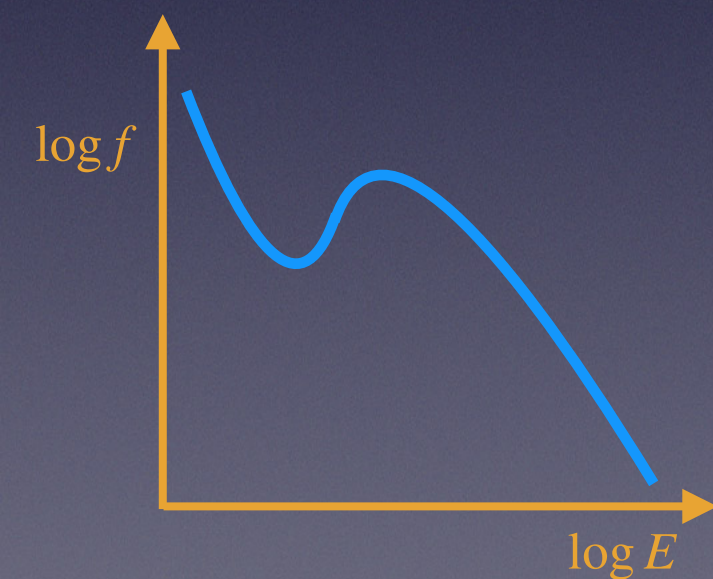


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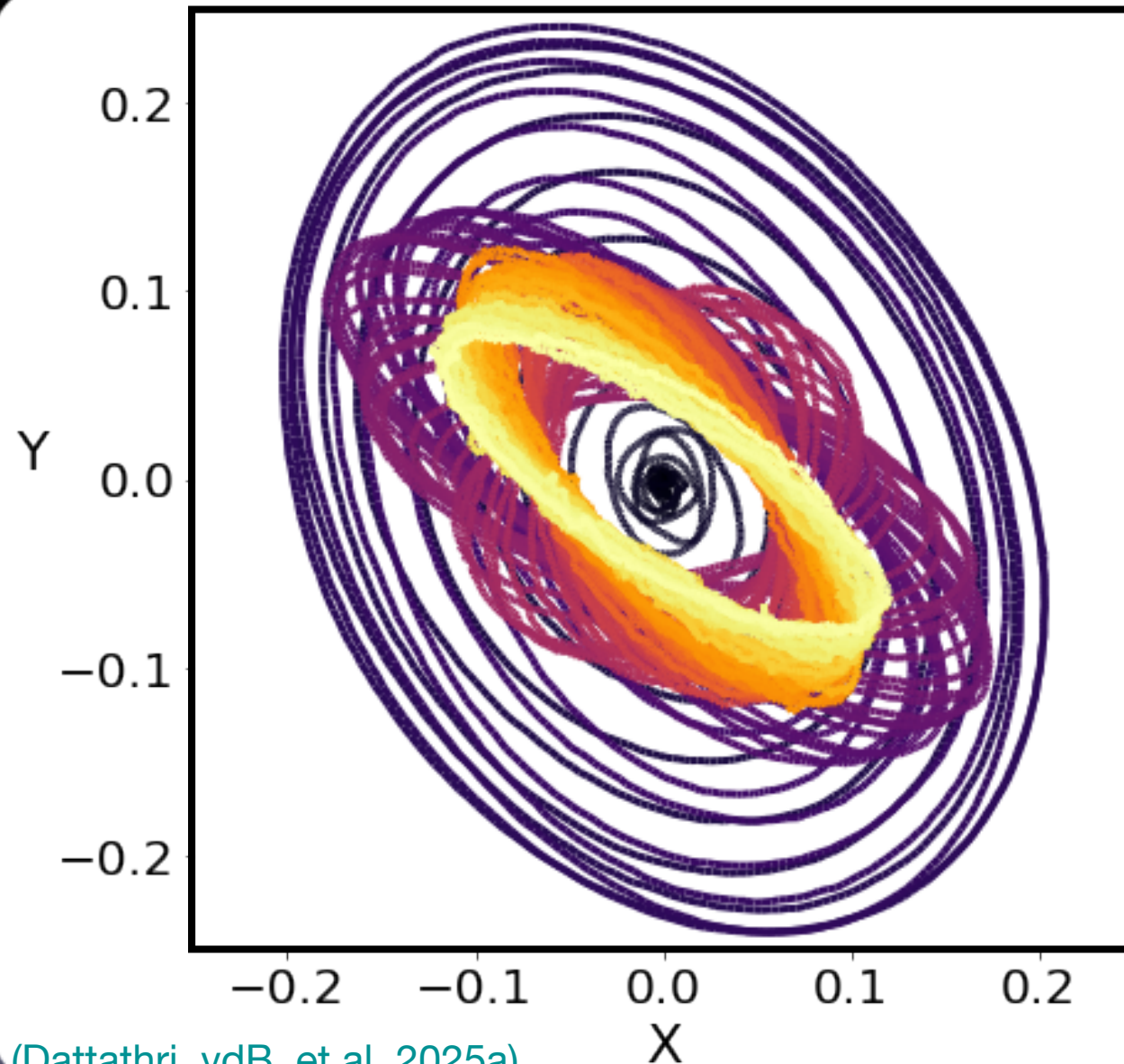


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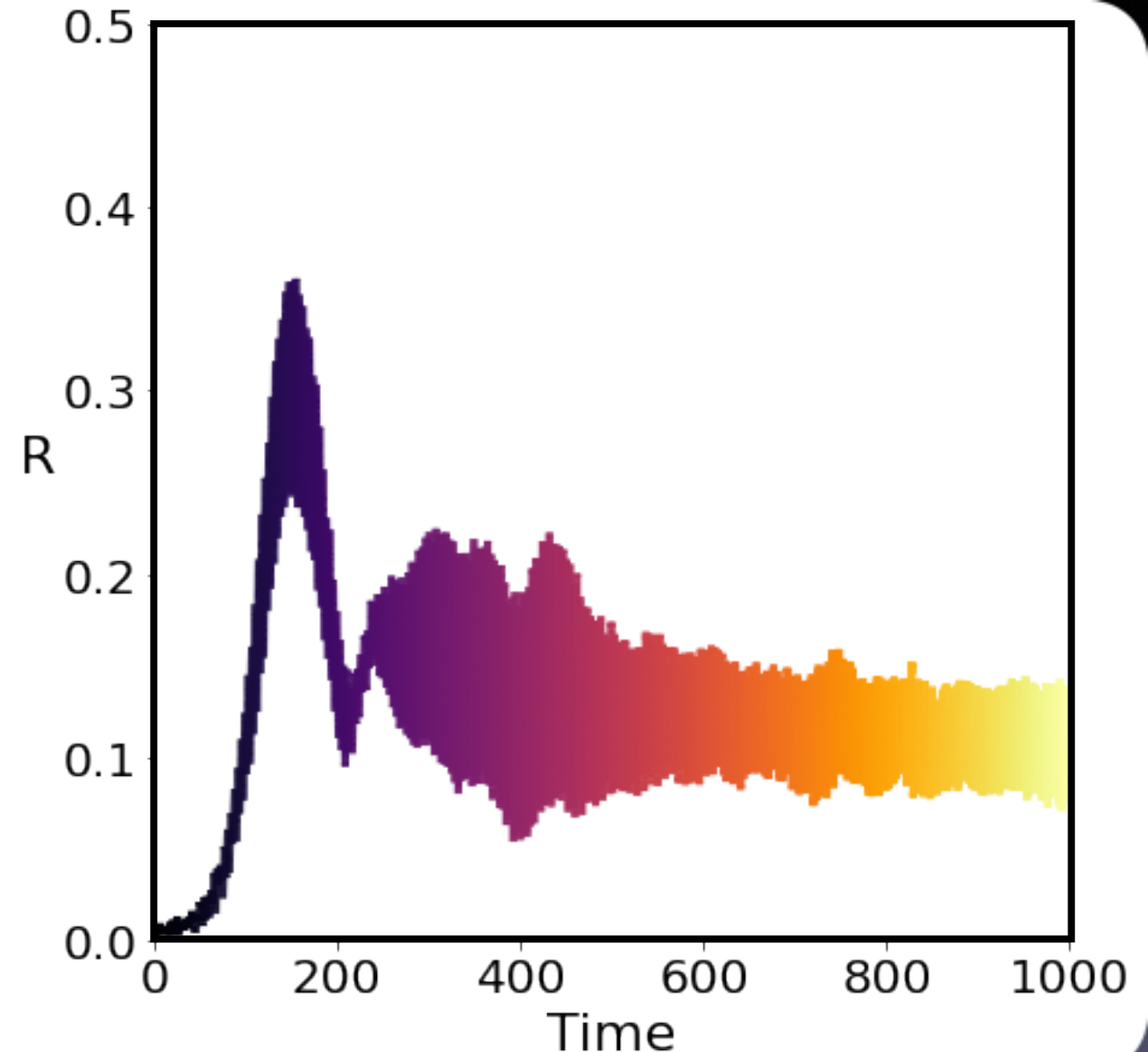
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Core Instability & Dynamical Buoyancy



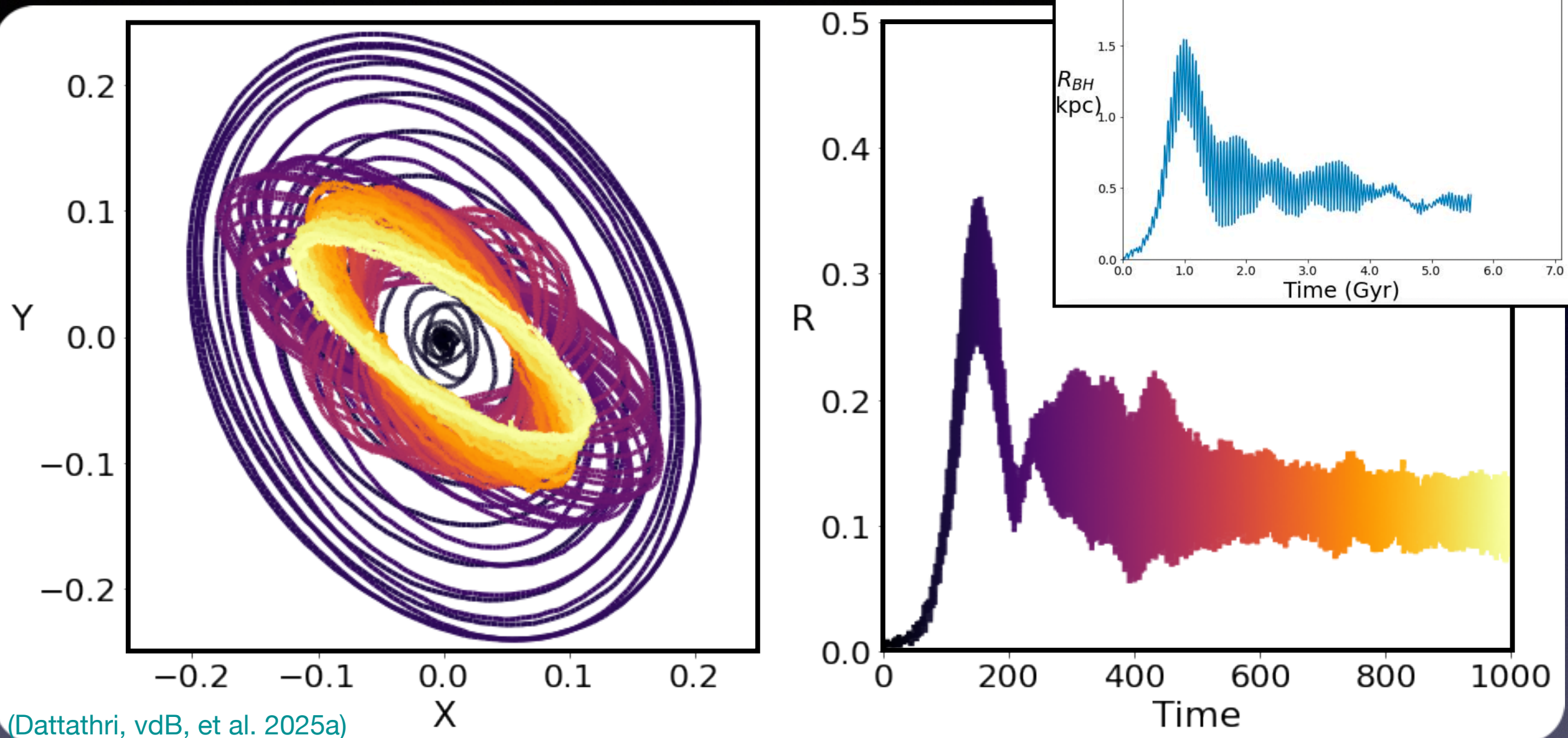
(Dattathri, vdB, et al. 2025a)



- Motion of 'cusp' (dislodged from center due to dipole mode) over time
- Similar to a beach ball sloshing in a swimming pool with a seiche
- BH at center of cusp would 'ride along'

Dattathri vdB et al, 2025a

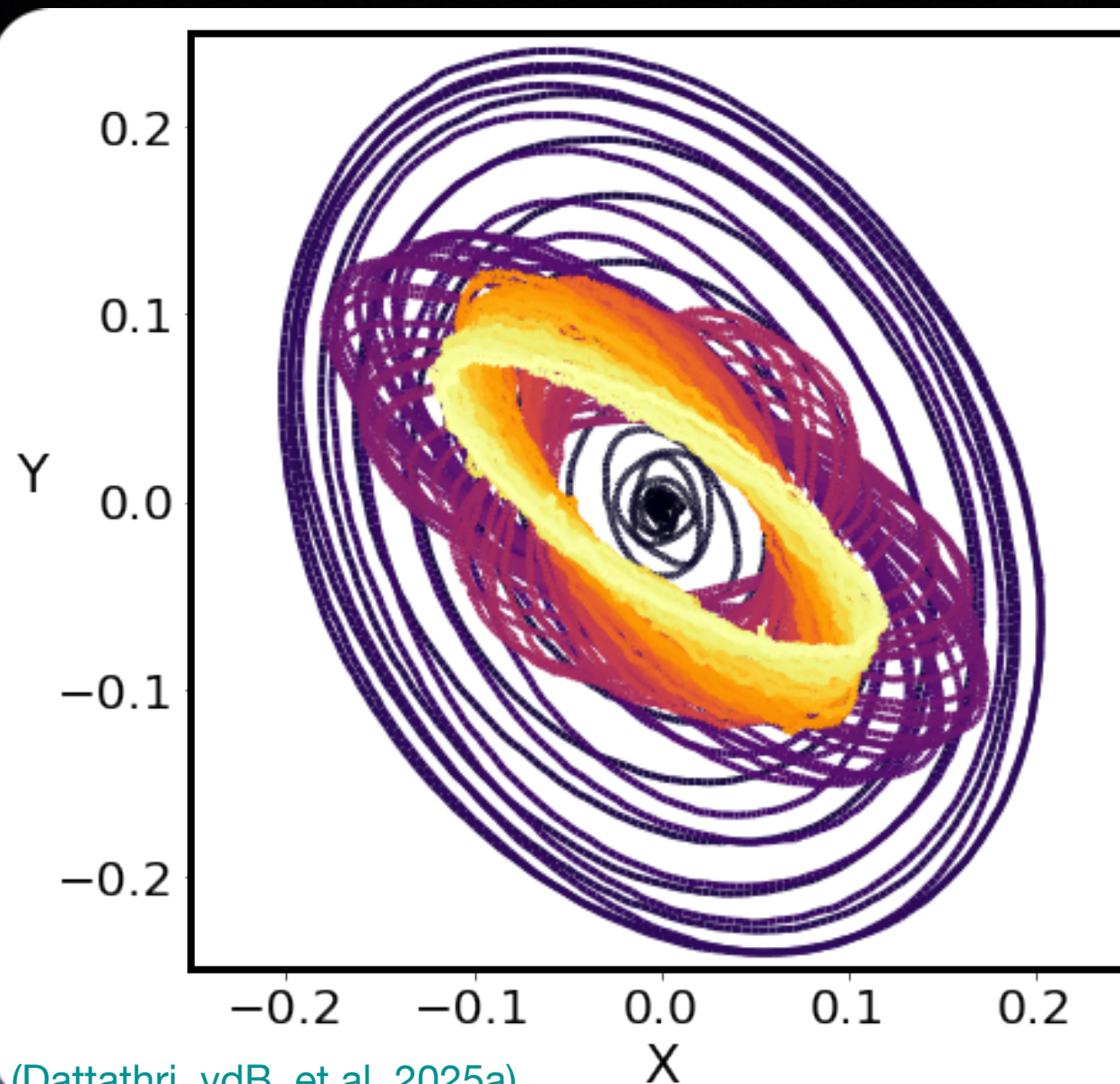
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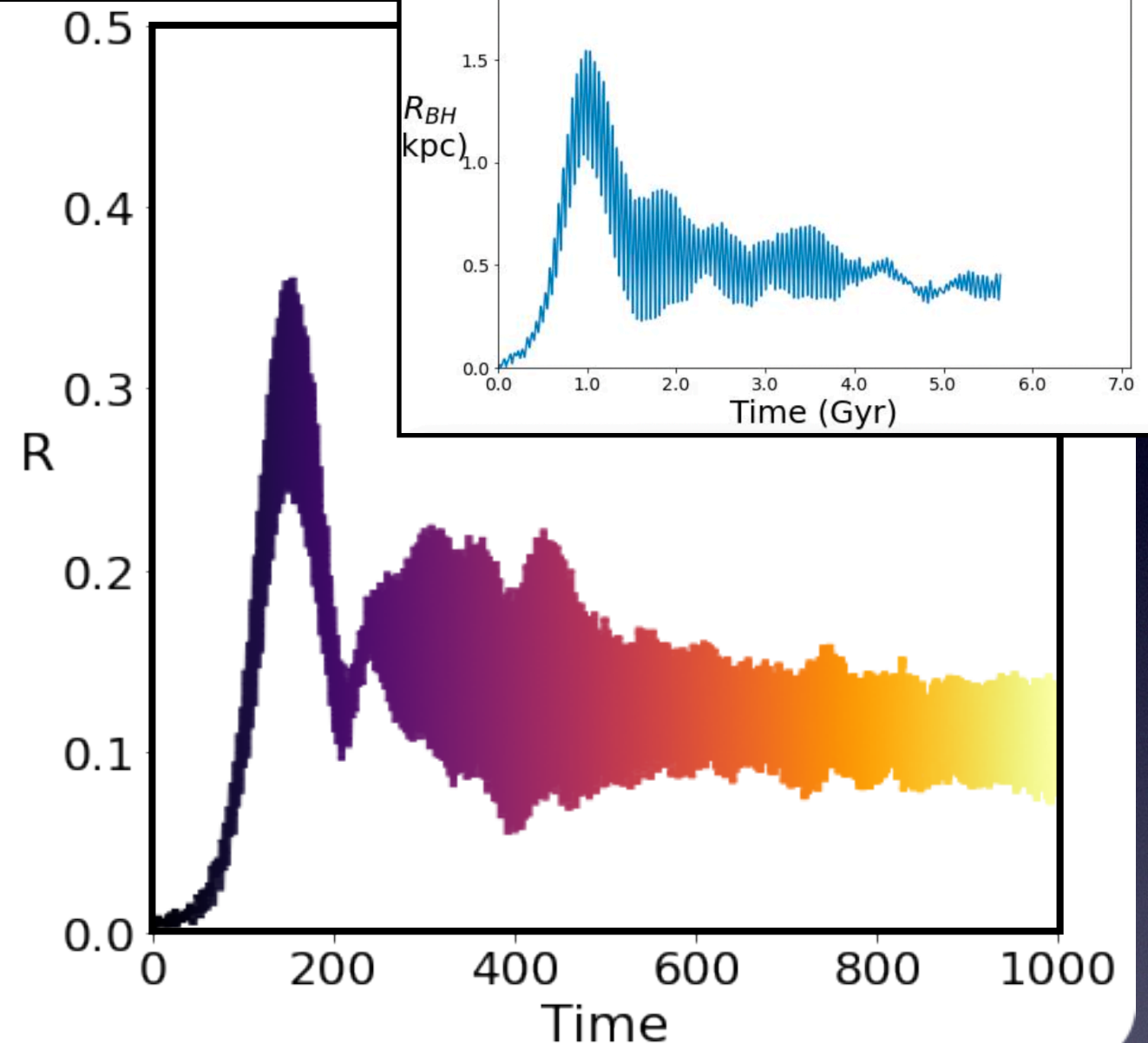
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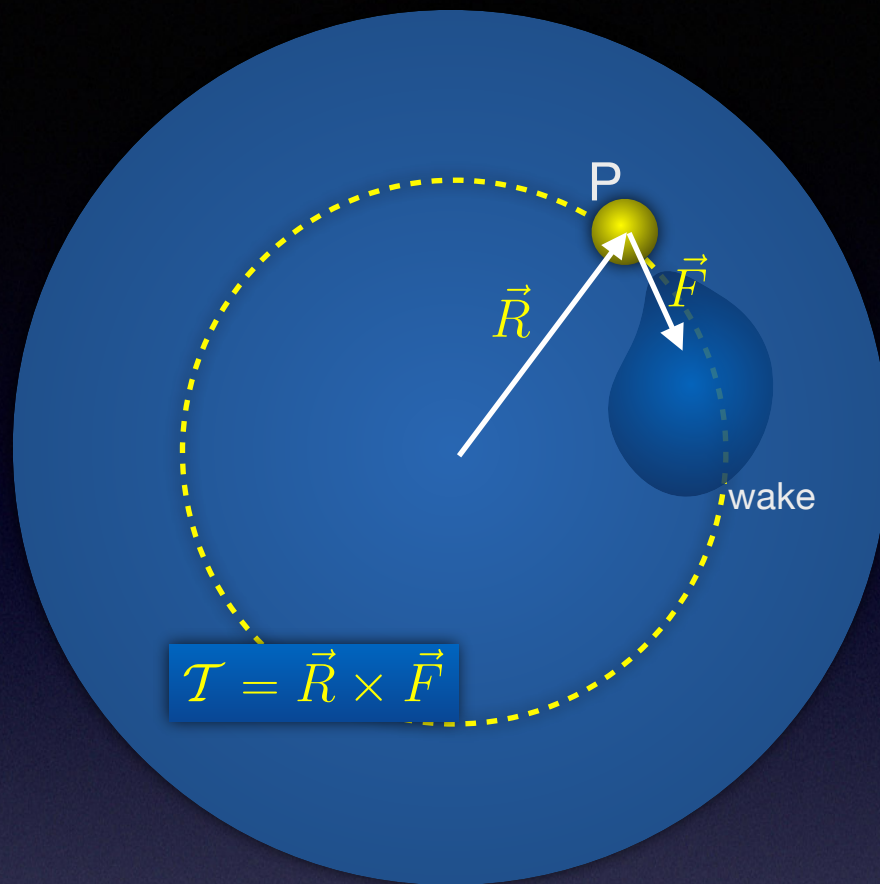
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dynamical buoyancy is manifestation of dipole instability

Dattathri vdB et al, 2025a

What is this all about?

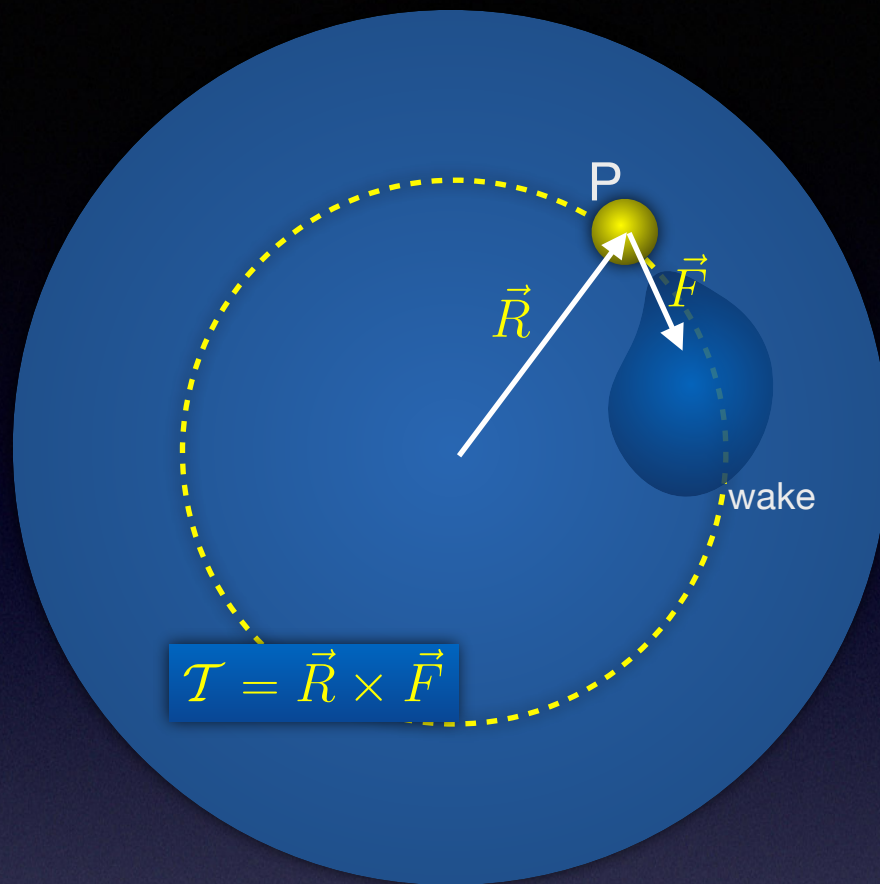


For isotropic DF:

$$\mathcal{T} = \mathcal{T}_{\text{LBK}} = 16\pi^4 \sum_{\ell} \ell_3^2 \Omega_p \int d\mathbf{J} \delta(\ell \cdot \boldsymbol{\Omega} - \ell_3 \Omega_p) \frac{\partial f}{\partial E} \left| \hat{\Phi}_{\ell}(\mathbf{J}) \right|^2$$

(Lynden-Bell & Kalnajs 1972, Tremaine & Weinberg 1984)

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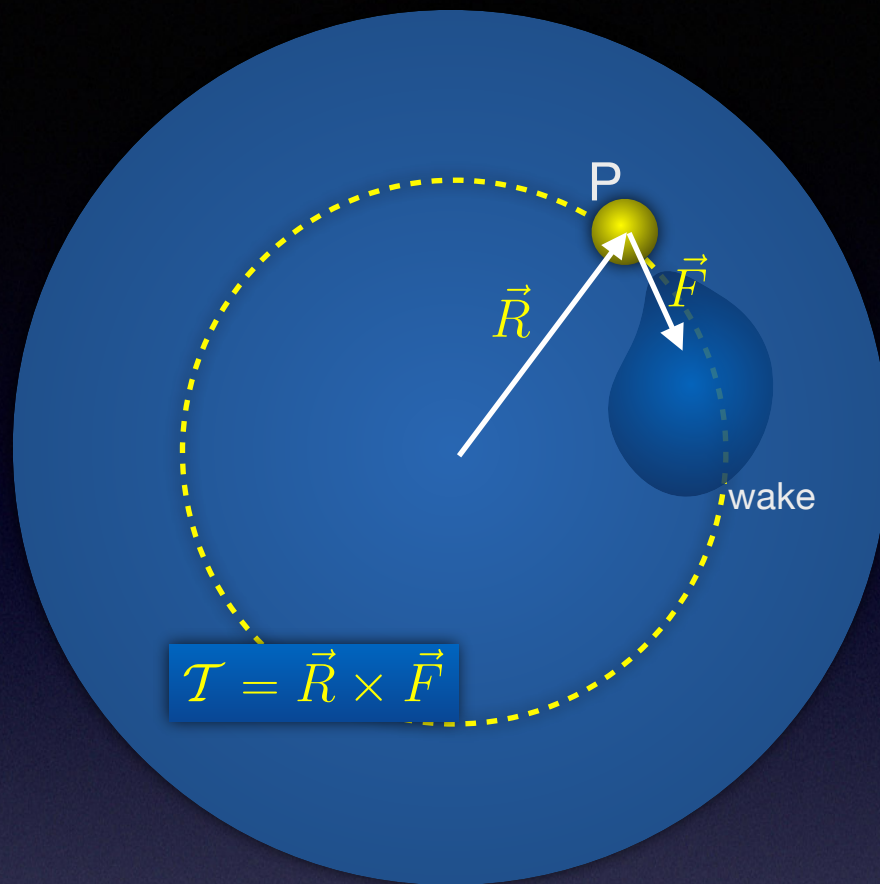


integral over
all orbits

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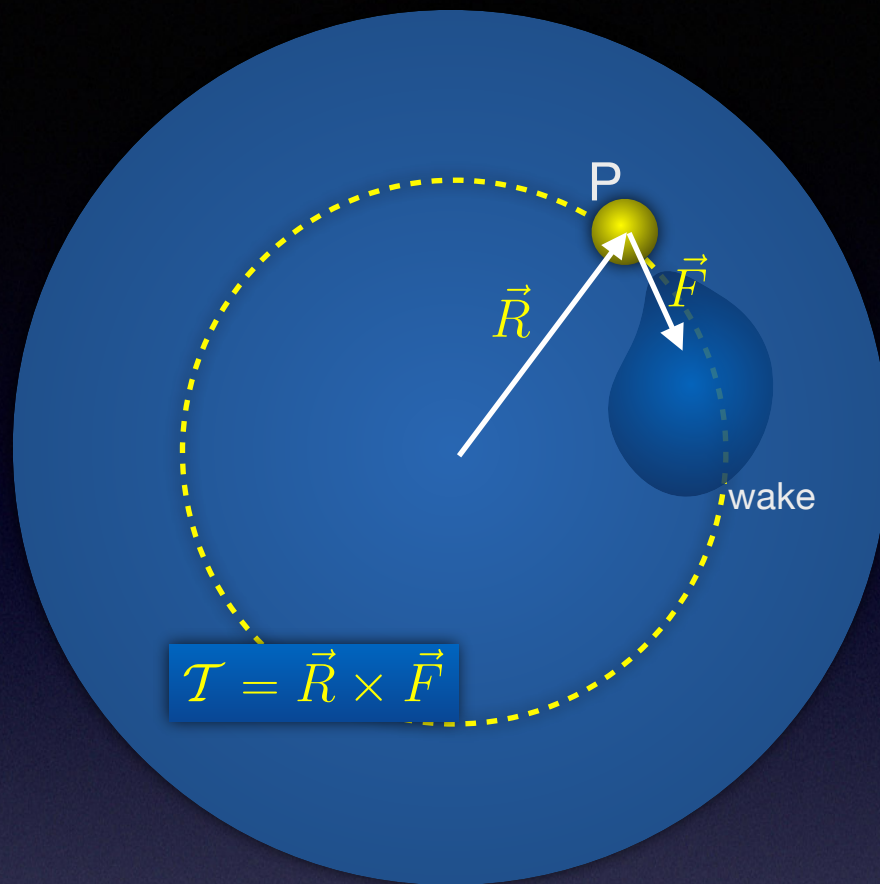


Dirac delta function
picks out resonances

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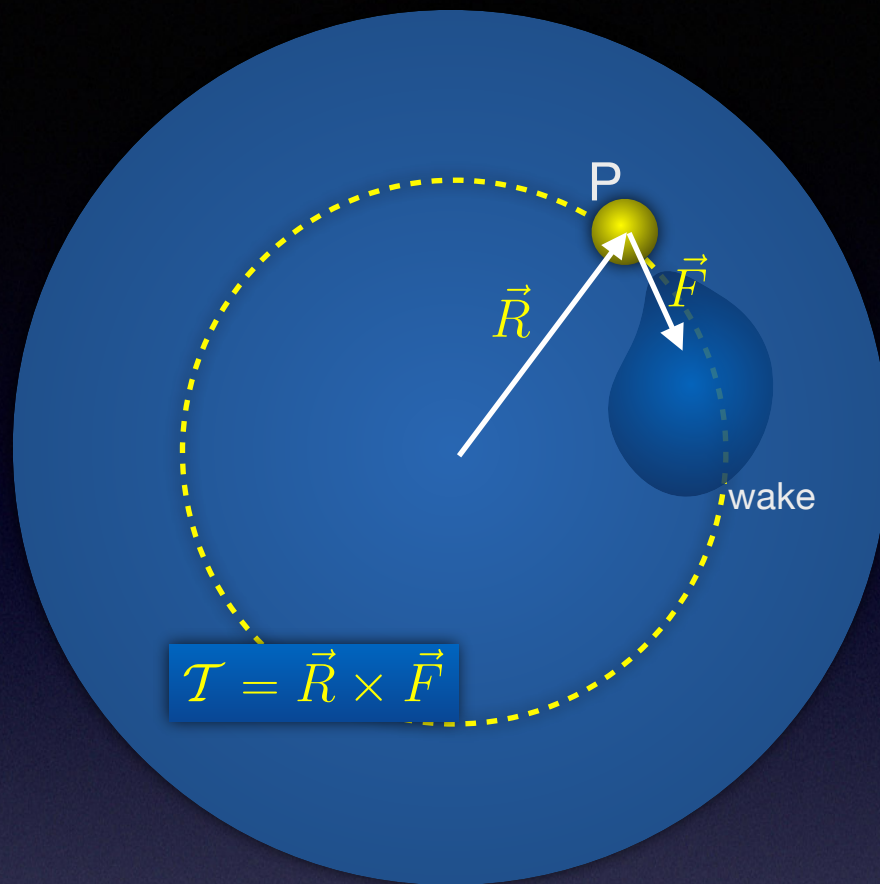


sum over
resonances

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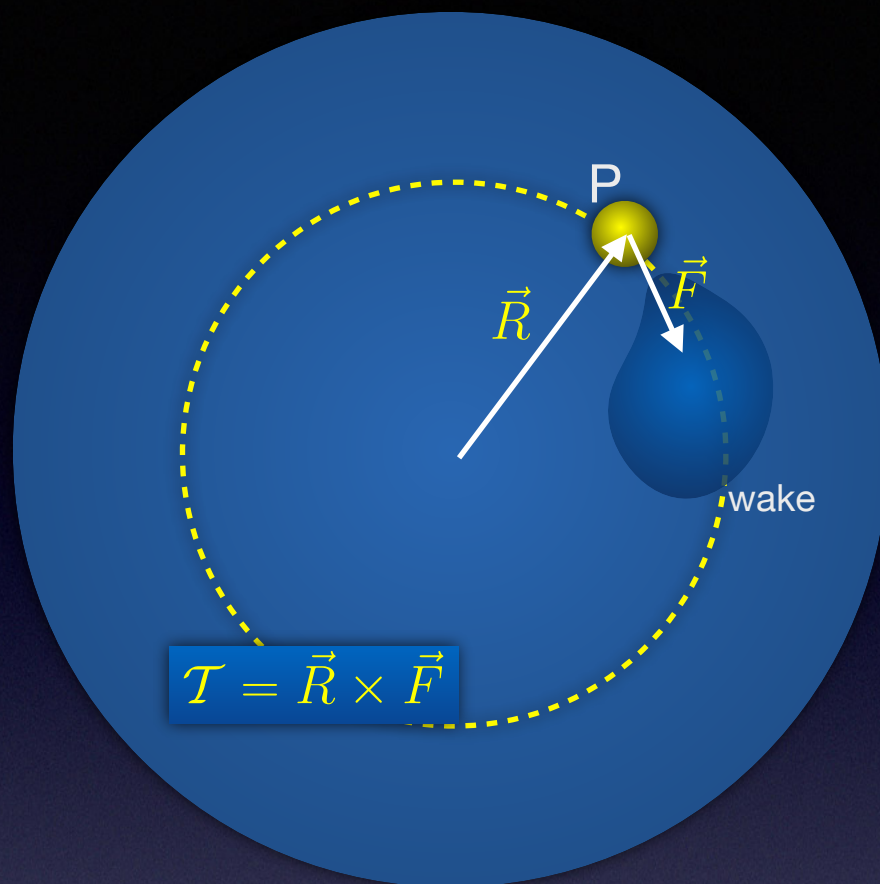


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gradient
of df

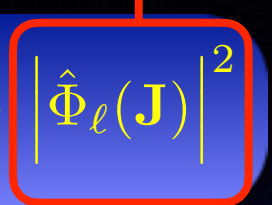
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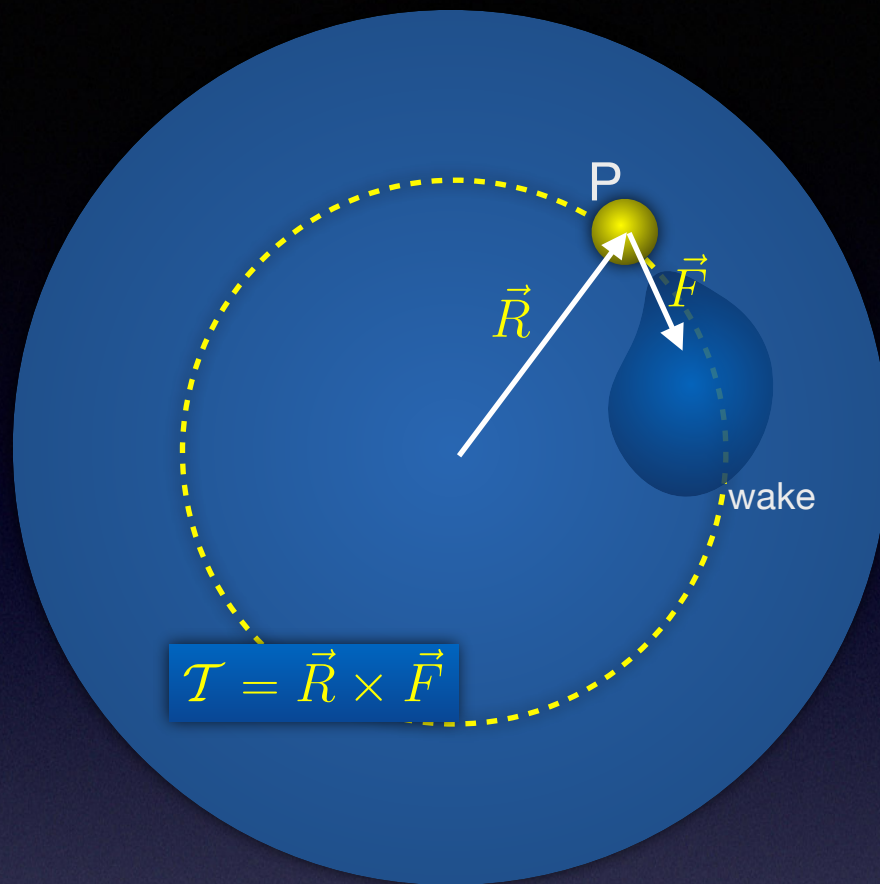
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perturber
potential



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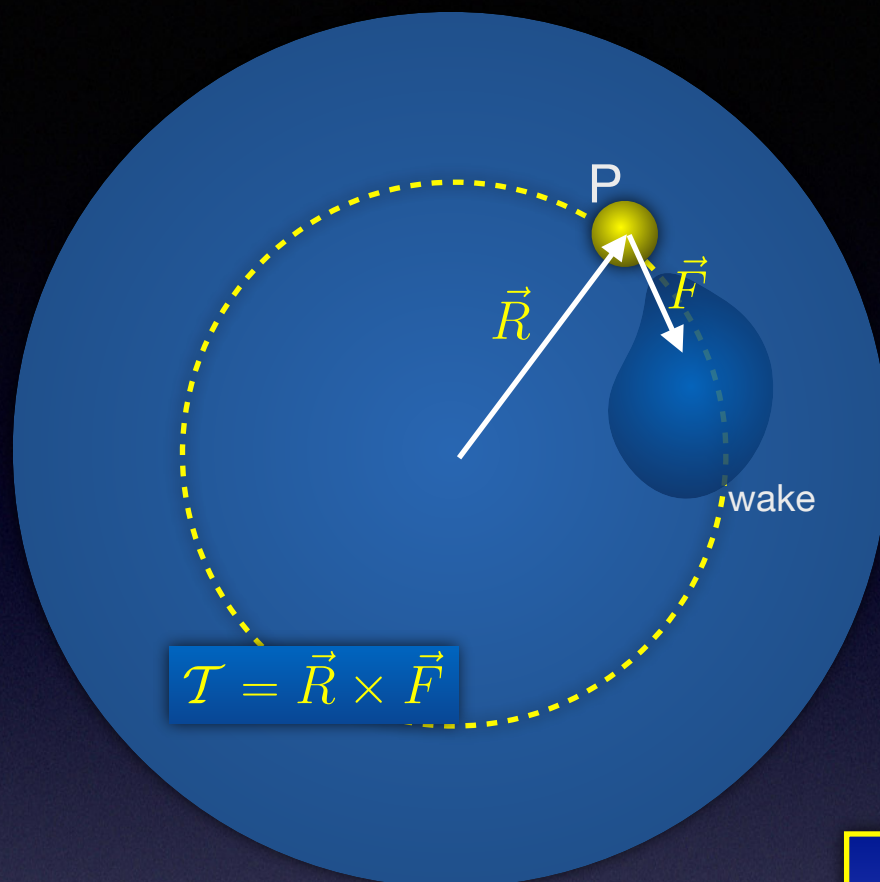
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In words:

Net torque arises from imbalance between gainers and losers, dictated by gradient in distribution function across resonances

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**A simple
picture
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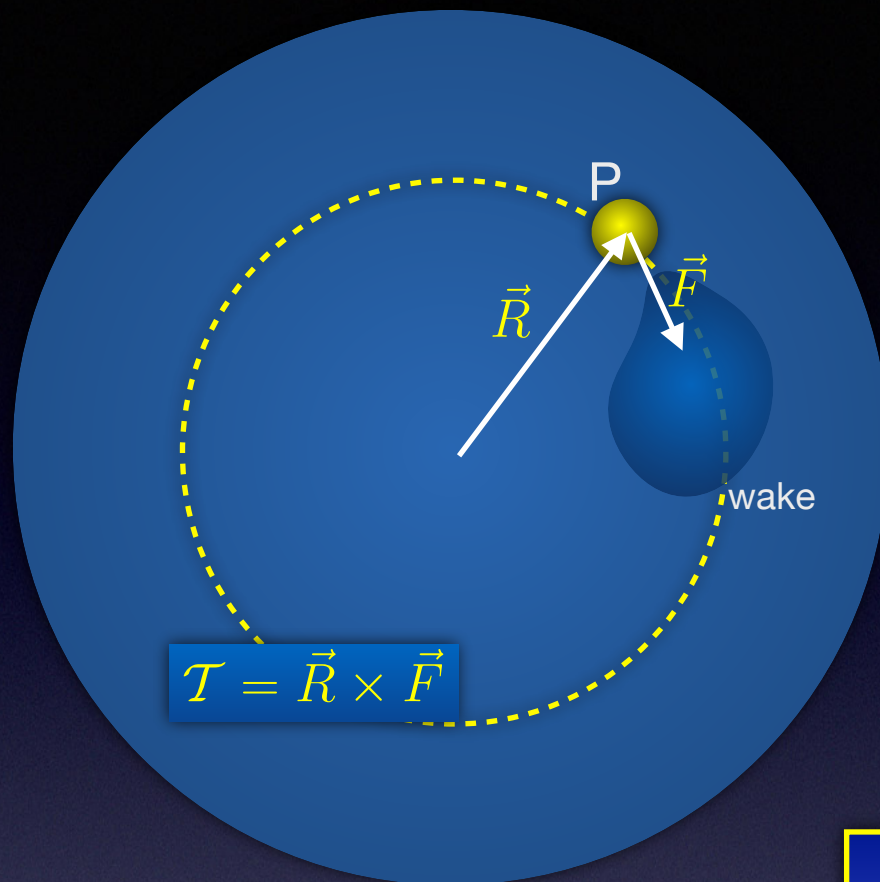
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$\partial f / \partial E < 0$	torque retarding	dynamical friction
$\partial f / \partial E = 0$	zero torque	core stalling
$\partial f / \partial E > 0$	torque enhancing	dynamical buoyancy

(Dattathri, vdB et al, 2025b, *in prep*)

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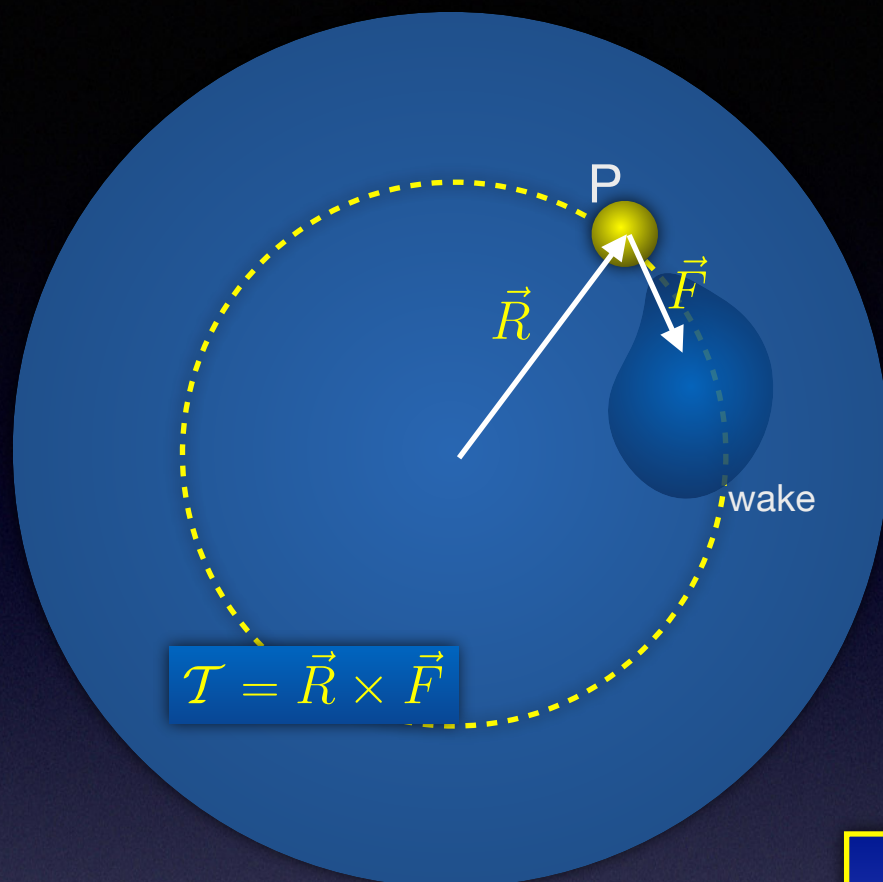
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A spherical symmetric, isotropic system is **stable** if $\partial f / \partial E < 0$ at all E (Antonov 1962)

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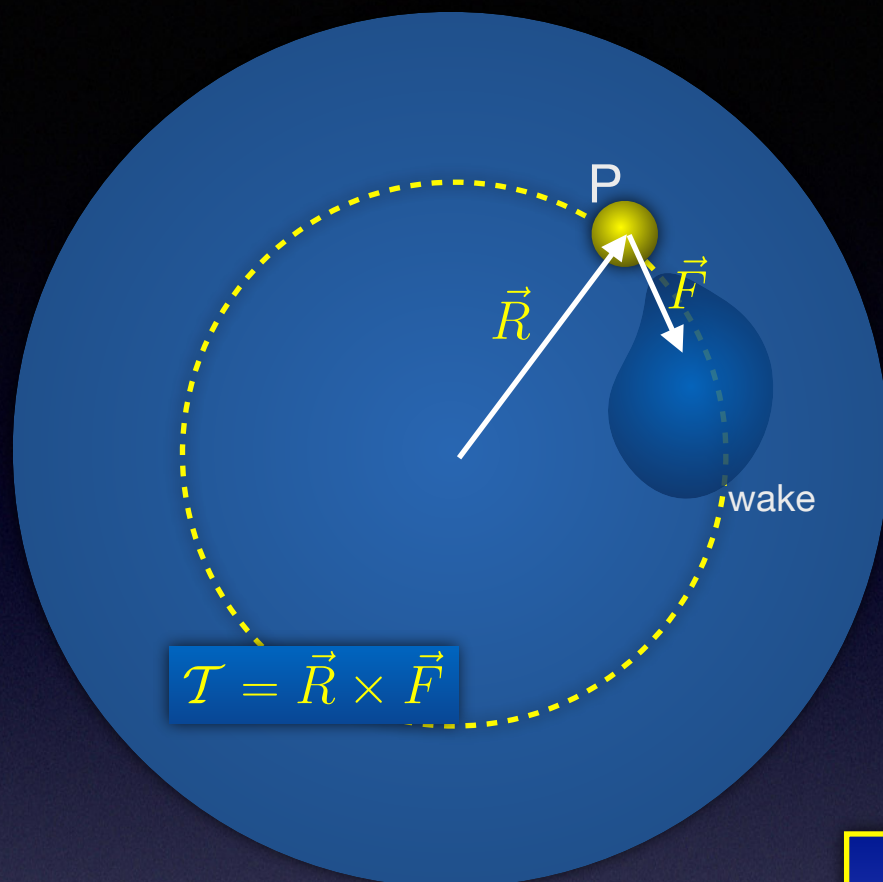
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A spherical symmetric, isotropic system is **stable** if $\partial f / \partial E < 0$ at all E (Antonov 1962)

A spherical symmetric, isotropic system is **unstable** if $\partial f / \partial E > 0$ at some E (conjecture)

What is this all about?



**A simple
picture
emerges**

$$\mathcal{T} = \mathcal{T}_{\text{LBK}} = 16\pi^4 \sum_{\ell} \ell_3^2 \Omega_p \int d\mathbf{J} \delta(\ell \cdot \boldsymbol{\Omega} - \ell_3 \Omega_p) \frac{\partial f}{\partial E} \left| \hat{\Phi}_{\ell}(\mathbf{J}) \right|^2$$

(Lynden-Bell & Kalnajs 1972, Tremaine & Weinberg 1984)

In words:

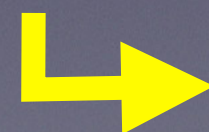
Net torque arises from imbalance between gainers and losers, dictated by gradient in distribution function across resonances

$\partial f / \partial E < 0$	torque retarding	dynamical friction
$\partial f / \partial E = 0$	zero torque	core stalling
$\partial f / \partial E > 0$	torque enhancing	dynamical buoyancy

(Dattathri, vdB et al, 2025b, *in prep*)

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dipole instability

(Dattathri, vdB et al. 2025a)

Analogy with Plasma Physics

Kinetic Theory of Gravitational Systems

dynamical friction

dynamical buoyancy

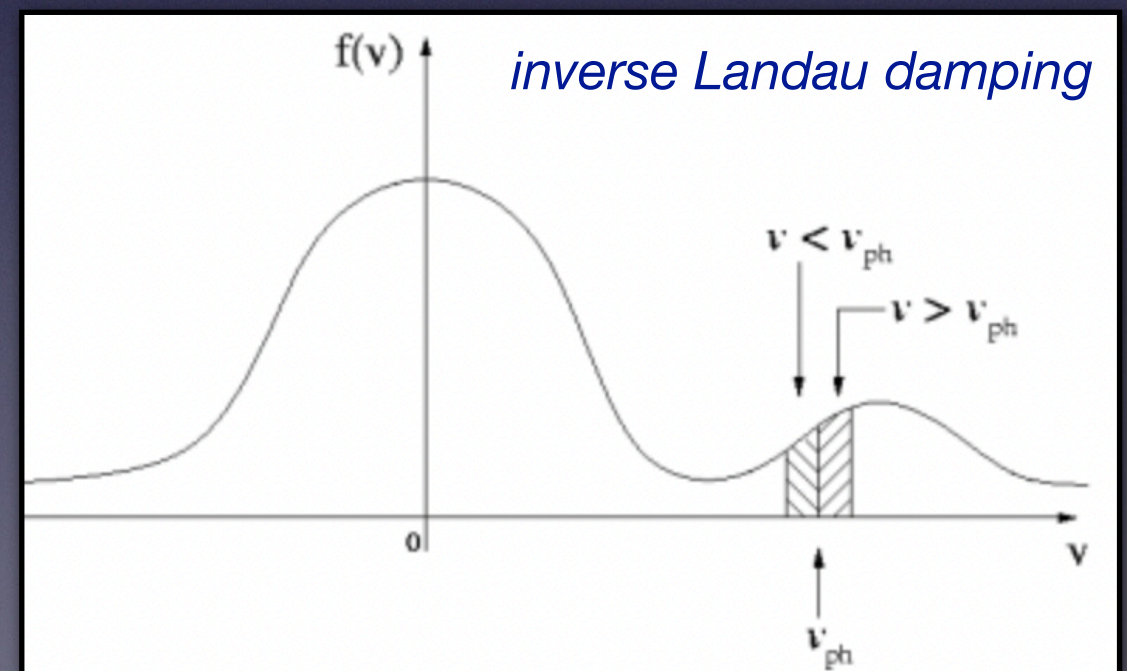
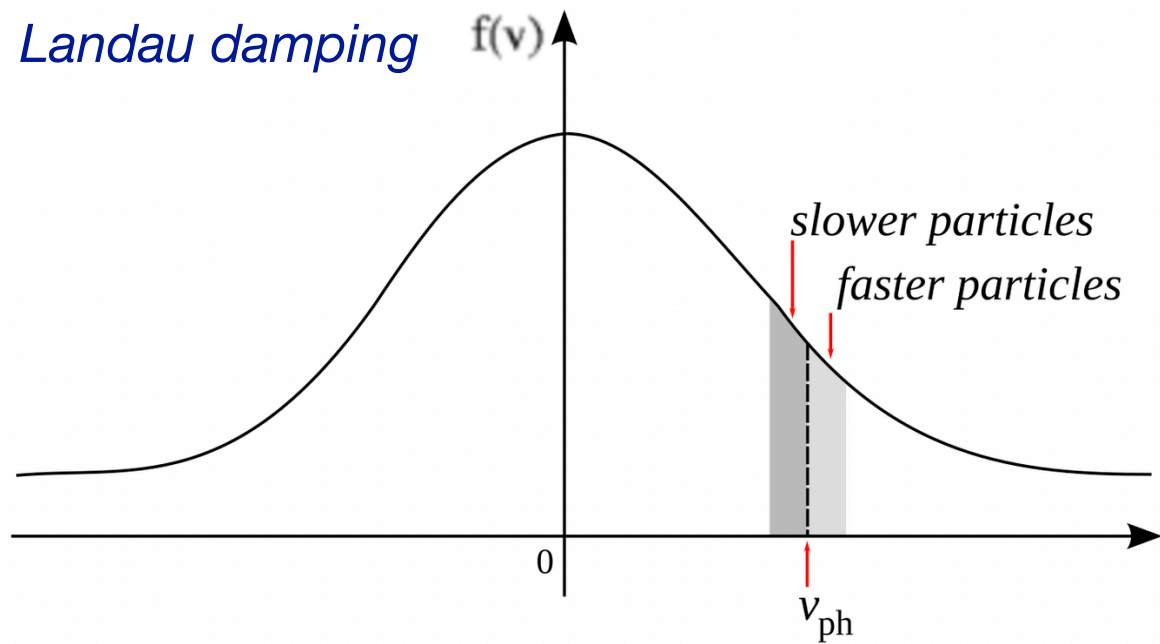
dipole instability

Kinetic Theory of Plasmas

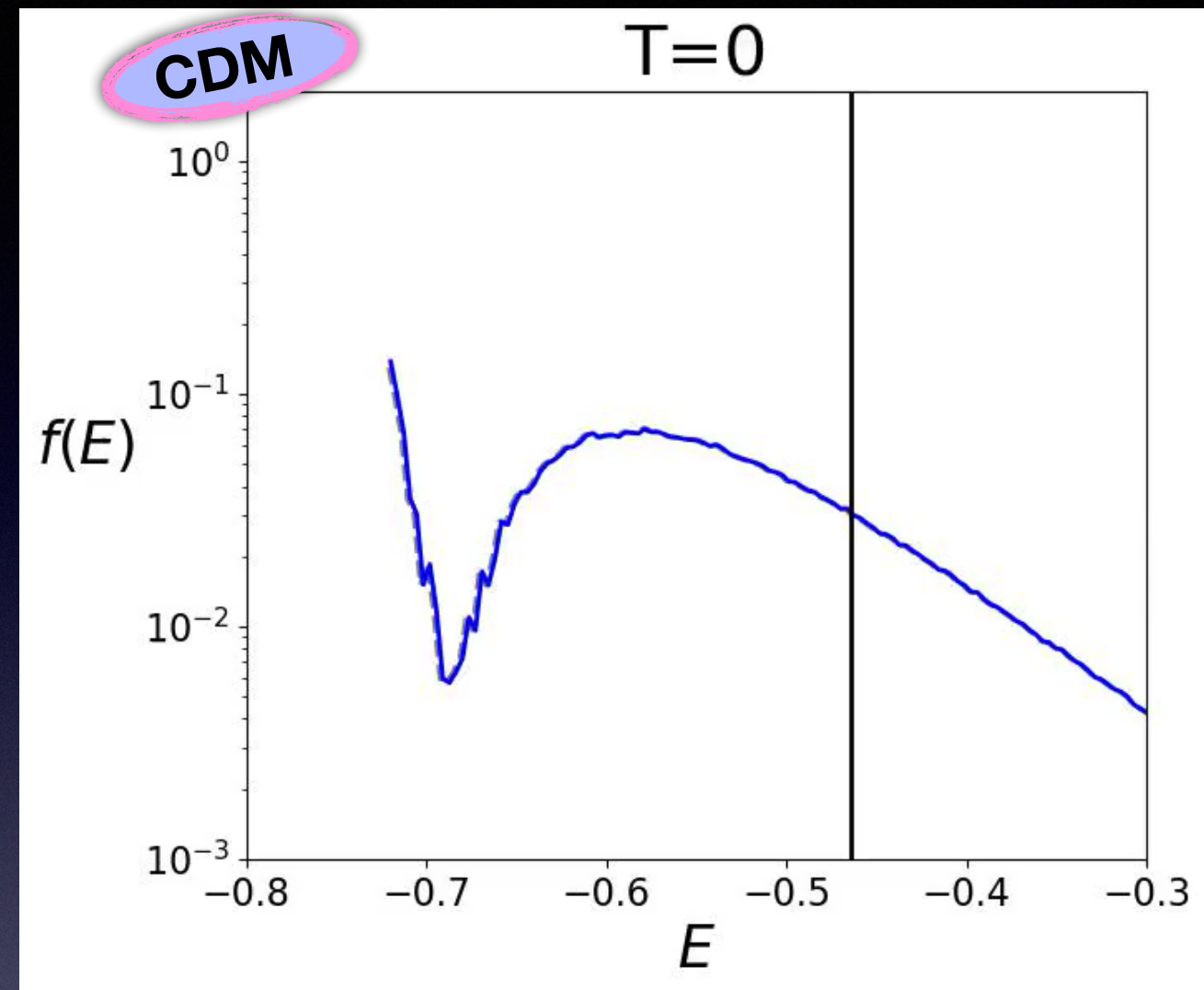
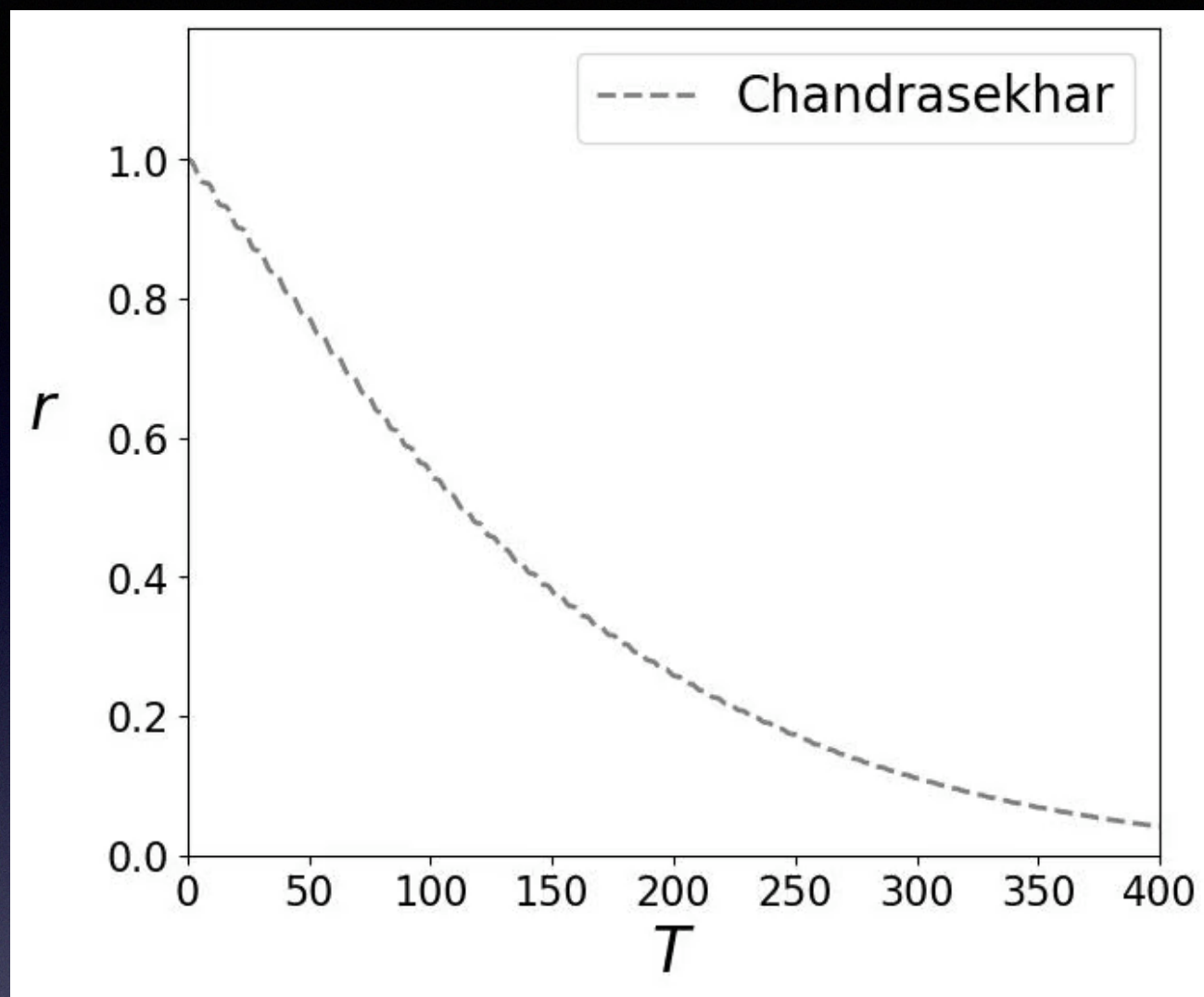
Landau damping

inverse Landau damping

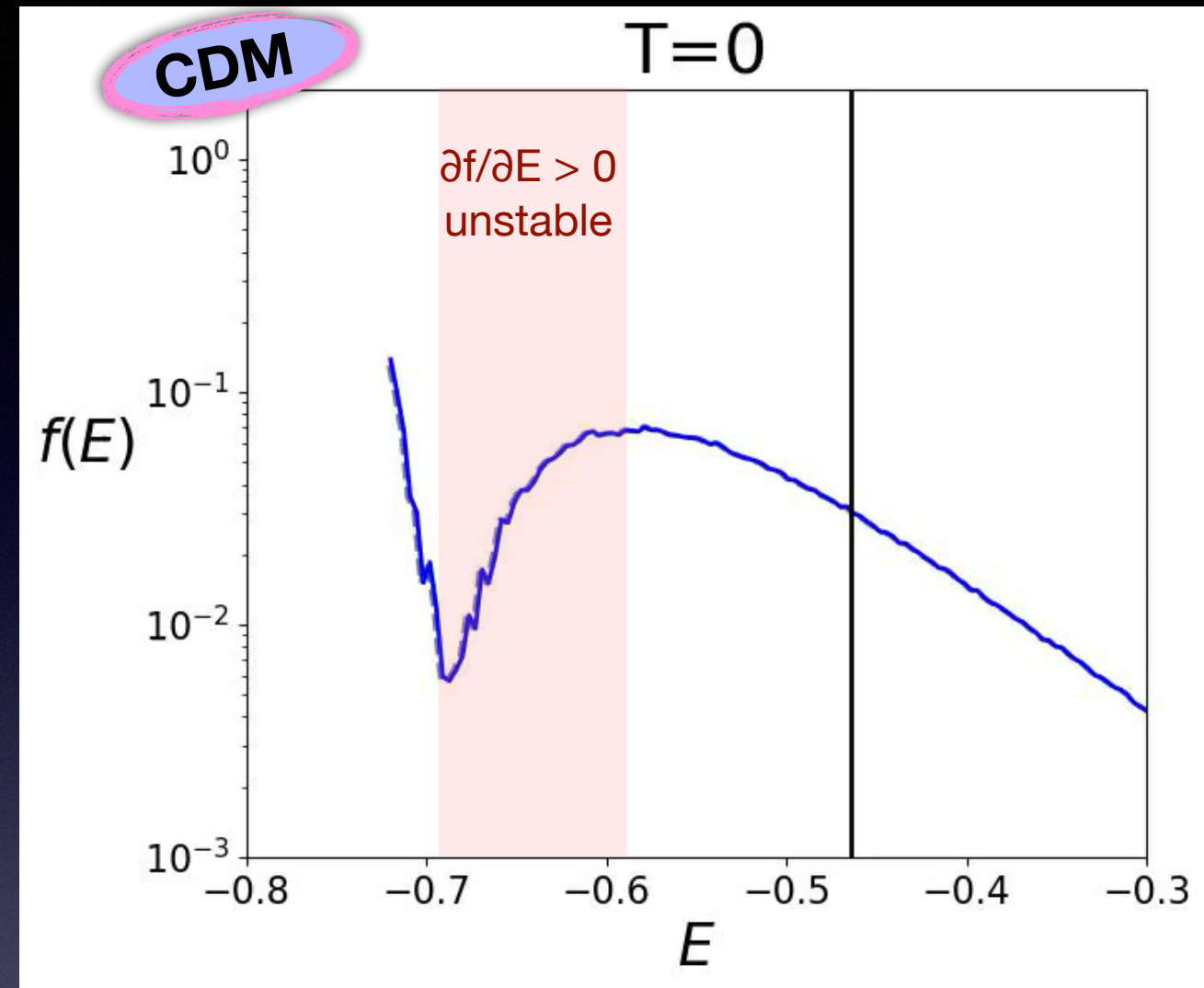
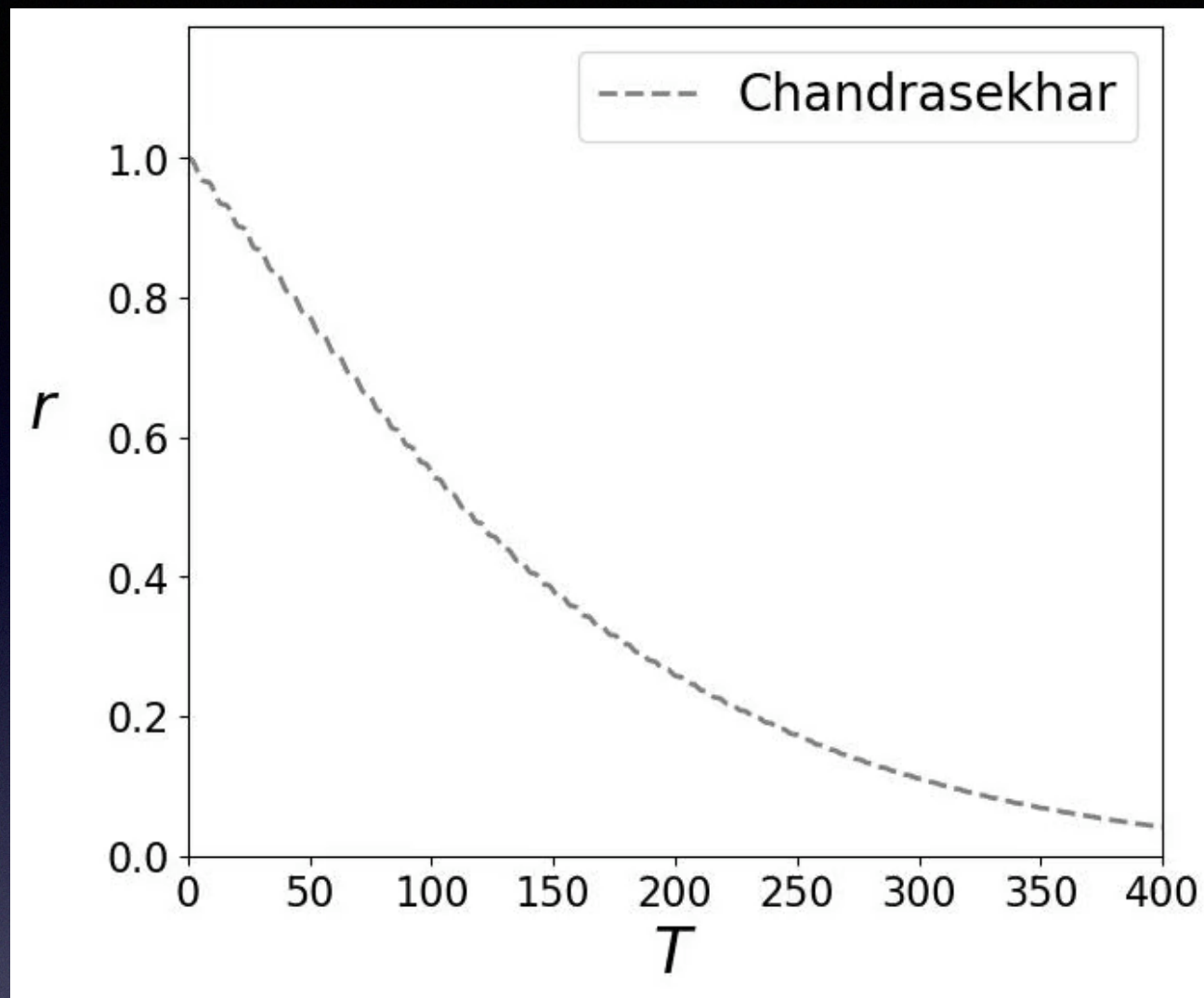
bump-on-tail instability



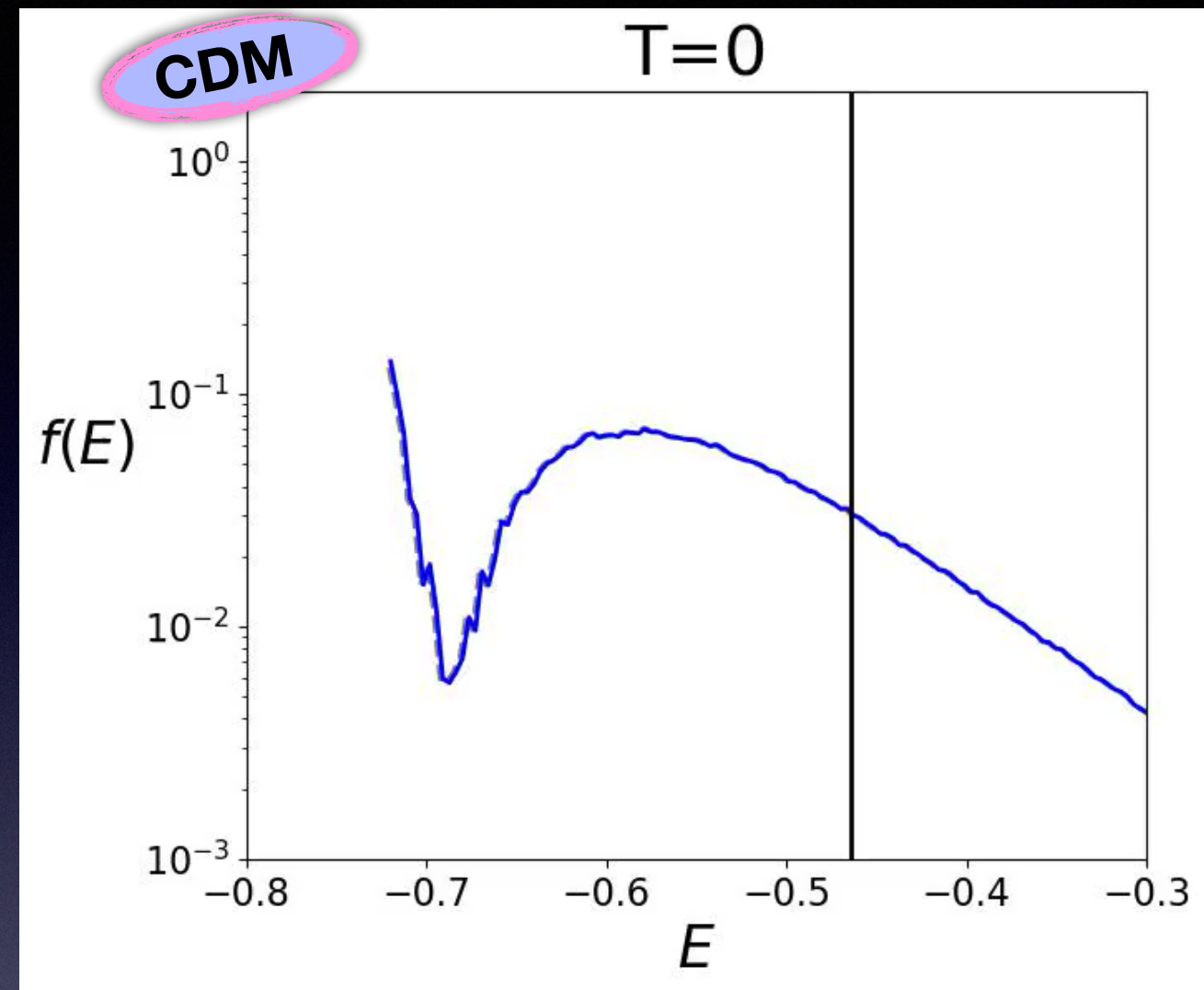
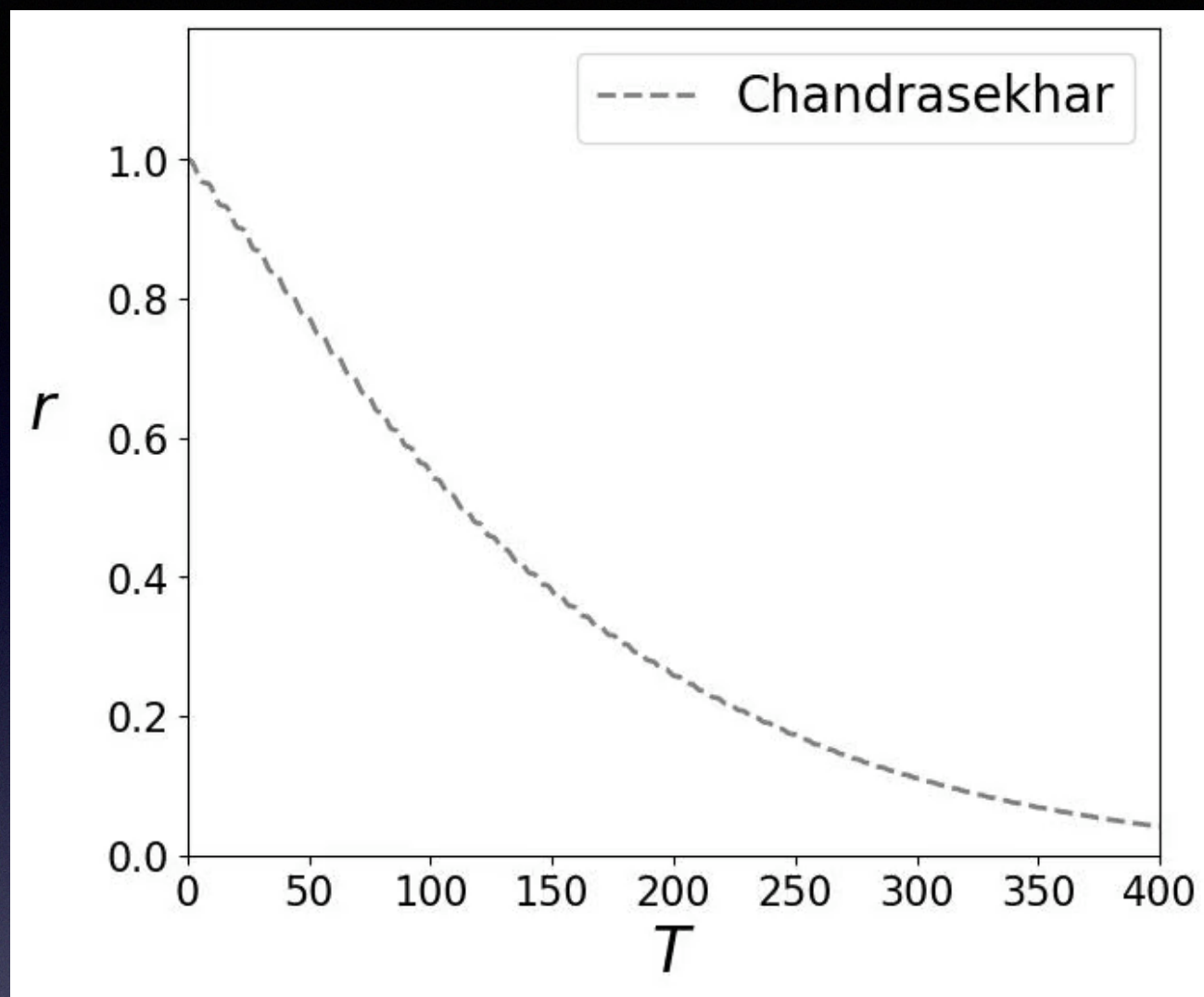
∇f ; the Linchpin of Core Dynamics



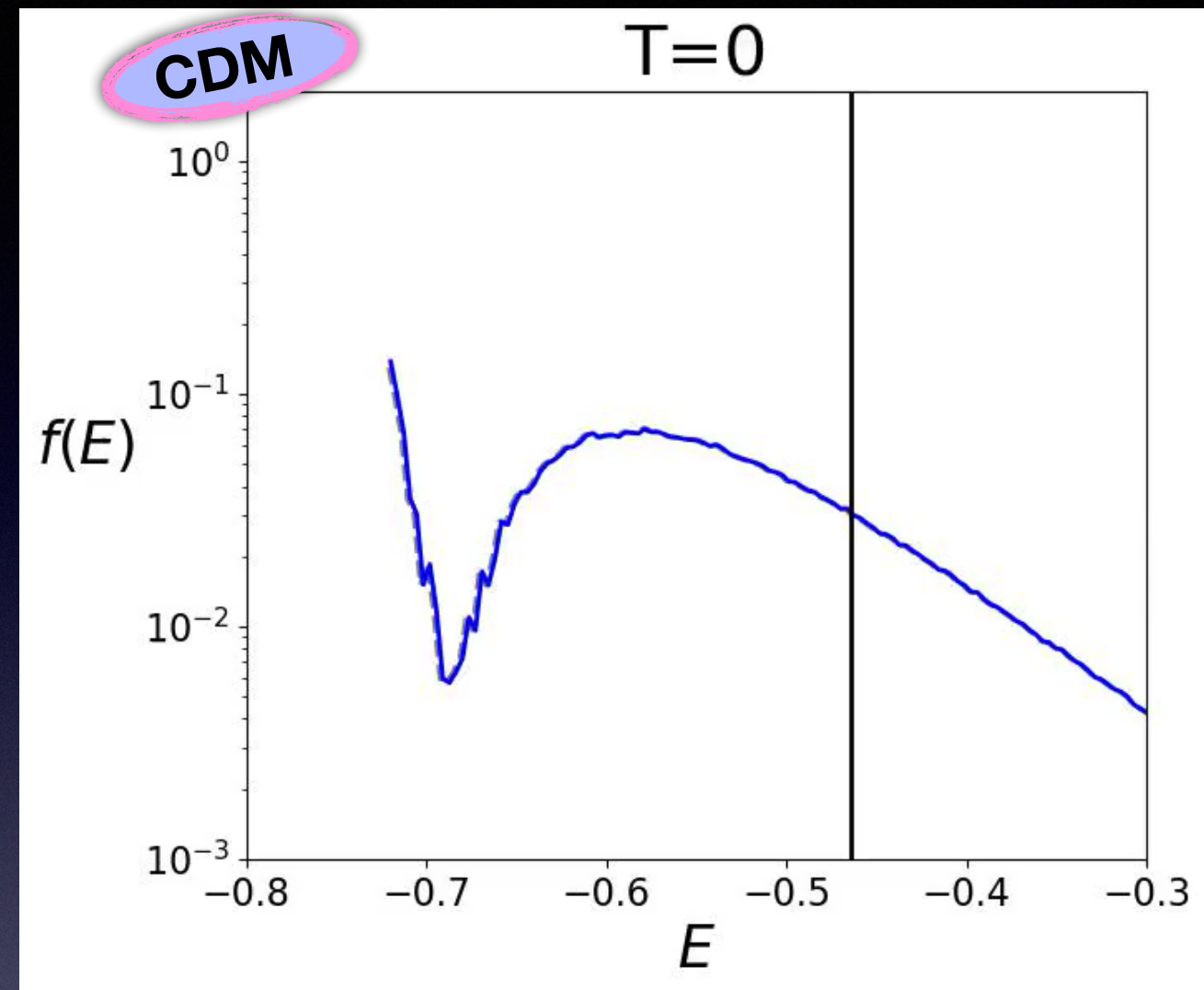
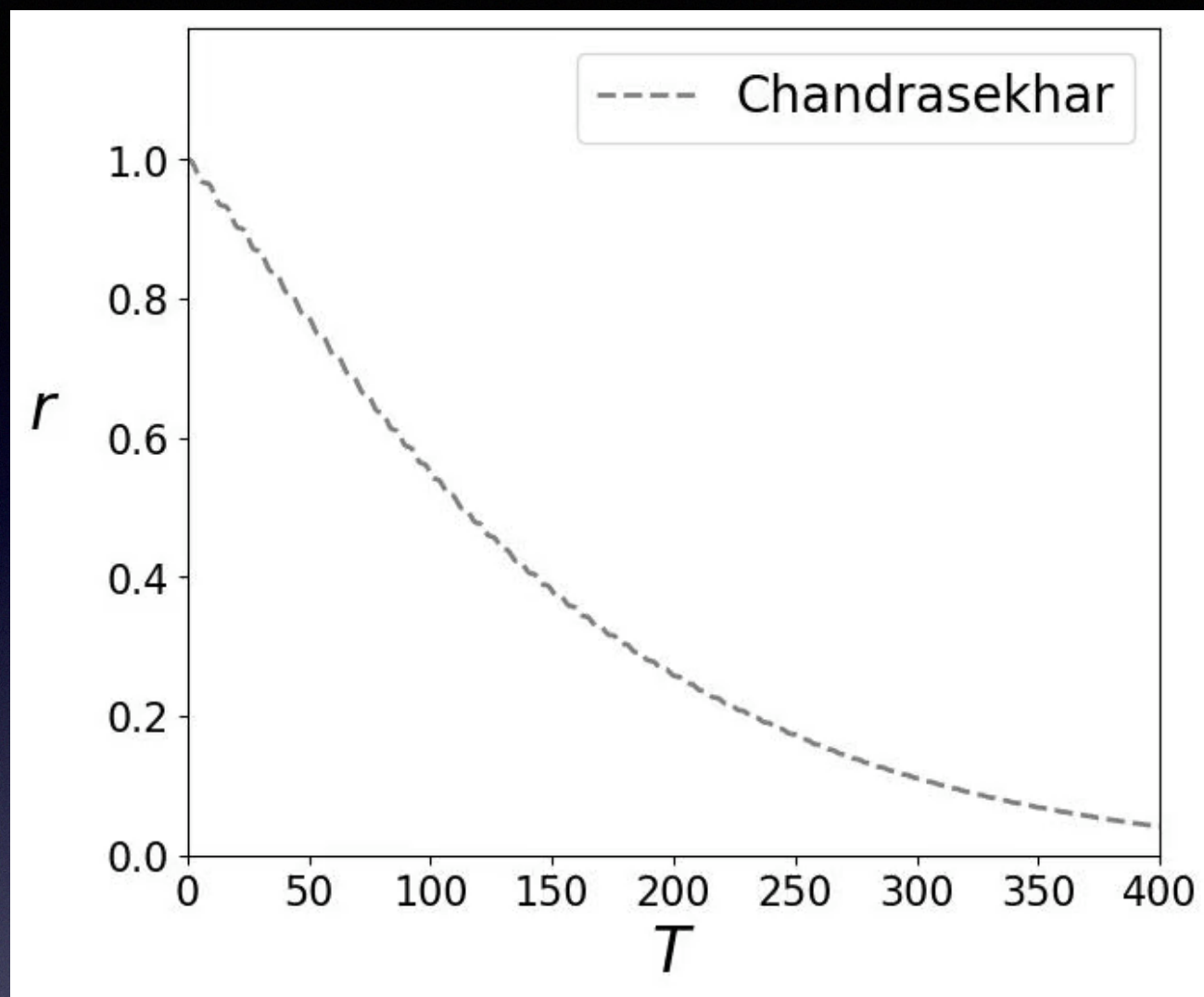
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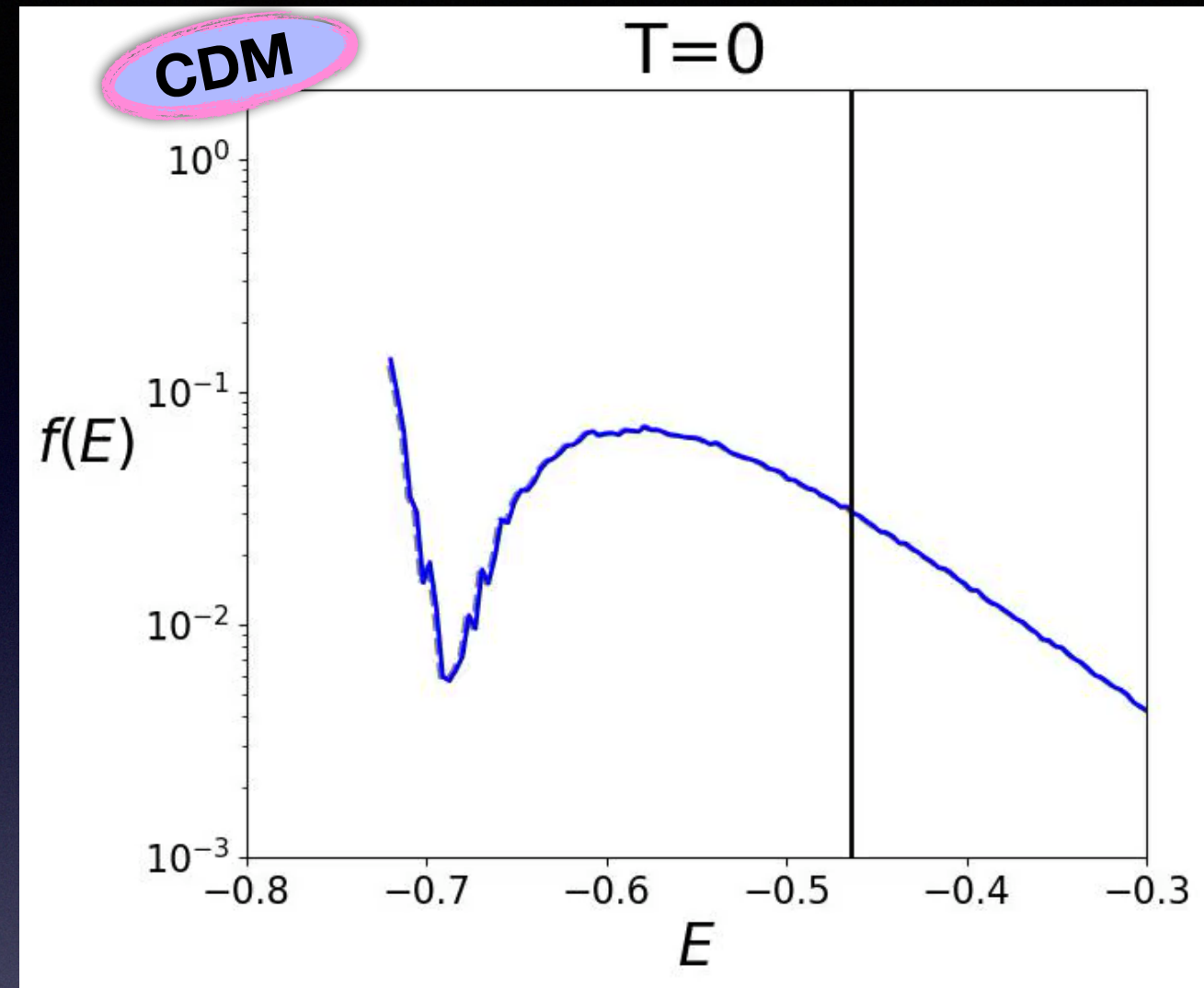
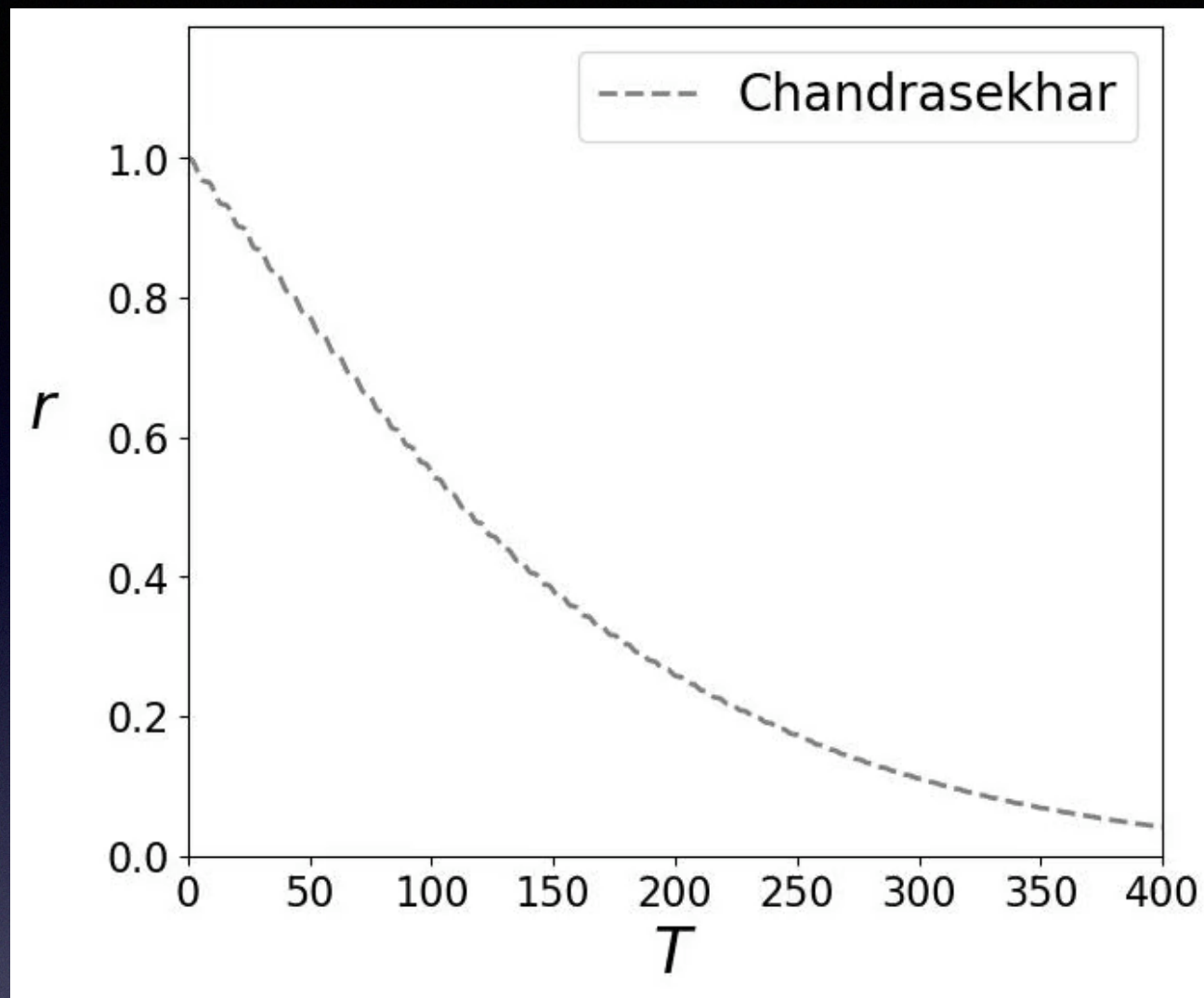
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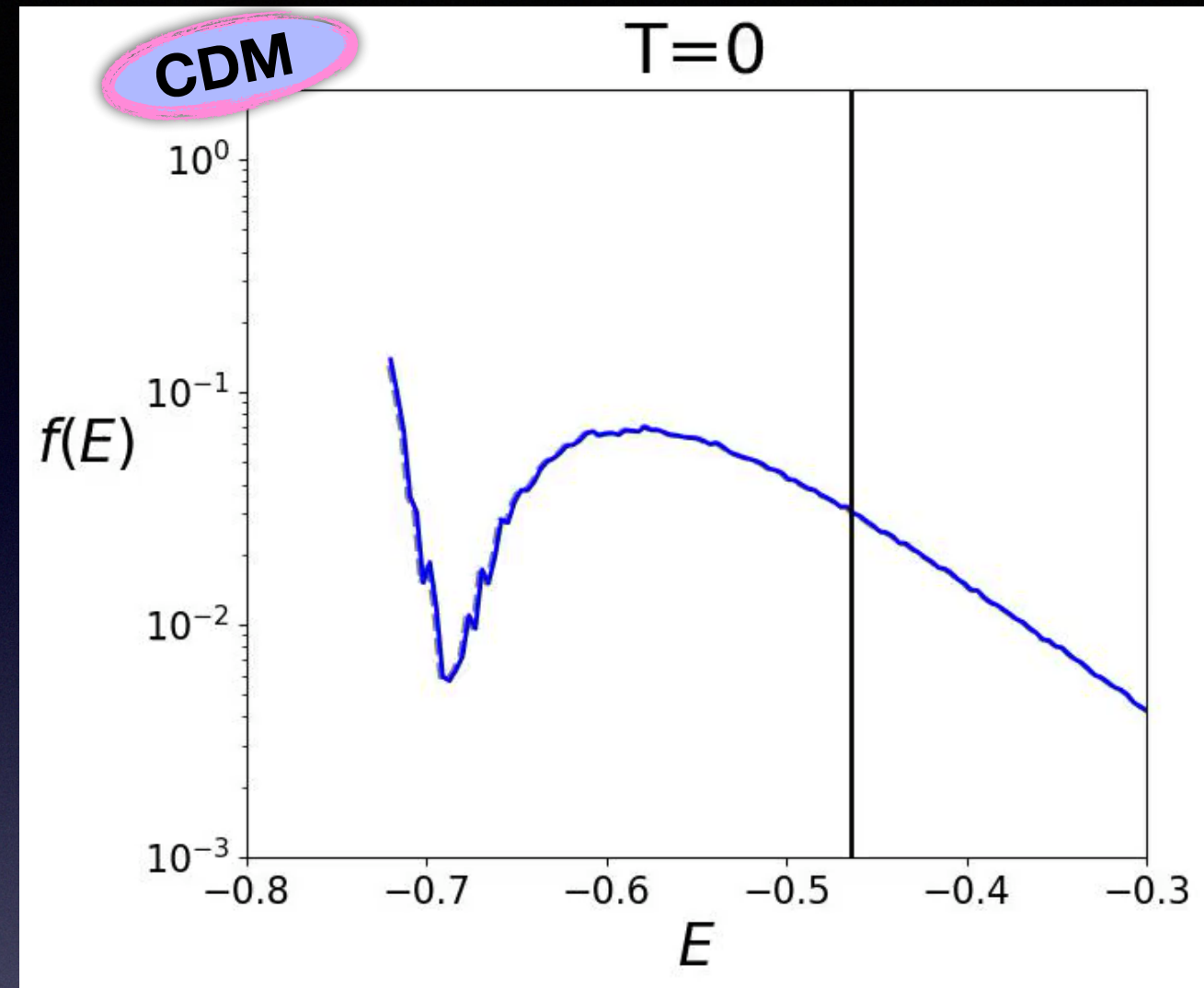
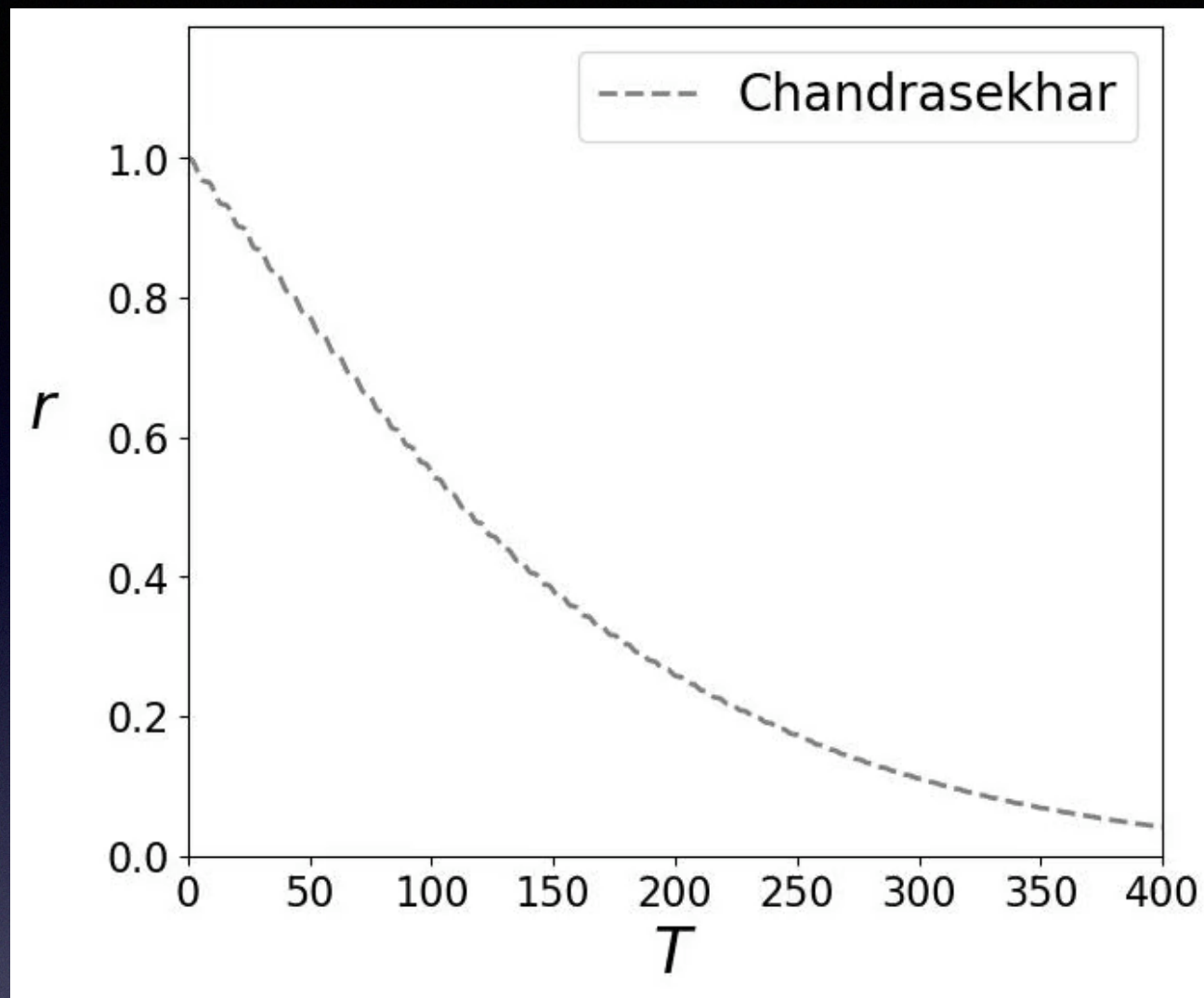


∇f ; the Linchpin of Core Dynamics



Perturber stalls where $\partial f / \partial E \sim 0$, in agreement with conjecture

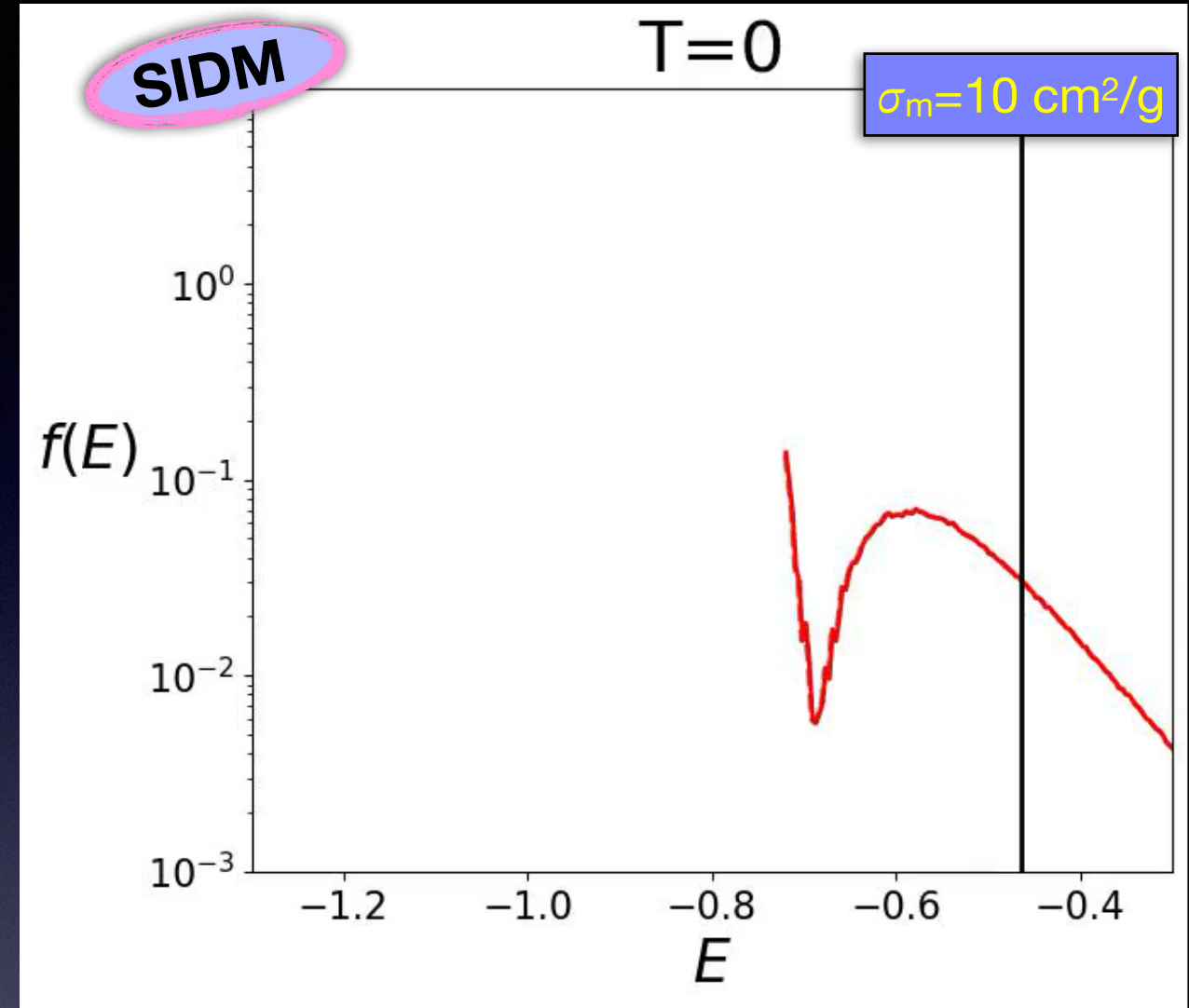
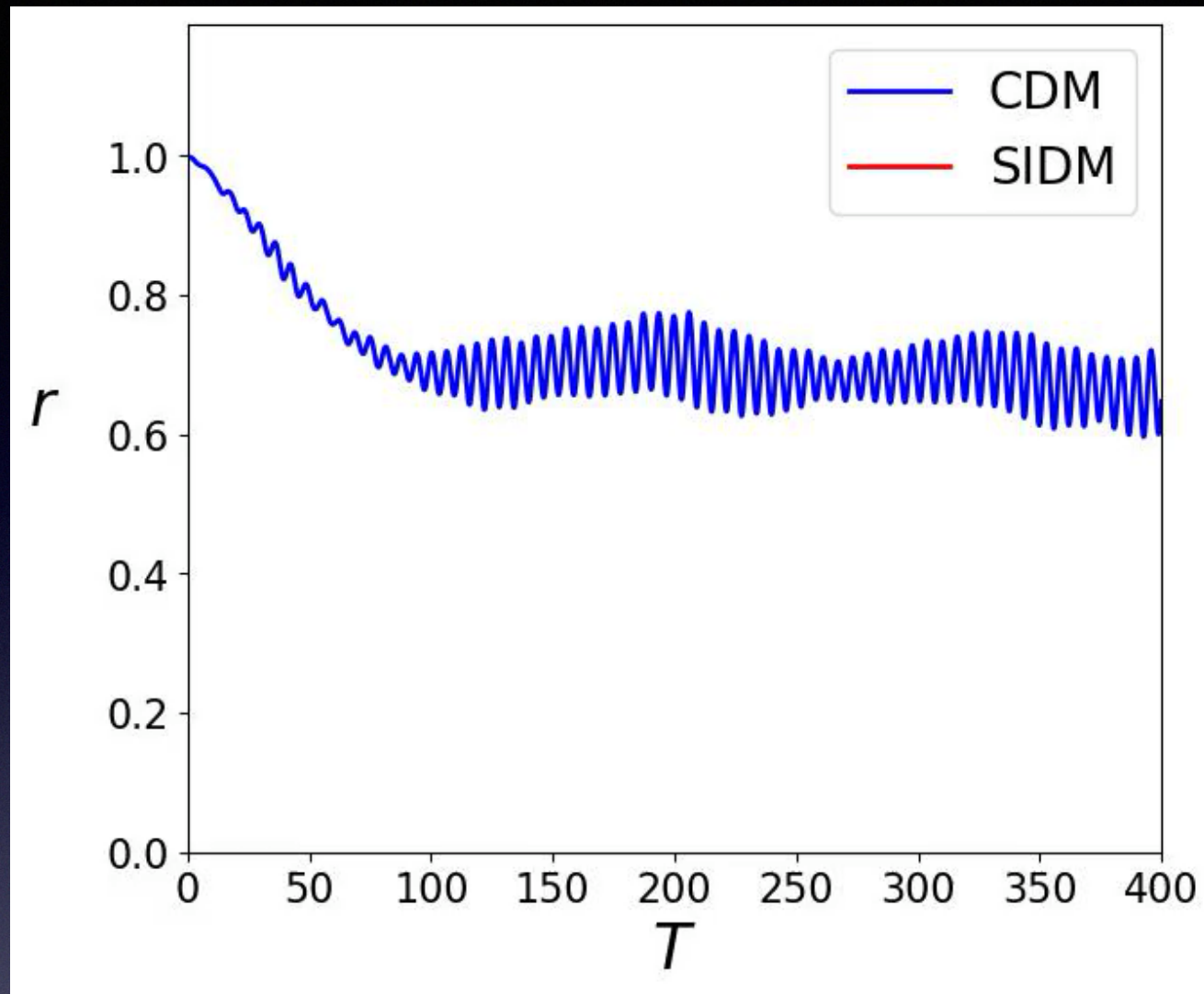
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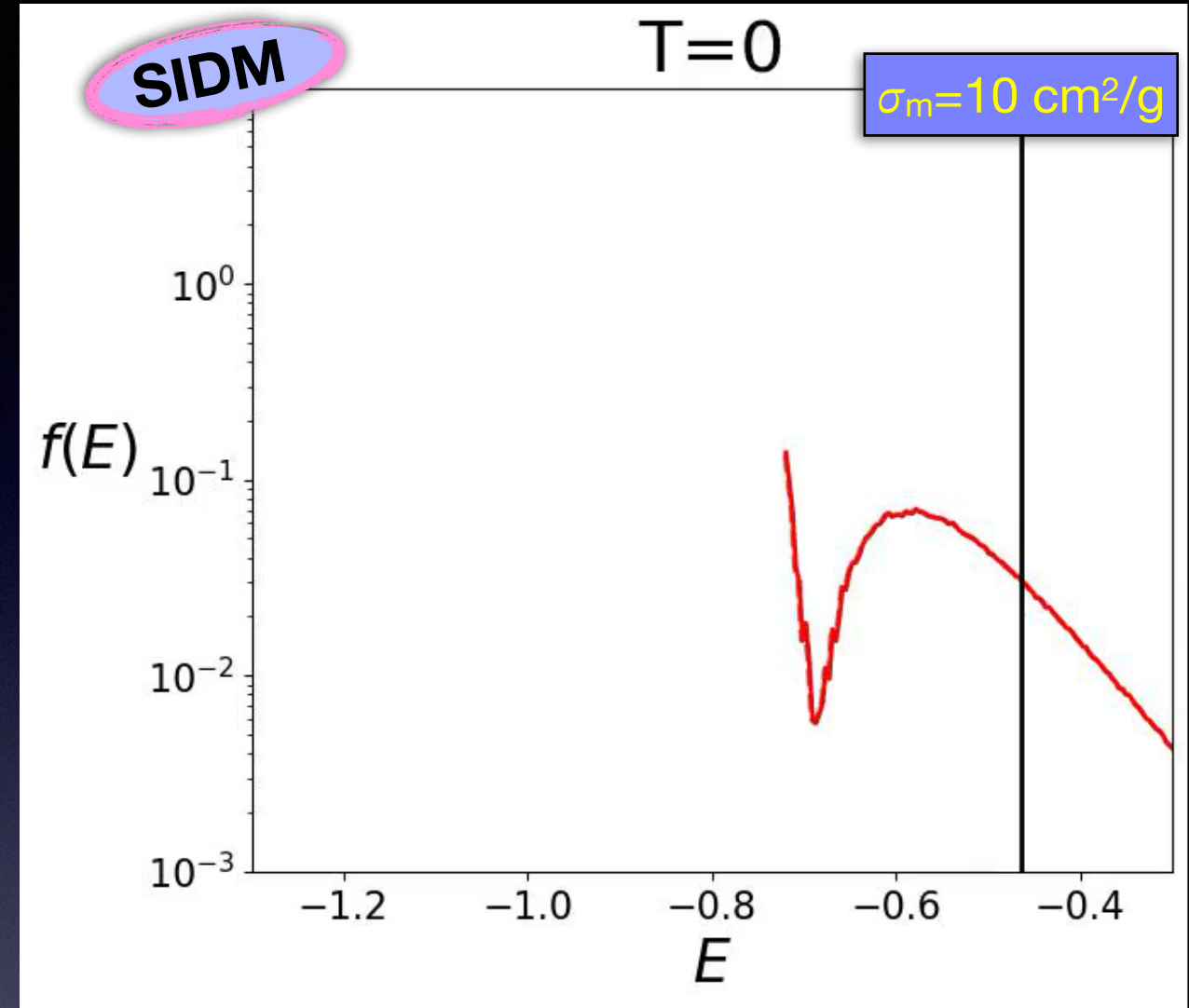
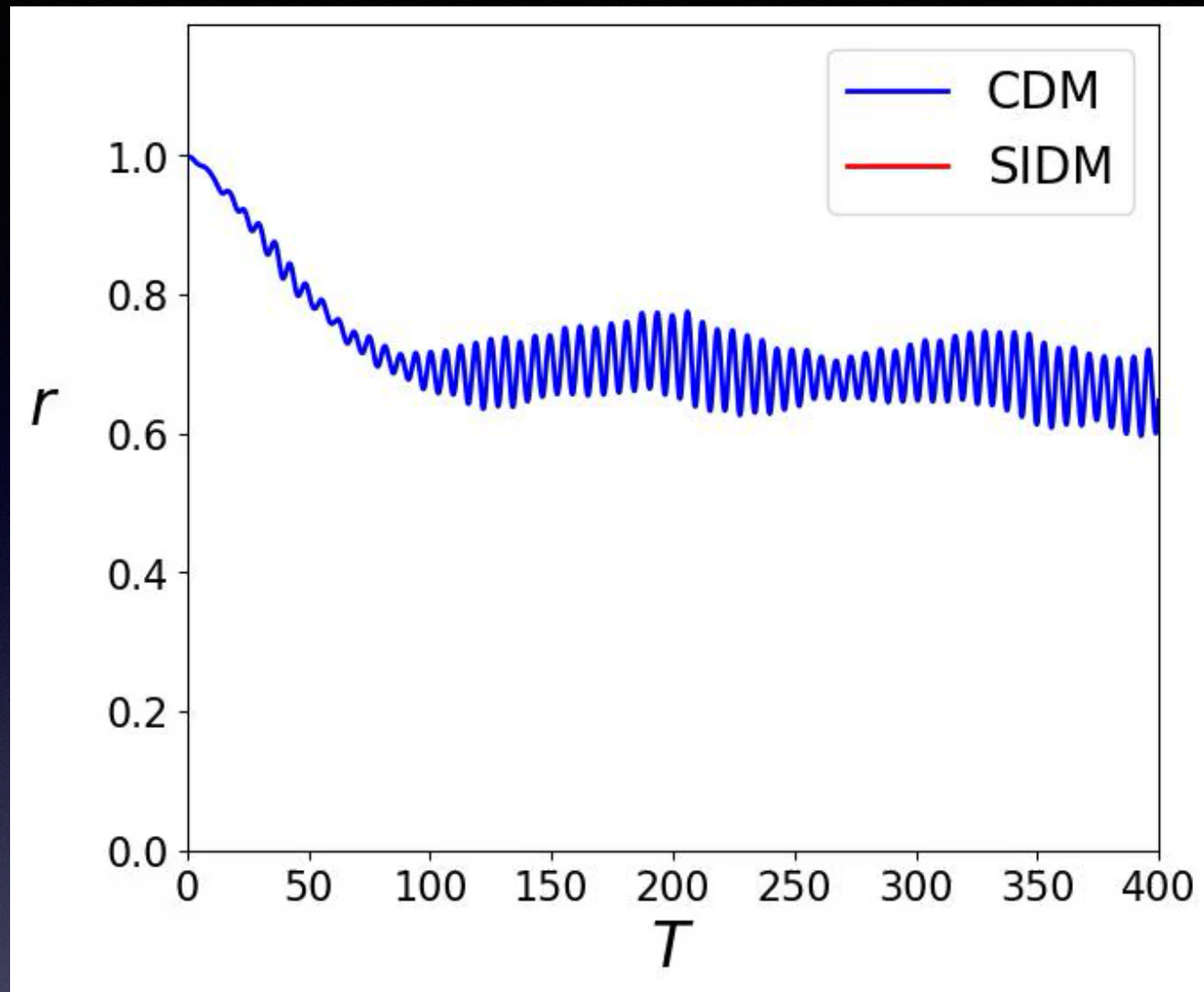
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Weak evolution in $f(E)$ due to dipole instability

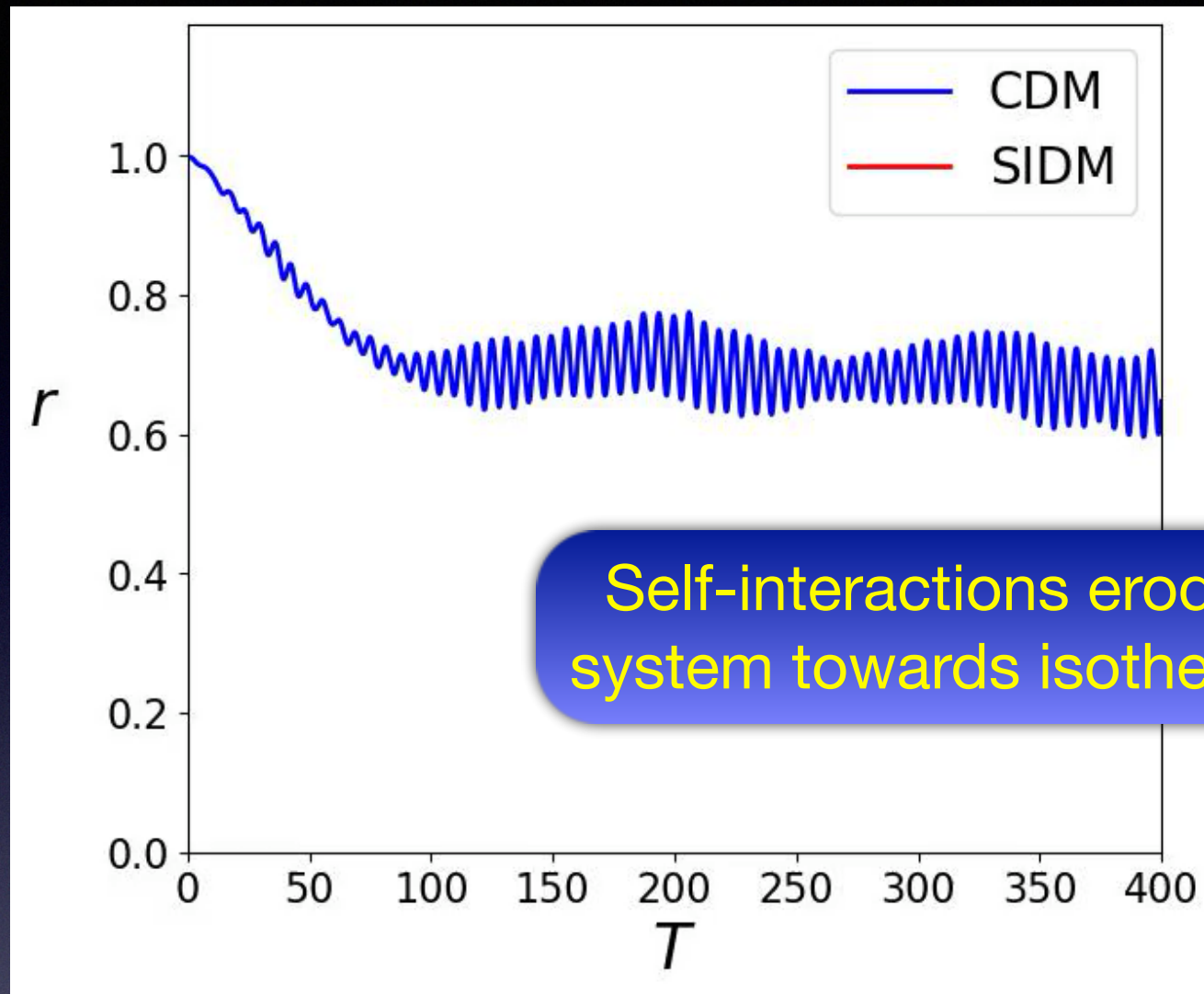
∇f ; the Linchpin of Core Dynamics



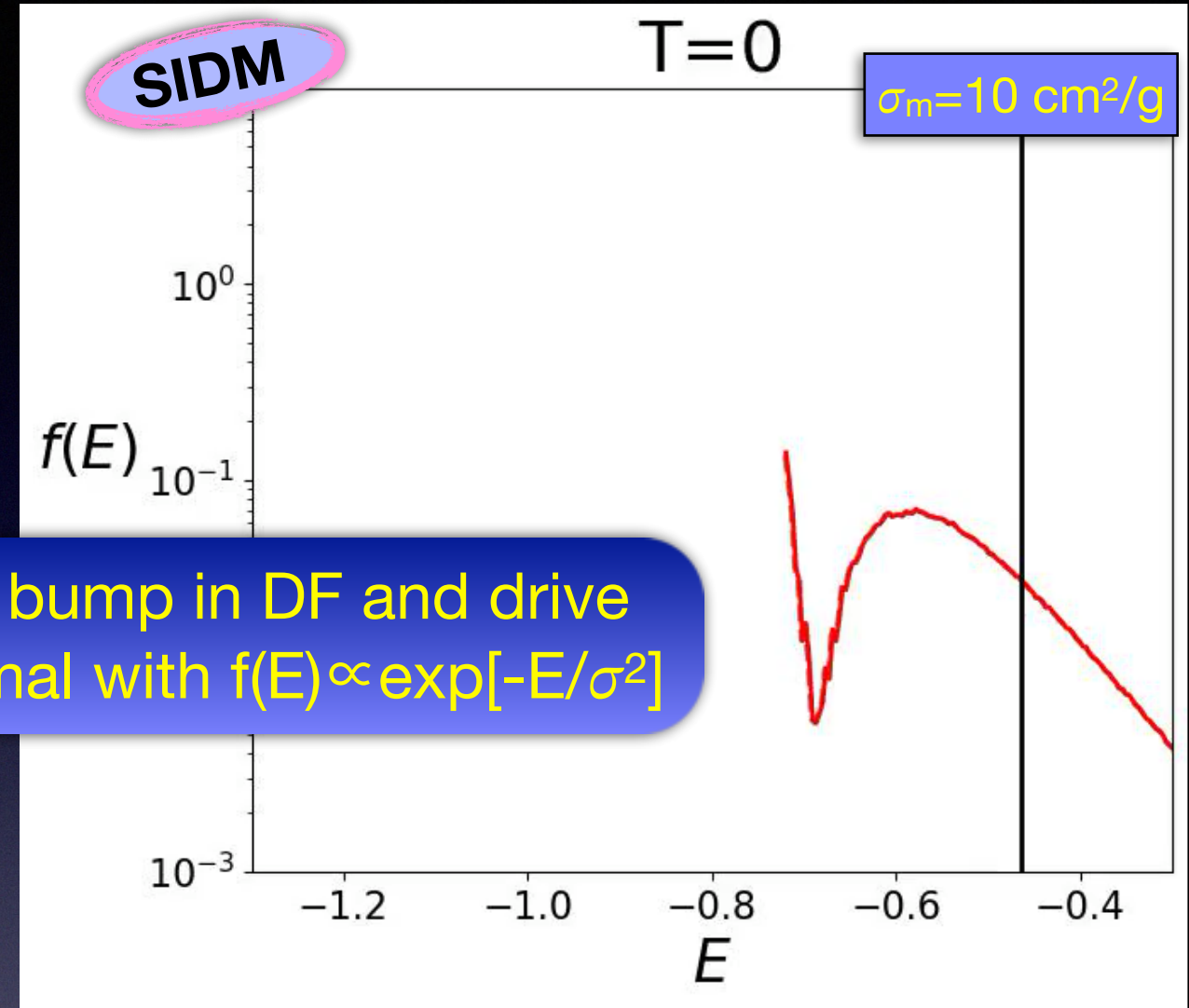
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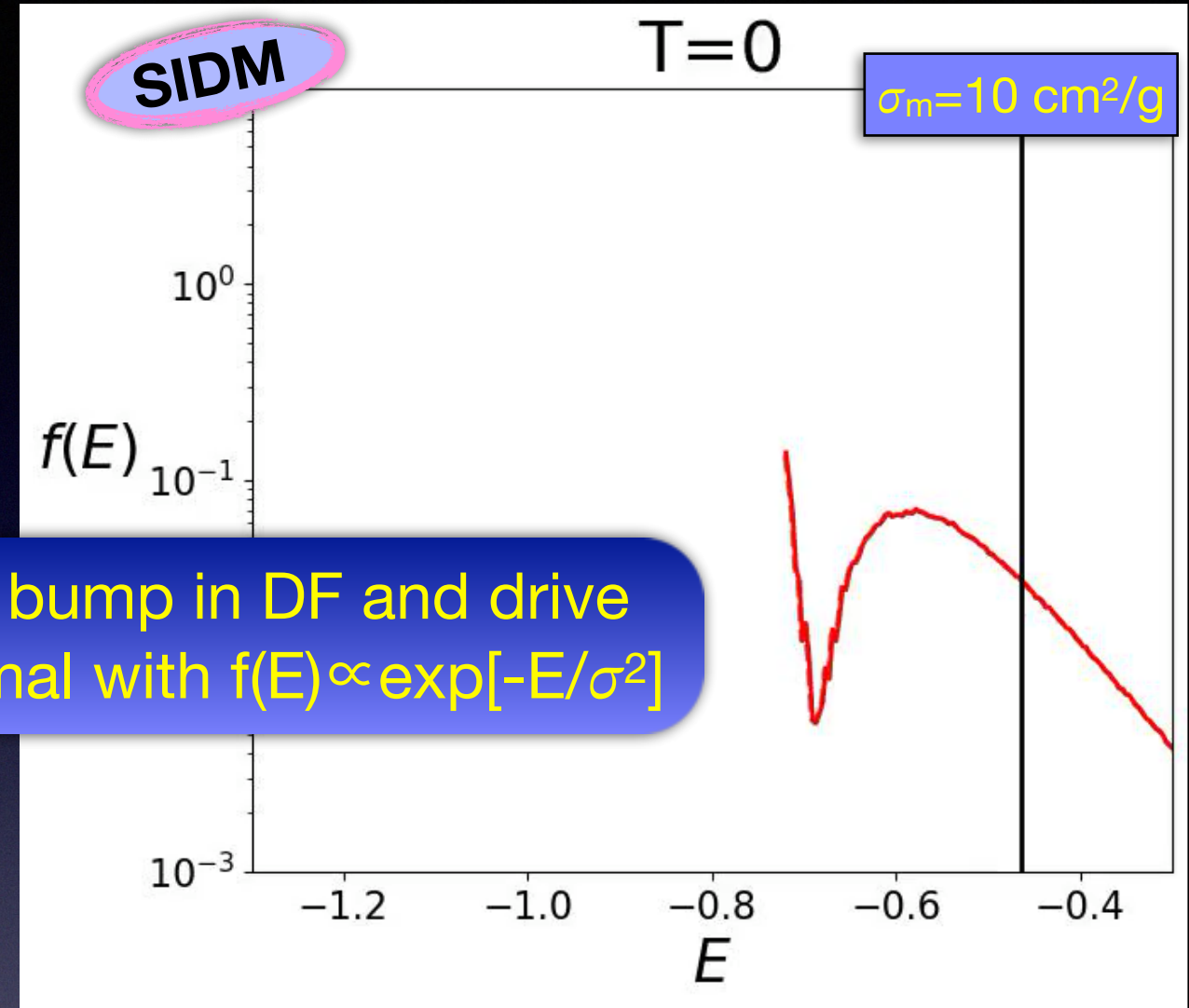
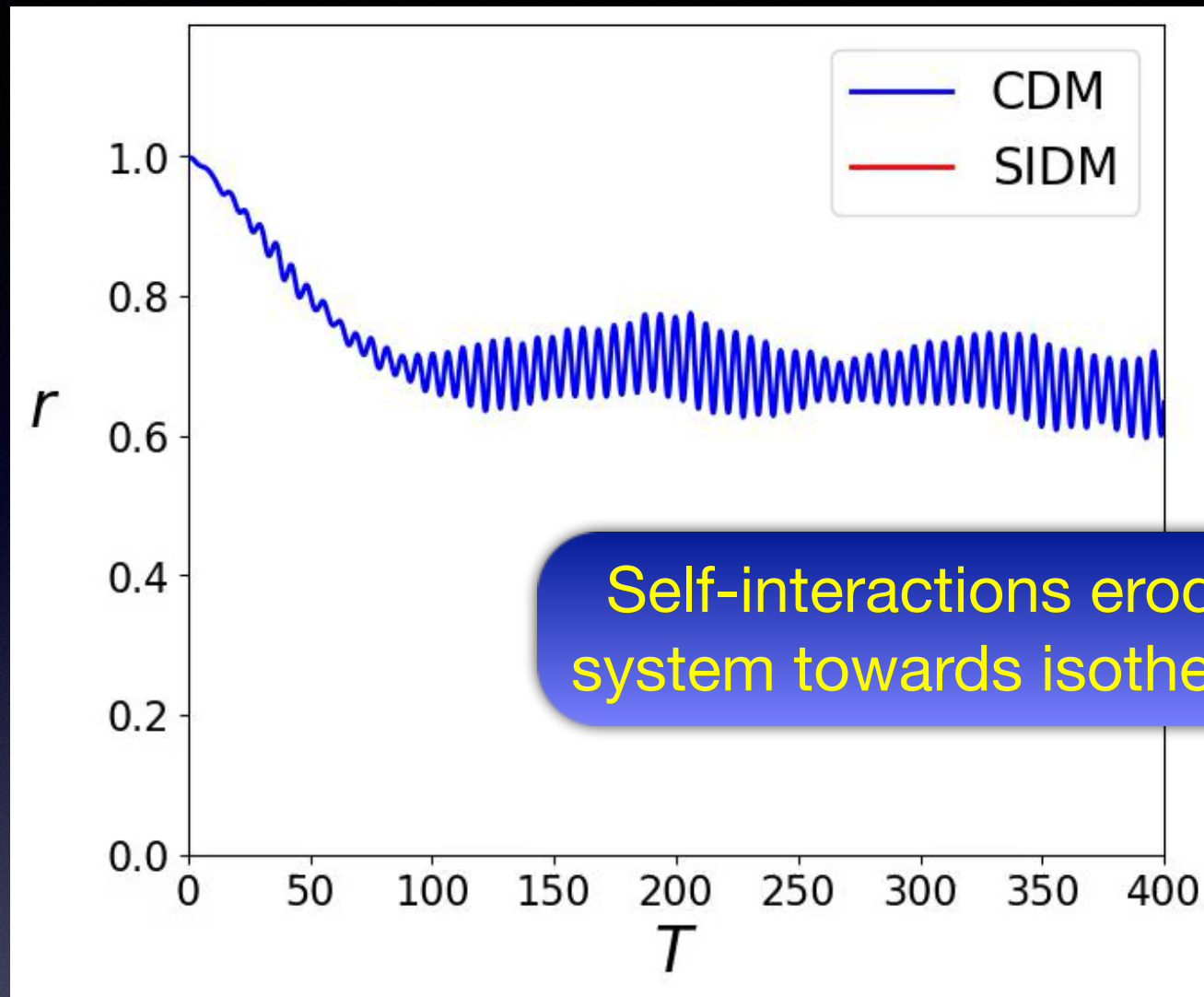
∇f ; the Linchpin of Core Dynamics



Self-interactions erode bump in DF and drive system towards isothermal with $f(E) \propto \exp[-E/\sigma^2]$



∇f ; the Linchpin of Core Dynamics



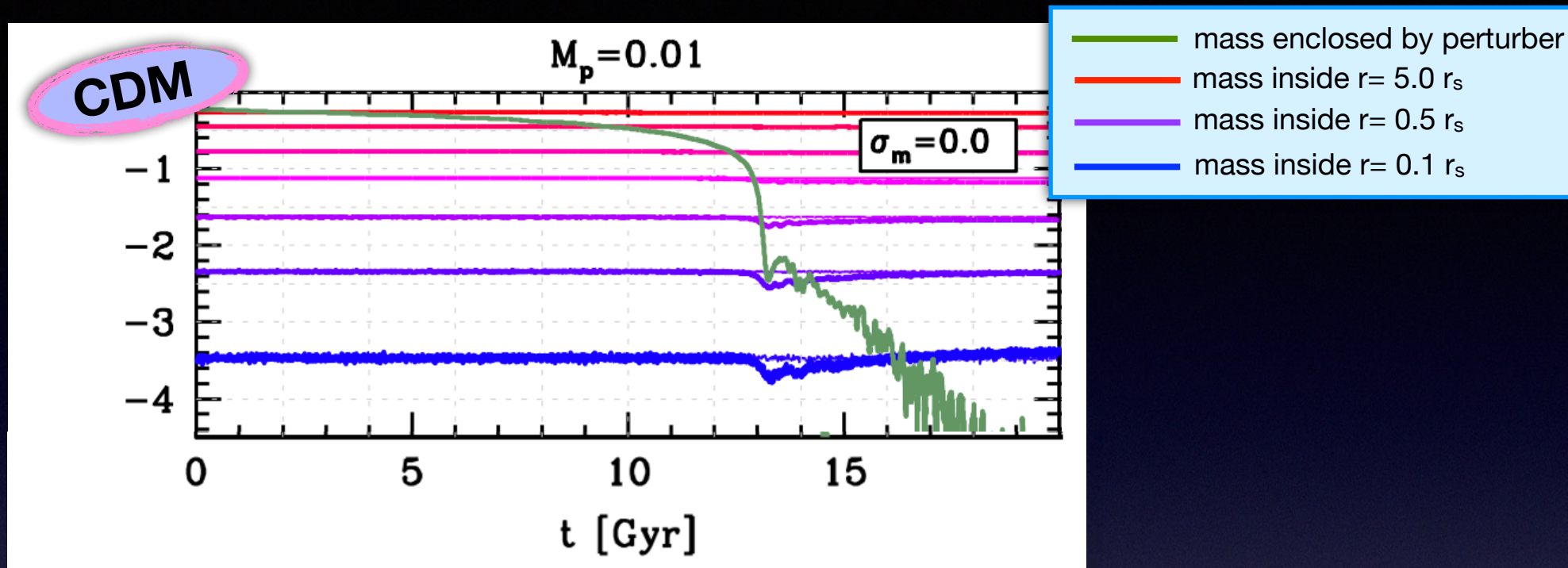
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No Core Stalling with SIDM

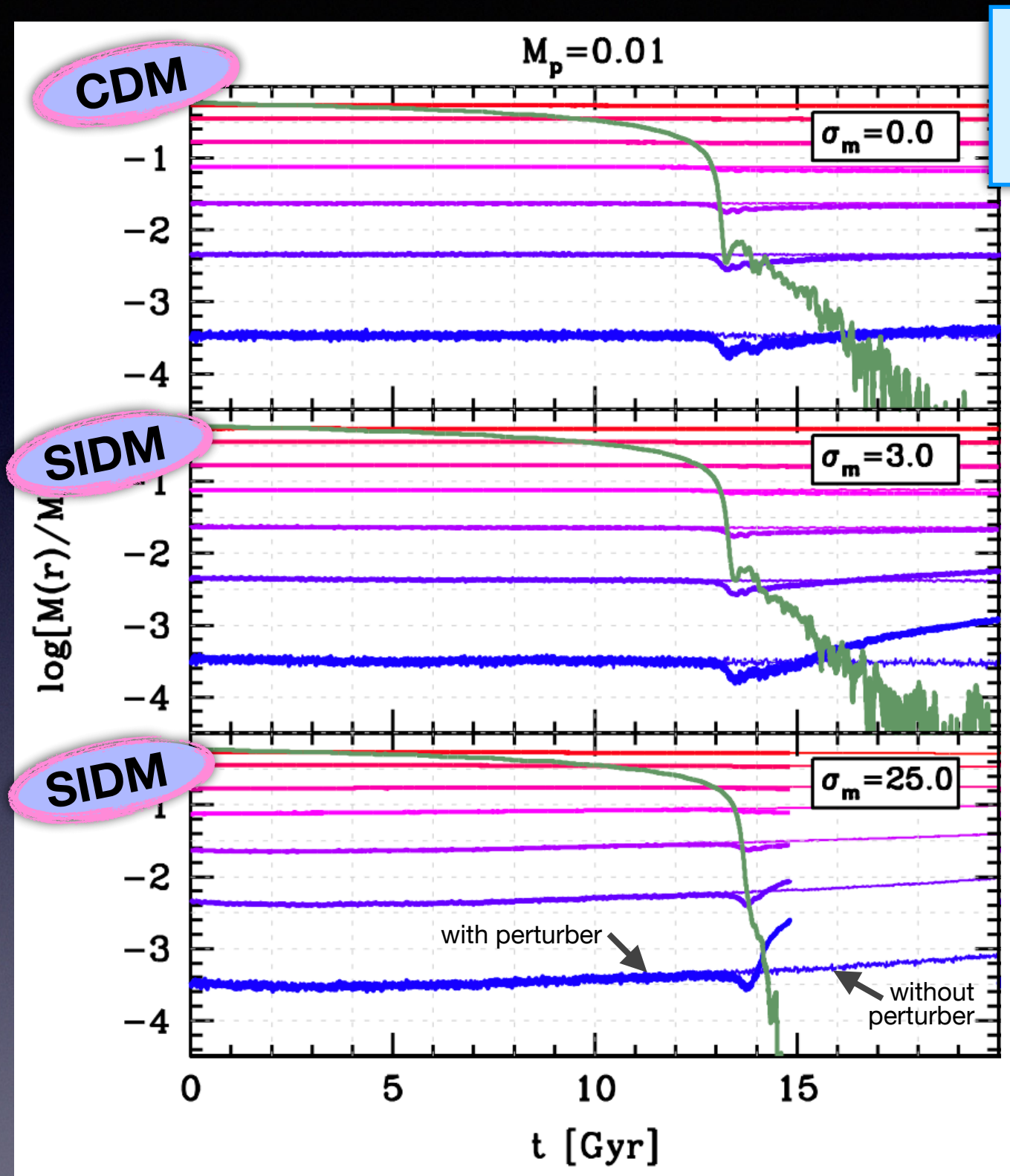
No dipole instability with SIDM

No buoyancy with SIDM

Dynamical Friction Catalyzes Core Collapse



Dynamical Friction Catalyzes Core Collapse

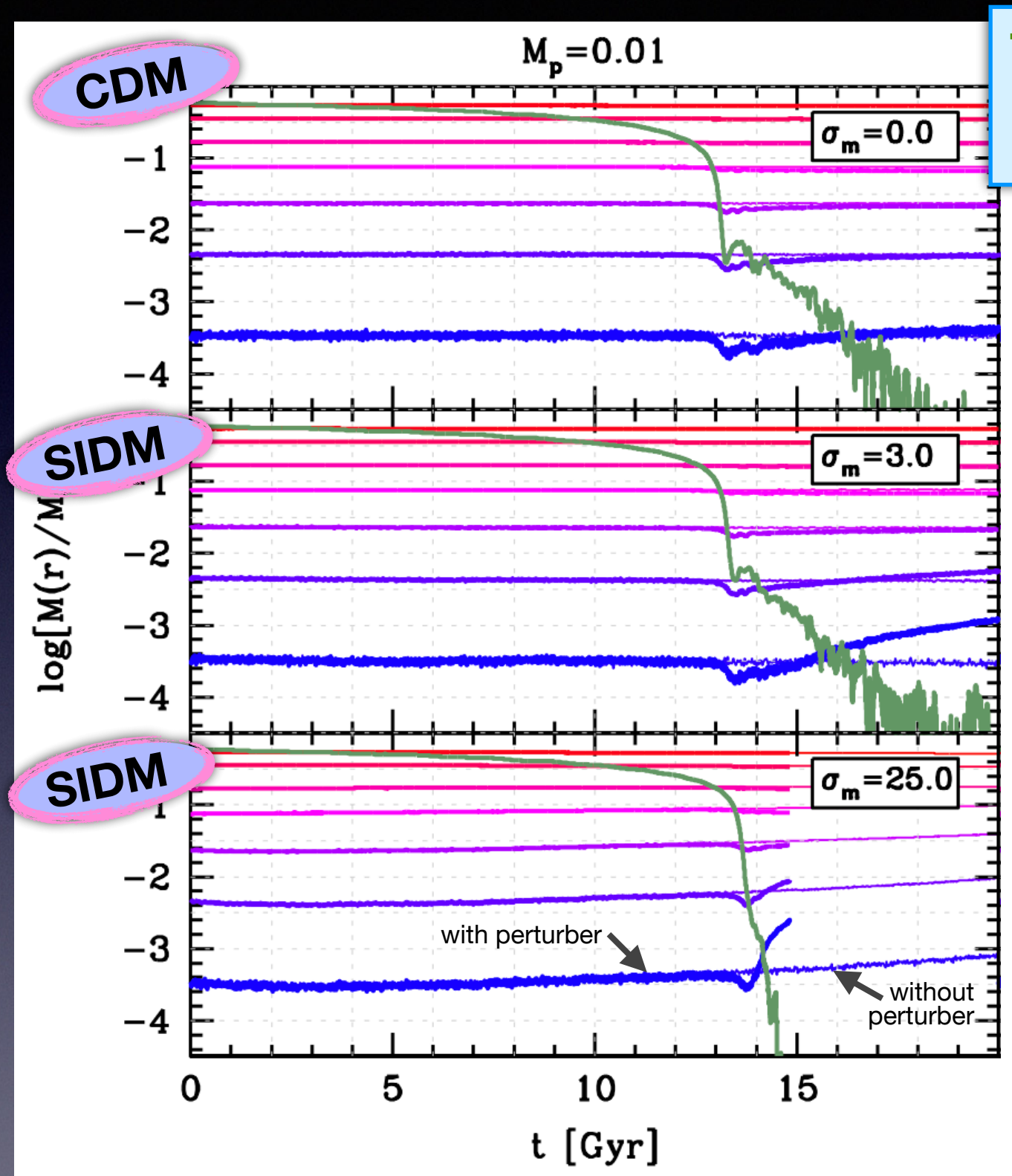


— mass enclosed by perturber
 — mass inside $r = 5.0 r_s$
 — mass inside $r = 0.5 r_s$
 — mass inside $r = 0.1 r_s$

DF of massive perturber triggers
 vastly accelerated core collapse

w/o perturber, $t_{cc} \sim 30$ (255) Gyr for $\sigma_m = 25$ (3) cm^2/g
 with perturber, $t_{cc} < 1$ (6) Gyr for $\sigma_m = 25$ (3) cm^2/g

Dynamical Friction Catalyzes Core Collapse

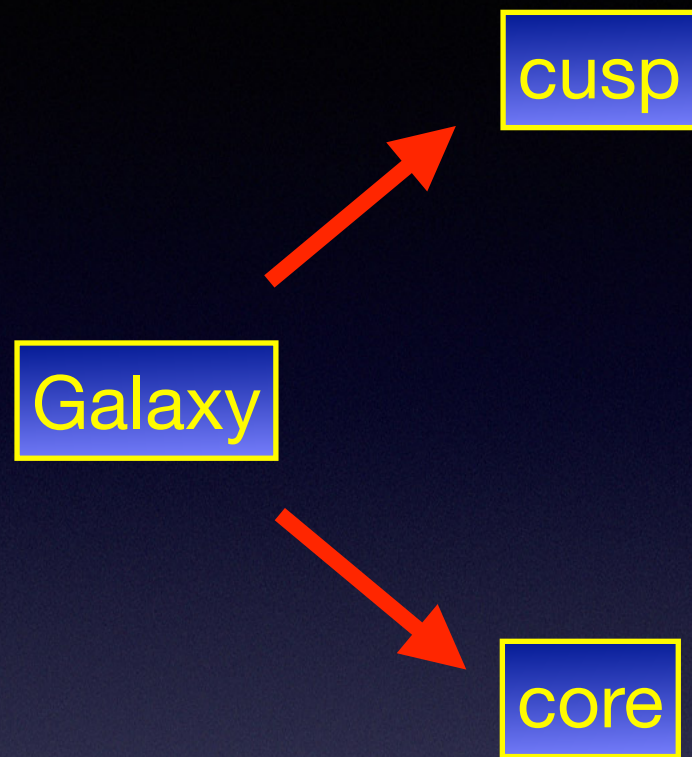


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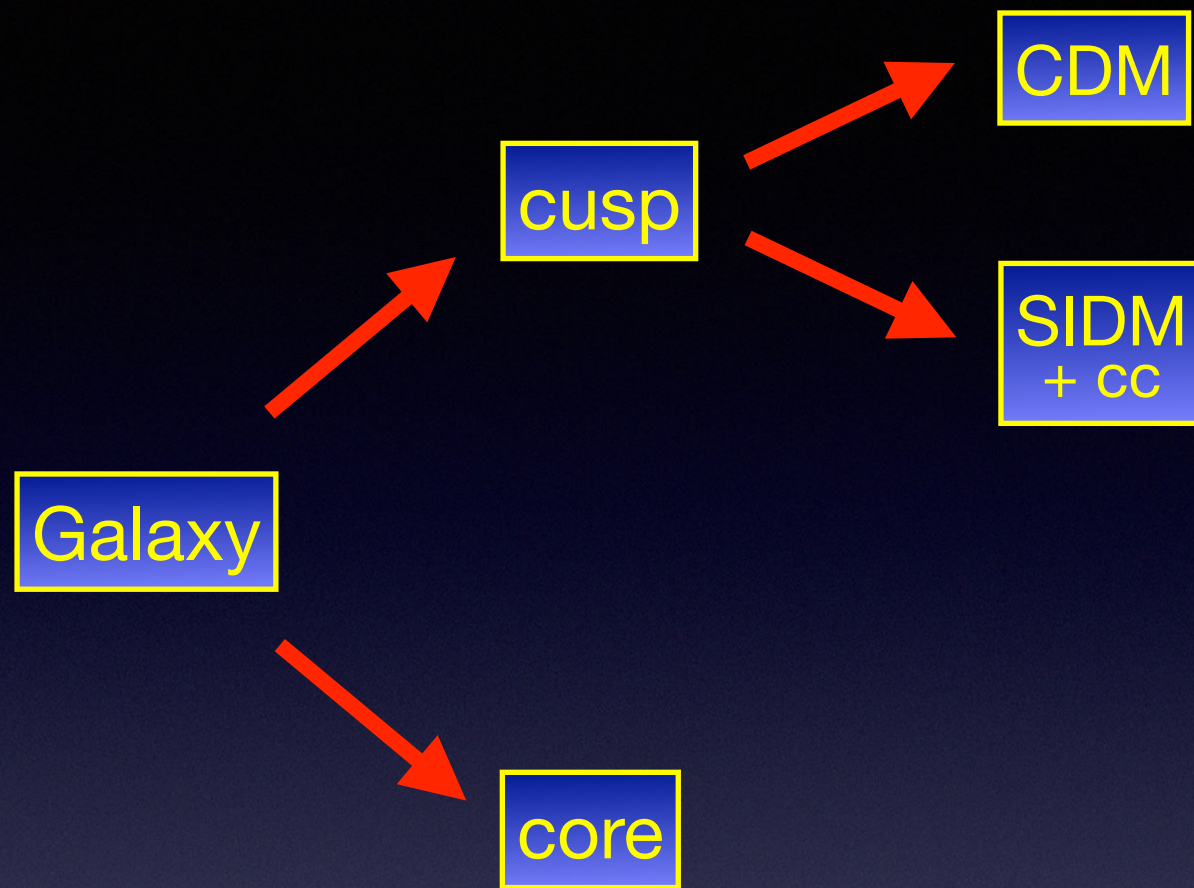
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Sinking of perturber towards center causes 'adiabatic contraction' that catalyzes core collapse

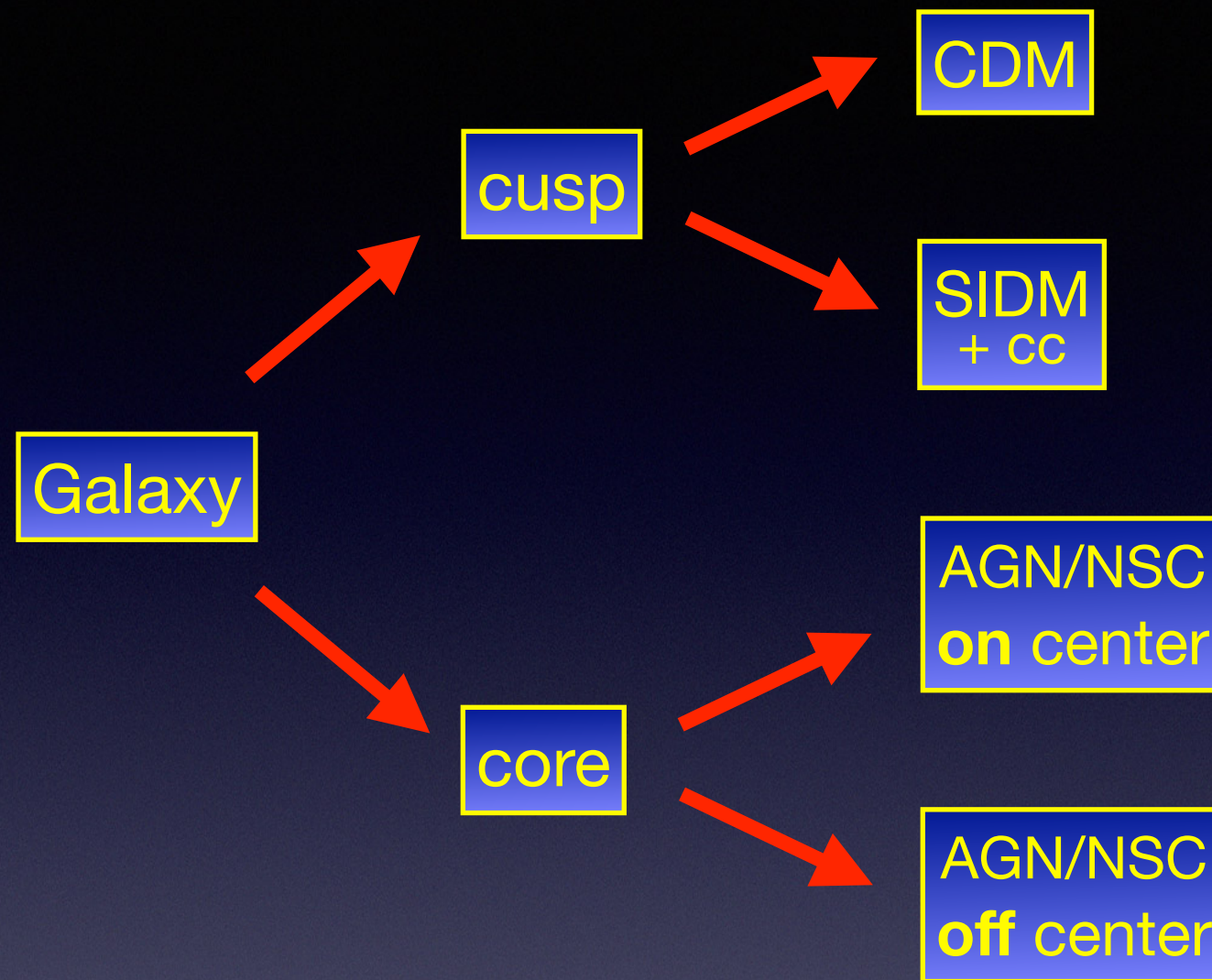
A Test-Bed for the Nature of Dark Matter



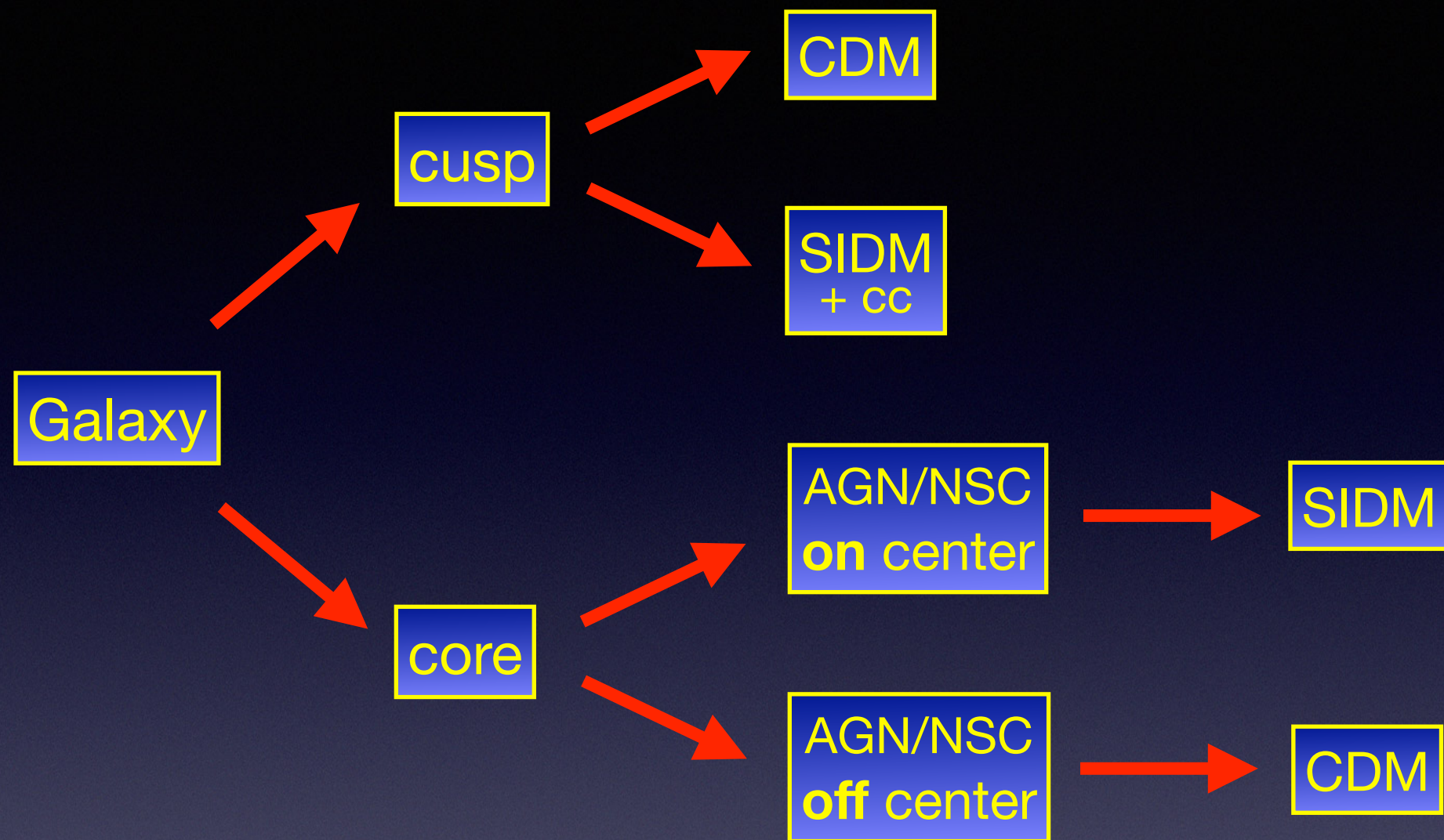
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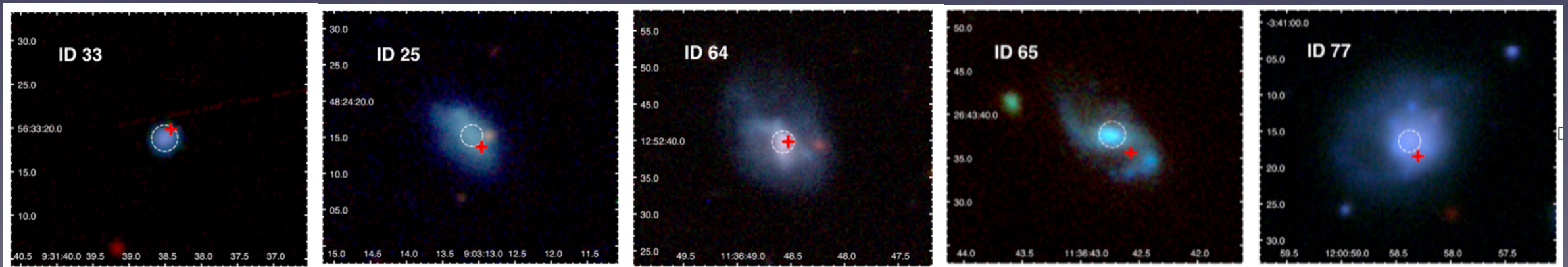
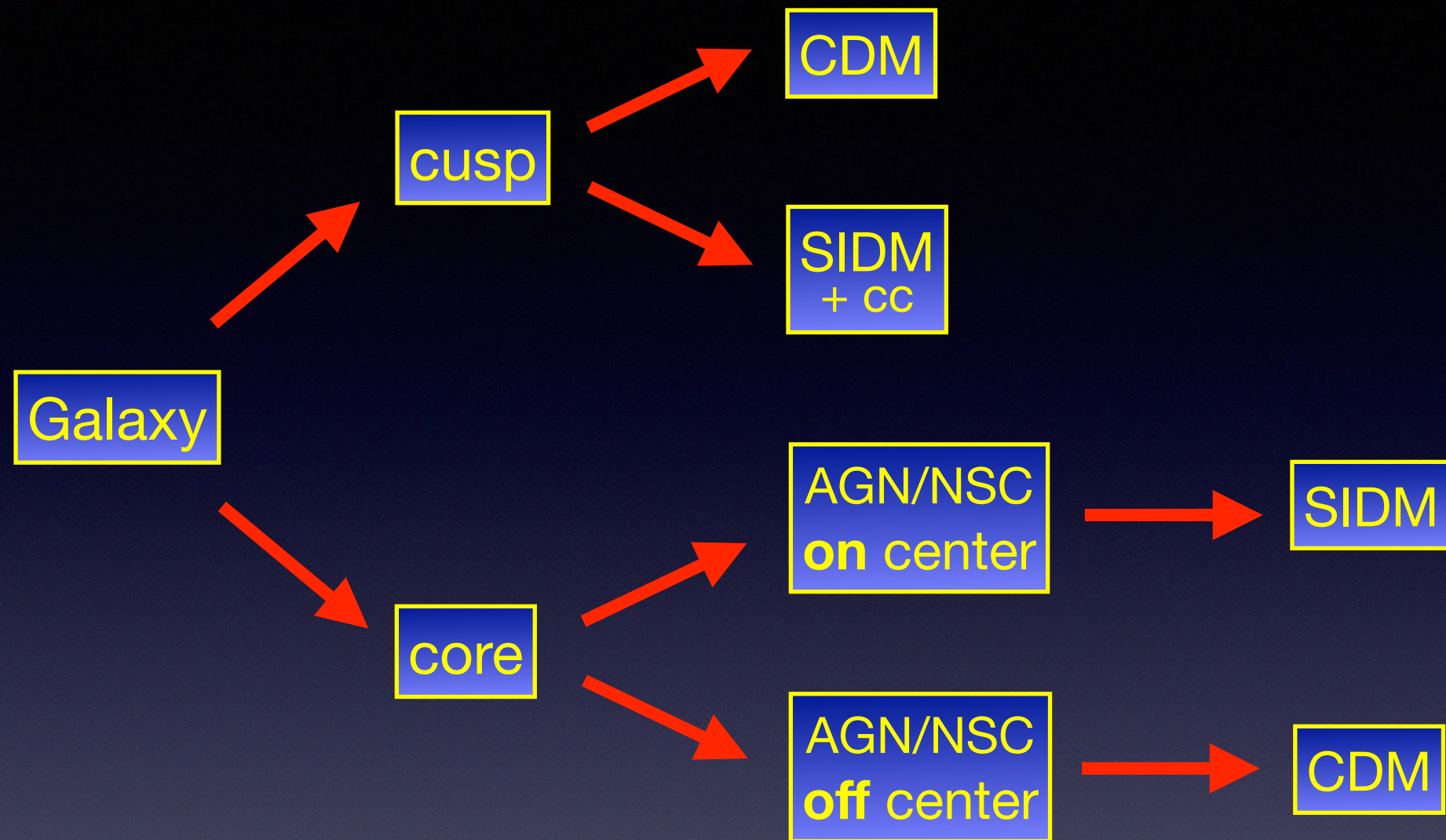
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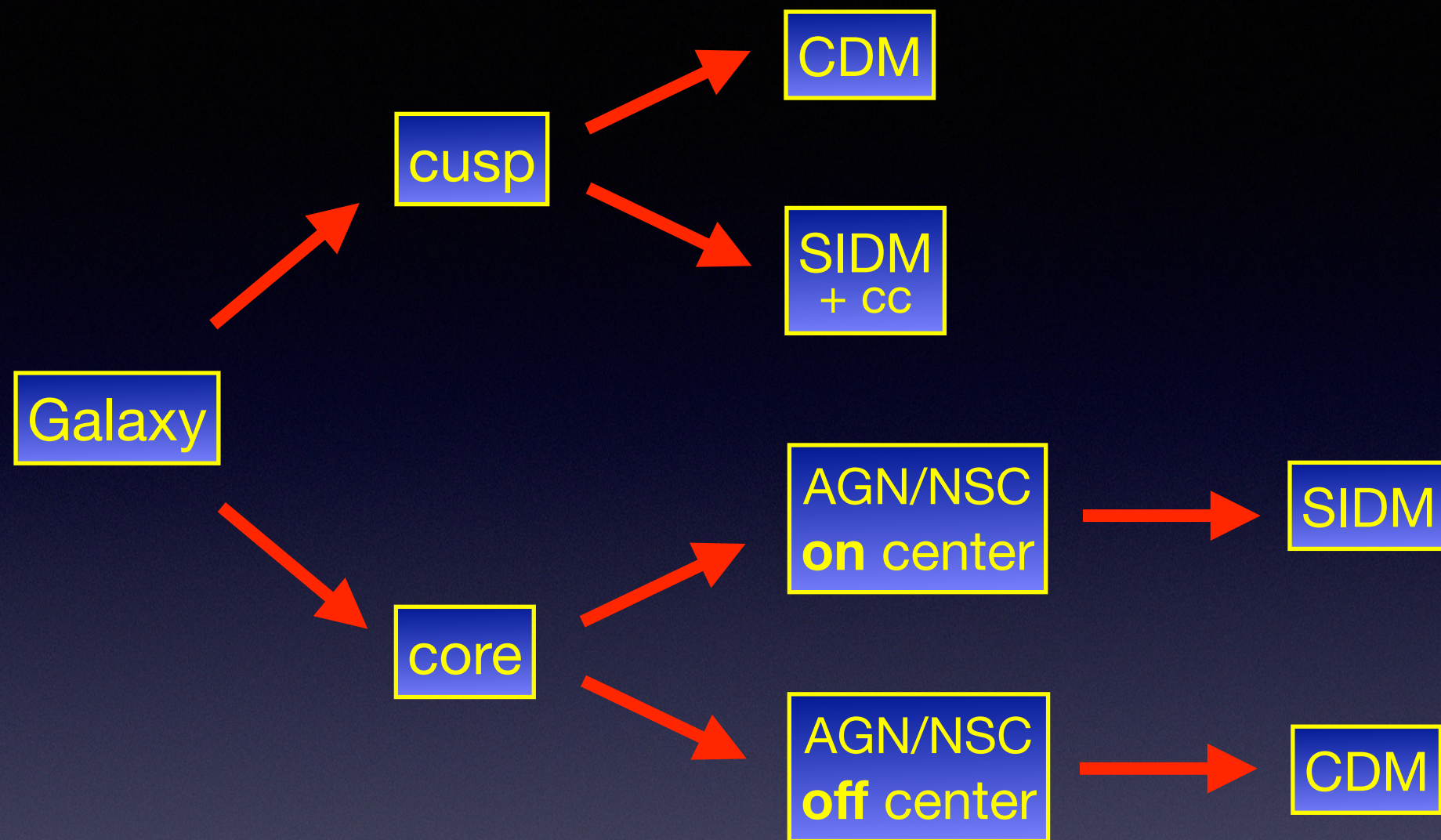


A Test-Bed for the Nature of Dark Matter



(Reines+20)

A Test-Bed for the Nature of Dark Matter



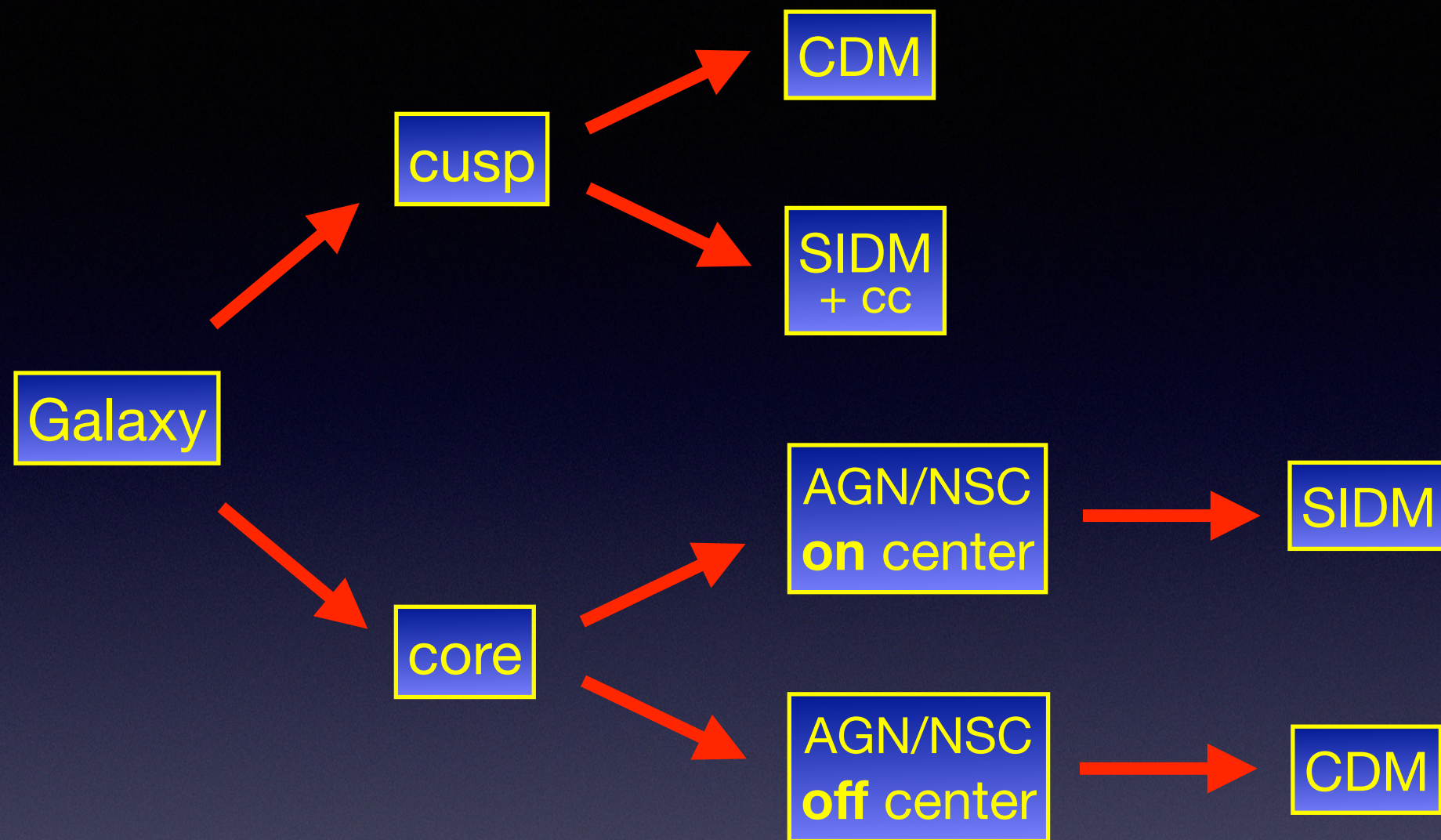
Roughly 50% of dwarf galaxies with radio AGN have offsets of 100 pc - 1 kpc*.

(Reines+20; Sargent+22)

More than 60% of 664 AGN dwarf galaxies in MaNGA data show offsets*.

(Mezcua & Domínguez-Sánchez 2024)

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*could also be due to mergers or BH recoil...

Summary

- Cores are common in galaxies; SIDM or CDM+baryonic processes?
- Cores reveal intriguing dynamics: core stalling & dynamical buoyancy
- These happen whenever $\partial f / \partial E \sim 0$ (stalling) or $\partial f / \partial E > 0$ (buoyancy)
- Cores with inflection in DF where $\partial f / \partial E > 0$ are dipole unstable
- Saturation of dipole mode into long-lived soliton explains buoyancy
- Core stalling is suppressed or absent in SIDM
- Dynamical friction inside core catalyzes core collapse
- Frequency of offset AGN/NSC can potentially inform nature of DM