Three-loop Jet Function for Boosted Heavy Quarks

(Soon to appear in arXiv)

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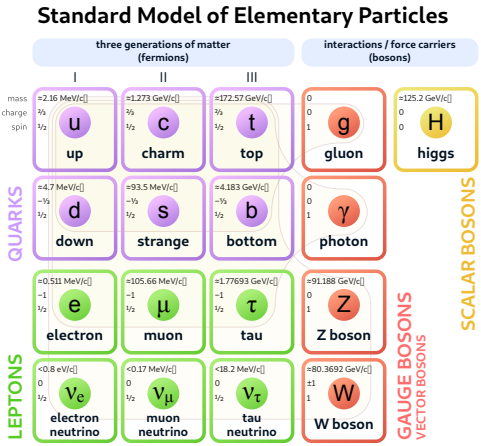
Overview

Introduction

Top quark mass as a test of the Standard Model

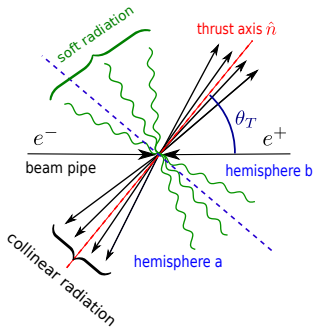


- Couples the strongest to the Higgs.
- Its mass (along with α_s) determines if the electroweak vacuum is stable [Degrassi et al., 2012].
- Uncertainty in Higgs and W mass determinations from electroweak fits sensitive to the top quark mass [Haller et al., 2018].
- For direct m_t measurements, calibrating parton-shower Monte Carlo event generators is crucial.

Using event shapes to measure the top quark mass

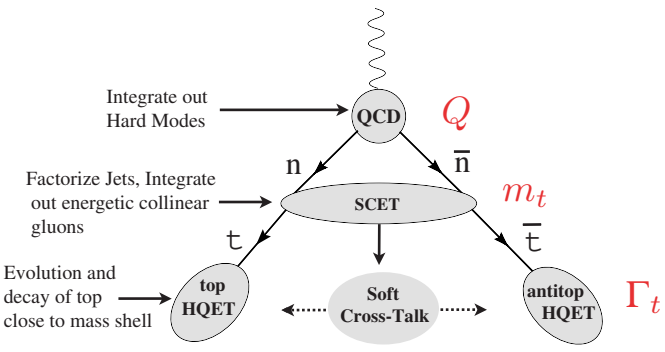
- Event shapes for massless quarks in e^+e^- colliders used to learn about hadronization, check group structure, determine α_s, etc.
- A class of event shapes can be used to determine the top mass with a precision $< \Lambda_{\rm QCD}$ at a future e^+e^- collider [Fleming et al., 2008a].
- The maximal sensitivity is attained in the peak of the distribution where bHQET [Manohar and Wise, 2000] and SCET [Bauer et al., 2002] are applicable.

$$M_1^2 = \left(\sum_{\rm hem.1} p_i^\mu \right)^2$$



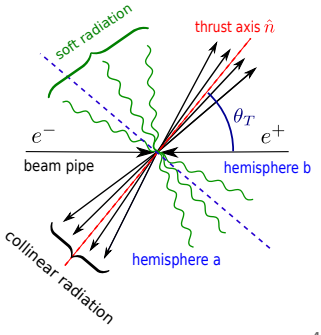
[Mateu and Rodrigo, 2013]

EFT approach in the peak region



[Fleming et al., 2008b]

$$\hat{s}_t \equiv \frac{M_i^2 - m_t^2}{m_t} \sim \Gamma_t \ll m_t$$



Factorization theorem

The double hemisphere invariant mass differential distribution reads

$$\frac{\mathrm{d}^2\sigma^{(\text{dijet})}}{\mathrm{d}M_1^2\mathrm{d}M_2^2} = \sigma_0 H_Q\big(Q,\mu_Q,\mu_m\big) H_m\bigg(m_t,\frac{Q}{m_t},\mu_m,\mu\bigg)$$
$$\times\int \mathrm{d}\ell^+\mathrm{d}\ell^- B_+\bigg(\frac{M_1^2-Q\ell^+}{m_t}-m_t,\Gamma_t,\mu\bigg)B_-\bigg(\frac{M_2^2-Q\ell^-}{m_t}-m_t,\Gamma_t,\mu\bigg)S\big(\ell^+,\ell^-,\mu\big),$$

where, for $\Gamma_t = 0$, $\delta m_t = 0$ and $v = r\cdot x$

$$B(\hat{s},\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int \mathrm{d}^d x\, e^{i\,r\cdot x}\,\langle 0|T\{\bar{h}_v(0)W_n(0)W_n^\dagger(x)h_v(x)\}|0\rangle\right].$$

The three-lop jet function

- The jet function is already known **analytically** at two loops [Jain et al., 2008].

  We increase the precision by one order reaching three-loop accuracy.

- Our results can be used to obtain the top quark mass at a future linear or circular e^+e^- collider with high precision and to improve the calibration between $m_t^{\rm MC}$ and $m_t^{\rm MSR}$ at N^{3}LL' [Hoang, 2020].

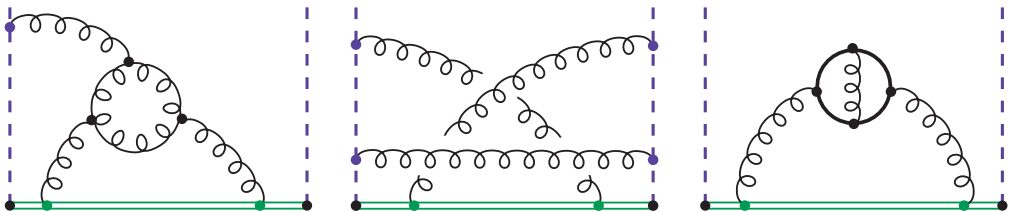
- Once H_m is known at three loops (and Γ_c at 5 loops), the thrust cross sections in the peak region will reach N^{3}LL' (N^{4}LL) precision.

Computation of the jet function

The bHQET jet-function

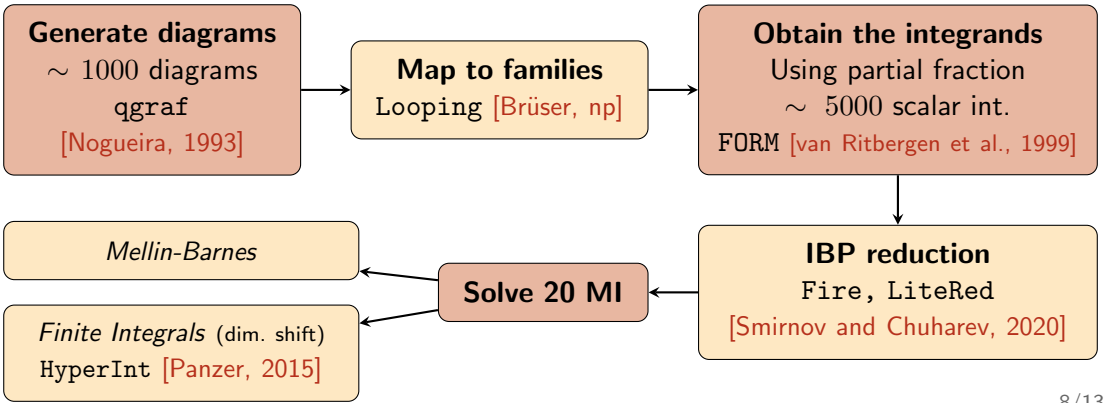
$$B(\hat{s},\mu) \equiv \mathrm{Im}\left[\frac{-i}{4\pi N_c m_t}\int \mathrm{d}^d x\, e^{i\,r\cdot x}\,\langle 0|T\{\bar{h}_v(0)W_n(0)W_n^\dagger(x)h_v(x)\}|0\rangle\right] \qquad (v\cdot r=\hat{s})$$

The momentum r only flows through the HQET propagators in the same direction as v.

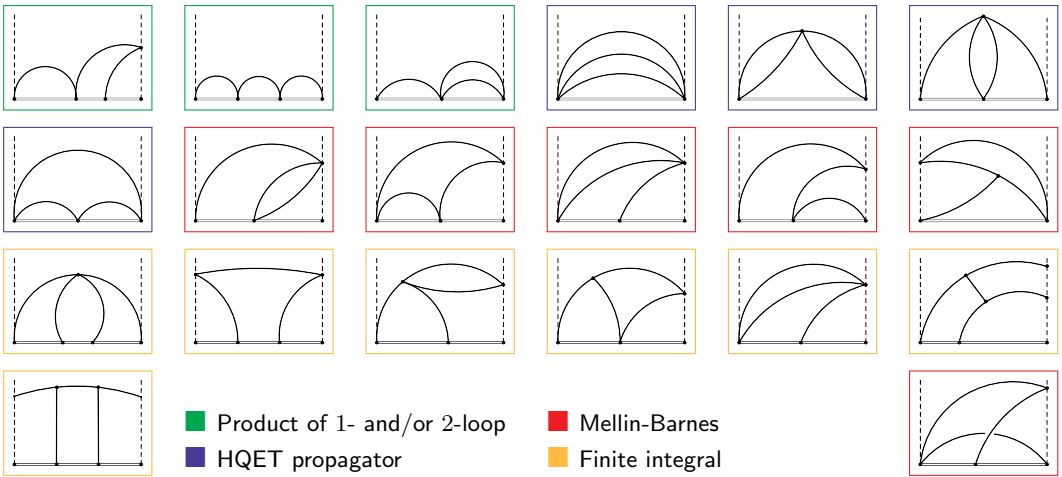


Strategy of a multiloop computation

The computation requires special techniques to handle the large number of diagrams.



Master integrals



Result for the three-loop jet function

Result for the three-loop jet function

The $\mathcal{O}(\alpha_s^3)$ non-logarithmic part of $\log(\mathrm{FT}[B])$ reads

$$
\begin{aligned}
&(\log \tilde{B})_{30} = \\
&\Big(\frac{203\pi^2\zeta_3}{27} - \frac{105398\zeta_3}{243} + \frac{236\zeta_3^2}{9} + \frac{902\zeta_5}{9} + \frac{31952191}{26244} + \frac{93821\pi^2}{8748} - \frac{3023\pi^4}{4860} + \frac{1031\pi^6}{10206}\Big) C_F C_A^2 \\
&+ \Big(\frac{3488\zeta_3}{243} + \frac{846784}{6561} - \frac{8\pi^2}{243} + \frac{52\pi^4}{1215}\Big) C_F T_F^2 n_\ell^2 + \Big(\frac{10760\zeta_3}{81} + \frac{8\pi^2\zeta_3}{9} + \frac{224\zeta_5}{9} - \frac{124717}{486} - \frac{55\pi^2}{54} + \frac{148\pi^4}{405}\Big) C_F^2 T_F n_\ell \\
&+ \Big(\frac{1664\zeta_3}{81} - \frac{76\pi^2\zeta_3}{27} - \frac{88\zeta_5}{3} - \frac{5273287}{6561} - \frac{12793\pi^2}{2187} - \frac{421\pi^4}{1215}\Big) C_F C_A T_F n_\ell \,.
\end{aligned}
$$

- ✓ All MI checked with `FIESTA`/`pySecDec`.
- ✓ Gauge parameter drops out.
- ✓ Agrees with the numerical estimate of [Gracia and Mateu, 2021].
- ✓ T_F^2 matches [Gracia and Mateu, 2021].
- ✓ Recovers the anomalous dimension.
- ✓ Verifies non-Abelian exponentiation th. [Gardi et al., 2013].

Other topics discussed in this paper

Coefficient relations

$$\tilde{B}_{in} = \frac{(-1)^n}{n!} \sum_{j=n-1}^{2i-1} B_{ij} |j|! \, \kappa_{j+1-n}$$

One- and two-loop results for arbitrary d

$$\mathcal{B}_1^{\text{bare}} = 4 \frac{d-2}{d-4} \mathsf{MI}^{1\mathsf{L}} C_F$$

Closed forms for Z factor

$$z(\gamma, \alpha_s) \equiv \int_0^{\alpha_s} \frac{\mathrm{d}x}{x} \frac{\gamma(x)}{\varepsilon + \hat{\beta}(x)}$$

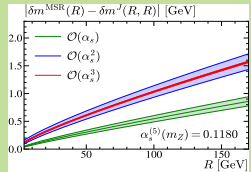
$$Z_\alpha(\alpha_s) = e^{-z(\hat{\beta}, \alpha_s)}$$

Jet function for unstable quarks

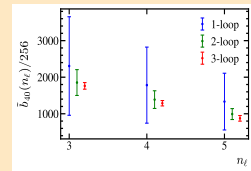
$$B(\hat{s}, \delta m, \Gamma_t, \mu) =$$

$$\exp\left(-2\delta m \partial_{\hat{s}}\right) \int \frac{\mathrm{d}\hat{s}'}{\pi} \frac{B(\hat{s}', \mu)\Gamma_t}{(\hat{s} - \hat{s}')^2 + \Gamma_t^2}$$

Short-distance masses



Fourth-loop estimate



Summary

- The jet function is a common ingredient in the factorization theorems for several observables in boosted heavy quark production.
- It takes a central role in determining the top quark mass in a renormalon-free scheme and in parton shower Monte Carlo generators.
- We outlined some of the techniques required to solve the problems that appear when studying processes at more than one loop.
- **We have analytically computed the 3-loop jet function**: to appear soon in arXiv.
- Paves the way to N^{3}LL' and N^{4}LL resummed cross sections.
- Outlook: Phenomenological analysis (see [Bachu et al., 2021] for 2-loop study).

Thank you for your attention!

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Closed forms for Z factor

$$z(\gamma,\alpha_s) \equiv \int_0^{\alpha_s} \frac{\mathrm{d}x}{x} \frac{\gamma(x)}{\varepsilon+\hat{\beta}(x)}, \qquad \beta_{\mathrm{QCD}}(\alpha_s) = -2\alpha_s\hat{\beta}(\alpha_s).$$

- For the strong coupling, $\alpha_s^{\mathrm{bare}} = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\varepsilon} Z_\alpha[\alpha_s(\mu)]\,\alpha_s(\mu)$:

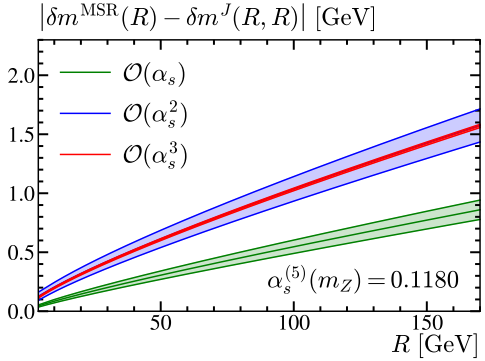
$$\mu\frac{\mathrm{d}\alpha_s}{\mathrm{d}\mu} \equiv -2\varepsilon\alpha_s + \beta_{\mathrm{QCD}}(\alpha_s), \qquad Z_\alpha(\alpha_s) = \exp\bigl[-z\bigl(\hat{\beta},\alpha_s\bigr)\bigr].$$

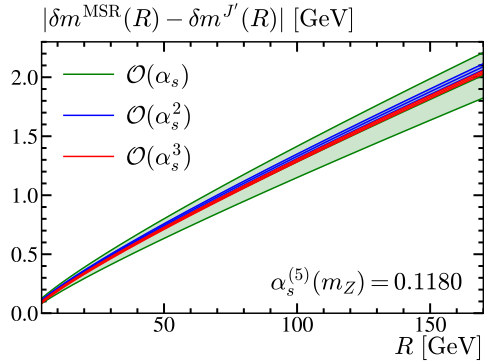
- For the position-space jet function, $\tilde{B}^{\mathrm{bare}}(x) = \tilde{Z}_B(x,\mu)\,\tilde{B}(x,\mu)$:

$$\frac{\mathrm{d}\log\tilde{Z}_B}{\mathrm{d}\log\mu} = -2\Gamma^{\mathrm{c}}(\alpha_s)\log(ix\mu e^{\gamma_E}) - \gamma_B^{\mathrm{nc}}(\alpha_s), \qquad \tilde{Z}_B(x,\mu) = C_B[\alpha_s(\mu)]\,(ix\mu e^{\gamma_E})^{z_B[\alpha_s(\mu)]}.$$

$$C_B(\alpha_s) = \exp\left[\frac{1}{2}z(\gamma_B^{\mathrm{nc}}+z_B,\alpha_s)\right], \qquad z_B(\alpha_s) = z(\Gamma^{\mathrm{c}},\alpha_s).$$

Short-distance masses





$$\delta m^{J}(\mu,R) = \frac{Re^{\gamma_E}}{2}\left\{\frac{\mathrm{d}}{\mathrm{d}\log(ix)}\log\left[m\tilde{B}(x,\mu)\right]\right\}_{ixe^{\gamma_E}=1/R}$$

$$\delta m^{J'}(R) = \frac{Re^{\gamma_E}}{2}\log\left[m\tilde{B}\left(\frac{1}{iRe^{\gamma_E}},R\right)\right]$$