Comparison of shape coexistence and Quantum Phase Transitions around A = 100 for even-even and odd-even cases

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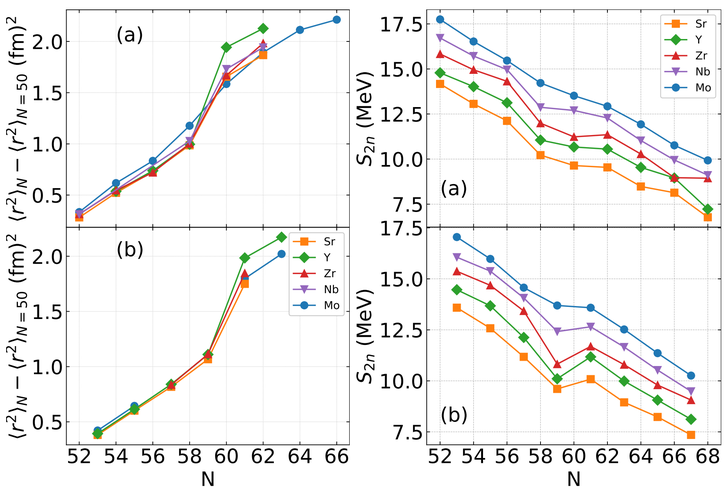
Shape of nuclei is determined by a fine balance between the stabilizing effect of closed shells and the pairing and quadrupole forces that tend to induce deformation [1]. In the mass region around A=100, there exist clear cut examples of the rapid appearance of deformation such as Zr (even-even) [2] and Nb isotopes (odd-even) [3], which can be understood in terms of the coexistence of two different configurations, i.e., shape coexistence. Sr [4] isotopes are also good candidates to study the onset of nuclear deformation and the influence of shape coexistence, while Ru and Mo [5] isotopes seem to be placed at the border of dilution of shape coexistence In addition, the structural evolution of odd-mass isotopes in this region is significant due to the diversity of configurations and coexisting shapes and to the enhancement of the onset of deformation [3]. In this work will be used as framework the Interacting Boson-Fermion Model [6] with Configuration Mixing (IBFM-CM) to introduce a mean-field view (intrinsic state) for studying the evolution of the nuclear deformation in A=100 region, focussing on the case of odd-even Nb isotopes. The objective is to compare the onset of deformation in Nb isotopes with the even-even cases, such as Sr and Zr, extracting information from the intrinsic state. To conclude, by applying the IBFM-CM framework and employing both algebraic and geometric approaches, we aim at providing insights into the evolution of nuclear shapes in even-even and odd-even nuclei in the mass region around A=100.

1 The onset of deformation: differences between even and odd nuclei

The region around N=60 and Z=40 is highly known for the sudden appearance of deformation.

- Nuclear deformation, though not directly observable, can be inferred through nuclear radii systematics and indirectly through S_{2n} analysis along an isotope chain.
- Strong pairing component lead to distinct behaviors in odd and even nuclei: results for radii and S_{2n} are presented separately for even and odd neutron numbers.

Experimental mean-square radii and S_{2n} for Sr, Y, Zr, Nb, and Mo isotopes as a function of the neutron number



2 The Interacting Boson Model

The Interacting Boson Model is a model for even-even nuclei in which the basic constituents are nucleon pairs treated as bosons.

- The building blocks of the model are bosons with angular momentum and parity $J^P = 0^+$ (called s-bosons) and $J^P = 2^+$ (d-bosons). Creation and annhilation operators are introduced as

$$s^\dagger, d^\dagger_m \quad (m = 0, \pm1, \pm2),$$
$$s, d_m \quad (m = 0, \pm1, \pm2).$$

- The simplified IBM Hamiltonian in the extendend consistent-Q formalism:

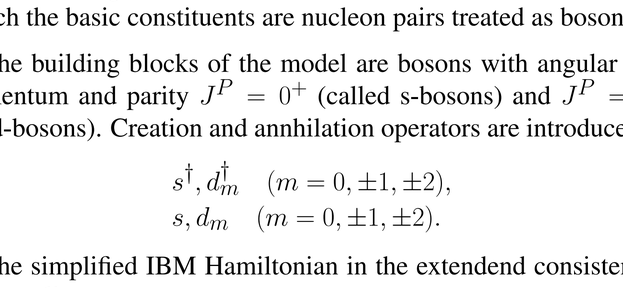
$$\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa' \hat{L}\cdot\hat{L} + \kappa' \hat{Q}\cdot\hat{Q}.$$

The Interacting Boson Model with Configuration Mixing

- Allows the simultaneous treatment of several boson configurations corresponding to particle-hole excitations across a shell or subshell closure.
- The model is based on a Hamiltonian made of two sectors, one corresponding to the regular part, $[N]$, another to the intruder one, $[N+2]$, and an interaction term among them, V_{mix}.

$$\hat{V}^{N,N+2}_{\rm mix} = \omega_0^{N,N+2}(s^\dagger\times s^\dagger + s\times s) + \omega_2^{N,N+2}(d^\dagger\times d^\dagger + \tilde{d}\times\tilde{d})^{(0)}$$

$$\hat{H} = \hat{P}^\dagger_N \hat{H}^N_{\rm ECQF}\hat{P}_N + \hat{P}^\dagger_{N+2}\left(\hat{H}^{N+2}_{\rm ECQF} + \Delta^{N+2}\right)\hat{P}_{N+2} + \hat{V}^{N,N+2}_{\rm mix},$$



3 The Interacting Boson Fermion Model with Configuration Mixing

The IBFM is an extension of the Interacting Boson Model (IBM), coupling bosons (L=0 and L=2) with an unpaired nucleon, either a neutron or proton. The Hamiltonian of the IBFM is written is terms of a boson, a fermion and a boson-fermion part,

$$\hat{H} = \hat{H}_B + \hat{H}_F + \hat{V}_{BF}.$$

The inclusion of 2p-2h excitations implies the use of a boson space $[N]\oplus[N+2]$ and, therefore, the IBFM-CM,

$$\hat{H} = \hat{P}^\dagger_N \hat{H}^N \hat{P}_N + \hat{P}^\dagger_{N+2}\left(\hat{H}^{N+2} + \Delta^{N+2}\right)\hat{P}_{N+2} + \hat{V}^{N,N+2}_{\rm mix},$$

4 Intrinsic state formalism for the IBFM-CM

This formalism is constructed in terms of the original intrinsic state of the IBM,

$$|N;\beta,\gamma\rangle = \frac{1}{\sqrt{N!}}\left(\frac{1}{\sqrt{1+\beta^2}}\left(s^\dagger + \beta\cos\gamma\, d^\dagger_0 + \frac{1}{\sqrt{2}}\beta\sin\gamma\,(d^\dagger_2 + d^\dagger_{-2})\right)\right)^N |0\rangle,$$

adding a fermion that can be in any of the available shells.

Therefore, taking into account the fermion degree of freedom, a matrix form is introduced,

$$\mathcal{H}_{IBFM}(N,\beta,\gamma) = \sum_{j,m}\left(E_B(N,\beta,\gamma) + \epsilon_j\right)a^\dagger_{j,m}a_{j,m} + \sum_{j_1,m_1,j_2,m_2} E_{BF}(N,\beta,\gamma)_{j_1,m_1,j_2,m_2} \times \left(a^\dagger_{j_1,m_1}a_{j_2,m_2} + a^\dagger_{j_2,m_2}a_{j_1,m_1}\right).$$

Where $a_{j,m}(a^\dagger_{j,m})$ are the fermion operators, ϵ_j the fermion single-particle energies, $E_B(N,\beta,\gamma)$ stands for the matrix element of $\hat{H}_B$ with the intrinsic state, and $E_{BF}(N,\beta,\gamma)_{m_1,m_2}$ is the matrix element of $\hat{V}_{BF}$.

Considering the regular and the intruder configurations:

FULL MATRIX TO BE DIAGONALIZED

$$\mathcal{H}^{CM}_{IBFM}(N,\beta,\gamma) = \begin{pmatrix} \mathcal{H}_{IBFM}(N,\beta,\gamma) & \Omega_{BF}(\beta) \\ \Omega_{BF}(\beta) & \mathcal{H}_{IBFM}(N+2,\beta,\gamma)\end{pmatrix},$$

where $\Omega_{BF}(\beta)$ is written as

$$\Omega_{BF}(\beta) = \sum_{j_1,m_1} V_{mix}(\beta)\, a^\dagger_{j_1,m_1}a_{j_1,m_1},$$

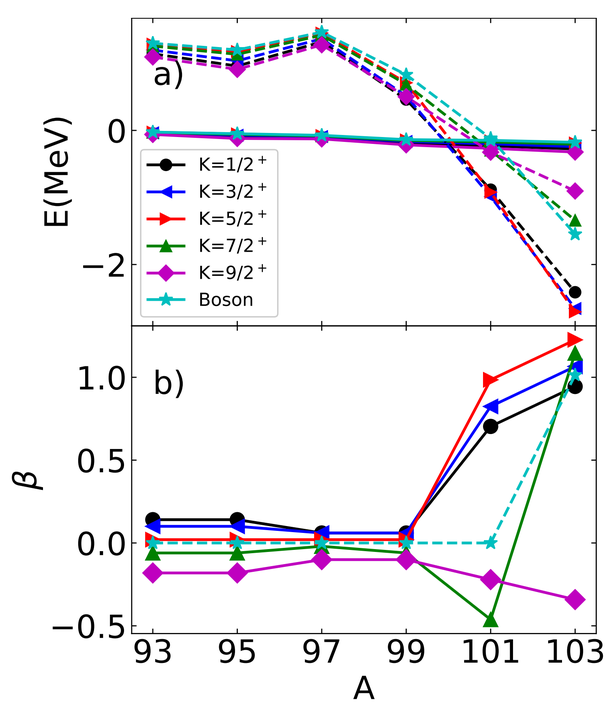
This new IBFM-CM intrinsic state formalism allows to analyze energy surfaces in nuclei with an odd number of nucleons including two different 2p-2h configurations, considering fermions in a single- or a multiple- j shell. Also the triaxial degree of freedom,γ, is explicitly taken into account.

5 Results for Niobium, Z = 41

The Nb isotopes ($^{93-103}$Nb) were analyzed within the IBFM-CM laboratory frame [7], determining the Hamiltonian parameters for this isotope chain, now used in this IBFM-CM intrinsic state description.

POSITIVE PARITY STATES: PROTON IN THE $1g_{9/2}$ SHELL

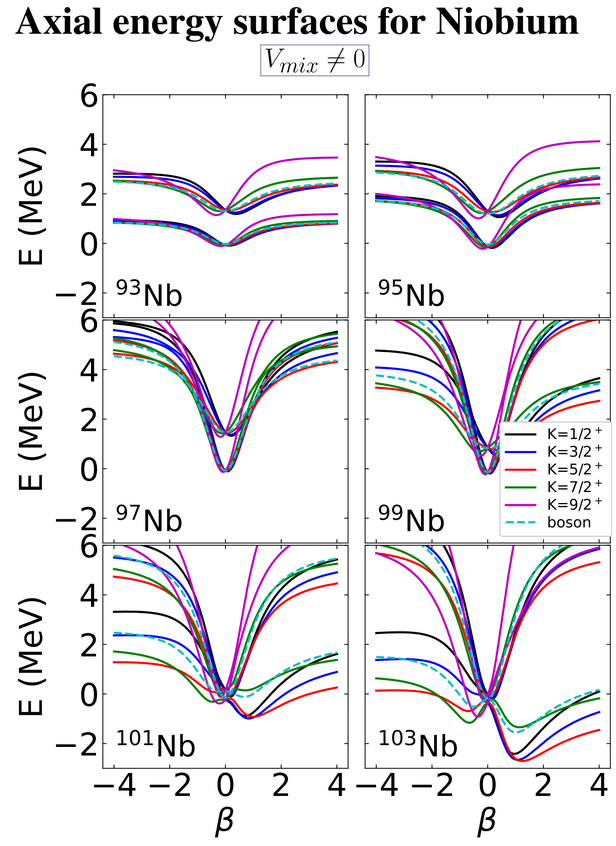
$V_{mix} = 0$



- We show energy of regular (solid lines) vs. intruder states (dashed lines) by angular momentum projection K, assuming **axial symmetry**.
- Regular and intruder boson energies plotted as a reference (cyan line with stars).
- The β value shown, minimizes the energy for each K, regardless of its regular or intruder nature.
- Sudden onset of deformation arises from this crossing, occurring **sooner for boson-fermion systems**.
- The lowest state for $^{93-99}$Nb has $K^\pi = 9/2^+$, which is in **agreement with the experimental ground state** $j = 9/2^+$. In $^{101-103}$Nb, the lowest state has $K^\pi = 5/2^+$, almost degenerated with $K^\pi = 3/2^+$ and $1/2^+$, once more in correspondence with the experimental ground state $j = 5/2^+$.

6 Axial energy surfaces for Niobium

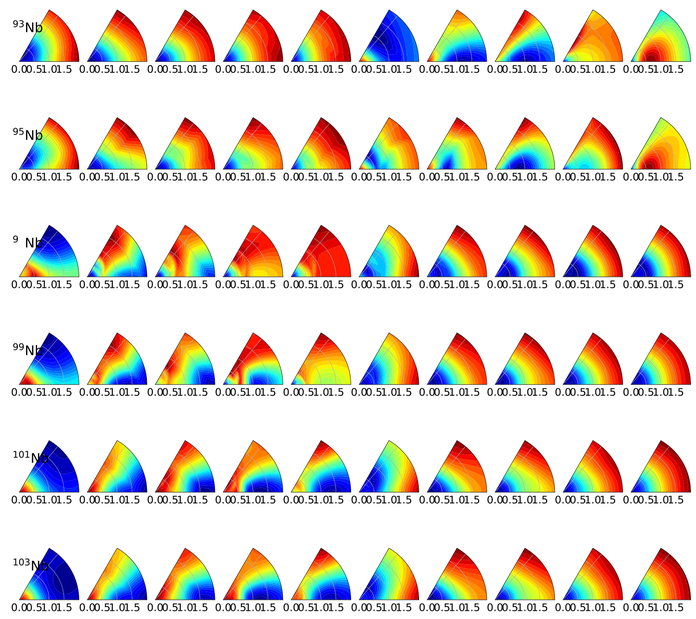
$V_{mix} \neq 0$



- In light isotopes, two distinct groups emerge: low-energy regular states and high-energy intruder states.
- In heavier nuclei, regular and intruder configurations start mixing and crossing.
- For the heaviest isotopes ($^{101-103}$Nb), the lowest state for each K has a deformed minimum with a strong intruder character.

7 Total energy surfaces for Niobium

Using the IBFM-CM mean-field formalism, the full $\beta-\gamma$ energy surface is presented for Nb isotopes.



Conclusions

- We examined nuclear radii and S_{2n} values for nuclei near Z=40 and N=60, analyzing even- and odd-neutron numbers separately.
- While nuclear radii show minimal difference between odd and even neutron cases, S_{2n} reveals notable changes at N=60 in odd-neutron nuclei.
- Additionally, using the new IBFM-CM intrinsic state formalism for Nb isotopes, we identify a configuration crossing at N=60, leading to a Type II quantum phase transition (QPT), in Nb also known in Zr.
- The odd Z value in nuclei seems to amplify the QPT's abrupt nature.

References

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Acknowledgments

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