A precise α_s determination from the R-improved QCD Static Energy

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XVI CPAN DAYS

Introduction

- α_s is the most important parameter in QCD $\rightarrow$ should be determined with high precision.
- Our strategy $\rightarrow$ comparing the QCD Static Energy obtained in lattice simulations with highly accurate perturbative results.
- We improve this method building on top of previous analyses [PRD 100 (2019) 11] in several ways (see also recent analysis [JHEP 09 (2020) 016]):
  - Leading renormalon subtraction $\rightarrow$ short-distance scheme (MSR).
  - Resummation of associated large logs with R-evolution.
  - Profiles functions for the renormalization scales.
- Thanks to these, can fit (for the first time) lattice data up to $r \sim 1\,\text{fm}$.

Static Energy

- It is defined as the potential energy between an infinitely massive quark anti-quark pair at a distance r, corrected by ultra-soft effects.
- Large ultra-soft logarithms can be summed up using an EFT framework.
- pNRQCD $\rightarrow$ Sum of a matching coefficient and an ultra-soft contribution.

$$E_{\mathrm{s}}(r) = V_{\mathrm{s}}(r, \mu, \mu_{\mathrm{us}}) + \delta_{\mathrm{us}}(r, \mu, \mu_{\mathrm{us}}) \equiv -C_F \frac{\alpha_{E_{\mathrm{s}}}(r)}{r},$$

Static potential

- Basic object to understand the behavior of QCD:

$$V_{\rm s}(r,\mu,\mu_{\rm us}) = V_{\rm s}^{\rm soft}(r) + V_{\rm s}^{\rm us}(r,\mu,\mu_{\rm us}) \equiv -C_F\frac{\alpha_{V_s}(r,\mu,\mu_{\rm us})}{r},$$

$$V_{\rm s}^{\rm soft}(r) = -C_F\frac{\alpha_s(\mu)}{r}\sum_{i=0}^{3}\left[\frac{\alpha_s(\mu)}{4\pi}\right]^i\sum_{j=0}^{i}a_{ij}\log^j\left(r\mu e^{\gamma_E}\right) \equiv -C_F\frac{\alpha_{V_s^{\rm soft}}(r)}{r}.$$

- Coefficients a_{i0} are known to four loops. $a_{i,j\geq 0}$ obtained with RGE.

- $V_{\rm s}$ known in perturbation theory up to $\mathcal{O}(\alpha_s^4)$ $\rightarrow$ ultra-soft contributions show up

$$V_{\rm s}^{\rm us}(r,\mu_s,\mu_{\rm us}) = -\frac{C_A^3C_F}{12\pi}\frac{\alpha_s^3(\mu_s)\alpha_s(\mu_{\rm us})}{r}\log\left(\mu_{\rm us}re^{\gamma_E}\right).$$

Ultra-soft term and resummation

$$\delta_{\mathrm{us}}\left(\mu_s, \mu_{\mathrm{us}}\right) = -\frac{C_A^3 C_F}{12\pi} \frac{\alpha_s^3\left(\mu_s\right) \alpha_s\left(\mu_{\mathrm{us}}\right)}{r} \log\left[\frac{C_A \alpha_s\left(\mu_s\right) e^{-5/6}}{\mu_{\mathrm{us}} r}\right]$$

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- One can see the necessity of resummation with these two (incompatible) choices that minimize either log.
  - $\mu_{us} \sim \alpha_s/r$ [use this one: no large logs in $\delta_{\rm us}$]
  - $\mu_{us} \sim 1/r$ [discarded: sum up logs in $V_s^{\rm us}$]

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$$\mu \frac{\mathrm{d}V_s(r,\mu_s,\mu)}{\mathrm{d}\mu} = -\frac{2C_F}{3r}\frac{\alpha_s(\mu)}{\pi}\left[1 + B\frac{\alpha_s(\mu)}{\pi}\right](rV_o - rV_s)^3 .$$

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- Integrating from $\mu = \mu_s \sim 1/r$ to $\mu = \mu_{us} \sim \alpha_s/r$

$$\begin{aligned} V_s\left(r,\mu_s,\mu_{\mathrm{us}}\right) =& V_s\left(r,\mu_s\right) + U_{\mathrm{us}}\left(r,\mu_s,\mu_{\mathrm{us}}\right), \\ U_{\mathrm{us}}\left(r,\mu_{s,},\mu_{us}\right) =& \frac{C_A^3 C_F}{6\beta_0 r}\alpha_s^3\left(\mu_s\right)\left\{\left(1+3\frac{\alpha_s\left(\mu_s\right)}{4\pi}\left[a_{1,0}+2\beta_0\log\left(r\mu_s e^{\gamma_E}\right)\right]\right)\log\left[\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\alpha_s\left(\mu_s\right)}\right]\right. \\ &\left. + \left(B-\frac{\beta_1}{4\beta_0}\right)\left[\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\pi}-\frac{\alpha_s\left(\mu_s\right)}{\pi}\right]\right\} \end{aligned}$$

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- Replacing in the static energy $\rightarrow$ no large logs for $\mu_s \sim 1/r$ and $\mu_{us} \sim \alpha_s/r$

$$\begin{aligned} E_{\mathrm{s}}(r) = & V_{\mathrm{s}}^{\mathrm{soft}}\left(r,\mu_s\right) + \frac{C_A^3 C_F}{12}\frac{\alpha_s^3\left(\mu_s\right)}{r}\left\{\frac{2}{\beta_0}\left(B-\frac{\beta_1}{4\beta_0}\right)\left[\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\pi}-\frac{\alpha_s\left(\mu_s\right)}{\pi}\right]\right. \\ & + \frac{2}{\beta_0}\left(1+3\frac{\alpha_s\left(\mu_s\right)}{4\pi}\left[a_{1,0}+2\beta_0\log\left(r\mu_s e^{\gamma_E}\right)\right]\right)\log\left[\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\alpha_s\left(\mu_s\right)}\right] \\ & \left. -\frac{\alpha_s\left(\mu_{\mathrm{us}}\right)}{\pi}\log\left[\frac{C_A\alpha_s\left(\mu_s\right)e^{-5/6}}{r\mu_{\mathrm{us}}}\right]-\frac{\alpha_s\left(\mu_s\right)}{\pi}\log\left(r\mu_s e^{\gamma_E}\right)\right\}. \end{aligned}$$

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- To cancel the renormalon we need a short-distance mass scheme $\rightarrow$ MSR mass [JHEP 04 (2018) 003]

$$\delta m_Q^{\rm MSR}(R) \equiv m_Q^{\rm pole} - m_Q^{\rm MSR}(R) = R\sum_{n=1}^{\infty}\delta_n^R\left[\frac{\alpha_s(R)}{4\pi}\right]^n$$
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- New type of large logs $\rightarrow$ resummation using R-evolution.

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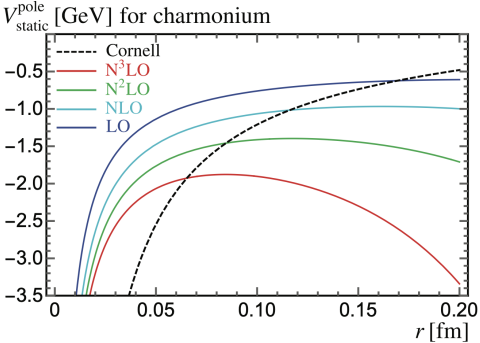

[Eur. Phys. J. C 79 (2019) 4, 323]

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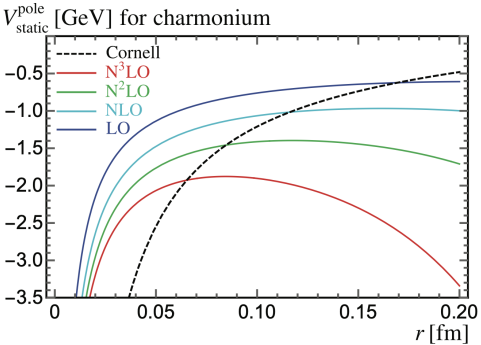

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- The next goal is to obtain a renormalon-free potential.

$$E_{Q\overline{Q}}(r) = 2m_Q^{\text{MSR}}(R_0) + 2\delta m_Q^{\text{MSR}}(R_0, \mu) + E_s(r)$$
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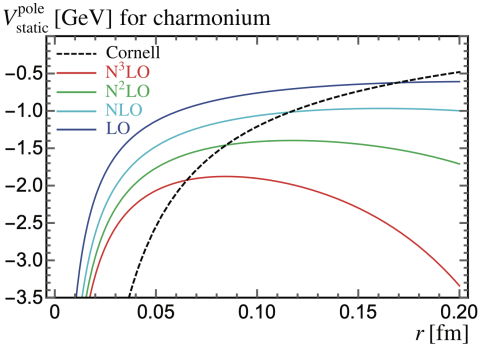

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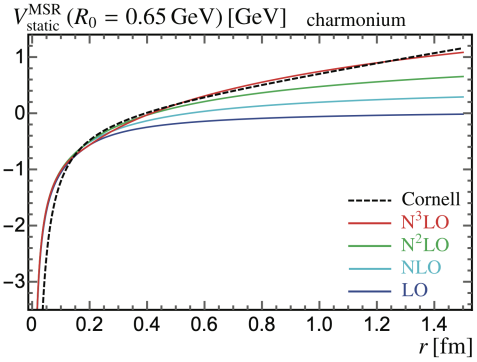

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- $\Delta^{\rm MSR}(R,R_0)$: solution to MSR mass R-RGE.
- We define the R-improved static potential:

$$V_s^{\rm MSR}(r,\mu,R_0) = V_s(r,\mu) + 2\delta^{\rm MSR}(R,\mu) + 2\Delta^{\rm MSR}(R,R_0).$$

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- Solution: use profile functions that ensure series convergence:

$$\mu_s = \sqrt{\left(\frac{\xi}{r}\right)^2 + \frac{b}{r} + a^2} + \mu_0 - a = \begin{cases} \frac{\xi}{r} & \text{for } r \to 0 \\ \mu_0 & \text{for } r \to \infty \end{cases}$$

with $\xi = \mathcal{O}(1)$ and $\mu_0 \sim 1\,\mathrm{GeV}$. This makes sure the series is stable and convergent over the entire spectrum.

Fits

- The static potential is defined up to an additive constant $\rightarrow$ each experiment n has a different offset A_n.
- Minimization of χ^2 function (this talk: consider only a single experiment).

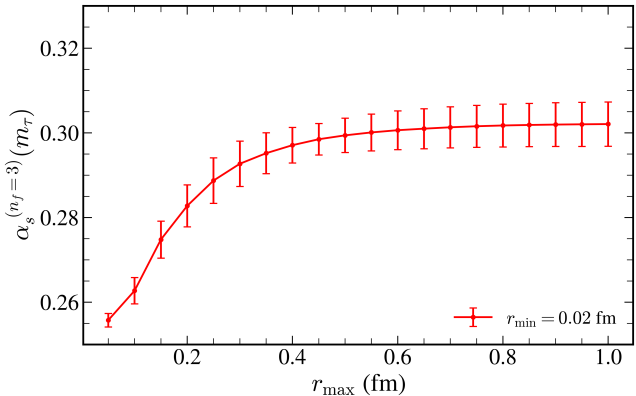
$$\chi^2 = \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_i} \left(\frac{V_{n,i}^{\text{pert}} + A_n - V_{n,i}^{\text{exp}}}{\sigma_{n,i}^{\text{exp}}} \right)^2 .$$

- We can marginalize first with respect to the offset.

$$\frac{\partial \chi^2}{\partial A} = 0 \Longrightarrow \tilde{A} = -\frac{\sum_i \frac{\left(V_i - V_i^{\text{exp}}\right)}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}} \Longrightarrow \tilde{\chi}^2 = \sum_i \frac{\left(V_i - V_i^{\text{exp}}\right)^2}{\sigma_i^2} - \frac{\left[\sum_i \frac{\left(V_i - V_i^{\text{exp}}\right)}{\sigma_i^2}\right]^2}{\sum_i \frac{1}{\sigma_i^2}}$$

Preliminary results

Fit for $r \in [r_{\min}, r_{\max}]$. Use combined dataset (no time to discuss combination!).



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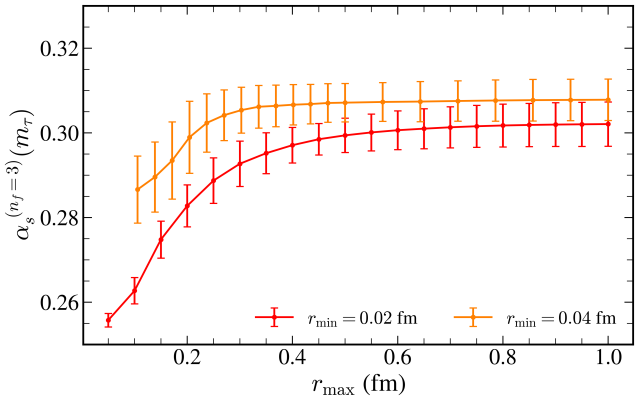

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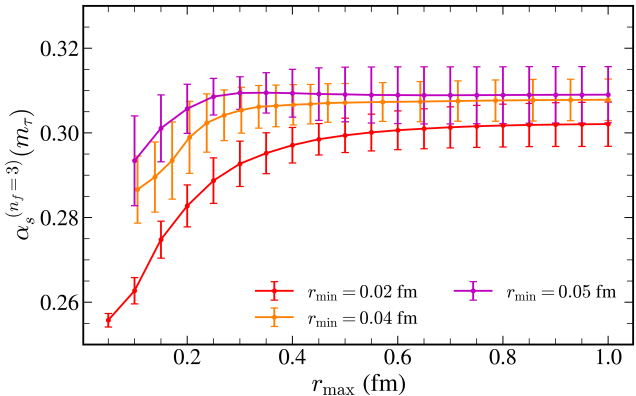

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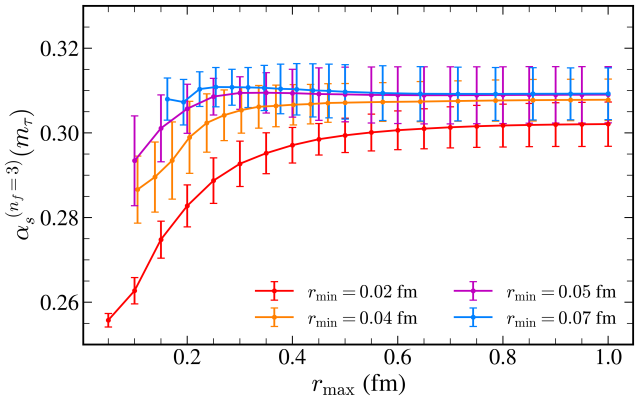

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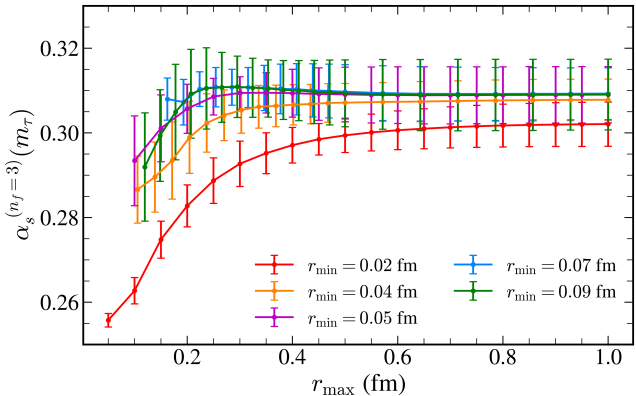

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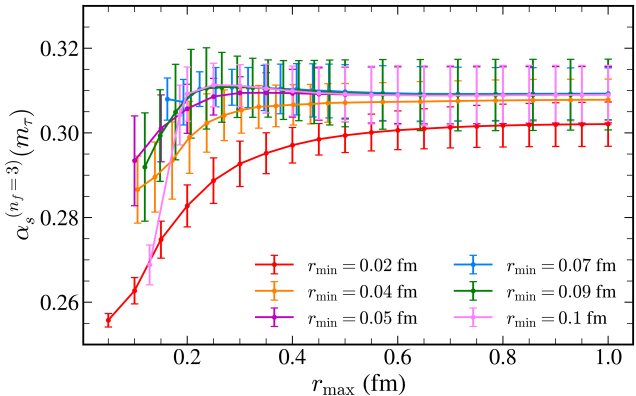

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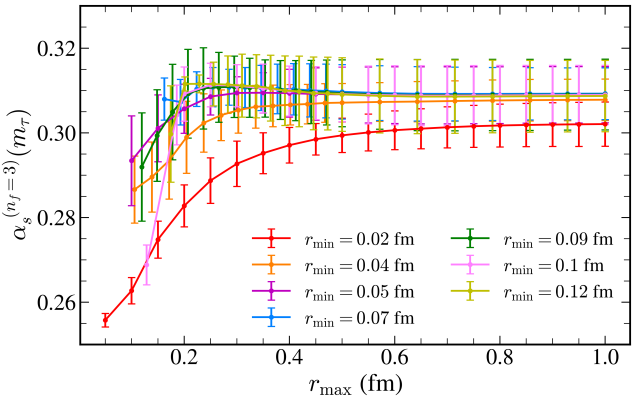

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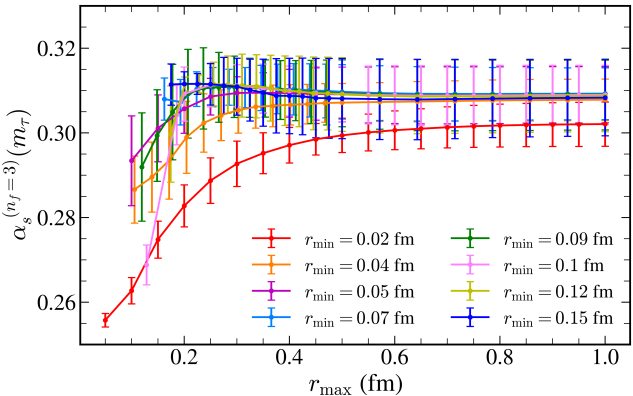

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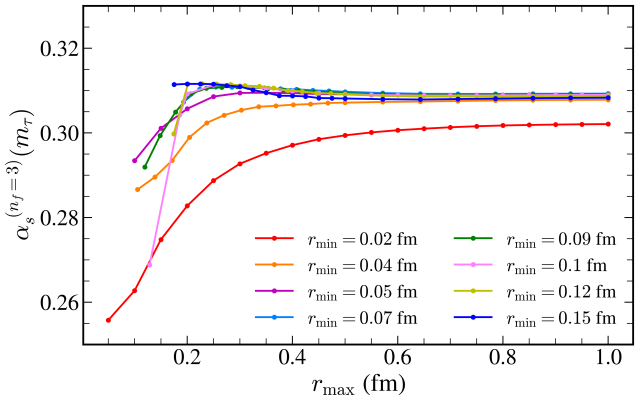

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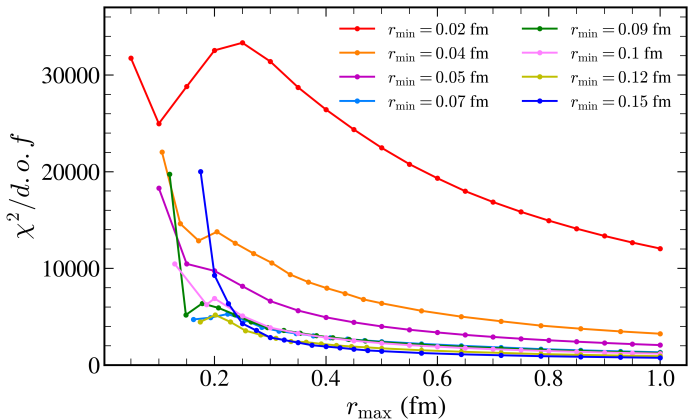
- Useful to check on minimal reduced χ^2 values. Large values caused by not including (highly-correlated) theoretical uncertainties in χ^2 (d'Agostini bias).



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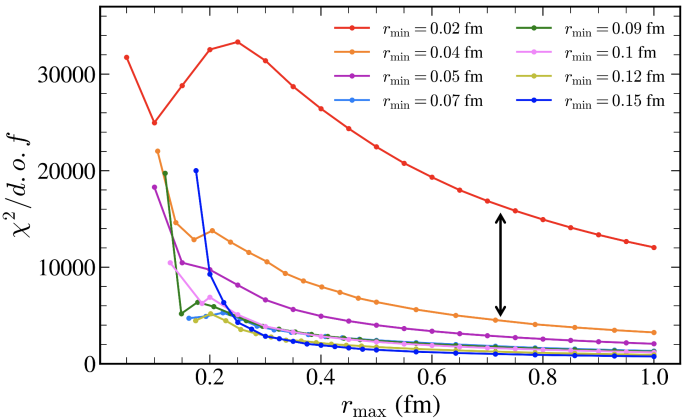

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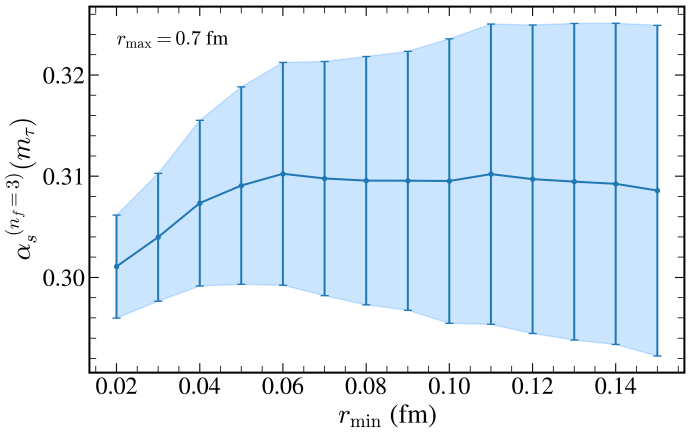

Preliminary results

- We have observed that data points with very small r appear to be biased (few points drive the fit to small values). Hence these are discarded.
- We perform a fit excluding them to obtain a preliminary value for α_s. Default dataset: $r \in [0.05, 0.7]$ fm. Error budget includes: perturbative (dominant), lattice (negligible), and dataset dependence (subdominant).

$$\alpha_s^{(n_f=3)}(m_\tau) = 0.3090 \pm 0.0098$$
$$\alpha_s^{(n_f=5)}(m_Z) = 0.1173 \pm 0.0012$$

Competitive with the w.a. $\alpha_s^{(5)}(m_Z) = 0.1180 \pm 0.0009$ and compatible at 0.5-σ level. Comparing with previous analyses:

$$\alpha_s^{(n_f=5)}(m_Z) = 0.11660^{+0.00110}_{-0.00056} \text{ [PRD 100 (2019) 11]}$$
$$\alpha_s^{(n_f=5)}(m_Z) = 0.1181 \pm 0.0009 \text{ [JHEP 09 (2020) 016]}$$

Conclusions

- Used the static energy to determine α_s, building (and improving) on previous analyses [PRD 100 (2019) 11].
- Performed ultra-soft large-log resummation up to $\mathrm{N^3LL}$.
- Employed the MSR mass and R-evolution to improve the static potential.
- Designed profile functions to increae the validity of the potential up to $r \sim 1\,\mathrm{fm}$.
- Carried out fits to lattice data to obtain a very competitive result for α_s.

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1173 \pm 0.0012 \qquad \text{[preliminary]}$$

BACKUP

Combined datasets

- We divided the r range in N bins.
- Combined r value

$$\tilde{r}_i = \frac{\sum_{j=1}^{n_i} \frac{r_{ij}}{\sigma_j^2}}{\sum_{j=1}^{n_i} \frac{1}{\sigma_j^2}}$$

- Combined static potencial value $\rightarrow$ minimization of:

$$\chi^2 = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left[\frac{V_{i,j} - \bar{V}_i + A_{\varepsilon(j)}}{\sigma_j} \right]^2$$