PN-pairing with Gogny interaction and Taurus code

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Acknowledgements

Tomás R. Rodríguez (UCM)
Luis. M. Robledo (UAM)
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Samuel. A. Giuliani (UAM)

Review of proton-neutron pairing

- Static correlations in nuclei : formation of Cooper pairs for like-particles $(T = 1, M_T = \pm 1)$.
- Pairs depend on the overlap, $Z \approx N$ nuclei requires extending the theory for *pn* pairs.
- *pn* pairs can be **isoscalar** $(T = 0, J = 1)$ and **isovector** $(T = 1, M_T = 0; J = 0)$.
- Current experimental facilities proved isovector effect on rotational bands. [1]
- Proposed to explain the origin of Wigner energy for isospin dependence in masses, outcomes in rotating nuclei, ... [2]

1. C.D.O'Leary *et al*, *Phys.Rev.C 67, 021301(R)* (2003)
2. A.V.Afanasjev, *Fifty Years of Nuclear BCS, pp.138-153* (2013)

Self-consistent Mean Field techniques, HFB method.

- The Hartree-Fock-Bogoliubov method is a self-consistent mean-field approach based on the variational method.
- The wave function is built by the product of Bogoliubov quasiparticles β

$$\beta_k^\dagger = \sum_l U_{lk} c_l^\dagger + V_{lk} c_l$$

$$U = \begin{pmatrix} U_{pp} & U_{pn} \\ U_{np} & U_{nn} \end{pmatrix} \qquad V = \begin{pmatrix} V_{pp} & V_{pn} \\ V_{np} & V_{nn} \end{pmatrix}$$

- HFB (and constrained-HFB) equations are solved with the gradient method

$$\delta \frac{\langle \Phi(a_i)|H|\Phi(a_i)\rangle}{\langle \Phi(a_i)|\Phi(a_i)\rangle} = \delta_{U,V}(E_{HFB}) = 0$$

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- The general HFB solutions normally break the Hamiltonian symmetries.

N, Z (particle number conservation)	Pairing condensate
J, J_z (rotational invariance)	Axial deformations
P (parity invariance)	Octupolar deformations
$\mathcal{T}$ (time-reversal invariance)	Cranking or odd-mass nuclei

EDF vs TAURUS code for HFB.

- Current EDF solvers are highly optimized to the underlying interaction, usually by assuming symmetry restrictions.
- TAURUS is a versatile code to solve the nuclear *many-body* problem **with almost no symmetry restrictions** [3] [4].
- It requires providing any generic two-body Hamiltonian for any nuclear interaction, via reduced matrix elements $\langle \hat{a}\hat{b} \,||V||\, \hat{c}\hat{d} \rangle_J$

3. B.Bally, T.R.Rodríguez, A.Sánchez-Fernández *The European Physical Journal A volume 57, Article number : 69*(2021)
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Advantages of TAURUS

Properties that are hard or impossible to compute with current EDF solvers.

- Freedom for the working basis,
  It can do both valence space and ***no-core*** **calculations**.
- ***proton-neutron*** **mixing in the wave function**.
- Exotic deformations : octupolar, hexadecupolar and triaxiality in-between.
- Odd nuclei, quasiparticle excitations, projections, etc.

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Gogny EDF

- The Gogny interaction is a phenomenological finite range force with a density-dependent term.

$$\hat{v}^{no\ dd}(\mathbf{r}_{12}, \sigma_{12}, \tau_{12}) + \ C_{dd}\ \delta(\mathbf{r}_1 - \mathbf{r}_2)\ \rho^{\alpha}\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right)$$

- The family of D1 interactions includes a finite range part, letting the treatment of the pairing and HF together (*ph* and *pp* parts).
- In this work we use D1S parametrization [5], (good description of nuclear properties involving pairing).
- Fitted without accounting for the *pn* mixing.

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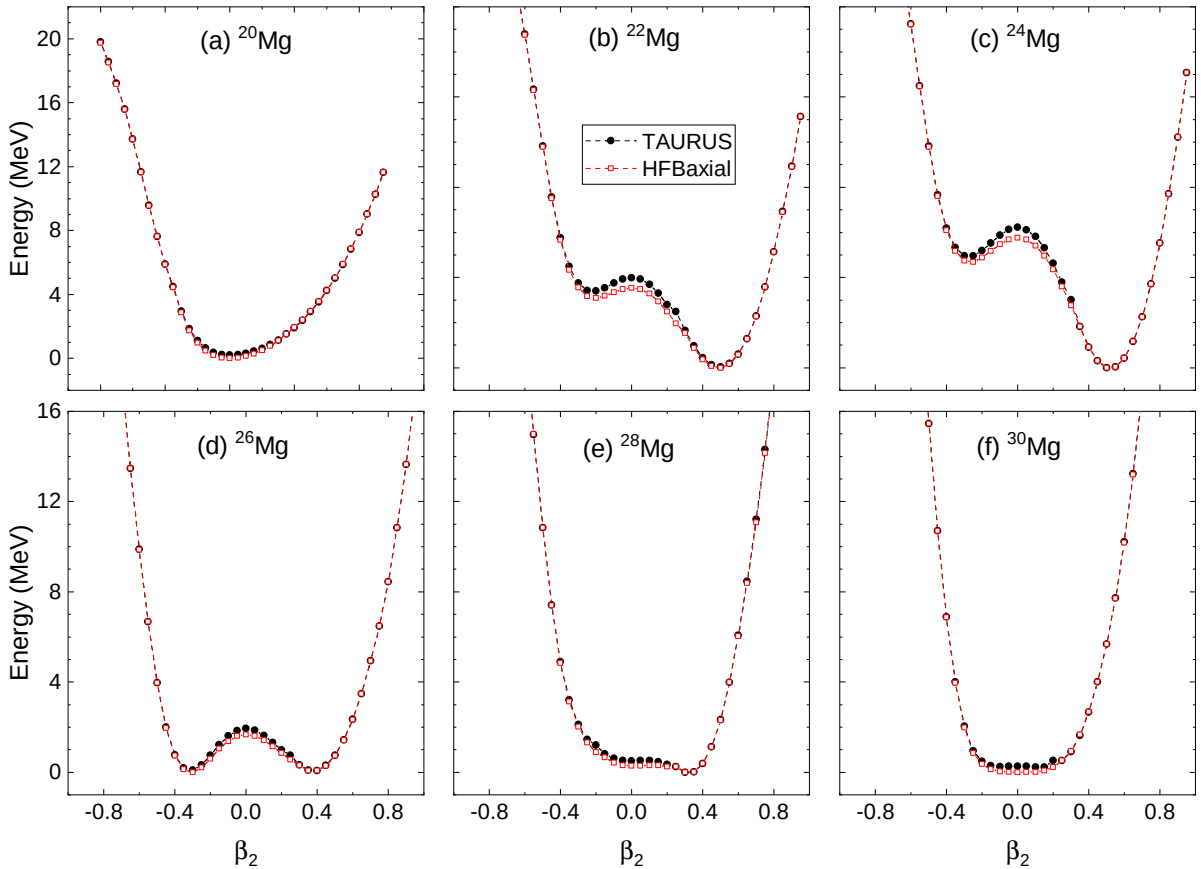
Benchmarking, HFBaxial code.

- Validation with the axial HFB code [7] (D1S interaction implemented).
- Mg energy deformation surfaces $E_{HFB}(\beta_{20})$



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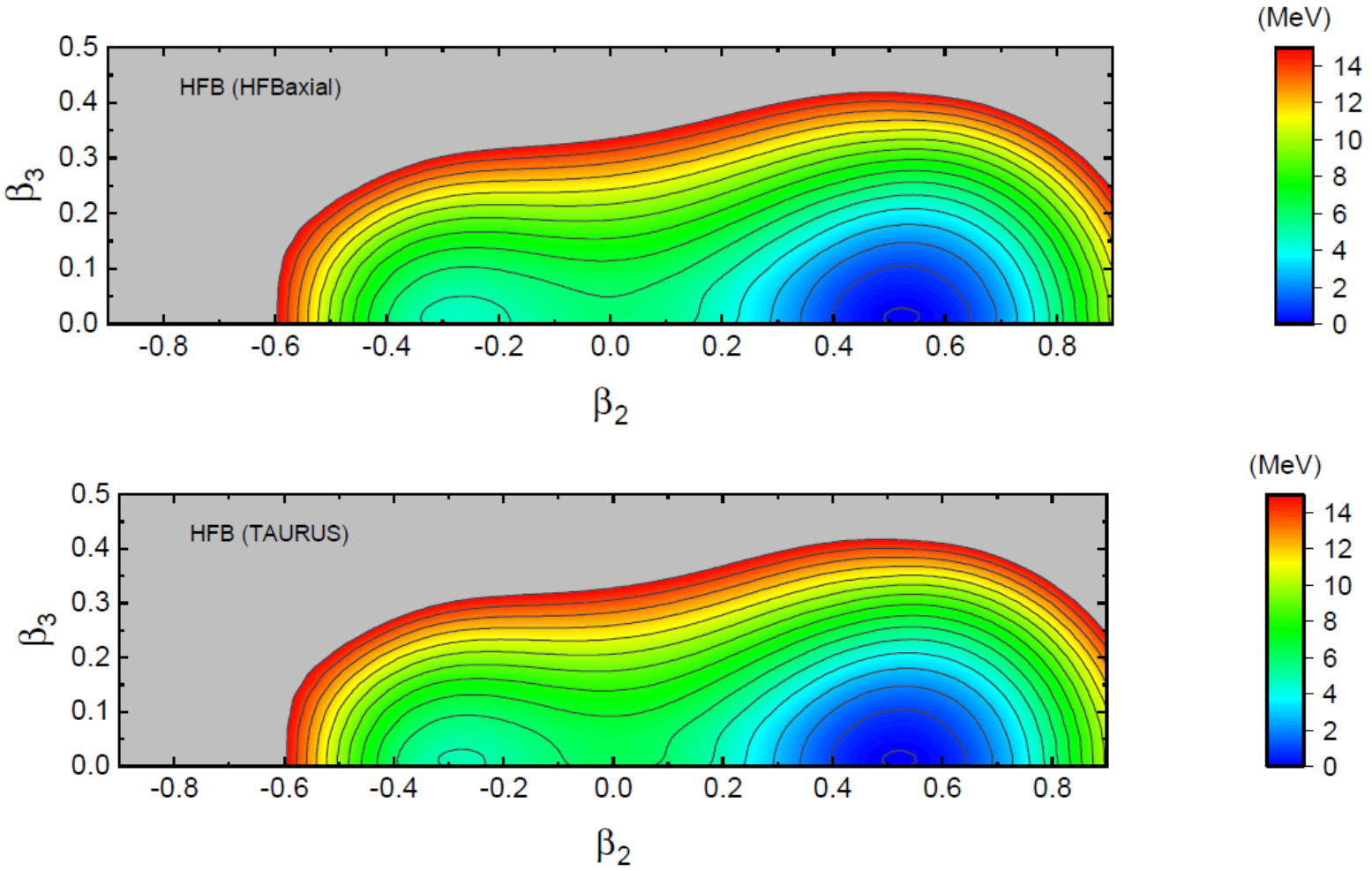

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Testing behavior with *pn*-mixing channels

Pair coupling operator

$$\delta^{JT}_{M_J M_T} = \langle \Phi | \frac{1}{2} \left([P^\dagger] + [P] \right) | \Phi \rangle$$

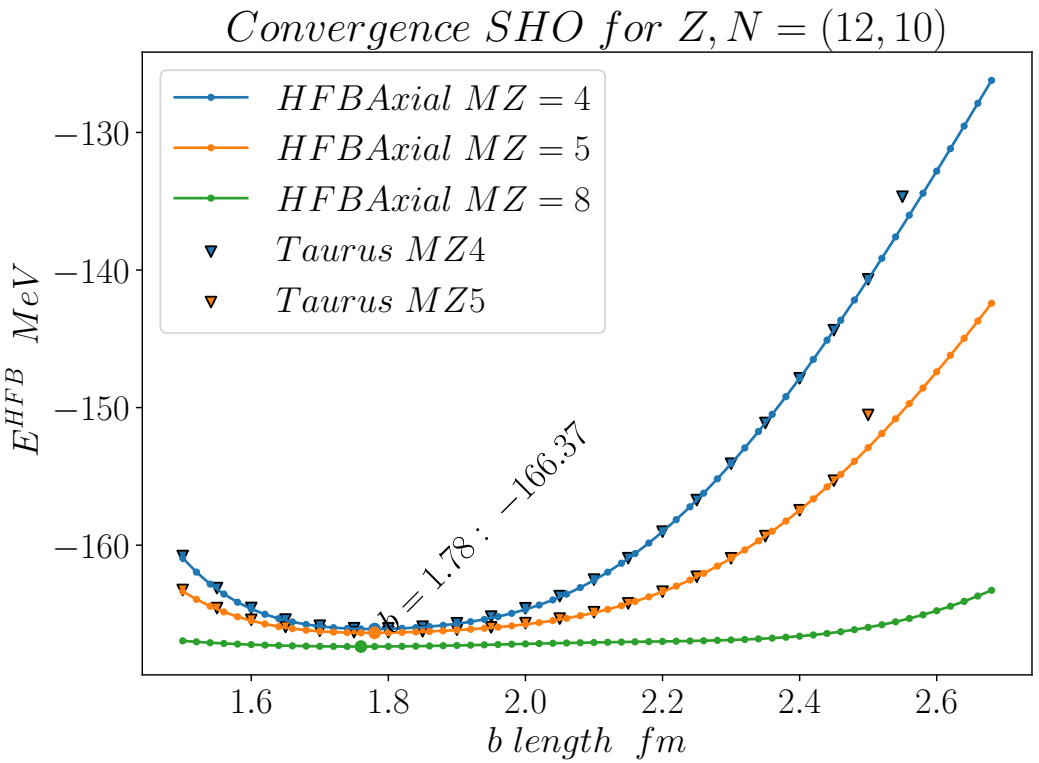
$$[P^\dagger]^{JT}_{M_J M_T} = \frac{1}{\sqrt{2}} \sum_a \sqrt{2j_a + 1} \, [c^\dagger_a c^\dagger_a]^{JT}_{M_J M_T}$$

What we expect ?

- Overall stability under the original D1S interaction.
- Behavior of the energy surfaces.
- Not having isovector/isoscalar minima.

Pairing Calculations

- No core calculation (COM, $\langle Q_{10} \rangle = 0$, Coulomb *on*), HO Shells$= 4,\ 5$
- Optimal oscillator length.
- Getting the unconstrained minimum.
- Constraint each $\delta^{TM_T}_{JM_J}$ from the minimum, while leaving the other δ and $\beta_{\lambda\mu}$ free



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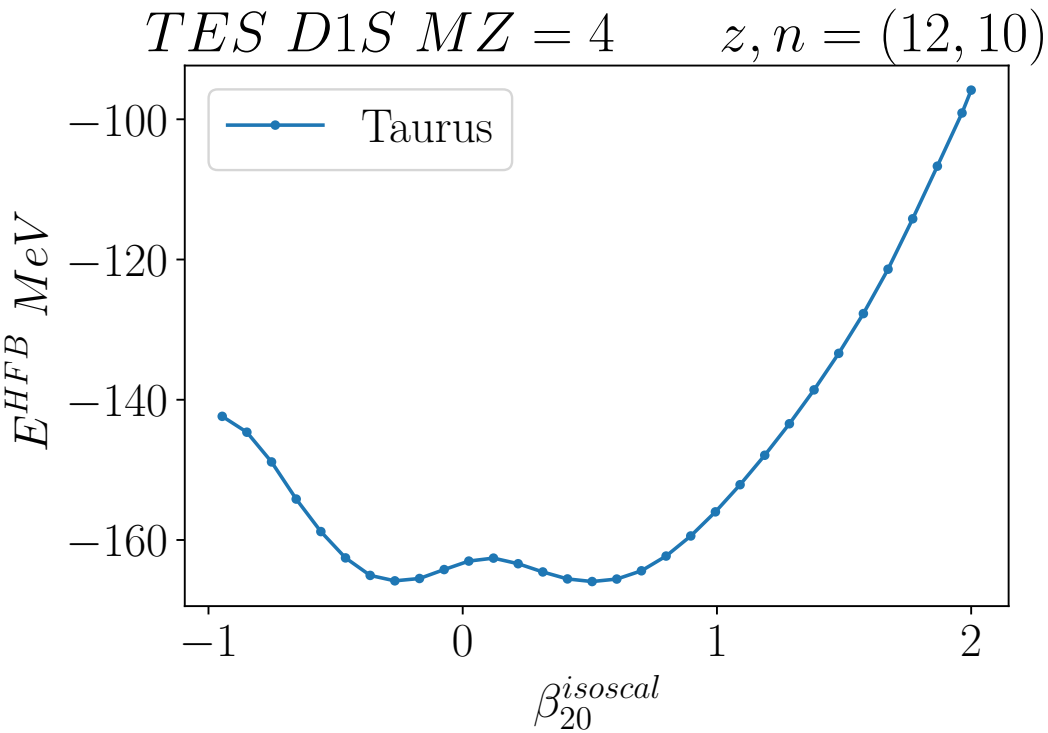
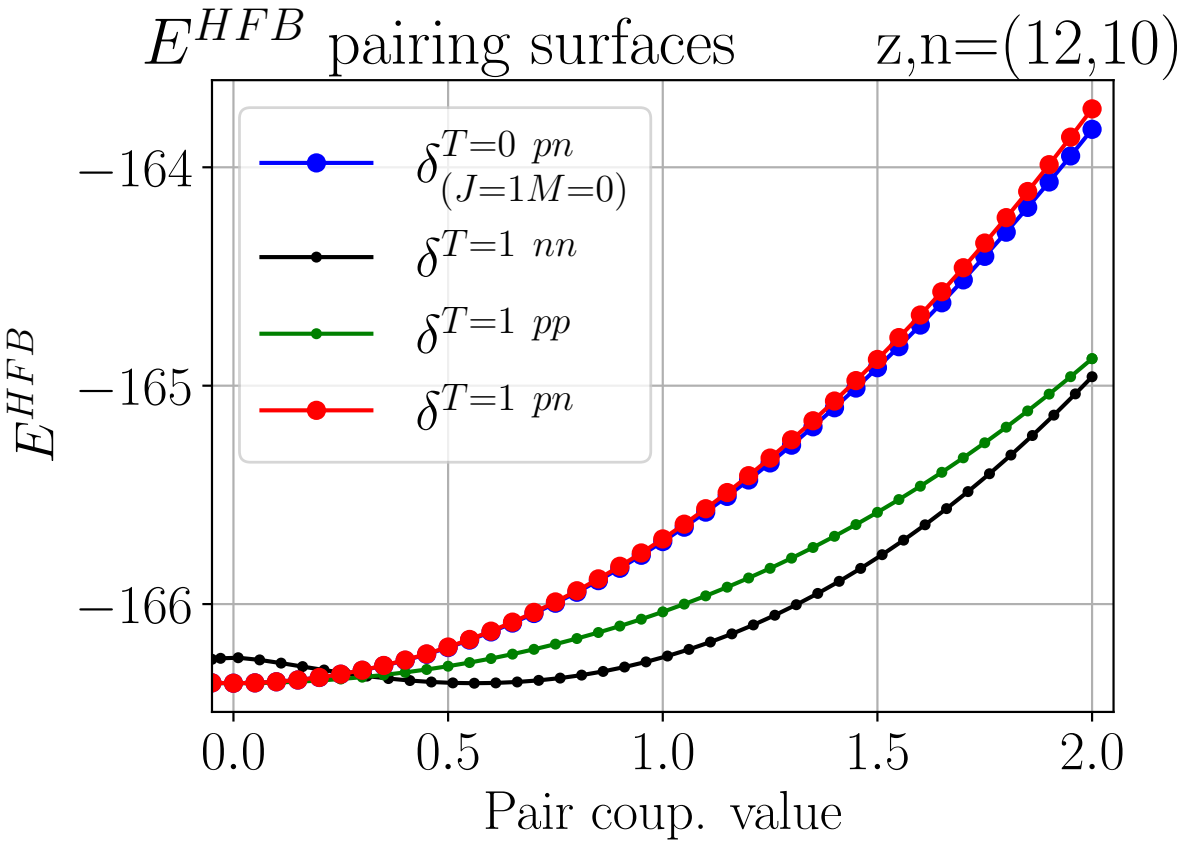


Analysis for Mg isotopes ($MZ_{max} = 5$, D1S)

General $N \neq Z$ case :

- E^{HFB} (isovector-pn/isoscalar) $> E^{HFB}$(pp/nn)
- E^{pair} (isovector-pn/isoscalar) $> E^{pair}$(pp/nn)



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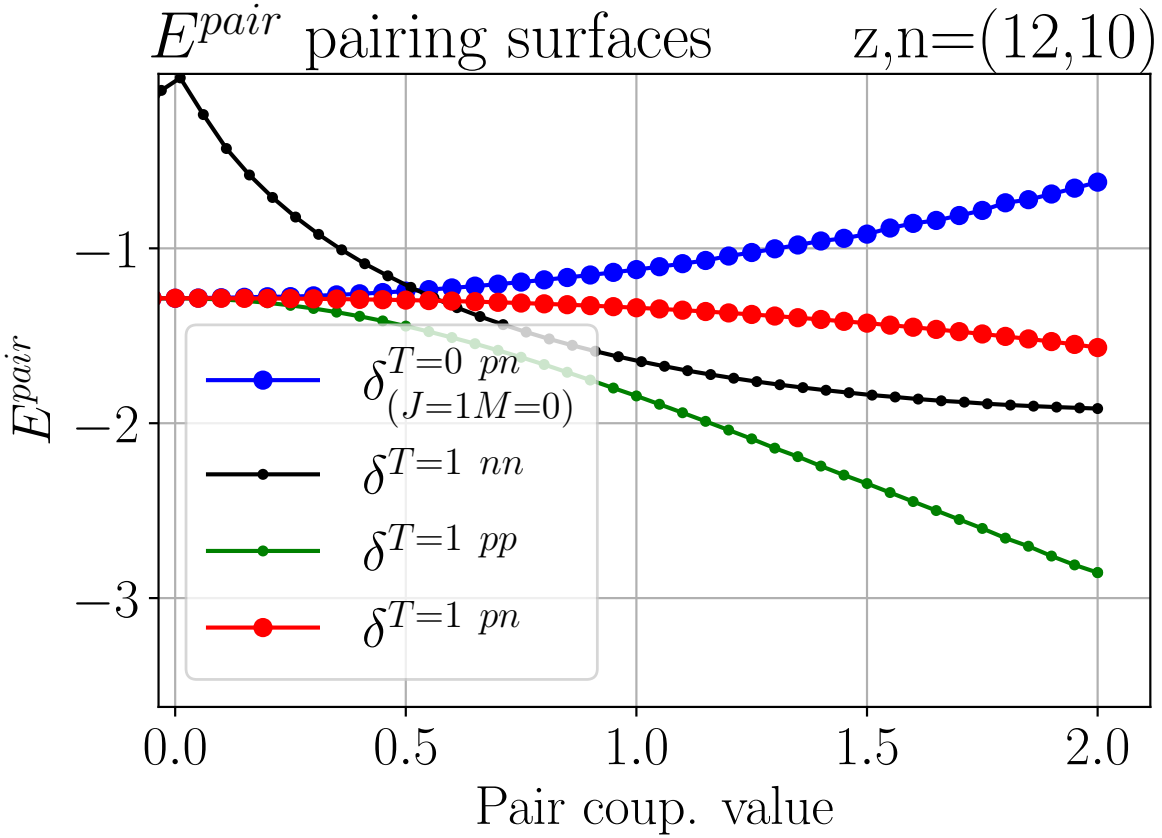

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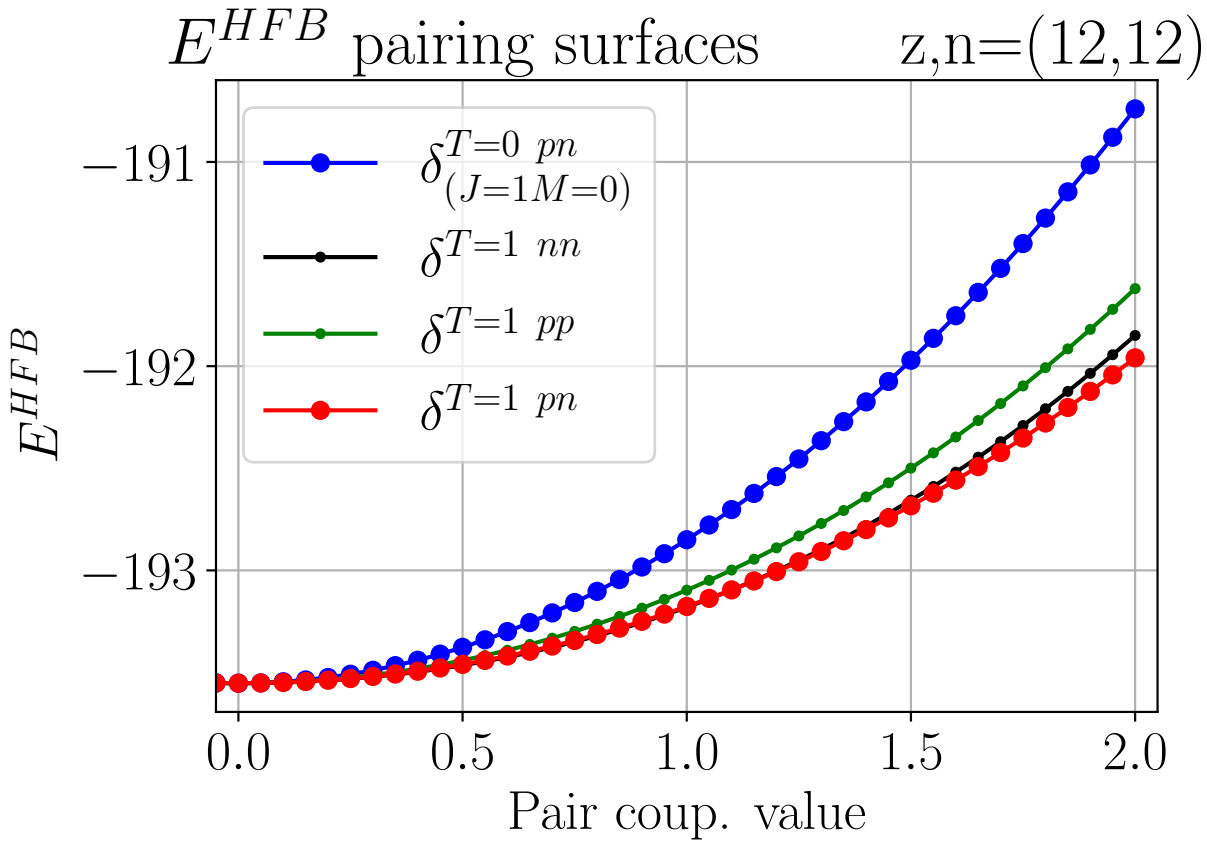

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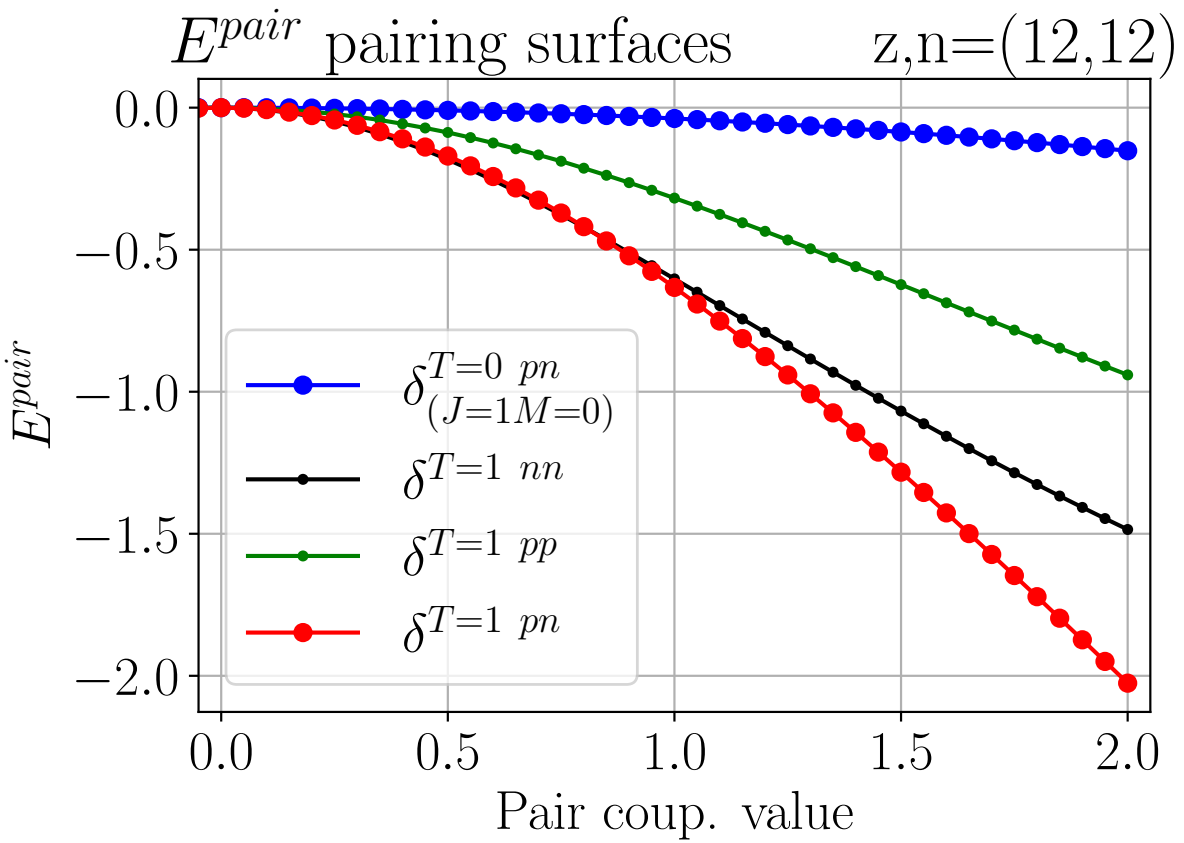

Analysis for even-even $N = Z$ nuclei ($MZ_{max} = 5$, D1S)

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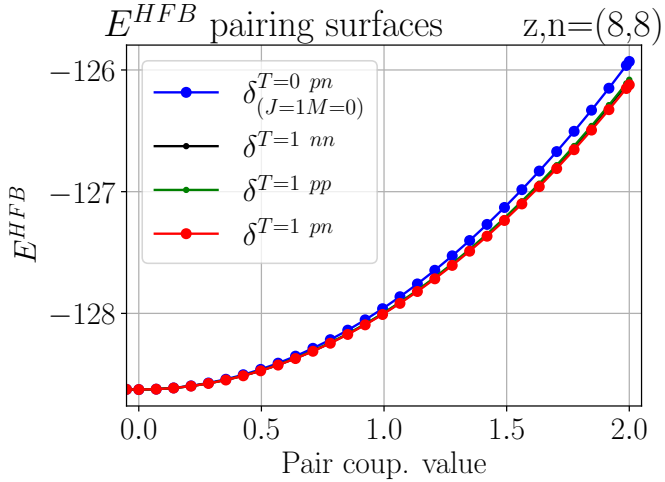

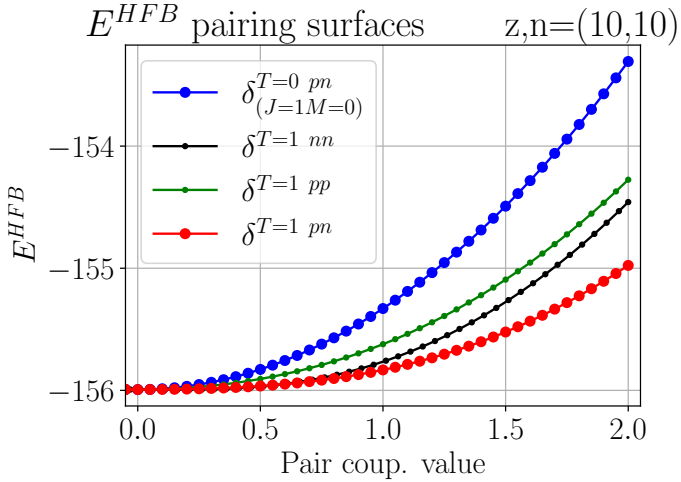

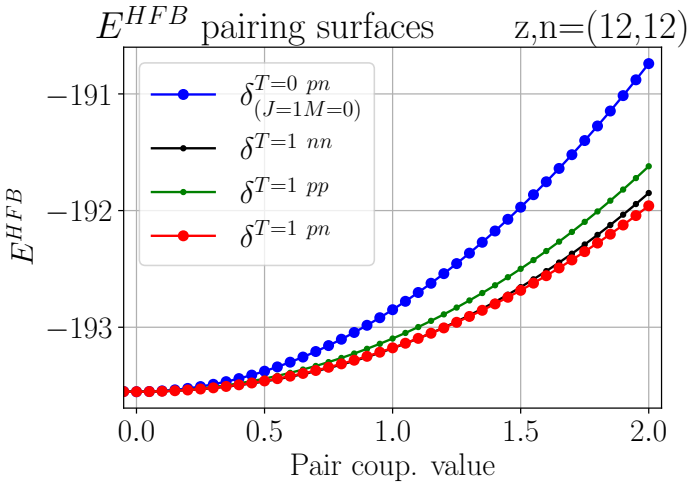

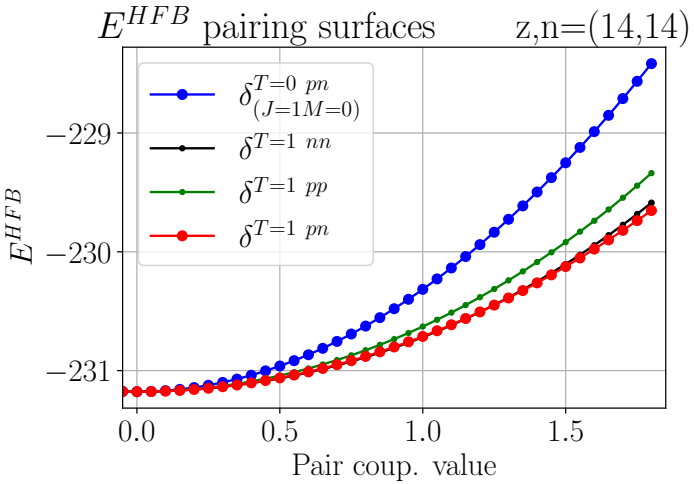

Ultraviolet Divergence and alternatives.

Divergence in the DD-pairing contribution.

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- Preliminary analysis shows E_{pn}^{pair} is also divergent even after removing the *pn*-DD part.

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- Applying a cutoff over unphysical divergent states [8].

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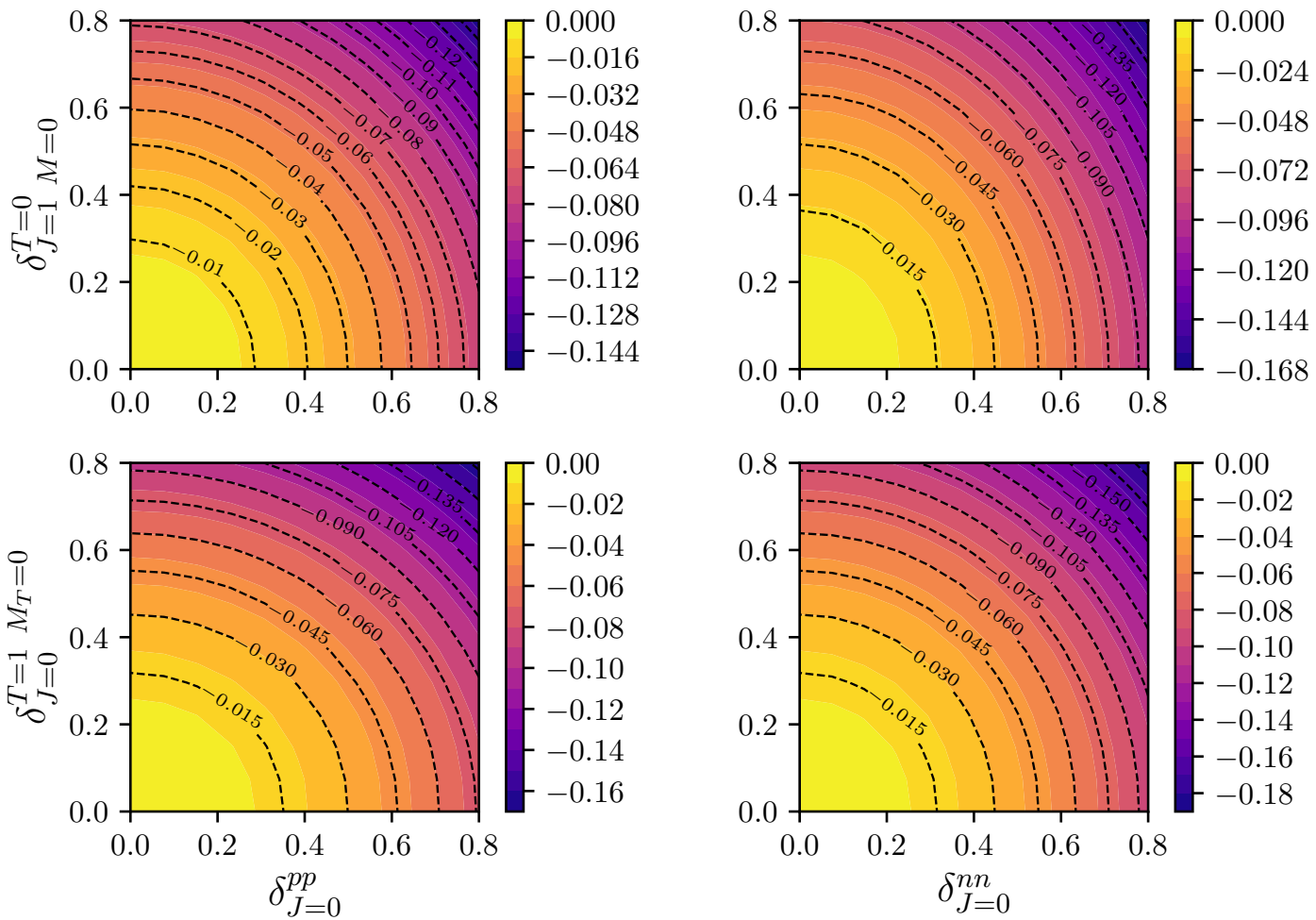
Similar trends : E^{pair} (isovector-pn) dominant for $N \approx Z$ case. E^{pair}(pp/nn) elsewhere.

$E^{(total)}_{pair}$ $B1^{(MZ5)}$ ^{24}Mg



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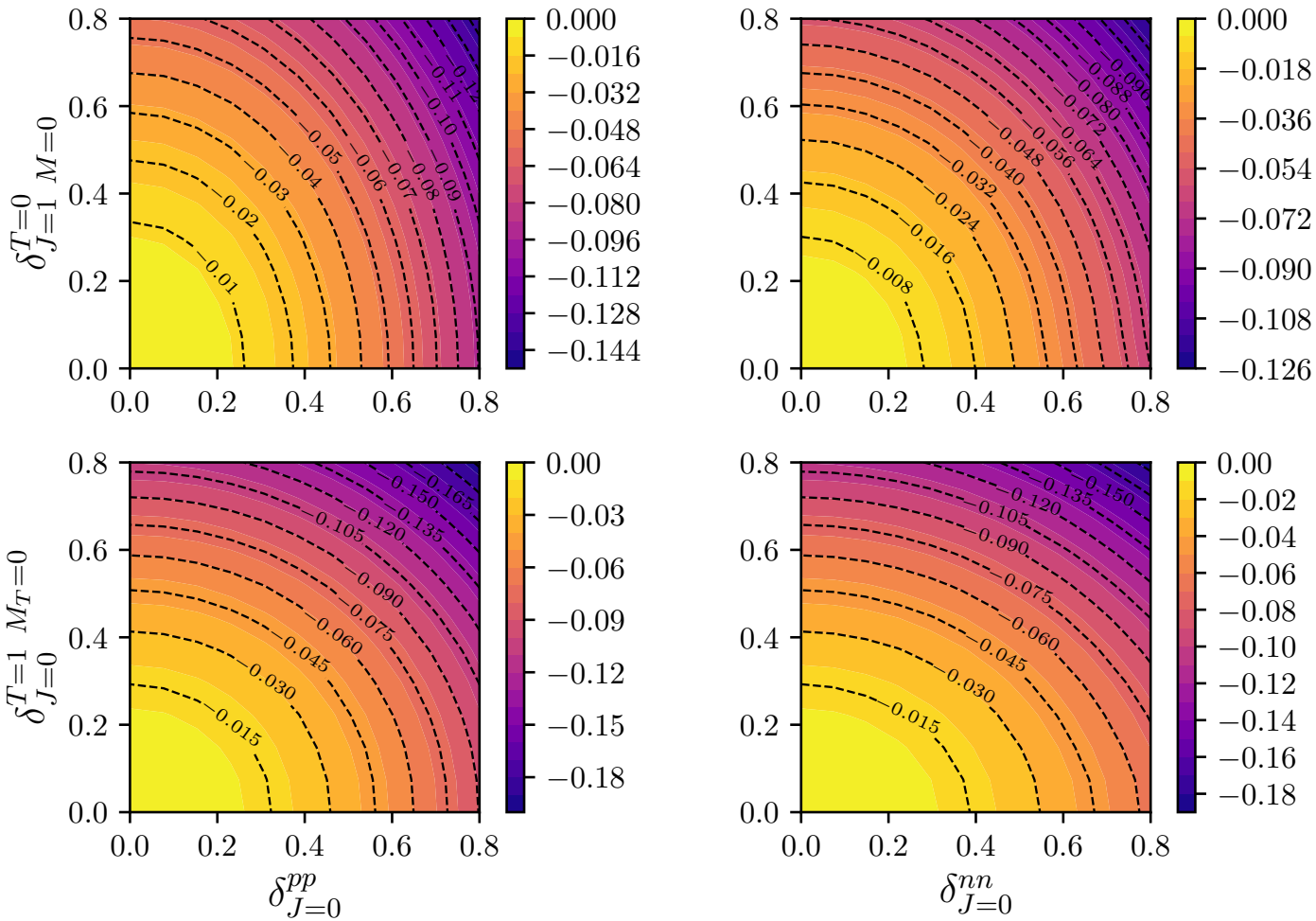

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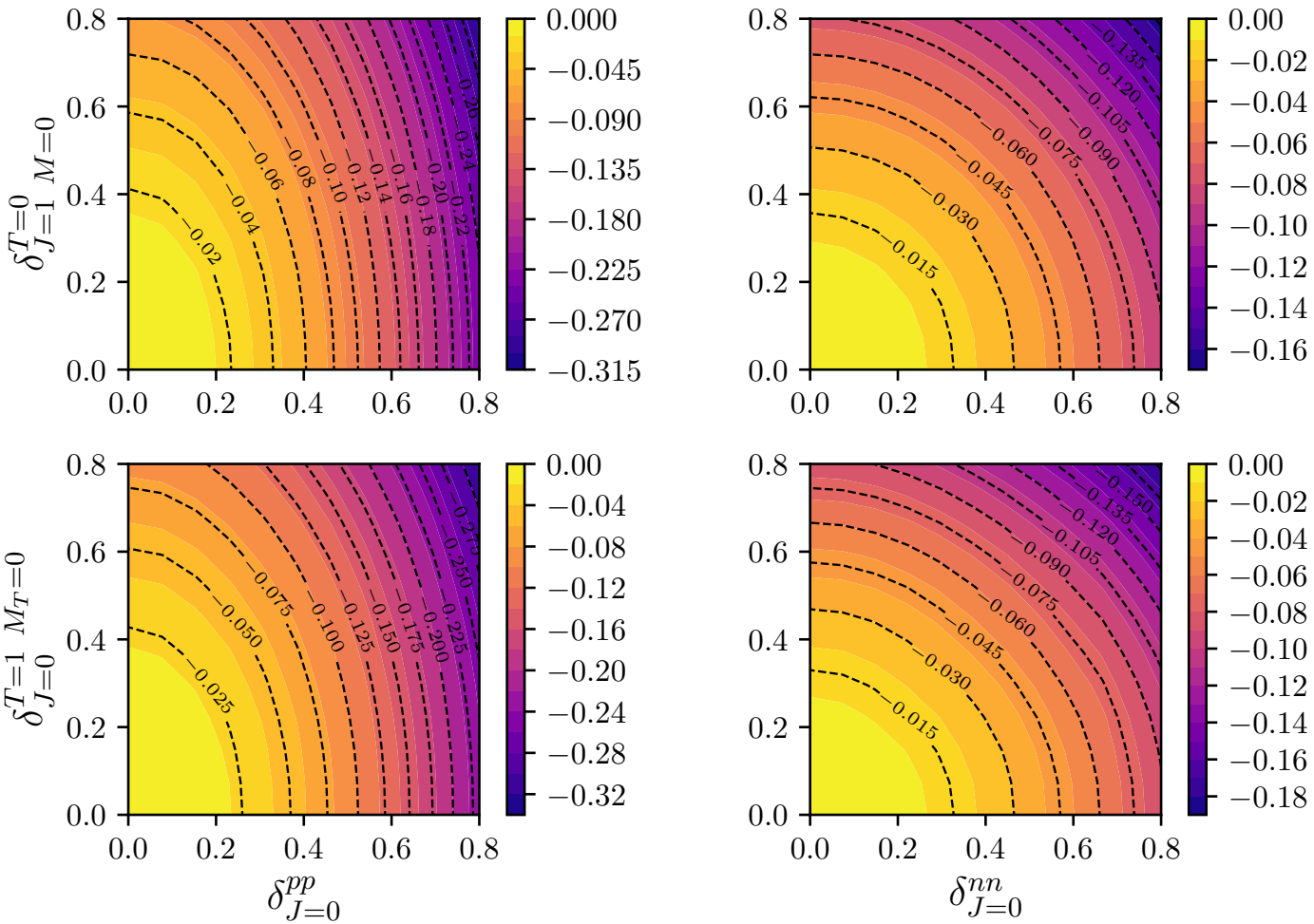

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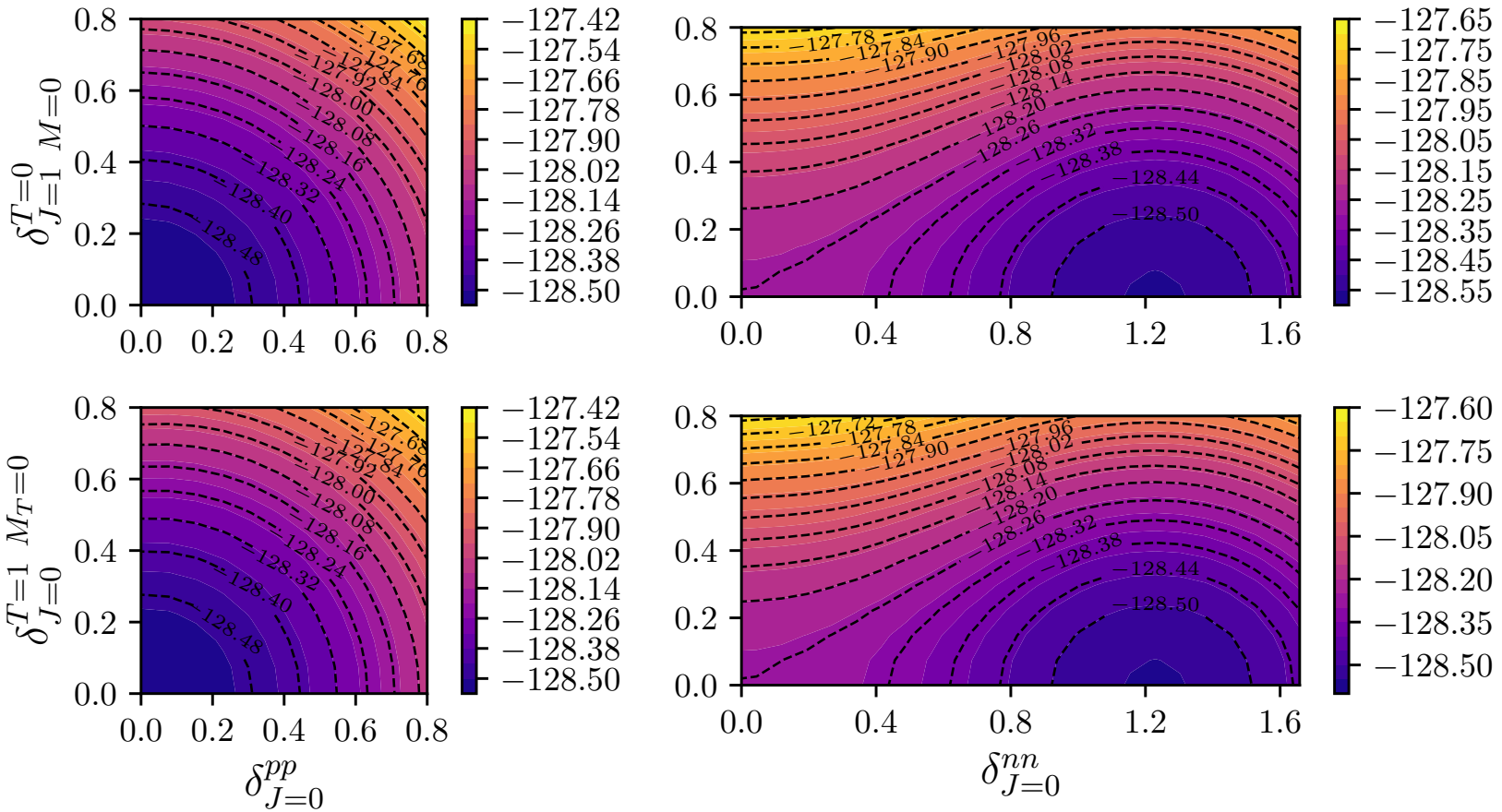

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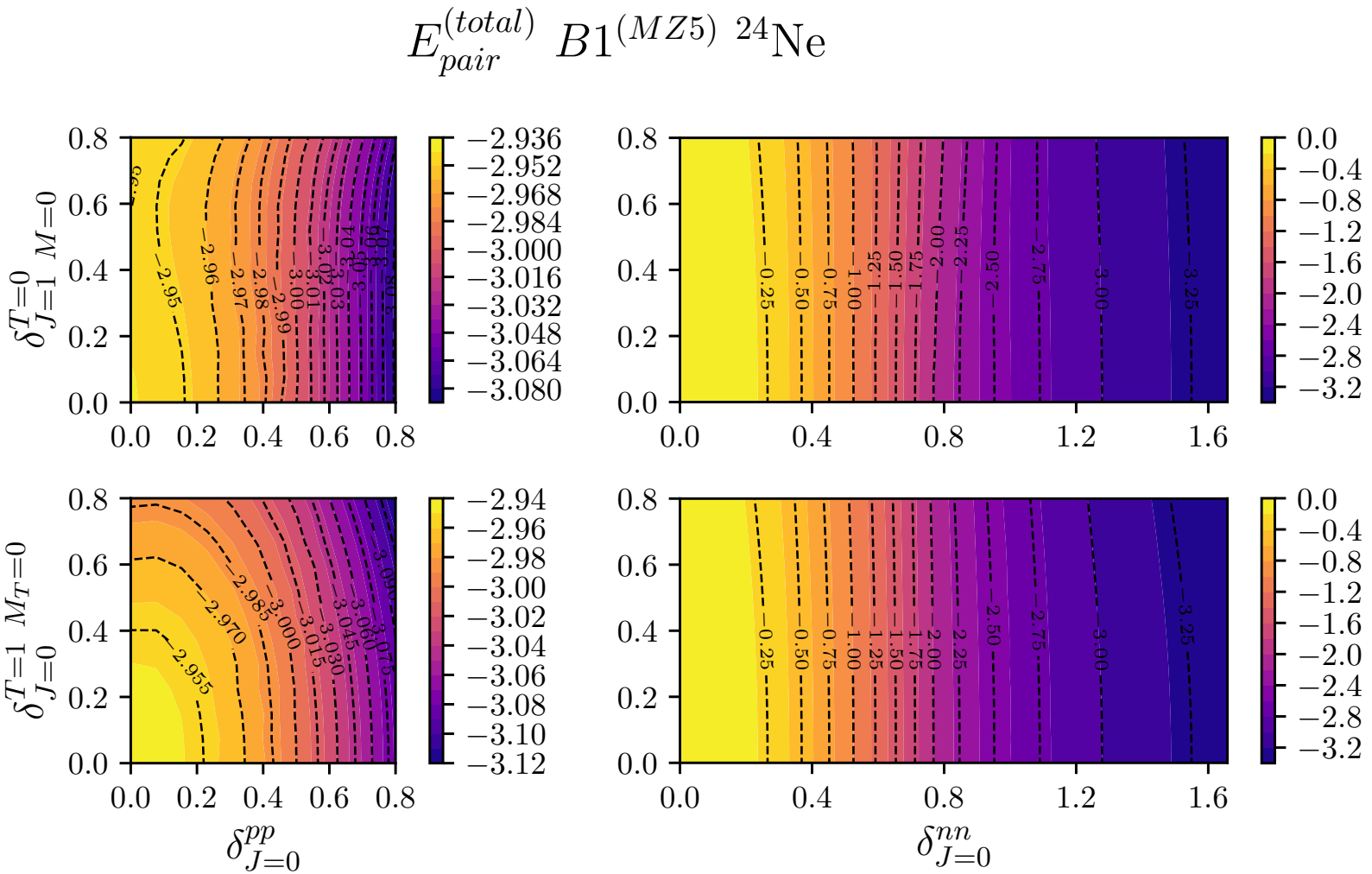

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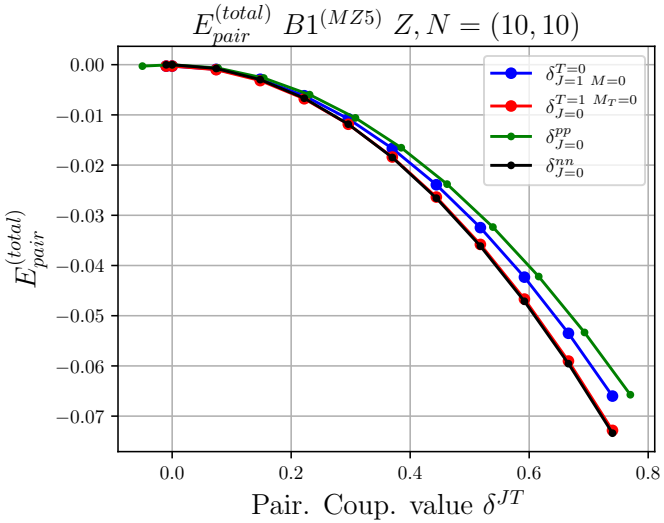

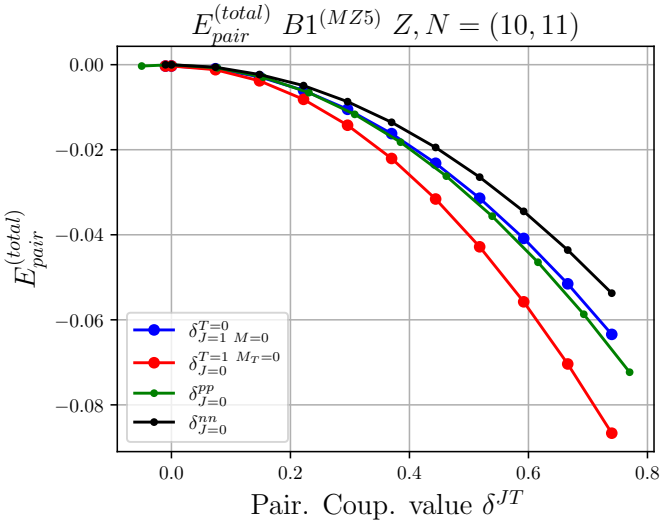

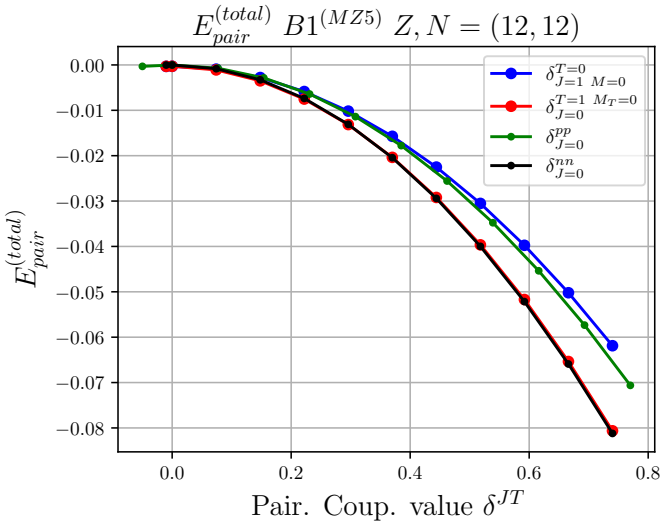

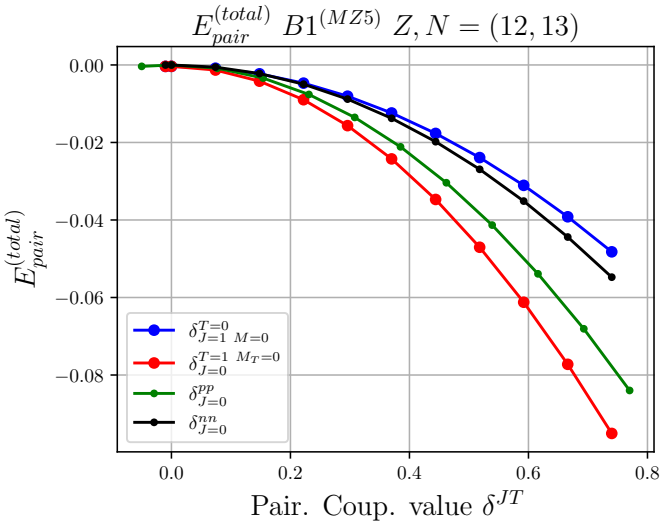

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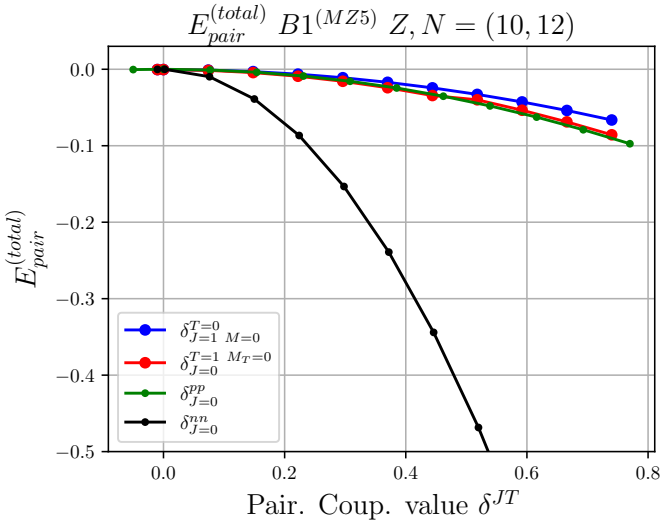

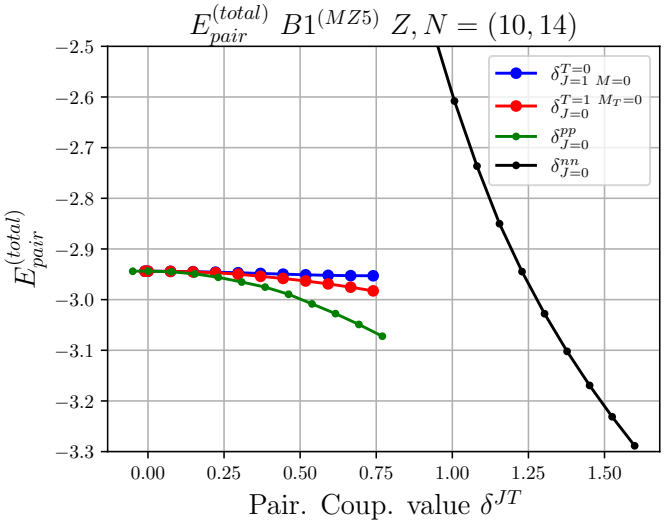

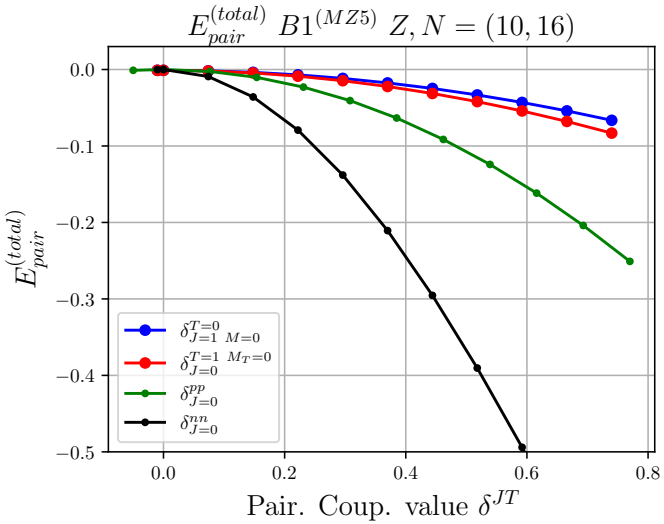

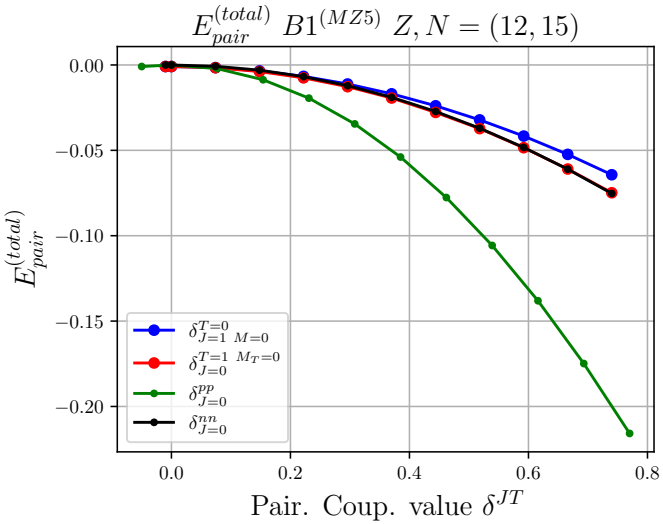

Conclusions

- We have explored the *pn* physics with the D1S/B1 interaction.
- Satisfactory outcome from the extension of the original D1S/B1 to *pn*-mixing wave functions at MF level.
- The interaction shows a consistent behavior when *pn* wave functions are employed in these light nuclei.
  - *pn* channels are not favoured for $N \neq Z$ (as expected).
  - *pn*-isovector channel is favored for $N = Z$ isotopes.

Perspectives

- Many *pn* pairing physics occurs in rotating nuclei, investigate cranking with D1S interaction.
- Explore other density-dependent interactions with this framework.
- Projection to particle number $P^N P^Z |\Phi\rangle$
  - Pairing condensates lose out the particle number as a good quantum number.
  - Restoring it might lead a real pn minimum.
  - It is necessary to access **odd-odd** nuclei at MF level.

Thank you !