Universidad de Granada

FTAE High Energy Theory

Efficient on-shell matching

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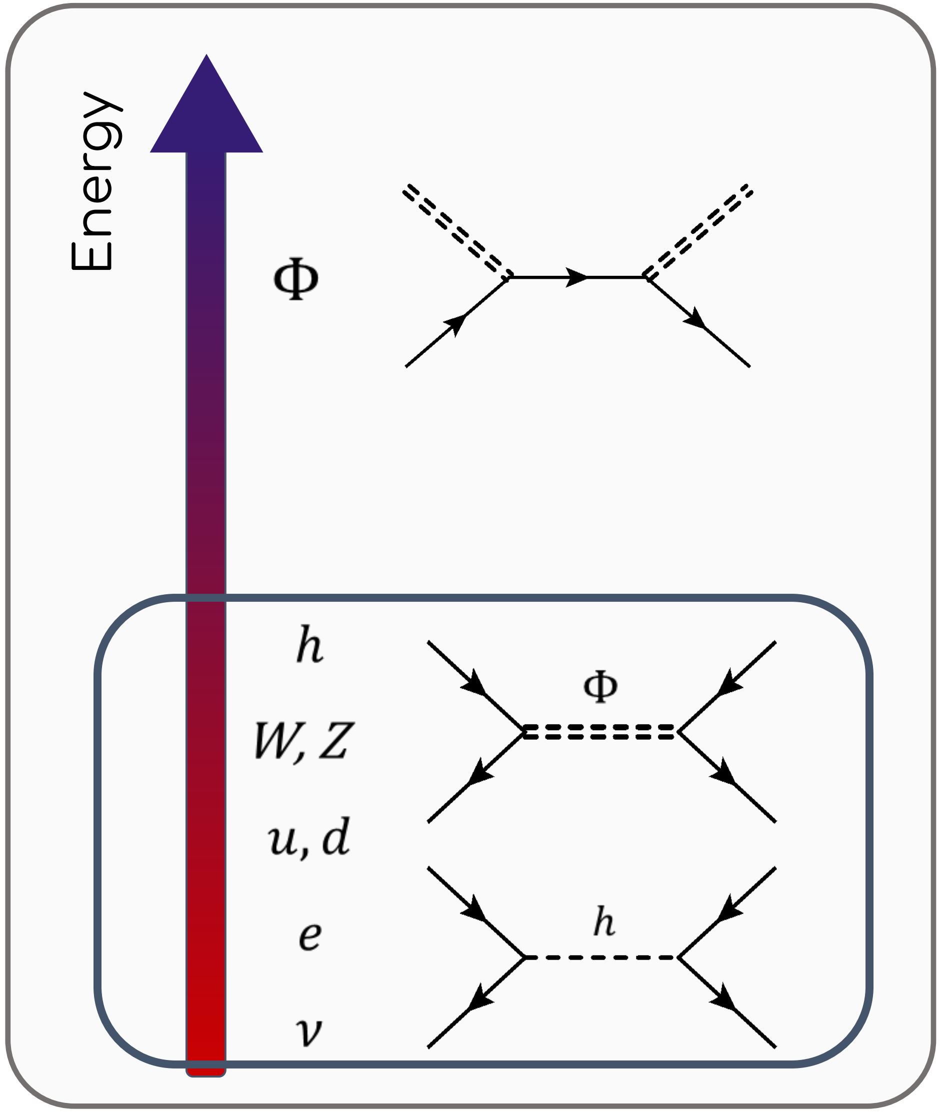
with M. Chala, J. Santiago and F. Vilches [2411.xxxxx]

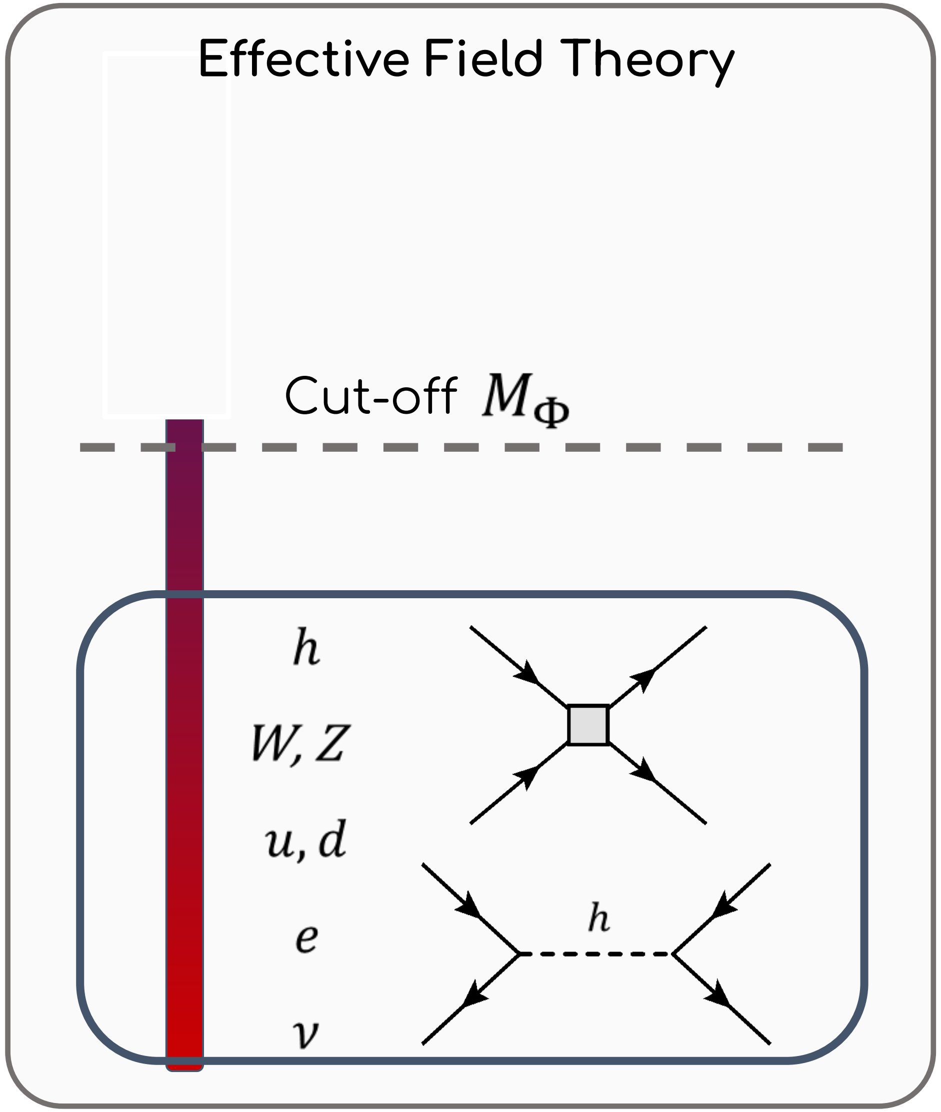
XVI CPAN DAYS

Junta de Andalucía

EFTs and matching



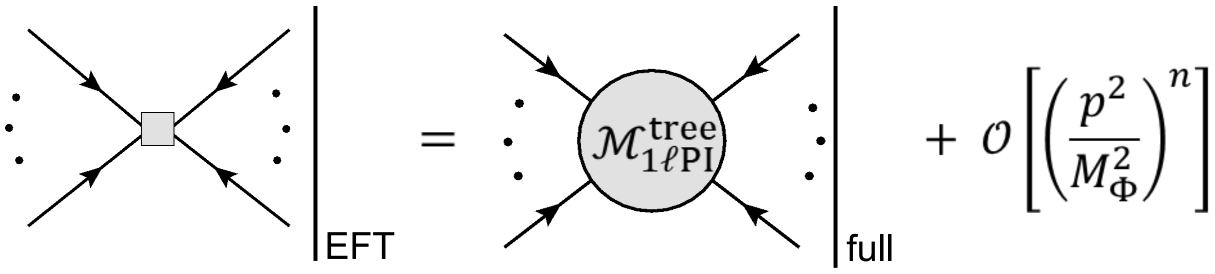
Matching

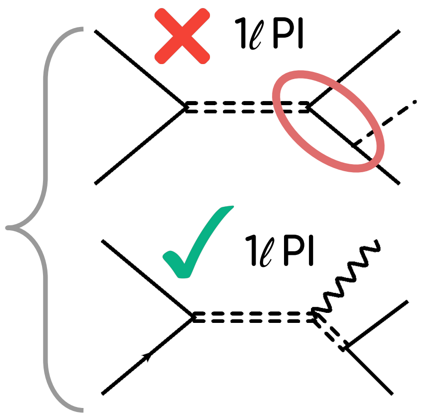


Off-shell matching

$$\boxed{\frac{p^2}{M_\Phi^2} \ll 1}$$

- Tree-level matching:

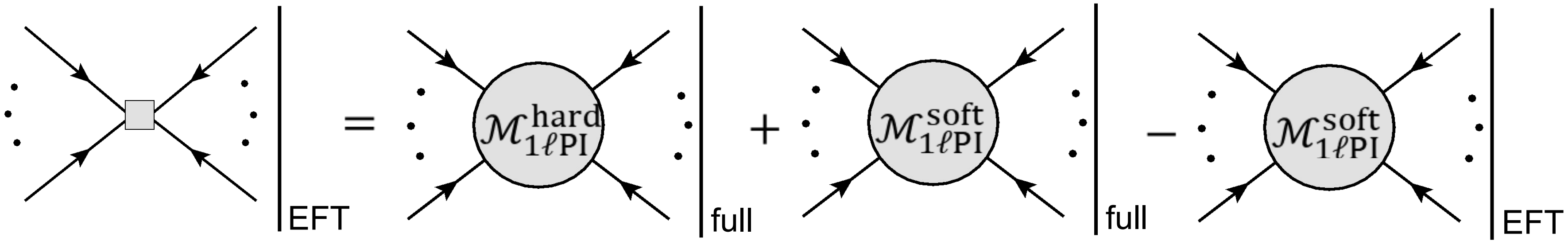




- One-loop matching: (method of regions)

  hard: $k \sim M_\Phi \gg p, m$

  soft: $M_\Phi \gg k \sim p, m$



Automatically cancel

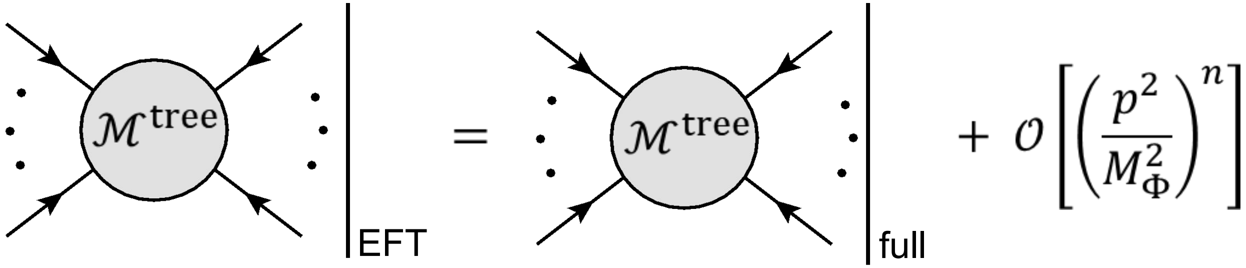
The matching results in values for couplings in the EFT ➡ $C_1, C_2, \ldots, C_i, \boxed{R_1, R_2, \ldots, R_j}$ ➡ $\boldsymbol{C_1, C_2, \ldots, C_i}$

Some correspond to redundant operators

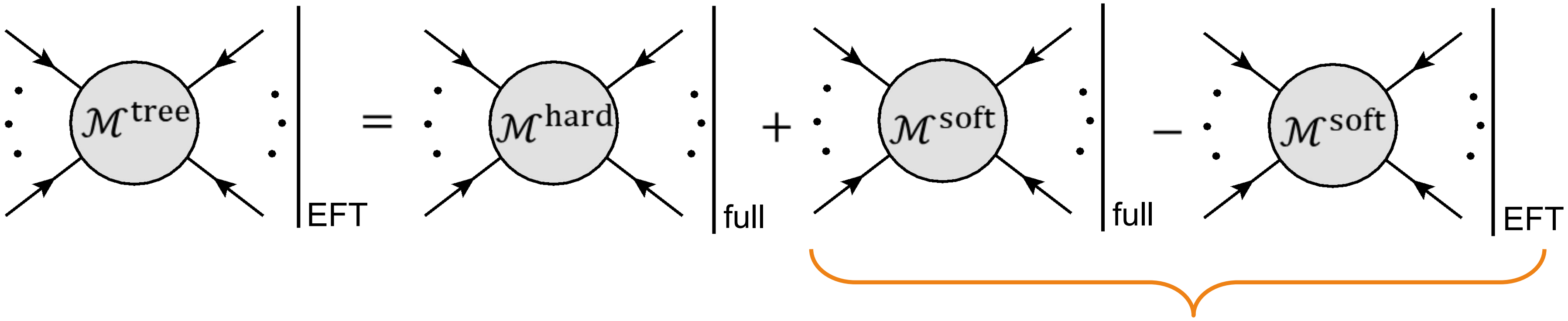
On-shell matching

$$\boxed{\frac{p^2}{M_\Phi^2} \ll 1}$$

- Tree-level matching:



- One-loop matching:
(method of regions)
hard: $k \sim M_\Phi \gg p, m$
soft: $M_\Phi \gg k \sim p, m$



Almost cancel
evanescent contributions

On-shell condition: $p^2 = m_{\text{phys}}^2$

Wavefunction factors: $\mathcal{M} = \left(\sqrt{Z}\right)^n \mathcal{M}_{\text{amp}}$

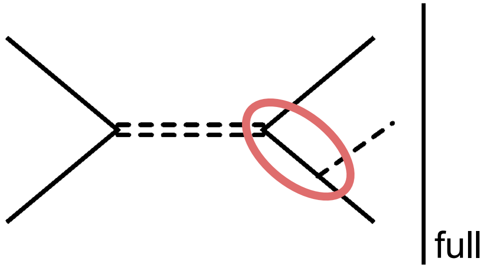
$\Rightarrow$ $C_1, C_2, \ldots, C_i$

(no redundancies!)

Slide 6

On-shell matching: non-localities

Light bridges



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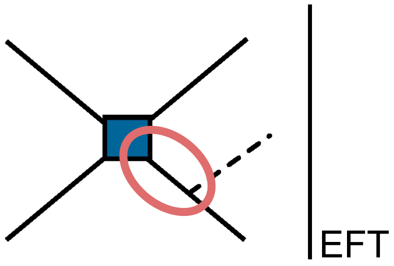


= **local** (polynomial in external momenta)

non-local   non-local

- Difficult to follow this cancelation analitycally → Substitution of **random-generated kinematics**
- The procedure is to be **numerical** but exact → **Rational kinematics**

Spinor Helicity Formalism
[*arXiv:2304.01589, arXiv:2202.02681*]

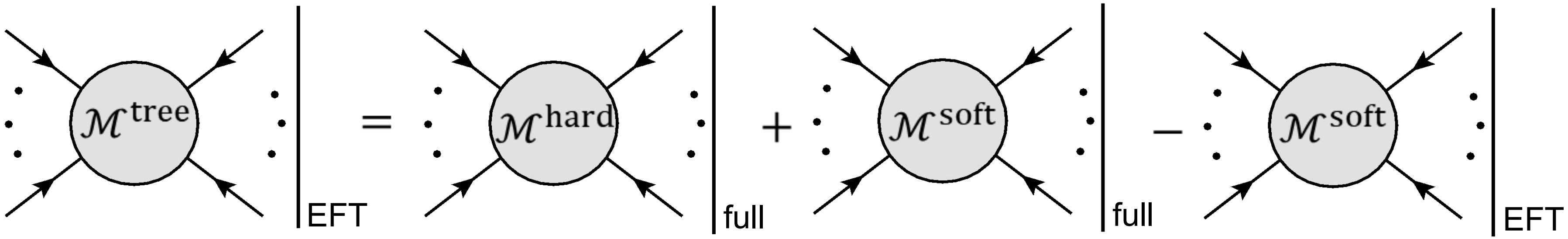
Rational values for $\left\{ \begin{array}{c} \text{momenta} \\ \text{polarizations} \\ \text{spinors} \end{array} \right\}$ with symbolic masses $\mathrm{m_i}$

Satisfying...
Momentum conservation
On-shell condition
Dirac equation
Transversality

$$\mathcal{M} = \alpha p_2 \cdot p_3 + \frac{\beta^2}{(p_1 + p_4)^2 - m^2} = \frac{9044503}{1681920} m^2 \alpha - \frac{840960}{8203543} \frac{\beta^2}{m^2}$$

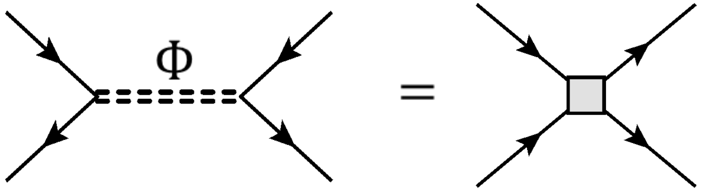
Evanescent shifts

- One-loop matching: (method of regions)
  - hard: $k \sim M_\Phi \gg p, m$
  - soft: $M_\Phi \gg k \sim p, m$

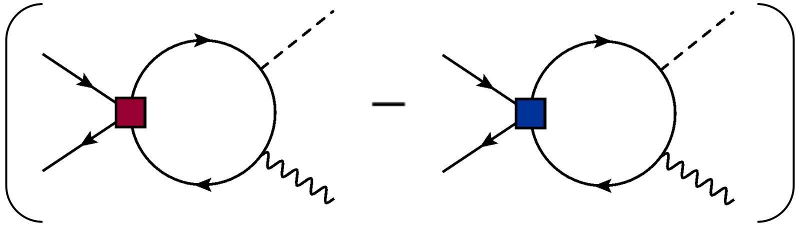


Almost cancel
evanescent contributions

Copy of SM Higgs: $\mathcal{L} = \mathcal{L}_{\text{SM}} + D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi - \left(\mathcal{Y}^{pr} \bar{\ell}^p \Phi e^r + \text{h.c.} \right)$



$$(\bar{\ell}^p e^r)(\bar{e}^s \ell^t) = -\frac{1}{2} (\bar{\ell}^p \gamma^\mu \ell^r)(\bar{e}^s \gamma_\mu e^t) \qquad \text{(d=4)}$$



$\neq 0$ $\longrightarrow$ ΔC_{eB}

Only finite parts coming from $\epsilon \times \dfrac{1}{\epsilon_{\text{UV}}}$

An example: one-loop matching to SMEFT

Copy of SM Higgs: $\mathcal{L} = \mathcal{L}_{\rm SM} + D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi - \left(\mathcal{Y}^{pr} \bar{\ell}^p \Phi e^r + \text{h.c.}\right)$

Operators to match…

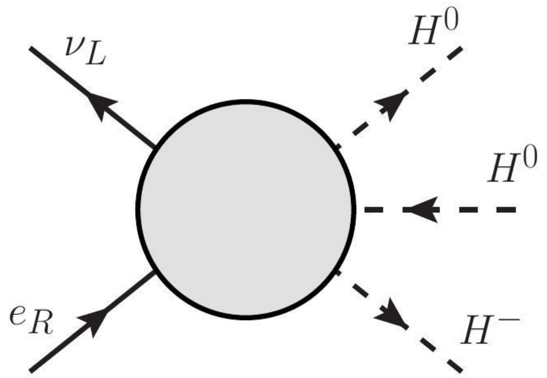
$$\mathcal{O}_\lambda = -(H^\dagger H)^2, \qquad \mathcal{O}_{He} = (\bar{e}\gamma^\mu e)(H^\dagger \mathrm{i} \overleftrightarrow{D}_\mu H),$$

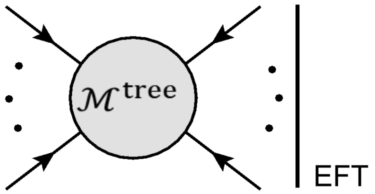
$$\mathcal{O}_{ye} = -\bar{\ell} e H, \qquad \mathcal{O}_{H\ell}^{(1)} = (\bar{\ell}\gamma^\mu \ell)(H^\dagger \mathrm{i} \overleftrightarrow{D}_\mu H),$$

$$\mathcal{O}_{eB} = (\bar{\ell}\sigma^{\mu\nu} e) H B_{\mu\nu}, \qquad \mathcal{O}_{H\ell}^{(3)} = (\bar{\ell}\gamma^\mu \sigma^I \ell)(H^\dagger \mathrm{i} \overleftrightarrow{D}^I_\mu H),$$

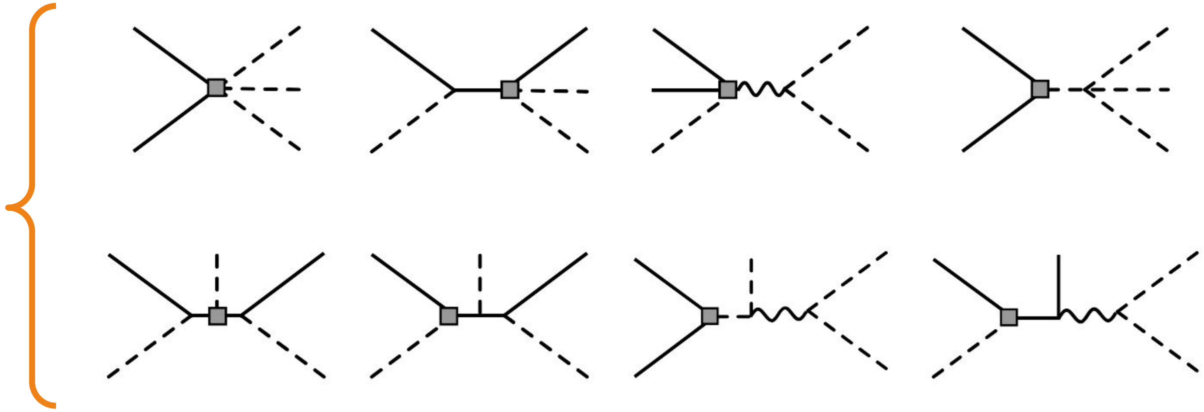
$$\mathcal{O}_{eW} = (\bar{\ell}\sigma^{\mu\nu} e)\sigma^I H W^I_{\mu\nu}, \qquad \mathcal{O}_{eH} = (H^\dagger H)\bar{\ell} e H,$$

…with the amplitude

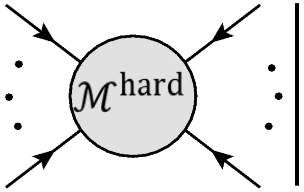


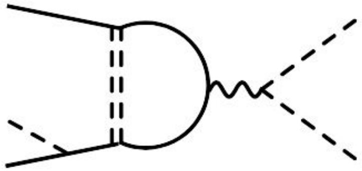
In the 

side:



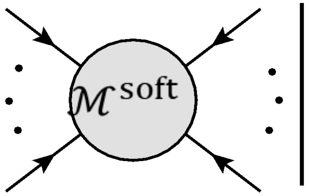
An example: one-loop matching to SMEFT

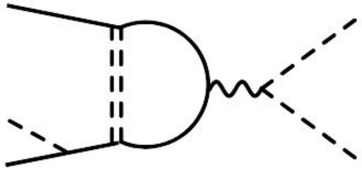
In the 

full side: 

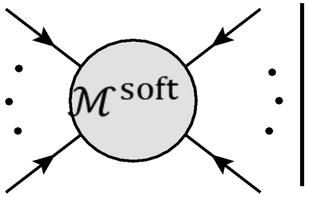
$$\sim \int_k \frac{1}{k^2 - M_\Phi^2} \frac{\not{k} + \not{p}_1}{(k+p_1)^2} \frac{\not{k} + \not{p}_2}{(k+p_2)^2}$$

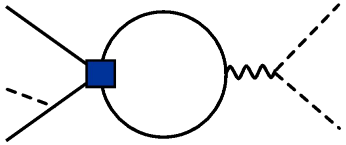
$$\sim \int_k \frac{(\not{k} + \not{p}_1)(\not{k} + \not{p}_2)}{k^4 (k^2 - M_\Phi^2)} \left(1 - \frac{p_1^2 + 2k \cdot p_1}{k^2} + \ldots\right) \left(1 - \frac{p_2^2 + 2k \cdot p_2}{k^2} + \ldots\right)$$

In the 

full side: 

$$\sim \frac{-1}{M_\Phi^2} \int_k \frac{\not{k} + \not{p}_1}{(k+p_1)^2} \frac{\not{k} + \not{p}_2}{(k+p_2)^2} \Bigg|_{\text{UV pole}}$$

In the 

EFT side: 

$$\sim C_{\ell e} \int_k \frac{\not{k}}{k^2} \frac{\not{k} + \not{p}_2}{(k+p_2)^2} \Bigg|_{\text{UV pole}}$$

An example: one-loop matching to SMEFT

We solve for effective couplings perturbatively in the EFT order:

$$
\begin{aligned}
\lambda &\to \lambda + \frac{g_2^4 m_H^2}{960\pi^2 M^2}\,, \\
y_e^{pr} &\to y_e^{pr} - \frac{1}{128\pi^2}\left(\mathcal{Y}\mathcal{Y}^\dagger y_e + 2 y_e \mathcal{Y}^\dagger \mathcal{Y}\right)^{pr} + \frac{m_H^2}{32\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,, \\
c_{HD} &\to -\frac{g_1^4}{1920\pi^2 M^2}\,, \\
c_{H\Box} &\to -\frac{1}{7680\pi^2 M^2}\left(g_1^4 + 3 g_2^4\right), \\
c_{He}^{pr} &\to \frac{g_1^4}{1920\pi^2 M^2}\delta^{pr} + \frac{7 g_1^2}{576\pi^2 M^2}\left(\mathcal{Y}^\dagger \mathcal{Y}\right)^{pr} + \frac{1}{192\pi^2 M^2}\left(6\mathcal{Y}^\dagger y_e y_e^\dagger \mathcal{Y} + y^\dagger \mathcal{Y}\mathcal{Y}^\dagger y_e\right)^{pr}\,, \\
c_{Hl}^{(1)pr} &\to \frac{g_1^4}{3840\pi^2 M^2}\delta^{pr} + \frac{17 g_1^2}{1152\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger\right)^{pr} - \frac{1}{192\pi^2 M^2}\left(6\mathcal{Y} y_e^\dagger y_e \mathcal{Y}^\dagger + y_e \mathcal{Y}^\dagger \mathcal{Y} y_e^\dagger\right)^{pr}\,, \\
c_{Hl}^{(3)pr} &\to -\frac{g_2^4}{3840\pi^2 M^2}\delta^{pr} + \frac{g_2^2}{1152\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger\right)^{pr} - \frac{1}{192\pi^2 M^2}\left(y_e \mathcal{Y}^\dagger \mathcal{Y} y_e^\dagger\right)^{pr}\,, \\
c_{eB}^{pr} &\to -\frac{g_1}{768\pi^2 M^2}\left(5\mathcal{Y}\mathcal{Y}^\dagger y_e + 2 y_e \mathcal{Y}^\dagger \mathcal{Y}\right)^{pr} + \frac{3 g_1}{128\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,, \\
c_{eW}^{pr} &\to -\frac{5 g_2}{768\pi^2 M^2}\left(\mathcal{Y}\mathcal{Y}^\dagger y_e\right)^{pr} - \frac{g_2}{128\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,, \\
c_{eH}^{pr} &\to -\frac{g_2^4}{3840\pi^2 M^2} y_e^{pr} + \frac{1}{192\pi^2 M^2}\left(y_e y_e^\dagger \mathcal{Y}\mathcal{Y}^\dagger y_e + 2 y_e \mathcal{Y}^\dagger \mathcal{Y} y^\dagger y - 3\mathcal{Y} y_e^\dagger y_e \mathcal{Y}^\dagger y_e - 3 y_e \mathcal{Y}^\dagger y_e y_e^\dagger \mathcal{Y}\right)^{pr} + \frac{1}{16\pi^2 M^2}\mathcal{Y}^{pr} y_e^{st}\left(y_e^\dagger\right)^{tu} y_e^{uv}\left(\mathcal{Y}^\dagger\right)^{vs} - \frac{\lambda}{32\pi^2 M^2}\mathcal{Y}^{pr}\left(\mathcal{Y}^\dagger\right)^{st} y_e^{ts}\,.
\end{aligned}
$$

Cross-check with

Matchete [arXiv:2212.04510]

and

MatchMakerEFT [arXiv:2112.10787]

A second example: reduction of Green's basis

Green's basis: minimal set of operators to match amplitudes **off-shell.**

Physical basis: minimal set of operators to match amplitudes **on-shell.**

Field redefinitions

Tedious
Non systematic
Hard to automate

$\mathcal{L}_{\text{full}} \supset$ operators in Green's basis

$\mathcal{L}_{\text{phys}} \supset$ operators in physical basis

Match both theories on-shell at tree level

» We find the couplings of $\mathcal{L}_{\text{phys}}$ in terms of those in $\mathcal{L}_{\text{full}}$. → *REDUCTION OF GREEN'S BASIS*

- **Example I:** Real scalar singlet with Z_2 symmetry up to dimension 8
- **Example II:** Bosonic sector in the SMEFT up to dimension 8

A second example: reduction of Green's basis

BOSONIC SECTOR

Cross-check with [2003.12525v5]

X^3	
$\mathcal{O}_{3G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{3\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{3W}$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$\mathcal{O}_{3\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
X^2D^2	
$\mathcal{O}_{2G}$	$-\frac{1}{2}(D_\mu G^{A\mu\nu})(D^\rho G^A_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$

X^2H^2	
$\mathcal{O}_{HG}$	$G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{G}}$	$\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{W}}$	$\widetilde{W}^I_{\mu\nu}W^{I\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{H\widetilde{B}}$	$\widetilde{B}_{\mu\nu}B^{\mu\nu}(H^\dagger H)$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$
$\mathcal{O}_{H\widetilde{W}B}$	$\widetilde{W}^I_{\mu\nu}B^{\mu\nu}(H^\dagger\sigma^I H)$
H^2XD^2	
$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$

H^2D^4	
$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
H^4D^2	
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\dagger(H^\dagger D_\mu H)$
$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
H^6	
$\mathcal{O}_H$	$(H^\dagger H)^3$

V. Gherardi, D. Marzocca y E. Venturini (2021) [2003.12525v5]

X^2H^2

$$c_{HB} \to c_{HB} \boxed{- 2m_0^2 c_{HB} r_{DH}}$$

$$c_{H\widetilde{B}} \to c_{H\widetilde{B}} \boxed{- 2m_0^2 c_{H\widetilde{B}} r_{DH}}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8}g'^2 r_{2B} + \frac{1}{2}g' r_{BDH} \boxed{- m_0^2(4c_{H\Box}r_{DH} + g' r_{BDH} r_{DH} + 2r_{DH}r'_{HD})}$$

$$c_{HD} \to c_{HD} - \frac{1}{2}g'^2 r_{2B} + 2g' r_{BDH} \boxed{- m_0^2(4c_{HD}r_{DH} + 4g' r_{BDH} r_{DH})}$$

Reduction of dim. 6 operators up to dim. 8!!

H^6

$$\begin{aligned} c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2 \Big(& \frac{1}{4}g'^2 c_{HD} r_{2B} - \frac{1}{16}g'^4 r_{2B}^2 - \frac{1}{2}g' c_{HD} r_{BDH} + \frac{1}{2}g'^3 r_{2B} r_{BDH} \\ & - \frac{3}{4}g'^2 r_{BDH}^2 - 6c_H r_{DH} - \lambda c_{HD} r_{DH} + 8\lambda c_{H\Box} r_{DH} + g'\lambda r_{BDH} r_{DH} - 11\lambda^2 r_{DH}^2 \\ & - \frac{1}{2}c_{HD} r'_{HD} + 4c_{H\Box} r'_{HD} + \frac{1}{2}g' r_{BDH} r'_{HD} - 9\lambda r_{DH} r'_{HD} - \frac{1}{4}r'^2_{HD} - r''^2_{HD} \Big) \end{aligned}$$

Conclusions

- On-shell matching is a **diagrammatic alternative** that allows to compute the **coefficients directly into the physical basis**.

- **More diagrams** to be computed in a single amplitude, but **several operators can be matched** at the **same time**.

- **No** need of defining **evanescent operators** or using **background field method** for finite matching.

- Useful for **renormalization** and computing **beta functions** (see [arXiv:2409.15408]).

- Can be used to compute the **reduction to any physical basis** from any redundant basis.

  ↳ Code in Mathematica in progress (***RGB***) based in Feynrules+FeynArts+FeynCalc (with F. Vilches)

FIXED BUGS

- $(p_i \cdot p_j)^n$ structures
- $\epsilon^{\mu\nu\rho\sigma}\gamma_\mu$ structures
- Duplicate components in 'FFSSS'

Universidad de Granada

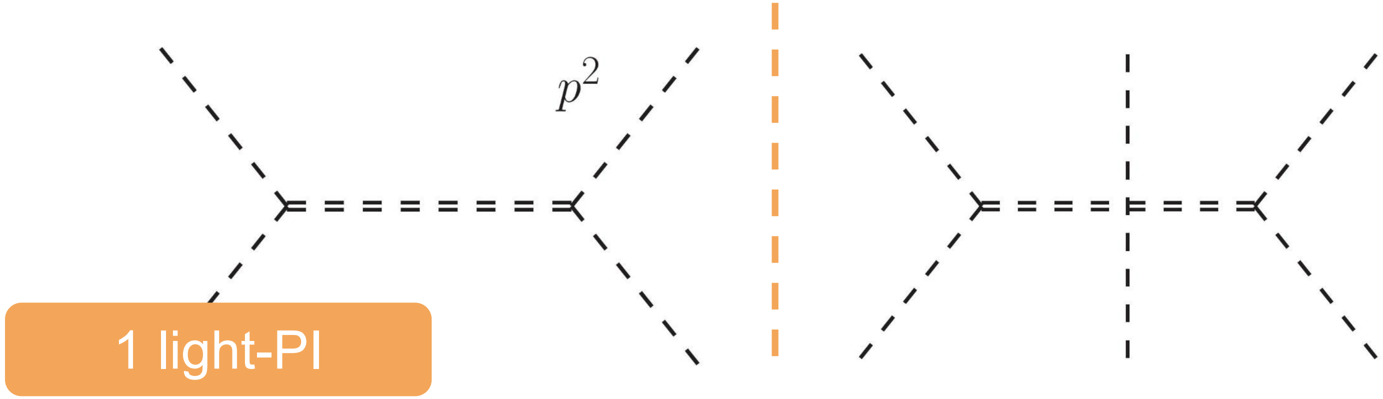
High Energy Theory

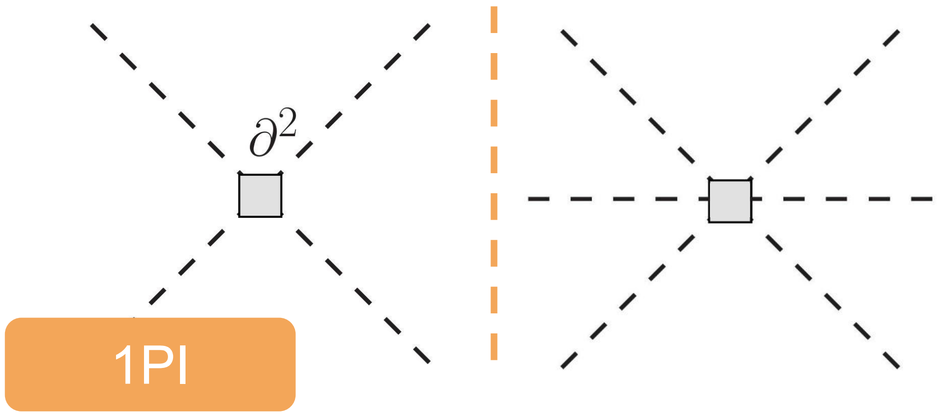
THANKS FOR YOUR ATTENTION !

Matching theories: Off-Shell vs On-shell

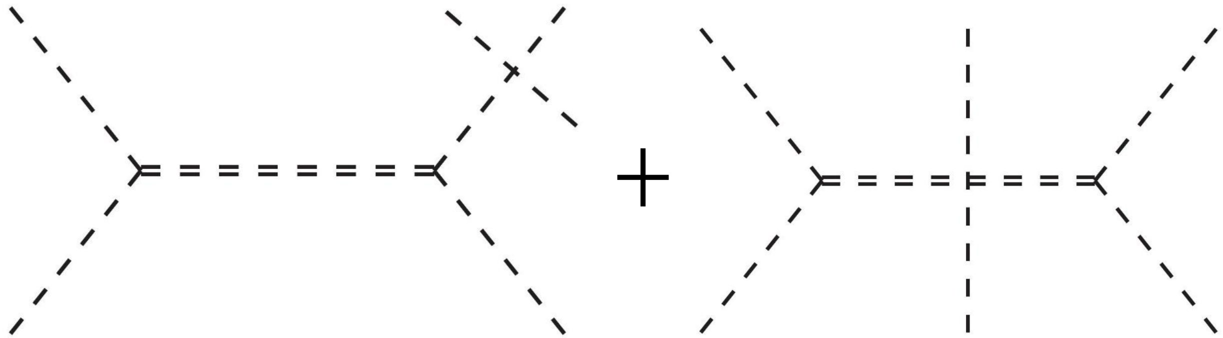
UV **IR**

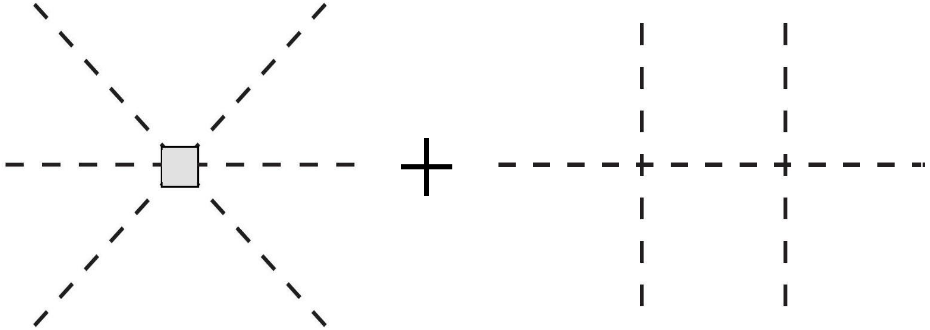
Off-shell





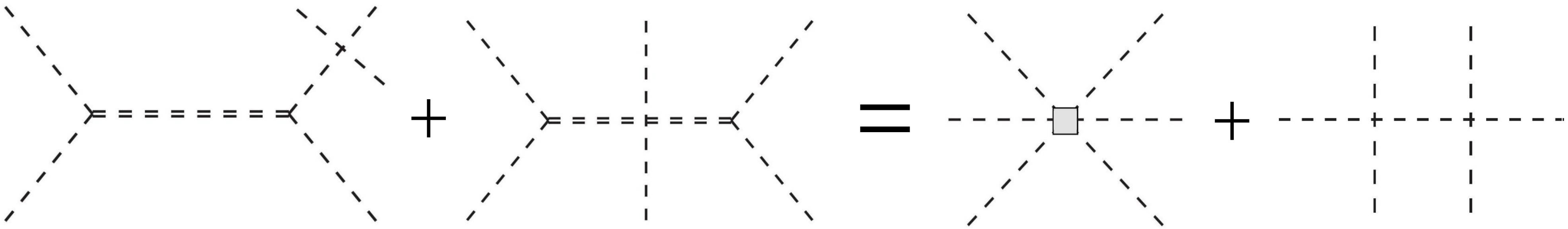
On-shell





Matching theories: Off-Shell vs On-shell

On-Shell:



Off-Shell	On-Shell
Large number of operators (Green's basis)	Smaller set of operators (physical basis)
Small number of diagrams (1lPI in UV, 1PI in IR)	All diagrams (light bridges too)
Contribution directly local in momenta	Delicate cancelation of non-local contributions
Need of Background Field Method	
Need of evanescent operators	

Some results in the SMEFT

Dimension 4 up to dimension 8 (H and B)

$$m_0^2 \to m_0^2 - m_0^4 r_{DH} + 2m_0^6 r_{DH}^2$$

$$\lambda \to \lambda - m_0^2(4\lambda r_{DH} + 2r'_{HD}) + m_0^4(16\lambda r_{DH}^2 + 10 r_{DH} r'_{HD})$$

$$y_E \to y_E(1 - m_o^2 r_{DH} + \frac{5}{2} m_0^4 r_{DH}^2)$$

Some results in the SMEFT

Dimension 6 up to dimension 8 (H and B)

X^2H^2

$$c_{HB} \to c_{HB} - 2m_0^2 c_{HB} r_{DH}$$
$$c_{H\tilde{B}} \to c_{H\tilde{B}} - 2m_0^2 c_{H\tilde{B}} r_{DH}$$

H^4D^2

$$c_{H\Box} \to c_{H\Box} - \frac{1}{8} g'^2 r_{2B} + \frac{1}{2} g' r_{BDH} - m_0^2 (4 c_{H\Box} r_{DH} + g' r_{BDH} r_{DH} + 2 r_{DH} r'_{HD})$$
$$c_{HD} \to c_{HD} - \frac{1}{2} g'^2 r_{2B} + 2 g' r_{BDH} - m_0^2 (4 c_{HD} r_{DH} + 4 g' r_{BDH} r_{DH})$$

H^6

$$\begin{aligned} c_H \to c_H + \lambda^2 r_{DH} + \lambda r'_{HD} + m_0^2 \Bigg(& \frac{1}{4} g'^2 c_{HD} r_{2B} - \frac{1}{16} g'^4 r_{2B}^2 - \frac{1}{2} g' c_{HD} r_{BDH} + \frac{1}{2} g'^3 r_{2B} r_{BDH} \\ & - \frac{3}{4} g'^2 r_{BDH}^2 - 6 c_H r_{DH} - \lambda c_{HD} r_{DH} + 8 \lambda c_{H\Box} r_{DH} + g' \lambda r_{BDH} r_{DH} - 11 \lambda^2 r_{DH}^2 \\ & - \frac{1}{2} c_{HD} r'_{HD} + 4 c_{H\Box} r'_{HD} + \frac{1}{2} g' r_{BDH} r'_{HD} - 9 \lambda r_{DH} r'_{HD} - \frac{1}{4} r'^2_{HD} - r''^2_{HD} \Bigg) \end{aligned}$$

Some results in the SMEFT

Dimension 8 (H and B)

XH^4D^2

$$c^{(1)}_{BH^4D^2} \to c^{(1)}_{BH^4D^2} - 4g' c_{HB} r_{2B} + \frac{1}{2} g'^3 r_{2B}^2 + 8 c_{HB} r_{BDH} - 2g'^2 r_{2B} r_{BDH} + 2g' r_{BDH}^2$$

$$c^{(2)}_{BH^4D^2} \to -4g' c_{H\widetilde{B}} r_{2B} + 8 c_{H\widetilde{B}} r_{BDH}$$

$X^2H^2D^2$

$$c^{(1)}_{B^2H^2D^2} \to 0$$

$$c^{(2)}_{B^2H^2D^2} \to 0$$

$$c^{(3)}_{B^2H^2D^2} \to 0$$

X^4

$$c^{(1)}_{B^4} \to 0$$

$$c^{(2)}_{B^4} \to 0$$

$$c^{(3)}_{B^4} \to 0$$

X^2H^4

$$c^{(1)}_{B^2H^4} \to -c_{HB} g'^2 r_{2B} + \frac{1}{16} g'^4 r_{2B}^2 + 2 c_{HB} g' r_{BDH} - \frac{1}{4} g'^3 r_{2B} r_{BDH} + \frac{1}{4} g'^2 r_{BDH}^2 - 2 c_{HB} \lambda r_{DH} - c_{HB} r'_{HD}$$

$$c^{(2)}_{B^2H^4} \to -g'^2 c_{H\widetilde{B}} r_{2B} + 2g' c_{H\widetilde{B}} r_{BDH} - 2\lambda c_{H\widetilde{B}} r_{BDH} - c_{H\widetilde{B}} r'_{HD}$$

Some results in the SMEFT

Dimension 8 (H and B)

H^4D^4

$$c_{H^4}^{(1)} \to \frac{1}{2}g'^2 r_{2B}^2 - 2g' r_{2B} r_{BDH} + 2r_{BDH}^2$$

$$c_{H^4}^{(2)} \to -\frac{1}{2}g'^2 r_{2B}^2 + 2g' r_{2B} r_{BDH} - 2r_{BDH}^2$$

$$c_{H^4}^{(3)} \to 0$$

H^6D^2

$$\begin{aligned} c_{H^6}^{(1)} \to & -\frac{3}{4}g'^2 c_{HD} r_{2B} + \frac{3}{16}g'^4 r_{2B}^2 + \frac{3}{2}g' c_{HD} r_{BDH} - \frac{3}{2}g'^3 r_{2B} r_{BDH} + \frac{9}{4}g'^2 r_{BDH}^2 - \lambda c_{HD} r_{DH} \\ & - 8\lambda c_{H\Box} r_{DH} - 3g'\lambda r_{BDH} r_{DH} + \lambda^2 r_{DH}^2 - \frac{1}{2}c_{HD} r'_{HD} - 4c_{H\Box} r'_{HD} - \frac{3}{2}g' r_{BDH} r'_{HD} \\ & - 3\lambda r_{DH} r'_{HD} - \frac{7}{4}r'^2_{HD} + r''^2_{HD} \end{aligned}$$

$$\begin{aligned} c_{H^6}^{(2)} \to & -\frac{1}{2}g'^2 c_{HD} r_{2B} + \frac{1}{8}g'^4 r_{2B}^2 + g' c_{HD} r_{BDH} - g'^3 r_{2B} r_{BDH} + \frac{3}{2}g'^2 r_{BDH}^2 - 2\lambda c_{HD} r_{DH} \\ & - 2g'\lambda r_{BDH} r_{DH} - c_{HD} r'_{HD} - g' r_{BDH} r'_{HD} \end{aligned}$$

Generation of random momenta

$$SL(2,\mathbb{C}) \cong SU(2)_L \times SU(2)_R \left\{ \begin{array}{c} \lambda \in SU(2)_L \\ \tilde{\lambda} \in SU(2)_R \end{array} \right\} \begin{array}{c} \lambda^\alpha = \varepsilon^{\alpha\beta}\lambda_\beta \\ \tilde{\lambda}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\tilde{\lambda}^{\dot{\beta}} \end{array}$$

Massless momenta :

$$P_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \Rightarrow P = p_\mu \sigma^\mu = \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix}$$

Massive momenta :

$$P^\mu := q^\mu + \frac{m^2}{2q \cdot k} k^\mu \quad \left| \begin{array}{rcl} q^2, k^2 & = & 0 \\ q_{\alpha\dot{\alpha}} & = & \lambda_\alpha \lambda_{\dot{\alpha}} \\ k_{\alpha\dot{\alpha}} & = & \mu_\alpha \tilde{\mu}_{\dot{\alpha}} \end{array} \right.$$

Evanescent operators

$$\mathcal{R} = \alpha\, \mathcal{O} \xrightarrow{d = 4 - 2\epsilon} \mathcal{R} = \alpha \mathcal{O} + \mathcal{E}$$

$\mathcal{O}(\epsilon)$

Additional finite local contributions in loop amplitudes

$$\boldsymbol{IR^{(0)} + IR^{(1)} = UV^{(0)} + UV^{(1)}}$$

$$\boldsymbol{IR^{(0)} + \boxed{IR^{(1)}_{soft}} = UV^{(0)} + UV^{(1)}_{hard} + \boxed{UV^{(1)}_{soft}}}$$

We take the hard region

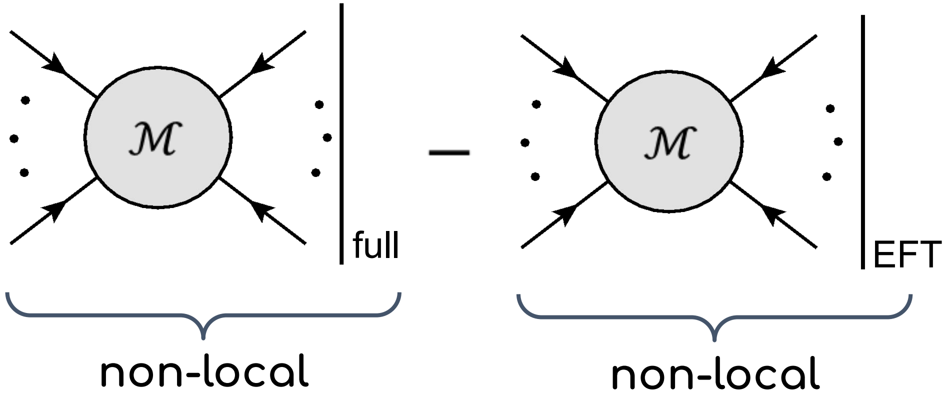
$$\int \mathcal{O} = \frac{1}{\epsilon}(a + b_{\mathcal{O}}\epsilon)$$

$$\int \mathcal{R} = \frac{1}{\epsilon}(a + b_{\mathcal{R}}\epsilon)$$

$$\int \mathcal{R} - \mathcal{O} = \frac{1}{\epsilon}(b_{\mathcal{R}}\epsilon - b_{\mathcal{O}}\epsilon) = b$$

On-shell matching: non-localities

Light bridges



= **local** (polynomial in external momenta)

- Difficult to follow this cancelation analitycally → Substitution of **random-generated kinematics**
- The procedure is to be **numerical** but exact → **Rational kinematics**

Spinor Helicity Formalism [*arXiv:2304.01589, arXiv:2202.02681*]

$$p^\mu = \frac{1}{2}\langle p\, \sigma^\mu\, p]$$

$$\varepsilon^\mu = \frac{1}{\sqrt{2}}\frac{\langle r\, \sigma^\mu\, p]}{\langle r\, p\rangle}$$

$$|p\rangle = P_L u(p)$$
$$|p] = P_R u(p)$$