Angular distributions of Drell-Yan leptons in the TMD factorization approach

Based on **S.Piloñeta** and **A.Vladimirov** **arXiv 2407.06277** (July 2024)

XVI CPAN DAYS

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Introduction

Hadron is a 3D object

- What do we do? We are interested in studying the structure of nucleons
  - Work with inelastic particle collisions involving hadrons
    - **TMD factorization theorem** → Cross-sections factorized into different blocks

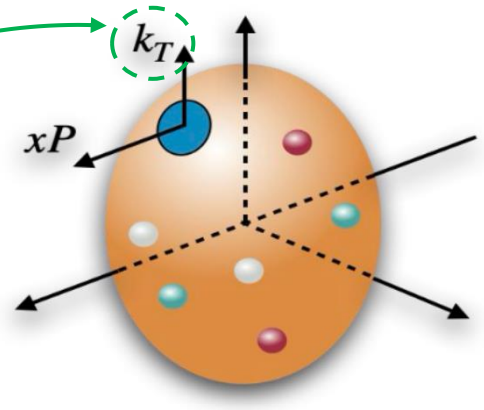
$$\frac{d\sigma}{dq_T} \sim \int \text{Perturbative} \times \text{Non-perturbative}$$

Perturbative → Feynman diagrams

Non-perturbative → **Transverse Momentum Dependent Distributions (TMDs)**

- **Generalization** of Parton Distributions Functions (PDFs)
  - Access to the 3D internal structure of the nucleon
- Extracted from the data → global analyses
- Predictive power → helpful for experiments

Parton's transverse momentum

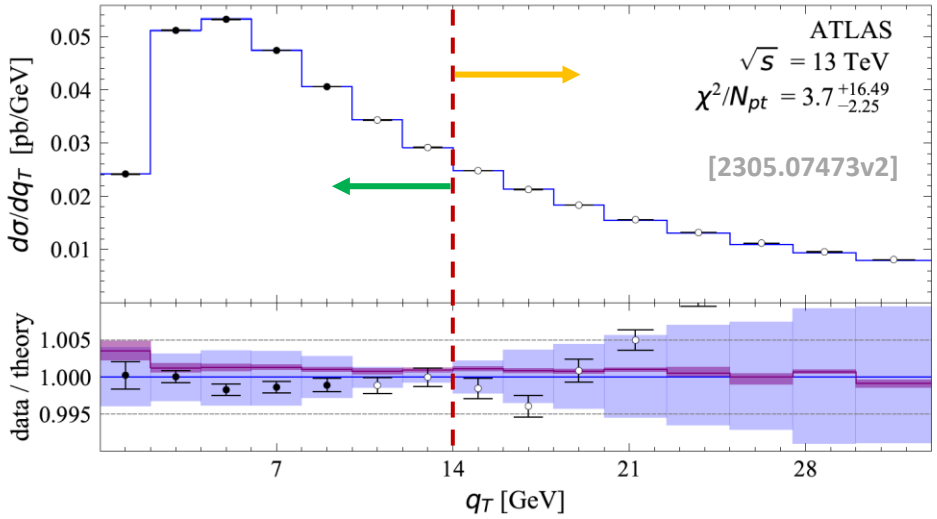


Introduction

Necessity for power corrections

- Although it is a very **powerful tool**, the **current form** of the **TMD factorization** has **limitations**

Z-boson production ATLAS measurement comparison



→ Very **precise description** at **low q_T**

→ **Prediction** systematically **lower** than the data at **larger q_T**

↓

Signal of the **necessity** for **power corrections**?

↓

TMD-with-KPCs factorization theorem

Develop a **theoretical framework** including them

For details, see [AV, 2307.13054v2]

↓

Focus on **Kinematic Power Corrections (KPCs)**

→ Restore **charge conservation** and **frame invariance**

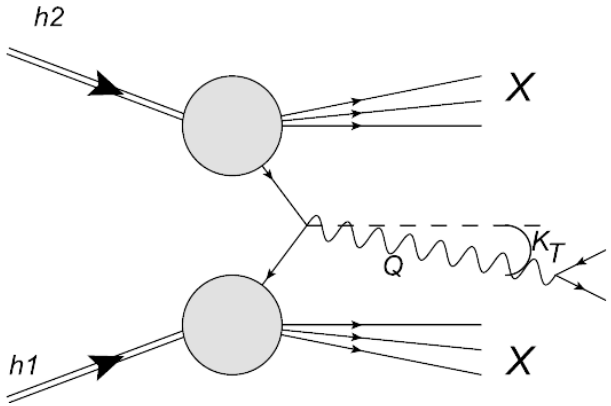
→ We **expect** that **they are the largest** in the TMD factorization regime

Unpolarized Drell-Yan as a starting point

Kinematics and Collins-Soper frame

- **Feasibility** of using the **TMD-with-KPCs factorization** ➡ **Drell-Yan reaction**

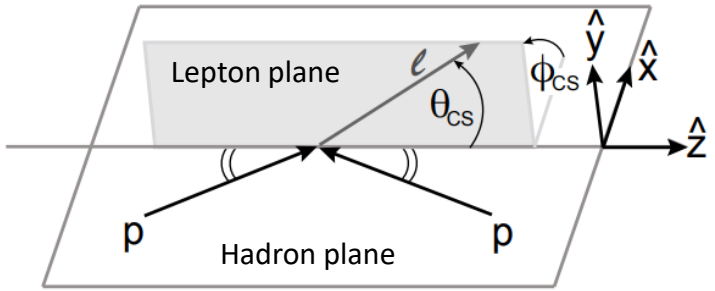
$$h_1(p_1) + h_2(p_2) \;\rightarrow\; \gamma^*/Z(q) + X \;\rightarrow\; \ell^-(l) + \ell^+(l') + X$$



Kinematics

- q^μ ➡ gauge boson momentum
  - $q^2 = Q^2$ ➡ large scale
- $p_1^\mu = \bar{n}^\mu p_1^+,\; p_2^\mu = n^\mu p_2^-$
- $p_1^2 = p_2^2 = 0$

- **Starting point** for **testing** the **TMD-with-KPCs** factorization theorem
  - ➢ **Angular distributions** of the **lepton pair**
    - **Collins-Soper (CS) frame**
      - Dilepton rest frame
      - Angles θ and ϕ



Unpolarized Drell-Yan as a starting point

Cross-section decomposition and angular distributions

- **Angular decomposition** of the **cross section**

$$\frac{d\sigma}{dp_T dy dQ d\Omega} = \frac{3}{16\pi}\frac{d\sigma^{U+L}}{dp_T dy dQ}\Big[A_U \;\rightarrow\; \text{Unpolarized angle-integrated cross-section}$$

$$(1+\cos^2\theta) + \frac{1-3\cos^2\theta}{2} A_0 + \sin 2\theta \cos\phi A_1$$

$$\frac{\sin^2\theta\cos 2\phi}{2} A_2 + \sin\theta\cos\phi A_3 + \cos\theta A_4$$

$$\sin^2\theta \sin 2\phi A_5 + \sin 2\theta \sin\phi A_6 + \sin\theta \sin\phi A_7 \Big]$$

$A_{0,\ldots,7}$ → **Angular distributions**

→ Only A_U described by the **TMD factorization theorem** so far

When we **incorporate power corrections,** we can **describe** them **all**!

But… its inclusion **complicates everything**

- Theoretical computation
- Numerical realization

Obtaining the A_n angular distributions

- Computation of the **unpolarized Drell-Yan cross-section** including **KPCs**
  - **Hadron tensor**
    - **Unpolarized** $f_{1;f/p}$ and **Boer-Mulders** $h^{\perp}_{1;f/p}$ distributions
    - Now it contains a more complicated **convolution integral**
      - Makes **numerical realization difficult**
  - **Lepton tensor** → parameterized in terms of the **CS-frame angles** θ and ϕ

- We have explicitly obtained the **theoretical expressions** for the various A_n
  - We can **codify** them in a **general way** as

$$A_n \sim \boxed{\text{Constants}} \times \boxed{\text{Gauge boson propagators}} \times \boxed{\text{Some combination of vector (v) and axial (a) coupling constants}}^{(1)} \times \boxed{\text{Convolution integral involving } f_1 \text{ or } h_1^{\perp} \text{ distributions}}$$

(1) The **inclusion** of **KPCs** leads to some **different coupling combinations**
  - (v^2+a^2) (v^2-a^2) (av) → Extra sensitivity to EW couplings

Obtaining the A_n angular distributions

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(2) Two possibilities
- $\mathcal{C}[A, f_1 f_2]$ → **Symmetric flavor** $f_{q_1} f_{\bar{q}_2} + f_{\bar{q}_1} f_{q_2}$
- $\mathcal{C}[A, \{f_1 f_2\}_A]$ → Extra sensitivity to **anti-symmetric flavor** $f_{q_1} f_{\bar{q}_2} - f_{\bar{q}_1} f_{q_2}$

Perturbative and non-perturbative set-up

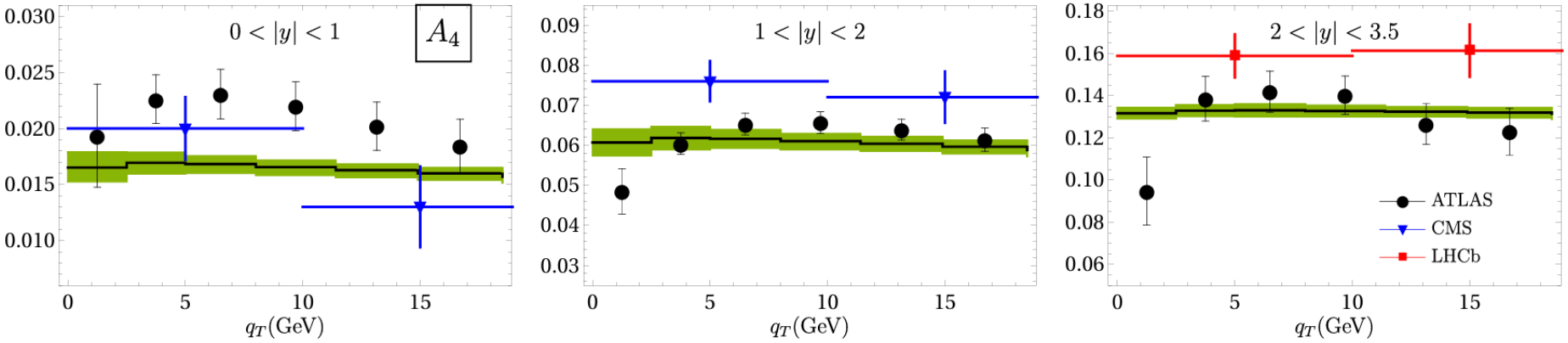
- The KPCs have been implemented in ***artemide*** github.com/VladimirovAlexey /artemide-development
- New global fit is required
  - Technical difficulties and ⇑ computation time → Complex task
- First exploratory study
  - We use the input of **ART23**
    - Perturbative elements (N^4LL) and evolution
    - Unpolarized TMD distribution f_1
  - ATLAS data for A_2 angular distribution → Boer-Mulders distribution $h_1^\perp$ extraction

    More details later!
  - We describe the data sufficiently well without significant modifications of the models for TMD distributions

Angular distribution A_4

- Leading power
- Proportional to the difference between quark and anti-quark distributions (anti-symmetric flavor)
- Inclusion in standard f_1 extractions

➢ Comparison as a function of q_T with **ATLAS**, CMS and LHCb measurements



- Theory prediction agrees very well with the measurements

Angular distribution A_4

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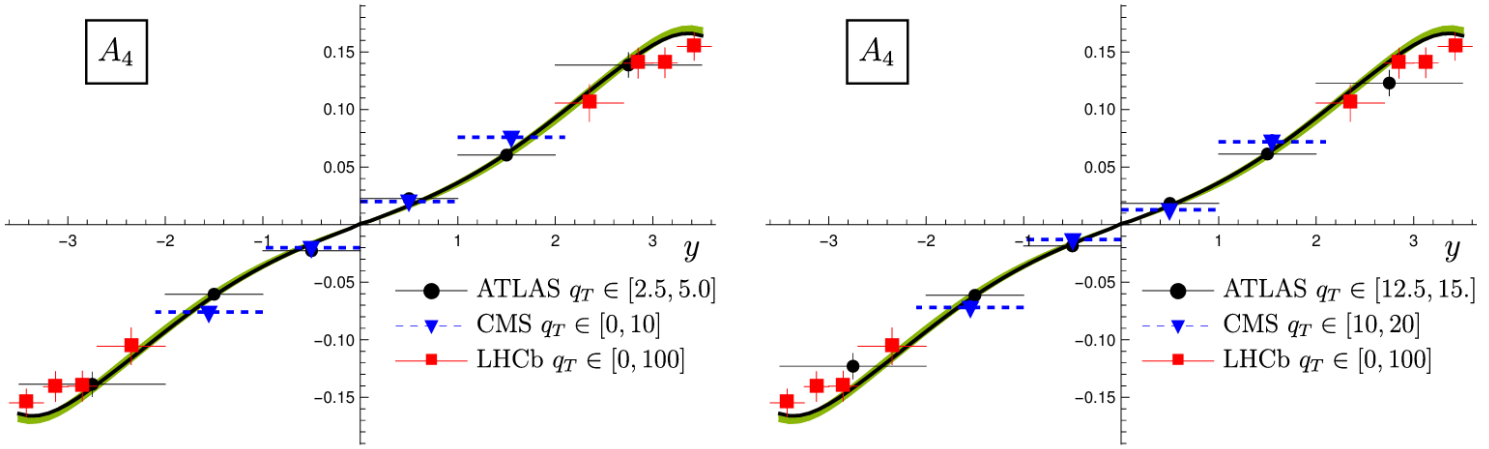
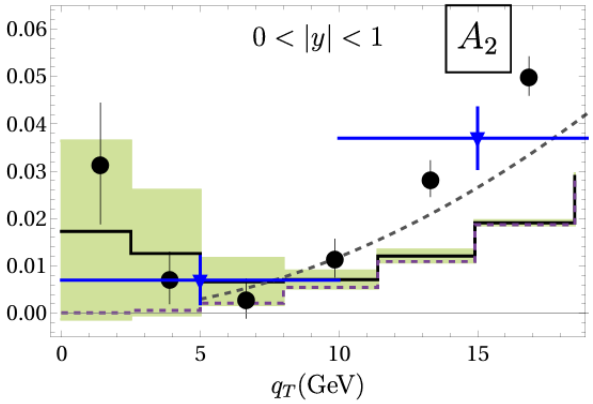
- Inclusion in standard f_1 extractions

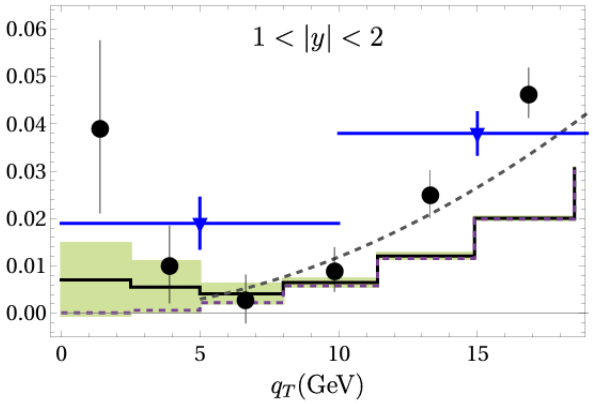
➢ Comparison as a function of y with **ATLAS**, CMS and LHCb measurements

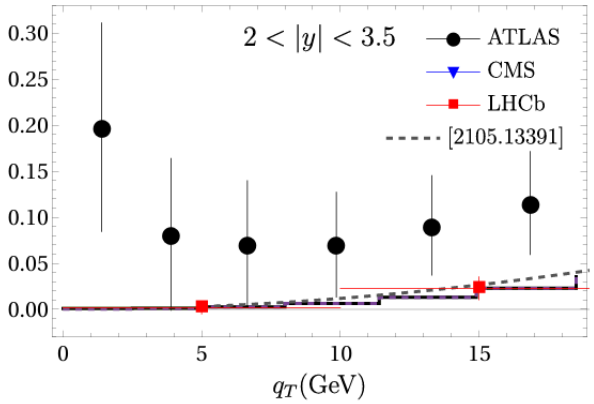


- The agreement is even more transparent

Angular distribution A_2



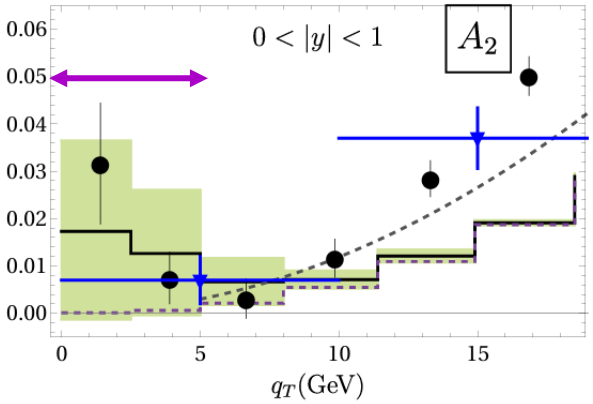


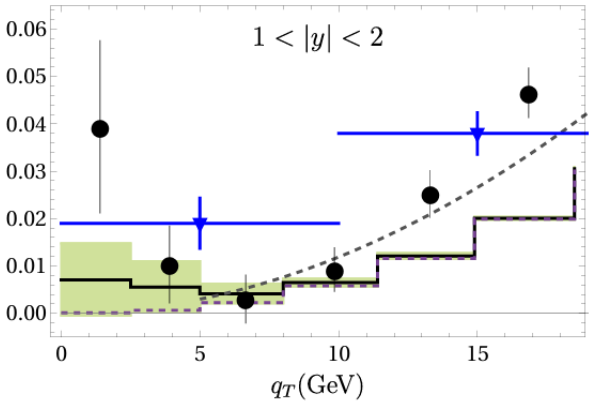


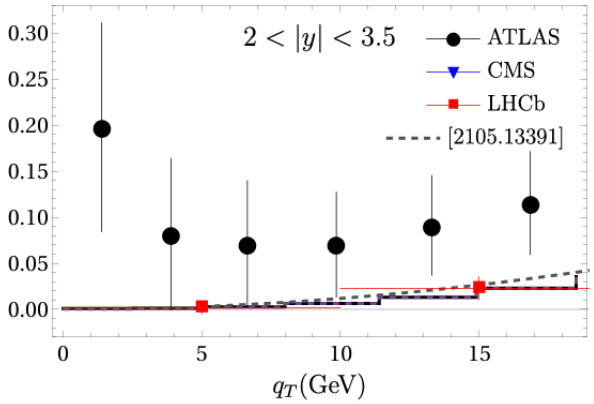
- Contains both **Boer-Mulders** and **unpolarized** distributions

$$A_2 \sim \frac{\boldsymbol{\Delta}^2 - 2(\boldsymbol{t}\boldsymbol{\Delta})^2}{2M^2}\left(h_1^\perp h_1^\perp - \frac{2M^2}{Q^2} f_1 f_1\right)$$

Angular distribution A_2







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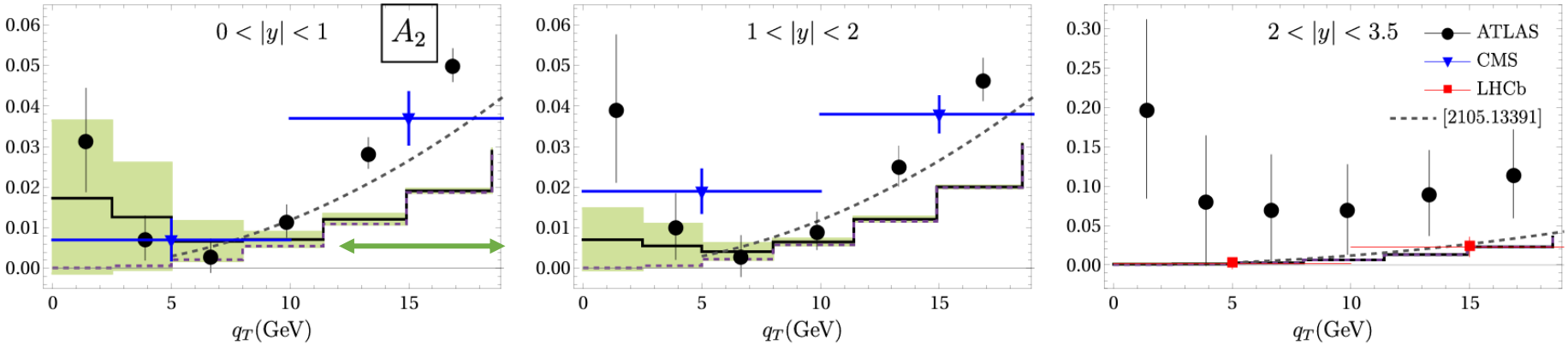
Dominant at $q_T \to 0$

Angular distribution A_2



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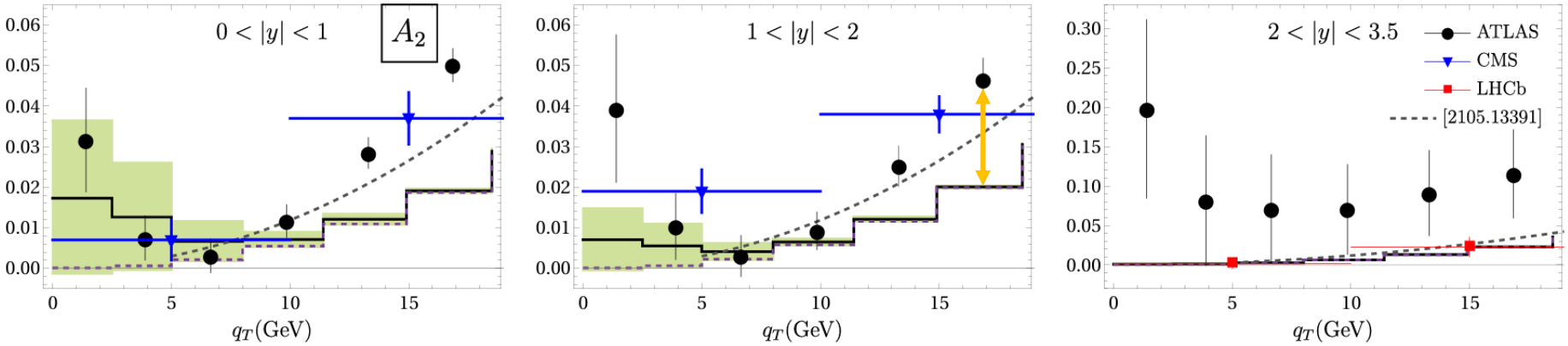

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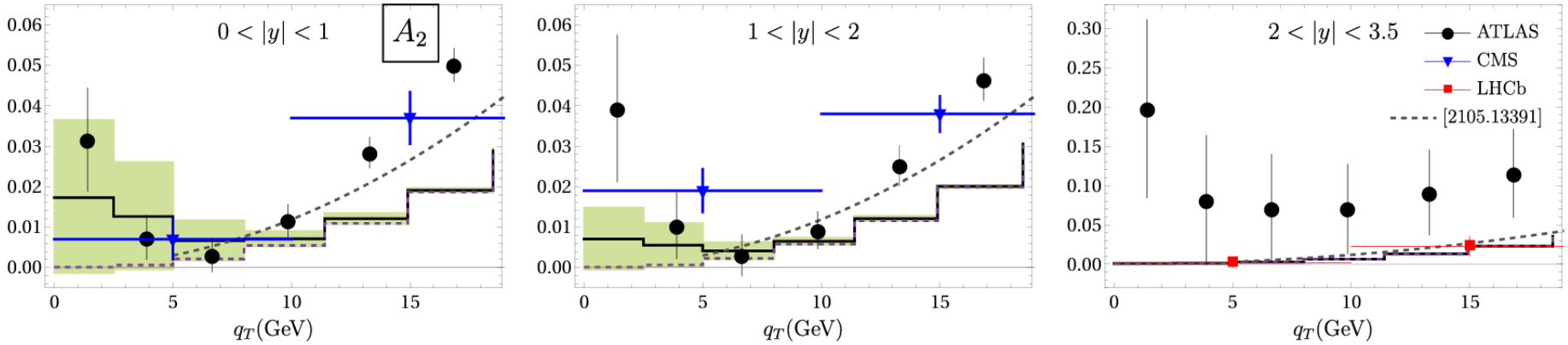
Dominant at $q_T \to 0$ Dominant at larger q_T

- **Additional contributions** from q_T^2/Q^2 corrections at large q_T

  - We can quantify them using ATLAS measurement
    - 2% - 4% at $q_T \sim 20\,\mathrm{GeV}$ → Explains **discrepancy** with the data

- We **extract** the **Boer-Mulders** function from the data for the A_2 **angular coefficient** measured at **ATLAS**



- We **model** it with the following form → $\tilde{h}_1^{\perp}(x,b) = \dfrac{2^{\alpha+1}}{\Gamma(\alpha+1)} \dfrac{N}{\cosh(\lambda b)}\, x \ln^{\alpha}(1/x)$

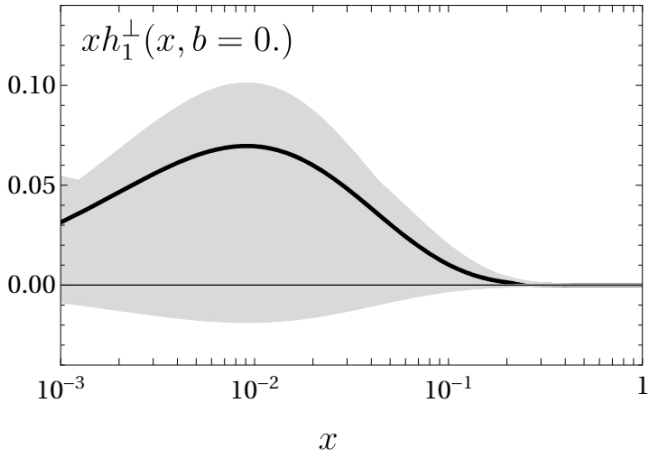
→ **Fix** the **parameters** → **Data points** with $q_T < 10\,\text{GeV}$ → $N_{pt} = 12$

→ **Insufficient** → Fix $\lambda = 0.2\text{GeV}$

Fit

$$\chi^2/N_{pt} = 1.16$$

$$N = -0.27^{+0.34}_{-0.12} \text{ and } \alpha = 9.4^{+5.4}_{-0.9}$$



Lam-Tung relation

$$A_{\rm LT} = A_0 - A_2$$

- If we use the **TMD factorization** theorem the **relation does not hold at LP**
- **TMD-with-KPCs** factorization theorem expression

$$A_{\rm LT} = \frac{4\pi\alpha_{\rm em}^2}{3N_c s}\sum_{q,G,G'} Q^4 \Delta_G^* \Delta_{G'} \Big\{ z_{+\ell}^{GG'} z_{+q}^{GG'} \mathcal{C}[2\frac{((\boldsymbol{tk}_1)-(\boldsymbol{tk}_2))^2-(\boldsymbol{k}_1-\boldsymbol{k}_2)^2}{Q^2}, f_1 f_1]$$

$$+ z_{+\ell}^{GG'} r_{+q}^{GG'} \mathcal{C}[\frac{\boldsymbol{k}_1^2+\boldsymbol{k}_2^2-((\boldsymbol{tk}_1)-(\boldsymbol{tk}_2))^2}{M^2} + \frac{\boldsymbol{k}_1^2+\boldsymbol{k}_2^2}{M^2}\frac{((\boldsymbol{tk}_1)-(\boldsymbol{tk}_2))^2-(\boldsymbol{k}_1-\boldsymbol{k}_2)^2}{Q^2}, h_1^\perp h_1^\perp]\Big\}$$

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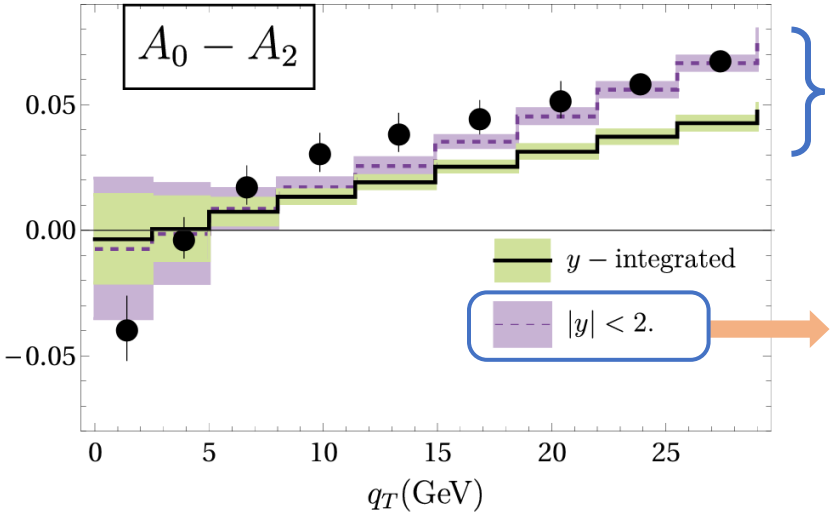

Prediction < ATLAS measurement → large x ✗

Theory drops to 0 at large y

Integrating to restrict the ***y*-region**

Good agreement

Lam-Tung relation

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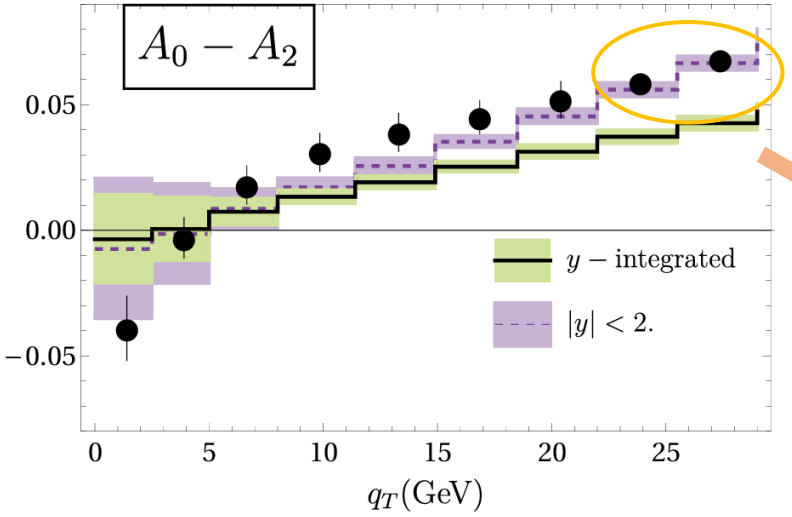
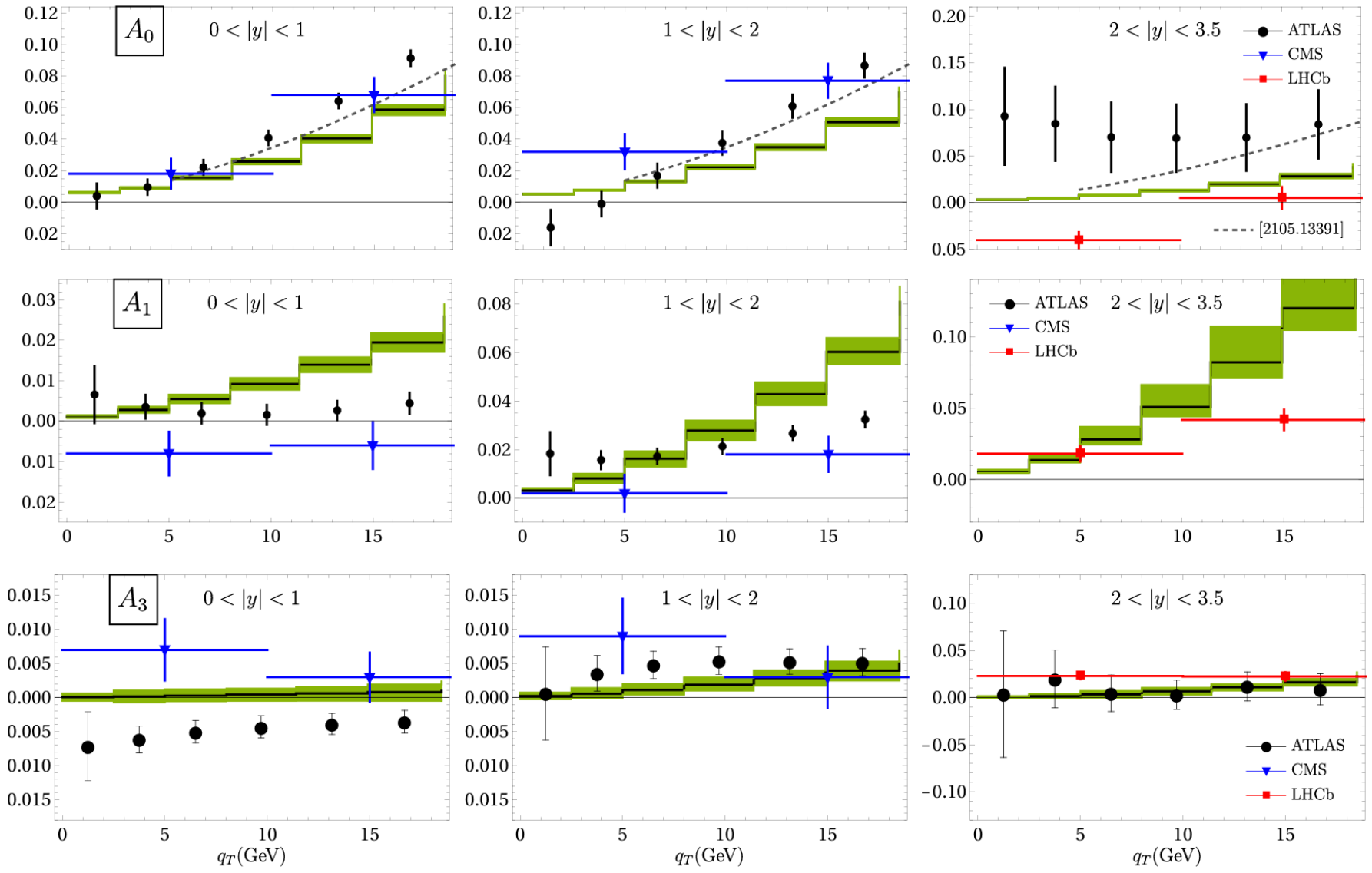


- **Suppressed** by an extra factor at larger q_T values
  - → **Smaller** q_T^2/Q^2 **corrections**

→ **Good agreement** in contrast to A_0 and A_2 independently

→ **Description** only posible **due to KPCs inclusion**

Angular distributions $A_{0,1,3}$



Conclusions

- First practical application of the TMD-with-KPCs factorization theorem
  - The angular distributions of the Drell-Yan leptons can be satisfactorily described
- The Boer-Mulders has been extracted using the A_2 data from ATLAS
  - The peculiar shape of A_2 at low-q_T is an evidence of the Boer-Mulders distribution
- We have tested the method proposed in [AV, 2307.13054v2]
  - It can be useful to describe some observables that are power-suppressed, like the Lam-Tung relation

But… this is not all! Next step: SIDIS process

- The same thing can be done with the SIDIS process
- Cross-section decomposition over structure functions

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} \sim \Big\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \ldots$$

We have already computed a theoretical expression in the TMD-with-KPCs factorization theorem framework

Currently working in the plots → We expect the effect of KPCs to be greater

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Thank you for your attention!

BACKUP

Power corrections

Motivation: a problem with normalization

- Why should we study **power corrections**?

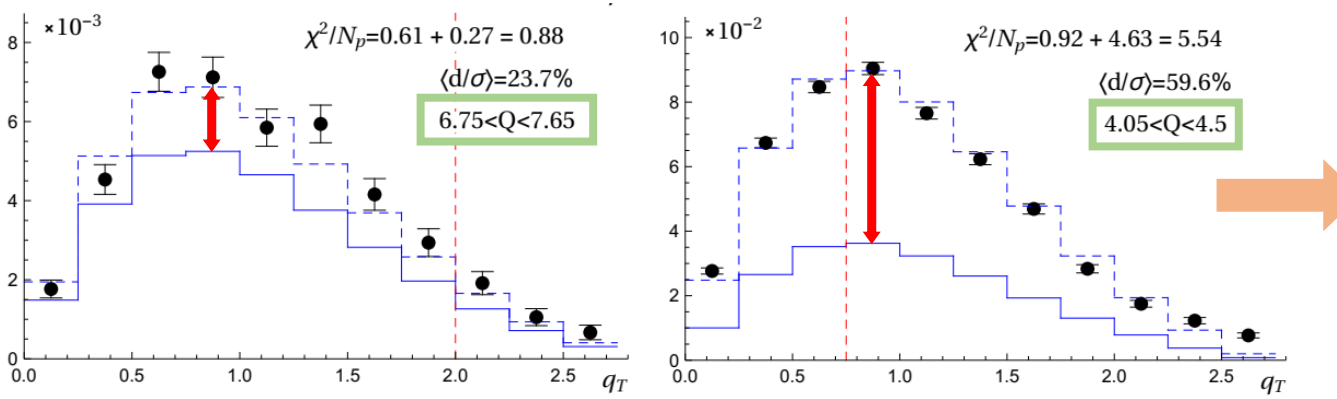
  → TMD factorization theorem

  - High energies ✓
  - Low energies → $Q \sim 10 - 15\,\mathrm{GeV}$ ✗

  Some **problems**…

  - Violation of EM-gauge invariance
  - Deficiencies in the data **normalization**

Same shape but different normalization



Evidence of power corrections?

JHEP 10 (2019) 090, A.V.

Power corrections

The four types

- **Corrections** to the **LP term** → **suppressed by** powers of $Q\,(\sim q^+ \sim q^-)$ **large scale**
- The power corrections can be categorized into **four** conceptual **types**
  - ❑ **Target-mass** corrections $\sim M/Q$ → hadron mass
  - ❑ ***Higher-twist*** power corrections $\sim (\Lambda/Q)^{n-2}$ → TMDs of larger **twist (*n = D – S*)**
  - ❑ q_T/Q power corrections

Position space $\sim \dfrac{1}{bQ}$ ⟷ **Momentum space** $\sim \dfrac{q_T}{Q}$

  - ❑ k_T/Q power corrections → **Kinematic Power Corrections (KPCs)**

Position space $\sim F_1(x_1, b)\,\dfrac{\partial}{\partial b^\mu}\,F_2(x_2, b)$ ⟷ **Momentum space** $\sim \dfrac{k_{2T}^\mu}{Q} F_1(x_1, k_{1T}) F_2(x_2, k_{2T})$

→ **Transverse derivatives** of TMD distributions

The hierarchy of the TMD factorization theorem

- General structure of the TMD factorization theorem

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Bigg\{ \Phi_2 \times \Phi_2 \quad \text{LP}$$

$$+\frac{1}{Q}\Big(D\ \ \Phi_2\times\Phi_2 + \quad \Phi_2\times\Phi_3\Big) \quad \text{NLP}$$

$$+\frac{1}{Q^2}\Big(D^2\,\Phi_2\times\Phi_2 + D\ \ \Phi_2\times\Phi_3 + \quad \Phi_3\times\Phi_3 + \quad \Phi_2\times\Phi_4 + \ \frac{\Phi_2\times\Phi_2}{b^2}\Big) \quad \text{NNLP}$$

$$+\frac{1}{Q^3}\Big(D^3\,\Phi_2\times\Phi_2 + D^2\,\Phi_2\times\Phi_3 + D\ \ \Phi_3\times\Phi_3 + D\ \ \Phi_2 + \Phi_4 + D\frac{\Phi_2\times\Phi_2}{b^2} + \ldots\Big)$$

$$+\frac{1}{Q^4}\Big(D^4\,\Phi_2\times\Phi_2 + \ldots\Big)$$

$$+\ \ldots\ \Bigg\}^{\mu\nu}$$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \frac{\partial}{\partial b^\mu}$ → transverse derivative

The hierarchy of the TMD factorization theorem

- General structure of the TMD factorization theorem

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Bigg\{ \Phi_2 \times \Phi_2 + \frac{1}{Q}\Big(D\ \Phi_2\times\Phi_2 + \Phi_2\times\Phi_3\Big) + \frac{1}{Q^2}\Big(D^2\,\Phi_2\times\Phi_2 + D\ \Phi_2\times\Phi_3 + \Phi_3\times\Phi_3 + \Phi_2\times\Phi_4 + \frac{\Phi_2\times\Phi_2}{b^2}\Big) + \frac{1}{Q^3}\Big(D^3\,\Phi_2\times\Phi_2 + D^2\,\Phi_2\times\Phi_3 + D\ \Phi_3\times\Phi_3 + D\ \Phi_2+\Phi_4 + D\frac{\Phi_2\times\Phi_2}{b^2} + \dots\Big) + \frac{1}{Q^4}\Big(D^4\,\Phi_2\times\Phi_2 + \dots\Big) + \dots \Bigg\}^{\mu\nu}$$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \frac{\partial}{\partial b^\mu}$ → transverse derivative

Each **column**

- Series of KPCs to the first term
- Unique non-perturbative content
- Independent contributions

The hierarchy of the TMD factorization theorem

- General structure of the TMD factorization theorem

$$W^{\mu\nu} = \frac{1}{N_c}\int \frac{d^2b}{(2\pi)^2}\, e^{-i(bq_T)} \Bigg\{ \Phi_2 \times \Phi_2$$

$$+ \frac{1}{Q}\left(D\ \Phi_2 \times \Phi_2 + \Phi_2 \times \Phi_3 \right) \quad (k_T/Q)$$

$$+ \frac{1}{Q^2}\left(D^2\ \Phi_2 \times \Phi_2 + D\ \Phi_2 \times \Phi_3 + \Phi_3 \times \Phi_3 + \Phi_2 \times \Phi_4 + \frac{\Phi_2 \times \Phi_2}{b^2} \right) \quad (k^2_T/Q^2)$$

$$+ \frac{1}{Q^3}\left(D^3\ \Phi_2 \times \Phi_2 + D^2\ \Phi_2 \times \Phi_3 + D\ \Phi_3 \times \Phi_3 + D\ \Phi_2 + \Phi_4 + D\frac{\Phi_2 \times \Phi_2}{b^2} + \ldots \right) \quad (k^3_T/Q^3)$$

$$+ \frac{1}{Q^4}\left(D^4\ \Phi_2 \times \Phi_2 + \ldots \right) \quad (k^4_T/Q^4)$$

$$+ \ldots \Bigg\}^{\mu\nu}$$

Notation

- Φ_n → TMD of twist-n
- $\times$ → Integral convolution
- $D \sim \frac{\partial}{\partial b^\mu}$ → transverse derivative

→ KPCs following the LP term

- Same TMDs and perturbative part as LP
- All power-suppressed terms containing twist-2 TMDs
- Restoration of gauge and frame invariance

Hadron tensor in TMD-with-KPCs factorization theorem

- Theory of KPCs for TMD factorization developed in [AV, 2307.13054v2]
  - KPCs series to the LP term derived and summed at all powers
  - Drell-Yan unpolarized cross-section computed in this framework

- Starting point → EW hadron tensor
  - → Unpolarized $f_{1;f/p}$ and Boer-Mulders $h^{\perp}_{1;f/p}$ distributions
  - → Same coefficient function as LP
  - → Main difference with LP → Convolution integral

$$W^{\mu\nu}_{GG'} = \frac{p_1^+ p_2^-}{N_c} C_0\left(\frac{Q^2}{\mu^2}\right) \int d^4k_1 d^4k_2 \delta(k_1^2)\delta(k_2^2)\delta^{(4)}(k_1+k_2-q) \qquad q_\mu W^{\mu\nu} = 0$$

$$\int d\xi_1 d\xi_2 \delta(k_1^+ - \xi_1 p_1^+)\delta(k_2^- - \xi_2 p_2^-) \sum_q \Big[$$

$$\mathrm{Tr}\,(\gamma_G^\mu \not{k}_2 \gamma_{G'}^\nu \not{k}_1) f_{1,q/h_1} f_{1,\bar{q}/h_1} + \mathrm{Tr}\,(\gamma_G^\mu \not{k}_1 \gamma_{G'}^\nu \not{k}_2) f_{1,\bar{q}/h_1} f_{1,q/h_1} - \frac{\epsilon_T^{\mu_1\alpha} k_1^\alpha k_1^{\nu_1} \epsilon_T^{\mu_2\beta} k_2^\beta k_2^{\nu_2}}{M^2} \Big($$

$$\mathrm{Tr}\,(\gamma_G^\mu \sigma^{\mu_2\nu_2} \gamma_{G'}^\nu \sigma^{\mu_1\nu_1}) h^{\perp}_{1,q/h_1} h^{\perp}_{1,\bar{q}/h_1} + \mathrm{Tr}\,(\gamma_G^\mu \sigma^{\mu_1\nu_1} \gamma_{G'}^\nu \sigma^{\mu_2\nu_2}) h^{\perp}_{1,\bar{q}/h_1} h^{\perp}_{1,q/h_1} \Big) \Big]$$

Angular decomposition of the lepton tensor

- Information about

  → Detected lepton pair + Measurement

  ↓

  Six-dimensional phase-space → $dPS = d^4q\, d\Omega$ → Lepton-pair angular part

  ↓

  Boson momentum $q = l + l'$

- Express lepton tensor via CS-frame angles θ and ϕ

  → Leptons momenta decomposition [Arnold, Metz, Schlegel, 0809.2262v2]

  → $l^\mu = \dfrac{q^\mu + \Delta_l^\mu}{2} \qquad l'^\mu = \dfrac{q^\mu - \Delta_l^\mu}{2}$

  ↓

  Parametrization

  ↓

$$\Delta_l^\mu = \bar{n}^\mu q^+ \frac{Q}{\tau}\left(\cos\theta - \frac{Q}{|\boldsymbol{q}_T|}\sin\theta\cos\phi\right) + n^\mu q^- \frac{Q}{\tau}\left(-\cos\theta - \frac{Q}{|\boldsymbol{q}_T|}\sin\theta\cos\phi\right)$$
$$+ q^\mu \frac{\tau}{|\boldsymbol{q}_T|}\sin\theta\cos\phi - \tilde{q}^\mu \frac{Q}{|\boldsymbol{q}_T|}\sin\theta\sin\phi$$

Angular decomposition of the lepton tensor

- Information about

  → Detected lepton pair + Measurement

  ↓

  Six-dimensional phase-space → $dPS = d^4q\, d\Omega$ → Lepton-pair angular part

  ↓

  Boson momentum $q = l + l'$

- Express lepton tensor via CS-frame angles θ and ϕ

  → Convenient decomposition for the lepton tensor

$$L^{\mu\nu}_{GG'} = (-Q^2)\left((v^2+a^2)_l \sum_{n=U,0,1,2,5,6} S_n(\theta,\phi)\mathfrak{L}^{\mu\nu}_n + (av)_l \sum_{n=3,4,7} S_n(\theta,\phi)\mathfrak{L}^{\mu\nu}_n \right)$$

Combination of **EW coupling constants**

Set of independent **angular polynomials**

Set of independent **tensors**

Angular structure functions

- **Differential cross-section** computation using the previous **hadron** and **lepton tensors**

  → The **result** is conveniently **expressed as**

$$\frac{d\sigma}{d^4q d\Omega} = \frac{3}{16\pi} \sum_{n=U,0,\ldots,7} S_n(\theta,\phi)\Sigma_n, \quad \text{Structure functions} \quad A_n = \frac{\Sigma_n}{\Sigma_U} \rightarrow \Sigma_U = \frac{d\sigma}{d^4q}$$

Normalization

- We have explicitly obtained the **theoretical expressions** for the various Σ_n

  → We can **codify** them in a **general way** as

$$\Sigma_n \sim \text{Constants} \times \text{Gauge boson propagator} \times \underset{1}{\text{Some combination of vector (v) and axial (a) coupling constants}} \times \underset{2}{\text{Convolution integral involving } f_1 \text{ or } h_1^\perp \text{ distributions}}$$

1 $\sim (v^2+a^2) \quad (v^2-a^2) \quad (av)$ → Some due to the Z-boson

2 Two possibilities

- $\mathcal{C}[A, f_1 f_2]$ → Symmetric flavor $f_{q_1} f_{\bar{q}_2} + f_{\bar{q}_1} f_{q_2}$
- $\mathcal{C}[A, \{f_1 f_2\}_A]$ → Anti-symmetric flavor $f_{q_1} f_{\bar{q}_2} - f_{\bar{q}_1} f_{q_2}$

Leading quark TMDs