Pauli blocking in deformed two-boy models applied to weakly-bound exotic nuclei.

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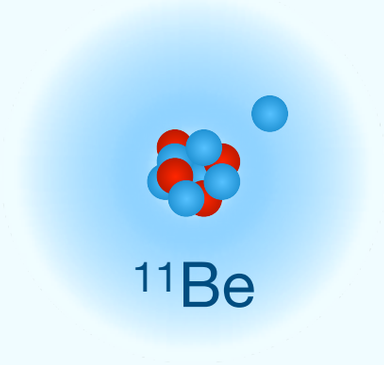
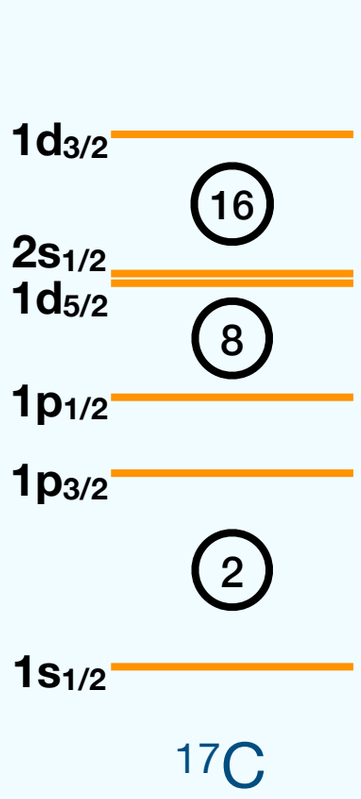
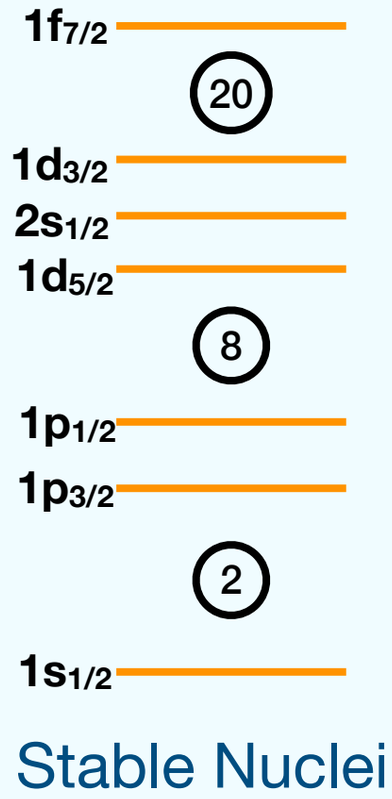
Weakly-bound exotic nuclei

- Different properties
  - Halo nature
  - Shell closure
- Reactions involving them
  - Advances in radioactive beam facilities
  - Realistic but manageable models
- Few-body models
  - Significant effect of deformation
  - Deformed core + neutron structure







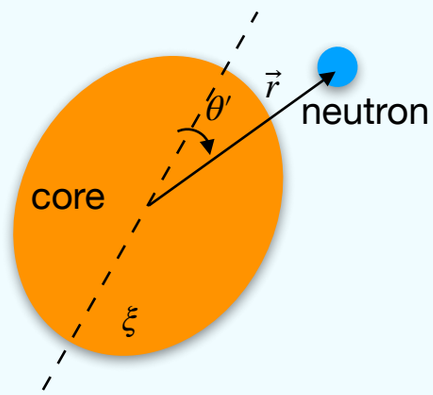
NAMD model

- Combination of Nilsson and PAMD

  [P. Punta *et al.* PRC**108** (2023) 024613]

  - Nilsson Hamiltonian

$$H_N = T + V_0(r) + V_{\ell s}(r)(\overrightarrow{\ell} \cdot \vec{s}) + V_2(r)Y_{20}(\theta')$$



  - Semi-microscopic coupling potentials with AMD transition densities

    [J. A. Lay *et al.* PRC**89** (2014) 014333]

$$V_\lambda(r) = \langle 0^+ || V_\lambda^{\mathrm{PAMD}}(\vec{r}, \xi) || \lambda^+ \rangle$$

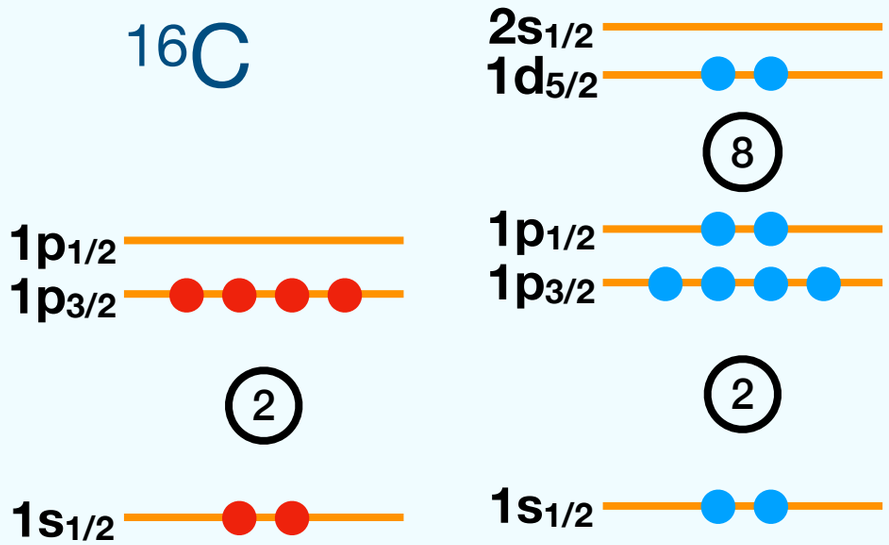
- Collective rotational term

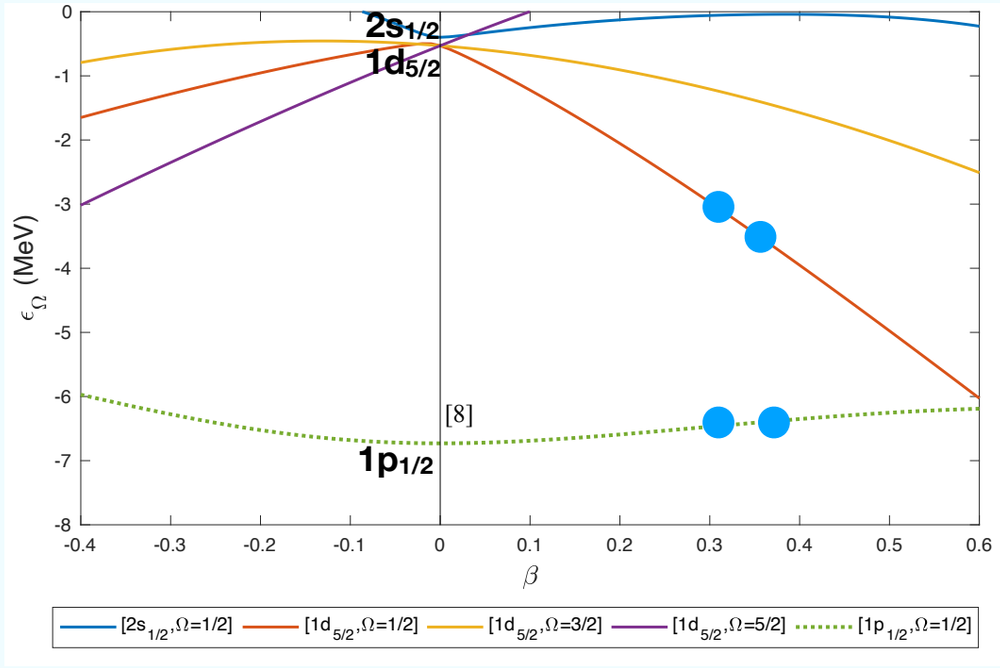
$$H = H_N + \frac{\hbar^2}{2\mathcal{J}}\vec{I}^2$$

- Eigenstates from diagonalization in THO basis

$$\phi_{nl}^{THO}(r) = \sqrt{\frac{ds}{dr}}\phi_{nl}^{HO}[s(r)]$$

Occupation of single-particle levels





- Valence spherical single-particle levels are partially filled
- Deformation gives rise to Nilsson levels with twofold degeneracy
- Are the Nilsson states fully occupied? **Pairing effects?**

BCS implementation

- Nilsson eigenstates

$$H_N|\nu J\rangle = \epsilon_\nu |\nu J\rangle$$

- Nilsson+rotational

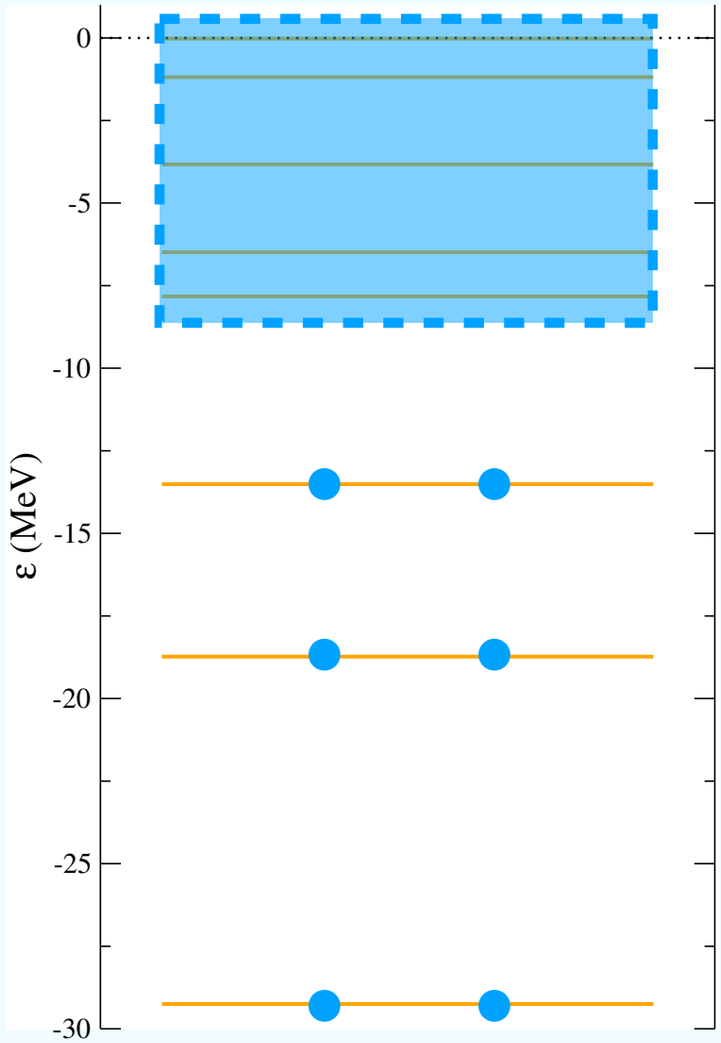
$$\langle \nu J|H|\nu' J\rangle = \epsilon_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle \nu J|\vec{I}^2|\nu' J\rangle$$

- Transformation to quasi-particles

$$\langle BCS,\nu J|H'|BCS,\nu' J\rangle = E_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}}\langle \nu J|\vec{I}^2|\nu' J\rangle F_{\nu\nu'}$$

- $F_{\nu\nu'}$ account for BCS occupation probabilities

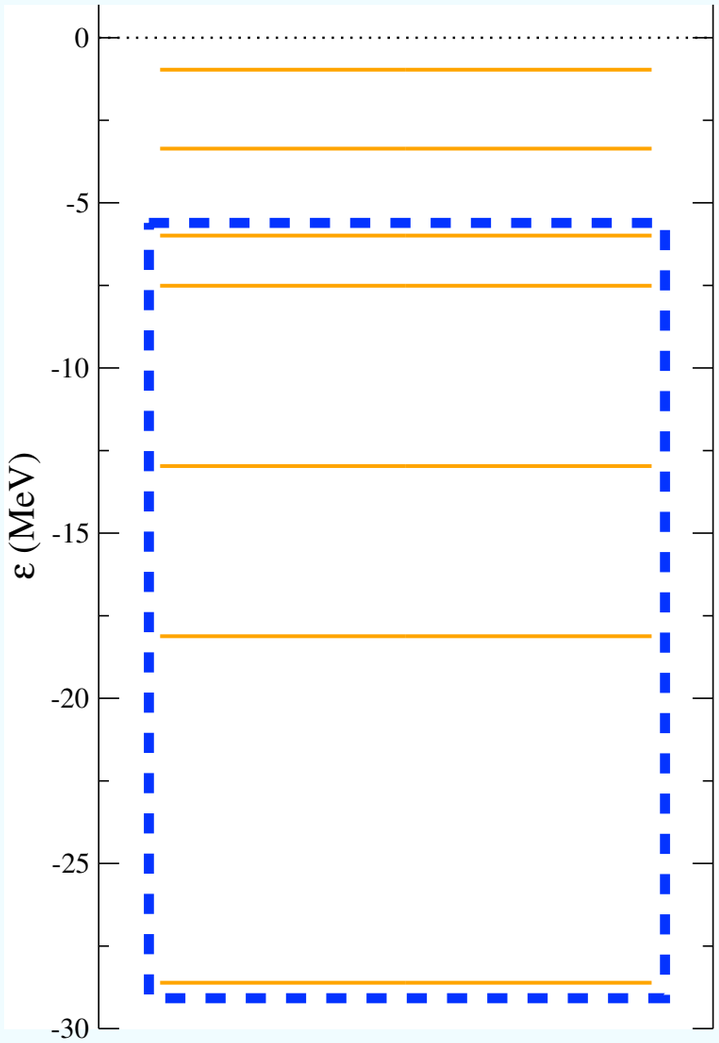
$$F_{\nu'\nu} = u_{\nu'}u_\nu - v_{\nu'}v_\nu + (1-2v_\nu)\delta_{\nu'\nu}$$



Pauli blocking methods

- Without Blocking (WB)

$$\langle \nu J | H | \nu' J \rangle = \epsilon_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}} \langle \nu J | \vec{I}^2 | \nu' J \rangle$$



Pauli blocking methods

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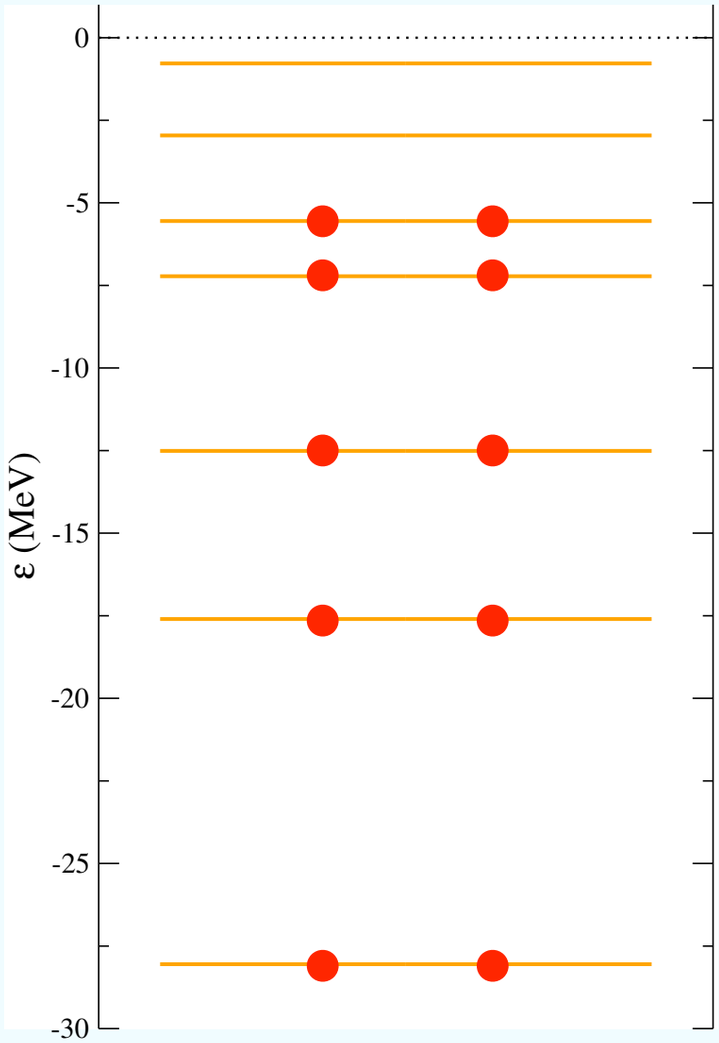
$$\langle \nu J | H | \nu' J \rangle = \epsilon_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}} \langle \nu J | \vec{I}^2 | \nu' J \rangle$$

- Total Blocking (TB)
  - Fully occupied levels ($u_\nu = 0, v_\nu = 1$)
  - Empty levels ($u_\nu = 1, v_\nu = 0$)



$$\langle BCS, \nu J | H' | BCS, \nu' J \rangle = E_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}} \langle \nu J | \vec{I}^2 | \nu' J \rangle F_{\nu\nu'} \qquad F_{\nu'\nu} = u_{\nu'} u_\nu - v_{\nu'} v_\nu + (1 - 2v_\nu)\delta_{\nu'\nu}$$

Pauli blocking methods

- Without Blocking (WB)

$$\langle \nu J | H | \nu' J \rangle = \epsilon_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}} \langle \nu J | \vec{I}^2 | \nu' J \rangle$$

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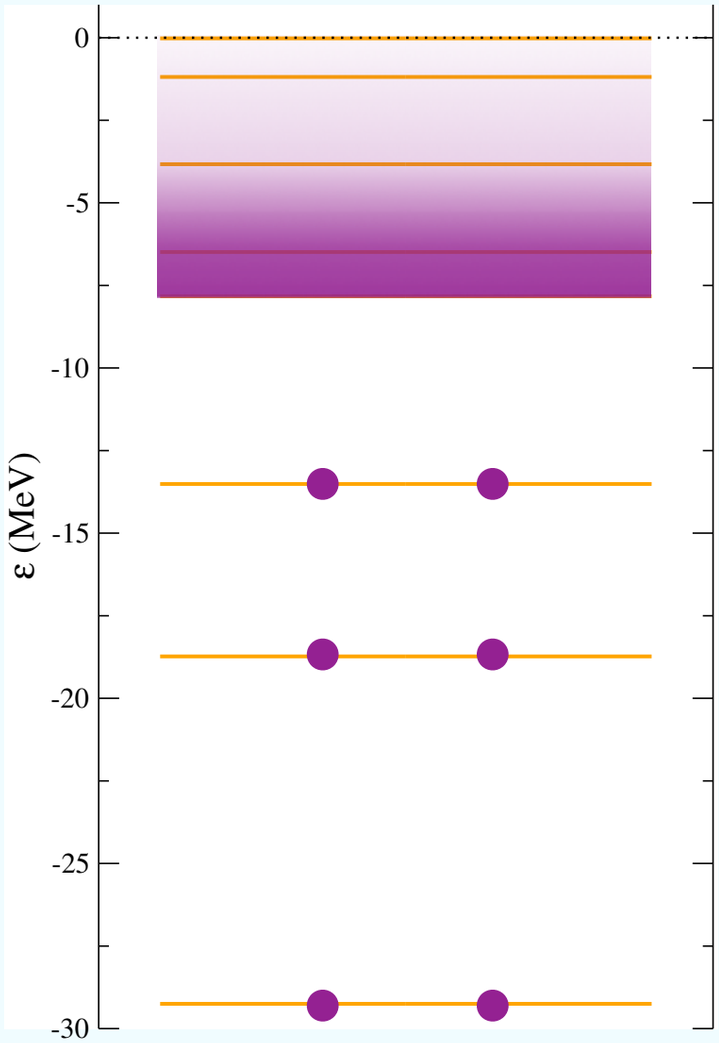


- Partial Blocking (PB)
  - BCS Partial Occupations

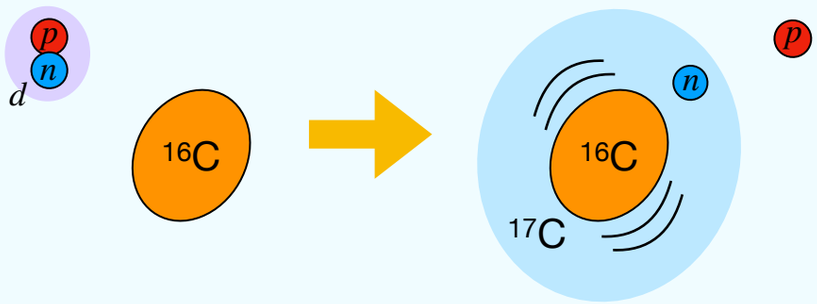
$$\langle BCS, \nu J | H' | BCS, \nu' J \rangle = E_\nu \delta_{\nu\nu'} + \frac{\hbar^2}{2\mathcal{J}} \langle \nu J | \vec{I}^2 | \nu' J \rangle F_{\nu\nu'} \qquad F_{\nu'\nu} = u_{\nu'} u_\nu - v_{\nu'} v_\nu + (1 - 2v_\nu)\delta_{\nu'\nu}$$

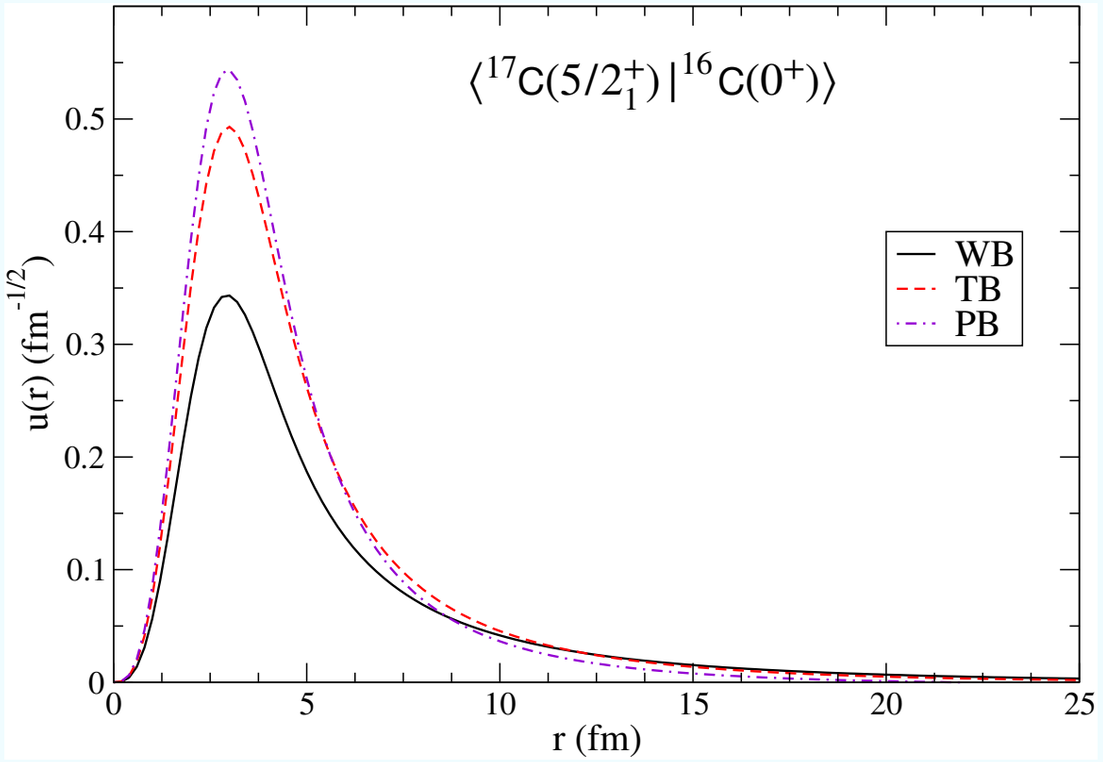
Application to neutron transfer reactions

- ^{16}C(*d*,*p*)^{17}C reaction is studied applying the Adiabatic Distorted Wave Approximation (ADWA).

- In the *post* form, these calculations require the $\langle {}^{17}\mathrm{C} \,|\, {}^{16}\mathrm{C}(0^+)\rangle$ overlaps.

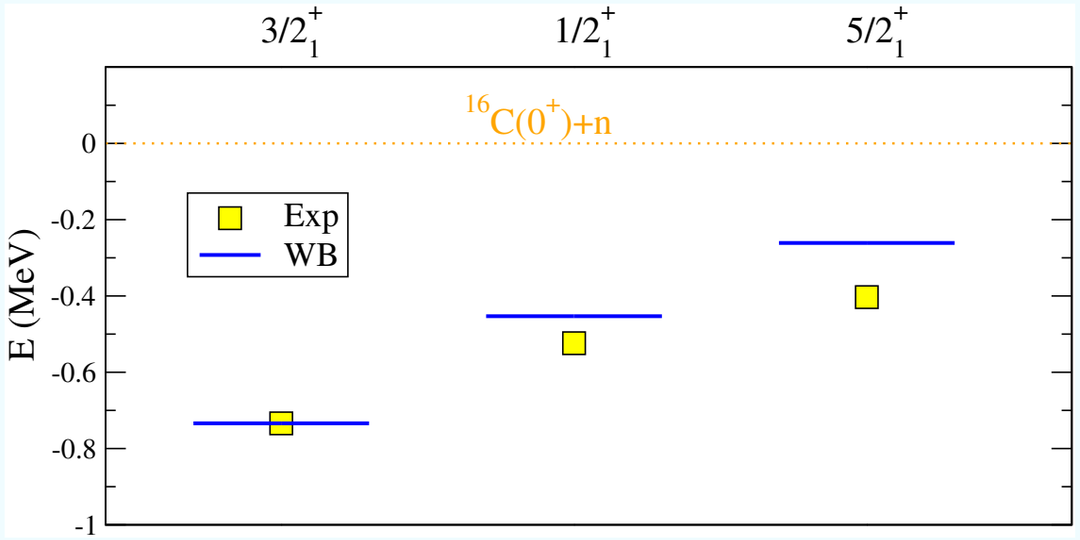
- Calculations are compared with experimental data from GANIL [X. Pereira-López *et al.* PLB**811** (2020) 135939]

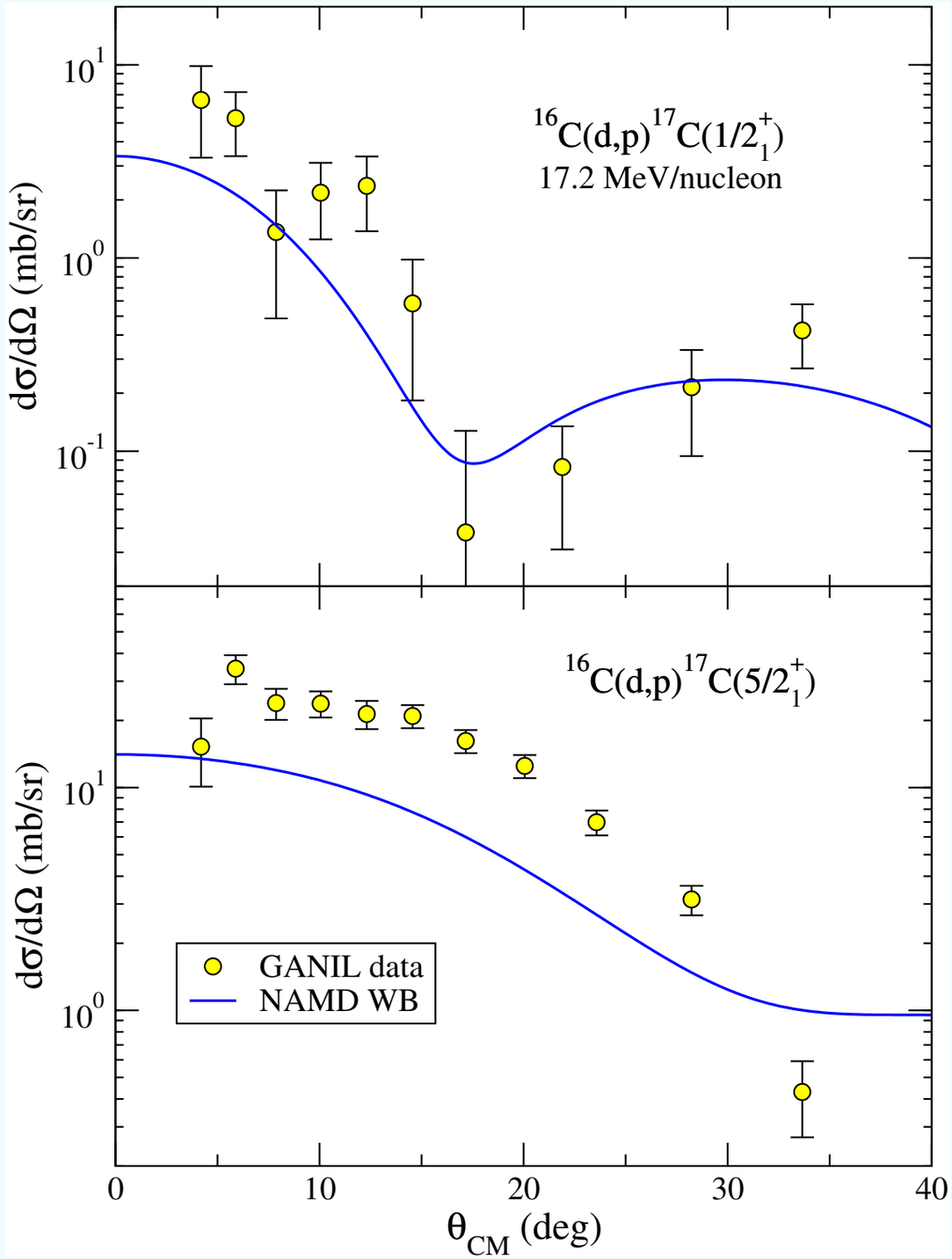




Application to ^{17}C and ^{16}C(*d*,*p*)^{17}C

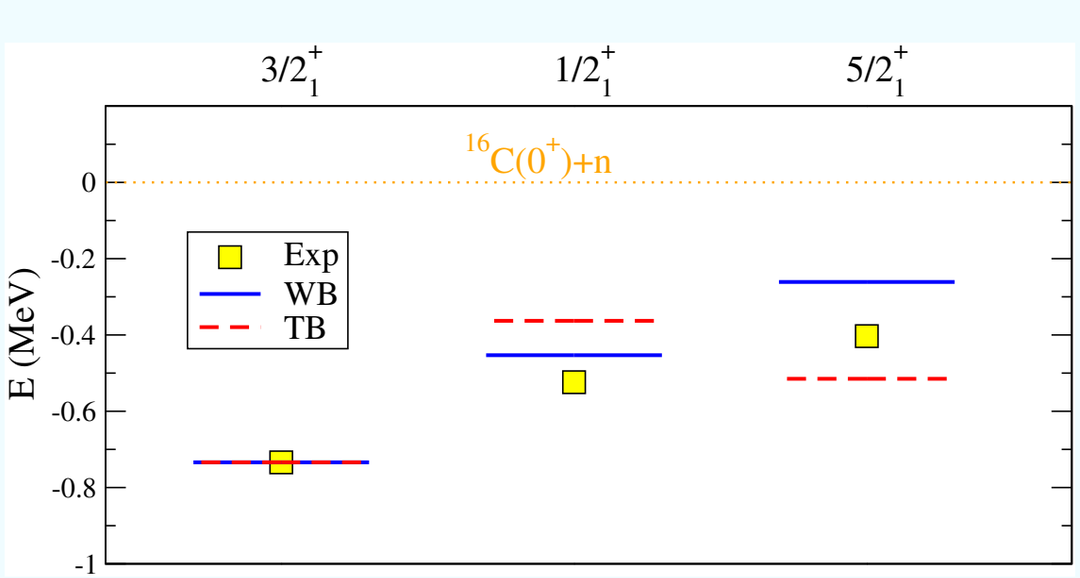
- Without Blocking (WB)

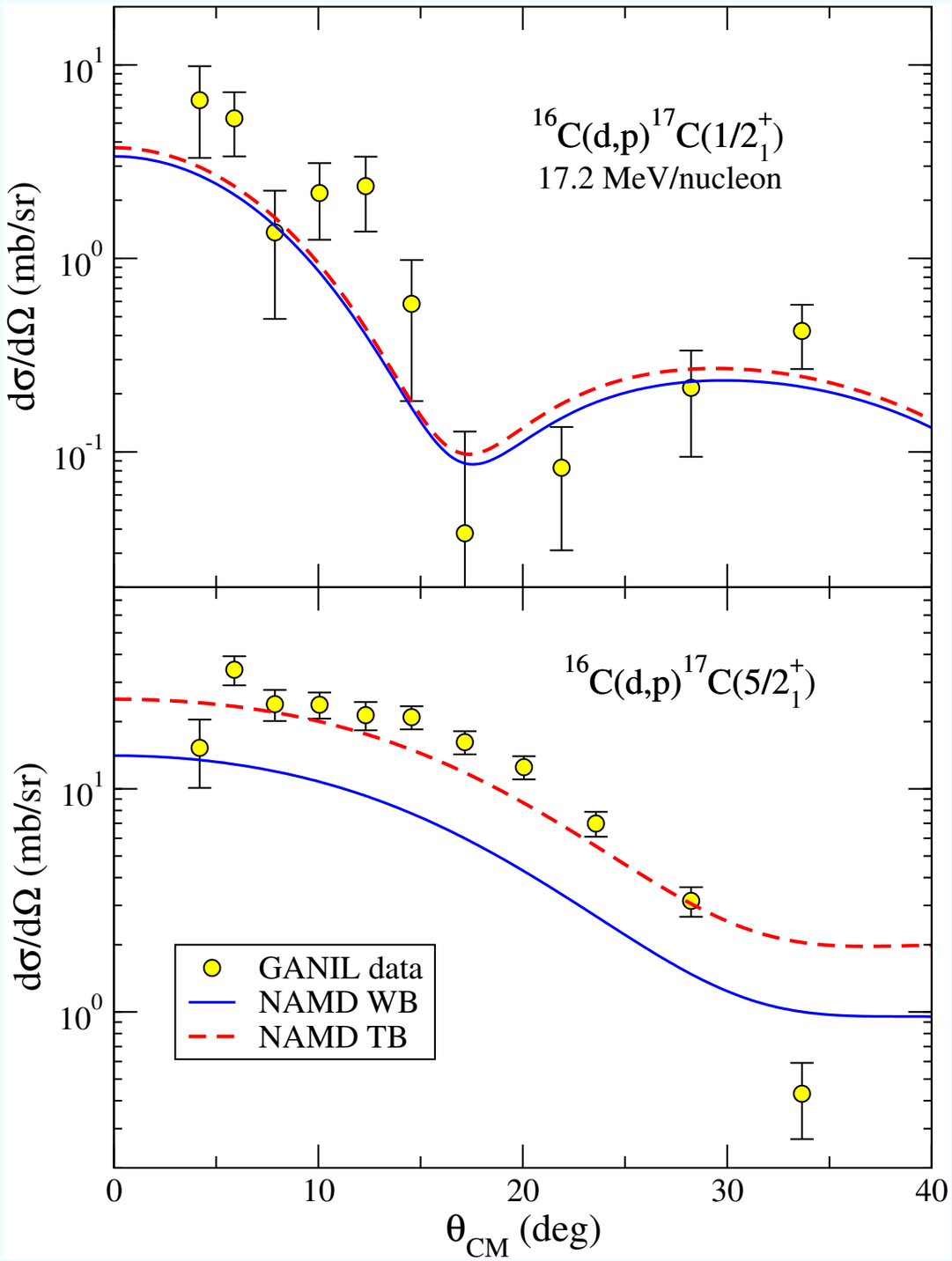




Application to ^{17}C and ^{16}C(*d*,*p*)^{17}C

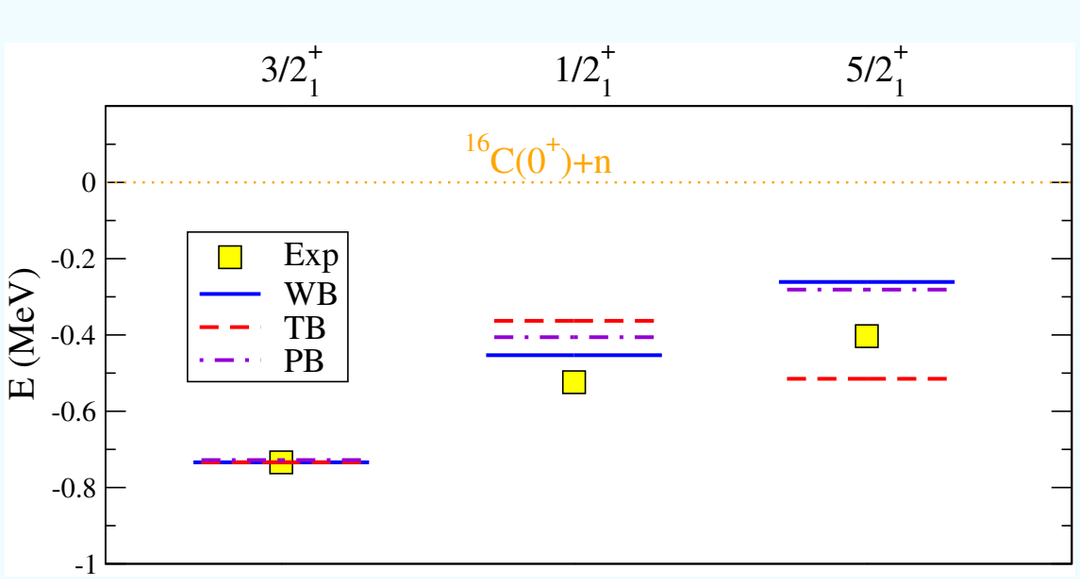
- Without Blocking (WB)
- Total Blocking (TB)





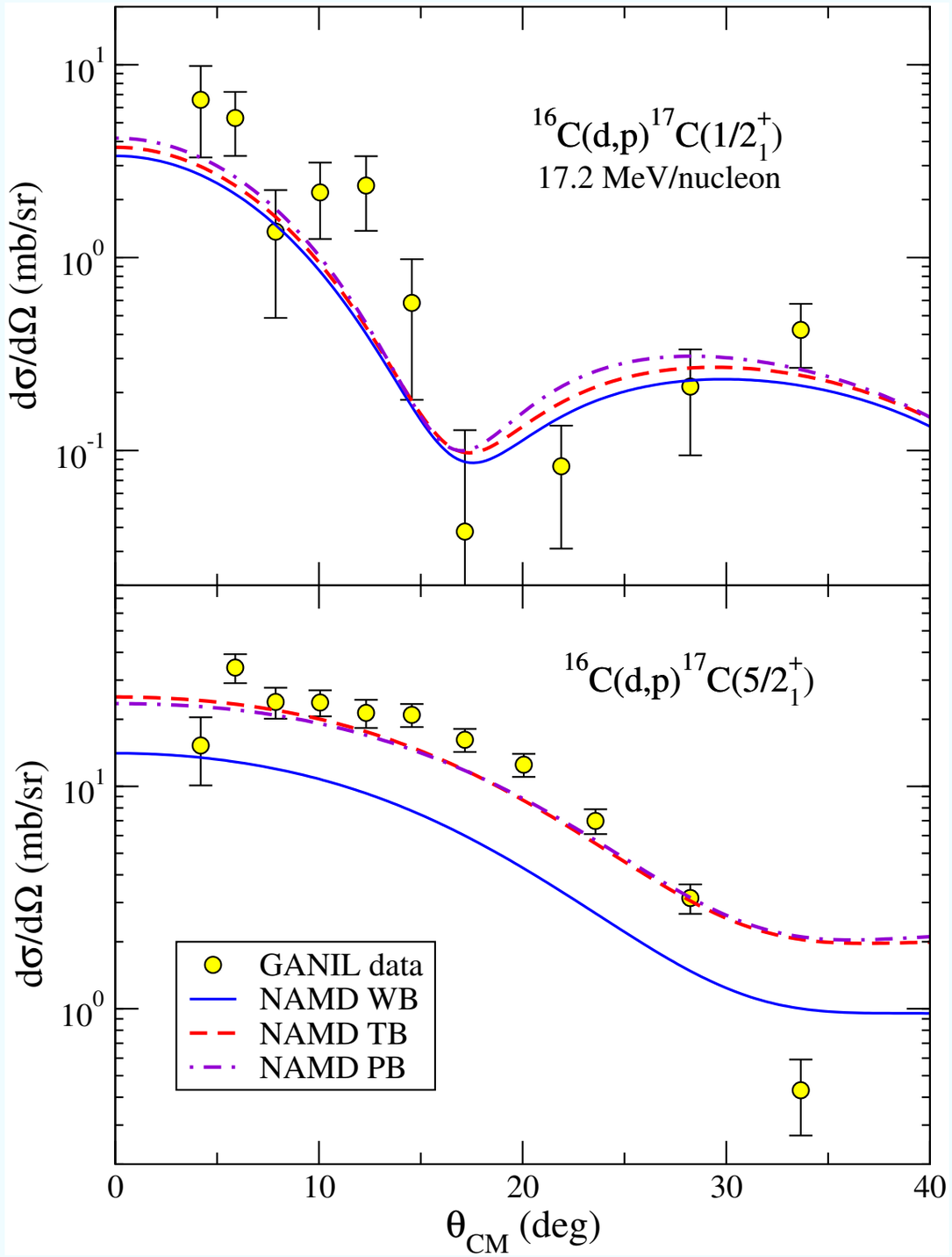
Application to ^{17}C and ^{16}C(*d*,*p*)^{17}C

- Without Blocking (WB)
- Total Blocking (TB)
- Partial Blocking (PB)



Pairing Gap

Three-point formula	1.76 MeV
NAMD PB	1.68 MeV

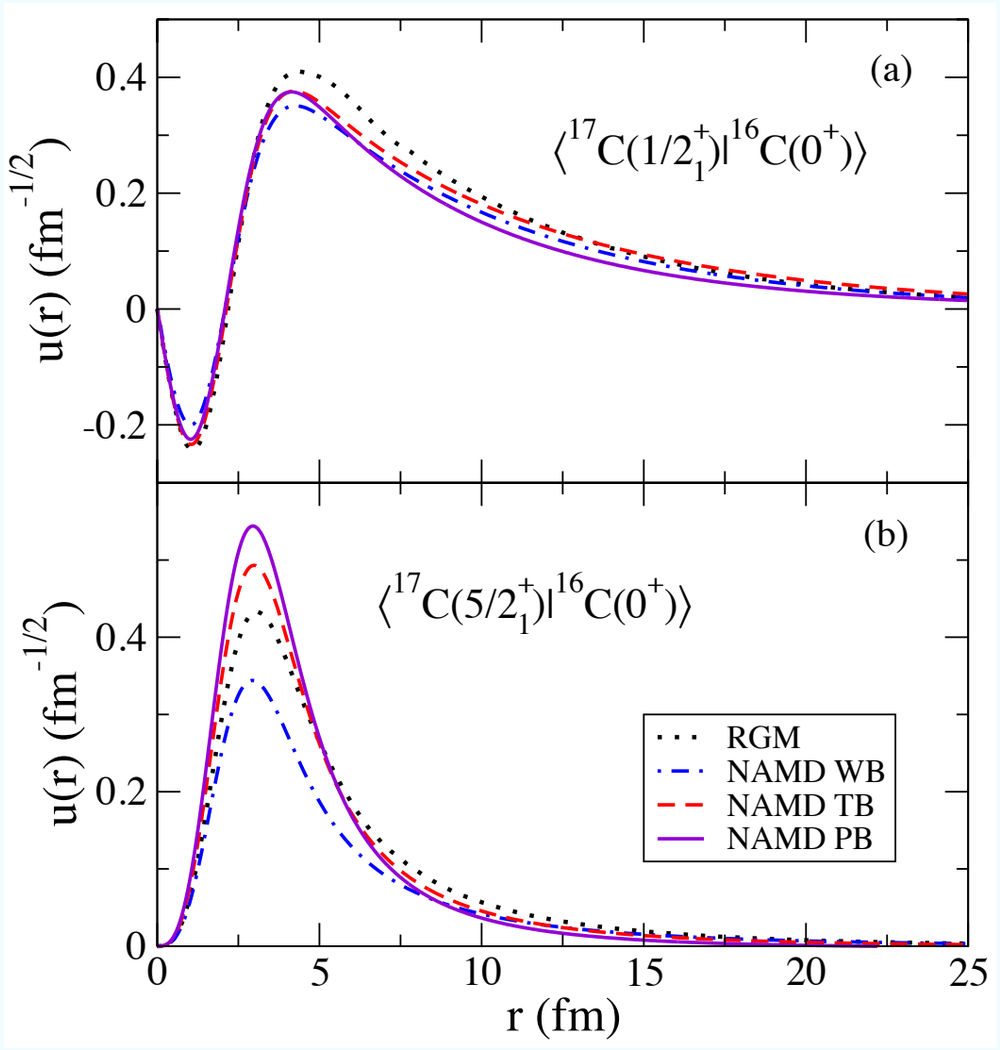


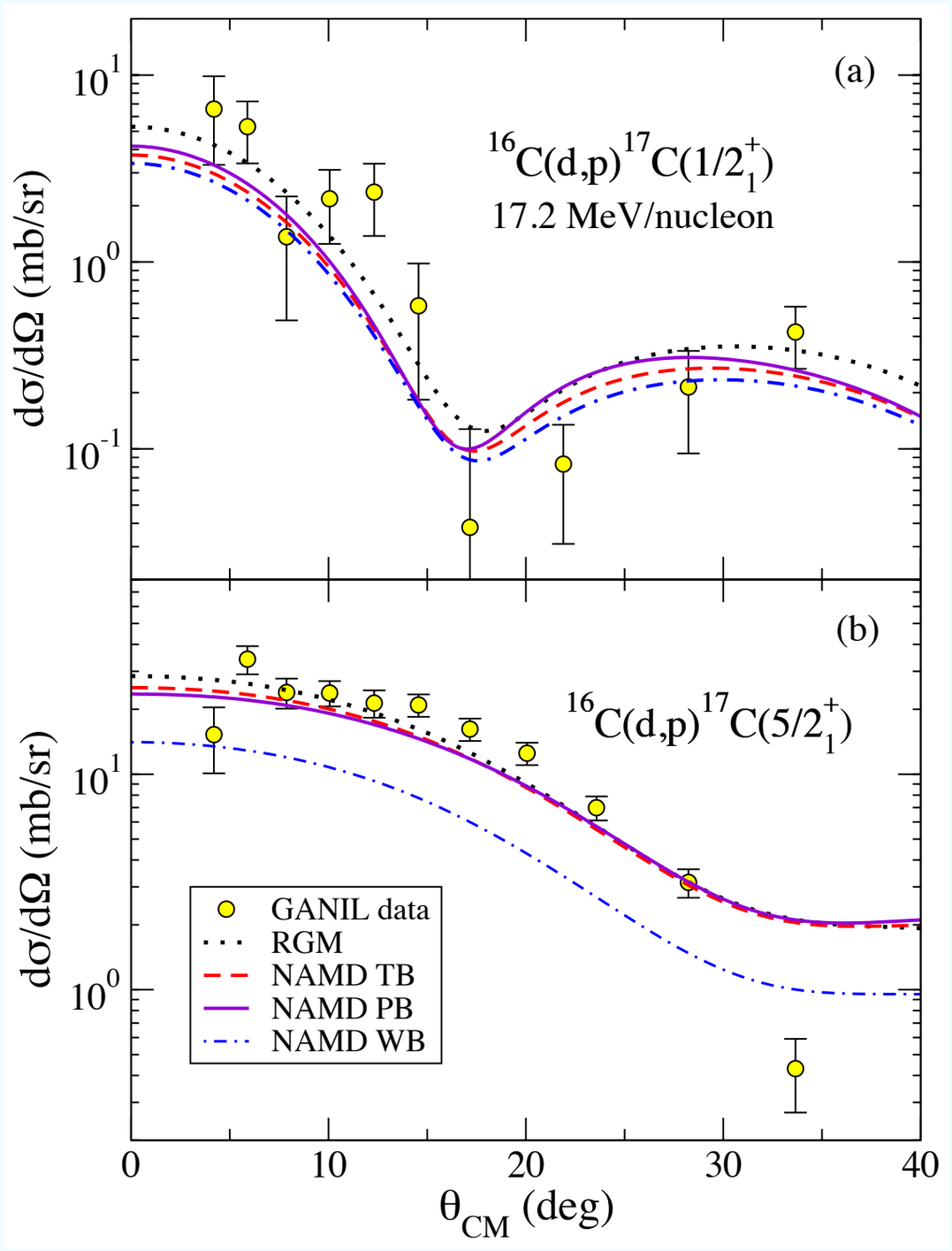
Comparison with RGM calculations

Resonating Group Method (RGM)

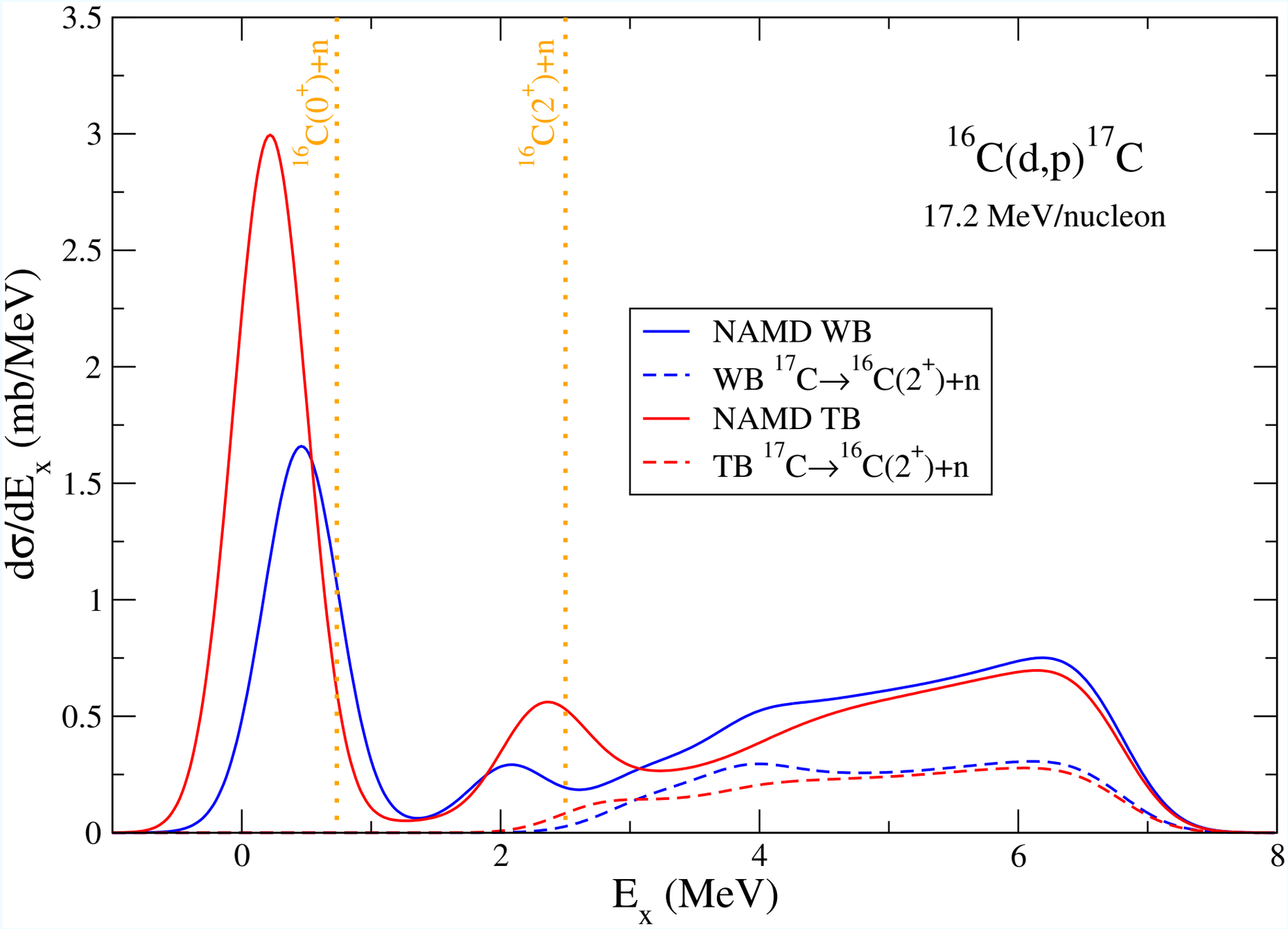
[Le Hoang Chien and P. Descouvemont

PRC**108** (2023) 044605]





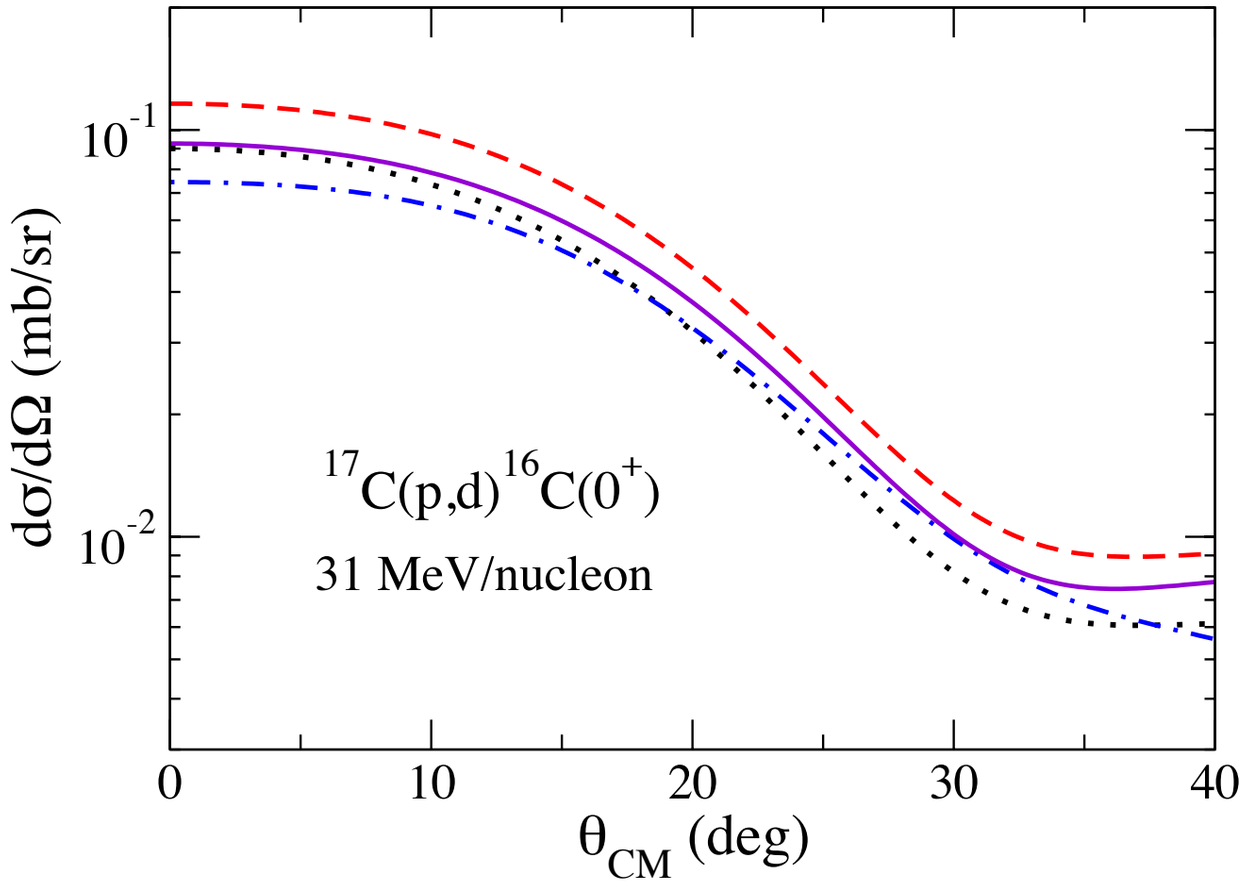
Transfer to the continuum: $^{16}C(d,p)^{17}C$

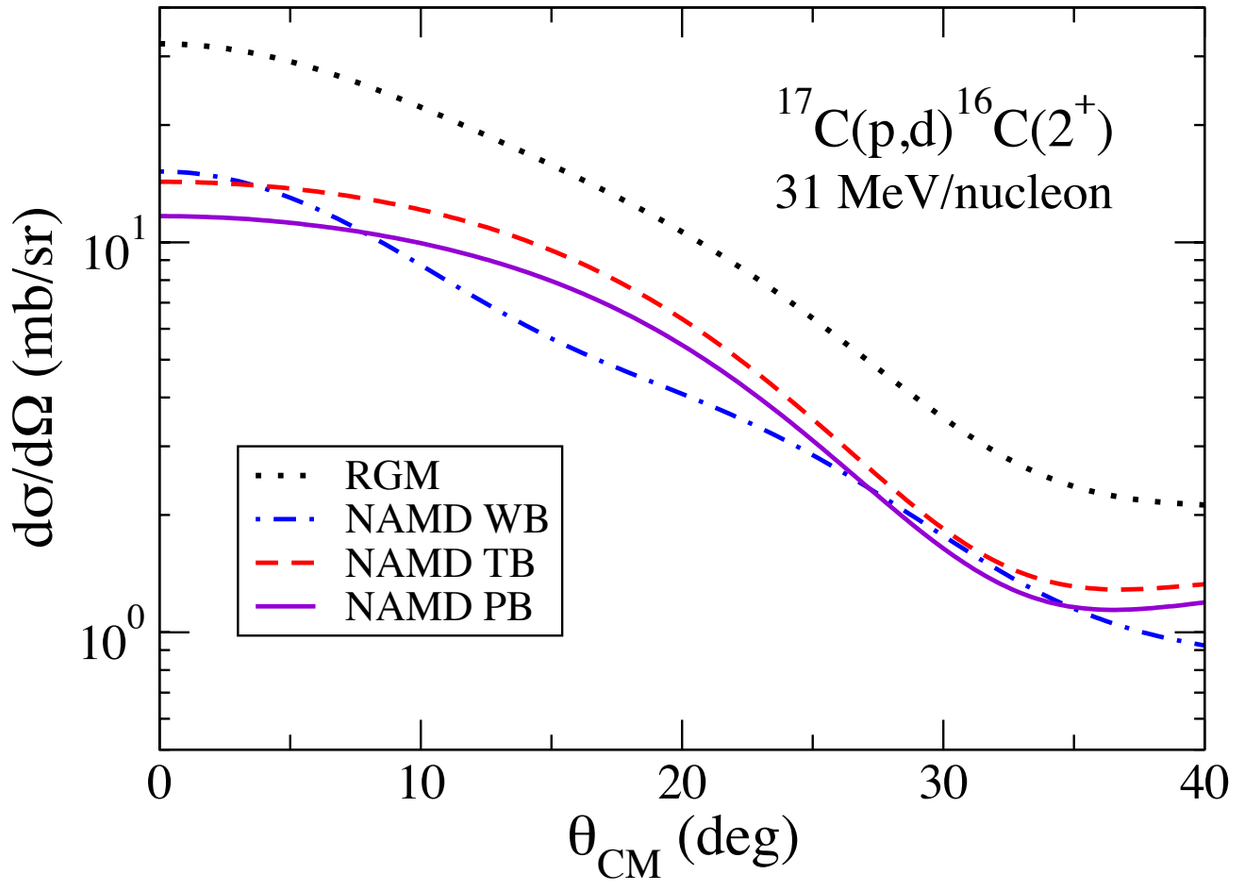


Application to ^{17}C(*p*,*d*)^{16}C

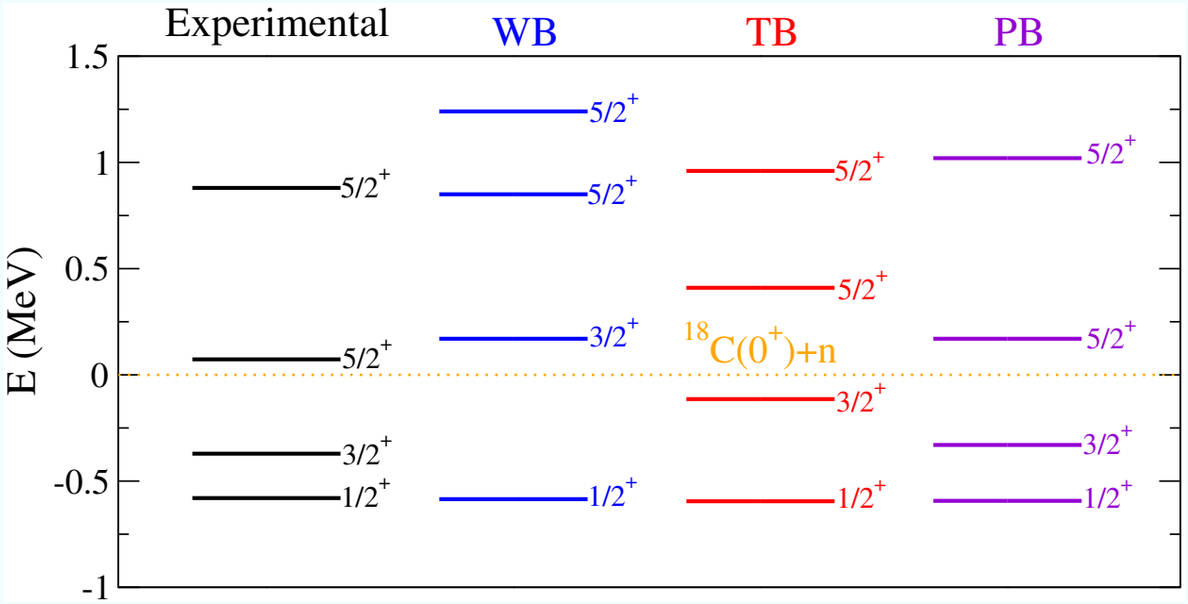
- ^{17}C(*p*,*d*)^{16}C reaction is studied applying the Adiabatic Distorted Wave Approximation (ADWA).

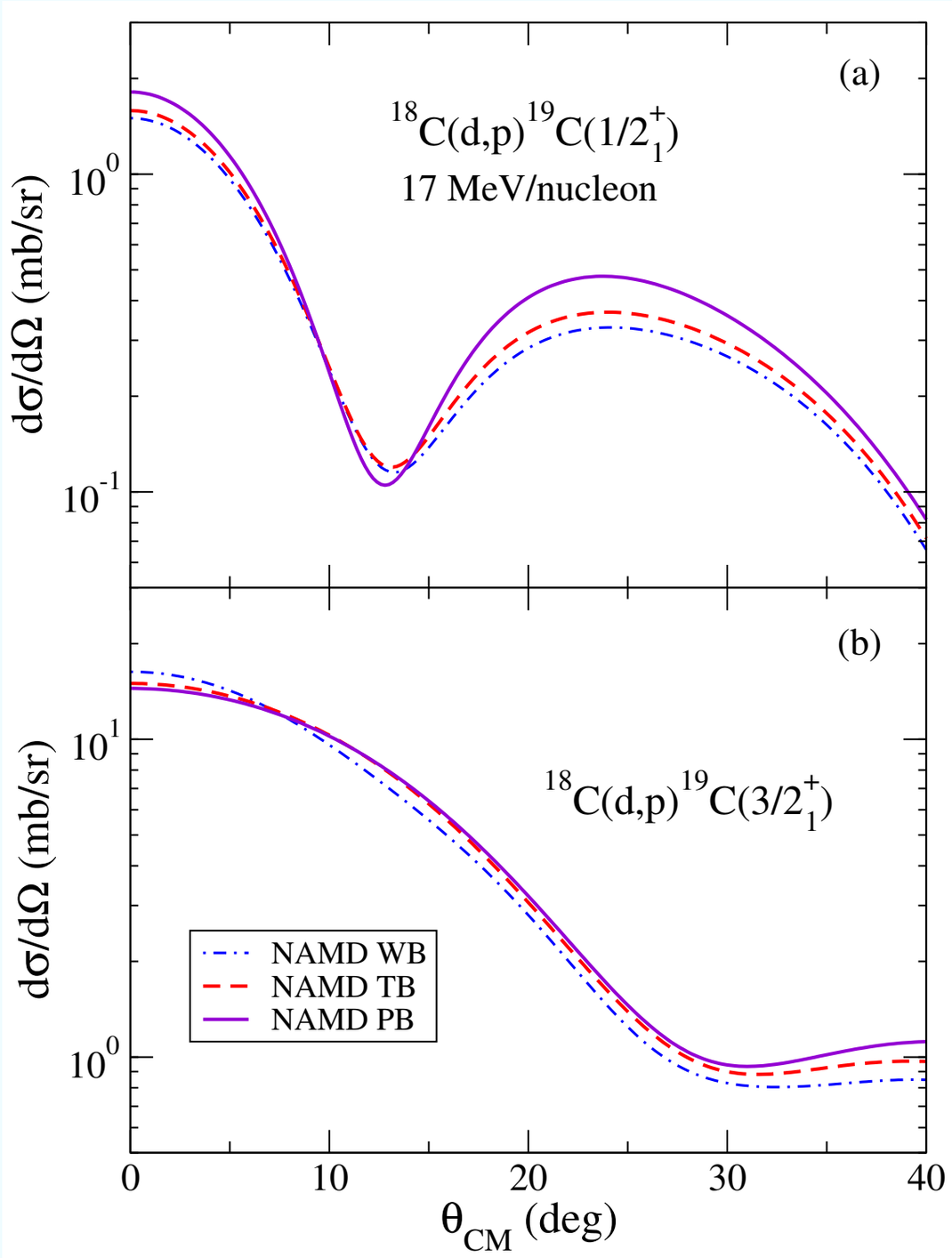
- In the *prior* form, these calculations require the $\langle {}^{16}\mathrm{C} \,|\, {}^{17}\mathrm{C}(3/2^+_1)\rangle$ overlaps.





Application to ^{19}C and ^{18}C(d,p)^{19}C





Conclusions

- The **NAMD** model shows **promising results** in the description of ^{17}C.

- **Significant effects** of **Pauli blocking** related to occupied Nilsson states are found.

- **Pauli blocking** methods are **needed** to reproduce **^{16}C(*d*,*p*)^{17}C($5/2^+_1$)** cross section.

- Transfer reaction **^{16}C(*d*,*p*)^{17}C** populating **unbound states** is being studied.

- Extensions to **breakup reactions** and **other weakly-bound nuclei** are in progress.

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AGENCIA
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INVESTIGACIÓN

Ph.D. grant FPU21/03931

Thanks for your attention

Backup

Application to ^{17}C

- Deformed two-body models
  - Nilsson
  - PAMD

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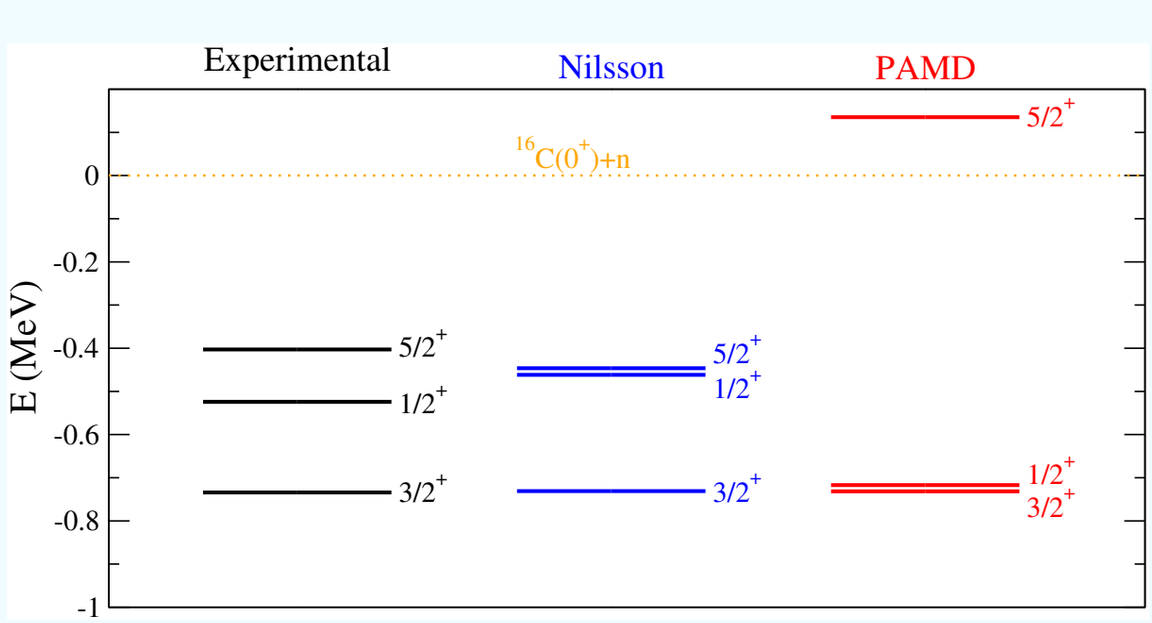
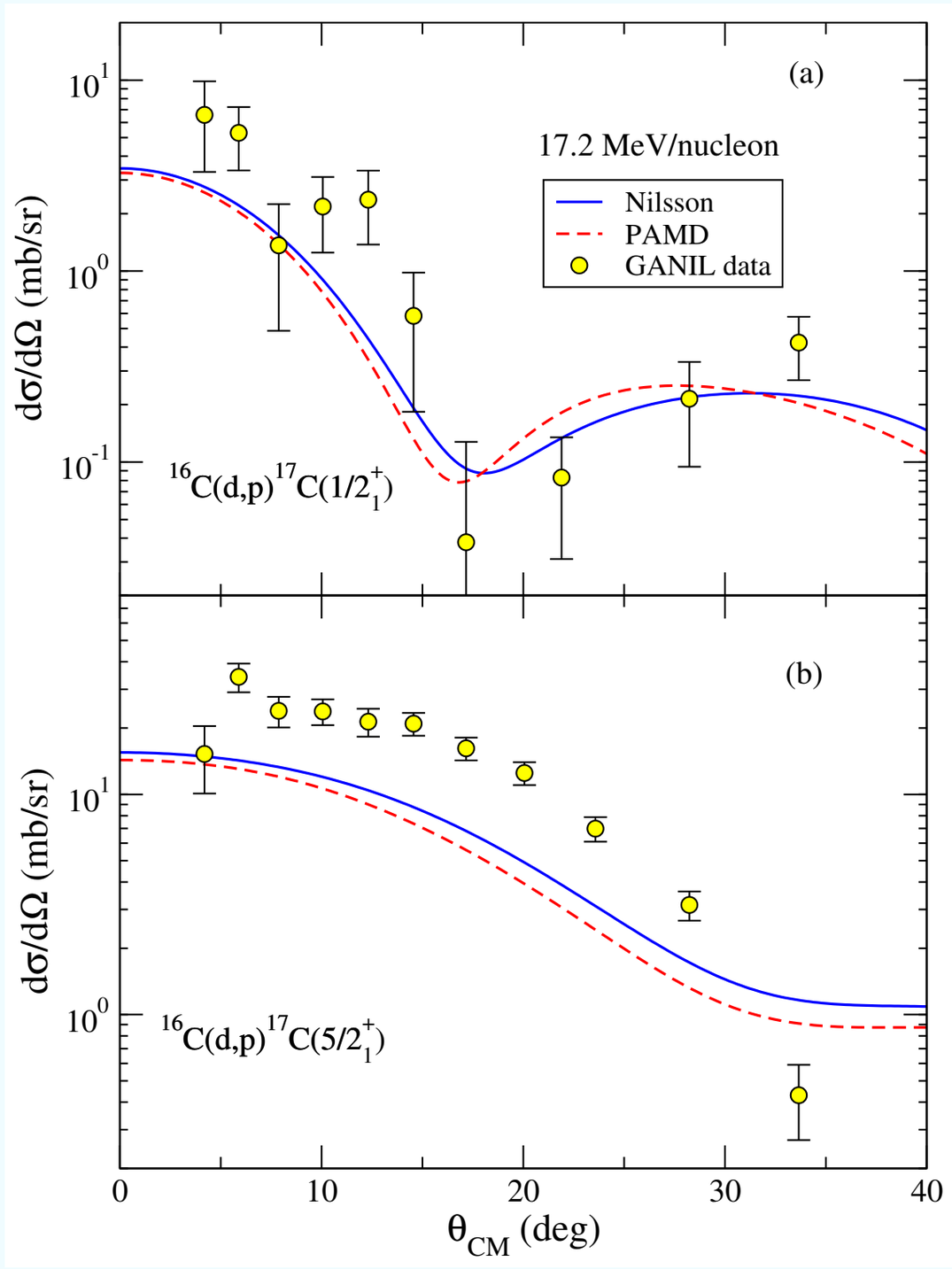
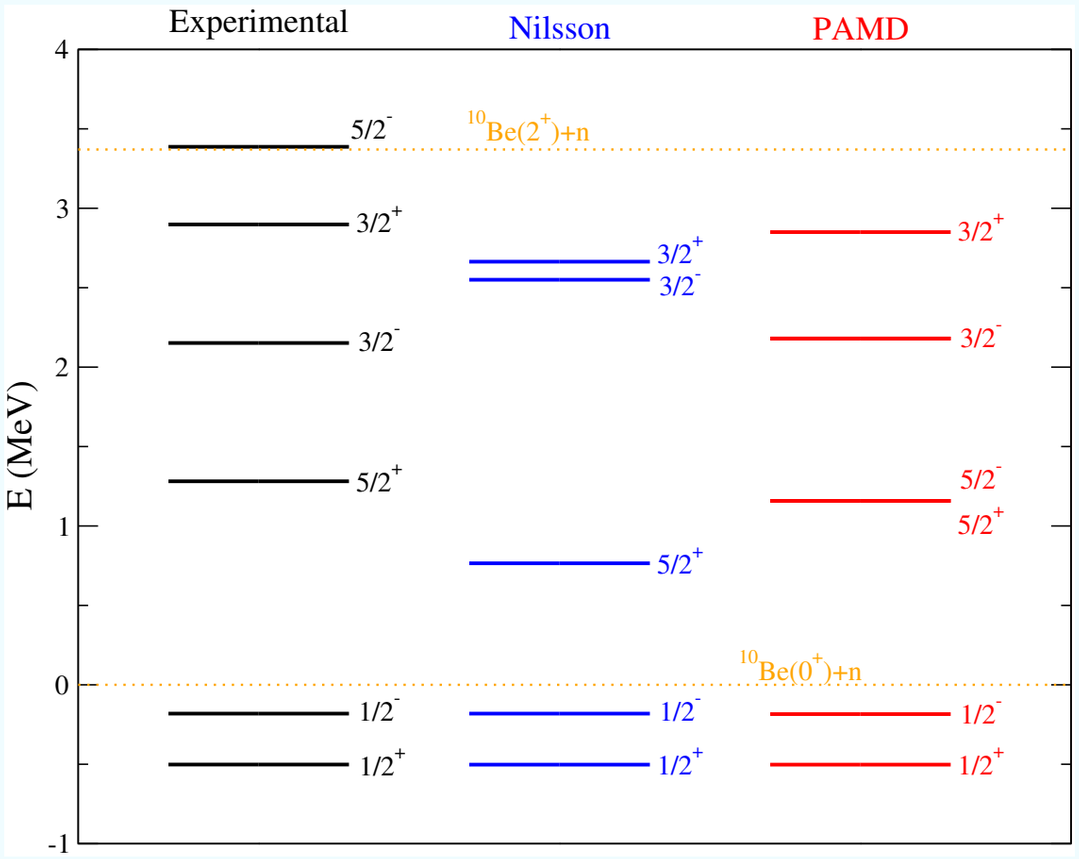


GANIL data [PLB**811** (2020) 135939]



Application to ^{11}Be



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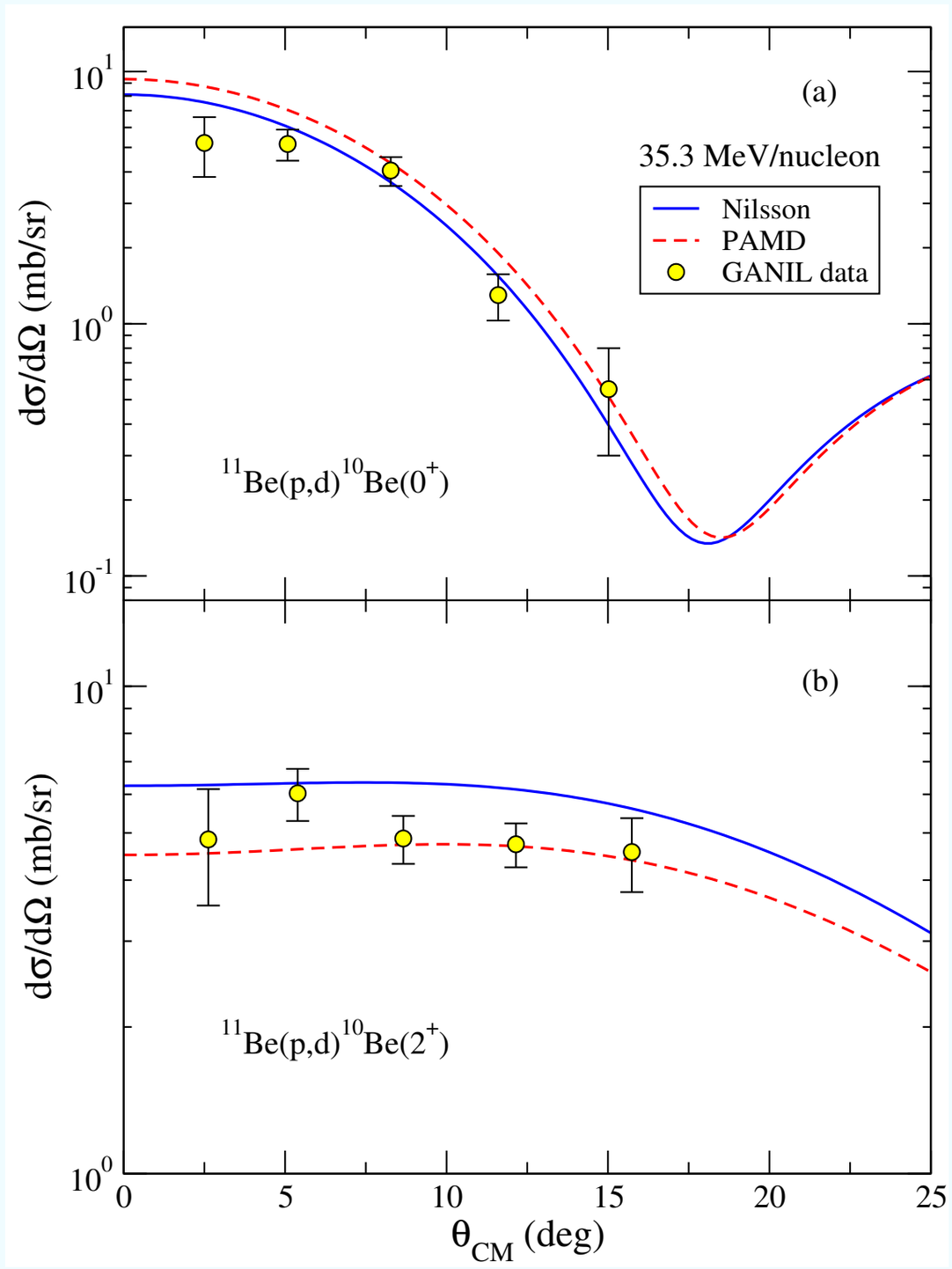
GANIL data [NPA**683** (2001) 48]



H' eigenstates

- Eigenvalues $H'|BCS, nJ^\pi\rangle = E_{nJ^\pi}|BCS, nJ^\pi\rangle$

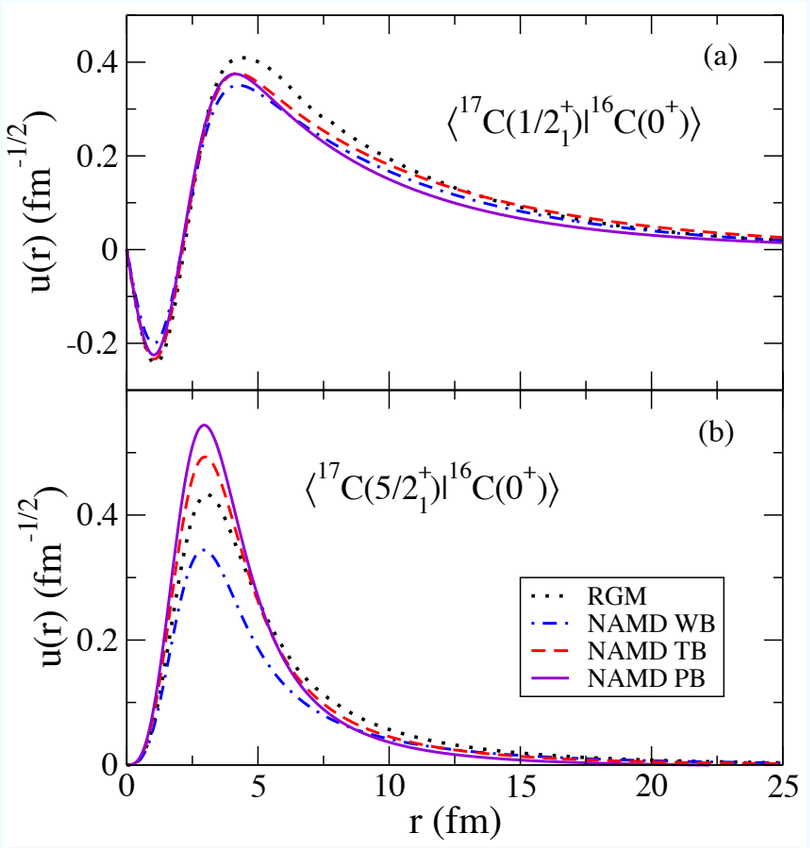
- Eigenfunctions $|BCS,\ nJ^\pi\rangle = \sum_\nu C_\nu^{nJ^\pi}|BCS,\ \nu J^\pi\rangle$

- Occupation $v^2_{nJ^\pi} = \sum_\nu (C_\nu^{nJ^\pi} v_\nu)^2$

- Particle energy ($v^2_{nJ^\pi} < 1/2$) $\epsilon_{nJ^\pi} = \lambda + E_{nJ^\pi}$

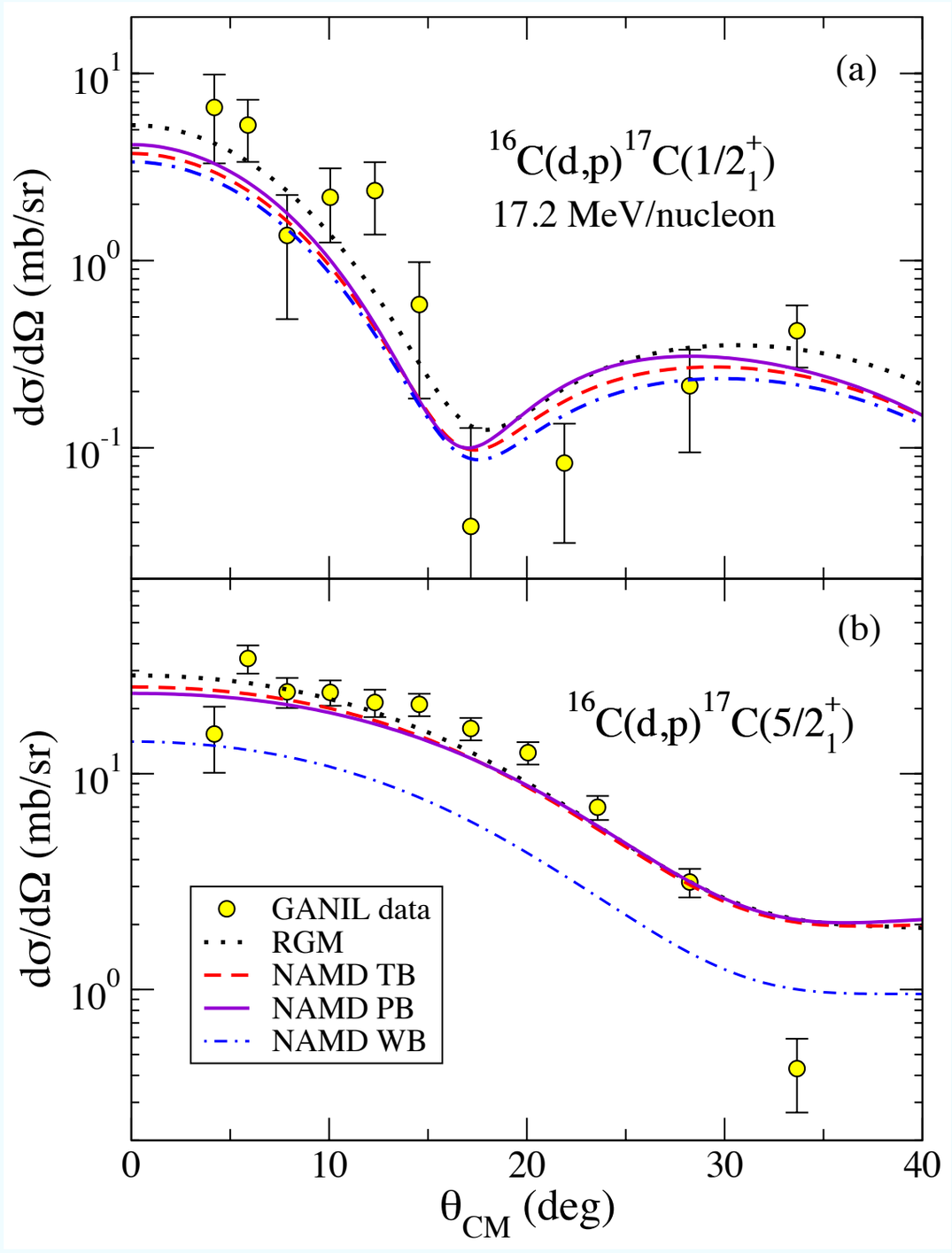
- Hole state ($v^2_{nJ^\pi} > 1/2$) $\epsilon^{(h)}_{nJ^\pi} = \lambda - E_{nJ^\pi}$

^{16}C(*d*,*p*)^{17}C (bound states)



Spectroscopic Factors for $(\ell s)j \otimes 0^+$

	$3/2^+_1$	$1/2^+_1$	$5/2^+_1$
Exp	$0.03\ ^{+0.05}_{-0.03}$	0.64±0.18	0.62±0.13
WB	0.00	0.68	0.33
TB	0.01	0.80	0.66
PB	0.01	0.69	0.76
RGM	0.01	0.94	0.56



Transfer to the continuum

- The *prior* form of the ADWA approximation is used

  X $\langle {}^{17}\mathrm{C} \,|\, {}^{16}\mathrm{C} \rangle$ → Too much spatial extension

  ✓ $\langle {}^{17}\mathrm{C} \,|\, V_{n-{}^{16}C} \,|\, {}^{16}\mathrm{C} \rangle$ → Little spatial extension

- The continuum is discretized with the pseudo-states method

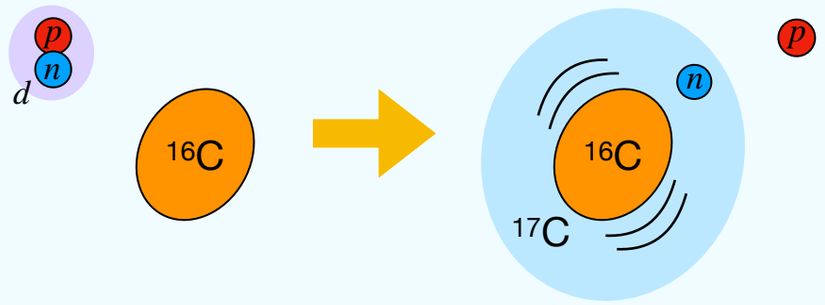
  - Scattering amplitudes for a discrete number of pseudo-states

  - Convolution with the *exact n*+core scattering states

$$\mathscr{F}(k,\theta) \approx \sum_n \langle \Psi(k,r) \,|\, \Psi_n^{THO}(r) \rangle \mathscr{F}_n^{THO}(\theta)$$

Pseudo-states discretization method

^{16}C(d,p)^{17}C



$$\mathscr{F}(k,\theta) = \sqrt{\frac{\mu_i \mu_f k_f}{(2\pi\hbar^2)^2 k_i}} \langle \chi_f^{(-)} \varphi_p \Psi_{^{17}C} \,|\, U \,|\, \chi_i^{(+)} \psi_d \phi_{^{16}C} \rangle$$

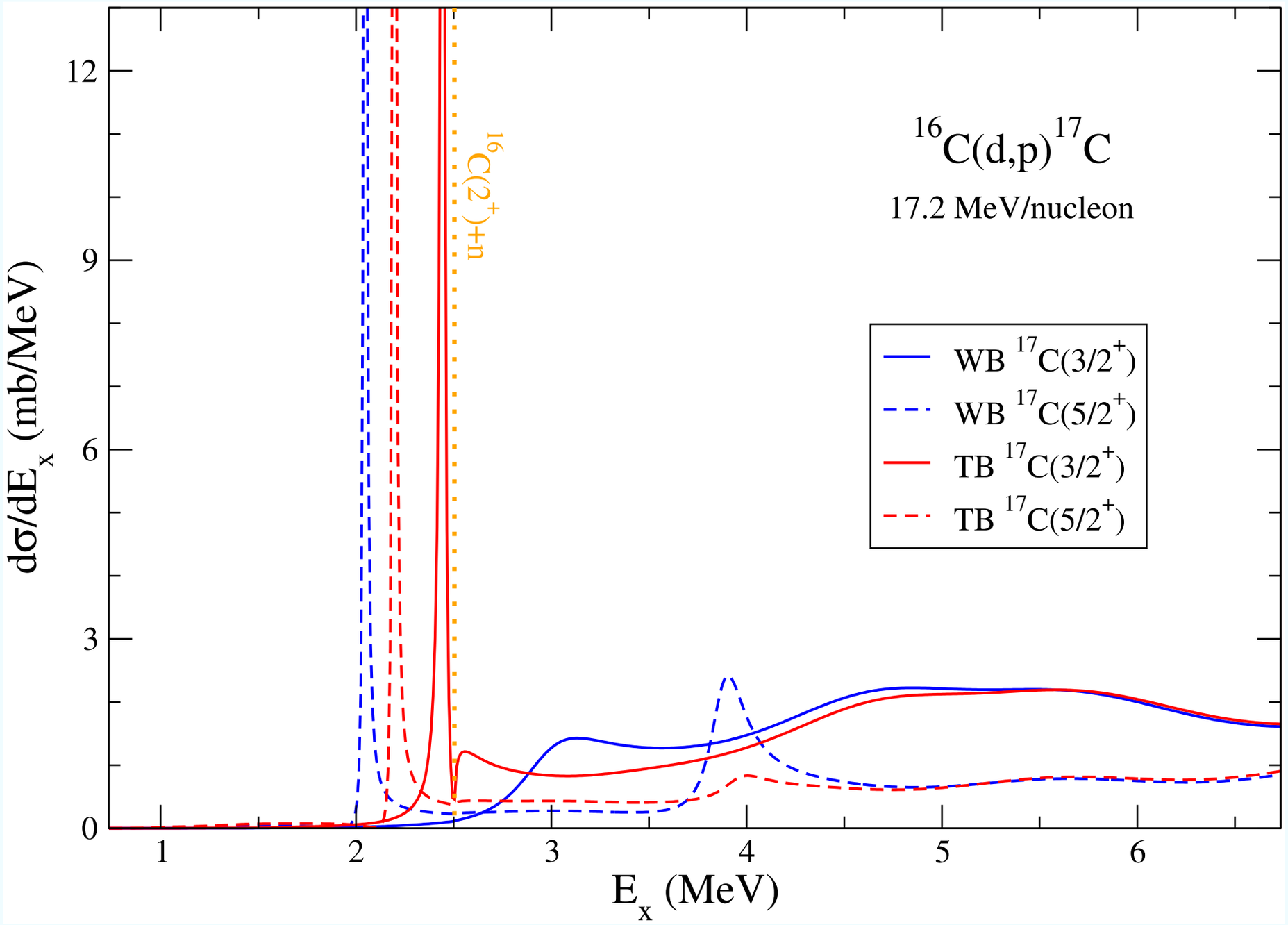
- Scattering amplitudes for a discrete number of pseudo-states

$$\mathscr{F}_n^{THO}(\theta) = \sqrt{\frac{\mu_i \mu_f k_n}{(2\pi\hbar^2)^2 k_i}} \langle \chi_f^{(-)} \varphi_p \Psi_n^{THO} \,|\, U \,|\, \chi_i^{(+)} \psi_d \phi_{^{16}C} \rangle$$

- Convolution with the *exact n*+core scattering states

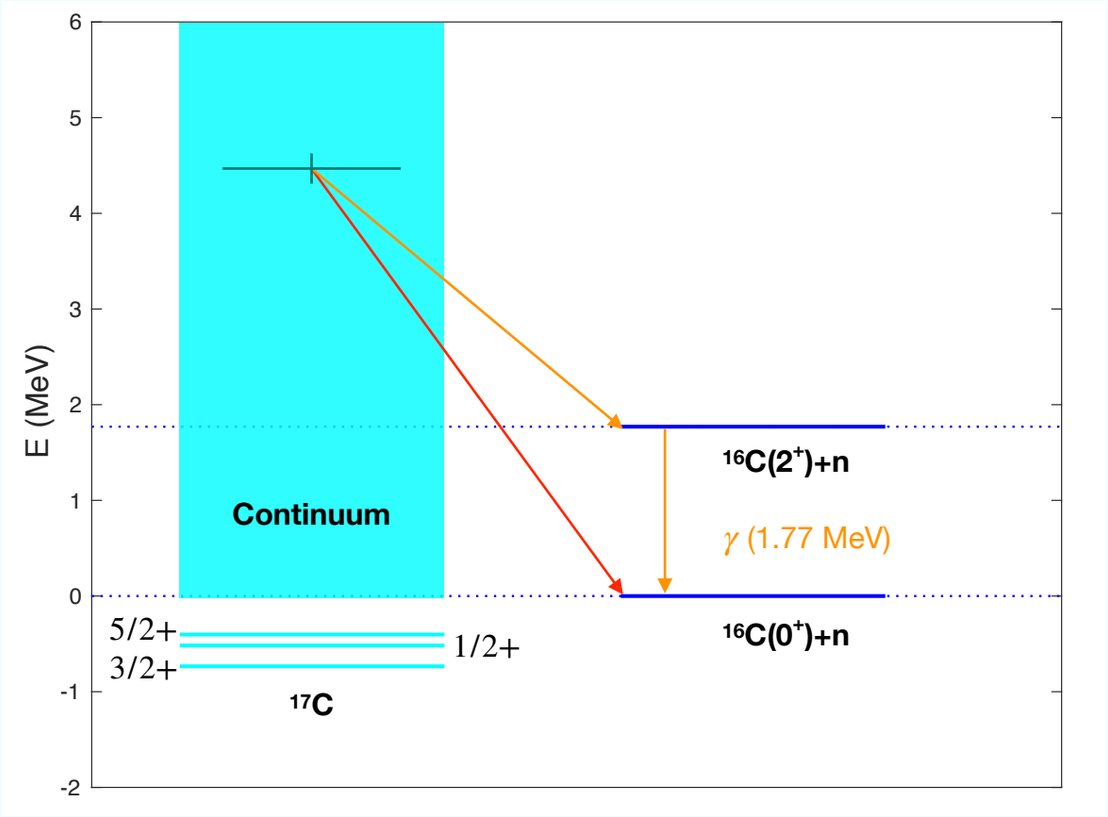
$$\mathscr{F}(k,\theta) \approx \sum_n \langle \Psi_{^{17}C}(k,r) \,|\, \Psi_n^{THO}(r) \rangle \mathscr{F}_n^{THO}(\theta)$$

Transfer to the continuum: $^{16}C(d,p)^{17}C$



Transfer to the continuum: Decay mode

$^{17}C \rightarrow {}^{16}C+n$

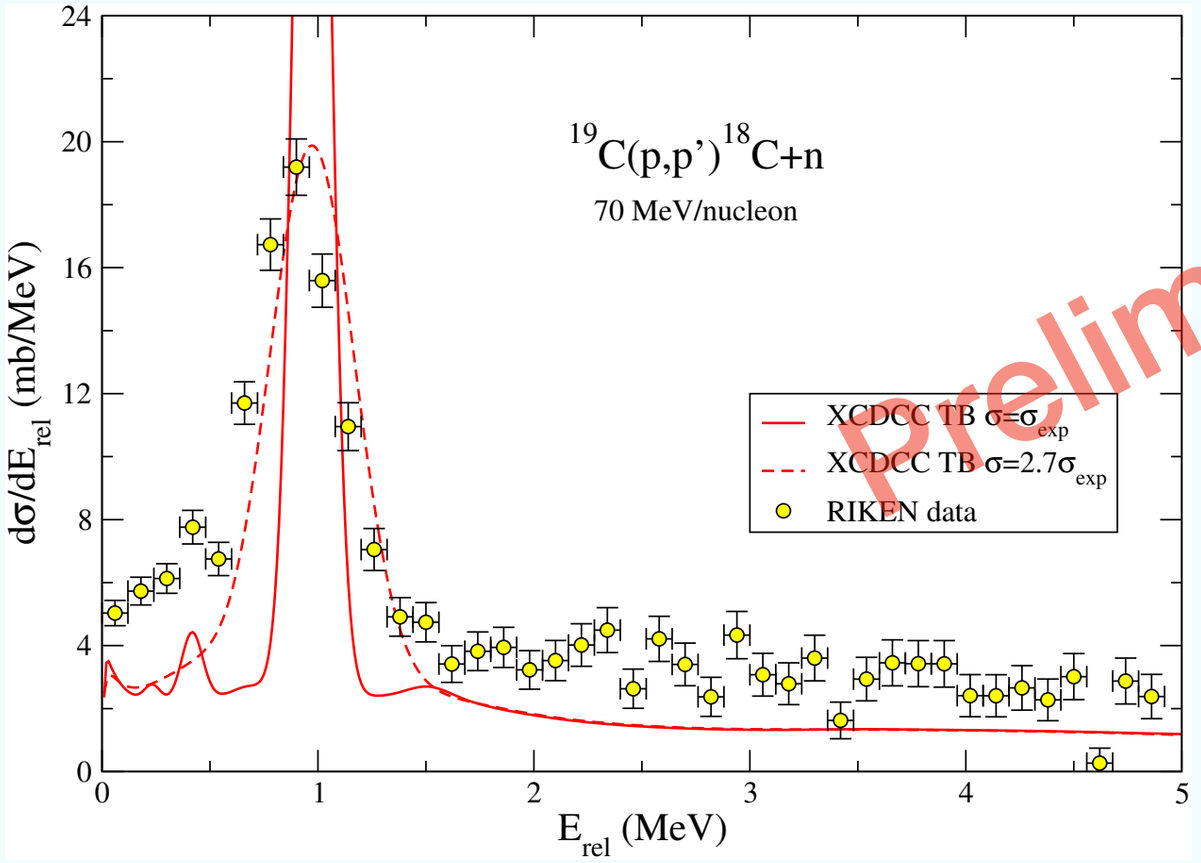


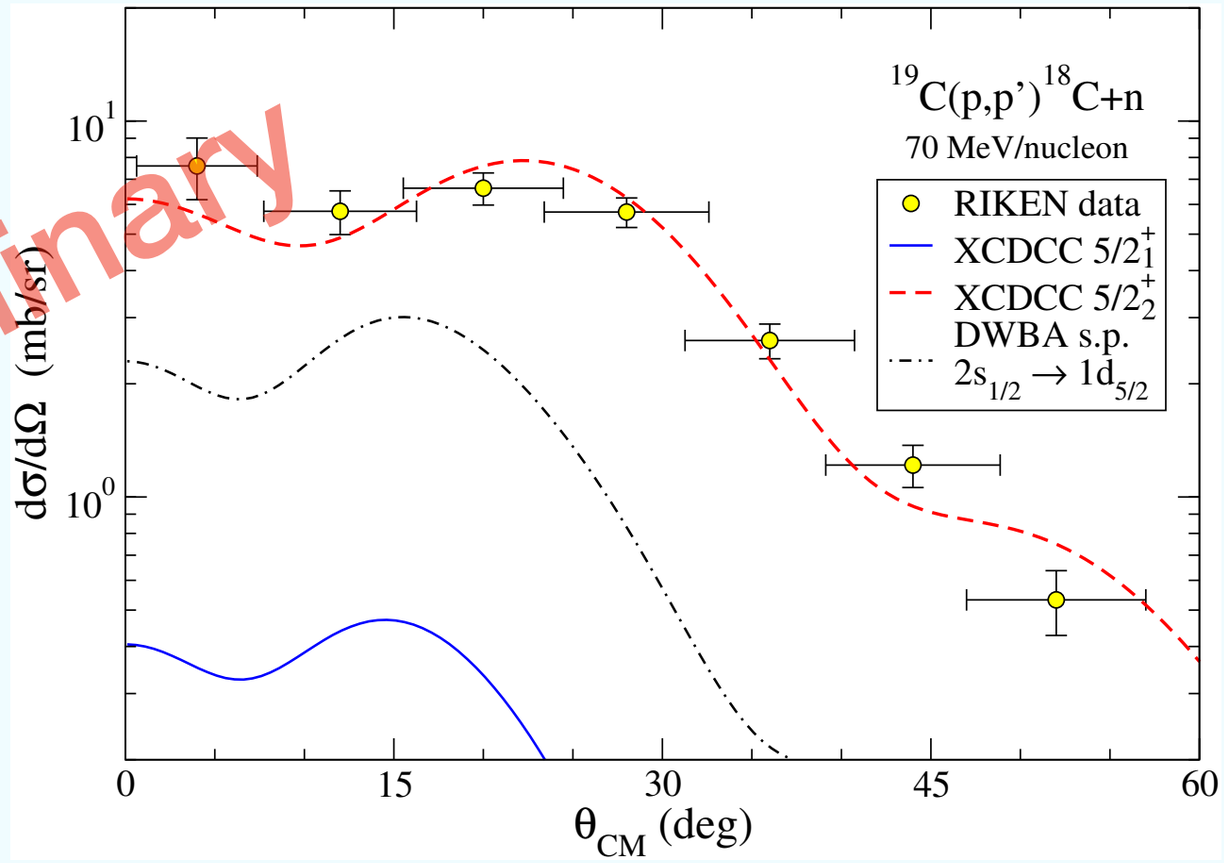
^{19}C Breakup: ^{19}C(p,p')^{18}C+n

- XCDCC calculations
  - n+^{18}C: NAMD model (Total Blocking)
  - p+^{18}C: Convolution of AMD densities with JLM interaction
  - Gaussian n-p potential

- Experimental data from RIKEN [Y. Satou *et al.* PLB**660** (2008) 320]

^{19}C Breakup: ^{19}C(*p*,*p*')^{18}C+n





- XCDCC calculations with NAMD (Total Blocking)
- Experimental data from RIKEN [Y. Satou *et al.* PLB**660** (2008) 320]
- Single-Particle calculations are not consistent with the data