J/ψ leptoproduction at small transverse momentum

[MGE, SFR, Taels, 2407.04793]
[MGE, Kishore, SFR, 2412.XXXXX]

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eman ta zabal zazu
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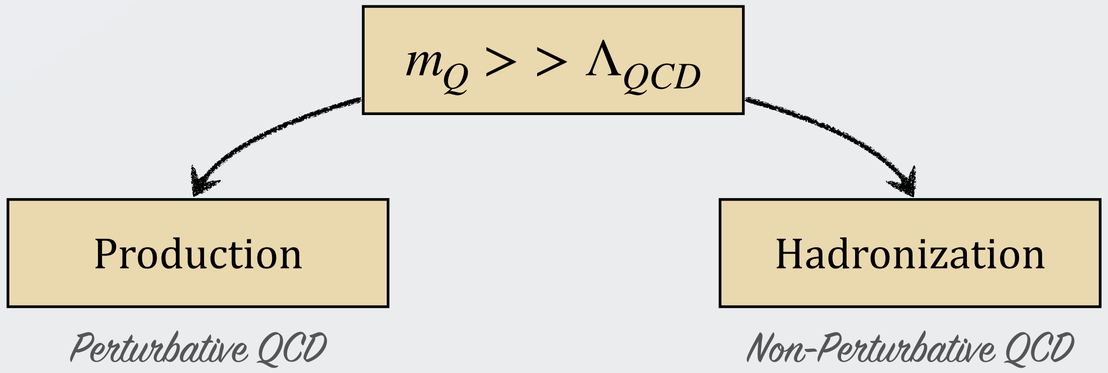
Setup of the talk

Introduction

- We are interested in **TMDs**. [R. A-Martinez, et al, 1507.05267]
- **Heavy quarks** have played a crucial role in the establishing and development of QCD:



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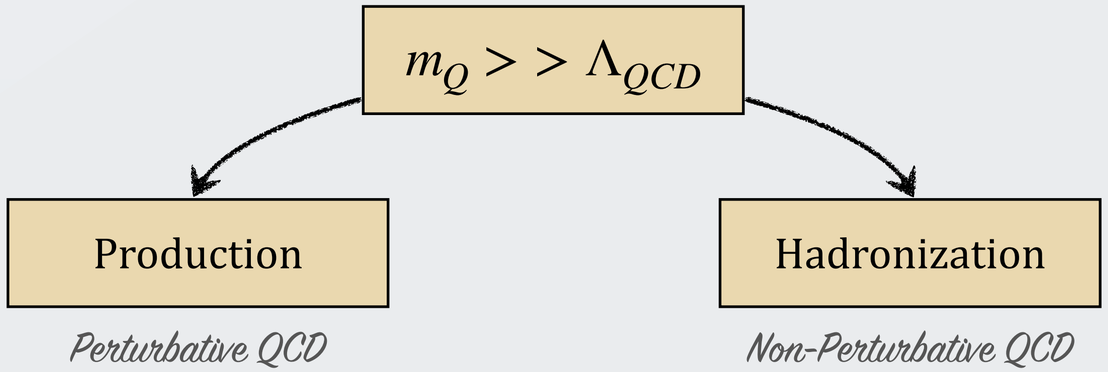

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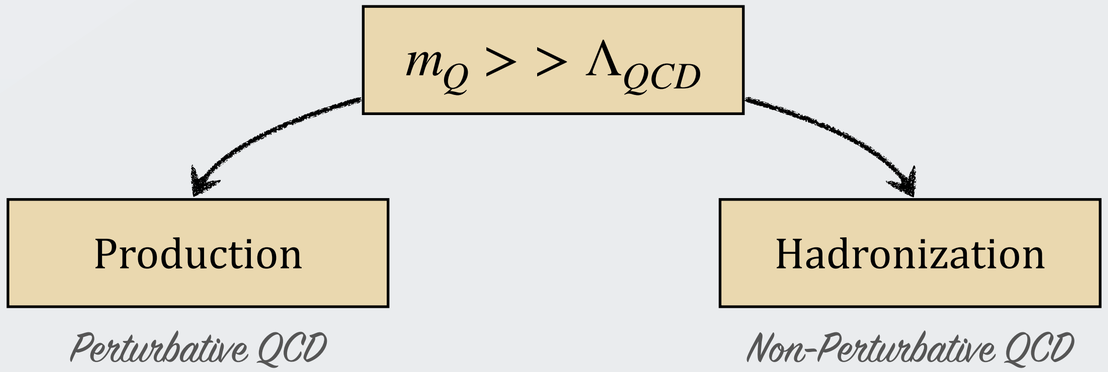

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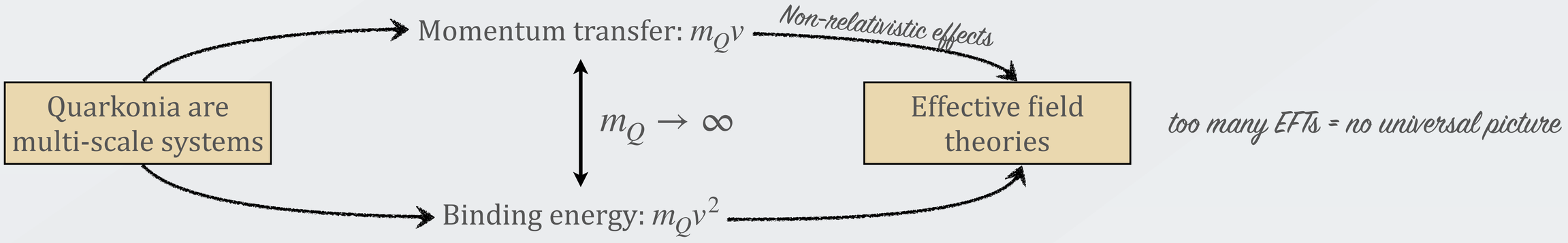
Outline

1. Recap
2. Factorization procedure
3. TMD Shape function
4. Conclusions & Outlook

Recap

- We do not know how is the mechanism of quarkonium production:



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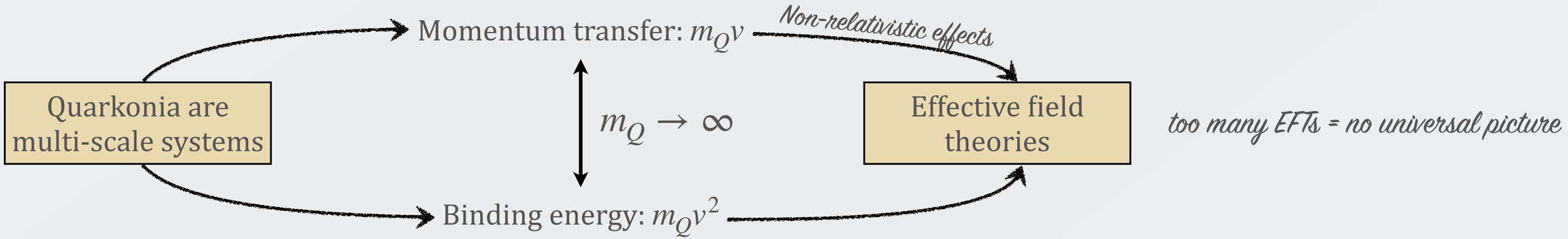

- Non-relativistic QCD (NRQCD) and Color-Singlet Model (CSM) are the most used EFTs. We use NRQCD:

[Bodwin, Braaten, Lepage, hep-ph/9407339]

In NRQCD, at each order of v^2, the long-distance part of the cross section is a number called LDME:

Semi-Inclusive DIS

$$d\sigma(\gamma^* + g \to J/\psi + X) = \sum_{[n]} d\sigma(\gamma^* + g \to c\bar{c}([n]) + X) \left\langle \mathcal{O}_{[n]\to J/\psi} \right\rangle$$

$$[n] = {}^{2S+1}L_J^{[col.]}$$

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[Bodwin, Braaten, Lee, hep-ph/0504014]

such that $$\left\langle \mathcal{O}_{[n]} \right\rangle = \sum_X \langle 0 | \mathcal{K}^\dagger_{[n]}(0) | J/\psi + X \rangle \langle J/\psi + X | \mathcal{K}_{[n]}(0) | 0 \rangle$$

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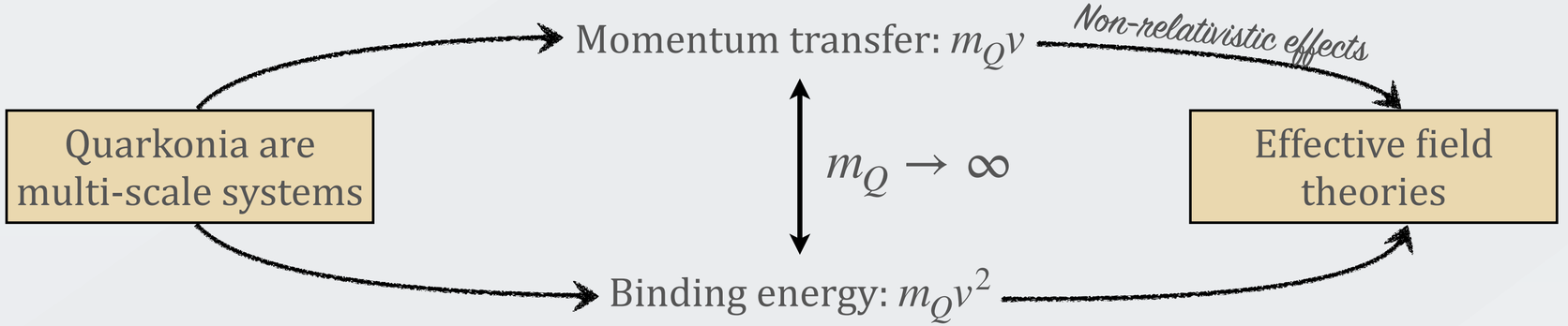

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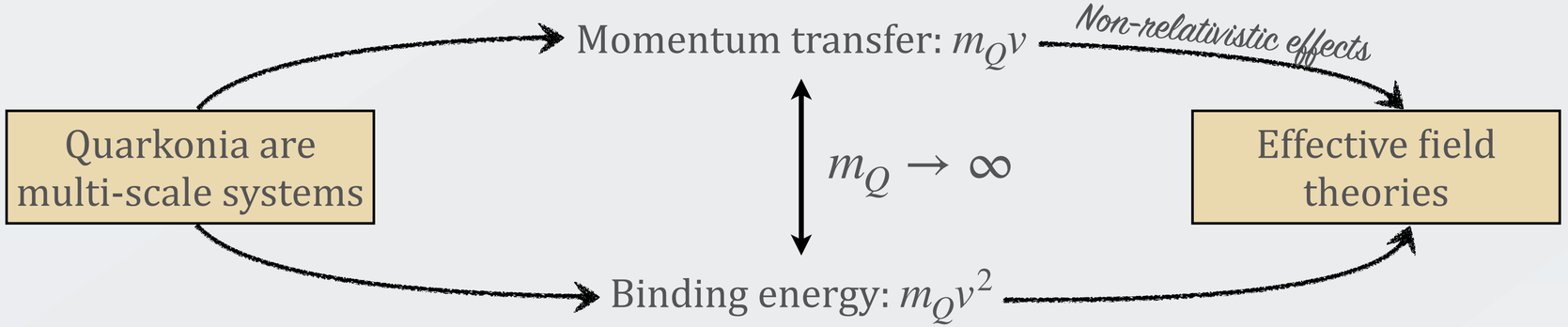

Reviews

[Boer, et. al., 2409.03691]
[Lansberg, 1903.09185]
[Andronic, et. al.,1506.03981]
[Brambilla, et. al., 1404.3723]
[Lansberg, hep-ph/0602091]

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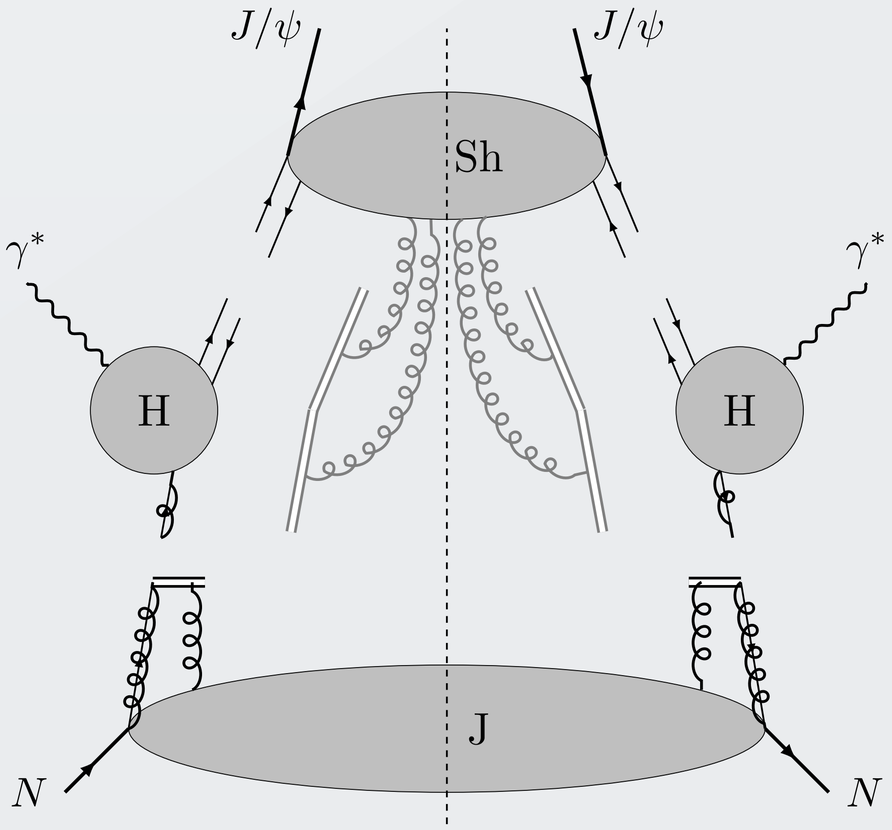

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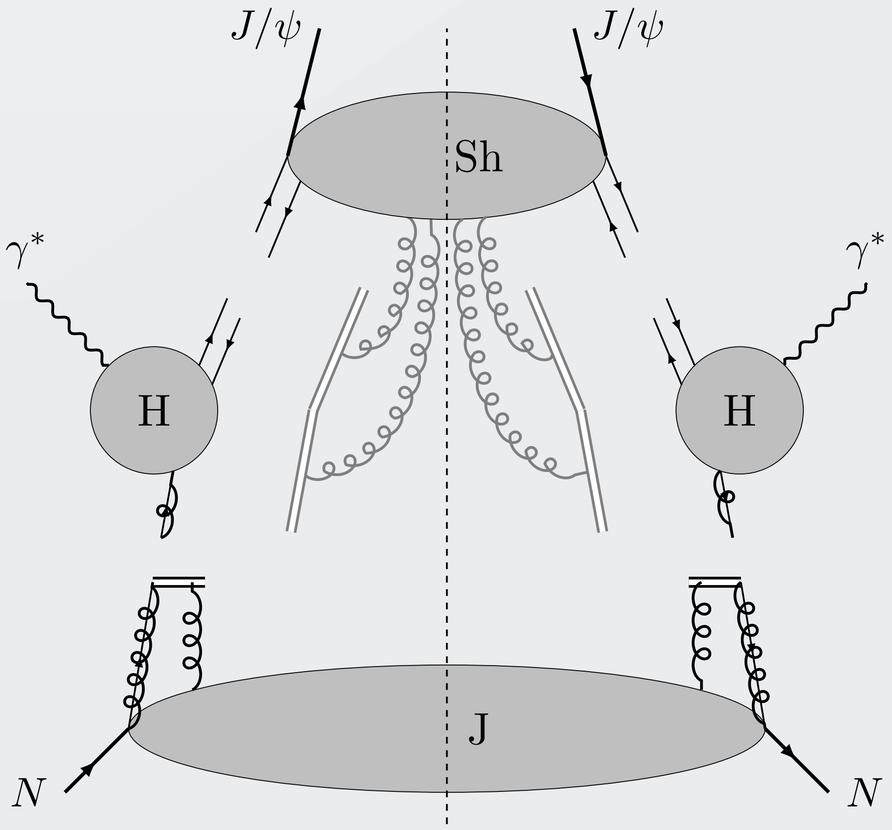

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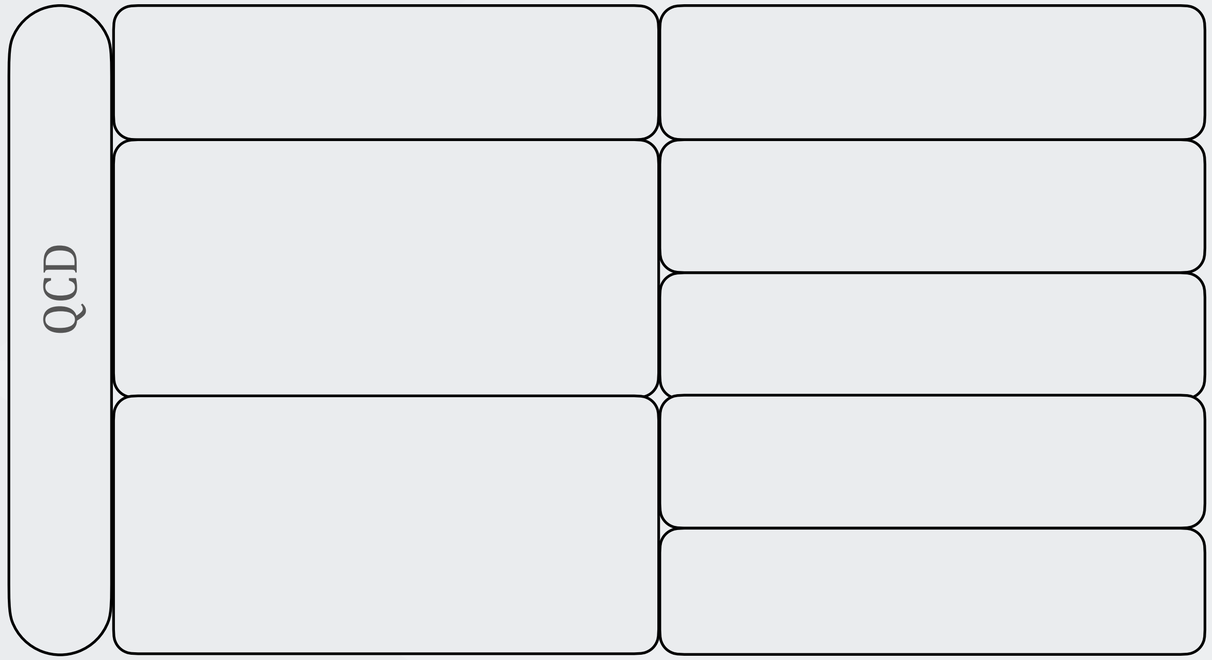

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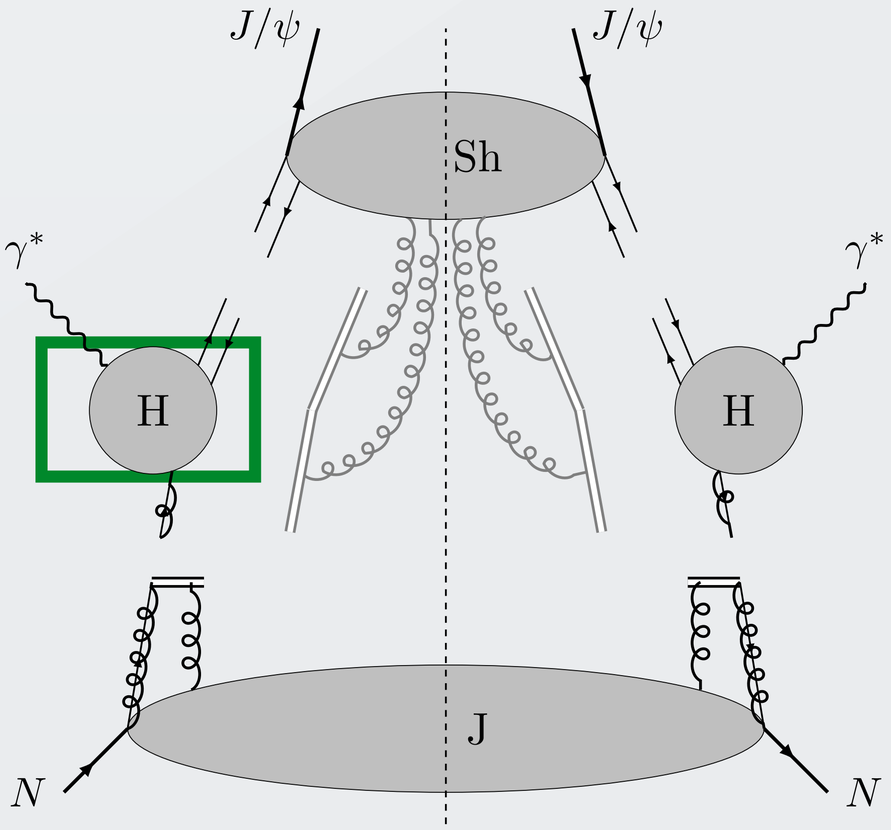

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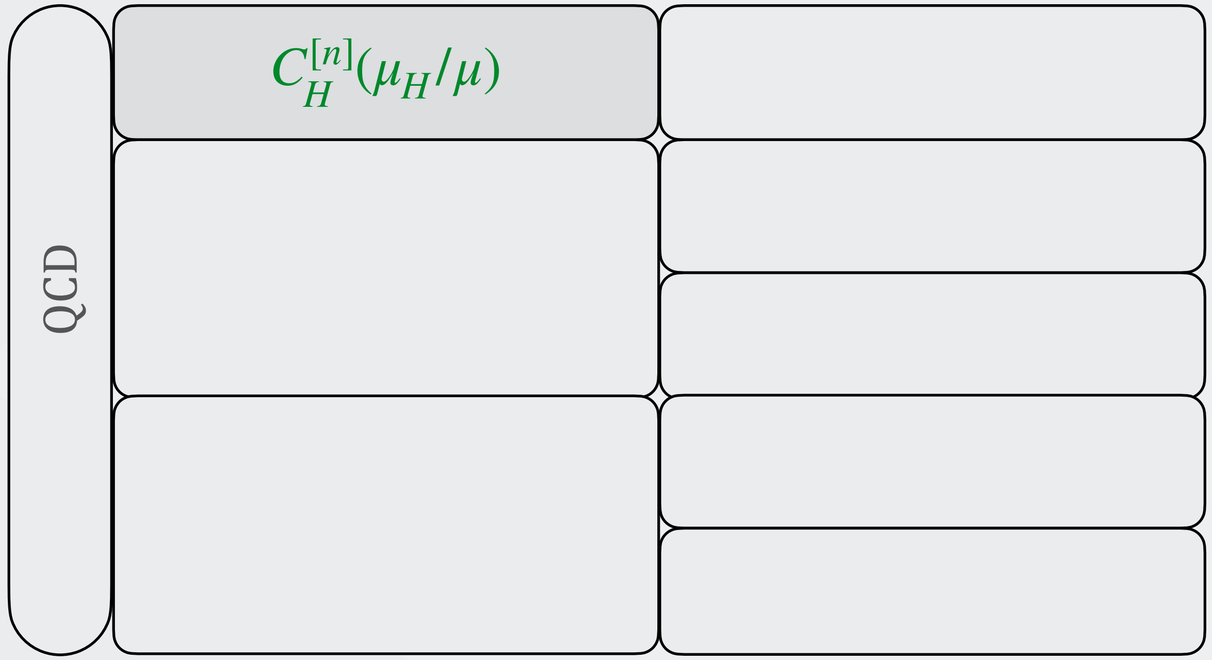

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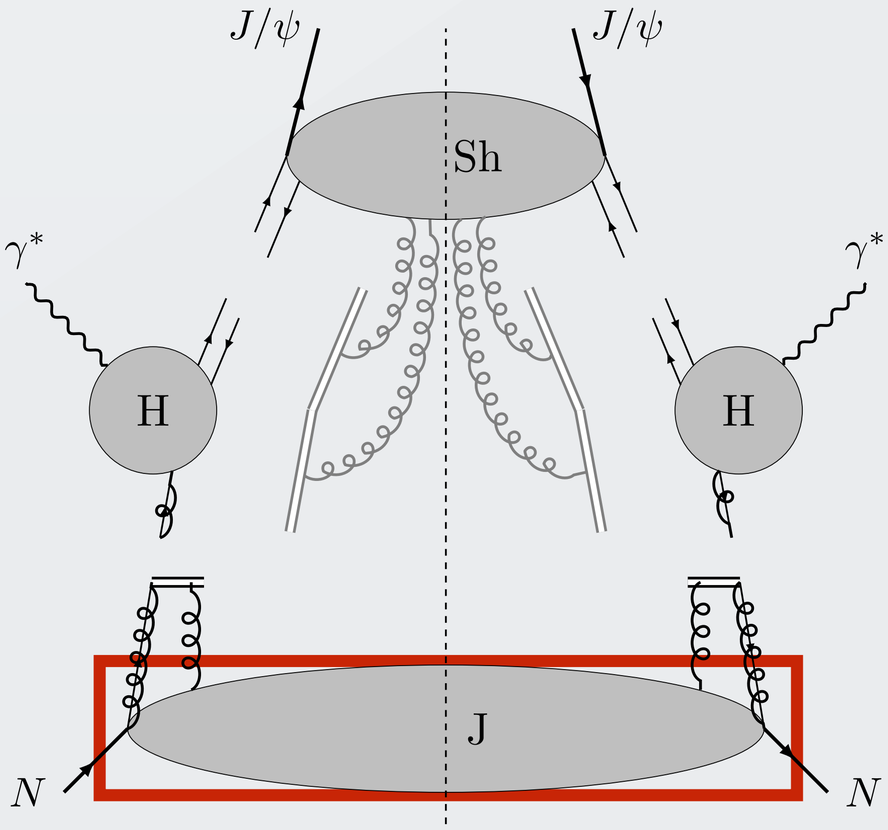

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$G_{g/N}(x, b_T; \mu, \mu_H)$

SCET qt

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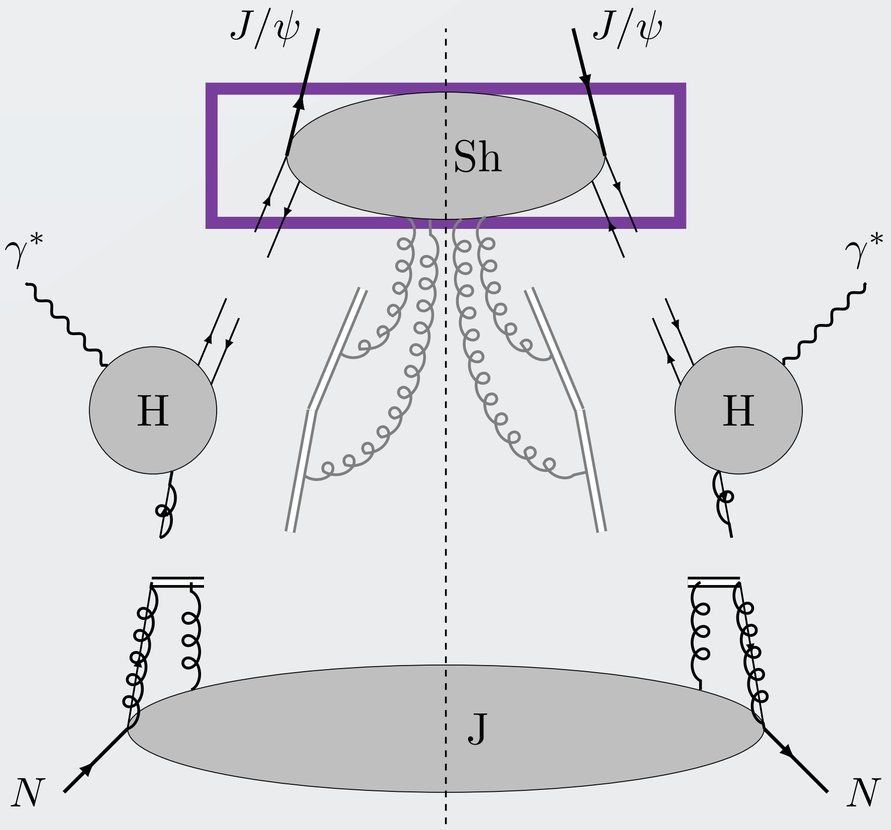

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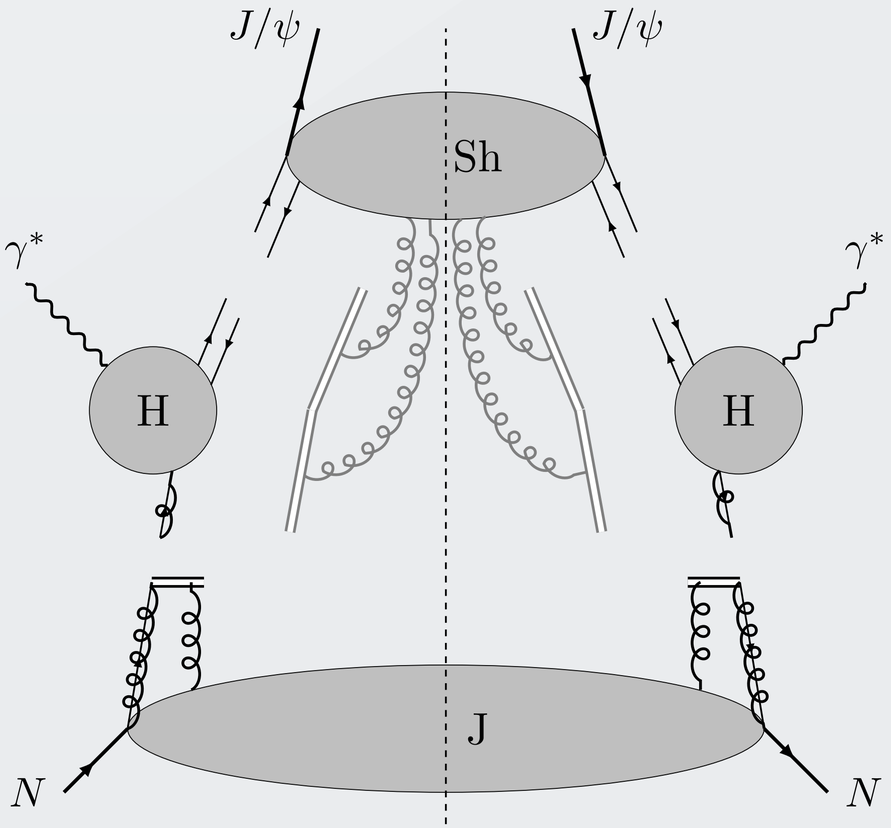

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		$f_{g/N}(x;\mu)$
	$S_{[n]\to Q}(b_T;\mu)$ SCET qt	$C^{[n]}(b_T^2\mu^2, \mu_s^2/\mu^2)$
		$\left\langle \mathcal{O}_{[n]\to J/\psi} \right\rangle(\mu)$

Factorization procedure: Hard part

- The hard part arises from matching QCD onto SCET at the hard scale. One integrates out the hard scale, leaving behind a Wilson coefficient, $C_H(\mu_H^2/\mu^2)$, that represent the part of the cross section containing the hard scale of the process:

$$J^{\mu}_{QCD} \longrightarrow C_H^{[n]}(\mu_H,\mu)\left[\mathcal{S}_v\mathcal{S}_n\,\chi^{\dagger}\left(\Gamma\cdot\mathscr{K}\right)^{\mu\alpha}_{[n]}B_{n\perp,\alpha}\,\psi\right]$$

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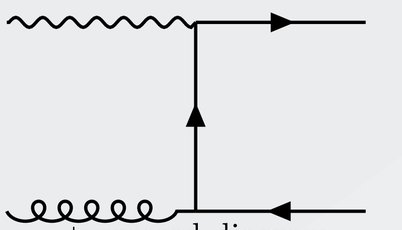

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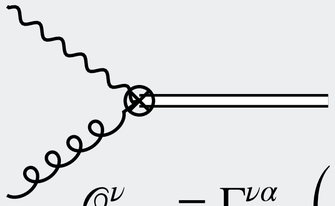

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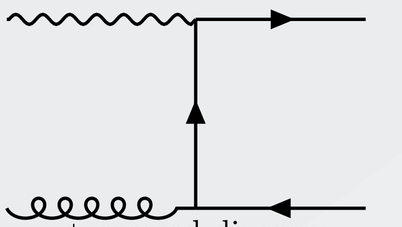

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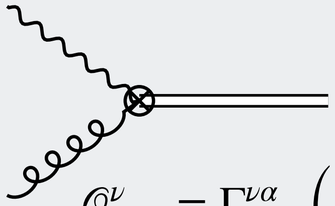

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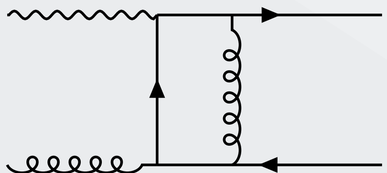

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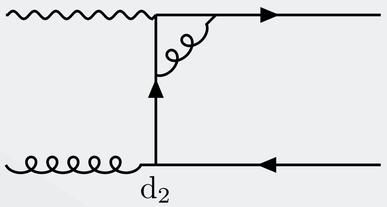

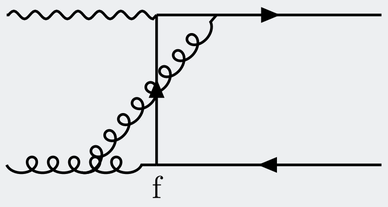

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$$\sigma^{v}_{[n]} = \sigma_{0[n]}\left(\frac{\alpha_s}{2\pi}\right)\left\{\frac{b_0}{\varepsilon_{UV}} - C_A\left(\frac{1}{\varepsilon_{IR}^2}+\frac{1}{\varepsilon_{IR}}\log\left(\frac{\mu^2}{\mu_H^2}\right)-\frac{b_0+C_A}{\varepsilon_{IR}}\right)+\mathscr{D}_{[n]}\right\}$$

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$$H^{[n]}(\mu_H^2/\mu^2) = |\,C_H^{[n]}(\mu_H^2/\mu^2)\,|^2 = 1+\left(\frac{\alpha_s}{2\pi}\right)\mathscr{D}^{[n]}(\mu_H^2/\mu^2)$$

$$= 1+\left(\frac{\alpha_s C_A}{2\pi}\right)\left\{-\frac{1}{2}\log^2\left(\frac{\mu_H^2}{\mu^2}\right)+\log\left(\frac{\mu_H^2}{\mu^2}\right)+\dots\right\}$$

$\mu_H(M,Q^2) = M(1+Q^2/M^2), M = 2m$

When Q goes to zero, this result coincides with photoproduction
[Maltoni, Mangano, Petrelli, hep-ph/9708349]

Factorization procedure: Collinear and Soft modes

- The hadronic tensor for the production of an unpolarized J/ψ is:

$$W^{\mu\nu} = \frac{x_B P_N^+}{Q^2}\left(1-\frac{x_B^2 M_N^2 M_{\psi\perp}^2}{\gamma_q^2 Q^4}\right)\delta\left(z-1-\frac{2x_B^2 M_N^2 (M_{\psi\perp}^2+Q_\perp^2)}{\gamma_q Q^4}\right)\sum_{[n]}\left|C_{[n]}\right|^2 \Gamma_{[n]}^{\dagger\mu\alpha'}\Gamma_{[n]}^{\nu\alpha}\int\frac{\mathrm{d}^2 b_\perp}{(2\pi)^2}e^{ib_\perp\cdot q_\perp}G_{g/N,\alpha\alpha'}(x,b_\perp)\,S_{[n]\to J/\psi}(b_\perp)$$

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Connection to gluon distributions (Motivation)

- Collinear matrix element: (unpolarized) Gluon TMDPDF: [e.g.: MGE, Kasements, Mulders, Pisano, 1502.05354]

$$G^{(0)}_{g/N}(x,b_\perp) = \frac{xP_N^+}{2}\int\frac{dy^- d^2 y_\perp}{(2\pi)^3}e^{-i(xP^+y^-/2-y_\perp\cdot k_{n\perp})}\sum_{X_n}\left\langle P_N\right|B^\mu_{n\perp}(y^-,y_\perp)\left|X_n\right\rangle\left\langle X_n\right|B^\mu_{n\perp}(0)\left|P_N\right\rangle$$

Factorization procedure: Collinear and Soft modes

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$$W^{\mu\nu} = \frac{x_B P_N^+}{Q^2}\left(1-\frac{x_B^2 M_N^2 M_{\psi\perp}^2}{\gamma_q^2 Q^4}\right)\delta\left(z-1-\frac{2x_B^2 M_N^2 (M_{\psi\perp}^2+Q_\perp^2)}{\gamma_q Q^4}\right)\sum_{[n]}\left|C_{[n]}\right|^2 \Gamma^{\dagger\mu\alpha'}_{[n]}\Gamma^{\nu\alpha}_{[n]}\int\frac{\mathrm{d}^2 b_\perp}{(2\pi)^2}e^{ib_\perp\cdot q_\perp} G_{g/N,\alpha\alpha'}(x,b_\perp)\, S_{[n]\to J/\psi}(b_\perp)$$

Connection to gluon distributions (Motivation)

- Collinear matrix element: (unpolarized) Gluon TMDPDF: [e.g.: MGE, Kasements, Mulders, Pisano, 1502.05354]

$$G^{(0)}_{g/N}(x,b_\perp) = \frac{xP_N^+}{2}\int\frac{dy^- d^2 y_\perp}{(2\pi)^3}e^{-i(xP^+y^-/2-y_\perp\cdot k_{n\perp})}\sum_{X_n}\left\langle P_N\right|B^\mu_{n\perp}(y^-,y_\perp)\left|X_n\right\rangle\left\langle X_n\right|B^\mu_{n\perp}(0)\left|P_N\right\rangle$$

- Soft matrix element: (unpolarized) TMD Shape function:

$$S^{(0)}_{{}^1S_0^{[8]}\to J/\psi}(b_\perp) = \frac{1}{N_c^2-1}\mathrm{tr}_c\left\langle 0\right|\left[(\mathcal{S}_v\mathcal{S}_n)^\dagger\chi^\dagger_{\bar{\mathbf{p}}}T^a\psi_{\mathbf{p}}\right](b_\perp)\mathcal{N}_\psi\left[\mathcal{S}_v\mathcal{S}_n\psi^\dagger_{\mathbf{p}}T^a\chi_{\bar{\mathbf{p}}}\right](0)\left|0\right\rangle\ ,$$

$$S^{(0)}_{{}^3P_J^{[8]}\to J/\psi}(b_\perp) = \frac{\Delta_j^{i'j'ij}}{(2J+1)M^2(N_c^2-1)}\sum_{\lambda'}\mathrm{tr}_c\left\langle 0\right|\left[(\mathcal{S}_v\mathcal{S}_n)^\dagger\chi^\dagger_{\bar{\mathbf{p}}}(q^{i'}\sigma^{j'})\,T^a\psi_{\mathbf{p}}\right](b_\perp)\mathcal{N}_{\psi(\lambda')}\left[\mathcal{S}_v\mathcal{S}_n\psi^\dagger_{\mathbf{p}}(q^i\sigma^j)\,T^a\chi_{\bar{\mathbf{p}}}\right](0)\left|0\right\rangle$$

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It is known at two-loops [MGE, Scimemi, Vladimirov, 1511.05590]

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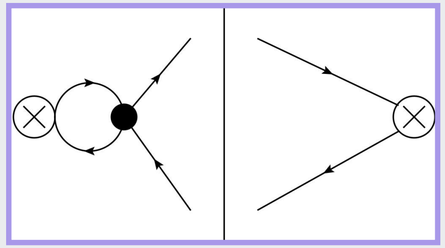
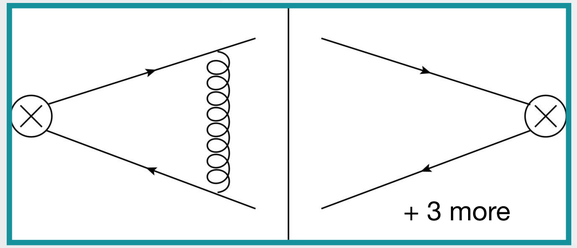
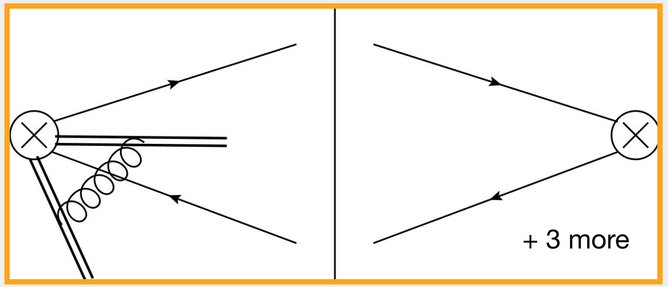
1S0[8] TMDShF at NLO (order α_s)

$$S^{(0)}_{{}^1S_0^{[8]}}(\mathbf{k}_\perp;\mu,\delta) = \frac{\alpha_s}{2\pi} \left\{ \delta^2(\mathbf{k}_\perp) \left((C_F - C_A/2)\frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right) \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} + \frac{8}{3\pi^2 m_c^2} \frac{1}{\mathbf{k}_\perp^2} \left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\mathrm{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\mathrm{LO}} \right) + \frac{S(\mathbf{k}_\perp;\delta)}{2} \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} \right\}$$







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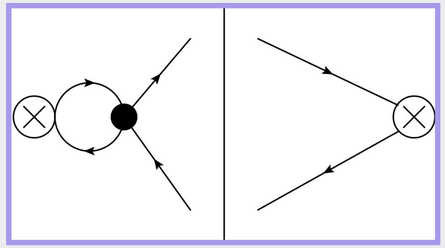

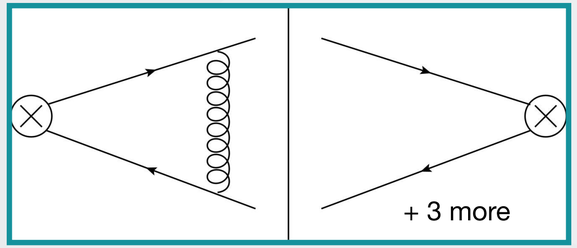

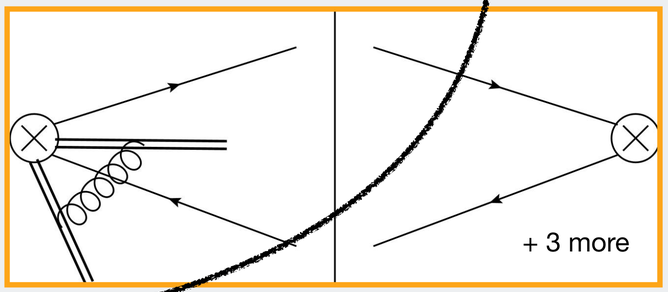

RDs have been cancelled

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- The coefficients C are obtained by matching the TMDShF and the LDMEs order-by-order:

There are not rapidity divergences

We need the LDMEs at NLO. They are known, but we performed the calculation for completeness:

$$\left\langle {}^1S_0^{[8]} \right\rangle(\mu) = \left(1 + (C_F - C_A/2)\frac{\pi\alpha_s(\mu)}{2\mathrm{v}}\right)\left\langle {}^1S_0^{[8]} \right\rangle^{LO} + \frac{4\alpha_s(\mu)}{3\pi m_c^2}\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{LO} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{LO}\right)\left(\frac{1}{\varepsilon_{\mathrm{UV}}} - \frac{1}{\varepsilon_{\mathrm{IR}}}\right)$$

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- The OPE coefficients, C, for the TMDShF are then:

$$C^S_{{}^1S_0^{[8]}}(b_T;\mu,\zeta_B)=1+\frac{\alpha_s(\mu)C_A}{2\pi}L_T(b_T;\mu)\left(1-\ln\zeta_B\right)$$

They are not dependent on the rapidity scale

$$C^S_{{}^1P_1^{[1]}}(b_T;\mu)=-\frac{\alpha_s(\mu)}{2\pi}\frac{8\,C_F}{3m^2}L_T(b_T;\mu)$$

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The tree-level (Born) amplitude is order α_s

… where the cross-section does not have any dependence on any scale:

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Universal — *Process(operator)-dependent*

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$$S_{[n]\to J/\psi}(b_T;\mu_f,\zeta_{B,f}) = S_{[n]\to J/\psi}(b_T;\mu_i,\zeta_{B,i})\, R(b_T;\mu_i,\zeta_{Bi},\mu_f,\zeta_{B,f})$$

$$R:(\mu_i,\zeta_{B,i})\to(\mu_f,\zeta_{B,f}) \qquad \text{such that} \qquad R = \exp\left\{\int_{\mu_i}^{\mu_f}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S\left(\alpha_s(\bar{\mu}),\ln\frac{\zeta_{B,f}}{\bar{\mu}^2}\right)\right\}\left(\frac{\zeta_{B,f}}{\zeta_{B,i}}\right)^{-D_g(b_t;\mu_i)}$$

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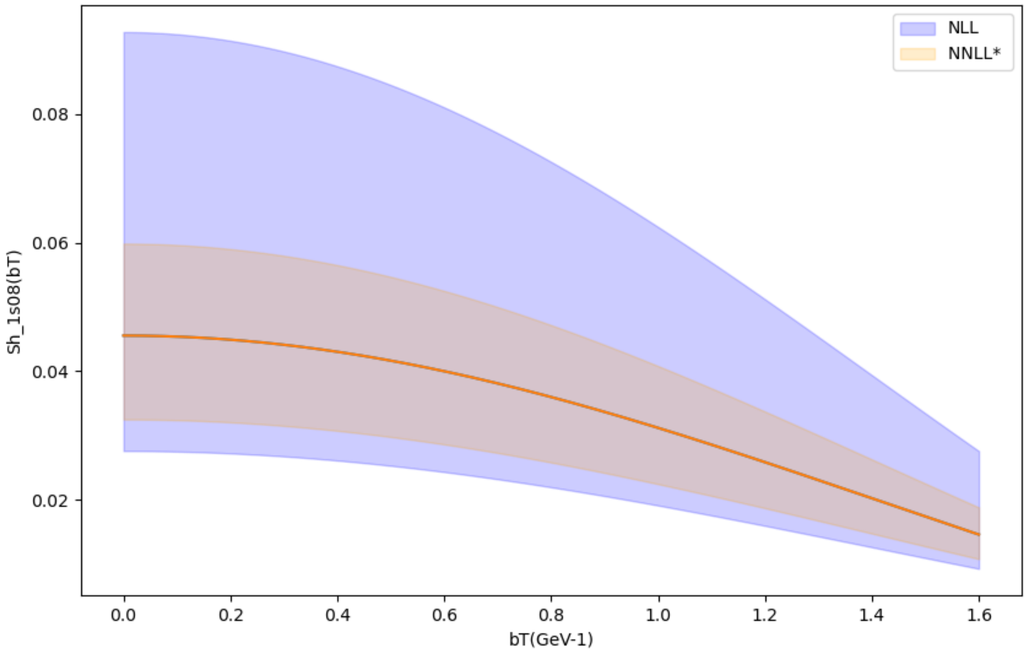
… and the resummation of large logs at the perturbative region will be:

$$S_{[n]\to J/\psi}(b_T;\mu,\zeta_B) = R(b_T;\mu_0,\zeta_{B,0},\mu,\zeta_B)\sum_{[m]} C^{[m]}_{[n]}(b_T;\mu_0,\zeta_{B,0})\times\frac{1}{N_{col.}N_{pol.}}\left\langle\mathcal{O}^{[m]}_{J/\psi}\right\rangle(\mu_0)$$

TMD Shape Function: evolution & resummation

- To finish the talk, I show the first plot of a TMDShF in terms of bT:



The bands come from varying independently both resummation scale μ_0 and rapidity scale ζ_0 by a factor of 2

$$Q = 10\,GeV, \quad \mu = \mu_H = 42\,GeV, \quad \zeta_B = 1$$

- It was plotted at NLL and NNLL* accuracy:

	Γ^A_{cusp}	γ^{nc}	$C^{[n]}_{[m]}$	D_g
LL	α_s	α_s^0	α_s^0	α_s^0
NLL	α_s^2	α_s	α_s^0	α_s
NNLL*	α_s^3	α_s	α_s	α_s^2

- The non-perturbative region is described by:

$$R_{NP} = \exp\left\{-A \ln\left(\frac{\mu_H}{Q_{NP}}\right) b_c^{*2}\right\} \qquad \begin{matrix} A = 0.04\,GeV^2 \\ Q_{NP} = 1\,GeV \end{matrix}$$

$$b_c(b_T) = \sqrt{b_T^2 + \frac{b_0^2}{\mu_H^2}} \qquad b_T^*(b_c) = \frac{b_c}{\sqrt{1 + \frac{b_c^2}{b_{Tmax}^2}}} \qquad b_{Tmax} = 1.5\,GeV^{-1}$$

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- We properly define the TMDShF = LDME + (u)soft contribution.
- We see the cancellation of the rapidity divergences, obtaining a well-defined TMD.
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- Calculation of the hard part (virtual contribution) at two-loops: $NNLL* \to NNLL$
- We are plotting the cross-section: prediction for the EIC
- Next order in α_s, which means LO in the velocity: $\gamma^* + a \to c\bar{c} + a$
- TMDShFs in other interesting processes: double J/ψ production at LHCb…

Conclusions & Outlook

- We derive the factorization theorem for the J/ψ leptoproduction at LP in α_s and NLP in the velocity.
- We properly define the TMDShF = LDME + (u)soft contribution.
- We see the cancellation of the rapidity divergences, obtaining a well-defined TMD.
- We get the behaviour of the TMDShF at the perturbative region in terms of LDMEs.
- We get the evolution for the TMDShF in terms of μ and ζ, and show for the first time its picture.

- Calculation of the hard part (virtual contribution) at two-loops: $NNLL* \to NNLL$
- We are plotting the cross-section: prediction for the EIC
- Next order in α_s, which means LO in the velocity: $\gamma^* + a \to c\bar{c} + a$
- TMDShFs in other interesting processes: double J/ψ production at LHCb...

Thank you!