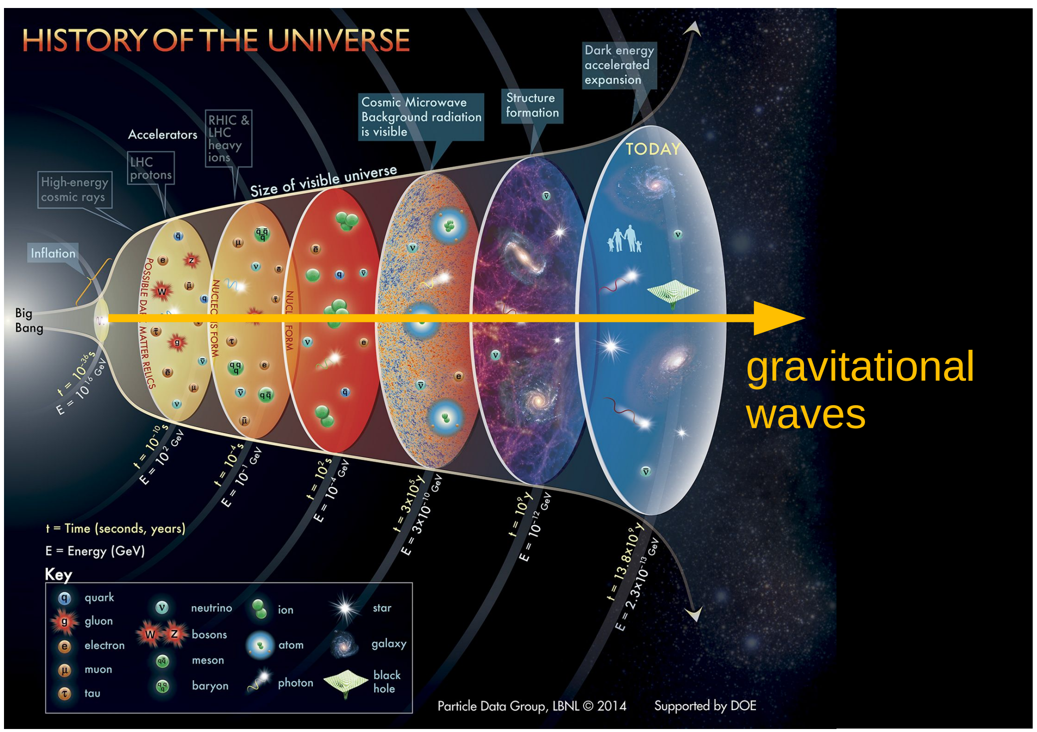
Searching for anisotropies with pulsar timing arrays



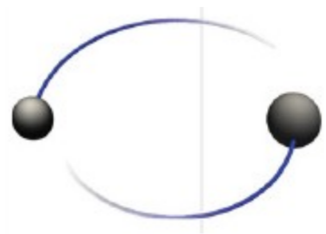
Valerie Domcke
CERN

Workshop “*Towards a realistic forecast detection of primordial gravitational wave backgrounds*”, *Valencia*
December 13, 2024

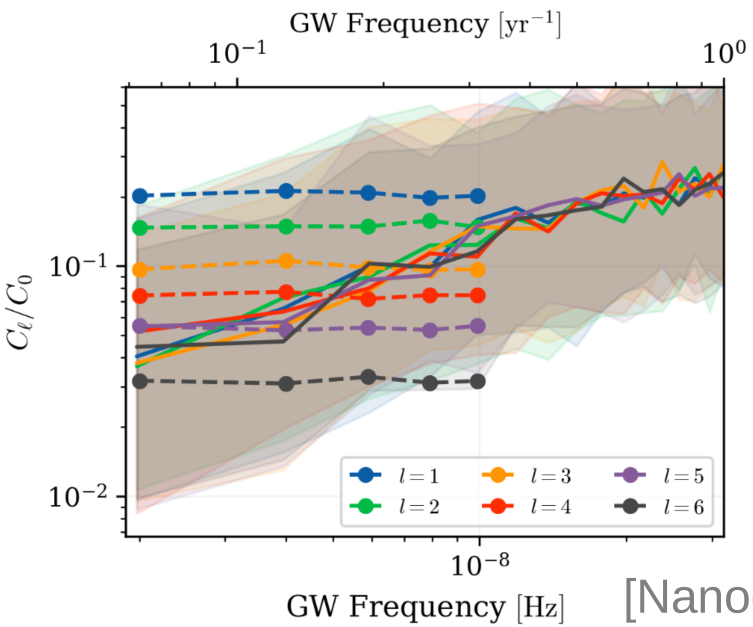
arxiv: 2407.14460
In collaboration with Frederik Depta, Gabriele Franciolini, Mauro Pieroni

Supermassive BHBs or Early Universe ?

Supermassive black hole mergers

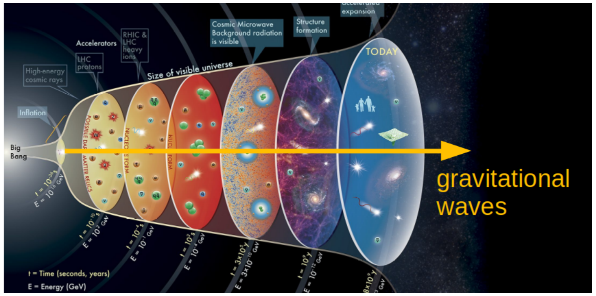


Expect ~ 2 – 20 % anisotropy from Poisson distributed SMBHBs

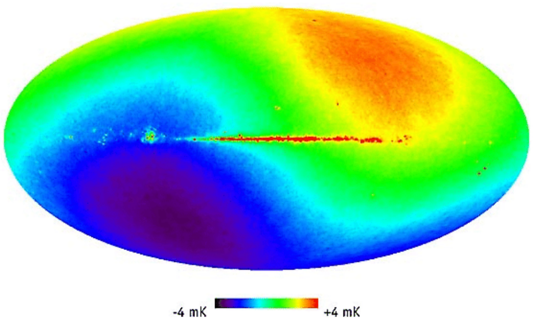


[Nanograv15]

Early Universe physics

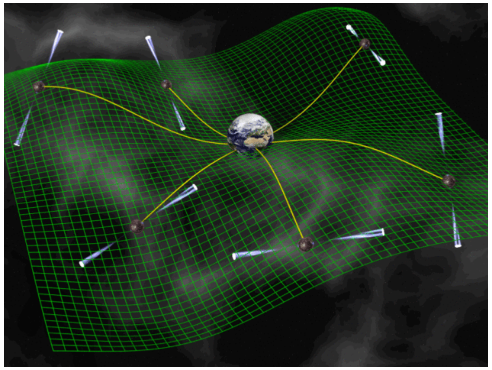


Expect ~ 0.1 % anisotropy aligned with kinematic CMB dipole



PTA response function

Time delay for pulse from pulsar I located at $D_I \hat{p}_I$:



$$\Delta t_I = \frac{1}{2} \hat{p}_I^a \hat{p}_I^b \int_0^{D_I} \mathrm{d}s \, h_{ab}\left(t(s), \vec{x}(s)\right)$$

$$\simeq \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}^2\hat{k} \sum_P \frac{1}{2\pi i f t} F^P_{\hat{p}_I}(\hat{k}) \, \tilde{h}_P(f;\hat{k}) \, e^{2\pi i f t}$$

„instrument" response

$$F^P_{\hat{p}_I}(\hat{k}) = \frac{1}{2} \frac{\hat{p}_I^a \hat{p}_I^b}{1 + \hat{p}_I \cdot \hat{k}} e^P_{ab}(\hat{k})$$

Stochastic gravitational wave background (SGWB)

$\hat{p}_I$ pulsar direction

$\hat{k}$ GW direction

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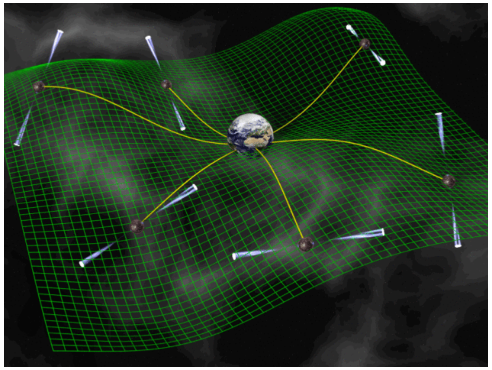

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Stochastic gravitational wave background (SGWB)

$$\langle \tilde{h}_P(f,\hat{k}) \tilde{h}^*_{P'}(f',\hat{k}') \rangle = \frac{1}{4} S_h(f) P(\hat{k}) \delta(f - f') \delta_{PP'} \delta^2(\hat{k},\hat{k}')$$

$\hat{p}_I$ pulsar direction

$\hat{k}$ GW direction

GW power spectral density.
For isotropic SGWB:

$$\Omega_{\mathrm{GW}} h^2 = \frac{h^2}{\rho_c} \frac{\mathrm{d}\rho_{\mathrm{GW}}}{\mathrm{d}\log f} \equiv \frac{2\pi^2 f^3}{3 H_0^2/h^2} S_h$$

angular distribution of
GW power on the sky

$$P(\hat{k}) = \sum_{\ell,m} c_{\ell m} Y_{\ell m}(\theta,\phi)$$

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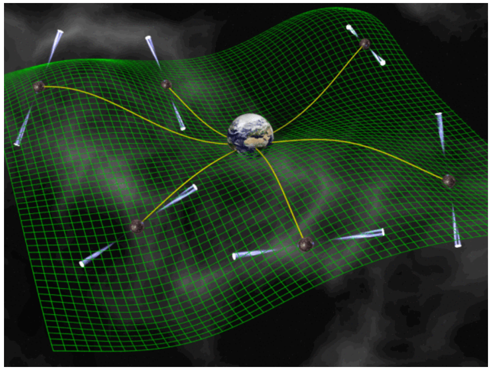

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Time delay with respect to flat space-time. Calibration of timing model

$$\simeq \int_{-\infty}^{\infty} \mathrm{d}f \int \mathrm{d}^2\hat{k} \sum_P \frac{1}{2\pi i f t} F^P_{\hat{p}_I}(\hat{k})\, \tilde{h}_P(f;\hat{k})\, e^{2\pi i f t}$$

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angular distribution of GW power on the sky

$$P(\hat{k}) = \sum_{\ell,m} c_{\ell m} Y_{\ell m}(\theta,\phi)$$

Measuring anisotropies

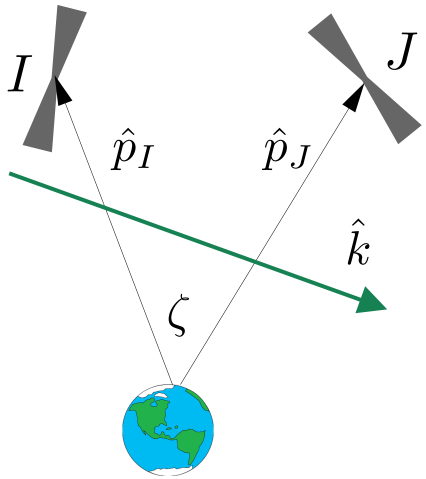
Cross-correlation:

$$\langle \Delta t_I \Delta t_J \rangle = \int_{-\infty}^{\infty} \mathrm{d}f \, \frac{S_h(f)}{24\pi^2 f^2} \frac{3}{2} \sum_P \int d^2\hat{k} \, F^P_{\hat{p}_I}(\hat{k}) F^P_{\hat{p}_J}(\hat{k}) \, P(\hat{k})$$

$$\equiv \Gamma_{IJ}(f) = \sum_{\ell m} c_{\ell m} \Gamma_{IJ,\ell m}$$

instrument response
GW background

parametrizes anisotropy in SWGB



Generalized overlap reduction functions:

$$\Gamma_{IJ,\ell m} = \frac{3}{2} \sum_P \int d^2\hat{k} F^P_{\hat{p}_I}(\hat{k}) F^P_{\hat{p}_J}(\hat{k}) Y_{\ell m}(\hat{k})$$

$\Gamma_{IJ,00}$ Hellings-Down correlation

$\Gamma_{IJ,\ell m}$ instrument response to ℓ, m-multipole

[Mingarelli et al `13]

Measuring anisotropies

Cross-correlation:

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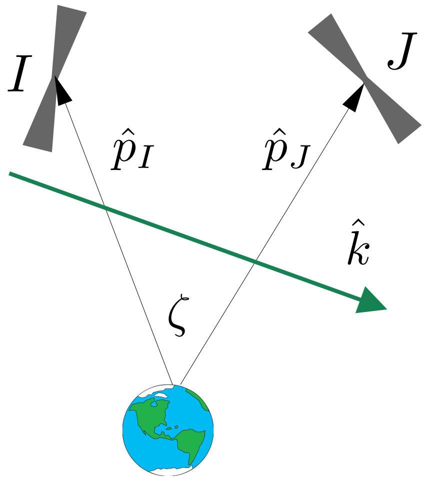
$$\equiv \Gamma_{IJ}(f) = \sum_{\ell m} c_{\ell m} \Gamma_{IJ,\ell m}$$

instrument response
GW background

parametrizes anisotropy in SWGB

No sum over pulsars I,J. Hellings-down curve is a very suboptimal probe of anistropies



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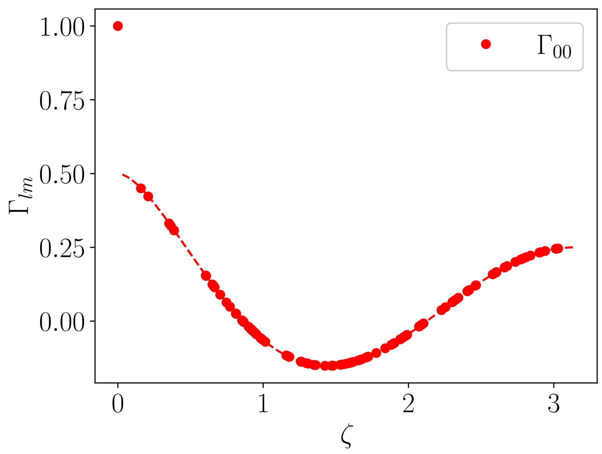
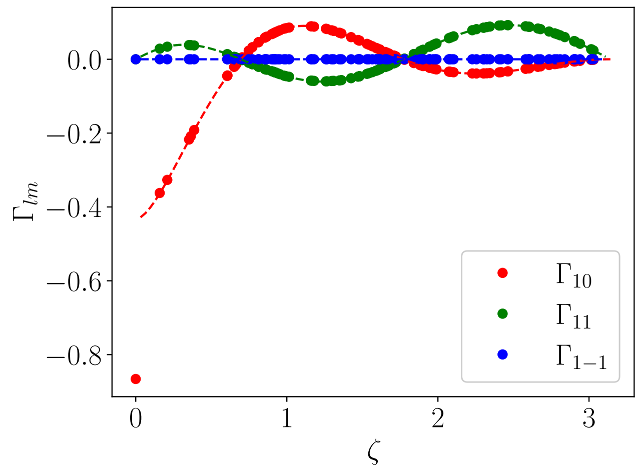
[Mingarelli et al `13]

Generalized overlap reduction functions

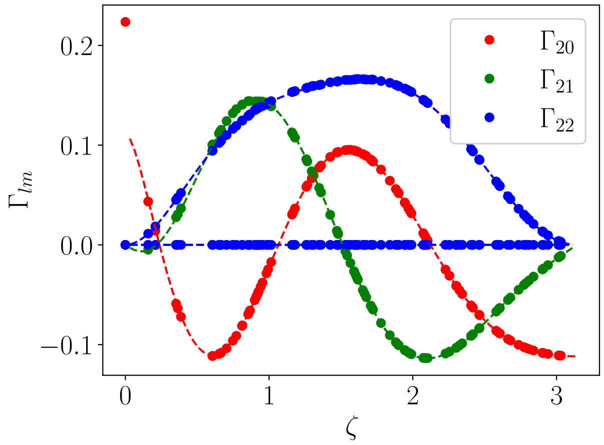
Generalized overlap reduction functions in computational frame (one pulsar on z axis):



Hellings-Down curve



dipole



quadrupole

Sensitivity to anisotropies

Covariance matrix

$$C_{IJ} = \langle \Delta t_I \Delta t_J \rangle \propto \sum_{\ell,m} c_{\ell m} \Gamma_{IJ,\ell m} + C_{\text{noise}}$$

Fischer information matrix

$$F_{\ell m,\ell' m'} = \sum_{f_k, IJKL} C^{-1}_{IJ} C^{-1}_{KL} \frac{\partial C_{JK}}{\partial c_{\ell m}} \frac{\partial C_{LI}}{\partial c_{\ell' m'}}$$

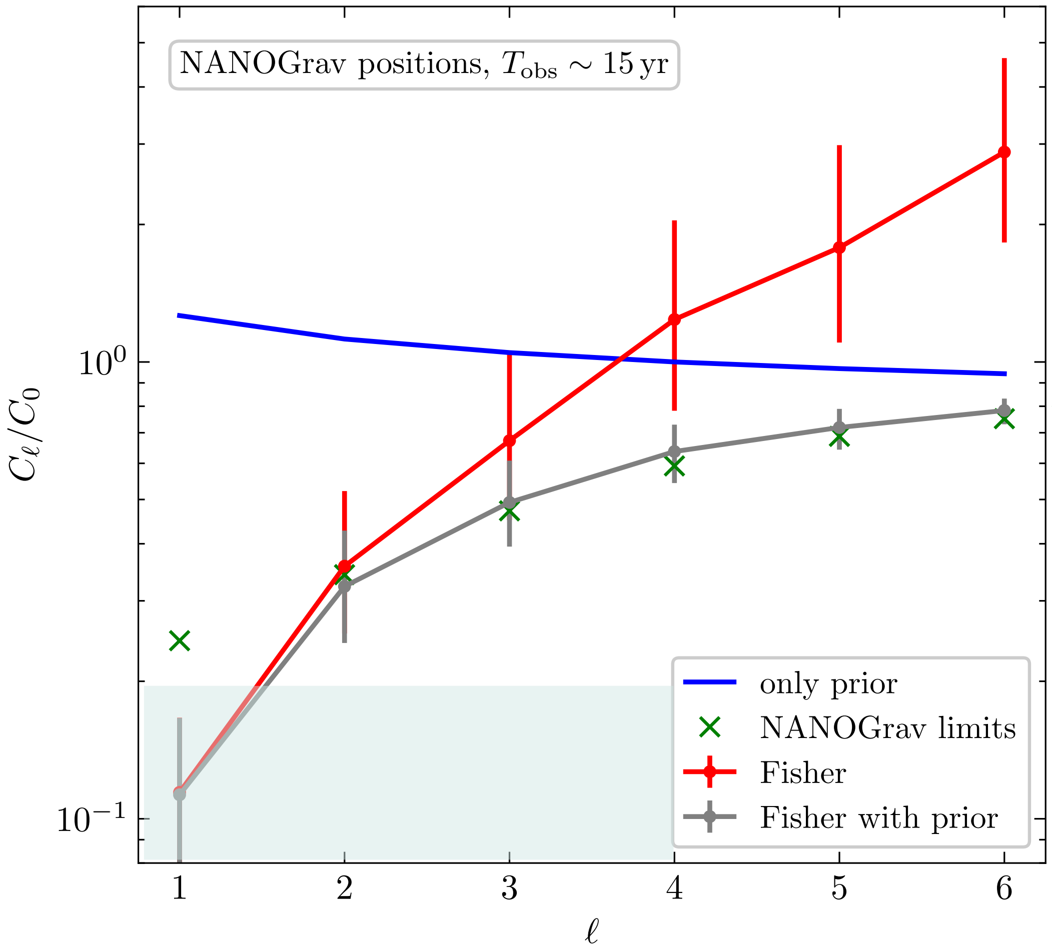
$$F^{-1}_{\ell m,\ell m} \quad \rightarrow \quad \text{sensitivity estimate for } c_{\ell m}$$

Assumptions:

- noise uncorrelated across pulsars, statistically isotropic
- Gaussian distributions for c_{lm}

Cross-check against NG15 results

[Depta, VD, Franciolini, Pieroni `24]



$$C_\ell = \frac{1}{2\ell+1}\sum_m |c_{\ell m}|^2.$$

Estimated sensitivity with 70 pulsars

Estimated reconstructed limits imposing NG prior

NG15 results

SMBHB background:

$$C_\ell/C_0 \sim 1 - 20\%$$

[Mingarelli et al `13]

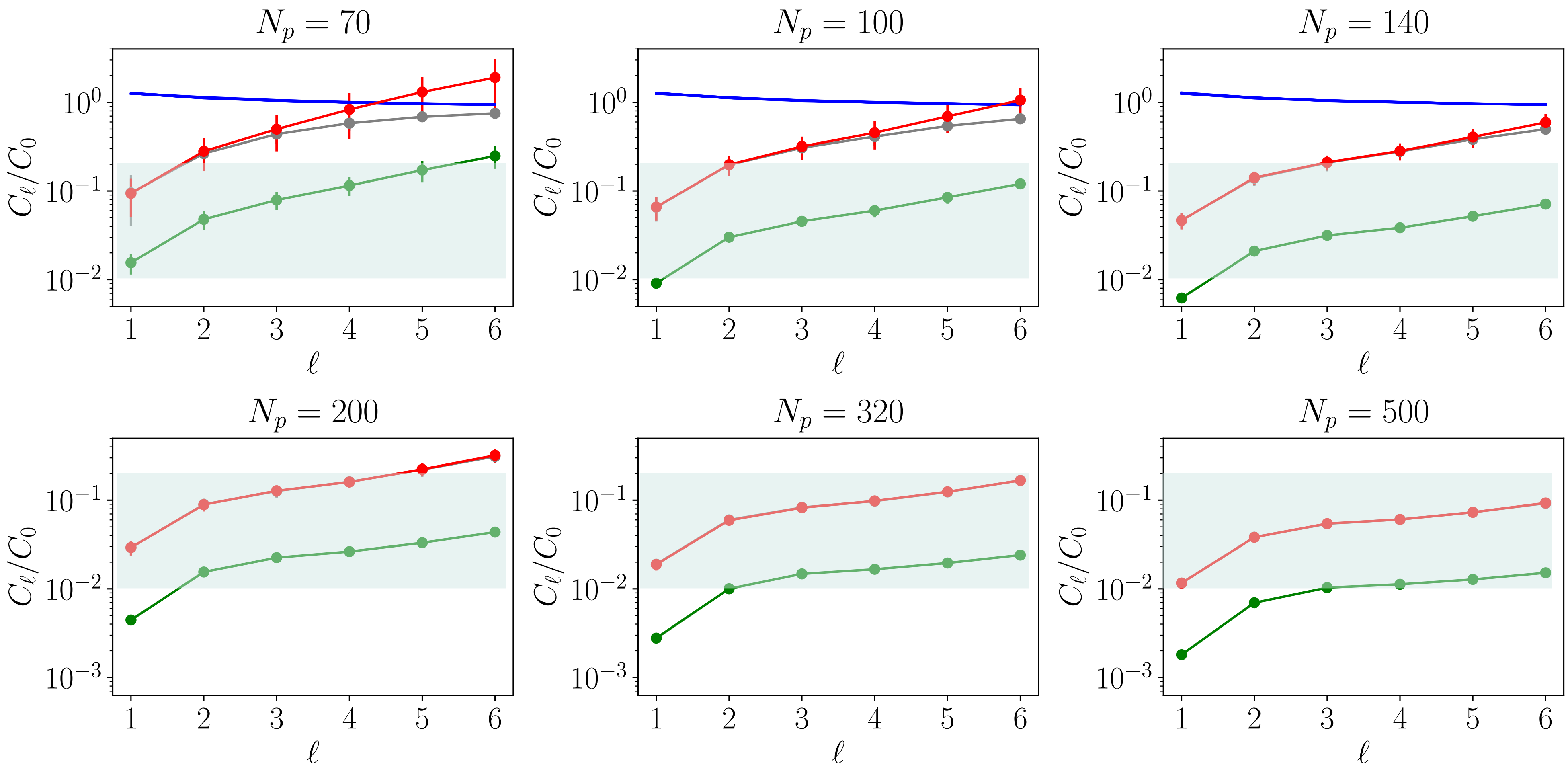
(weakly) informative constraints only for low multipoles

Prospects: anisotropy searches with PTAs

See also Meerkat `24
noise levels: 2412.01148

[Depta, VD, Franciolini, Pieroni `24]

Prospects with upcoming PTAs, assuming EPTA-like and SKA-like noise:



Anisotropies of SMBHB background within reach of upcoming PTAs.

Note on „square root basis“

$$P(\hat{k}) = \left[\sum_{L=0}^{L_{\max}} \sum_{m=-L}^{L} g_{LM} Y_{LM}\right]^2$$

$c_{\ell,m}$

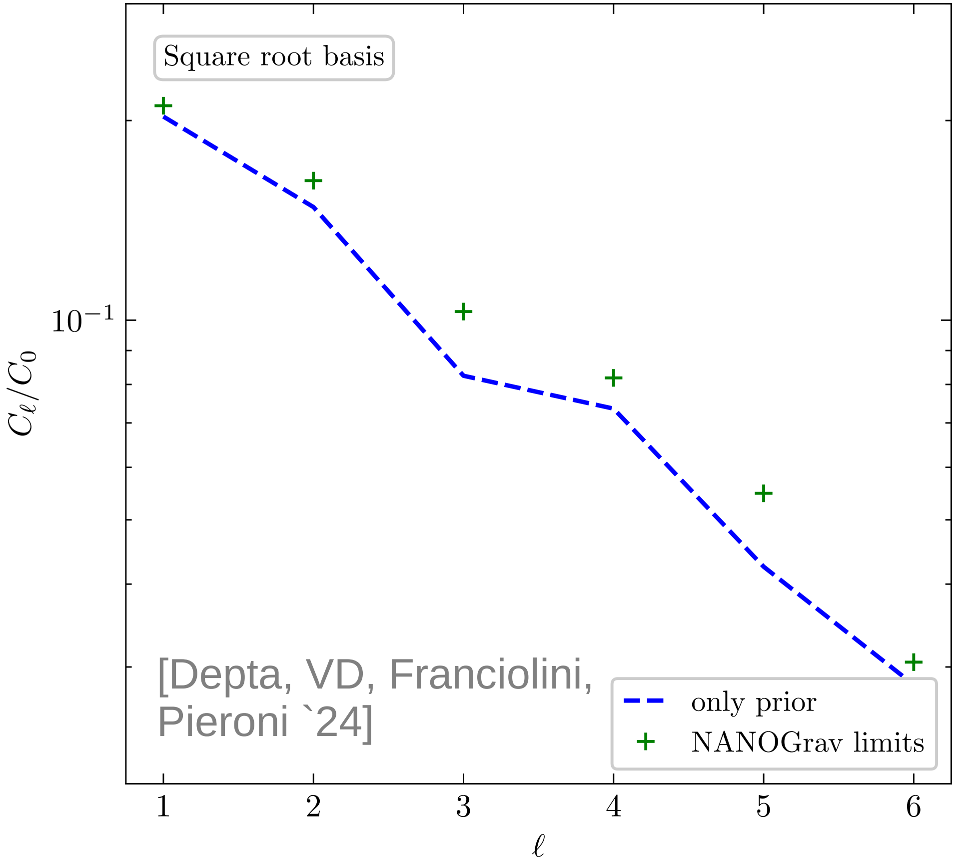
Clebsch-Gordan

Note on „square root basis“

$$P(\hat{k}) = \left[\sum_{L=0}^{L_{\max}} \sum_{m=-L}^{L} g_{LM} Y_{LM}\right]^2$$

$c_{\ell,m}$

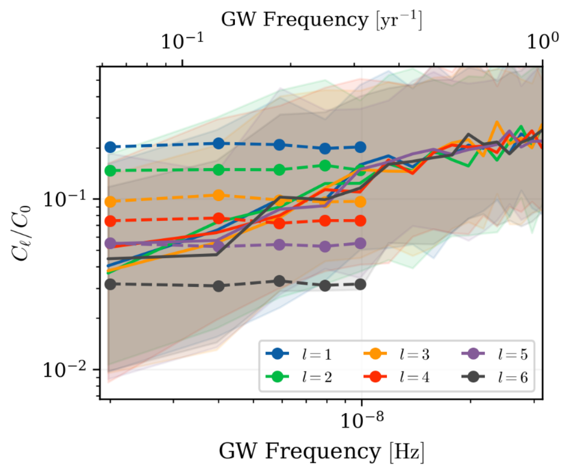
Clebsch-Gordan



[Depta, VD, Franciolini, Pieroni `24]

- Fully prior dominated
- Don’t use this with current data levels

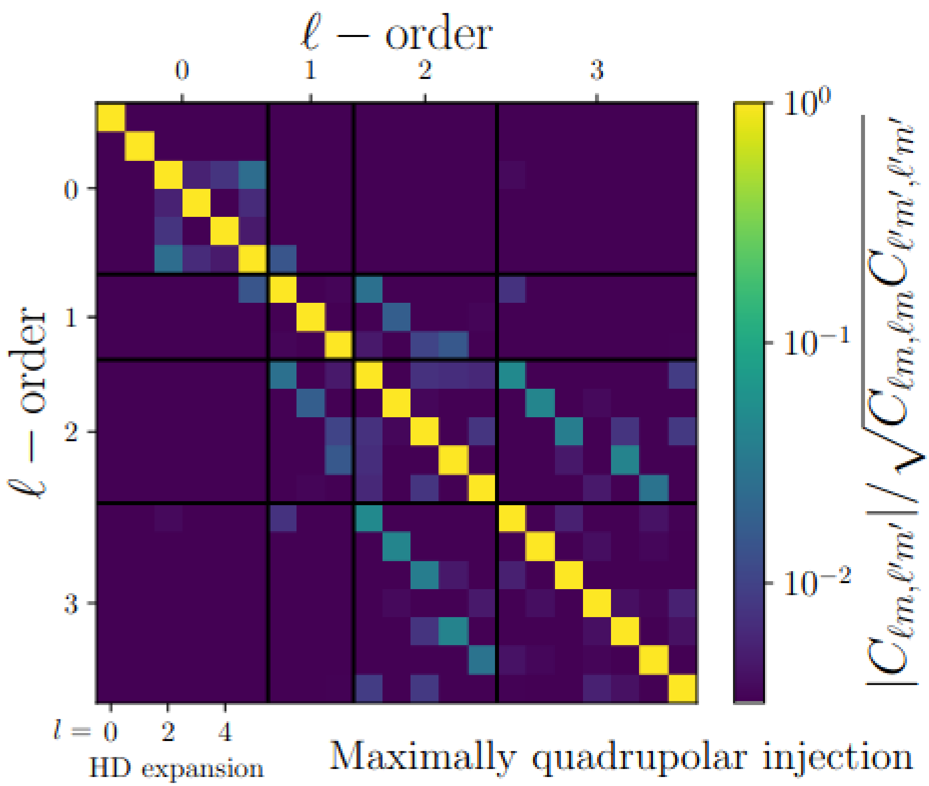
X



Impact of anisotropies on HD reconstruction

$$\langle \Delta t_I \Delta t_J \rangle = \int_{-\infty}^{\infty} \mathrm{d}f \, \frac{S_h(f)}{24\pi^2 f^2} \frac{3}{2} \sum_P \int d^2\hat{k} \, F^P_{\hat{p}_I}(\hat{k}) F^P_{\hat{p}_J}(\hat{k}) \; P(\hat{k})$$

Expand in ℓ, m (anisotropies) and Legendre polynomials $P_l(\zeta)$ (HD curve)

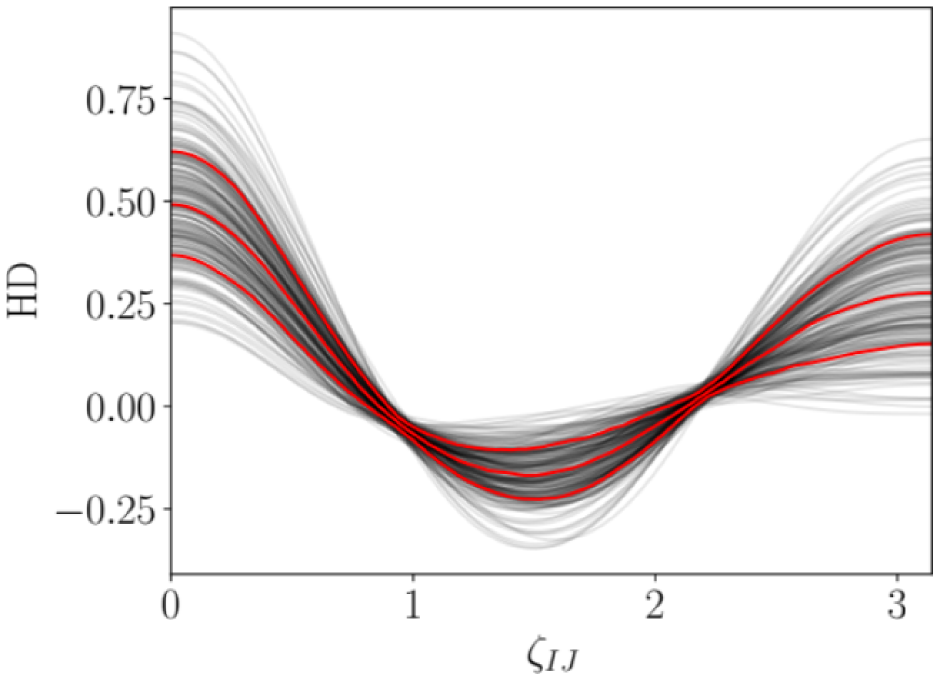


Maximally quadrupolar injection

- Small (< 1 %) mixing of the two expansions
- HD reconstruction not significantly biased by anisotropies if accounted for in fit

Cosmic variance

[see also Allen `22]



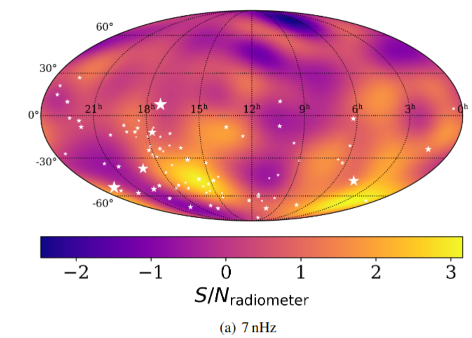
1/T ~ nHz ~ 1/(10 pc) , D ~ kpc

Observations limited by observation time

→ snapshot of GWB instead of ensemble average

→ cosmic variance

e.g. Meerkat `24:
“hotspot with p = 0.015, not accounting for CV“
2412.01214



Cosmic variance impacts **interpretation** of locally measured anisotropies:
local measurement versus ensemble average / fundamental model parameters

[see also Konstandin et al `24 / Andrea’s talk]

Conclusions

Anisotropies are key to distinguishing astrophysical from cosmolgical GWBs in the PTA band

With current PTA data, only constraints on dipole and possibly quadrupole are (mildly) informative

About 150 pulsars (~ current IPTA configuration) with SKA-like noise (demonstrated by Meerkat) required to probe %-level anisotropies

Anisotropy expected from SMBHB GWB: detection possible very soon, exclusion possible within less than 10 years.

Caution: - forecast vs real data analysis
- interpretation of local anisotropies subject to cosmic variance

... and an advertisement:

CERN TH visitor program https://theory.cern/visitor-info

short-term visits typically O(week)

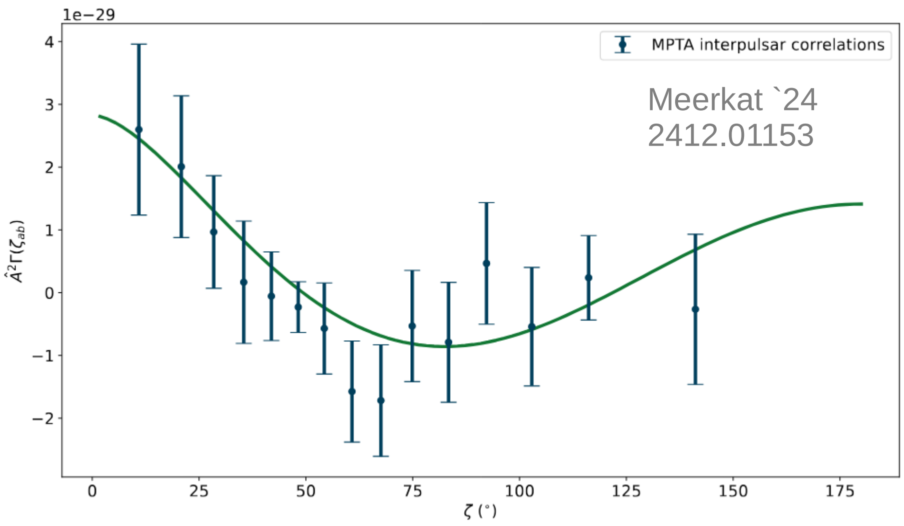
long term visits (> 3 months, usually sabbaticals)

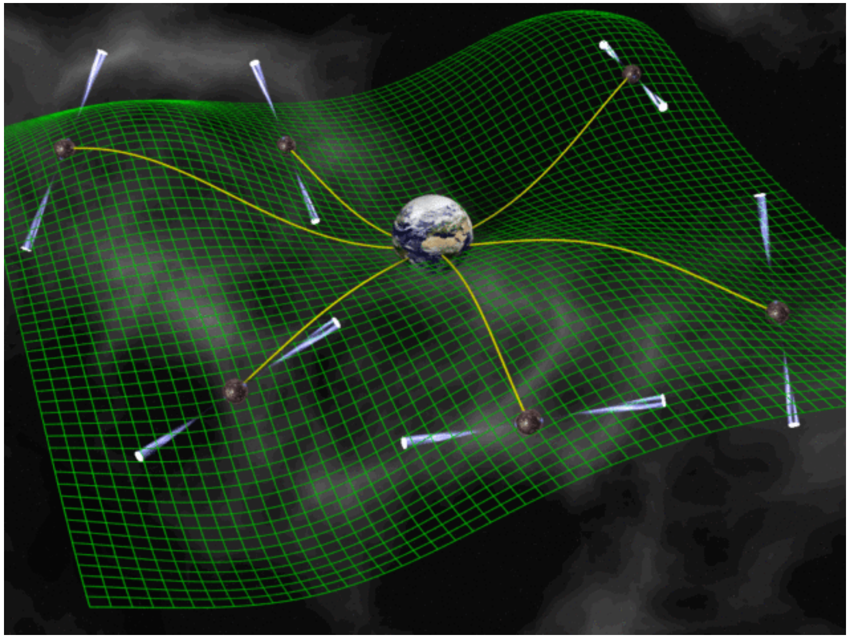
consider applying!

Backup slides

Pulsar timing arrays

- search for delays in pulse arrivals
- 2020: evidence for common stochastic noise component across all pulsars
- 2023: evidence for Hellings-Down correlation (i.e. gravitational waves)



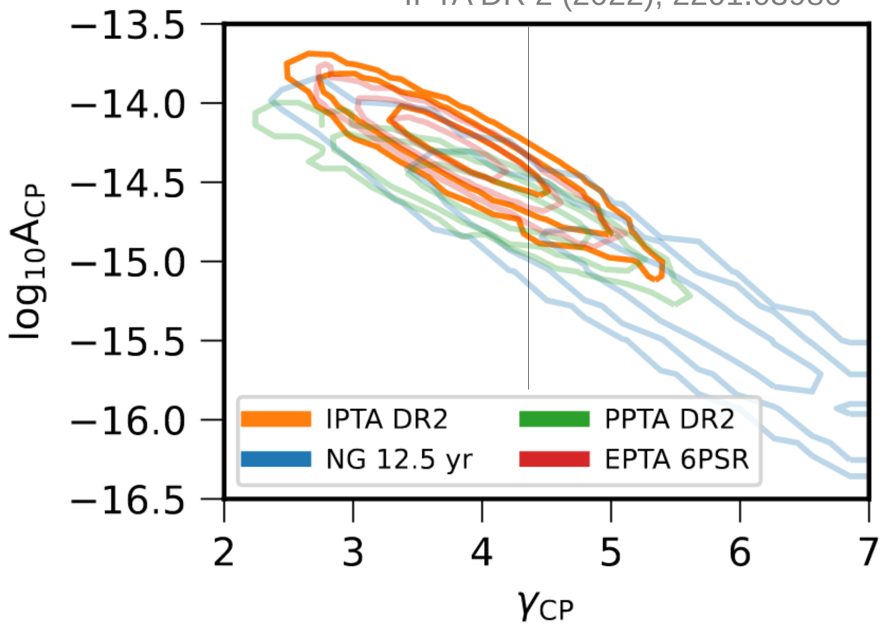


- likely origin: supermassive BH binaries
- SGWB or individual source?
- cosmological or astrophysical?

Information on spectral shape

power-law fit $h_c(f) = A_{\rm CP} \left(\frac{f}{f_{\rm yr}} \right)^{(3-\gamma_{\rm CP})/2}$

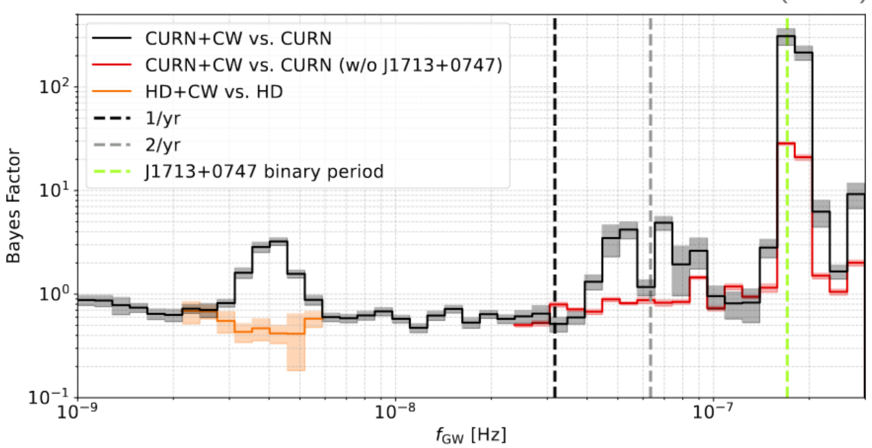
IPTA DR 2 (2022), 2201.03980



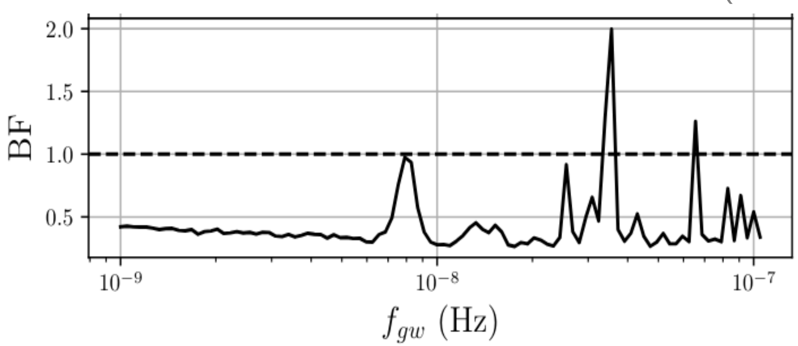
No conclusive indication of origin of GW signal

search for individual source

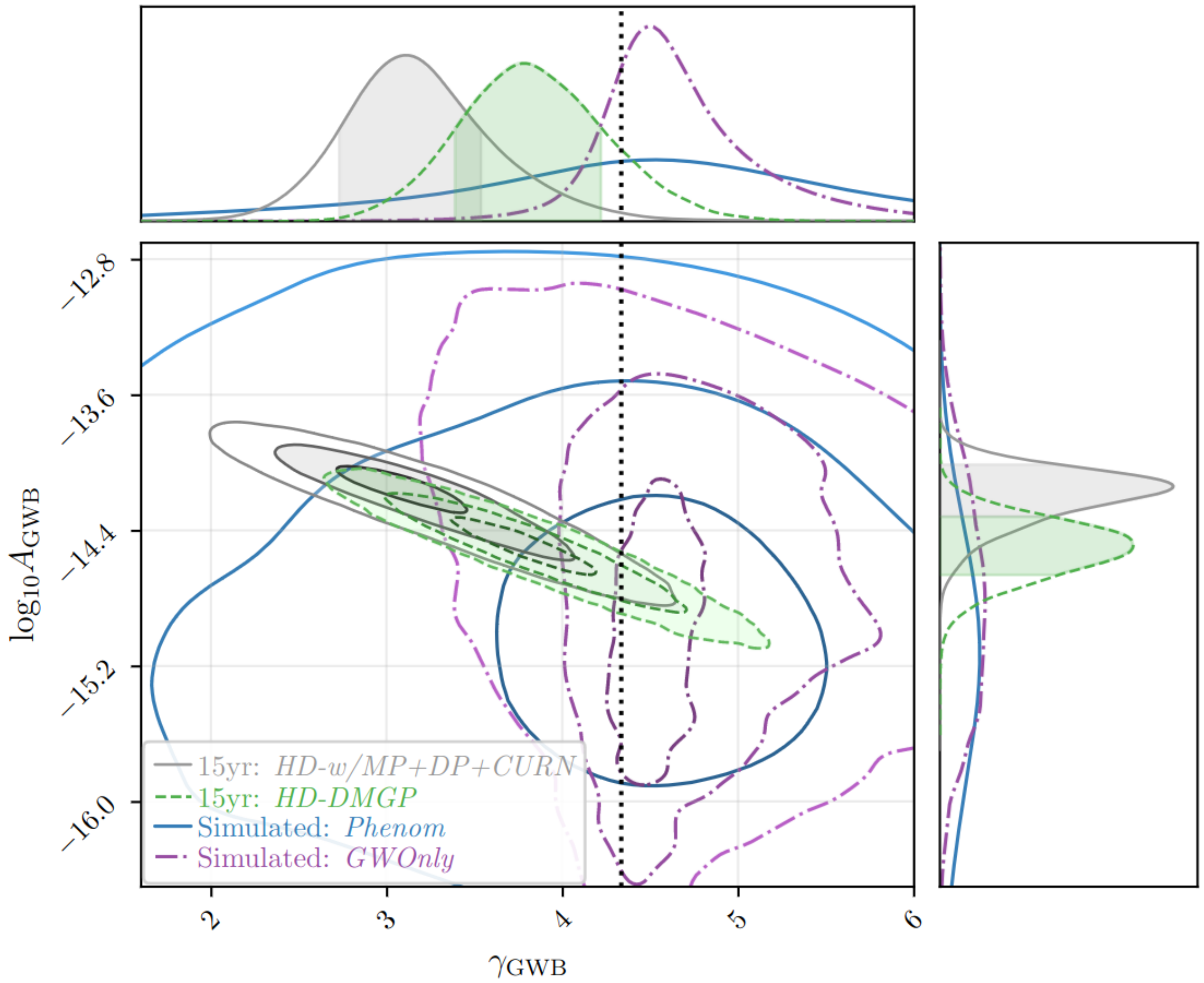
NANOGrav 15 individual source search (2023)



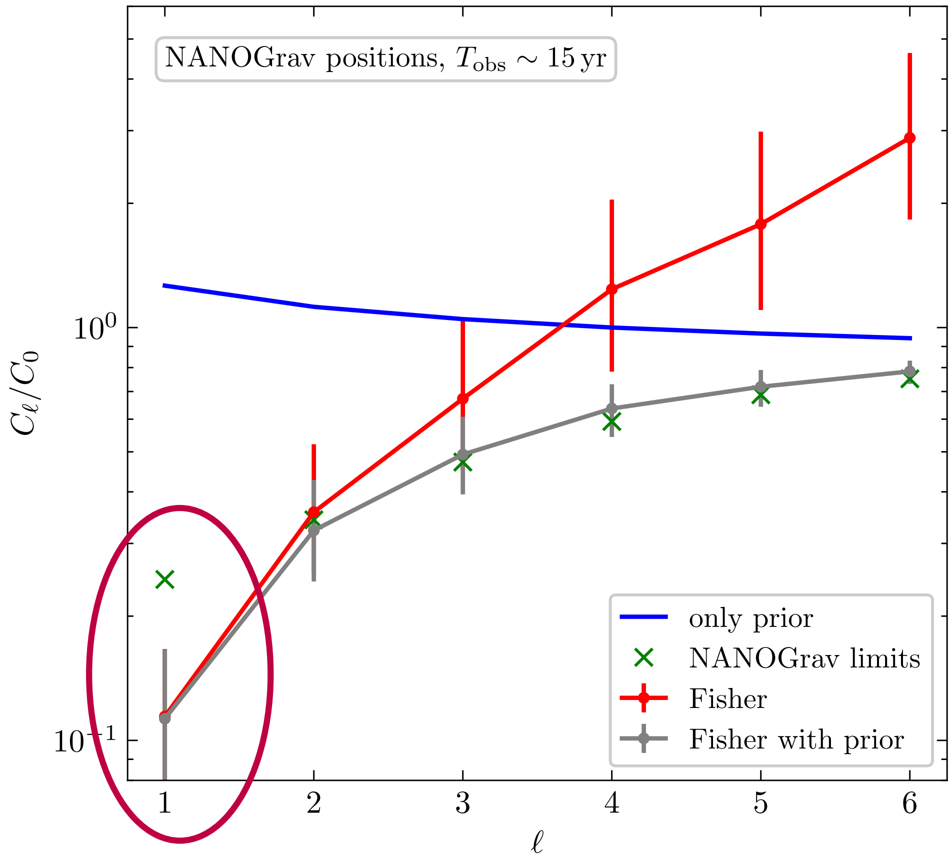
IPTA DR2 individual source search (2023)



NANOGrav: SMBHB interpretation



What's going on with the dipole ?



Possible explanations:

- Noise underestimated (higher multipoles less effected due to prior effect)
- Noise not uncorrelated across pulsars
- Non-gaussian noise / signal distributions
- Unequal noise levels across pulsars: only few high quality pulsars available to test large scales
- Forecast vs actual data analysis
- Detection of a dipole with C_1/C_0 ~ 2 % ?