Gravsphere2: An extended higher-order Jeans modeling approach

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Mass contents from stellar kinematics

- Stellar velocity dispersions trace mass contents from the Jeans equations:

$$\frac{1}{\nu_\star}\frac{\partial \nu_\star \sigma_r^2}{\partial r} + \frac{2\beta\sigma_r^2}{r} = -\frac{GM}{r^2},$$

where: $$\sigma^2_{r/t} = \langle \mathrm{v}^2_{r/t} \rangle - \langle \mathrm{v}_{r/t} \rangle^2, \qquad \beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

- Solve for the radial velocity dispersion for a given mass profile:

$$\sigma_r^2(r) = \frac{1}{\nu_\star(r) g(r)} \int_r^\infty \frac{GM(r')\nu_\star(r')}{r'^2} g(r') dr', \qquad g(r) \equiv \exp\left(2\int \frac{\beta(r)}{r} dr\right)$$

Stellar kinematics observables (LOS + PMs)

- Can't measure $\sigma_r(r)$ and $\beta(r)$ directly, but can constrain them with projected quantities (line-of-sight + proper motions):

$$\sigma^2_{\rm LOS}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty \left(1 - \frac{R^2}{r^2}\beta(r)\right) \frac{\nu_\star(r)\sigma_r^2(r) r}{\sqrt{r^2 - R^2}} dr,$$

$$\sigma^2_{\rm PM,\,t}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty \left(1 - \beta(r)\right) \frac{\nu_\star(r)\sigma_r^2(r) r}{\sqrt{r^2 - R^2}} dr.$$

$$\sigma^2_{\rm PM,\,R}(R) = \frac{2}{\Sigma_\star(R)} \int_R^\infty \left(1 - \beta(r) + \frac{R^2}{r^2}\beta(r)\right) \frac{\nu_\star(r)\sigma_r^2(r) r}{\sqrt{r^2 - R^2}} dr$$

The mass/density - anisotropy degeneracy

- Full consideration of these observables is important to avoid degeneracies with mass models

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How to break the density-anisotropy degeneracy in spherical stellar systems

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Beyond dispersions: Fourth-order moments

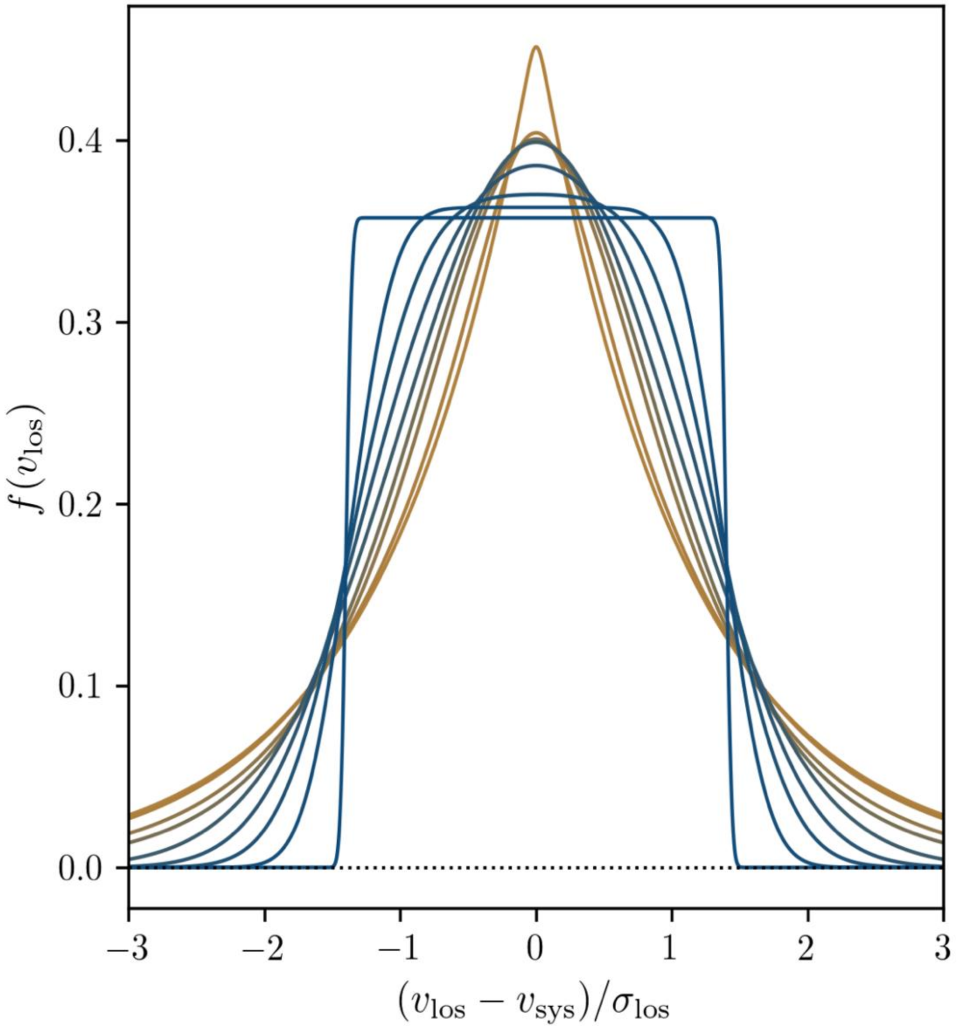
- Fourth-order Jeans equations give us information about velocity distribution:

$$\frac{1}{\nu_\star}\frac{\partial \nu_\star \langle v_r^4 \rangle}{\partial r} - \frac{3}{r}\langle v_r^2 v_t^2 \rangle + \frac{2}{r}\langle v_r^4 \rangle = -3\sigma_r^2 \frac{GM}{r^2},$$

$$\frac{1}{\nu_\star}\frac{\partial \nu_\star \langle v_r^2 v_t^2 \rangle}{\partial r} - \frac{1}{r}\langle v_t^4 \rangle + \frac{4}{r}\langle v_r^2 v_t^2 \rangle = -\sigma_t^2 \frac{GM}{r^2}$$

- Kurtosis tells us the tailedness:

$$\kappa \equiv \langle v^4 \rangle / \sigma^4$$



Wardana et al. (2025)

Why should we look at higher moments?

- Helps us break the mass-anisotropy degeneracy

- Ensures our assumed velocity distribution is physically consistent (e.g. Gaussianity $\neq$ equilibrium, avoid negative moments)

- Flexibility to capture deviations from Gaussianity in data

→ Reduces biases and uncertainities in mass modeling

Solving the fourth-order problem

- Richardson & Fairbairn (2013) introduce new anisotropy term to solve at 4th order:

$$\frac{1}{\nu_\star}\frac{\partial \nu_\star \langle v_r^4\rangle}{\partial r} + \frac{2\beta'}{r}\langle v_r^4\rangle = -3\sigma_r^2\frac{GM}{r^2}, \qquad \beta' \equiv 1 - \frac{3}{2}\frac{\langle v_r^2 v_t^2\rangle}{\langle v_r^4\rangle}$$

$$\langle v_r^4\rangle(r) = \frac{1}{\nu_\star(r)g'(r)}\int_r^\infty 3\sigma_r^2(r')\frac{GM(r')\nu_\star(r')}{r'^2}g'(r')dr'$$

- Problem: Does this create a new degeneracy? Do we need to make assumptions about β' ?

New approach: general anisotropy + PMs

- Solve for β and β' as independent general functions:

$$\beta(r) = \beta_0 + (\beta_\infty - \beta_0)\frac{1}{1 + \left(\frac{r_t}{r}\right)^{\eta}},$$

- Introduce new observable: fourth-order proper motions, solving along the full 3Ds (not just LOS):

$$\langle v^4_{\rm PM,t}\rangle(R) = \frac{2}{\Sigma_\star(R)}\int_R^\infty \frac{F_{\rm PM,t}(r,R)\nu_\star(r)r}{\sqrt{r^2-R^2}}dr,$$

$$F_{\rm PM,t}(r,R) \equiv \frac{1}{2}\left[(1-\beta')(2-\beta') - \frac{r}{2}\frac{\partial\beta'}{\partial r}\right]\langle v_r^4\rangle + \frac{3}{4}(\beta-\beta')\sigma_r^2\frac{GM}{r},$$

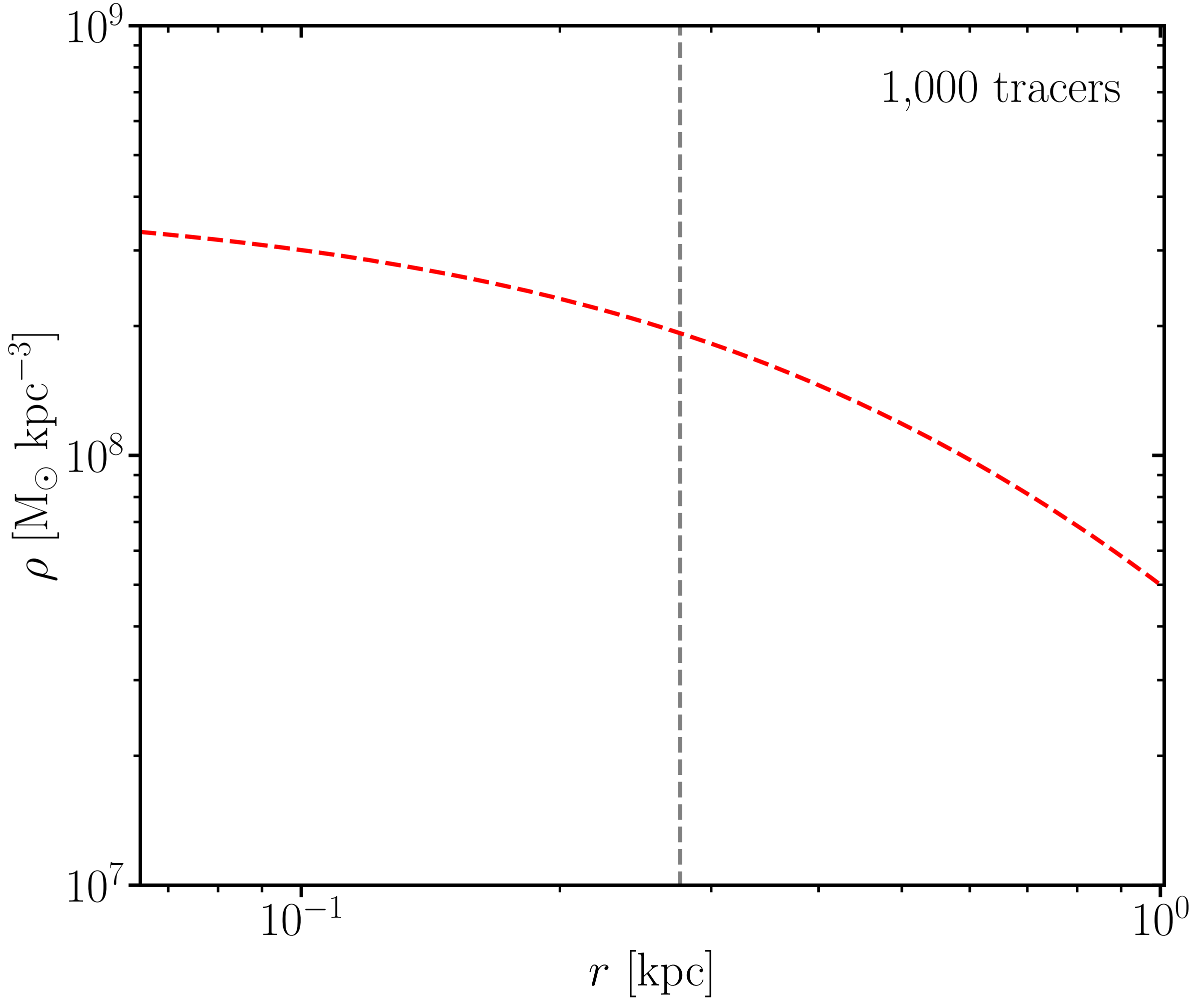
$$\langle v^4_{\rm PM,R}\rangle(R) = \frac{2}{\Sigma_\star(R)}\int_R^\infty \frac{F_{\rm PM,R}(r,R)\nu_\star(r)r}{\sqrt{r^2-R^2}}dr,$$

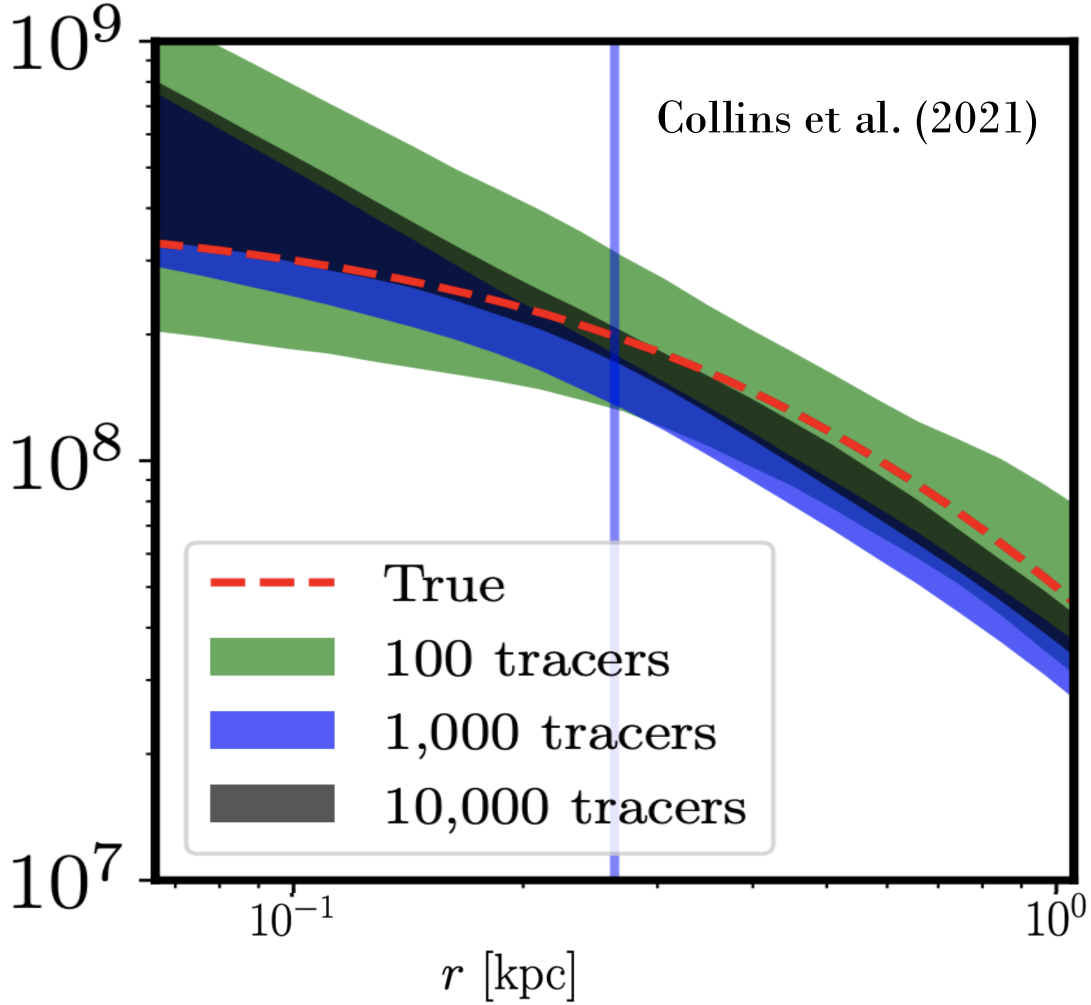
$$F_{\rm PM,R}(r,R) \equiv \left(1 - 2\frac{R^2}{r^2} + \frac{R^4}{r^4}\right)F_{\rm PM,t}(r,R) + \left(2(1-\beta')\frac{R^2}{r^2} - (1-2\beta')\frac{R^4}{r^4}\right)\langle v_r^4\rangle.$$

Other updates in Gravsphere2

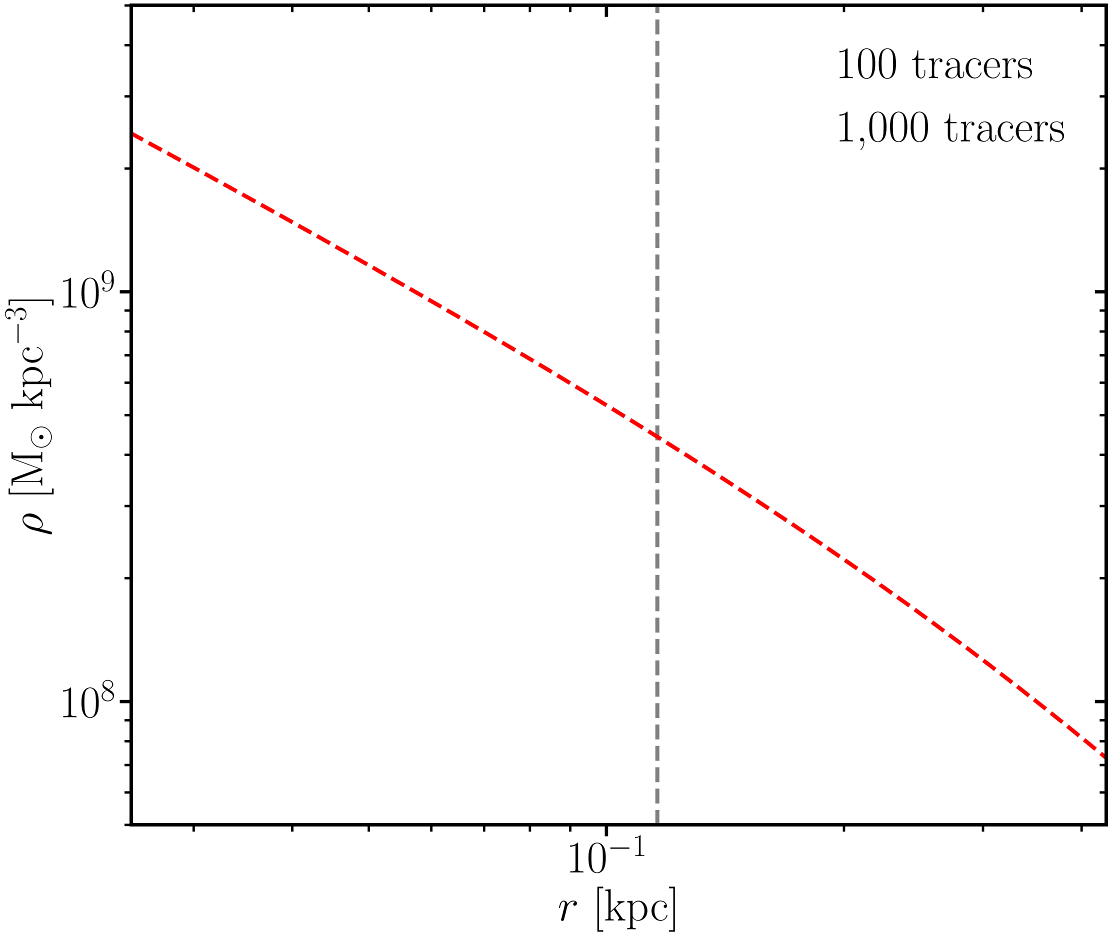
- Individual velocities, no binning
- General PDF with analytic error convolution (Sanders & Evans 2020)
- More general stellar tracer $\alpha\beta\gamma$ profile
- Robust posterior with nested sampling (**dynesty**)

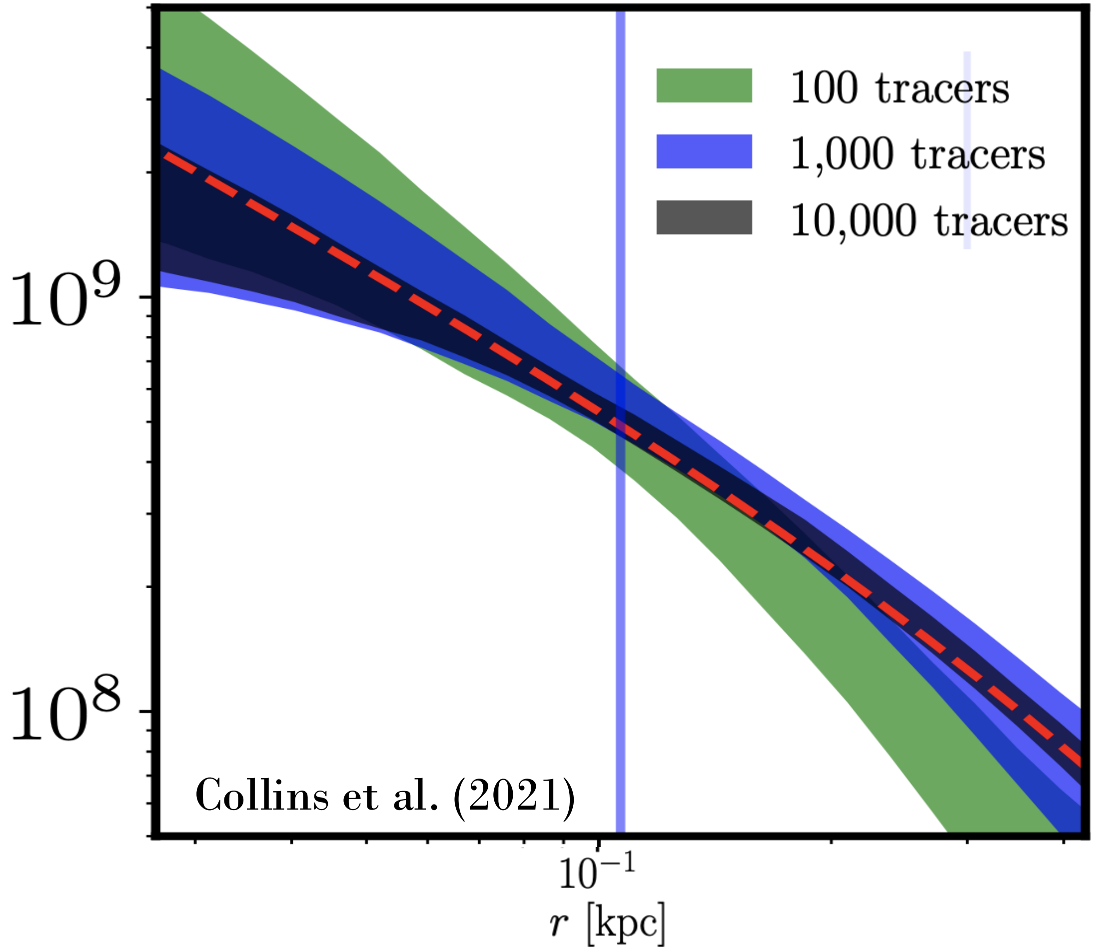
Mock data results: density profiles (core)



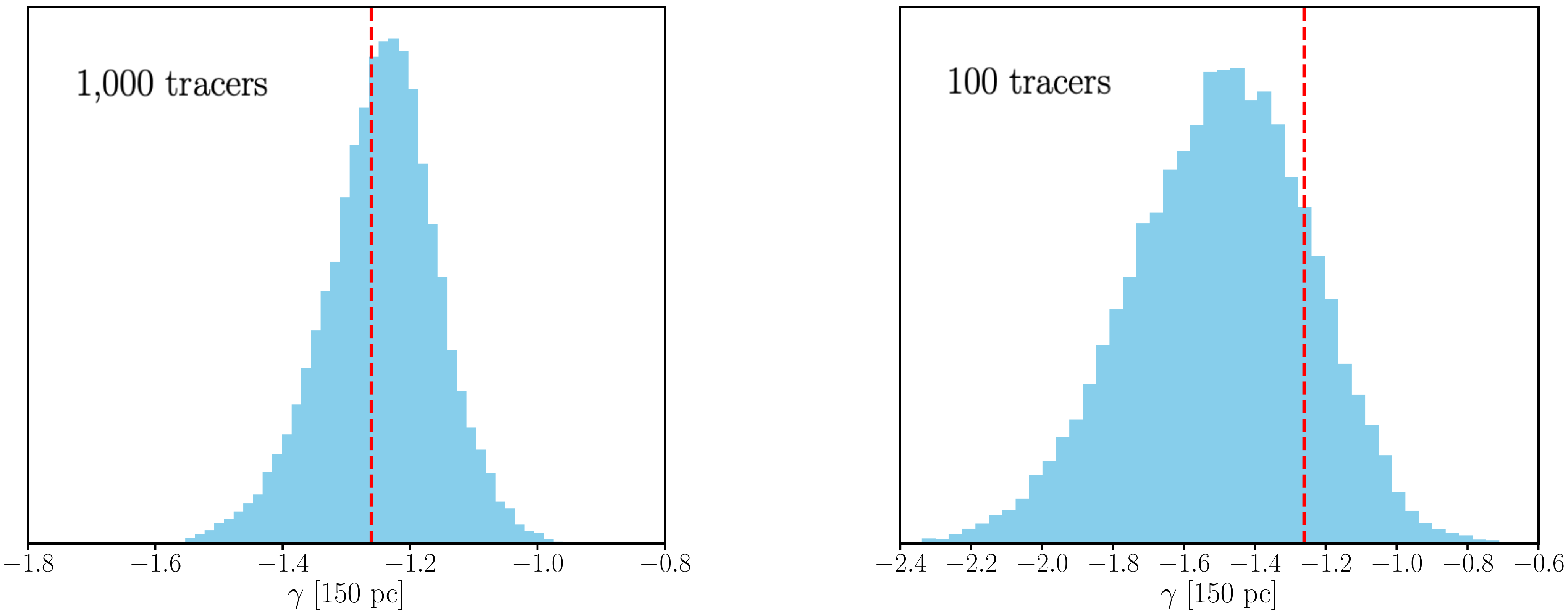


Mock data results: density profiles (cusp)





Mock data results: inner slopes (cusp)



Conclusions / comments

- Developed Gravsphere2, a substantially improved version of the publicly available Jeans solver Gravsphere
- Introduced fourth-order moments in proper motions and general solution to fourth-order Jeans equations
- Tested with mock data, finding substantial improvements in mass profile recovery
- Preliminary but exciting!: Cuspier-than-NFW cusp in Draco + IMBH

Thank you!

Credit: Ève Barlier 2025