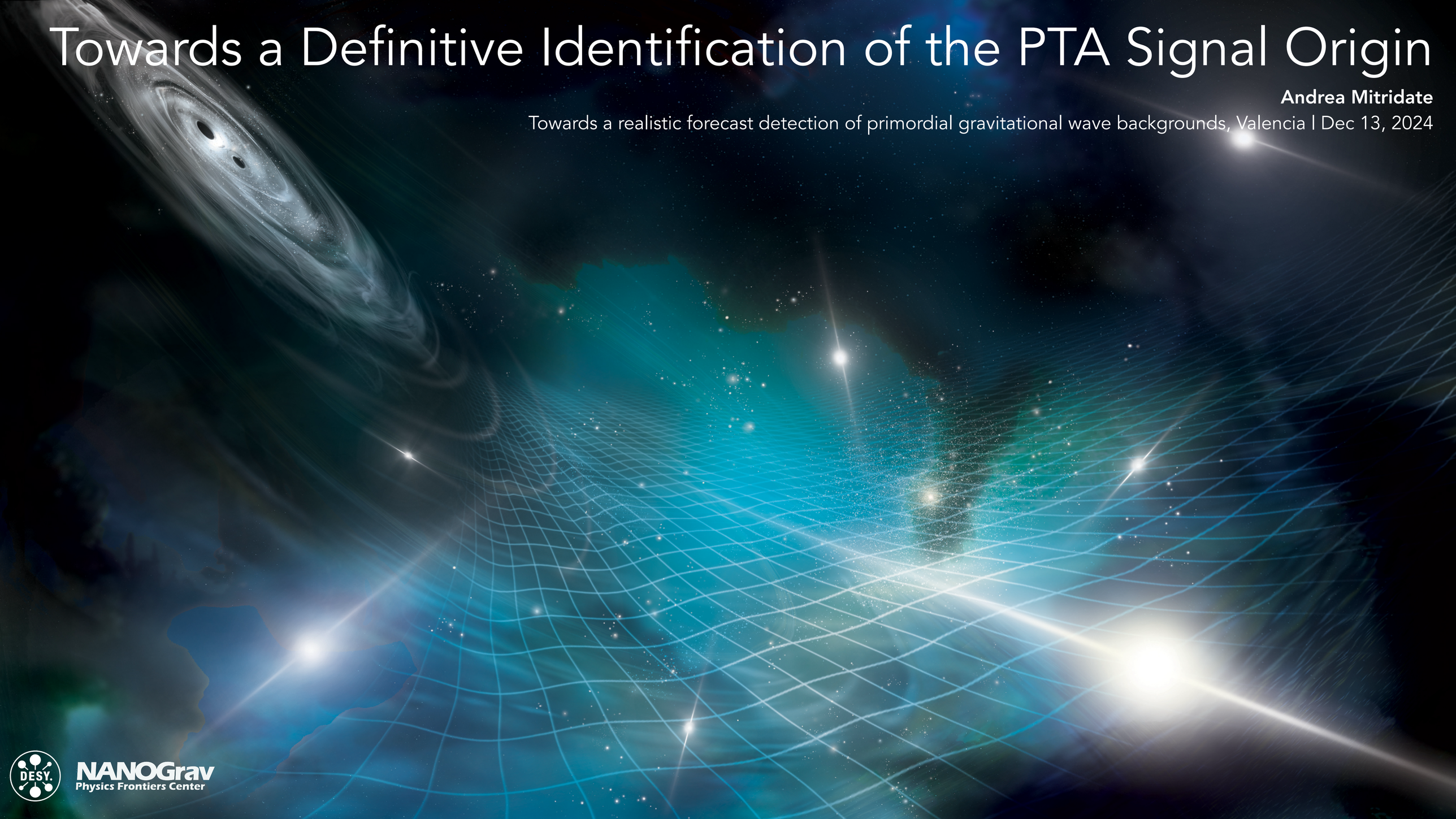
Towards a Definitive Identification of the PTA Signal Origin

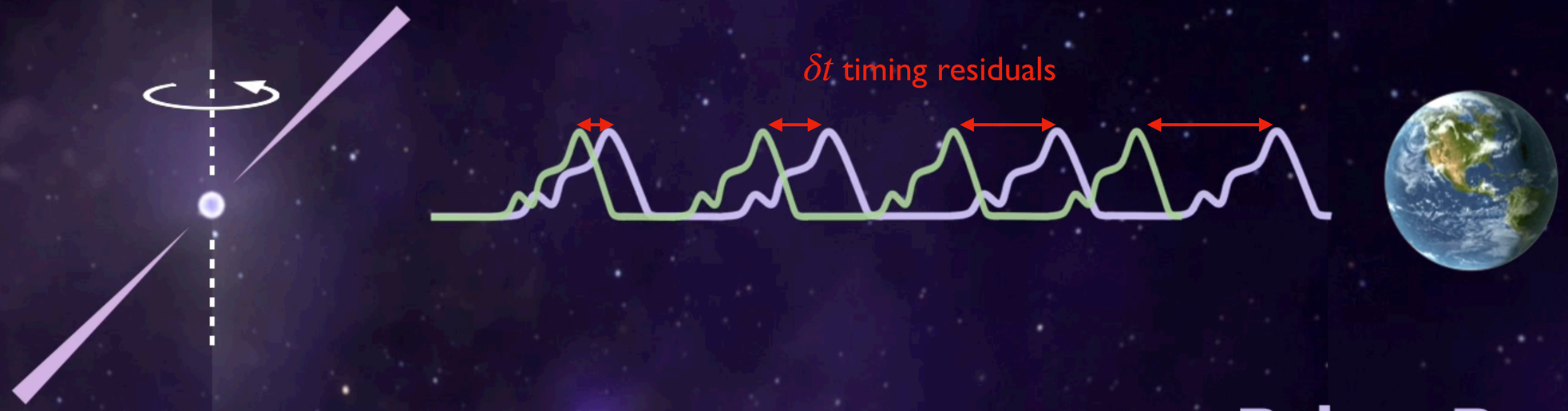
Andrea Mitridate

Towards a realistic forecast detection of primordial gravitational wave backgrounds, Valencia | Dec 13, 2024



DESY.
NANOGrav
Physics Frontiers Center

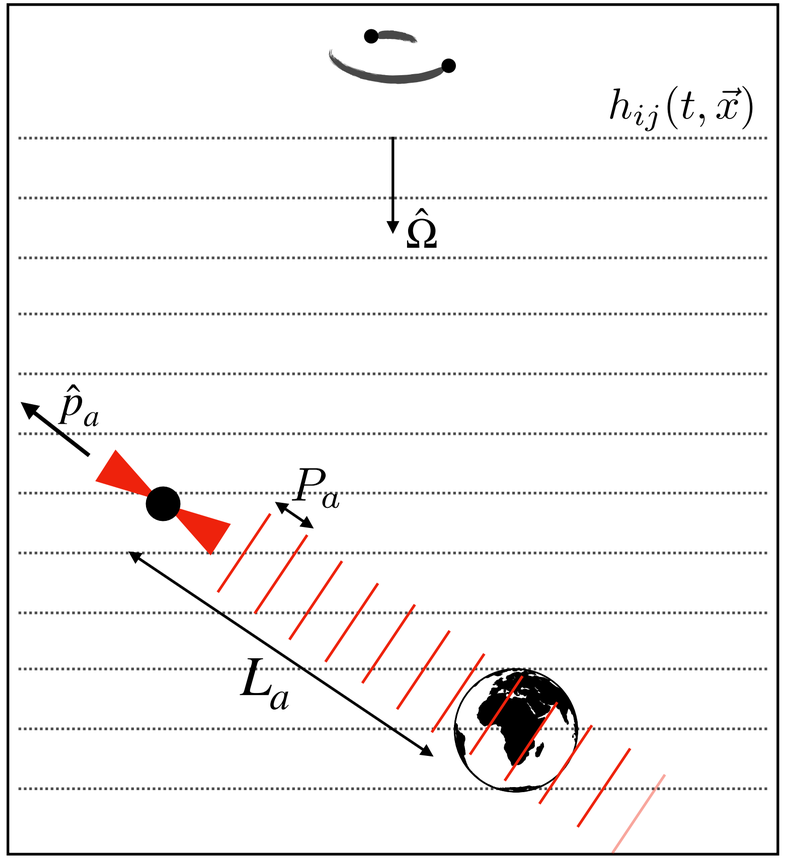
TIMING RESIDUALS



Pulses Recorded by Radio Telescope

GW SIGNALS

$$\frac{\delta P_a(t)}{P_a} = \frac{\hat{p}_a^i \hat{p}_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)}\left[h_{ij}(t,\vec{0}) - h_{ij}(t-L_a, L_a\hat{p}_a)\right]$$



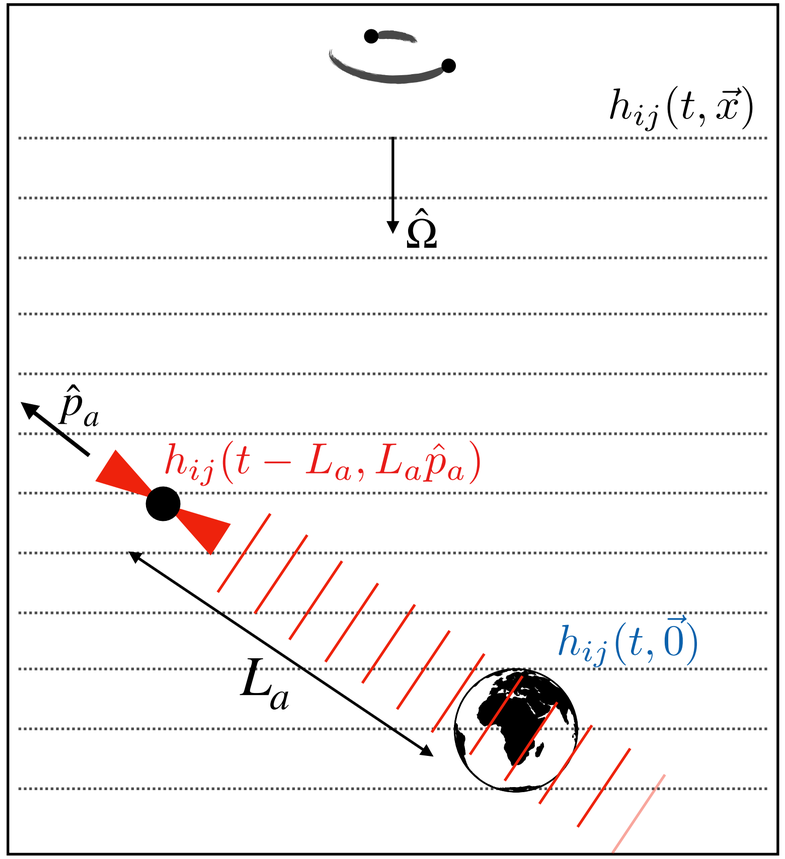
GW SIGNALS

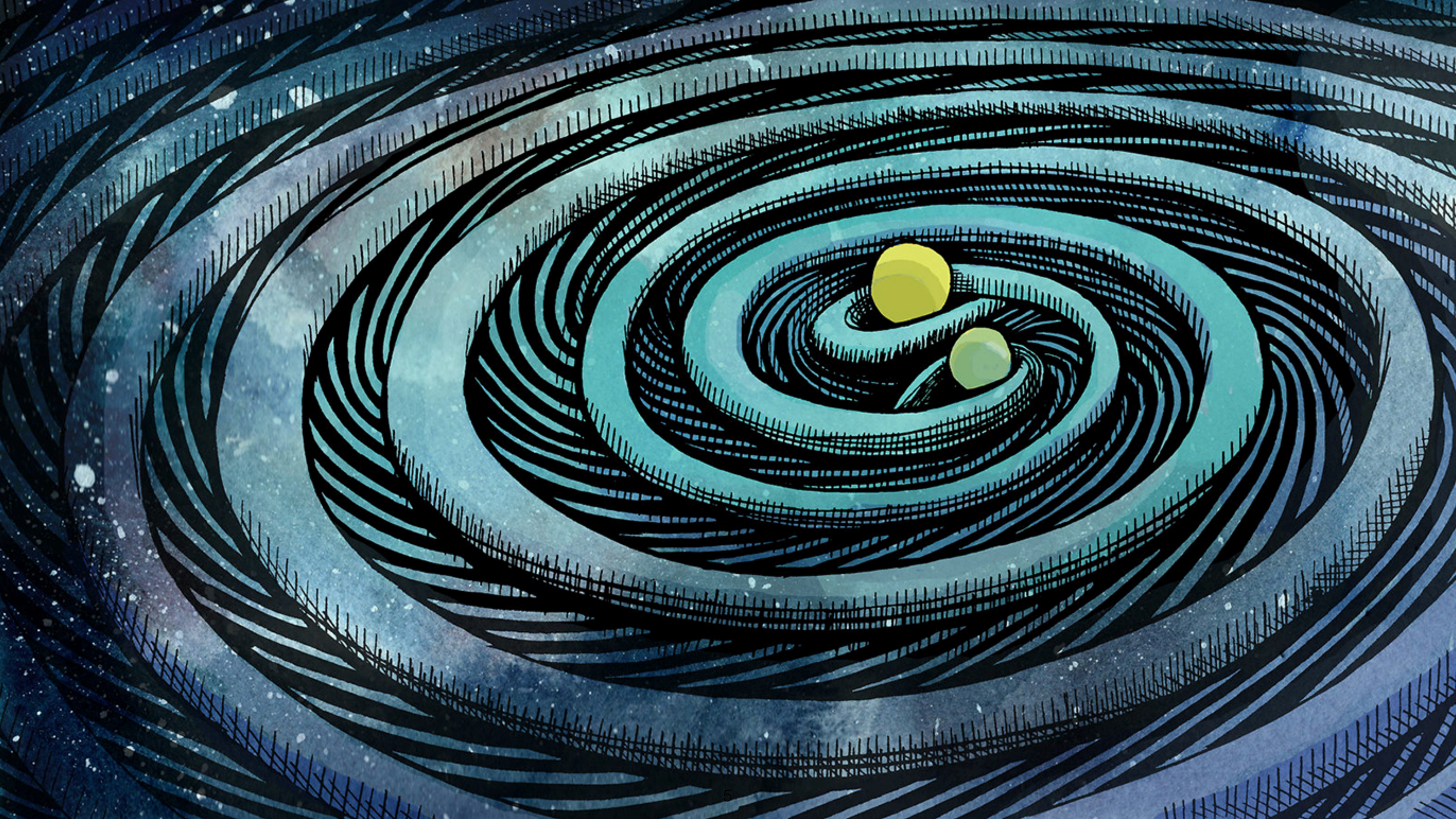
$$\frac{\delta P_a(t)}{P_a} = \frac{\hat{p}_a^i \hat{p}_a^j}{2(1+\hat{\Omega}\cdot\hat{p}_a)} \left[h_{ij}(t,\vec{0}) - h_{ij}(t-L_a, L_a\hat{p}_a) \right]$$

Earth term pulsar term

geometric response

$$\delta t_a(t) = \int^t dt' \, \frac{\delta P_a(t')}{P_a}$$





CONTINUOUS WAVE

$$h_{ij}(t, \vec{x}) = \sum_A h^A(t - \hat{\Omega} \cdot \vec{x})\, e^A_{ij}(\hat{\Omega})$$

assuming:

- constant orbital frequency, ω
- plane of the orbit is ⊥ to the line of sight

$$h^+(x) = h_0 \cos(\omega x + \phi) \qquad h^\times(x) = h_0 \sin(\omega x + \phi)$$

CONTINUOUS WAVE

$$\delta t \propto F_A(h_A^e - h_A^p)$$

$$h_{ij}(t, \vec{x}) = \sum_A h^A(t - \hat{\Omega} \cdot \vec{x})\, e_{ij}^A(\hat{\Omega})$$

assuming:

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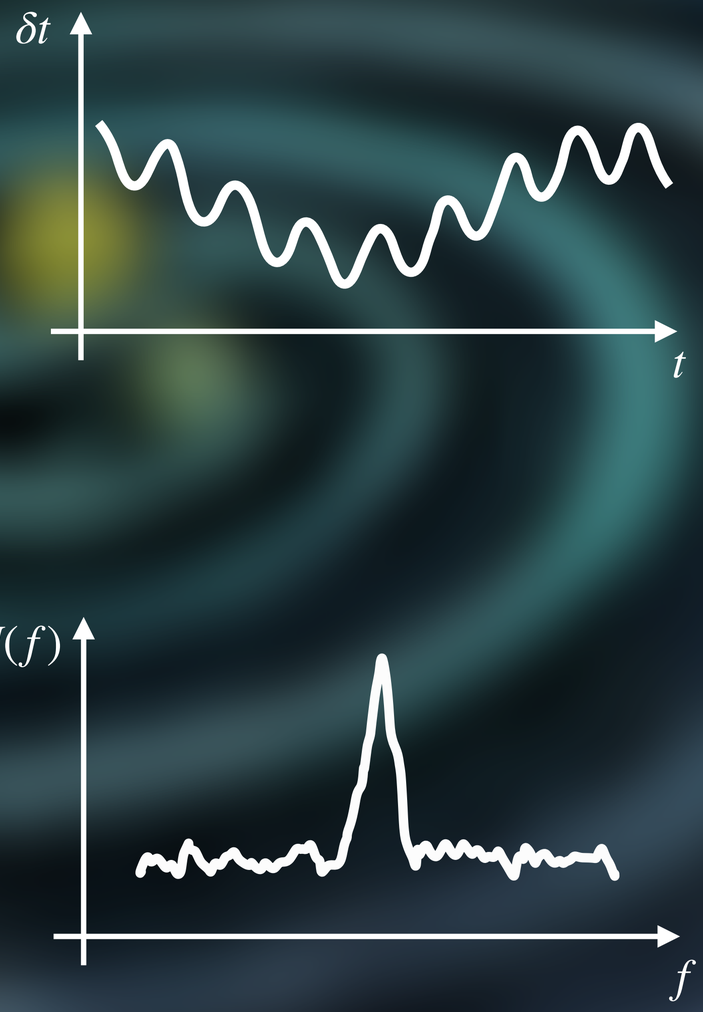

GW BACKGROUND

$$h_{ij}(t,\vec{x}) = \sum_A \int df \int d\hat{\Omega}\; \tilde{h}_A(f,\hat{\Omega})\, e^A_{ij}(\hat{\Omega})\, e^{-2\pi i f (t - \hat{\Omega}\cdot\vec{x})}$$

GW BACKGROUND

polarization tensors

$$h_{ij}(t,\vec{x}) = \sum_A \int df \int d\hat{\Omega}\, \underbrace{\tilde{h}_A(f,\hat{\Omega})}_{\text{“Fourier” components}}\, \overbrace{e^A_{ij}(\hat{\Omega})}^{\text{polarization tensors}}\, \underbrace{e^{-2\pi i f(t-\hat{\Omega}\cdot\vec{x})}}_{\text{plane waves}}$$

GW BACKGROUND

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the complex functions $\tilde{h}^A(f,\hat{\Omega})$ are treated as random variables

GW BACKGROUND

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$$\langle \tilde{h}_A(f,\hat{\Omega}) \rangle = 0$$

GW BACKGROUND

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$$\langle \tilde{h}_A(f,\hat{\Omega})\rangle = 0$$

$$\langle \tilde{h}_A(f,\hat{\Omega})^* \tilde{h}_A(f',\hat{\Omega}')\rangle = \delta_{AA'}\delta(\hat{\Omega},\hat{\Omega}')\delta(f-f')\underbrace{H(f)}_{\text{GWB power spectrum}}$$

GW BACKGROUND

polarization tensors

$$h_{ij}(t,\vec{x}) = \sum_A \int df \int d\hat{\Omega}\, \underbrace{\tilde{h}_A(f,\hat{\Omega})}_{\text{“Fourier” components}}\, \overline{e^A_{ij}(\hat{\Omega})}\, \underbrace{e^{-2\pi i f(t-\hat{\Omega}\cdot\vec{x})}}_{\text{plane waves}}$$

the complex functions $\tilde{h}^A(f,\hat{\Omega})$ are treated as random variables

$$\langle \tilde{h}_A(f,\hat{\Omega})\rangle = 0$$

$$\langle \delta t_a(t)\rangle = 0$$

$$\langle \tilde{h}_A(f,\hat{\Omega})^* \tilde{h}_A(f',\hat{\Omega}')\rangle = \delta_{AA'}\delta(\hat{\Omega},\hat{\Omega}')\delta(f-f')\underbrace{H(f)}_{\text{GWB power spectrum}}$$

$$\langle \delta t_a(t_i)\delta t_b(t_j)\rangle \propto \Gamma_{ab} \int df\; H(f)\; e^{2\pi i f(t_i - t_j)}$$

GW BACKGROUND

$$\langle \delta t_a(t_i) \delta t_b(t_j) \rangle \propto \Gamma_{ab} \int df \; H(f) \; e^{2\pi i f (t_i - t_j)}$$

GW BACKGROUND

$$\langle \delta t_a(t_i) \delta t_b(t_j) \rangle \propto \Gamma_{ab} \underbrace{\int df \; H(f) \; e^{2\pi i f (t_i - t_j)}}_{\text{the signal is time-correlated}}$$

GW BACKGROUND

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the **power spectrum** of the GWB-induced signal is **common across pulsars**

GW BACKGROUND

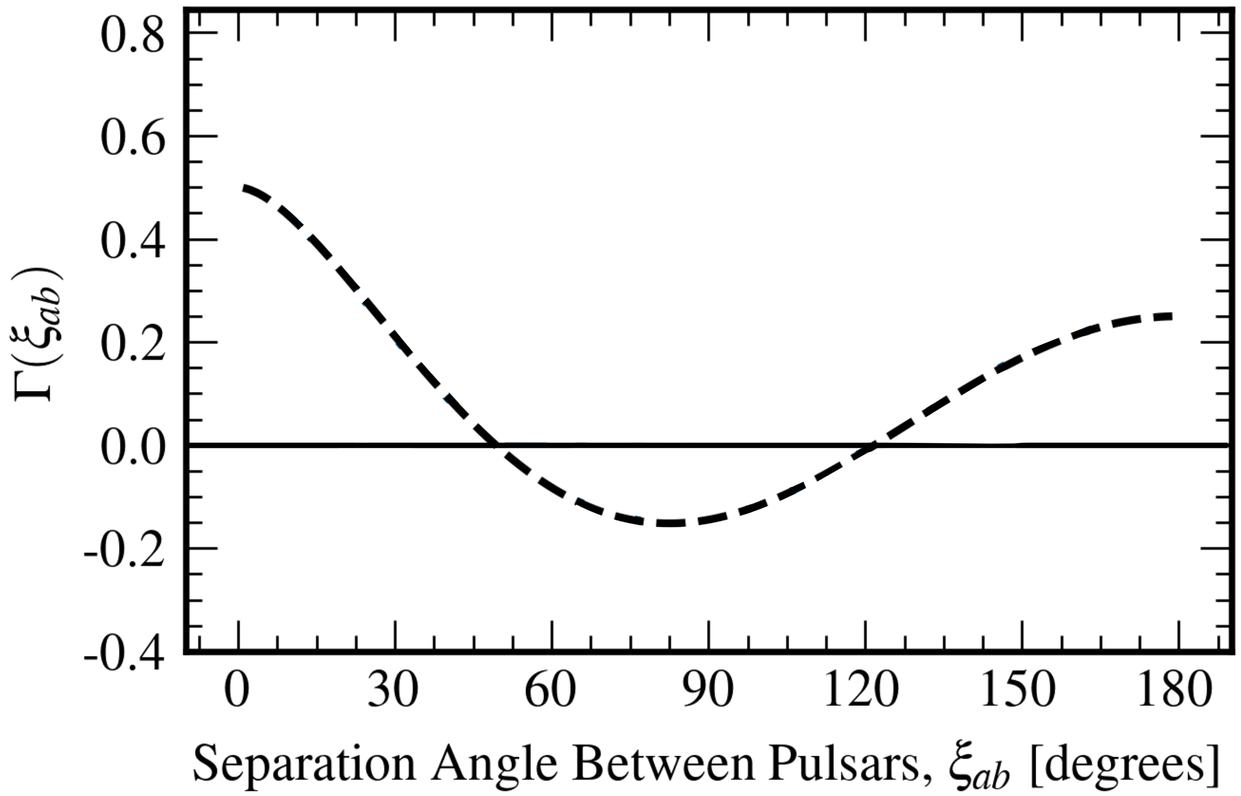
$$\langle \delta t_a(t_i) \delta t_b(t_j) \rangle \propto \Gamma_{ab} \int df \; H(f) \; e^{2\pi i f (t_i - t_j)}$$

the signal is **correlated among pulsars**

$$\Gamma_{ab} = \int d\hat{\Omega} \sum_A F_a^A(\hat{\Omega}) F_b^A(\hat{\Omega}) [1 + \delta_{ab}]$$

$$F_a^A(\hat{\Omega}) = \frac{\hat{p}_a^i \hat{p}_a^j}{2(1 + \hat{\Omega} \cdot \hat{p}_a)} e_{ij}^A(\hat{\Omega})$$

HELLINGS & DOWNS CURVE

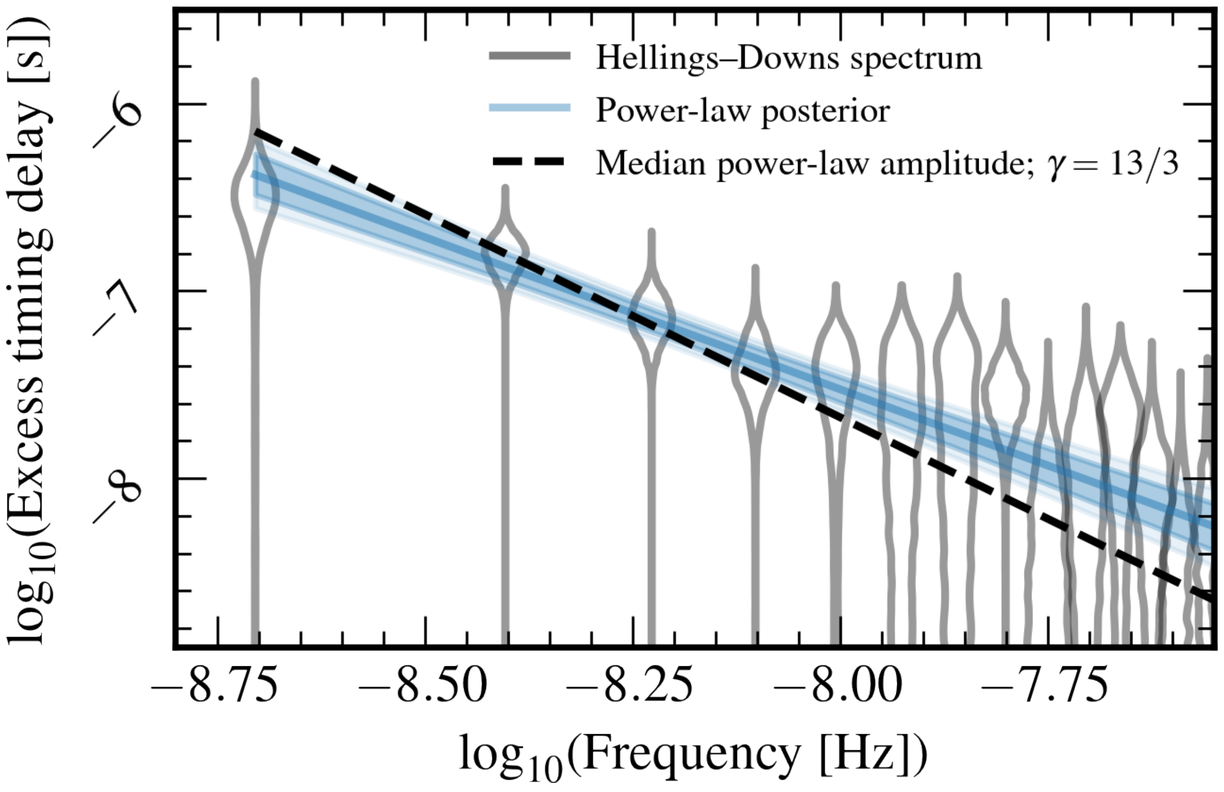


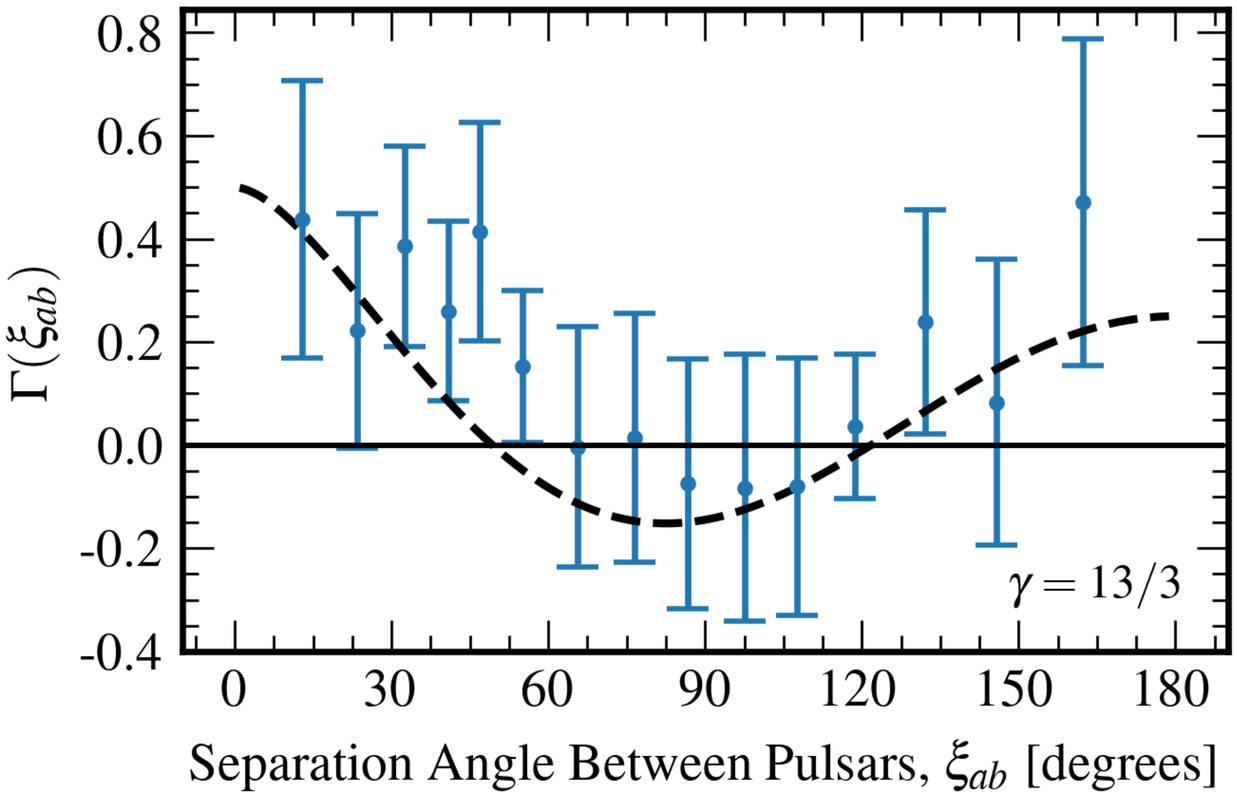
searching for a GWB means searching for **stochastic delays** in the TOAs that are **common across pulsars** and **spatially correlated**

EVIDENCE FOR GWB

NANOGrav 15-year

Agazie et al. [2306.16213]





we find evidence for Hellings & Downs correlation with a p-value of $5 \times 10^{-5} - 1.9 \times 10^{-4}$ (approx. $3.5 - 4\sigma$)

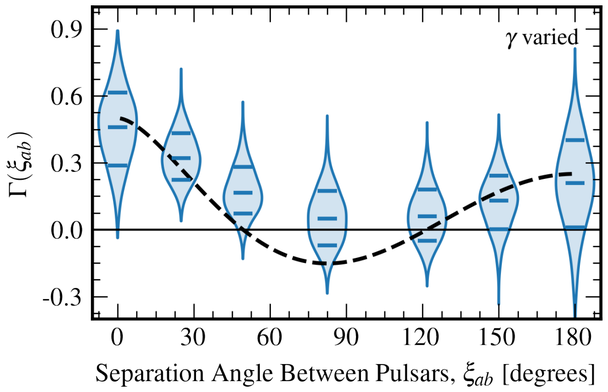
EVIDENCE FOR GWB

NANOGrav:

68 pulsars, 16yr of data

~3-4σ significance

Agazie et al. [2306.16213]

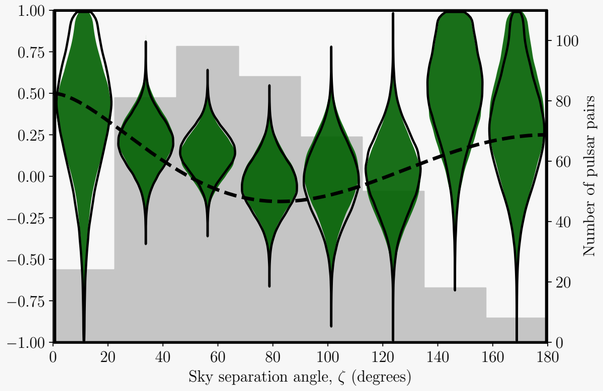


PPTA:

32 pulsars, 18yr of data

~2σ significance

Reardon et al. [2306.16215]

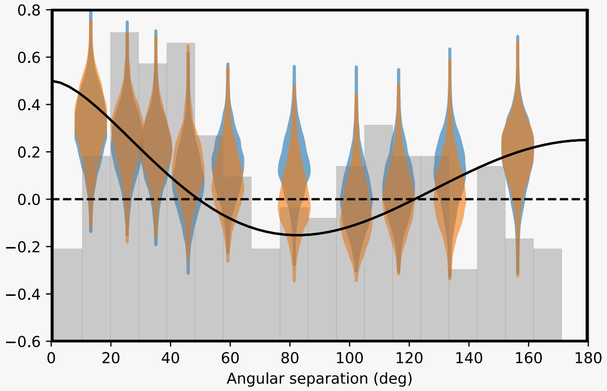


EPTA + InPTA:

25 pulsars, 24yr of data

~3σ significance

Antoniadis et al. [2306.16214]

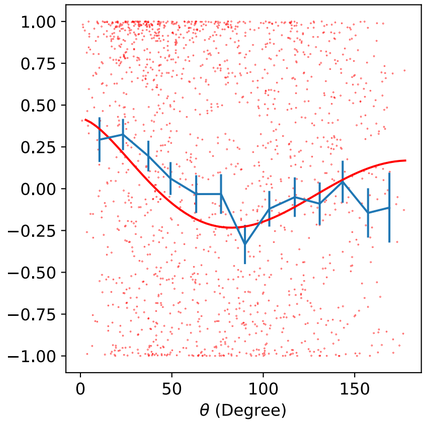


CPTA:

57 pulsars, 3yr of data

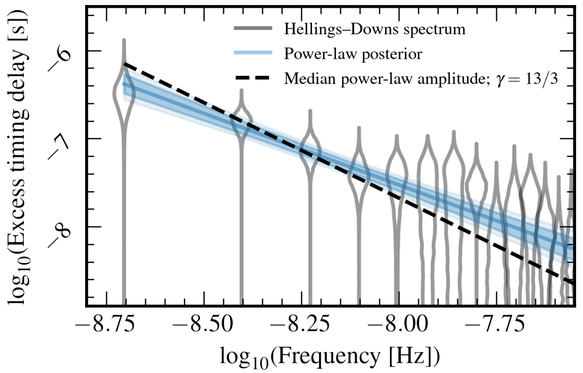
~4.6σ significance

Xu et al. [2306.16216]



SPECTRUM

NANOGrav



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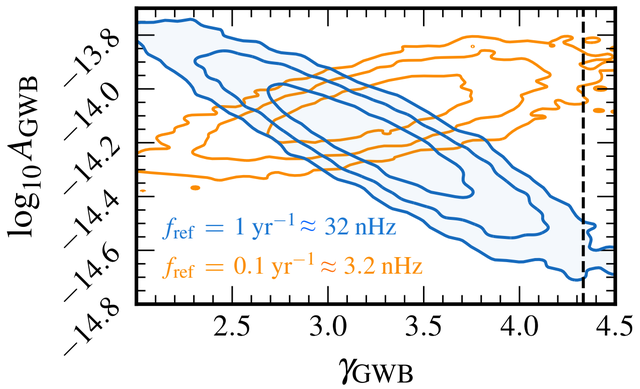
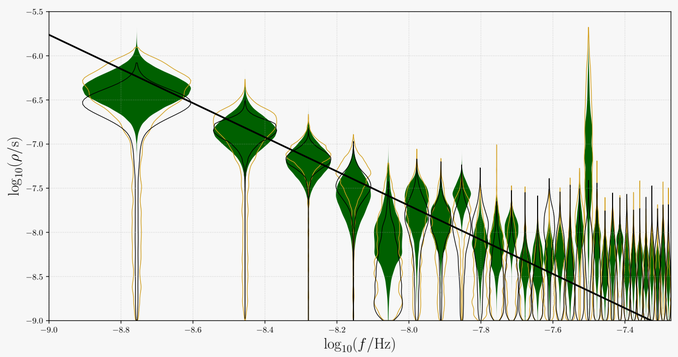
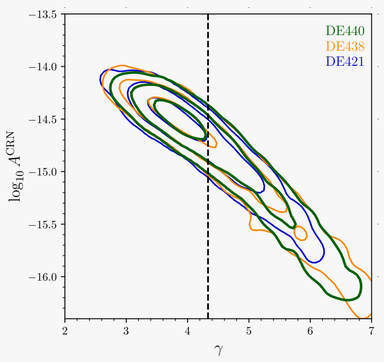


PPTA

Reardon et al. [2306.16215]





EPTA + InPTA

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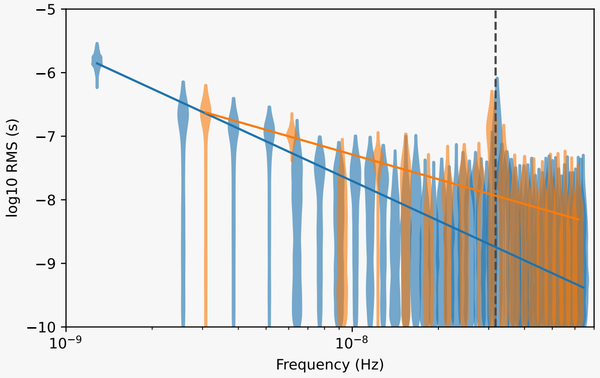
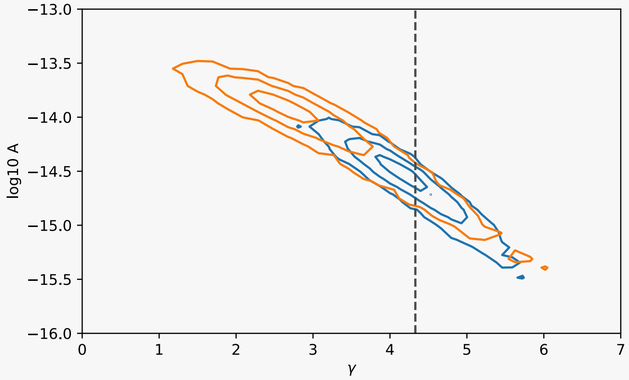
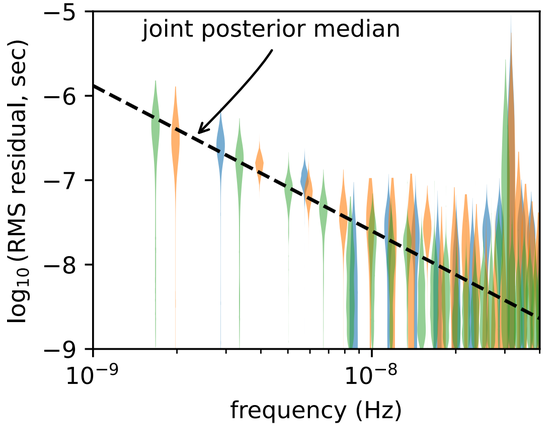
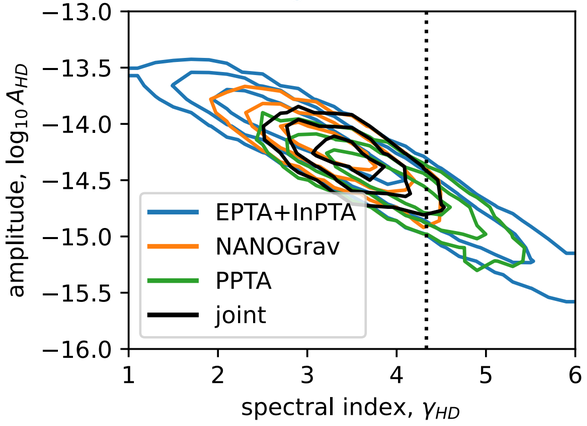




IPTA early data combination

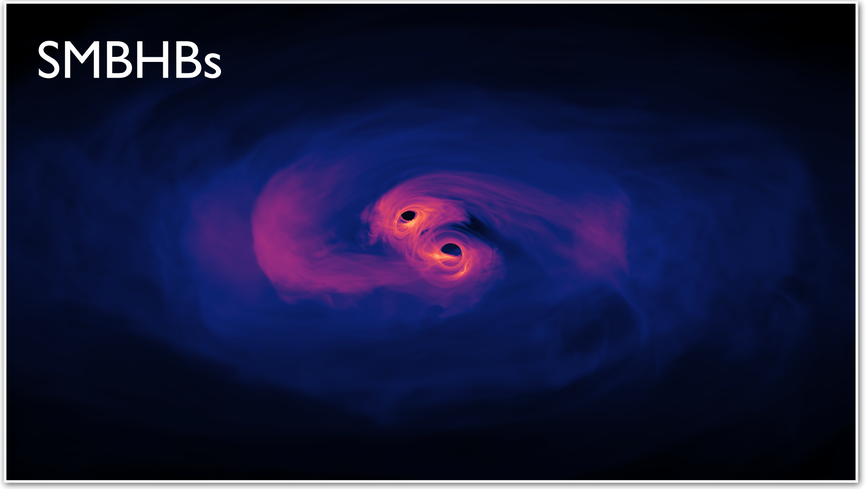
Agazie et al. [2309.00693]





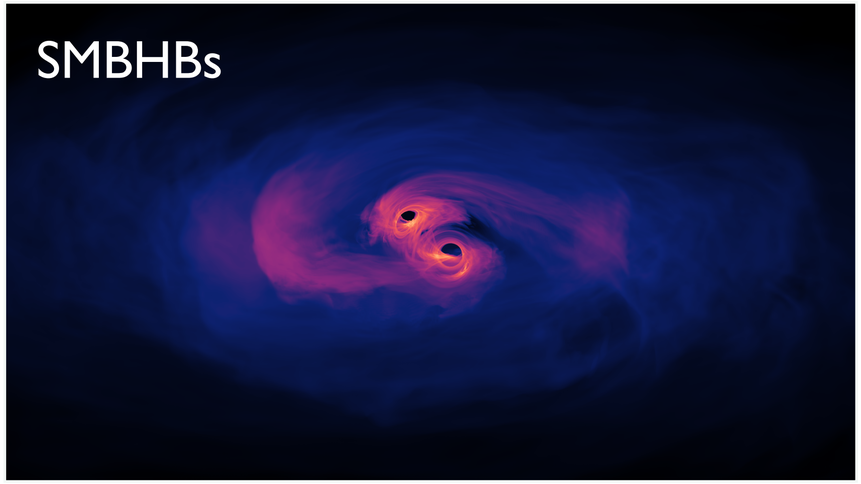
what is the source?

ASTRO OR COSMO?

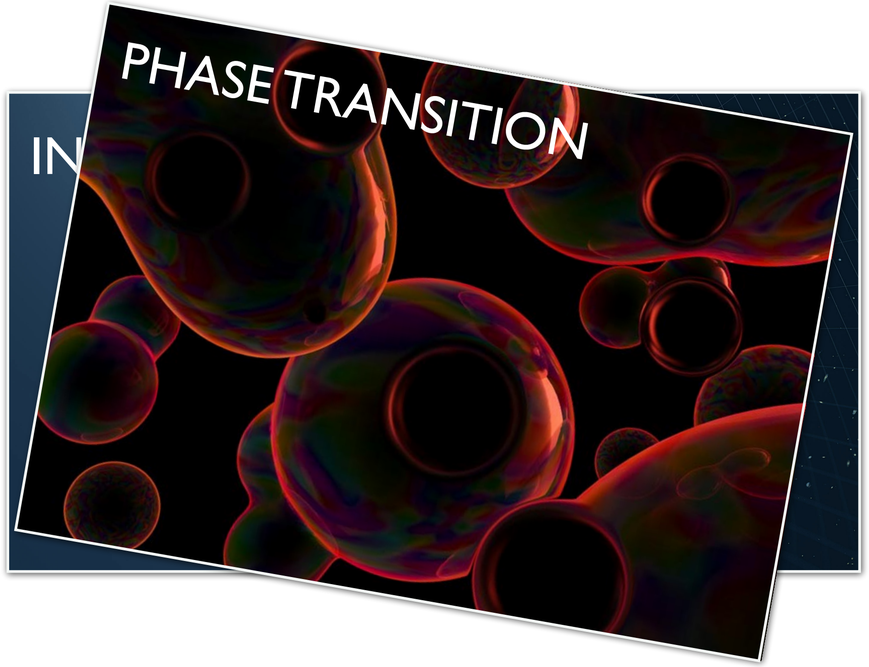


VS

ASTRO OR COSMO?



VS



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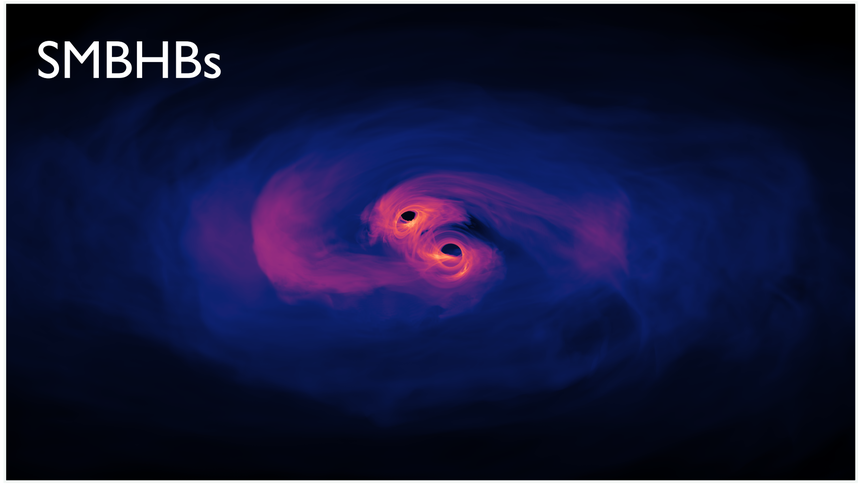

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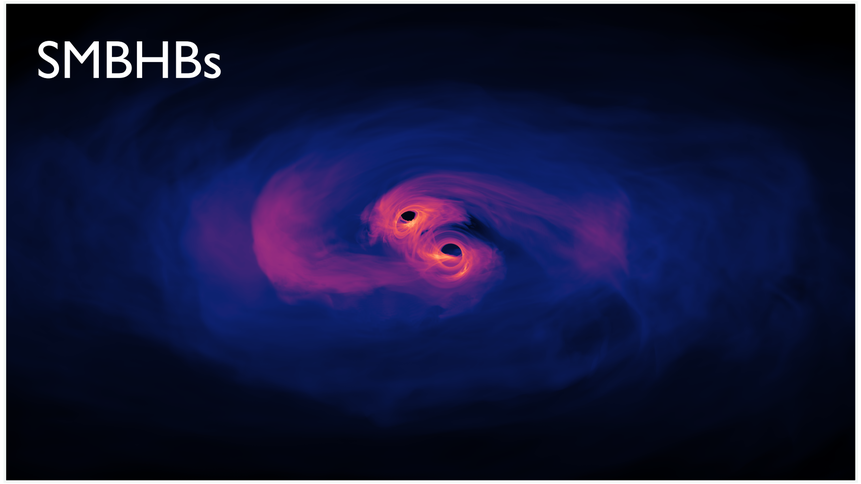

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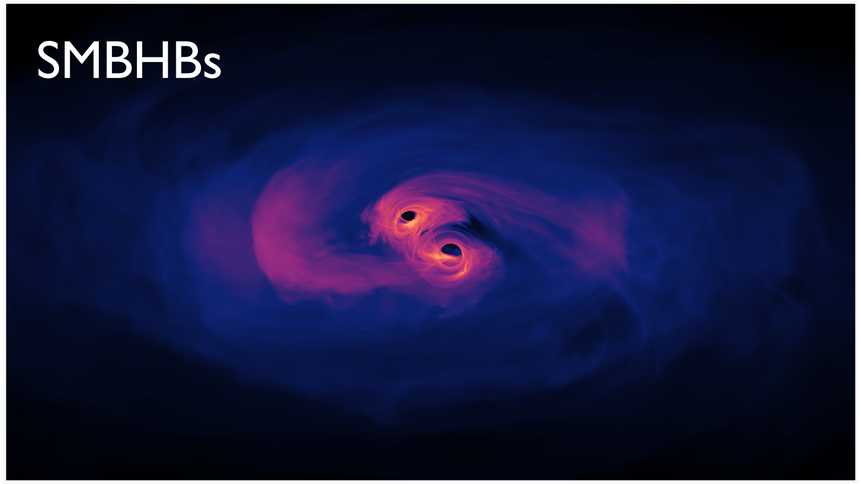


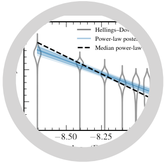
VS

HOW TO TELL THE DIFFERENCE

what are the distinguishing features of astrophysical and cosmological signals?

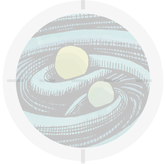
astrophysical ← VS → cosmological

$\Omega_{GW} \propto f^{2/3}$ at intermediate frequencies
turn over at low and high frequencies



$\Omega_{GW} \propto f^{3}$ at low frequencies (?)
???

GW signal from individual sources can be resolved from the background

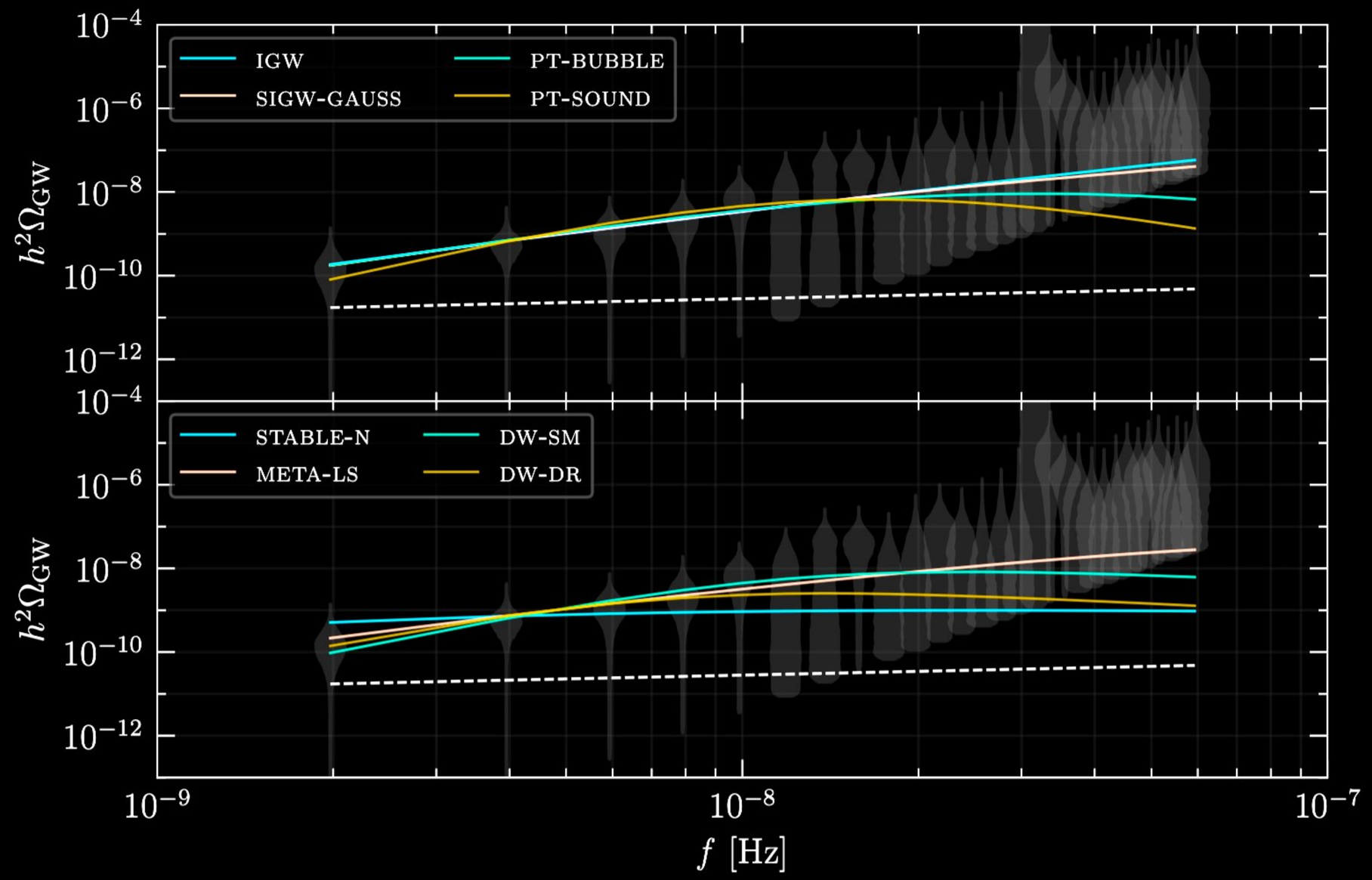


no resolvable sources

sizable anisotropies in the GWB sky

~ isotropic

Afzal et al. (**NANOGrav**) [2306.16219]



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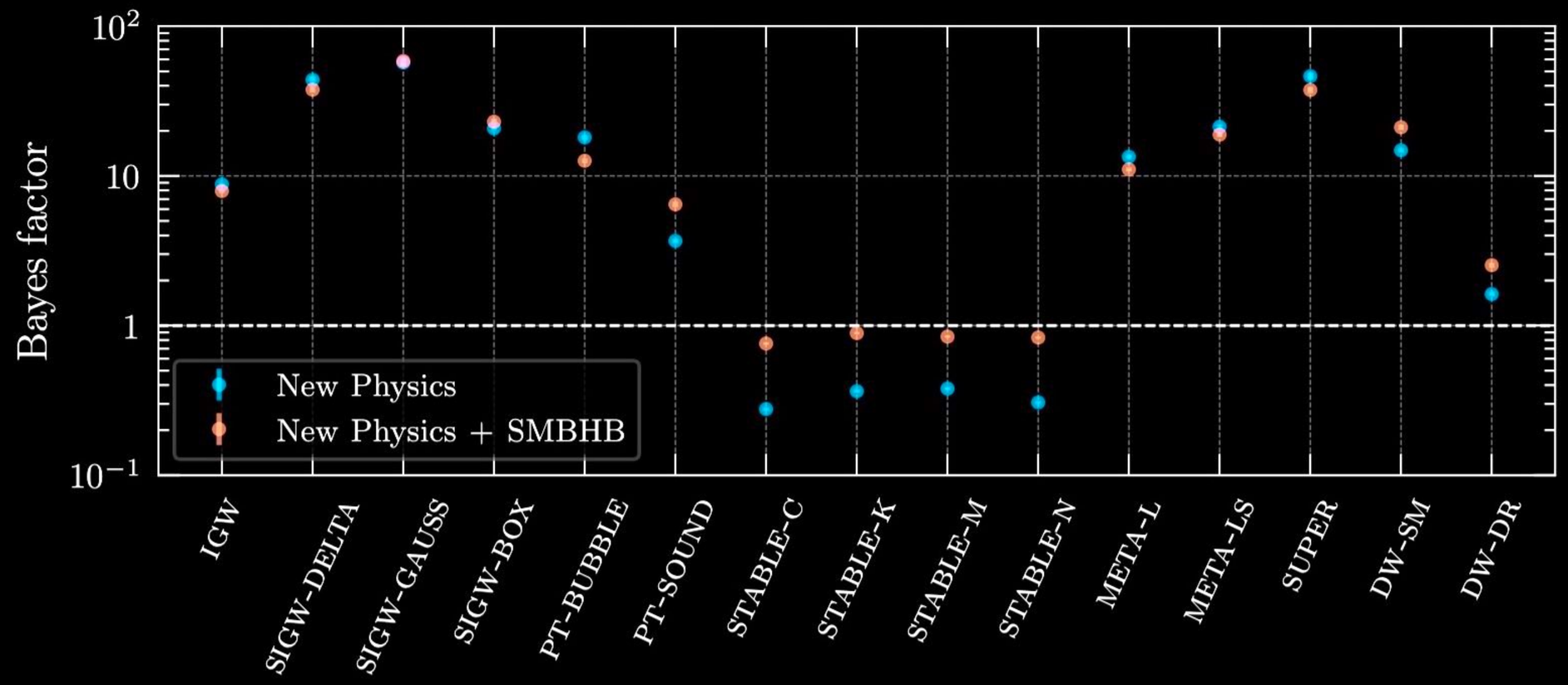

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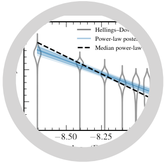

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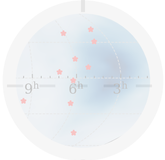
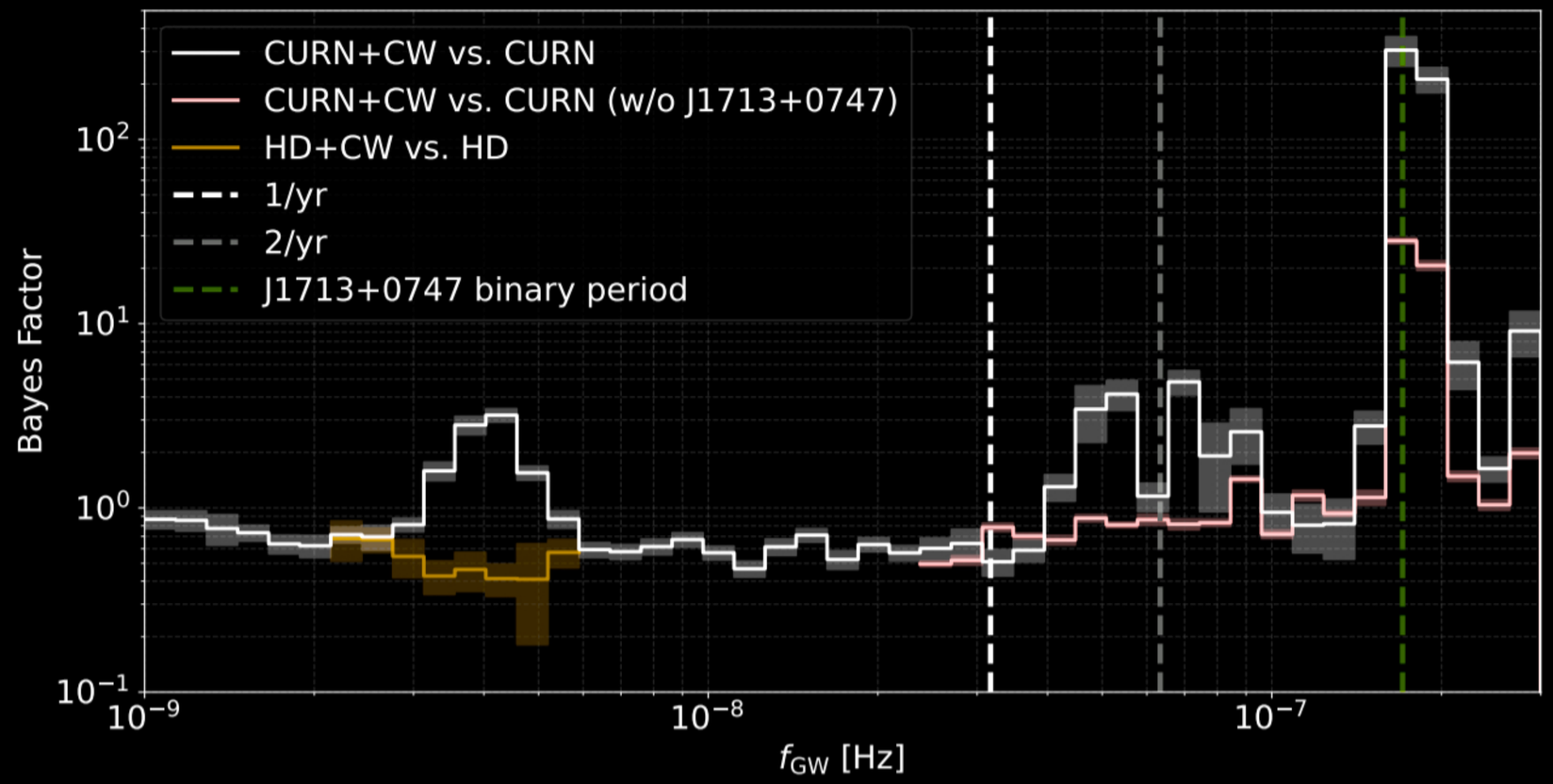


~ isotropic

Agazie et al. (**NANOGrav**) [2306.16222]



HOW TO TELL THE DIFFERENCE

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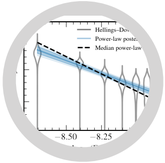
astrophysical	VS	cosmological
$\Omega_{GW} \propto f^{2/3}$ at intermediate frequencies turn over at low and high frequencies		$\Omega_{GW} \propto f^{3}$ at low frequencies (?) ???
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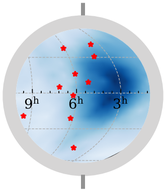
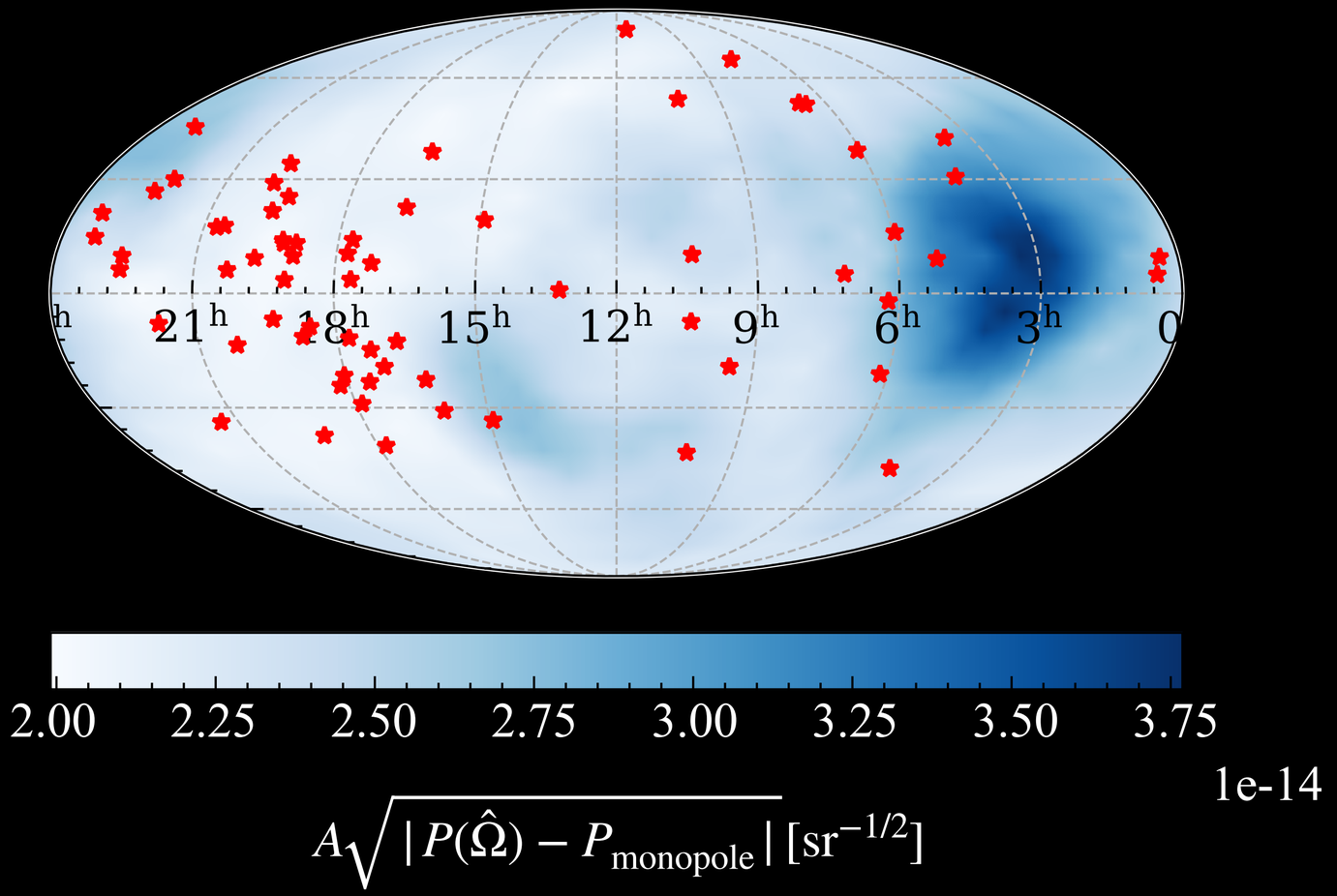
sizable anisotropies in the GWB sky		~ isotropic

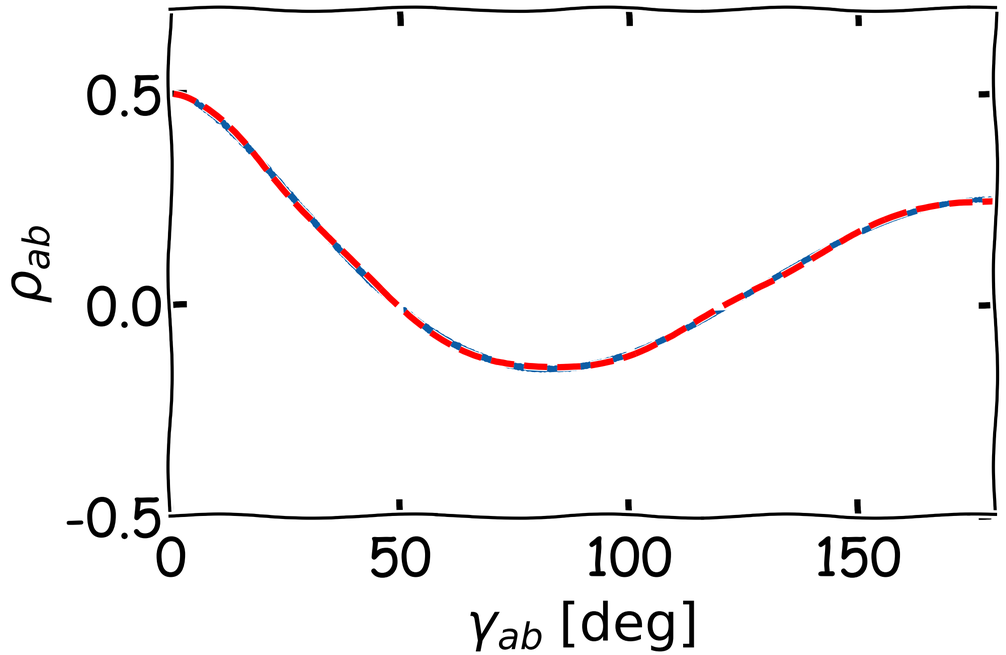

21h
18h
15h
12h
9h
6h
3h
0
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
1e-14
$A\sqrt{|P(\hat{\Omega}) - P_{\mathrm{monopole}}|}\,[\mathrm{sr}^{-1/2}]$

a deep dive on anisotropies

THE BASIC IDEA

isotropic GWB

$$\langle \tilde{h}_A^*(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle \propto \delta(\hat{\Omega},\hat{\Omega}')$$



THE BASIC IDEA

anisotropic GWB

$$\langle \tilde{h}_A^*(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle \propto \delta(\hat{\Omega},\hat{\Omega}')P(\hat{\Omega})$$

$$\langle \rho_{ab}\rangle \propto \sum_A \sum_k F^A_{a,k} F^A_{b,k}\, P_k$$

GWB power

pulsars correlations

geometric response

$$F^A_{a,k} = \frac{\hat{p}^i_a \hat{p}^j_a}{2(1+\hat{\Omega}_k\cdot\hat{p}_a)} e^A_{ij}(\hat{\Omega}_k)$$

THE BASIC IDEA

anisotropic GWB

$$\langle \tilde{h}^*_A(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle \propto \delta(\hat{\Omega},\hat{\Omega}')P(\hat{\Omega})$$

$$\langle \rho_{ab}\rangle \propto \sum_A \sum_k F^A_{a,k} F^A_{b,k}\ P_k$$

GWB power

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geometric response

$$F^A_{a,k} = \frac{\hat{p}^i_a \hat{p}^j_a}{2(1+\hat{\Omega}_k\cdot\hat{p}_a)} e^A_{ij}(\hat{\Omega}_k)$$

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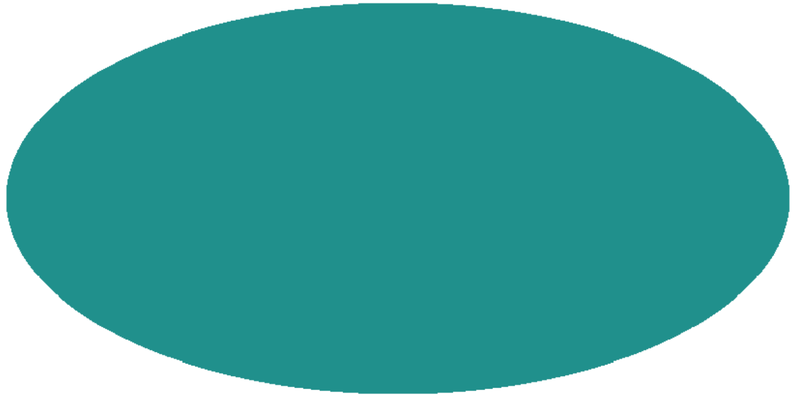

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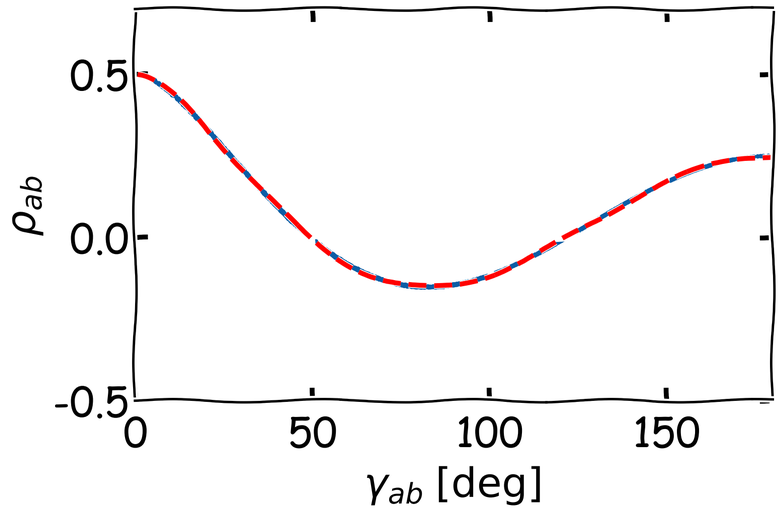

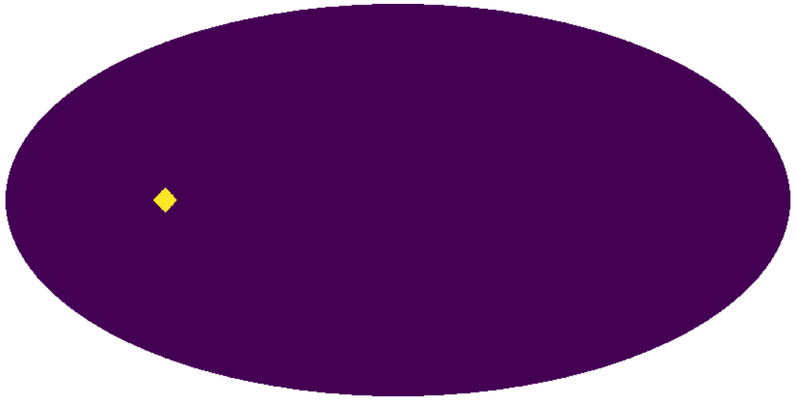

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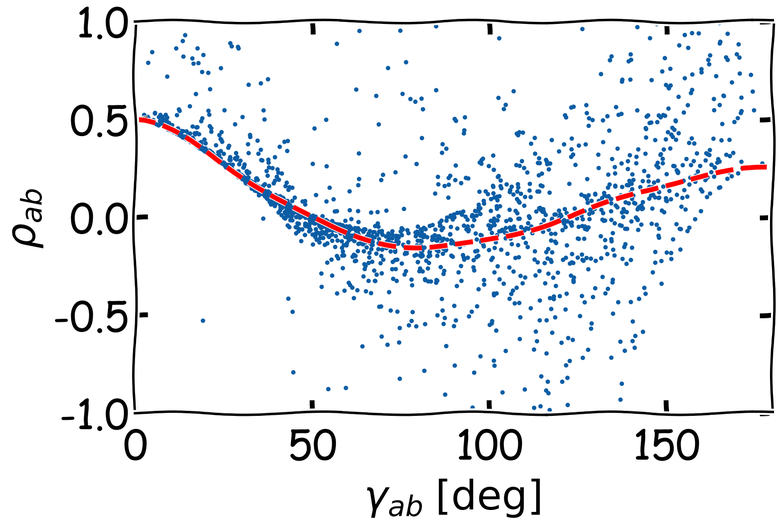

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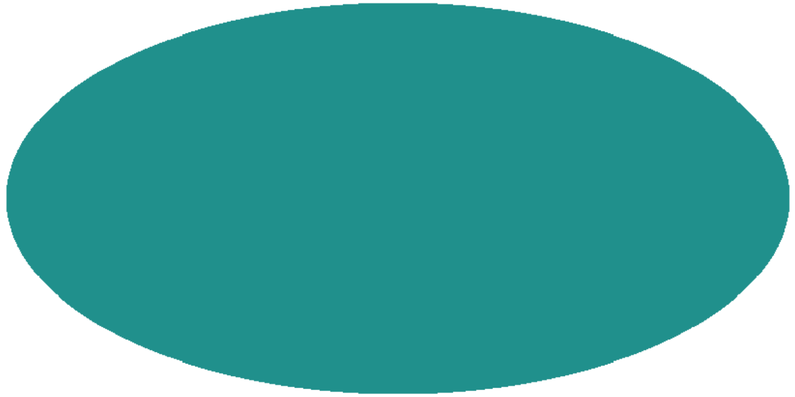

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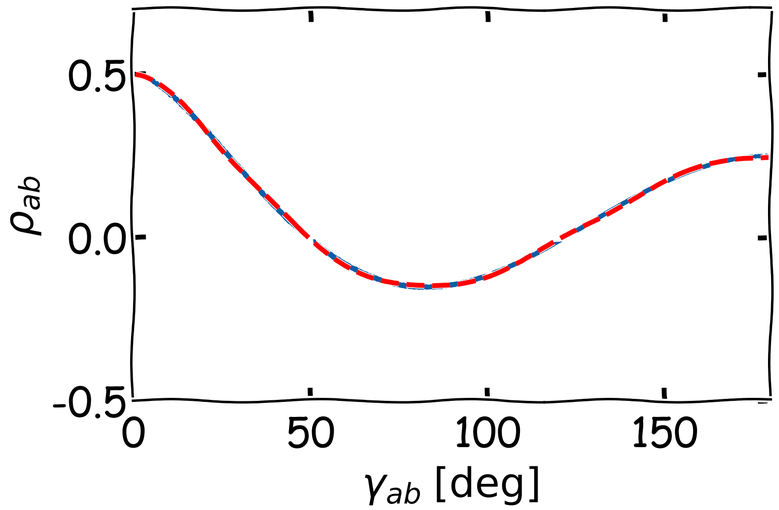

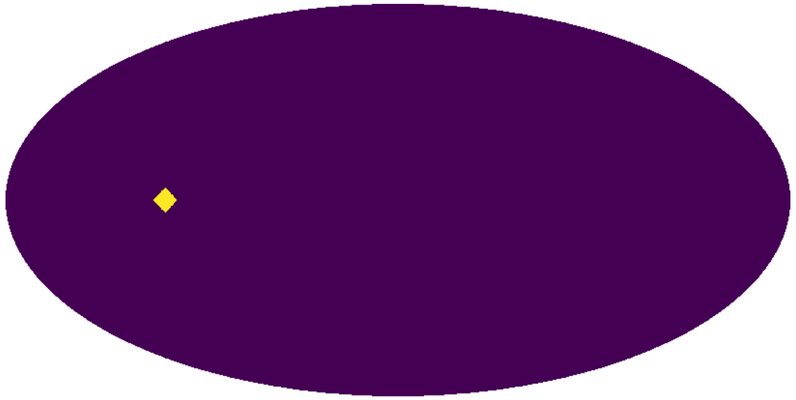

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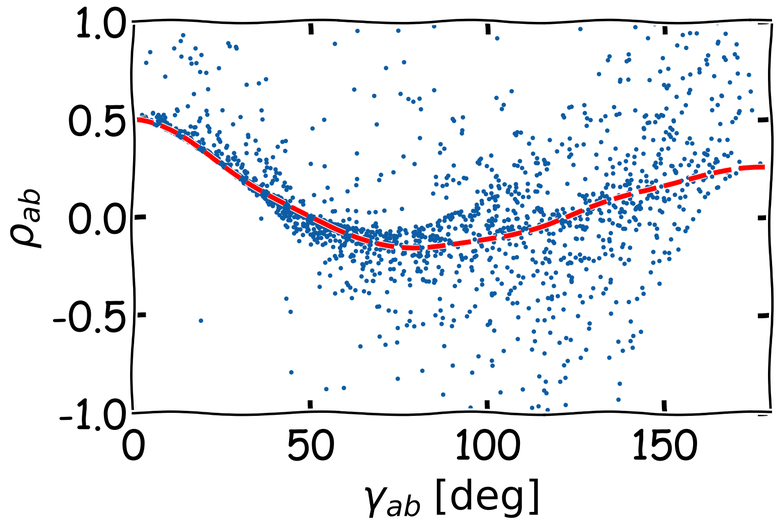

FREQUENTIST SEARCHES FOR ANISOTROPIES

three ingredients

the data

measured cross correlations

ρ_{ab}

the noise

cross correlations uncertainties

Σ_{ab}

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→

given $\{\rho, \mathbf{\Sigma}\}$ maximize p with respect to $\mathbf{P}$

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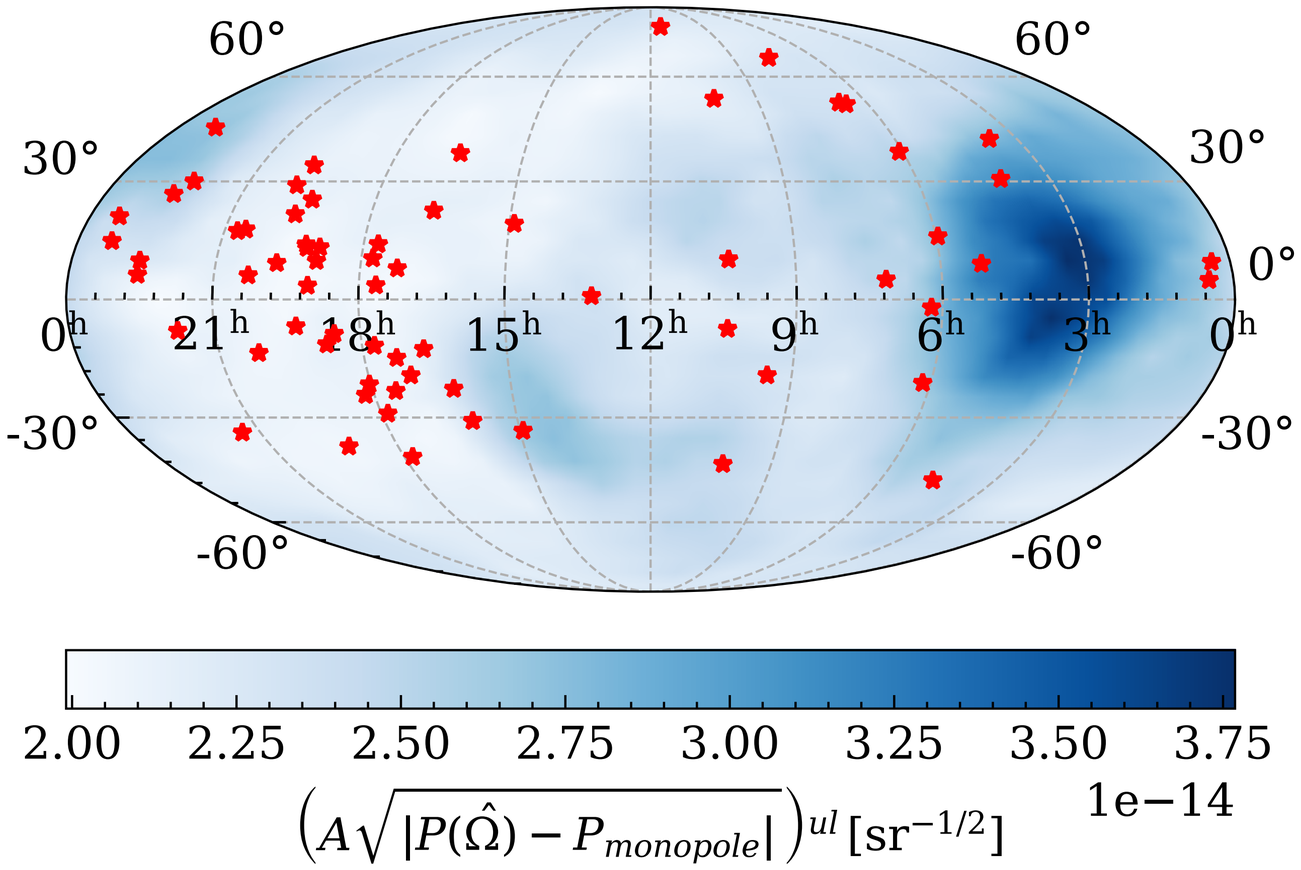

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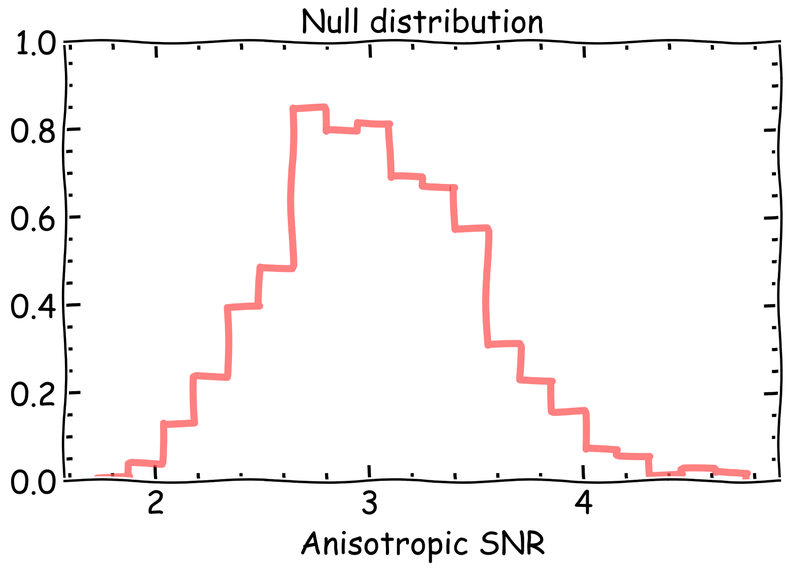

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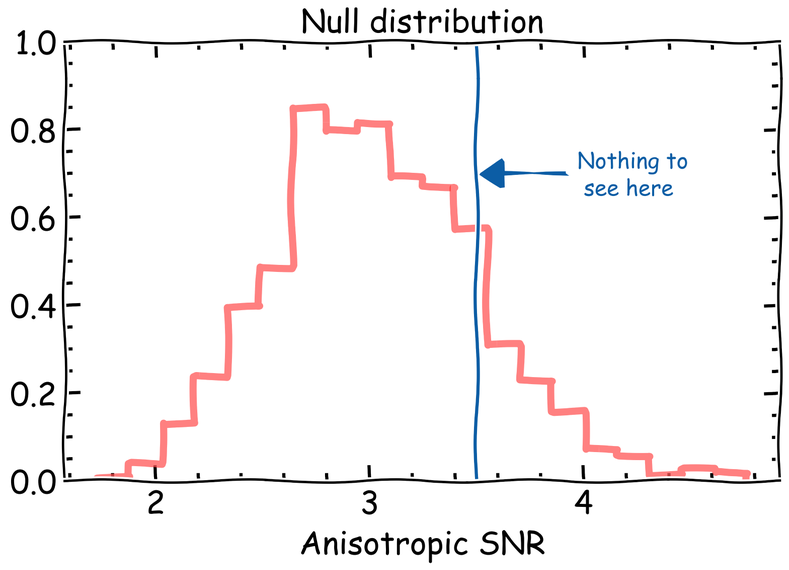

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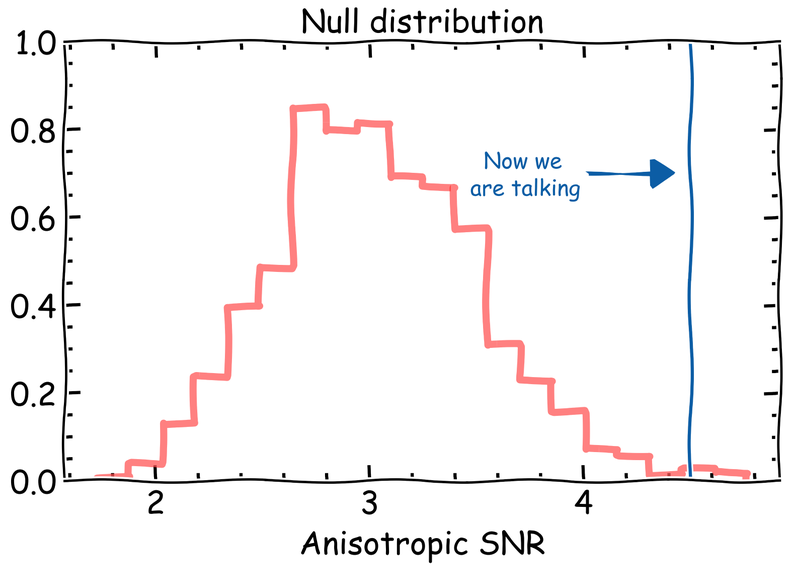
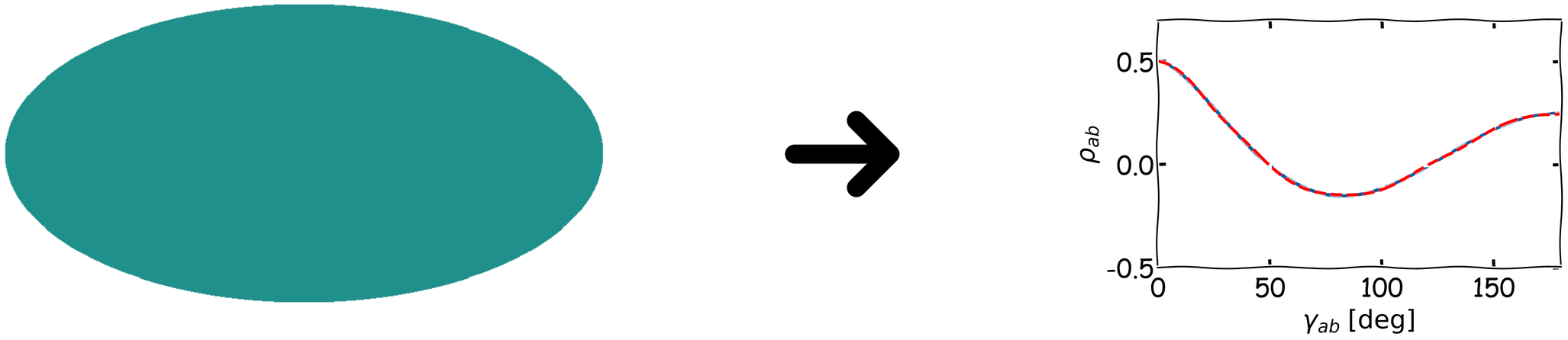


GENERATING NULL DISTRIBUTIONS



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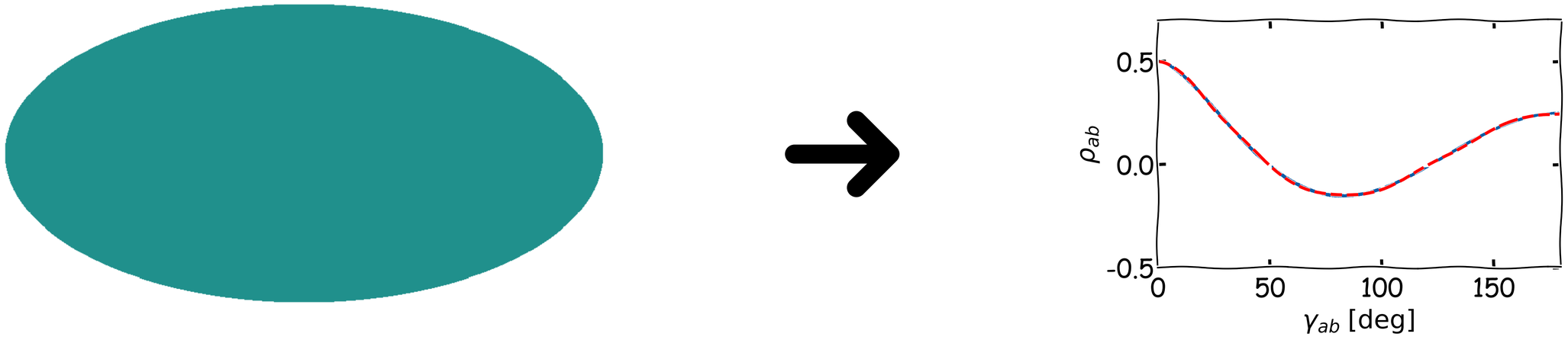

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$$\rho_{ab} = \mathrm{HD}_{ab} + \mathcal{N}(0, \Sigma_{ab})$$

"true value"

noise

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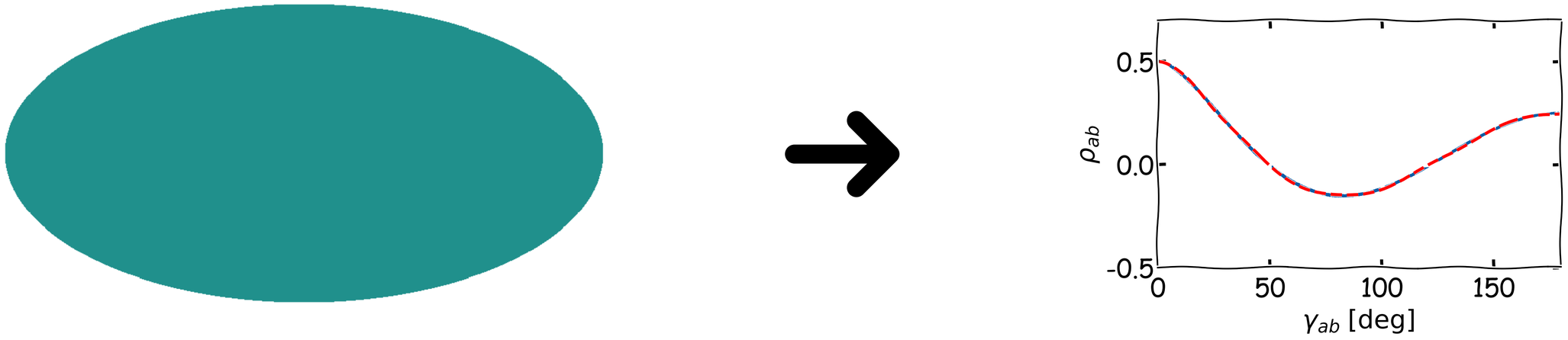

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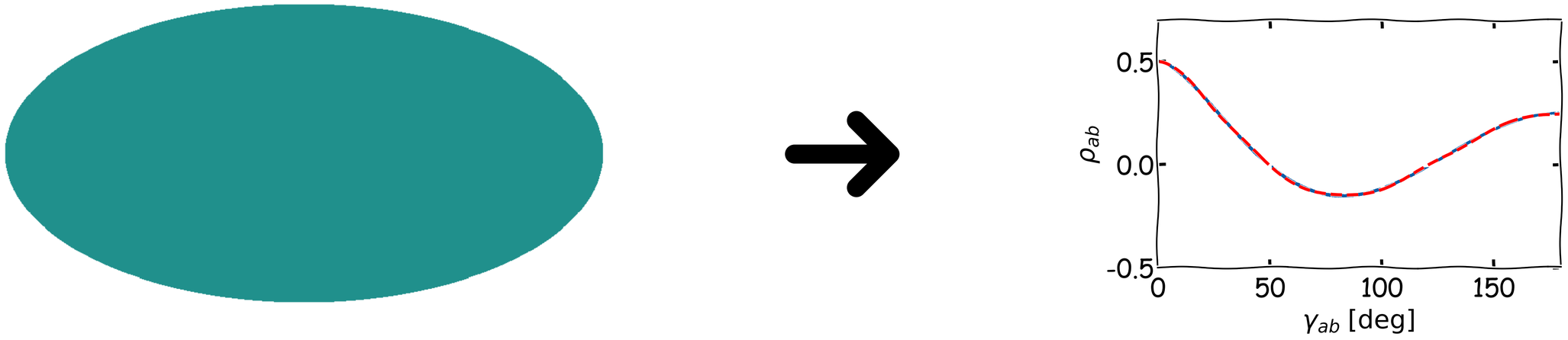

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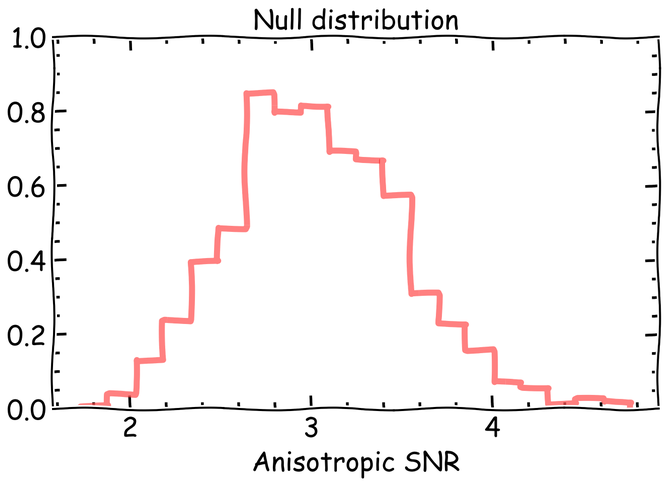

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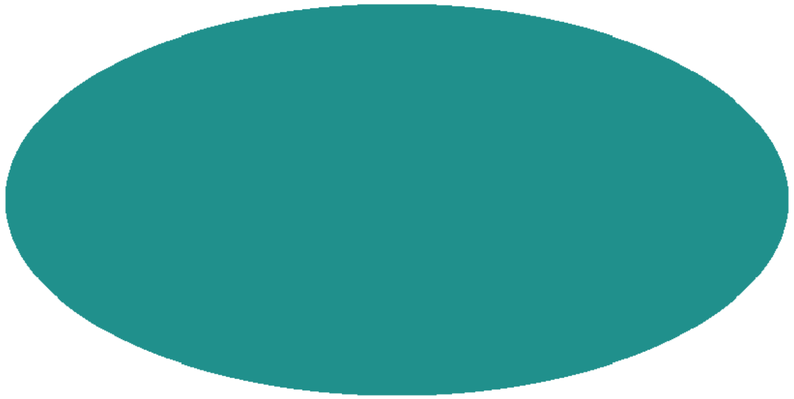

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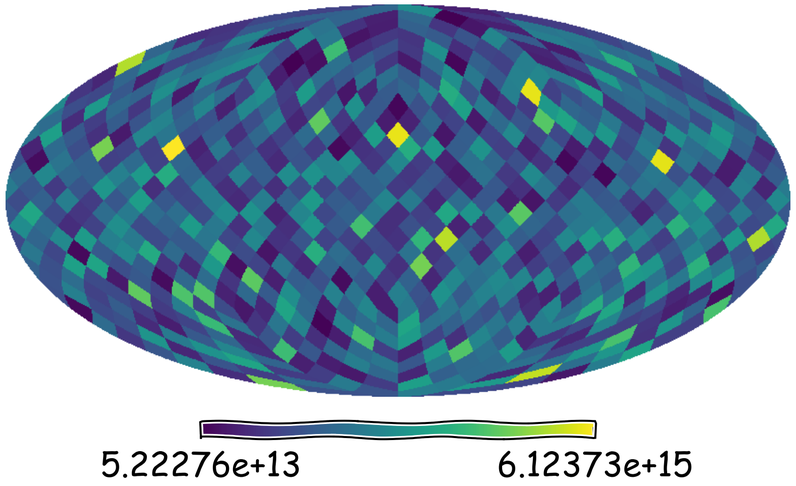

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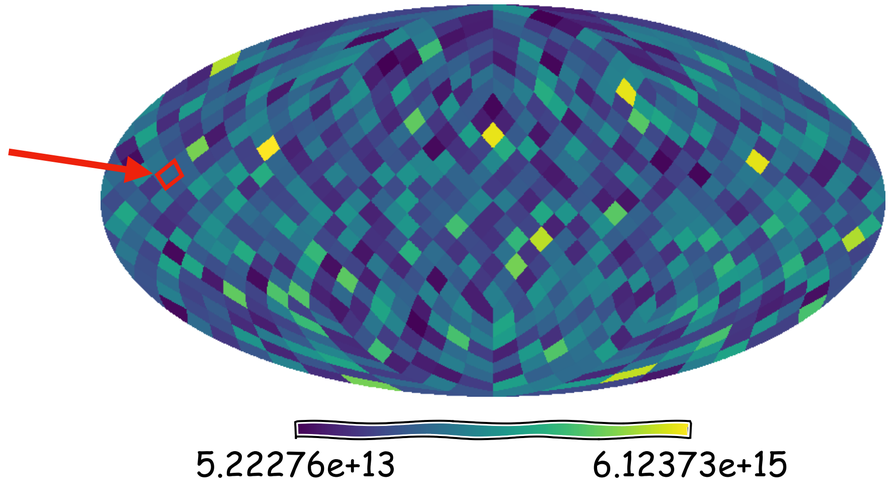

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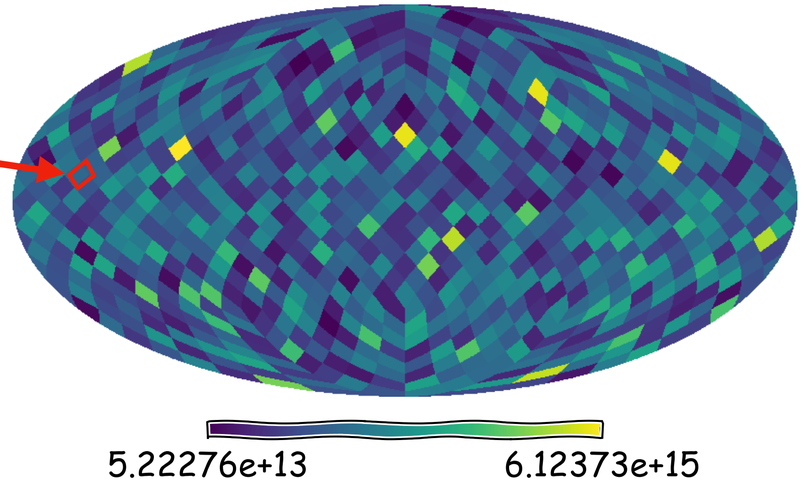

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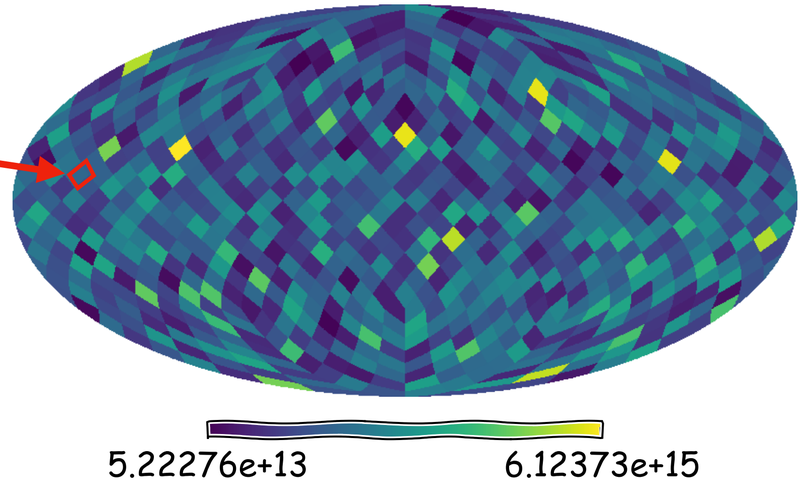

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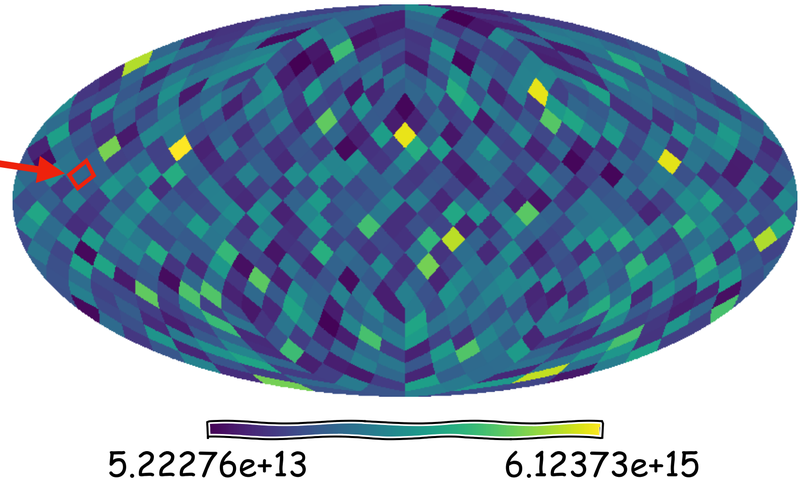

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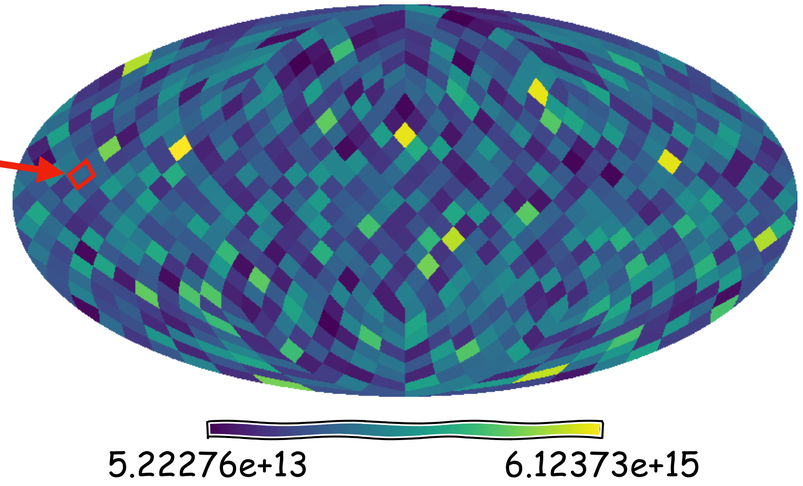

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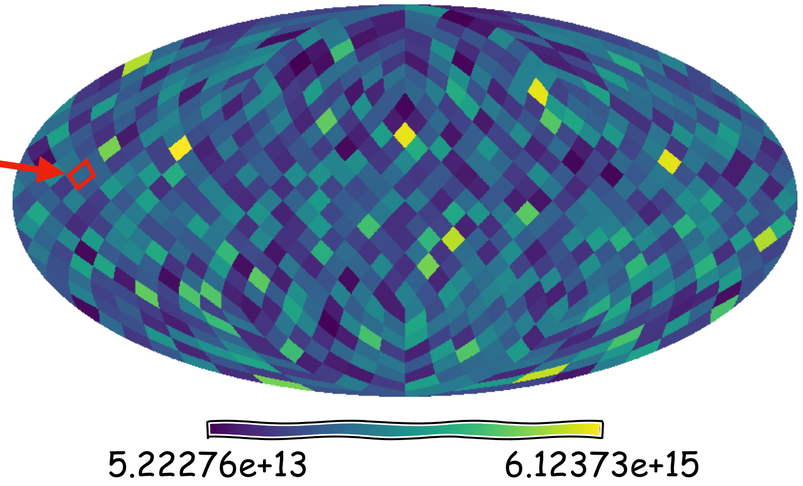

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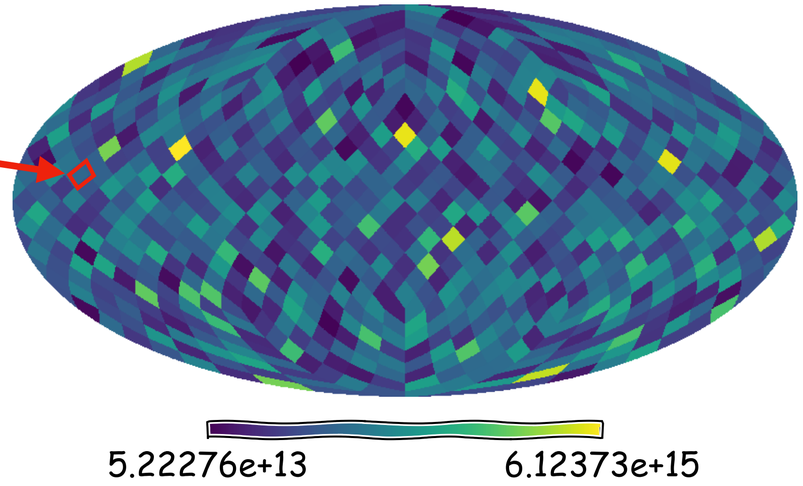

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sum over frequencies, pixels, and polarizations

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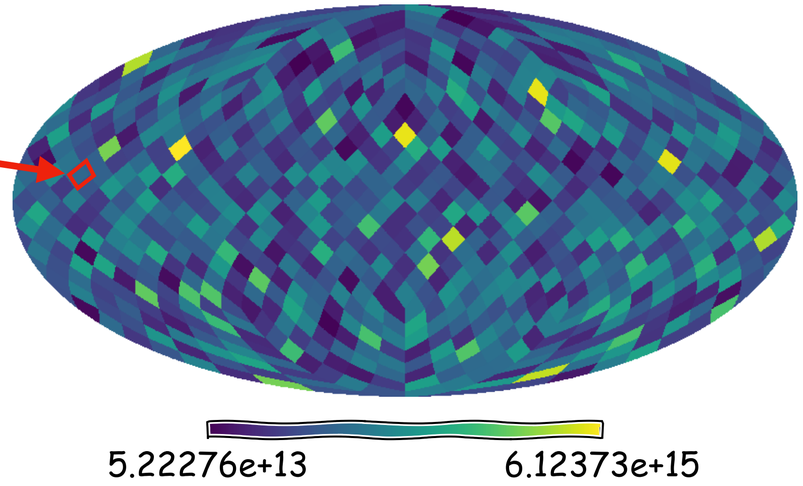

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response functions for pulsars a and b

$$R^A_a(f,\hat{\Omega}) \equiv F^A_a(\hat{\Omega})\left[1 - e^{2\pi i f L_a(1+\hat{p}_a\cdot\hat{\Omega})}\right]$$

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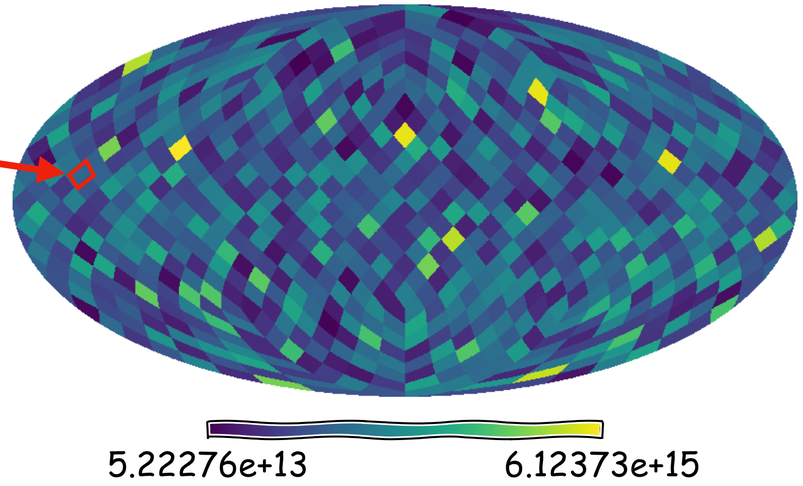

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pixel amplitudes

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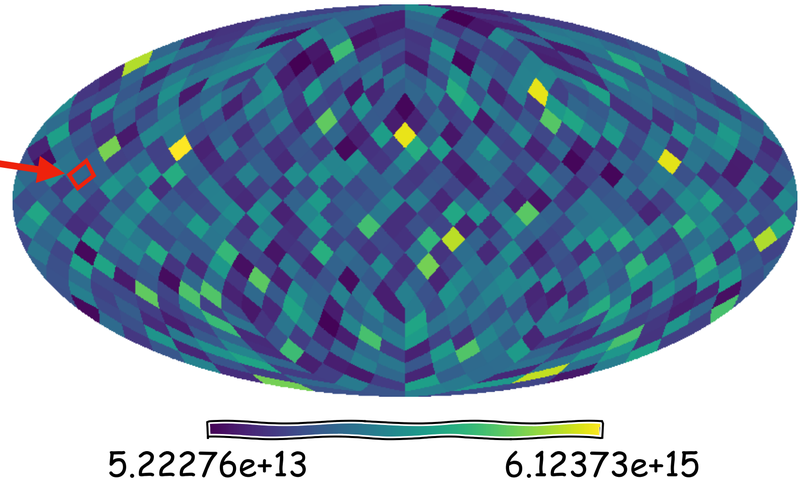

one realization of a GWB can be thought as a collection of

$h^A_{k,i}$ $\phi^A_{k,i}$

one **amplitude** and **phase** for each, pixel, frequency, and polarization

given a set of $\{h^A_{k,i}, \phi^A_{k,i}\}$, the cross correlation coefficients can be uniquely derived

$$\rho_{ab} = \Delta f^2 \Delta\hat{\Omega}^2 \sum_{jj'}\sum_{kk'}\sum_{AA'} R^{A*}_{akj} R^{A'}_{bk'j'}\, h^A_{kj} h^{A'}_{k'j'} \exp(i\Delta\phi^{AA'}_{kk',jj'}) \mathrm{sinc}(\pi(j-j')) + \mathrm{c.c.}$$

interference between different pixels

WHY IS LIFE SO HARD?!

each of this pixel can be thought as emitting a GW plane wave

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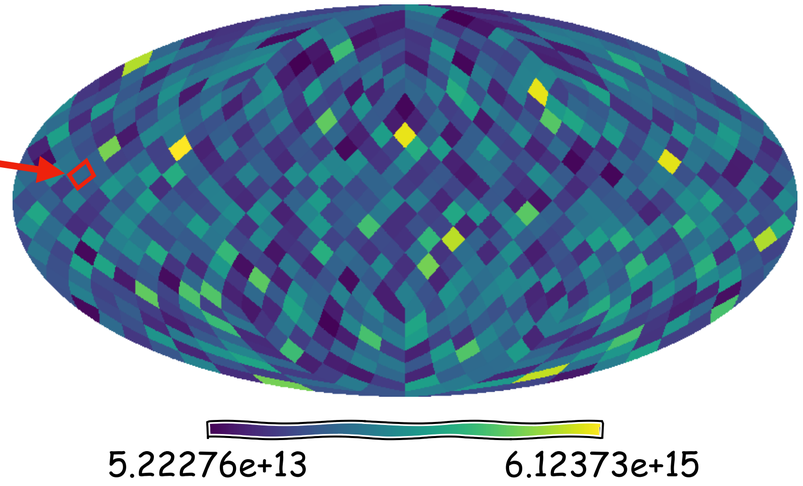

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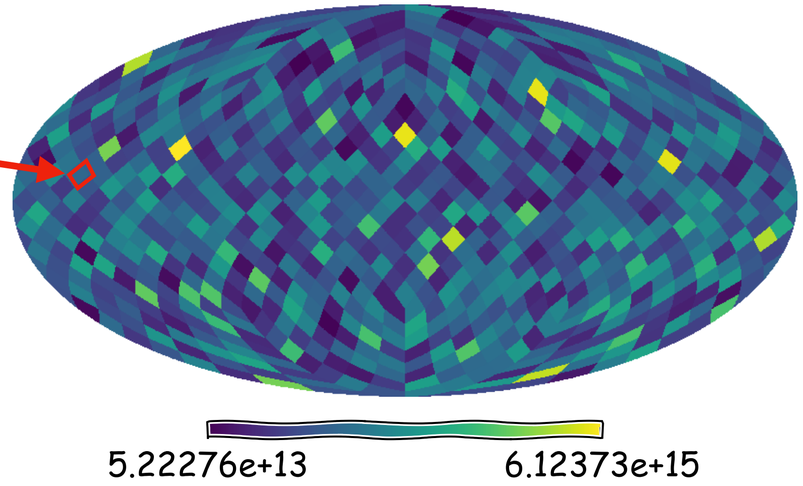

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$$\langle \rho_{ab} \rangle \propto \sum_A \sum_k F^A_{a,k} F^A_{b,k} \; P_k$$

if we average over many realizations of the GWB this reduces to the usual expression

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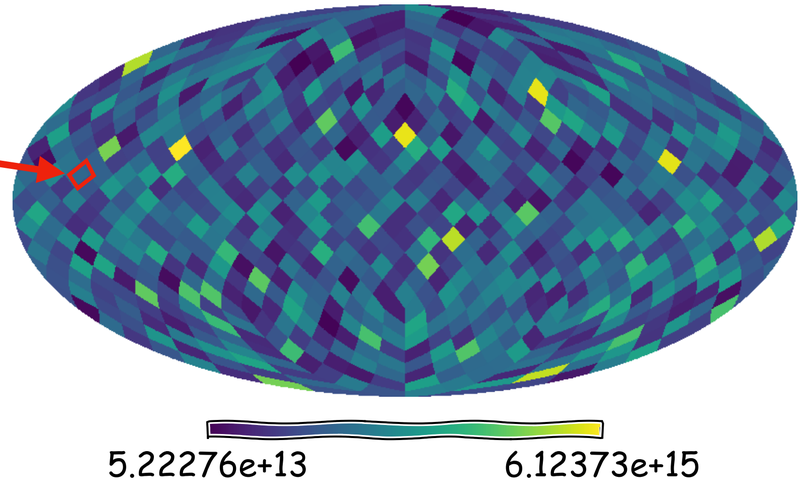

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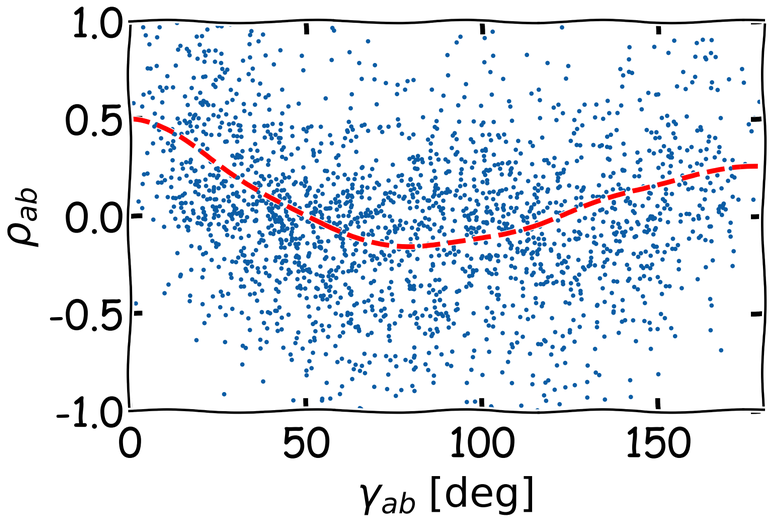

each realization of an isotropic GWB will produce cross correlation that fluctuate around the HD value

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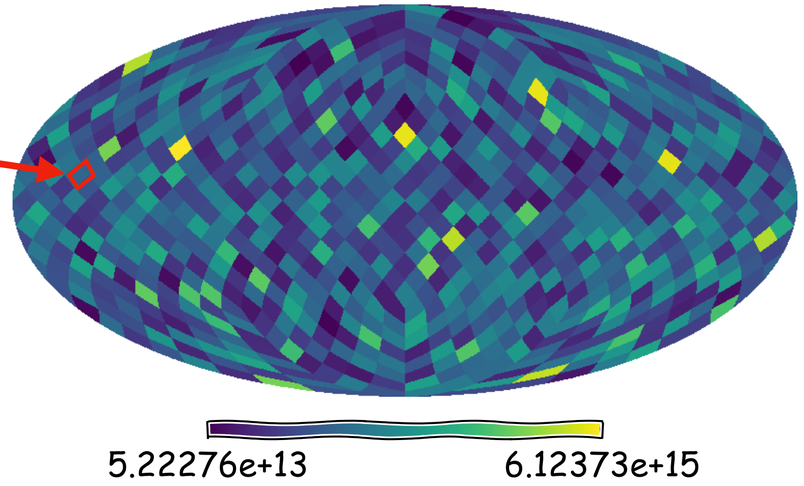

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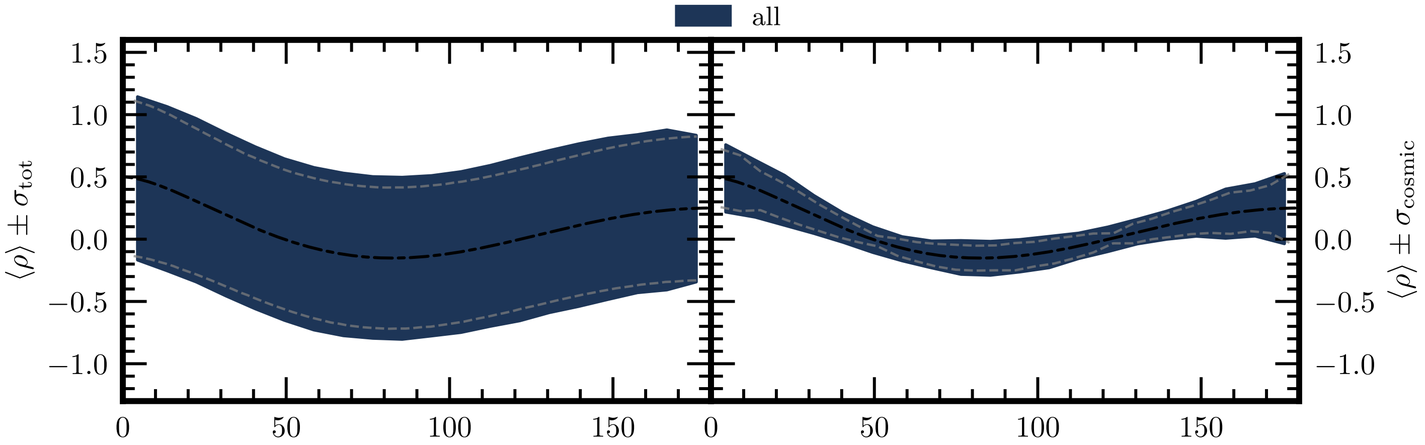

Konstandin, Lemke, AM, Perboni *"The impact of cosmic variance on PTAs anisotropy searches"*, [2498.07741]

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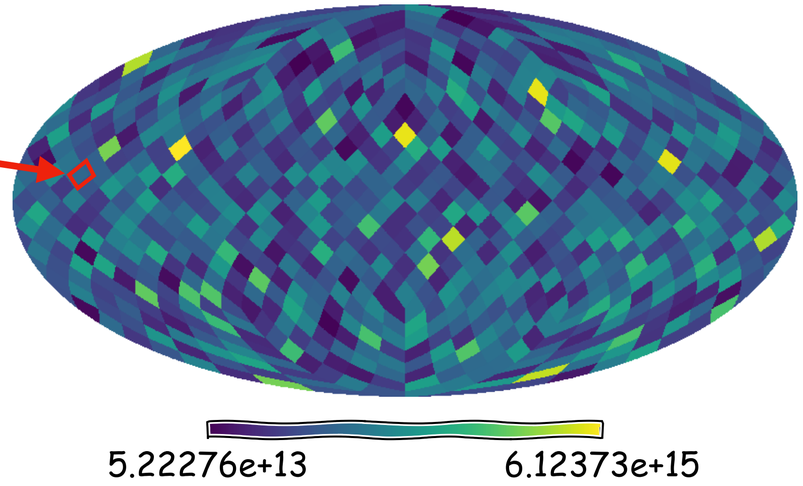

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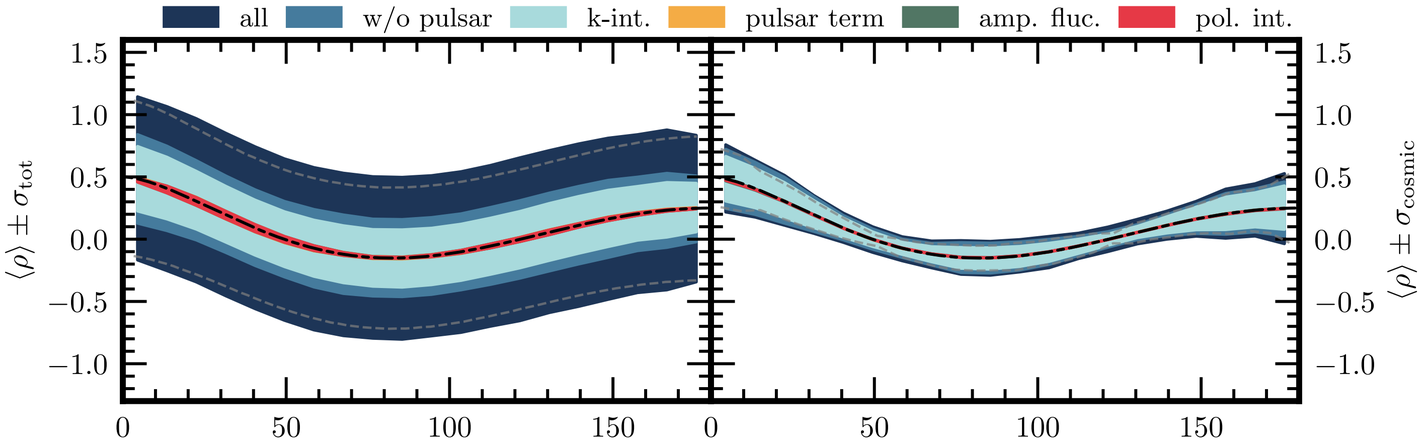

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NULL DISTRIBUTIONS v2.0

given an isotropic GWB as null hypothesis

$$\langle \tilde{h}^*_A(f,\hat{\Omega})\tilde{h}_A(f',\hat{\Omega}')\rangle = \delta_{AA'}\delta(f-f')\delta(\hat{\Omega},\hat{\Omega}')H(f)$$

take one realization of the null hypothesis

$$\{h^A_{k,i}, \phi^A_{k,i}\}$$

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$$\rho_{ab} = \hat{\rho}_{ab} + \mathcal{N}(0, \Sigma_{ab})$$

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run anisotropy search on these mock correlations and get an reconstructed sky

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↓

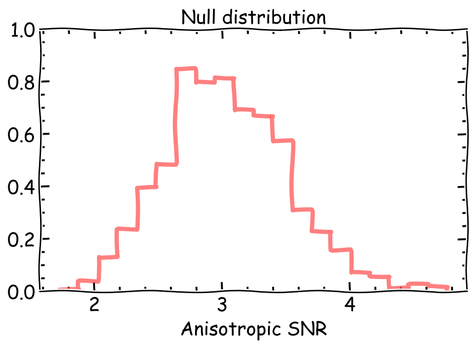
run anisotropy search on these mock correlations and get an reconstructed sky

←

compute anisotropic SNR

repeat many times

←



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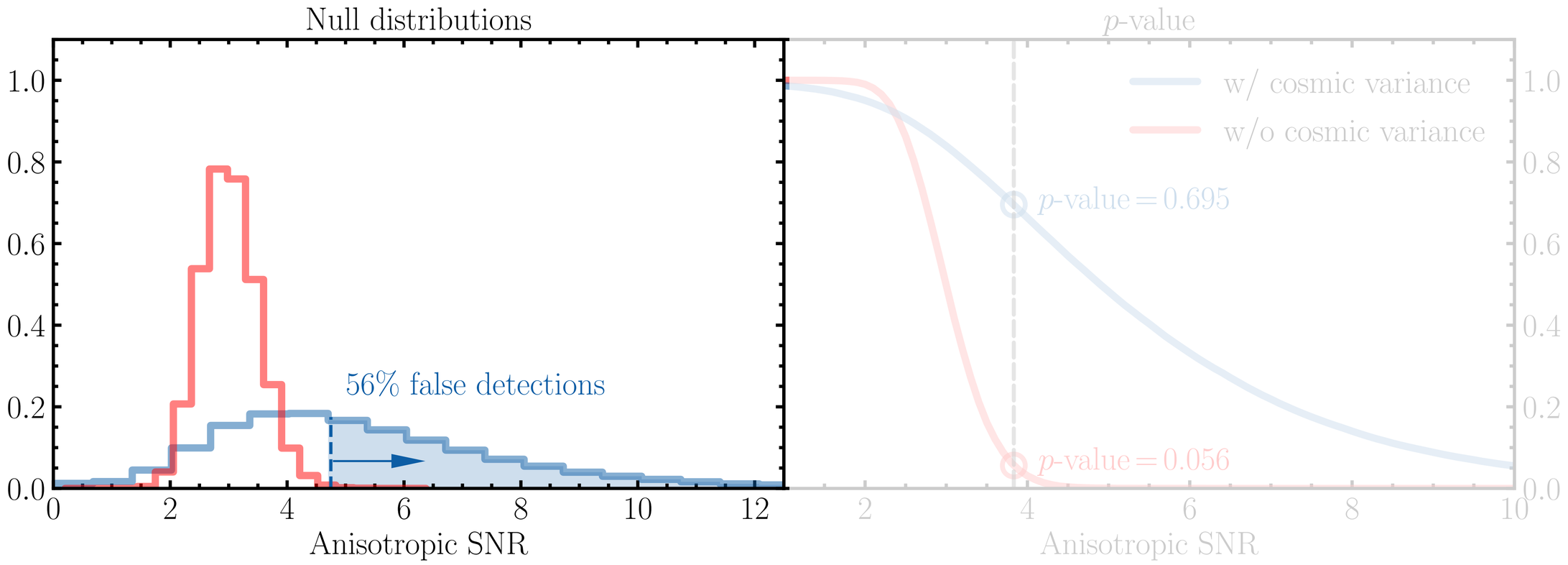

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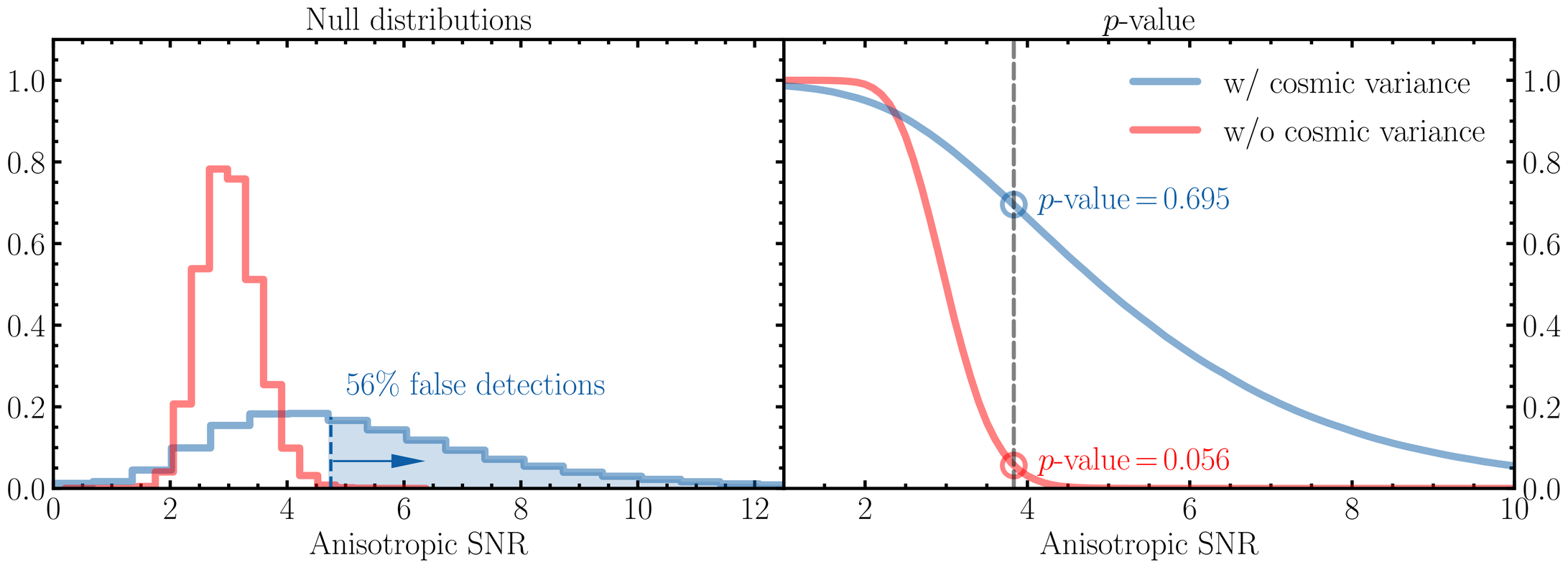

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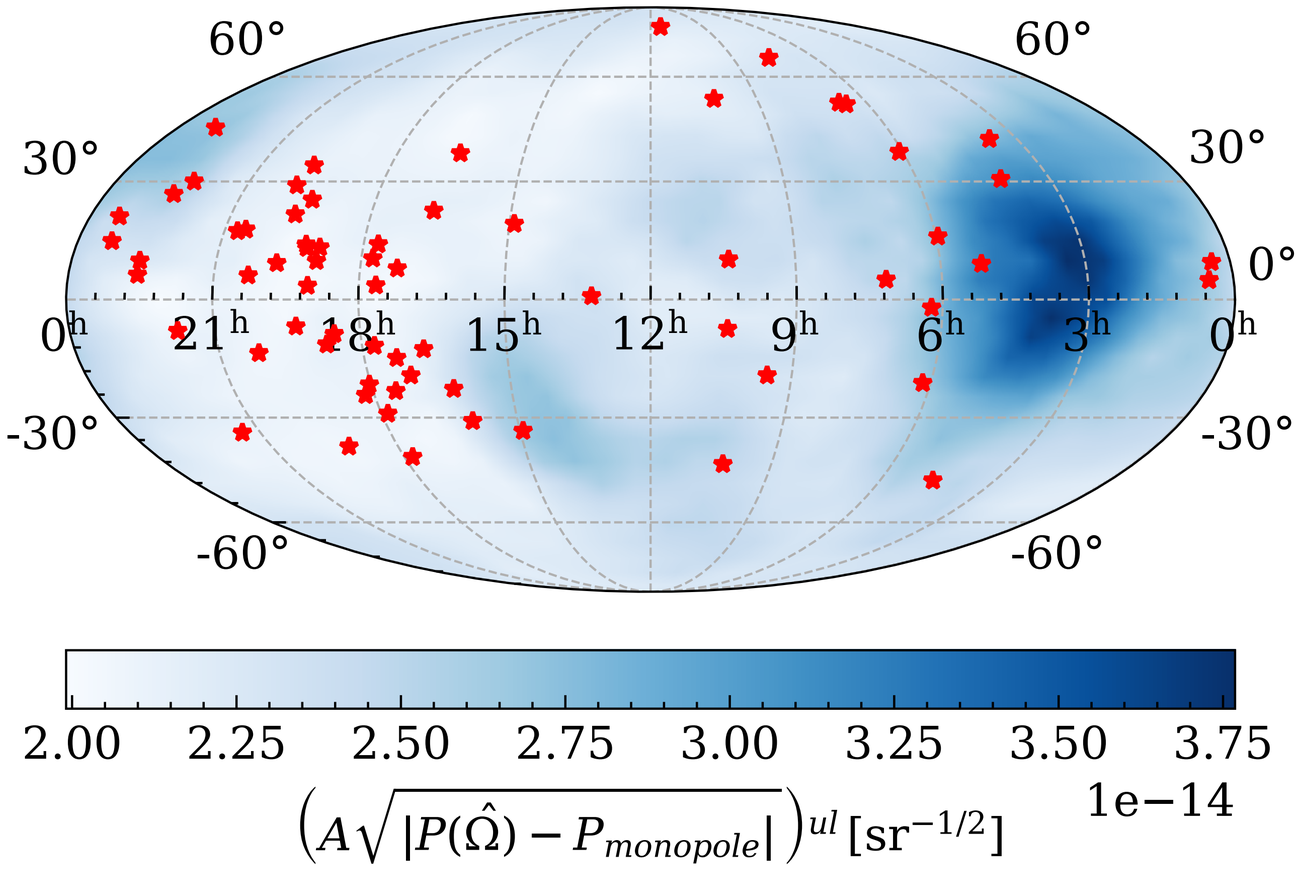
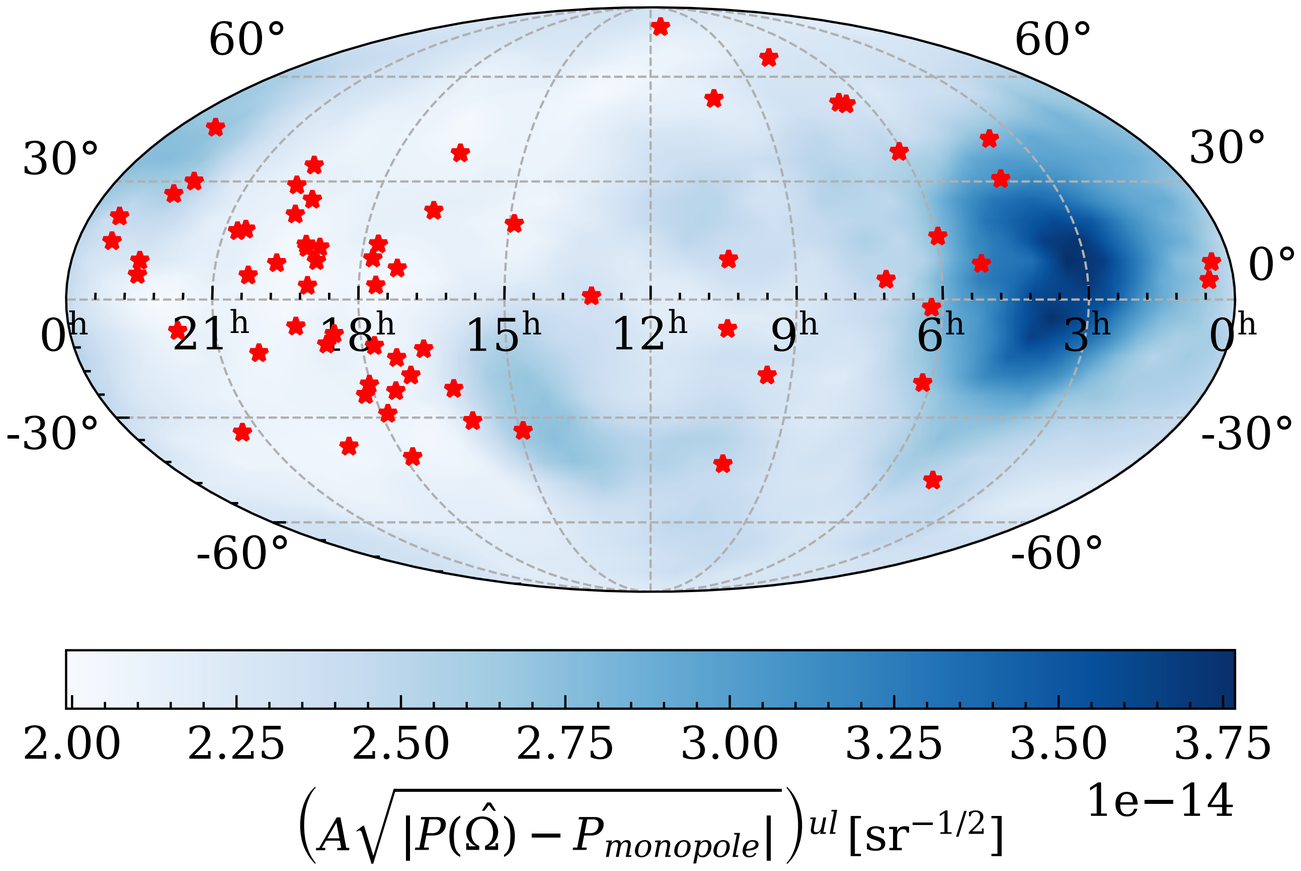
SO, HAVE WE DETECTED ANISOTROPIES?!



for the reconstructed GWB map in the NG15 data

SNR~4 $\Rightarrow p \sim 0.05$

SO, HAVE WE DETECTED ANISOTROPIES?!

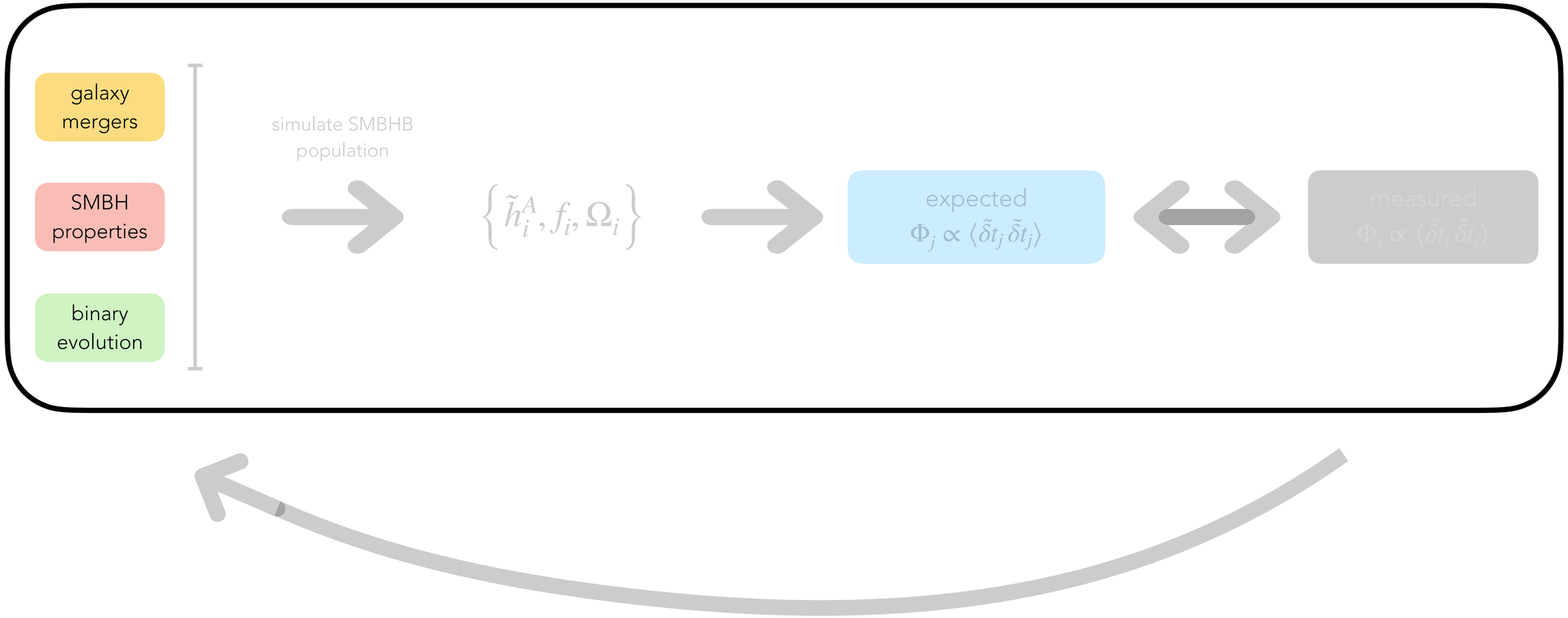


for the reconstructed GWB map in the NG15 data

SNR~4 $\Rightarrow p \sim 0.5$ 0.7

INTERFERENCE, INTERFERENCE EVERYWHERE

how astrophysical inference analyses are performed



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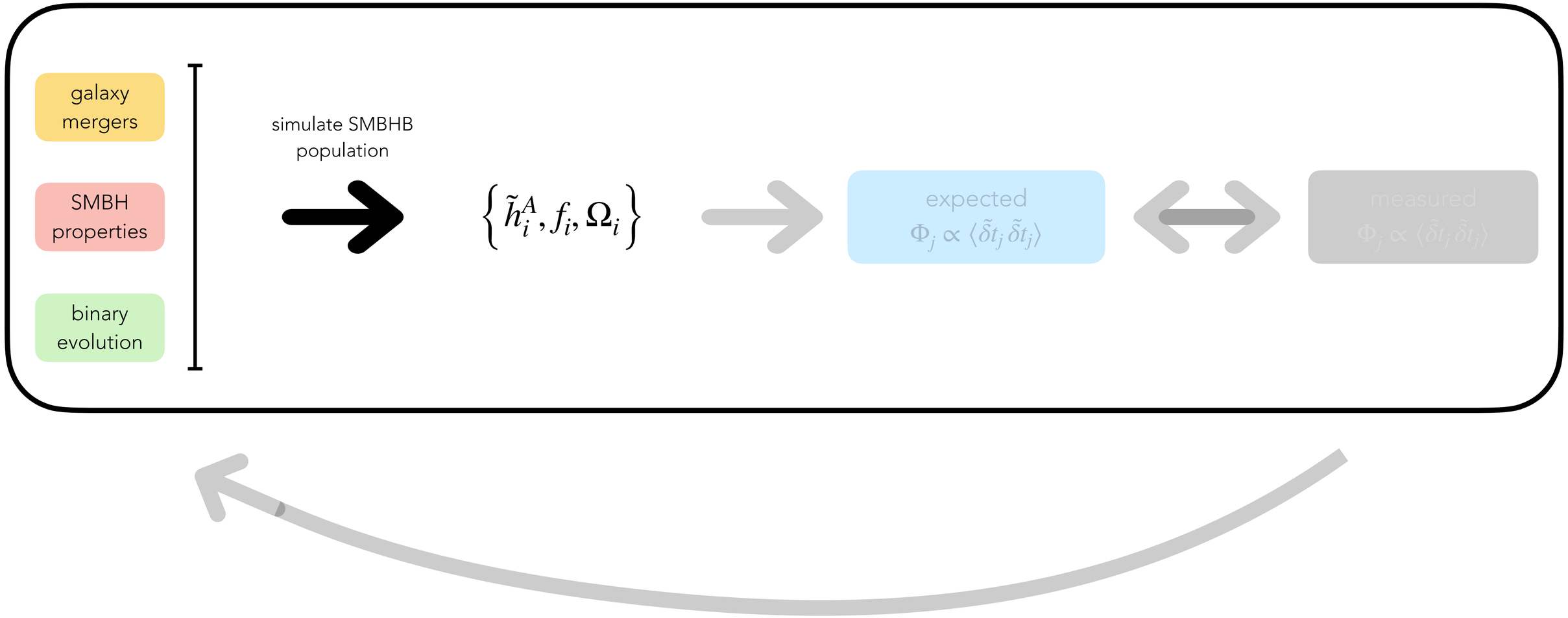

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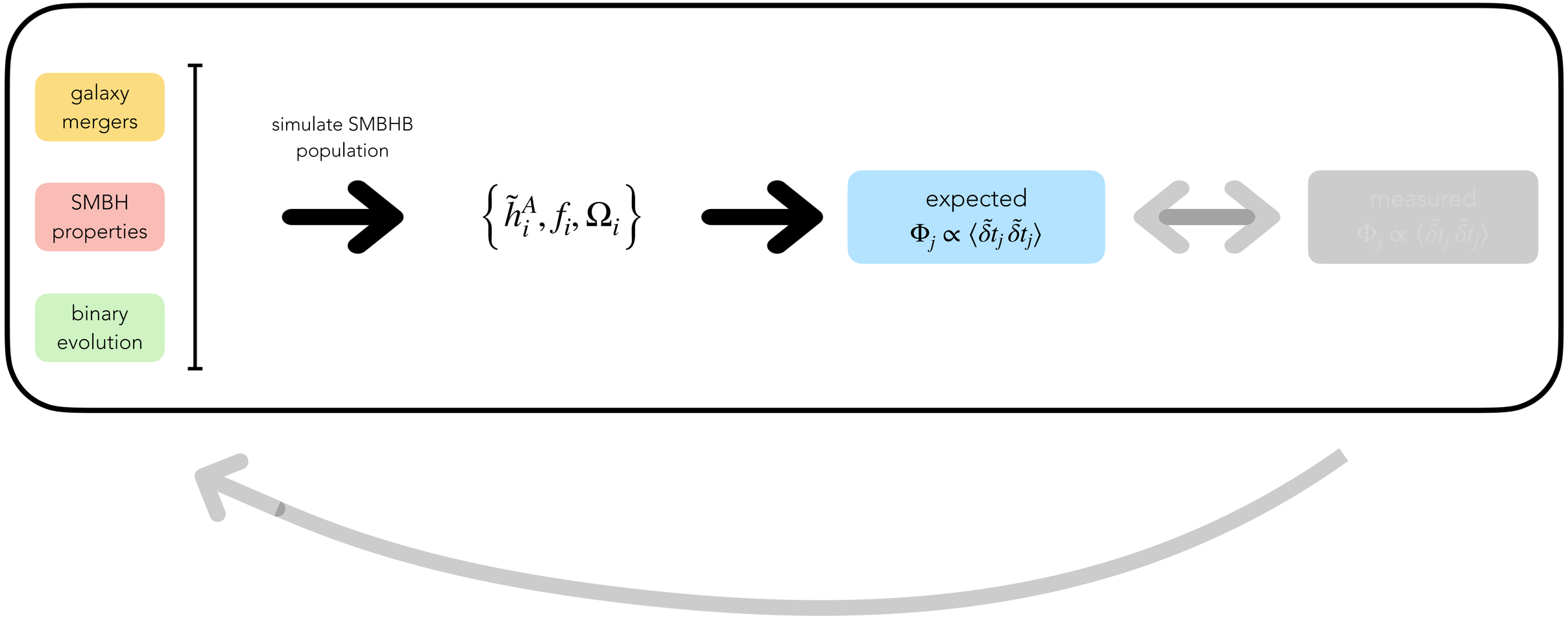

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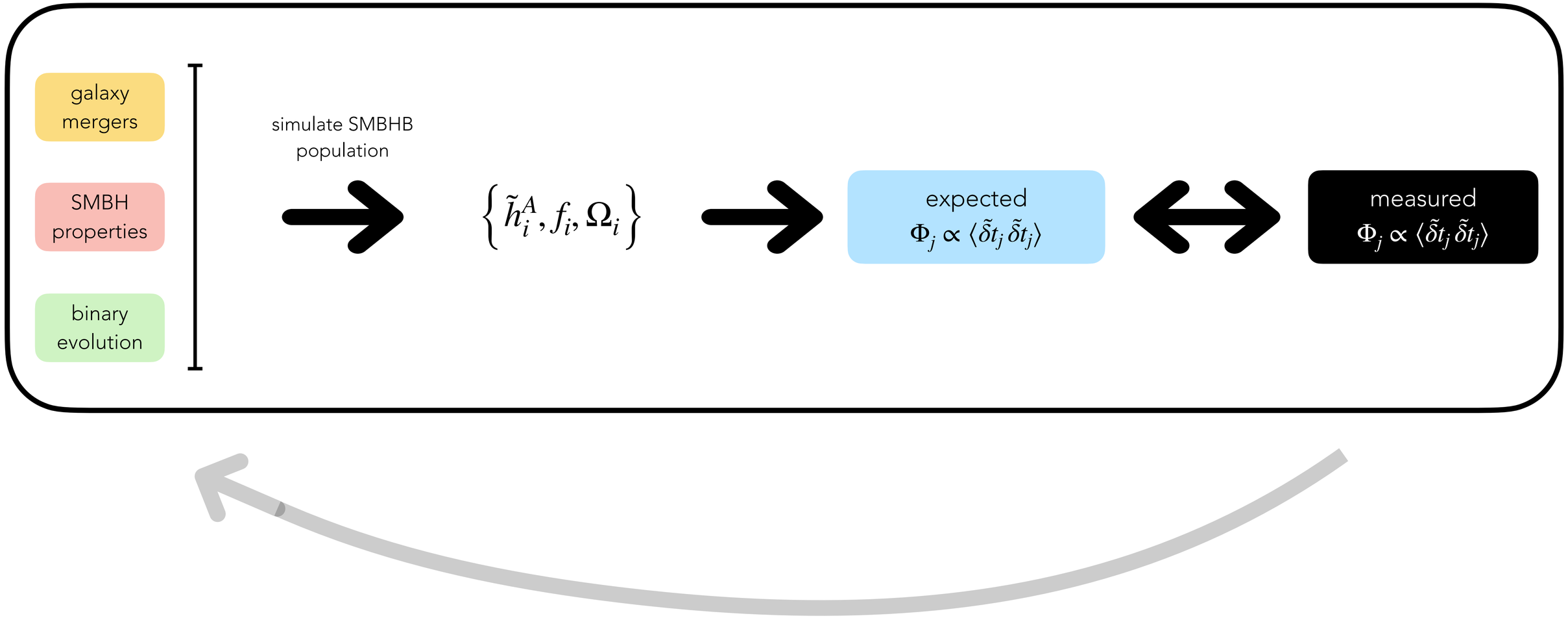

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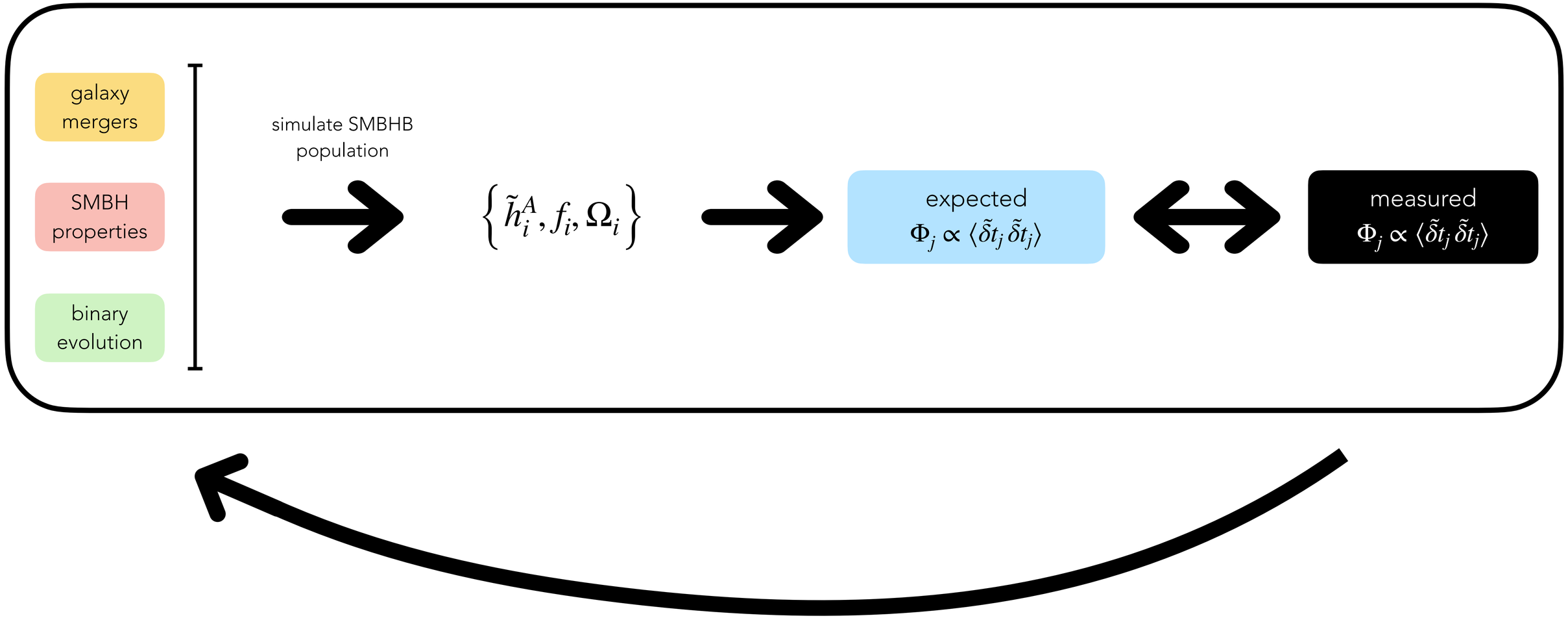
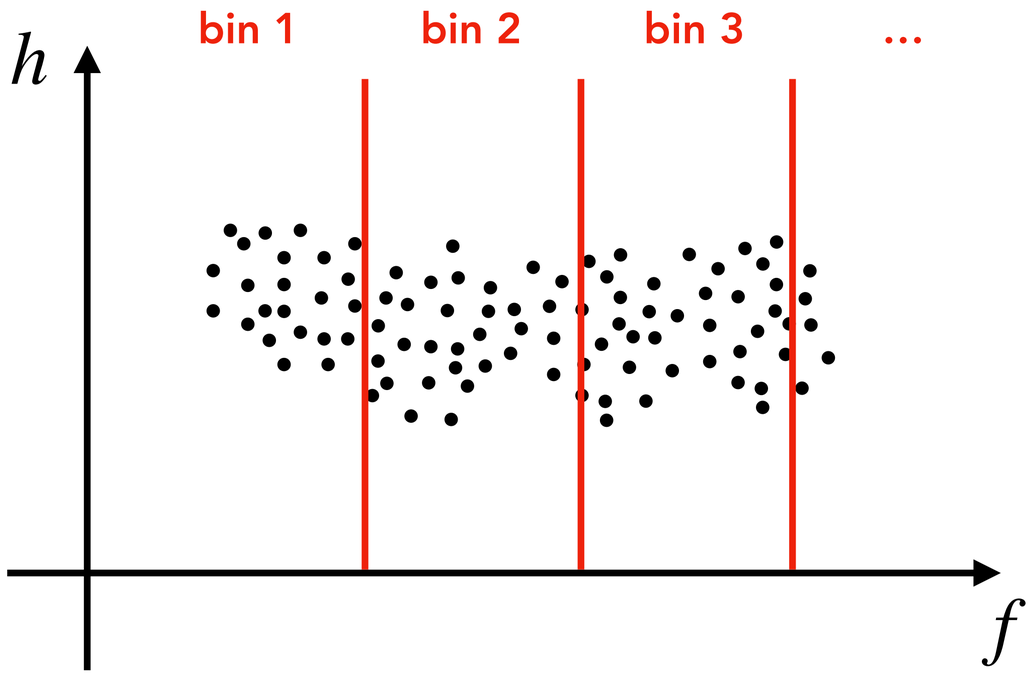


INTERFERENCE, INTERFERENCE EVERYWHERE

$$\left\{\tilde{h}_i^A, f_i, \Omega_i\right\} \rightarrow$$

expected
$\Phi_j \propto \langle \tilde{\delta t}_j \tilde{\delta t}_j \rangle$



$$\Phi_j \propto \sum_{i \in \text{bin } j} |\tilde{h}_i|^2$$

INTERFERENCE, INTERFERENCE EVERYWHERE

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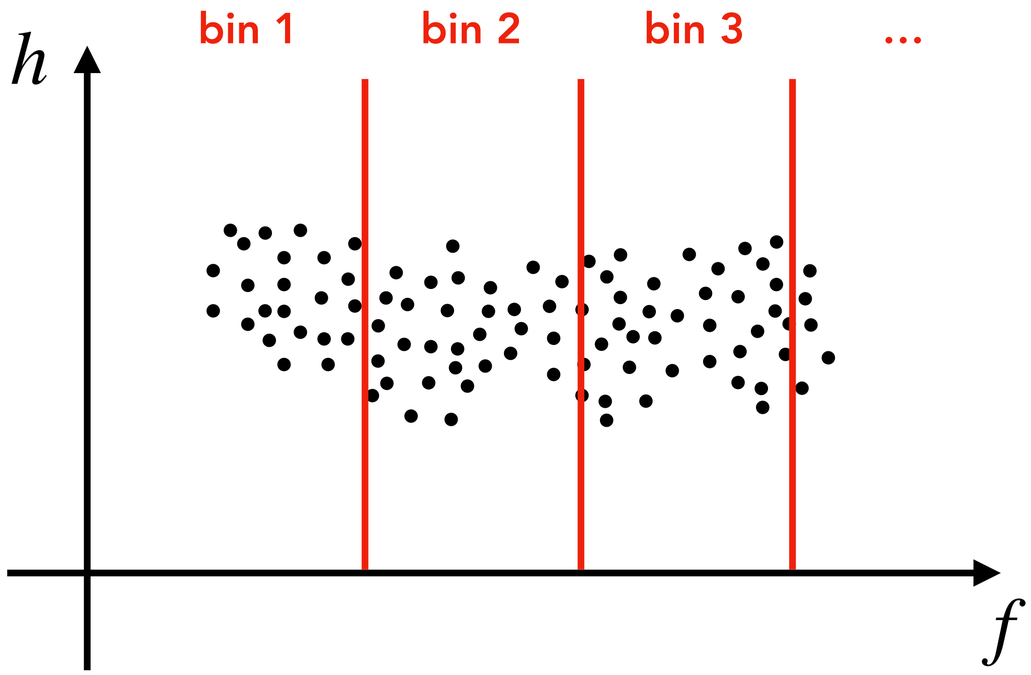
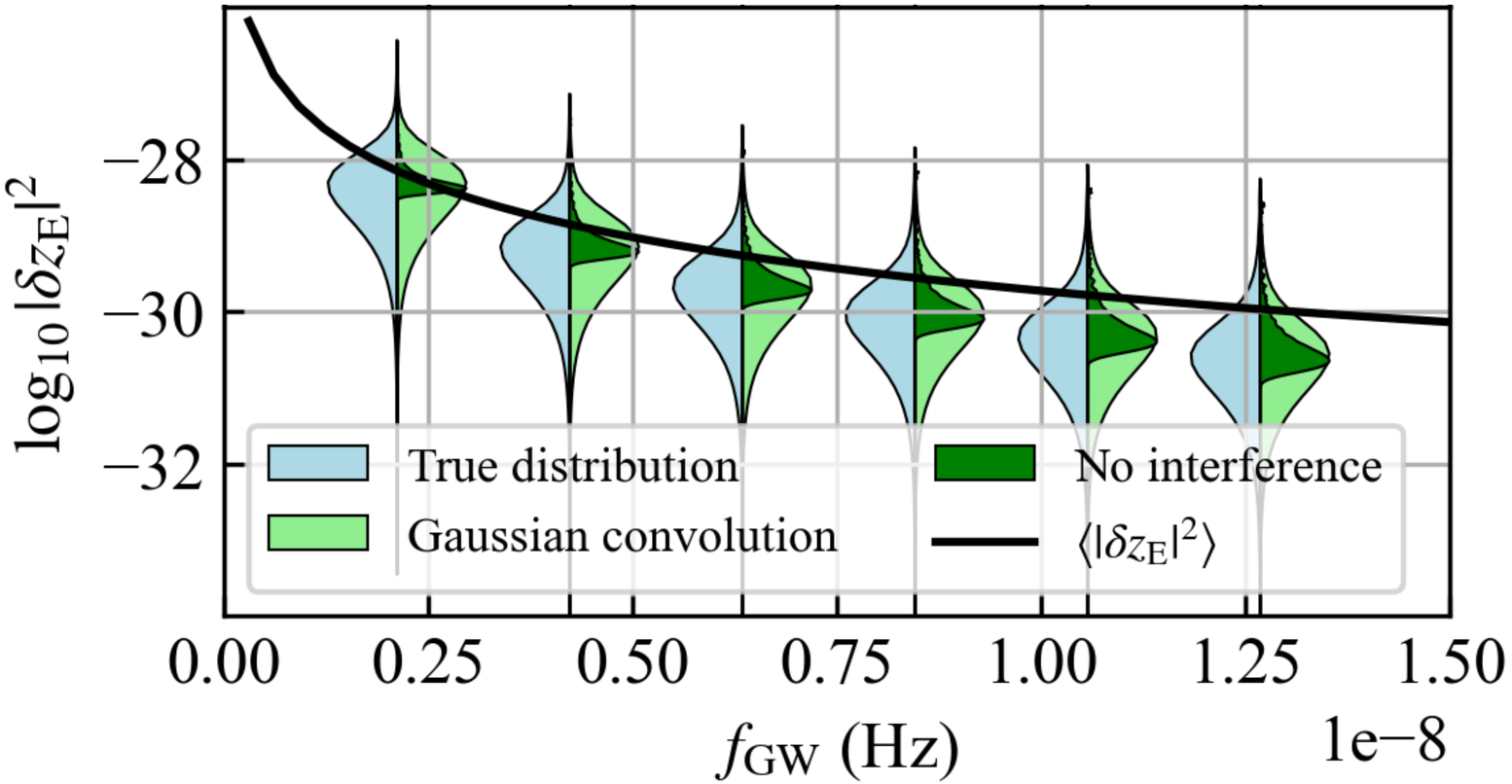


$$\Phi_j \propto \sum_{ii'} \tilde{h}_i \tilde{h}_{i'} w_{ji} w_{ji'}$$

INTERFERENCE, INTERFERENCE EVERYWHERE

how astrophysical inference analyses are performed

Xue et al. [2409.19516]



strong evidence for a GWB in the nHz band

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cosmology or astrophysics?

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cosmology or astrophysics?

CW and anisotropies will help us discriminating

strong evidence for a GWB in the nHz band

cosmology or astrophysics?

CW and anisotropies will help us discriminating

we need to develop tools to characterize the GWB

backup

HOW TO SAMPLE A GWB

$$\langle \tilde{h}^{A*}_{kj} \tilde{h}^{A'}_{k'j'} \rangle = \delta_{AA'} \delta_{kk'} \delta_{jj'} \frac{H_j}{\Delta\hat{\Omega}\,\Delta f}$$

$$\langle \tilde{h}^{A}_{kj} \tilde{h}^{A'}_{k'j'} \rangle = \langle \tilde{h}^{A*}_{kj} \tilde{h}^{A'*}_{k'j'} \rangle = 0$$

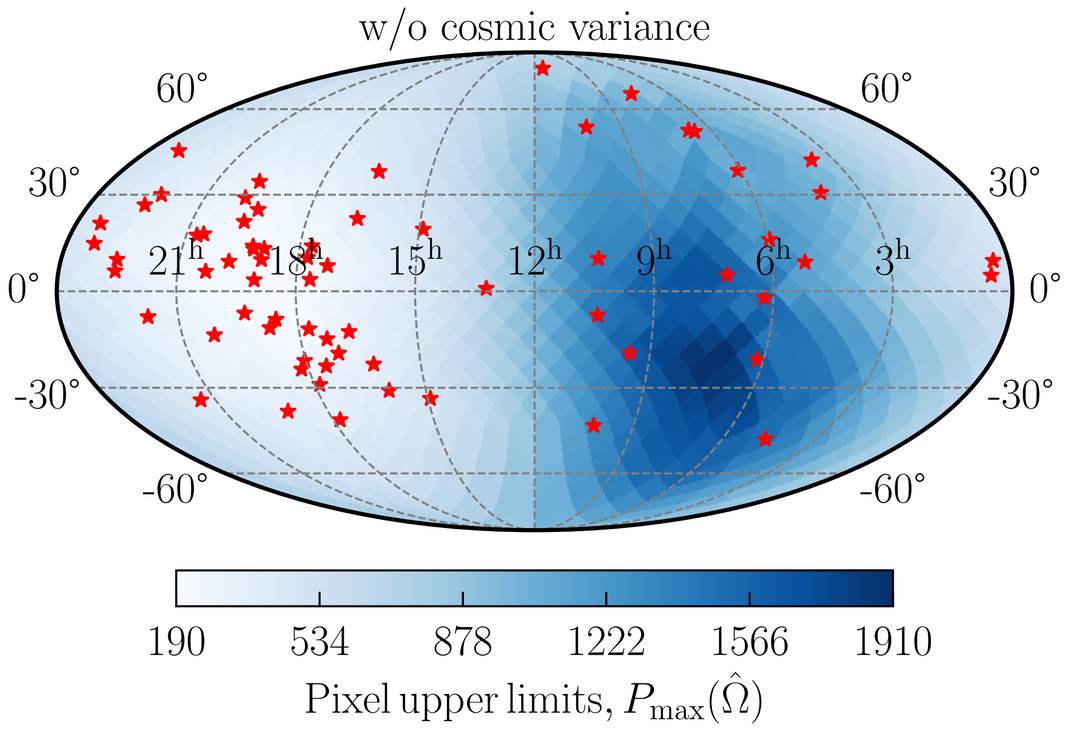
$$\tilde{h}^{A}_{kj} \equiv h^{A}_{kj} e^{i\phi^{A}_{kj}}$$

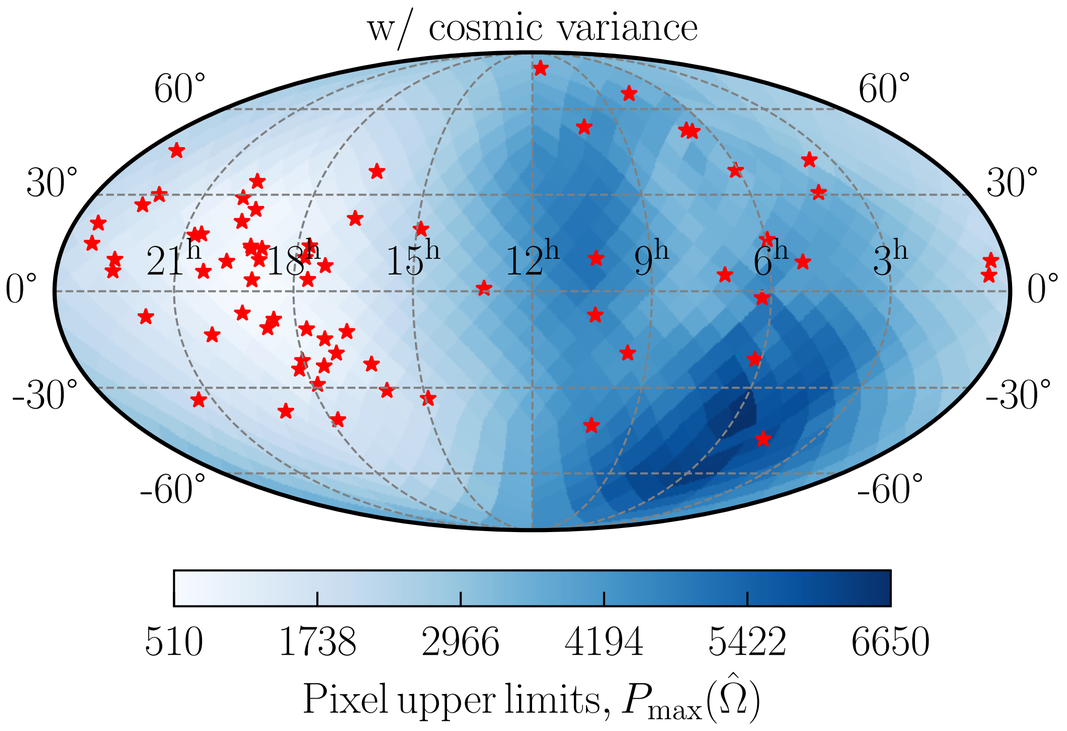
$$p(h) = \frac{h}{2\pi\sigma_j^2} e^{-h^2/2\sigma_j^2}$$

$$\sigma_j^2 = H_j/(2\Delta\hat{\Omega}\Delta f)$$

$$p(\phi) = \mathrm{Uniform}[0,\, 2\pi]$$

RESULTS IN THE PIXEL BASIS





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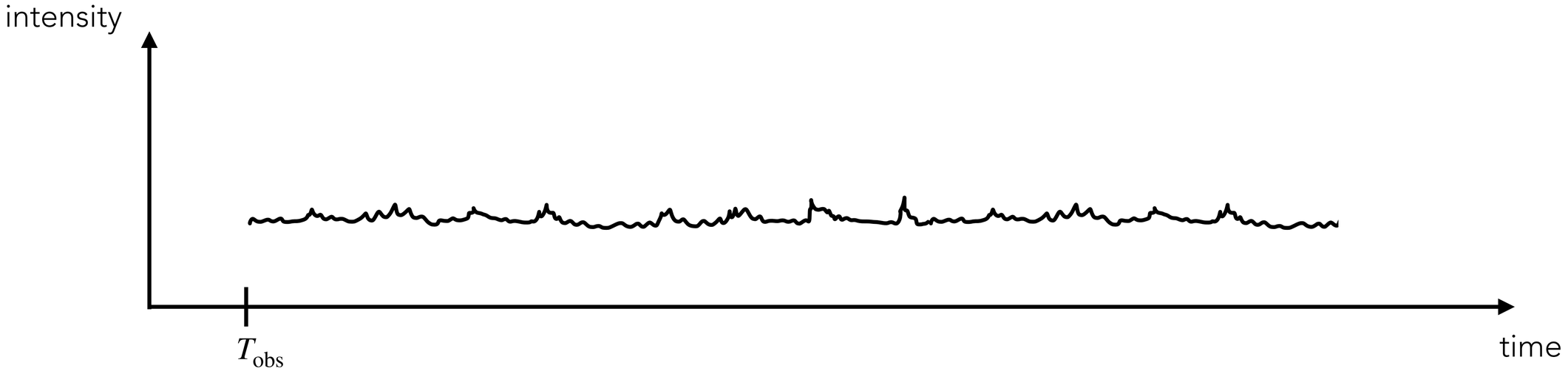
PULSAR TIMING



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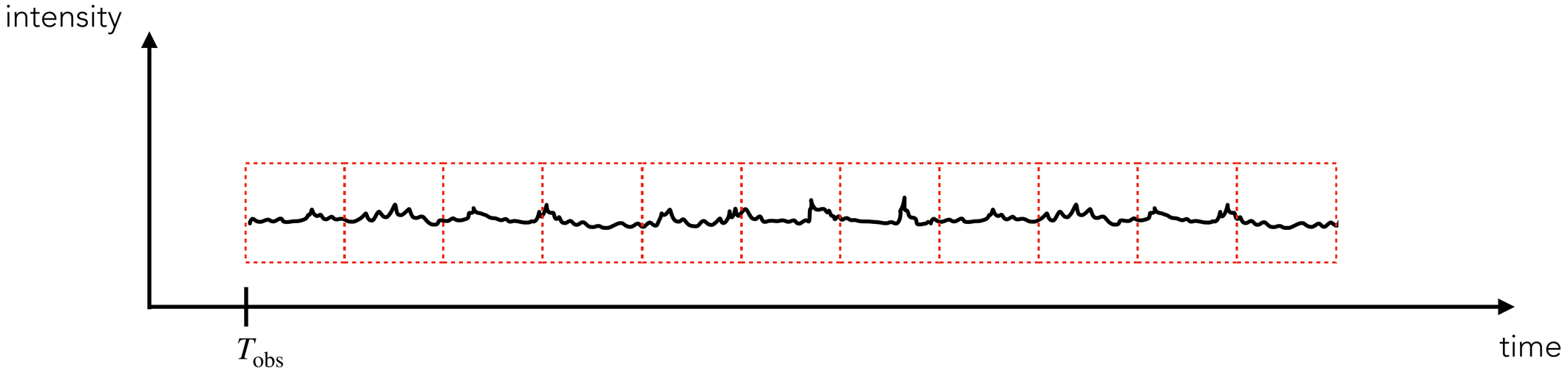

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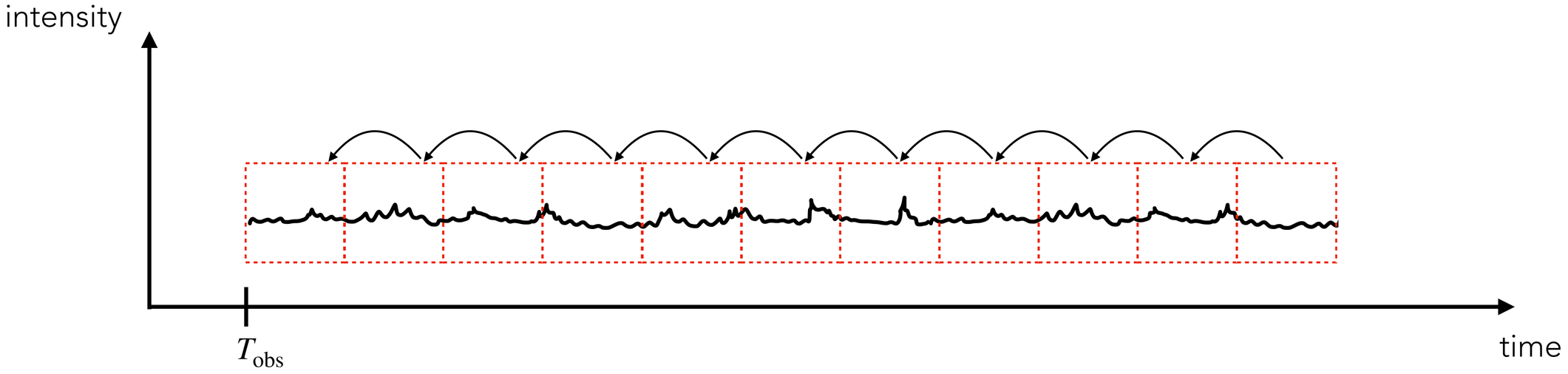

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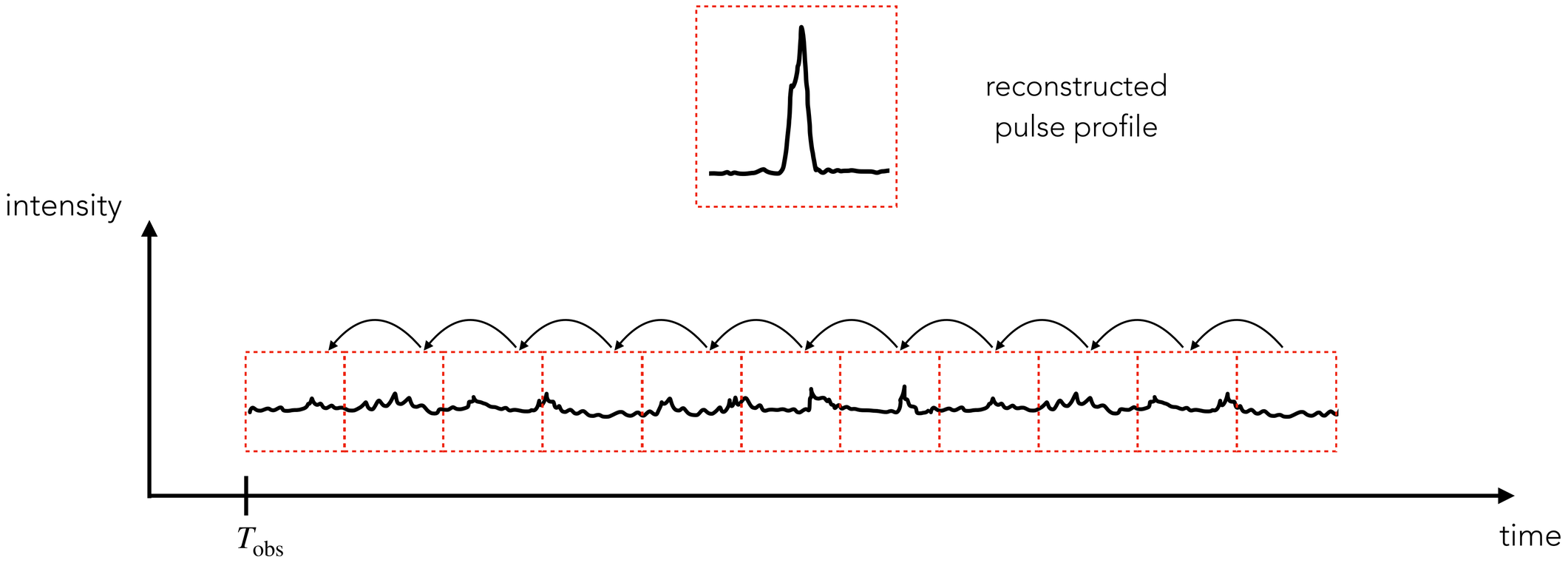

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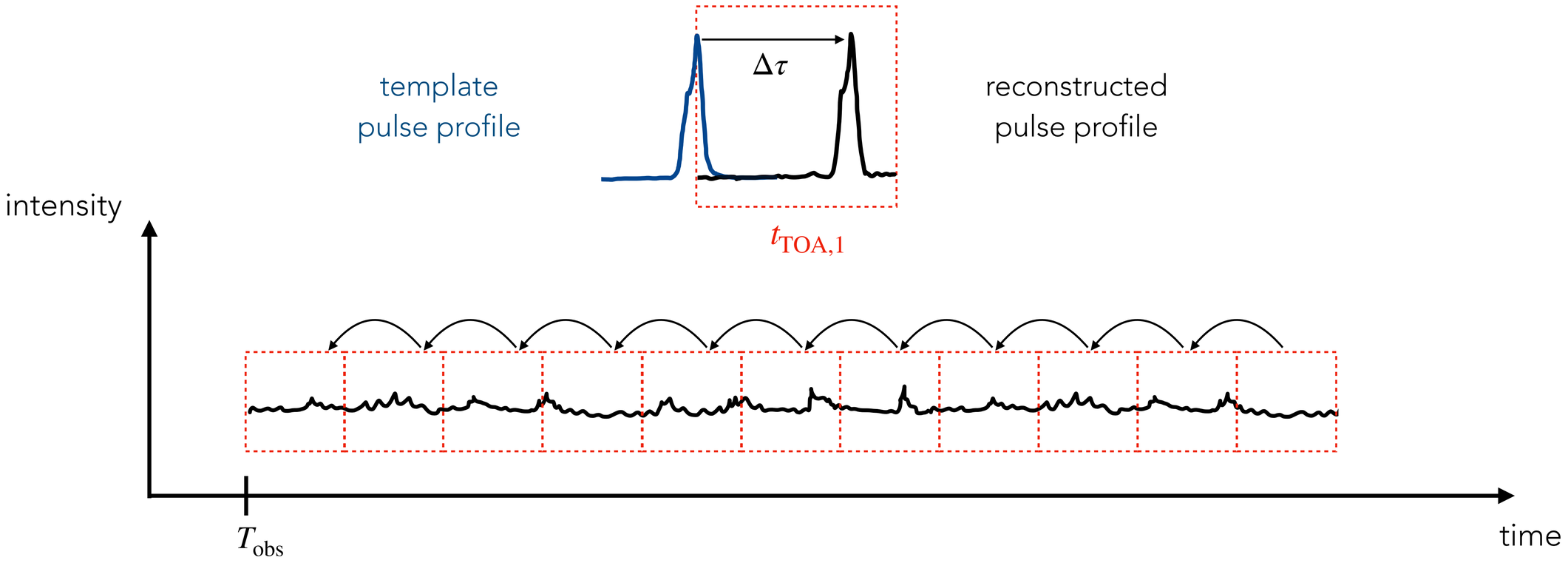

$$t_{\mathrm{TOA}} = T_{\mathrm{obs}} + \Delta\tau$$

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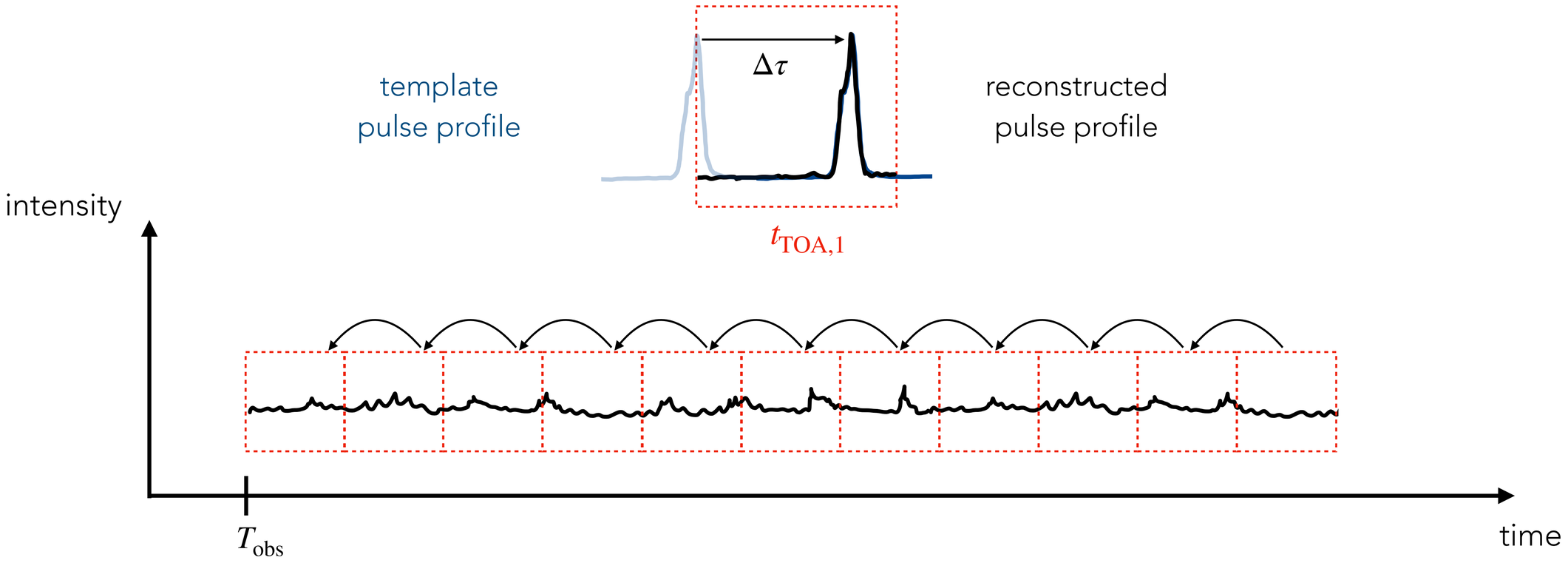


$$t_{\mathrm{TOA}} = T_{\mathrm{obs}} + \Delta\tau$$

BUILDING A TIMING MODEL

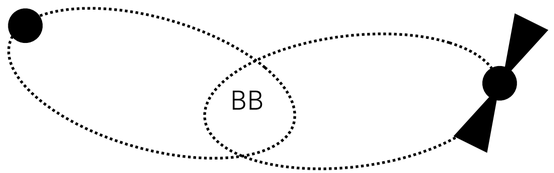
$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$

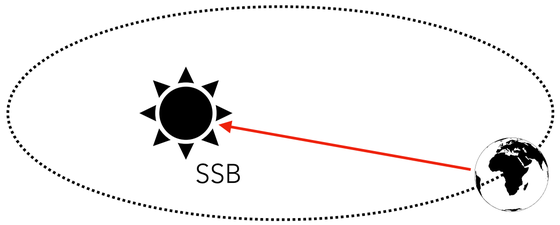
the goal is to relate the arrival time in Terrestrial Time (TT) to the emission time in the pulsar reference frame

BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$

first step transform from the from the TT time to the SSB time





$$\Delta_{\odot} = \Delta_{\mathrm{A}} + \Delta_{\mathrm{R}\odot} + \Delta_{\mathrm{p}} + \Delta_{\mathrm{D}\odot} + \Delta_{\mathrm{S}\odot} + \Delta_{\mathrm{E}\odot}$$

atmospheric delay: group velocity of radio waves in the atmosphere differs from the vacuum speed of light

Roemer delay: difference in the arrival time of the pulse at the observatory and at the SSB

Parallax term: delay due to the curvature of spherical wavefront

dispersion term: due to radio waves propagation in interplanetary medium

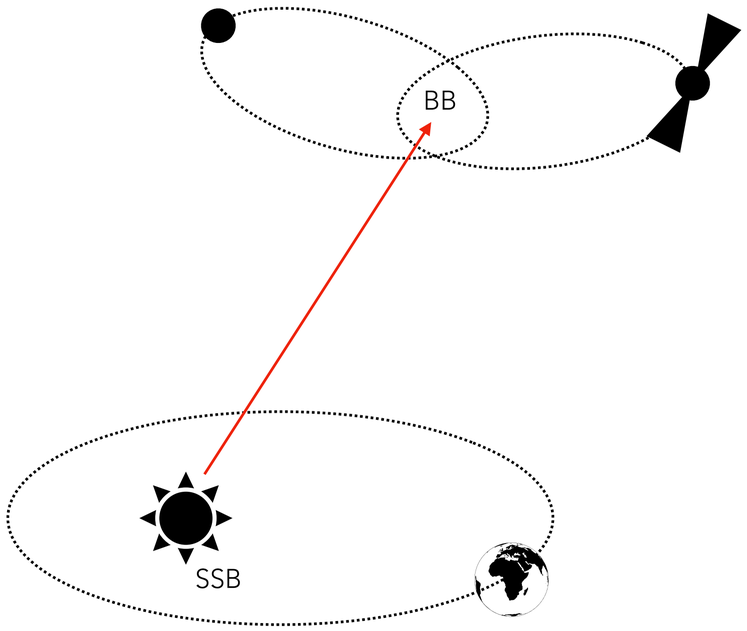
Shapiro delay: due to propagation of radio pulses through potential wells of solar systems bodies

Einstein delay: difference in the arrival time due to acceleration and potential of the Earth

BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$

second step transform from the from SSB to the BB time



$$\Delta_{\mathrm{IS}} = \Delta_{\mathrm{VP}}$$ **vacuum propagation delay:** vacuum light travel time

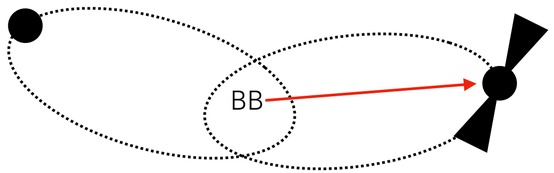
$$\Delta_{\mathrm{ISD}}$$ **dispersion term:** due to propagation through the interstellar medium

$$\Delta_{\mathrm{ES}}$$ **Einstein delay:** due to the secular motion of the SSB

BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$

last step transform from the from BB to the pulsar time



$$\Delta_{\mathrm{B}} = \Delta_{\mathrm{RB}}$$

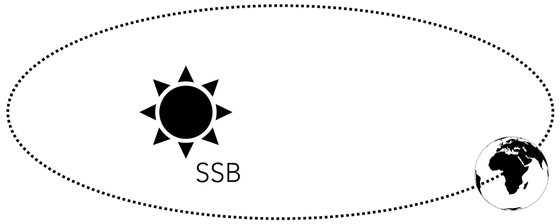
Roemer delay: delay due to the orbital motion of the pulsar

$$\Delta_{\mathrm{EB}}$$

Einstein delay: difference in the emission time from the pulsar inertial reference frame and the binary barycenter

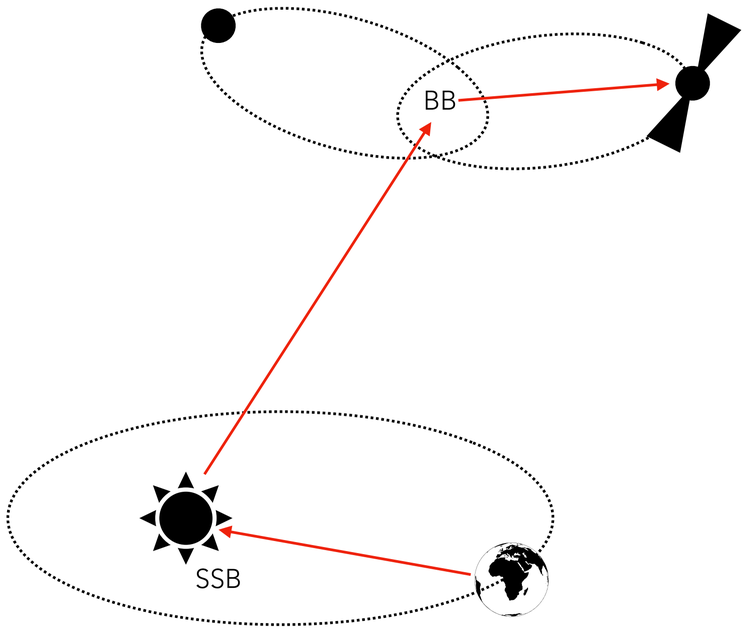
$$\Delta_{\mathrm{SB}}$$

Shapiro delay: effect of the gravitational potential of the binary companion



BUILDING A TIMING MODEL

$$t_{\mathrm{e}}^{\mathrm{psr}} = t_{\mathrm{a}}^{\mathrm{obs}} - \Delta_{\odot} - \Delta_{\mathrm{IS}} - \Delta_{\mathrm{B}}$$



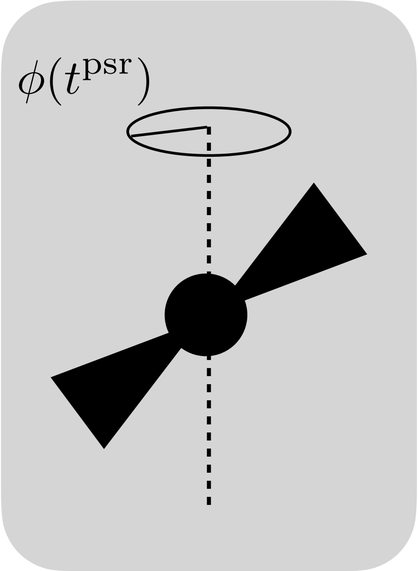
arXiv: 0603381

Correction	Typical value/range
Observatory clock to TT	1 μs
Hydrostatic tropospheric delay	10 ns
Zenith wet delay	1.5 ns
IAU precession/nutation	~5 ns
Polar motion	60 ns
ΔUT1	1μs
Einstein delay	1.6 ms
Roemer delay	500 s
Shapiro delay due to Sun	112 μs
Shapiro delay due to Venus	0.5 ns
Shapiro delay due to Jupiter	180 ns
Shapiro delay due to Saturn	58 ns
Shapiro delay due to Uranus	10 ns
Shapiro delay due to Neptune	12 ns
Second order Solar Shapiro delay	9 ns
Interplanetary medium dispersion delay	100 ns^b
Interstellar medium dispersion delay	~1 s^b

BUILDING A TIMING MODEL

$$\phi(t^{\rm psr}) = \phi_0 + \nu_0(t^{\rm psr} - t_0) + \frac{1}{2}\dot{\nu}_0(t^{\rm psr} - t_0)^2$$

unknown quantities



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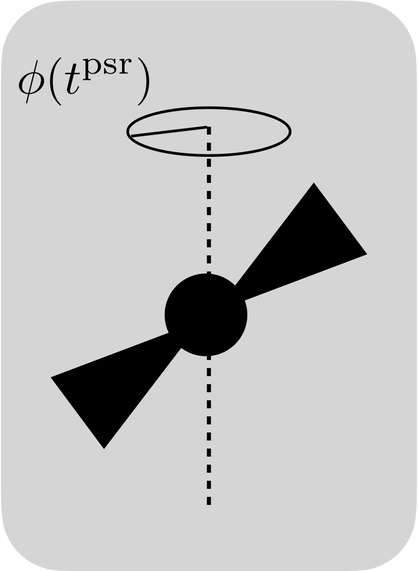

unknown timing models parameters are fitted to the TOAs

$$\chi^2 = \sum_i \left(\frac{\phi(t_{{\rm e},i}^{\rm psr}) - N_i}{\sigma_i} \right)^2$$

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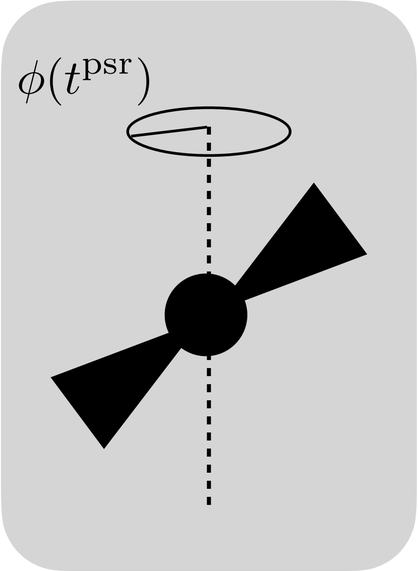

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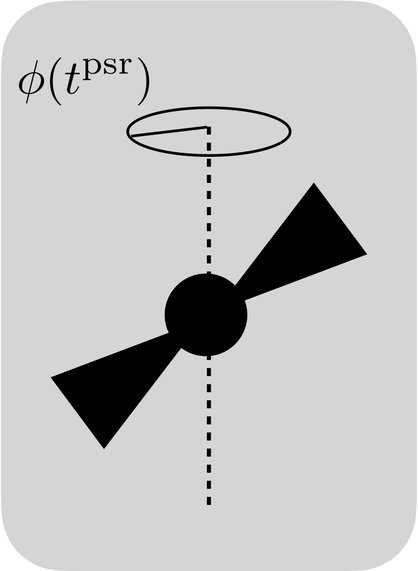
closest integer

BUILDING A TIMING MODEL



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closest integer

time of emission in the pulsar reference frame

FACE-OFF

free parameters

$$h^2\Omega_{\rm GW} \propto \frac{A^2}{H_0^2}\left(\frac{f}{\rm yr^{-1}}\right)^{5-\gamma} {\rm yr}^{-2}$$

VS

free parameters

$$h^2\Omega_{\rm GW}(f;\ \Theta)$$

FACE-OFF

$$\mathcal{B} = \frac{\mathcal{Z}_{\mathrm{NP}}}{\mathcal{Z}_{\mathrm{BHB}}}$$

$$\mathcal{Z} = \int d\Theta \; P(\mathcal{D}|\Theta, \mathcal{H}) \times P(\Theta|\mathcal{H})$$

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likelihood function

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prior distributions

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prior distributions

Afzal et al. [2306.16219]