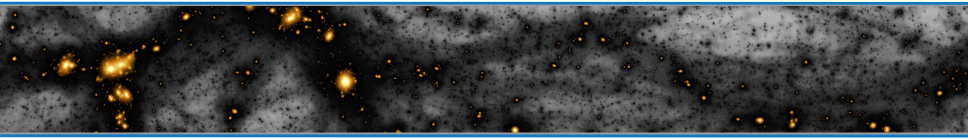
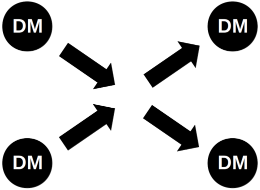


The state of the art of self-interacting dark matter simulations

Moritz S. Fischer (Sternwarte München - LMU)

Self-Interacting Dark Matter

- Self-Interacting Dark Matter (SIDM): Class of particle physics models that assume dark matter to be self-interacting.



- Self-interactions appear to be natural from particle physics.
- SIDM is promising and can solve or at least mitigate small-scale problems.

How Can We Model SIDM?

Approximative:

- Gravothermal fluid model
- Jeans approach

} assumes relaxed halo

From first principles:

- N-body simulations
- and more ...

} computational expensive

Modelling Dark Matter Self-Interactions

- SIDM is neither collisionless (like CDM) nor fully collisional (like a fluid)
- Requires 6D phase-space information
- We have to solve the collisional Vlasov-Poisson / Boltzmann equation:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_x f - \nabla_x \Phi \cdot \nabla_v f = \left(\frac{\partial f}{\partial t} \right)_{\mathrm{coll}}$$

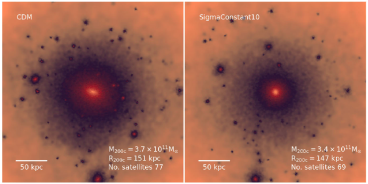
- Self-interactions are described by a collision term

Places to look for SIDM

Cosmological Simulations

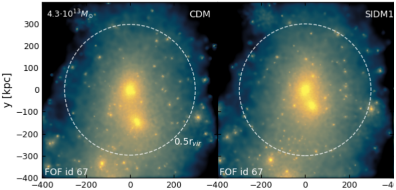
Giulia Despali

TangoSIDM



Correa et al. 2022

AIDA-TNG



Despali et al. 2025

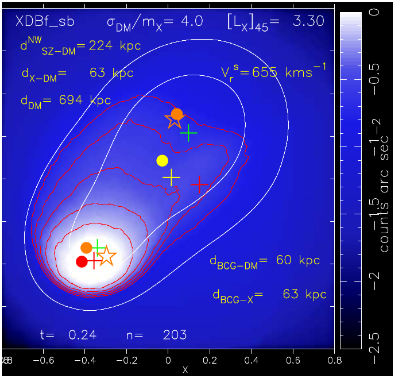
There are many more cosmological simulations, for a full box or as a zoom-in simulation at various mass scales, as DM-only or "full physics" run.

Merging Galaxy Cluster

Riccardo Valdarnini

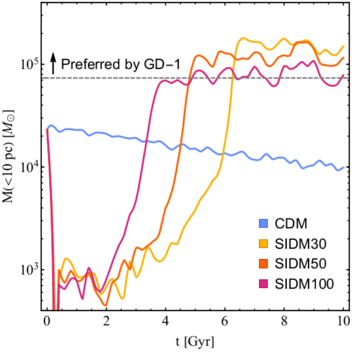
Merging galaxy clusters as probe of SIDM

- Simulations of El Gordo analogues by Riccardo Valdarnini
- More merger simulations by various authors (e.g. Stacy Kim, Andrew Robertson, Moritz Fischer, V. M. Sabarish)

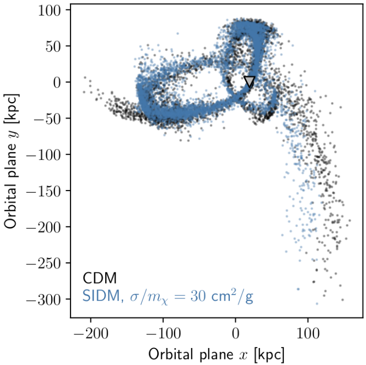


Valdarnini 2024

Stellar Streams as Probes of Dark Matter



Zhang et al. 2025

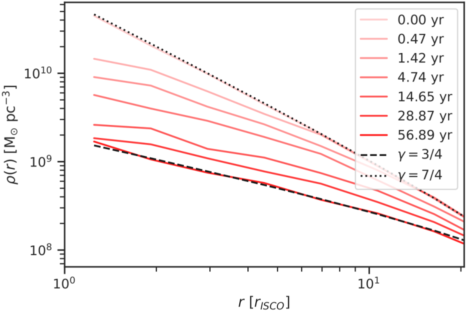


Hainje et al. 2025

Review talks by Ana Bonaca and Ethan Nadler.

SIDM Density Spike

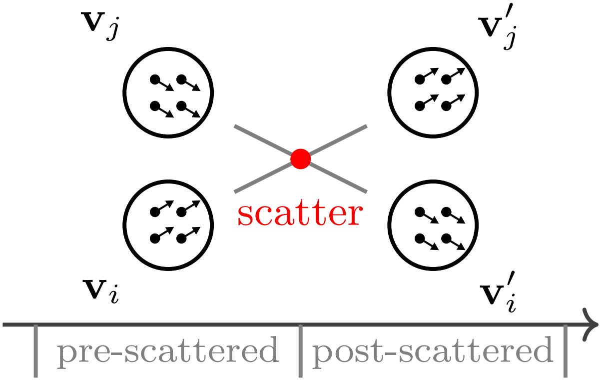
- Black holes might host a DM spike
- They accrete the DM
- Steady-state solution for SIDM spike
- Slope depends on velocity-dependence (Shapiro & Paschalidis 2014)



Sabarish et al. 2025

Numerical schemes for N-body codes

Numerical Scheme for Self-Interacting Dark Matter



A Brief History

- First SIDM N-body simulation by Burkert (2000) based on pairs of particles.
- Improvement by Kochanek & White (2000) using the phase space distribution and nearest neighbours for the kernel size.
- First simulation of the limit of a strongly forward-dominated cross-section by Fischer et al. (2020).
- Further implementations: Koda & Shapiro 2011; Vogelsberger et al. 2012; Rocha et al. 2013; Fry et al. 2015; Robertson et al. 2017; Vogelsberger et al. 2019; Banerjee et al. 2020; Correa et al. 2022; Yang & Yu 2022; Valdarnini 2024; and more.

Monte Carlo Scheme for SIDM

- Interactions of numerical particles are treated analogously to collisions of physical particles
- Probability that two particles interact:

$$P_{ij} = \frac{\sigma(|\Delta \vec{v}_{ij}|)}{m_\chi} \, m \, |\Delta \vec{v}_{ij}| \, \Delta t \, \Lambda_{ij}$$

$\rightarrow$ Impracticable for frequent scattering, because $\Delta t \rightarrow 0$

Geometric Factor

Different formulations for the geometric factor, Λ_{ij}, are used

$$\Lambda_{ij} = W(|\mathrm{x}_j - \mathrm{x}_i|, h_i)$$

(e.g. Vogelsberger et al., Robertson et al., Yang & Yu, Valdarnini)

$$\Lambda_{ij} = \int W(|\mathrm{x} - \mathrm{x}_i|, h_i)\, W(|\mathrm{x} - \mathrm{x}_j|, h_j)\, \mathrm{d}^3\mathrm{x}$$

(e.g. Rocha et al., Banerjee et al., Fischer et al., Correa et al.)

Different kernel functions, W, are employed, e.g. top-hat kernel or spline kernel. They fulfil $1 = \int W(\mathrm{x}, h)\, \mathrm{d}^3\mathrm{x}$, with a kernel size h often set adaptively using the nearest neighbours.

Time step criterion

Aim: Interaction probability should be kept small, $P_{ij} \ll 1$.

- Need estimator for P during next time step.
- P of last time step, or formulation based on local velocity dispersion is used.
- More robust estimator can be constructed when considering the full velocity distribution, with v_e being the velocity for which $v\,\sigma(v)$ becomes maximal.

$$\Delta t_i = \tau \frac{2}{v_\mathrm{e}} \frac{1}{m\,\Lambda_{ii}} \left(\frac{\sigma(v_\mathrm{e})}{m_\chi} \right)^{-1}$$

Λ_{ii} is the maximal possible value for the geometric factor.

Time Integration

Our codes typically use a symplectic leap-frog integrator in the kick-drift-kick (KDK) formulation.

$$\begin{aligned}
&\text{I.} \quad \mathrm{v}_i^{n+1/2} &&= \mathrm{v}_i^n + \mathrm{a}_i^n\,\Delta t_i/2 && \text{(half-step kick)}\\
&\text{II.} \quad \mathrm{x}_i^{n+1} &&= \mathrm{x}_i^n + \mathrm{v}_i^{n+1}\,\Delta t_i && \text{(drift)}\\
&\text{III.} \quad \mathrm{a}_i^{n+1} &&= \mathrm{a}_{\mathrm{grav},ij}^{n+1}(\mathrm{x}_j^{n+1}) && \text{(compute accelerations)}\\
&\text{IV.} \quad \mathrm{v}_i^{n+1} &&= \mathrm{v}_i^{n+1/2} + a_i^{n+1}\,\Delta t_i/2 && \text{(half-step kick)}
\end{aligned}$$

Self-interactions are implemented between different steps: Robertson et al. (I.-II.), Rocha et al. (II.-III.), Fischer et al. (IV.-I.), Correa et al. (II., in between half-step drifts).

Multiple Scatterings

Problem:

- Particles may scatter multiple times per time step
- Energy conservation is violated when using the same velocities (from the beginning of the time step)

Solutions:

- Reduce Δt such that multiple scatters become negligible

  or

- Update velocities after every interaction and ensure execution in a consecutive manner

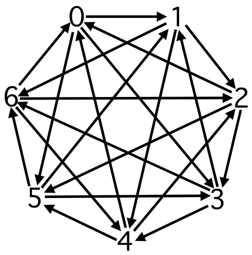
Parallelisation

Message passing interface (MPI)

- distributed memory, communication between processes

Open multi-processing (OpenMP)

- shared memory accessible to multiple threads



Robertson 2017

Problem of multiple scatters for energy conservation is more difficult for parallelised computations. Huge improvements by Robertson et al. 2017, fully resolved by Fischer et al. 2020, Valdarnini 2024.

Tests

How to test?

Aim: Make sure that code produces accurate solution.

Consistency:

- $\lim_{\Delta x, \Delta t \to 0} \{\text{differential eq.} - \text{difference eq.}\} = 0$

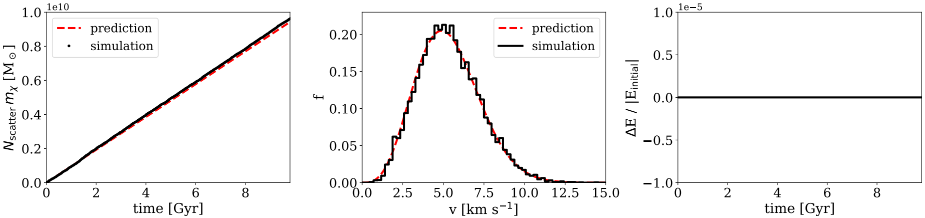
Convergence:

- $\lim_{\Delta x, \Delta t \to 0} \{\text{exact solution} - \text{num. solution}\} = 0$

$\rightarrow$ **Test problem with a known solution necessary!**

Need to test various aspects, e.g. parallelisation, variable time steps, ...

Example: Scattering Rate



- Periodic box without gravity, constant density, velocities follow a Maxwell-Boltzmann distribution, $N = 10^4$.
- Run with MPI and OpenMP parallelisation.

Velocity- and angular dependence

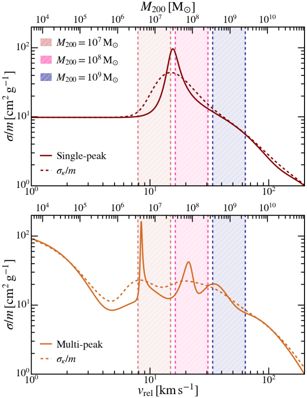
Velocity Dependence

Daniel Gilman

Various velocity-dependencies can be simulated

Including models with resonant scattering

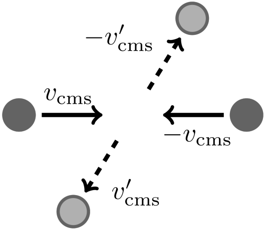
- Simulations by Tran et al. (2024, 2025)
- Time step criterion should be able to handle the resonant peaks



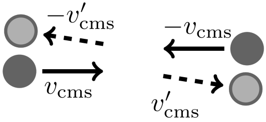
Tran et al. 2025

Angular Dependence

One can distinguish two regimes:



large-angle scattering
– rare –



small-angle scattering
– frequent –

Frequent Self-Interacting Dark Matter (fSIDM)

Collision term can be reformulated (Fischer et al. 2020):

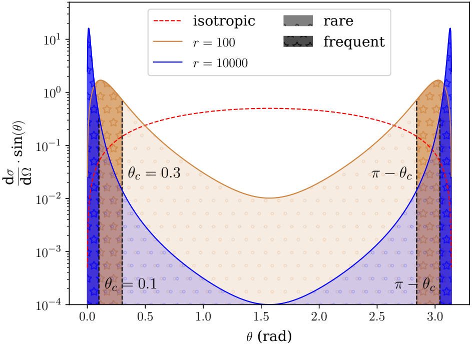
- Interactions of numerical particles are **NOT** treated as collisions of physical particles
- Effective description (drag force) is used for the collision term

$$F_{\mathrm{drag}} = \frac{1}{2} \frac{\sigma_{\mathrm{T}}(|\Delta \vec{v}_{ij}|)}{m_\chi} m^2 \, |\Delta \vec{v}_{ij}|^2 \, \Lambda_{ij}$$

- If numerical particles are close, they interact (no probability)

Combining Schemes of rSIDM and fSIDM

- Realistic cross-section may have small and large-angle scattering
- θ_c to distinguish small and large angles
- fSIDM scheme for $\theta < \theta_c$
- rSIDM scheme for $\theta > \theta_c$



Arido et al. 2025

Gravothermal collapse

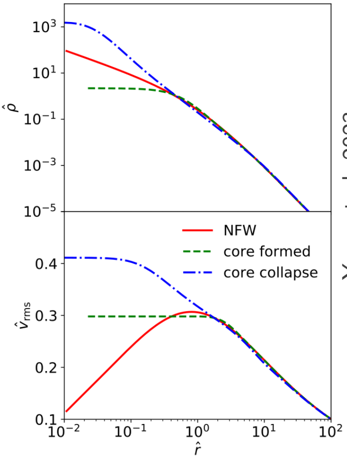
The Gravothermal Collapse of SIDM Halos

1. **Core formation phase:**

   Heat flows to the centre and decreases the density.

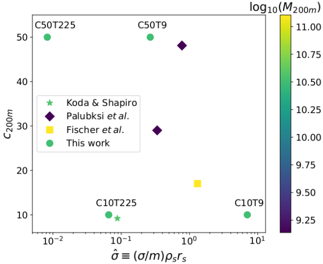
2. **Collapse phase:**

   Heat is flowing outwards only, driving the collapse.

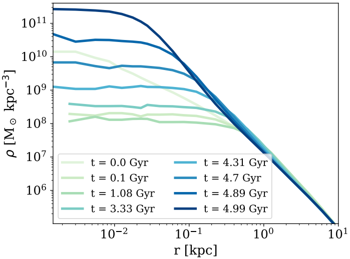


Yang et al. 2023

Collapse Simulations



Mace et al. 2024



Fischer et al. 2025b

Several studies of numerical properties (e.g. Mace et al., Palubski et al., Fischer et al.)

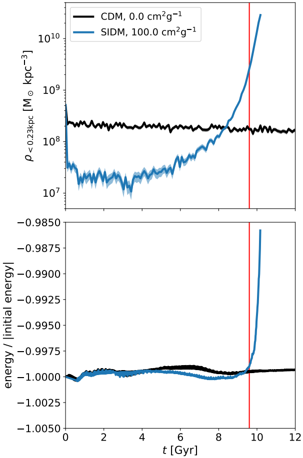
Isolate NFW halo with $N = 5 \times 10^7$ particles, data is public

There is more to come, see talk by Frank van den Bosch.

Numerical Challenge: SIDM Collapse

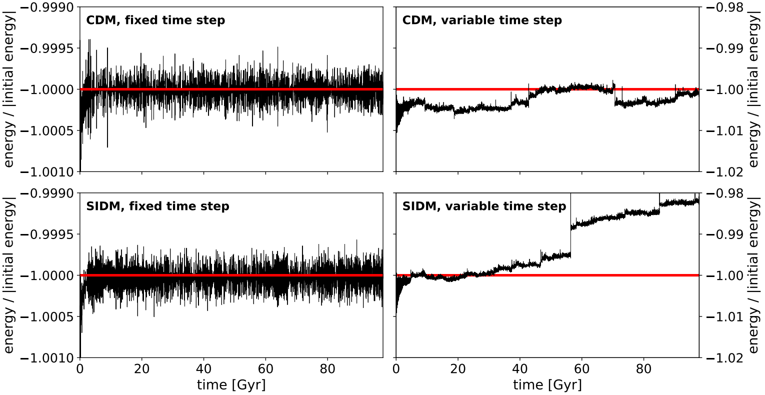
Difficult to conserve total energy:

- Such dense objects are challenging even without SIDM.
- Time reversibility is lost when particles change time step.
- Asymmetric gravitational force evaluation can lead to increase in total energy.



Fischer et al. 2024b

Numerical Challenge: Energy Drift



Fischer et al. 2024b

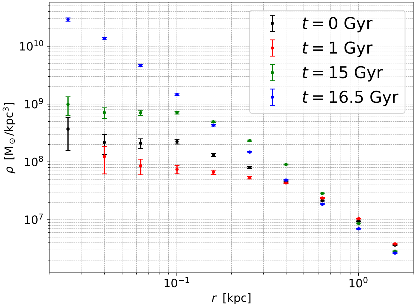
Alternatives and extensions to traditional N-body codes

Exploiting spherical symmetries

Oren Slone

Motivation: Study gravothermal collapse in spherical symmetric halo

- Can use symmetries:
  $\rightarrow$ 6D phase space reduces to 3D problem
- *N* particles in spherical shells
- NSphere code (Kamionkowski et al. 2025)



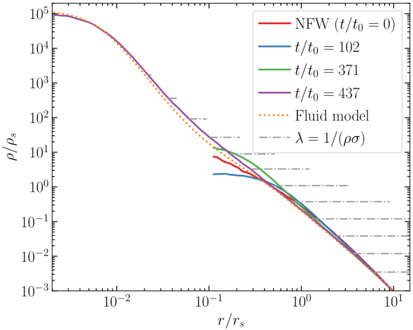
Kamionkowski et al. 2025

Direct Simulation Monte Carlo

James Gurian

Idea: Use Direct Simulation Monte Carlo (DSMC) methods for SIDM

- Particles in cells are scattered, simpler "neighbour" search, no kernel function
- First simulations for NFW halo by Gurian & May (2025), 3D phase space



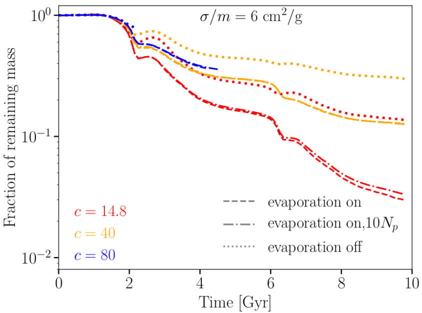
Gurian & May 2025

Satellites and Evaporation

Annika Peter

Problem: Simulating satellites while resolving the host is expensive

- Describe the host potential analytically
- Additional interactions for satellite-host scattering (evaporation)
- Semi-analytic description of dynamical friction

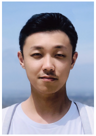


Zeng et al. 2021

Simulating various SIDM models

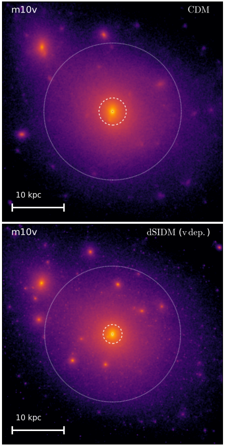
Simulations of Dissipative Interactions

Xuejian Shen



DM could have dissipative processes

- Simulation of a simple model by Shen et al. (2022, 2024), fraction of E_{kin} is lost when particles interact
- Simulations with a threshold velocity by Huo et al. (2019)
- Atomic dark matter simulations by Roy et al. (2023), subcomponent can be described as a fluid

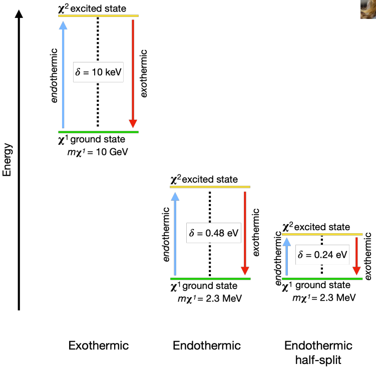


Shen et al. 2021

Simulations with Excited States

Stephanie O'Neil

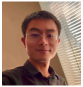
- Motivated by particle physics (e.g. Schutz & Slayter 2015, Medvedev 2014)
- Implementation in N-body codes (e.g. Vogelsberger et al. 2019)
- Rich phenomenology (O'Neil et al. 2023, Leonard et al. 2024, Low et al. 2025)



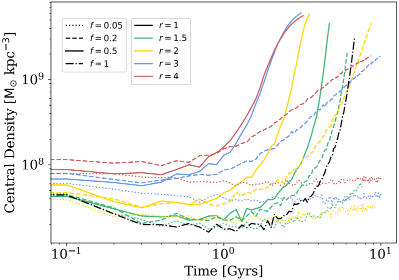
O'Neil et al. 2023

Unequal-Mass Scattering

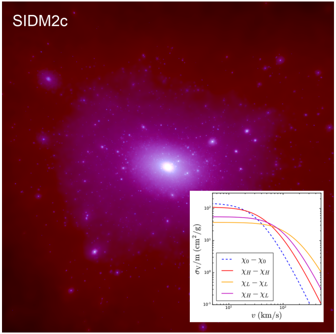
Daneng Yang



Simulations for a two-species model with unequal masses



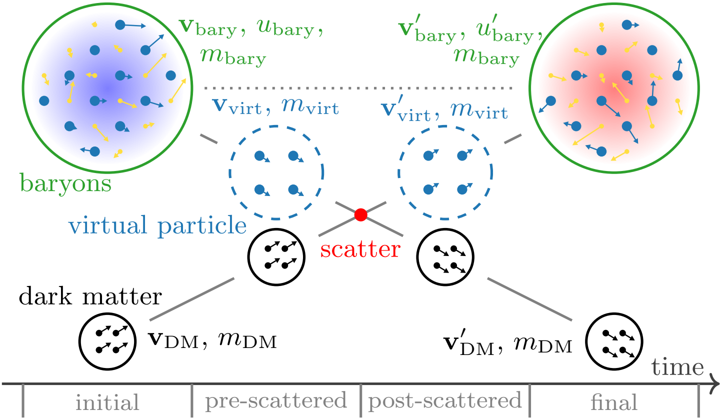
Patil & Fischer 2025



Yang et al. 2025

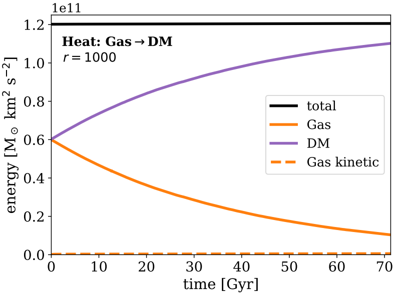
Scattering beyond self-interactions

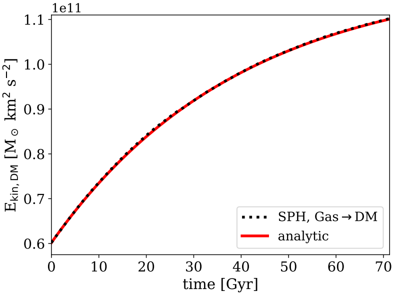
Dark Matter–Baryon Scattering



Fischer et al. 2025a

Heat Conduction – Unequal Masses





Fischer et al. 2025a

- Simulation with unequal masses (1:1000) and forward-dominated cross-section

Summary

1. **Tests are important**
   - Of all parts of the code!
   - Convergence against any solution is not enough
2. **Gravothermal collapse is challenging to simulate**
   - Because of computational costs
   - Exploiting symmetries is promising
3. **We "can" simulate many interesting models**
   - Multi-state models and dissipative interactions
   - Unequal-mass scattering (in multi-component models)
   - Models with DM-baryon interactions

Backup Slides

Adaptive Gravitational Softening

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\,\mathrm{G} \sum_{j=1}^{N} m_j \left[\frac{\Phi'_{ij}(h_j) + \Phi'_{ij}(h_i)}{2} \right] \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

$$- \frac{\mathrm{G}}{2} \sum_{j=1}^{N} m_j \left[\frac{\zeta_i}{\Omega_i} \frac{\partial W_{ij}(h_i)}{\partial \mathbf{r}_i} + \frac{\zeta_j}{\Omega_j} \frac{\partial W_{ij}(h_j)}{\partial \mathbf{r}_i} \right]$$

$$\zeta_i \equiv \frac{\partial h_i}{\partial \rho_i} \sum_{k=0}^{N} m_k \frac{\partial \Phi_{ik}(h_i)}{\partial h_i}$$

$$\Omega_i \equiv 1 - \frac{\partial h_i}{\partial \rho_i} \sum_{k=0}^{N} m_k \frac{\partial W_{ik}(h_i)}{\partial h_i}$$

(Price & Monaghan 2007; Springel 2010; Barnes 2012; Iannuzzi & Dolag 2011; Hopkins et al. 2023)