Foreground and resolved stellar mass black hole binaries in LISA

- Stochastic gravitational wave background from stellar origin binary black holes in LISA
  S. Babak et al, arXiv:2304.06368
  Corresponding authors: M. Pieroni and J. Torrado

- Stellar-mass black-hole binaries in LISA: characteristics and complementarity with current-generation interferometers
  R. Buscicchio et al, arXiv:2410.18171
  Corresponding authors: R. Buscicchio and J. Torrado

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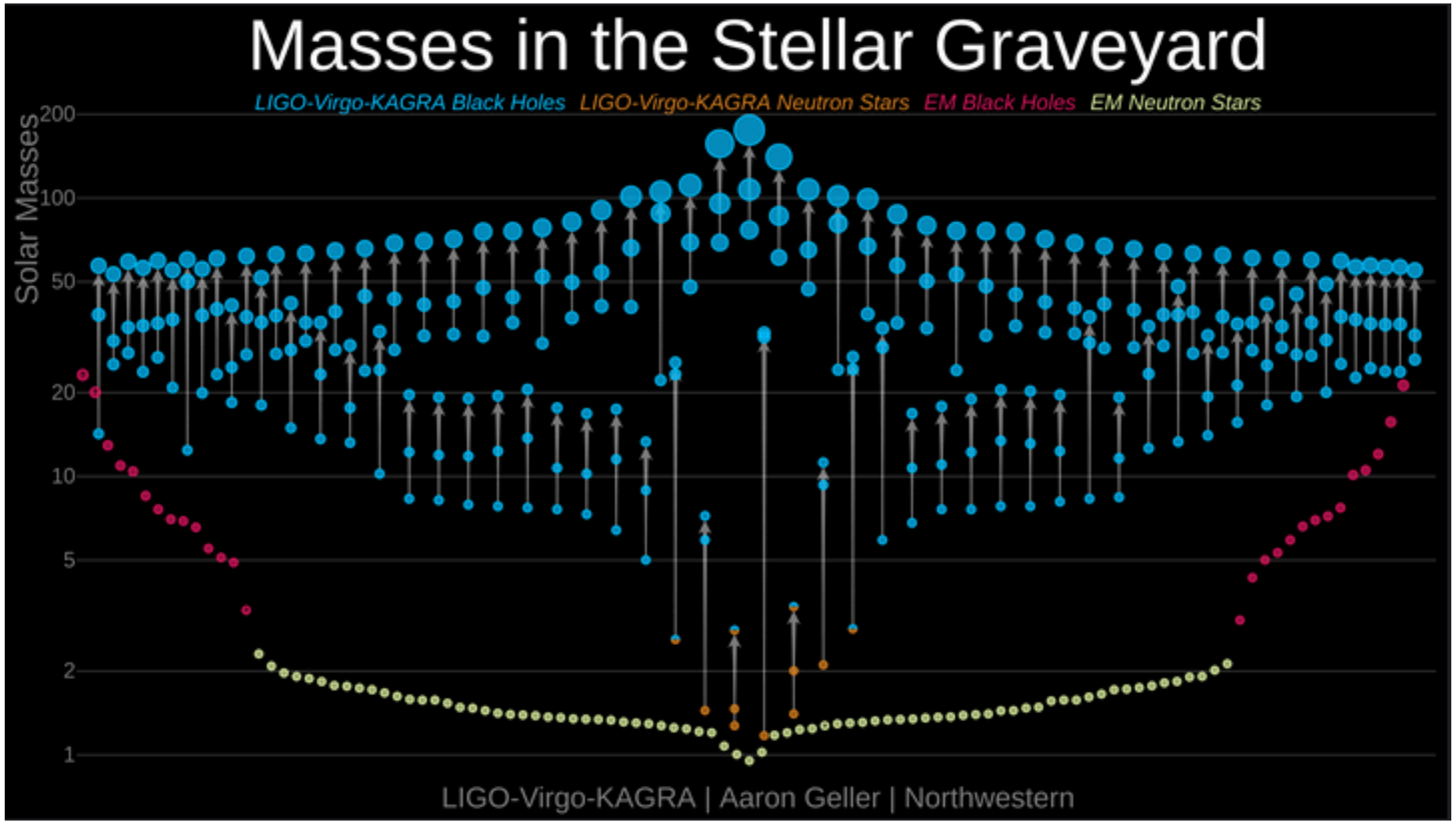
M. Pieroni, A. Ricciardone, A. Sesana, J. Torrado

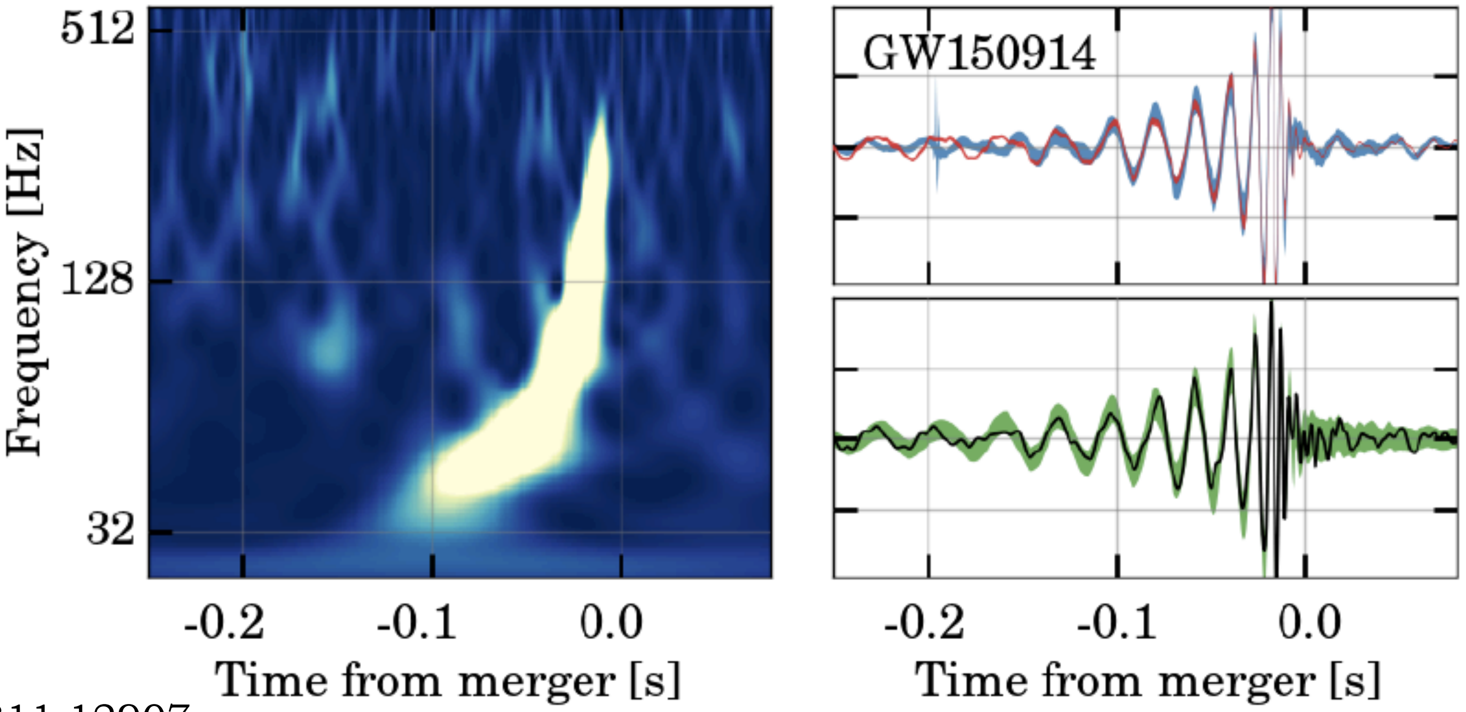
Stellar mass black hole binaries

$$5\, M_\odot < m < 140 M_\odot$$

Progenitors are most probably stars (but can be primordial)



LVK measure the last fraction of second before merger



LIGO/Virgo arXiv:1811.12907

Stellar mass black hole binaries

However these binaries can be formed even Gyrs ago

GW emission at twice the angular frequency $f = \omega_S/\pi$

$$M_c = \frac{(m_1\, m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

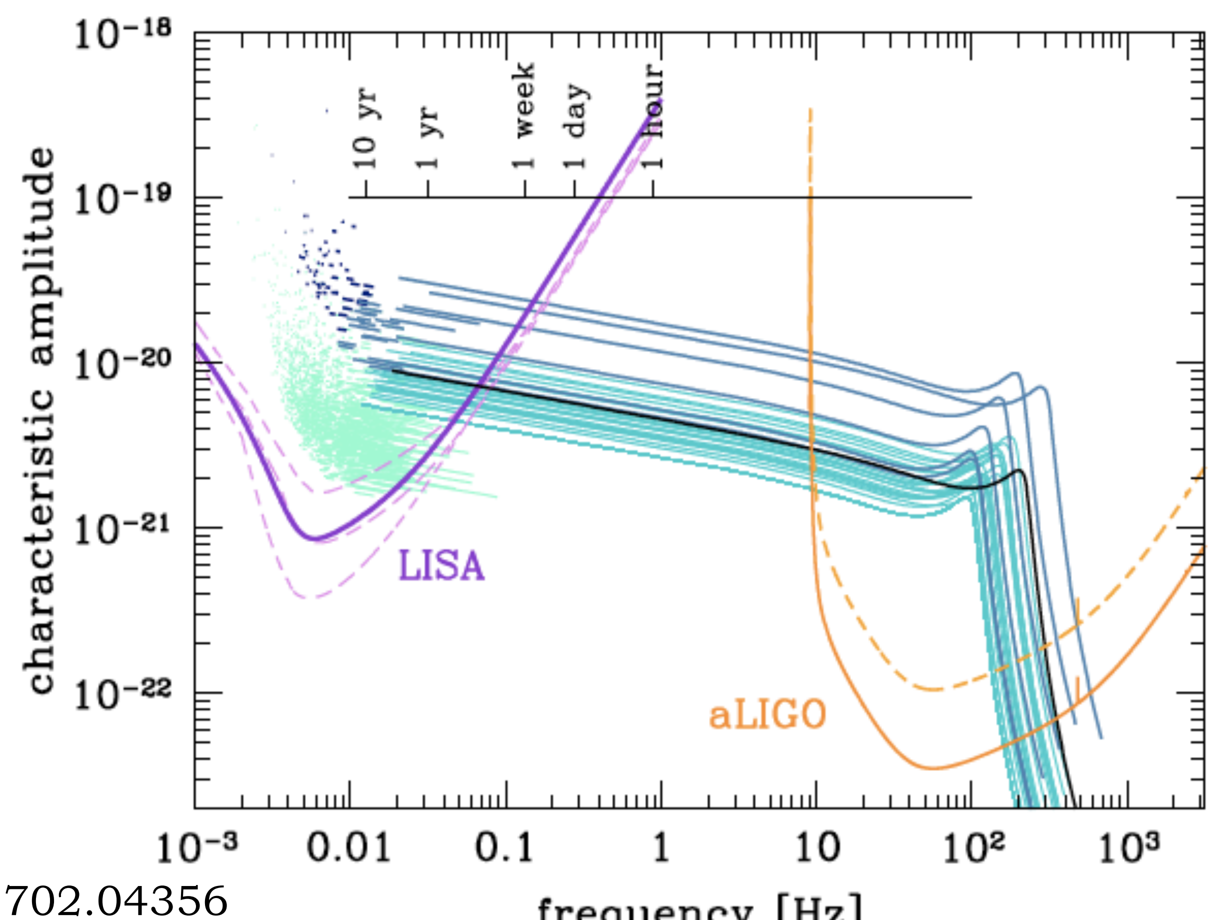
Relation with the time to coalescence for circular Newtonian binary

$$f(\tau) = \frac{1}{\pi}\left(\frac{G\, M_c}{c^3}\right)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8}$$

$M_c = 25\,\mathrm{M}_\odot \quad \tau = 0.2\,\mathrm{sec} \quad \longrightarrow \quad f = 37\,\mathrm{Hz}$ LVK band

$M_c = 25\,\mathrm{M}_\odot \quad \tau = 10\,\mathrm{year} \quad \longrightarrow \quad f = 0.01\,\mathrm{Hz}$ LISA band

- How many of these sources will LISA detect?
- How many will be detectable in the Earth-based interferometer band a reasonable amount of time after detection in LISA? (multi-band sources)
- Is there an associated a foreground?



A Sesana arXiv:1702.04356

Our analysis in a nutshell

- Take as input the population models of these binaries inferred by LVK GWTC-3
- Produce synthetic catalogues based on that model
- Simulate the GW signal from several realisations in LISA
- Adopt an SNR threshold to distinguish binaries that can be confidently detected from those that produce the confusion noise
- Iteratively subtract binaries that pass the detection threshold from the simulated LISA signal
- Infer both the foreground and the detected sources (given SNR threshold)

Our results in a nutshell

- Most of the binaries are sub threshold and lead to a foreground
- Because of this, the foreground follows the predicted $f^{2/3}$ spectral shape
- High redshift population characteristics play a role in the foreground estimation
- The uncertainty on the foreground measurement is much lower than the one from the population (for now)
- An average of 5 sources individually resolved, with one or two multi band
- These are much closer than those observed by LVK
- They will be extremely well characterised by LISA
- LISA is complementary to LVK design sensitivity for higher mass sources $m > 150\ M_{\odot}$

smBHB population model and catalogues production

Assumptions:
1. no eccentricity
2. τ_c uniformly distributed over the population
3. no redshift dependence of the population parameters

$$\frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}z \mathrm{d}\tau_c} = R(z,\tau_c)\left[\frac{\mathrm{d}V_c}{\mathrm{d}z}(z)\right] p(\xi|\theta)$$

↓

Number of binaries with given redshift, time to coalescence,
other binary parameters ξ (masses, spins…)
that are in LISA at a given time

smBHB population model and catalogues production

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$$\frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi\mathrm{d}z\mathrm{d}\tau_c} = R(z,\tau_c)\left[\frac{\mathrm{d}V_c}{\mathrm{d}z}(z)\right]p(\xi|\theta)$$

2. Binary formation and coalescence are in a *steady state*, since changes in the demographics are on a much longer timescale than LISA: the rate per unit coming volume and τ_c is such that $R(z,\tau_c) = R(z,\tau_c = 0)$

we can use LVK *merger* rate! $R(z) = R_0(1+z)^\kappa$

However! Tested by LVK only until $z \simeq 1$ while the foreground is due in majority to low-SNR binaries that are far away: need to *extend* the power law for the foreground computation (not relevant for the resolved sources)

smBHB population model and catalogues production

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$$\frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}z \mathrm{d}\tau_c} = R(z,\tau_c)\left[\frac{\mathrm{d}V_c}{\mathrm{d}z}(z)\right] p(\xi|\theta)$$

$$R(z) = R_0 C \frac{(1+z)^\kappa}{1+\frac{\kappa}{r}\left(\frac{1+z}{1+z_{\mathrm{peak}}}\right)^{\kappa+r}}$$

R_0, κ from LVK posterior

$$R_0 = 17.3^{+10.3}_{-6.7}\,\mathrm{Gpc}^{-3}\mathrm{yrs}^{-1}$$

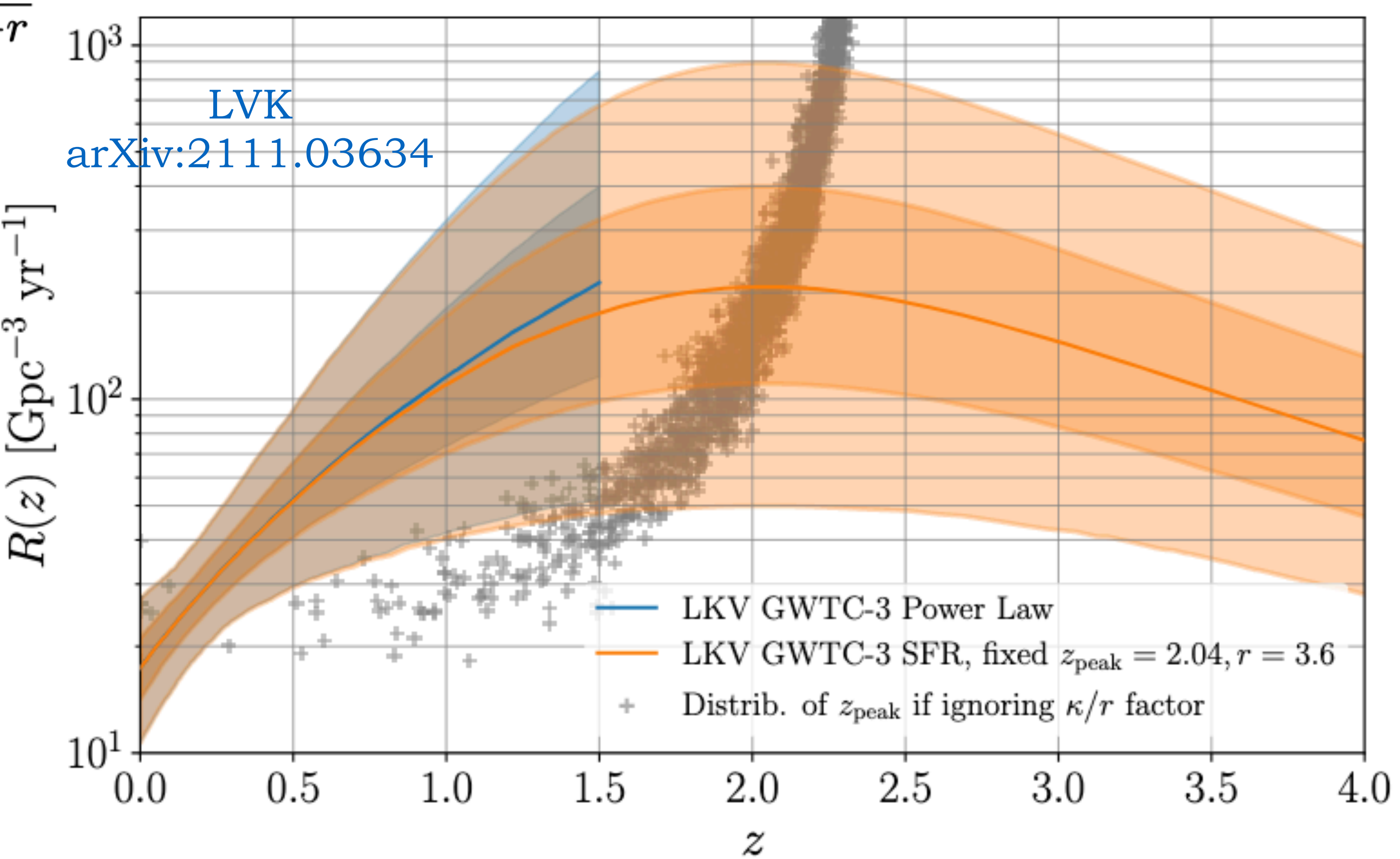
$$\kappa = 2.7^{+1.8}_{-1.9}$$

r, z_{peak} from star formation rate

$$z_{\mathrm{peak}} = 2.04 \qquad r = 3.6$$

Madau & Fragos arXiv:1606.07887

LVK arXiv:2111.03634



smBHB population model and catalogues production

Assumptions: 1. no eccentricity

2. τ_c uniformly distributed over the population

3. no redshift dependence of the population parameters

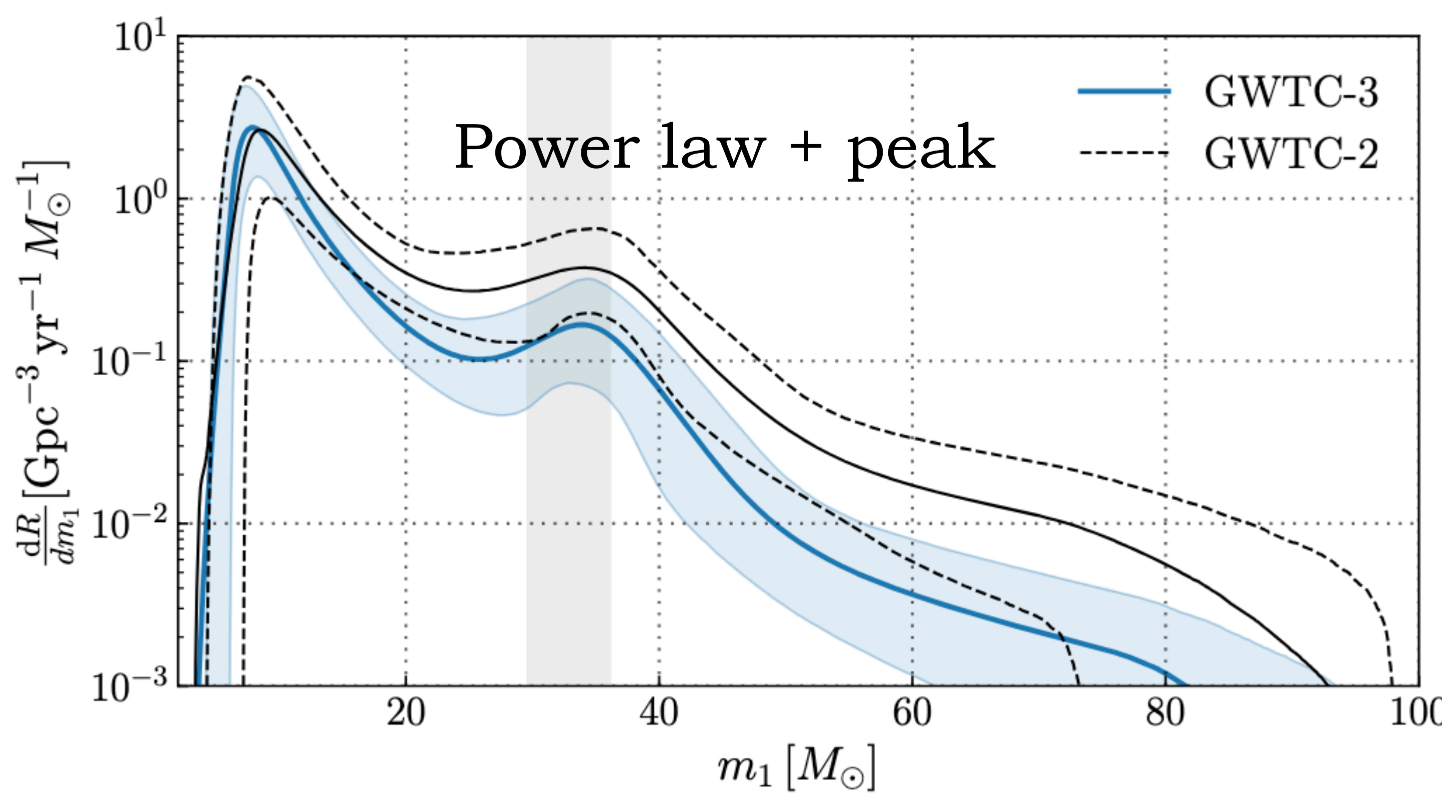
$$\frac{d^3N(z,\tau_c,\xi,\theta)}{d\xi dz d\tau_c} = R(z,\tau_c)\left[\frac{dV_c}{dz}(z)\right] p(\xi|\theta)$$

Probability density function of binary and population parameters

Masses and spins distributions as in LVK arXiv:2111.03634

Time to coalescence: $\tau_c \leq 10^4$ years
Isotropic distribution, uniform polarisation…

Parameter	Prior
Time-to-coalescence (source frame)	$U[0, \tau_{c,\max}^{(\mathrm{det})}/(1+z)]$ yrs
Ecliptic Longitude	$U[0, 2\pi]$ rad
Ecliptic Latitude	$\arcsin(U[-1,1])$ rad
Inclination	$\arccos(U[-1,1])$ rad
Polarization	$U[0, 2\pi]$ rad
Initial Phase	$U[0, 2\pi]$ rad



From which one calculates the frequency of entry in LISA band, the in-band time, the luminosity distance…

smBHB population model and catalogues production

Assumptions: 1. no eccentricity
2. τ_C uniformly distributed over the population
3. no redshift dependence of the population parameters

THE CATALOGUES:

1. Ten realisations of the same “universe” with fixed parameters, $z_{max} = 1$, $\tau_C \leq 10^4$ yrs, used to perform the “iterative subtraction” method (one with $\tau_C = 1.5\ 10^4$ yrs)

Rate of events $R(z)$	**Mass distribution**	**Spin distribution**
$R_{0.2} = 28.1\,\mathrm{Gpc}^{-3}\mathrm{yrs}^{-1}$ $\kappa = 2.7$ $z_{\rm peak} = 2.04$ $r = 3.6$	$[m_{\rm min}, m_{\rm max}] \in [2.5, 100]\,M_\odot$ $\delta_{\rm min} = 7.8\,M_\odot$ $\alpha = 3.4$ $\lambda_{\rm peak} = 0.039$ $\mu_m = 34\,M_\odot$ $\sigma_m = 5.1\,M_\odot$ $\beta_q = 1.1$	$\mathrm{E}[a] = 0.25$ $\mathrm{Var}[a] = 0.03$ $\zeta = 0.66$ $\sigma_t = 1.5$

2. 1000 realisation of the above “universe” but with $z_{max} = 5$, to perform the “Monte Carlo” method

3. one realisation per each of the posterior samples from the GWTC-3 catalogue, with initial SNR cut, used to infer resolved sources by “iterative subtraction”

Computation of the foreground from the smBHBs

First method: analytical calculation, detection not taken into account

$$\Omega_{\rm GW}(f) = \Omega_{\rm GW}(f_{\rm ref}) \left(\frac{f}{f_{\rm ref}}\right)^{2/3}$$

$$\frac{\rho_{\rm GW}^{\rm (tot)}}{\rho_c} = \int_0^\infty \frac{{\rm d}f}{f}\,\Omega_{\rm GW}(f) = \int {\rm d}\xi \int {\rm d}V_c \int {\rm d}\tau_c \,\frac{{\rm d}^3 N(z,\tau_c,\xi,\theta)}{{\rm d}\xi {\rm d}V_c {\rm d}\tau_c}\,\frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c}$$

$$\downarrow$$

GW energy emitted by a single event

$$\frac{\rho_{\rm GW}^{\rm (event)}}{\rho_c} = \frac{1}{16\pi G\rho_c}\,\frac{\langle \dot h_+^2 + \dot h_\times^2\rangle}{(1+z)^4}$$

$$\dot h_+(t_S) = \frac{4\pi^{2/3}}{a(t_S) r}(G\,M_c)^{5/3}\left(\frac{1+\cos^2\theta}{2}\right)\underbrace{\frac{{\rm d}[f^{2/3}(t_S)\cos(2\Phi(t_S))]}{{\rm d}t_S}}$$

In the limit of circular orbit with slowly varying radius

$$\dot f_S \ll f_S^2$$

$$\simeq -f^{2/3}(t_S) 2\,\underbrace{\dot\Phi(t_S)}_{\pi f_S}\sin(2\Phi(t_S))$$

$$\langle \dot h_+^2(t_S)\rangle = \frac{32}{a_S^2 r^2}(\pi G\,M_c)^{10/3}\left(\frac{1+\cos^2\theta}{2}\right) f_S^{10/3}$$

Computation of the foreground from the smBHBs

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$$\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW}}(f_{\mathrm{ref}}) \left(\frac{f}{f_{\mathrm{ref}}} \right)^{2/3}$$

$$\mathrm{d}V_c = \frac{d_M^2}{H(z)} \mathrm{d}\Omega \, \mathrm{d}z$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \int_0^\infty \frac{\mathrm{d}f}{f} \, \Omega_{\mathrm{GW}}(f) = \int \mathrm{d}\xi \int \mathrm{d}V_c \int \mathrm{d}\tau_c \, \frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}V_c \mathrm{d}\tau_c} \, \frac{\rho_{\mathrm{GW}}^{(\mathrm{event})}}{\rho_c}$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \int \mathrm{d}\xi \int \mathrm{d}\tau_c \int \mathrm{d}z \, \frac{d_M^2}{H(z)} \, \frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}V_c \mathrm{d}\tau_c} \, \frac{1}{16\pi G \rho_c (1+z)^4}$$

$$\frac{32}{a_S^2 r^2} (\pi G M_c f_S)^{10/3} \int \mathrm{d}\Omega \left[\left(\frac{1+\cos^2\theta}{2} \right)^2 + \cos^2\theta \right] = 16\pi/5$$

$$d_M = a_0 \, r$$

Extra factor $(1+z)^2$

$$\frac{\mathrm{d}f_S}{\mathrm{d}\tau_c} = \frac{96\pi^{8/3}}{5} (G\,M_c)^{5/3} f_S^{11/3}$$

$$\frac{\rho_{\mathrm{GW}}^{(\mathrm{tot})}}{\rho_c} = \frac{\pi^{2/3}}{3\,G\rho_c} \int \frac{\mathrm{d}f}{f} f^{2/3} \int \mathrm{d}\xi \int \frac{\mathrm{d}z}{H(z)(1+z)^{4/3}} (GM_c)^{10/3} \frac{\mathrm{d}^3 N(z,\tau_c,\xi,\theta)}{\mathrm{d}\xi \mathrm{d}V_c \mathrm{d}\tau_c}$$

Signal amplitude determined by the population characteristics and the cosmology

Computation of the foreground from the smBHBs

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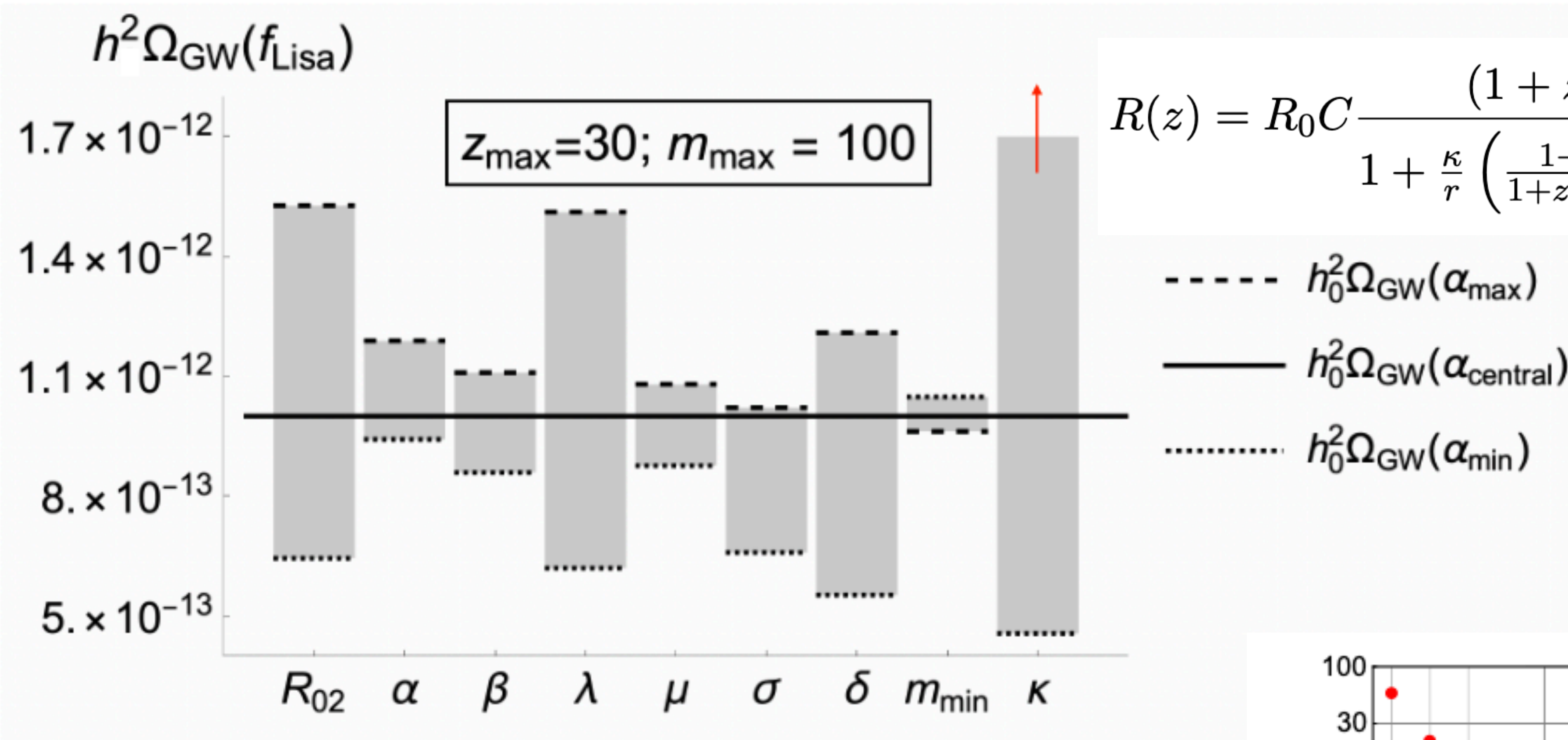
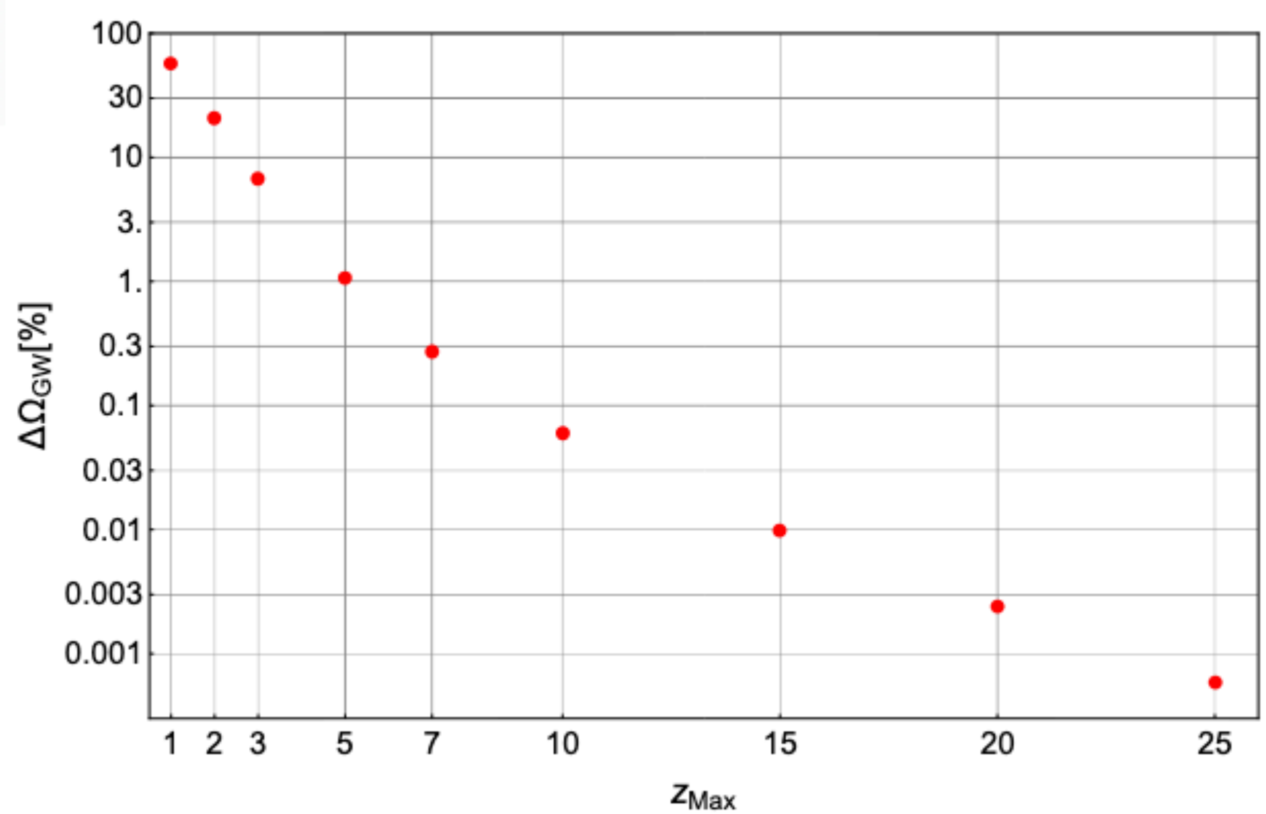


$$R(z) = R_0 C \frac{(1+z)^{\kappa}}{1 + \frac{\kappa}{r}\left(\frac{1+z}{1+z_{\mathrm{peak}}}\right)^{\kappa+r}}$$

Of all the population model parameters, κ influences the most the foreground amplitude



As LISA errors are 1-5%, it is OK to integrate (or produce catalogues) up to z_{max} = 5 (z_{min}=10^{-5})

Computation of the foreground from the smBHBs

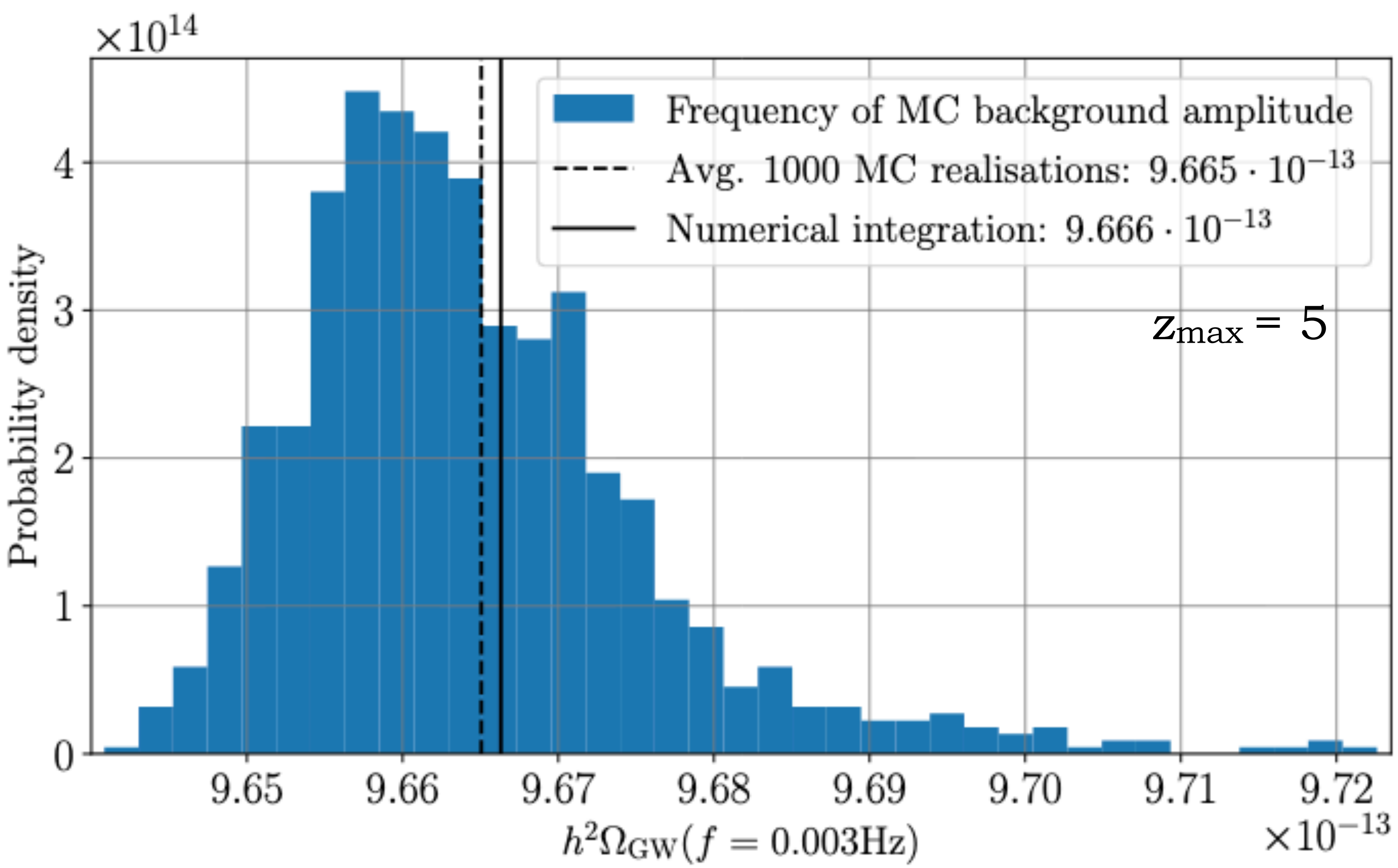
Second method: Monte Carlo sum with two similar approaches, detection not taken into account

$$\Omega_{\rm GW}(f) \approx \frac{2\pi^{2/3}}{9}\frac{G^{5/3}}{c^3 H_0^2}\frac{1}{\tau_{c,\max}^{(\rm det)}}\left(\sum_{i\in{\rm pop}}\frac{\mathcal{M}_i^{5/3}}{d_{M,i}^2(1+z_i)^{1/3}}\right) f^{2/3}$$

OR

$$\approx \frac{64\pi^{10/3}}{15}\frac{G^{10/3}}{c^8 H_0^2}\frac{1}{\delta_f}\sum_{i\in N_j}\frac{(1+z_i)^{4/3}}{d_{M,i}^2}\mathcal{M}_i^{10/3} f_i^{13/3}$$

Verify $f^{2/3}$ prediction



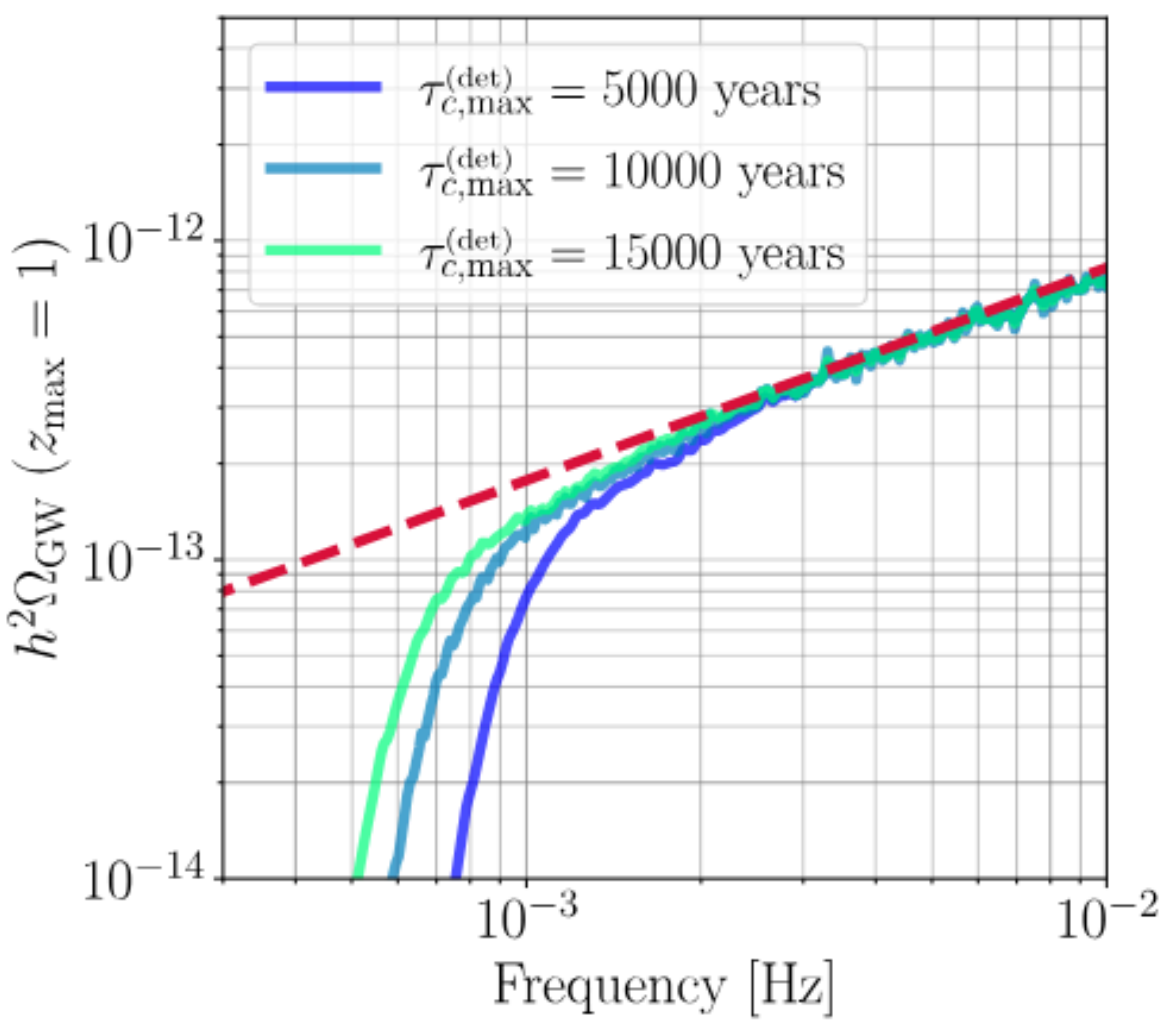
One doesn't need to worry about the variance from realisation (1000 draws of the same universe here): only 0.2%

Computation of the foreground from the smBHBs

Third method, the best: iterative approach, LISA detection taken into account
IMPORTANT: detector sensitivity can impact foreground shape and amplitude

- To each binary in the catalogues, associate waveform: spinning, non-precessing binary
- Summing them leads to simulated LISA data stream (assume 4 years data taking)
- Construct total PSD (noise+binaries) and evaluate SNR w.r.t. that PSD
- Subtract binaries with SNR > threshold and reevaluate PSD
- Continue until either all sources are subtracted or the PSD doesn't vary any longer
- Get both noise+foreground PSD (stable w.r.t. realisations) and resolvable sources according to chosen SNR threshold (depend on realisation)

Our catalogues aren't complete (τ_c too small) because of computational limitations, but still capture the relevant information



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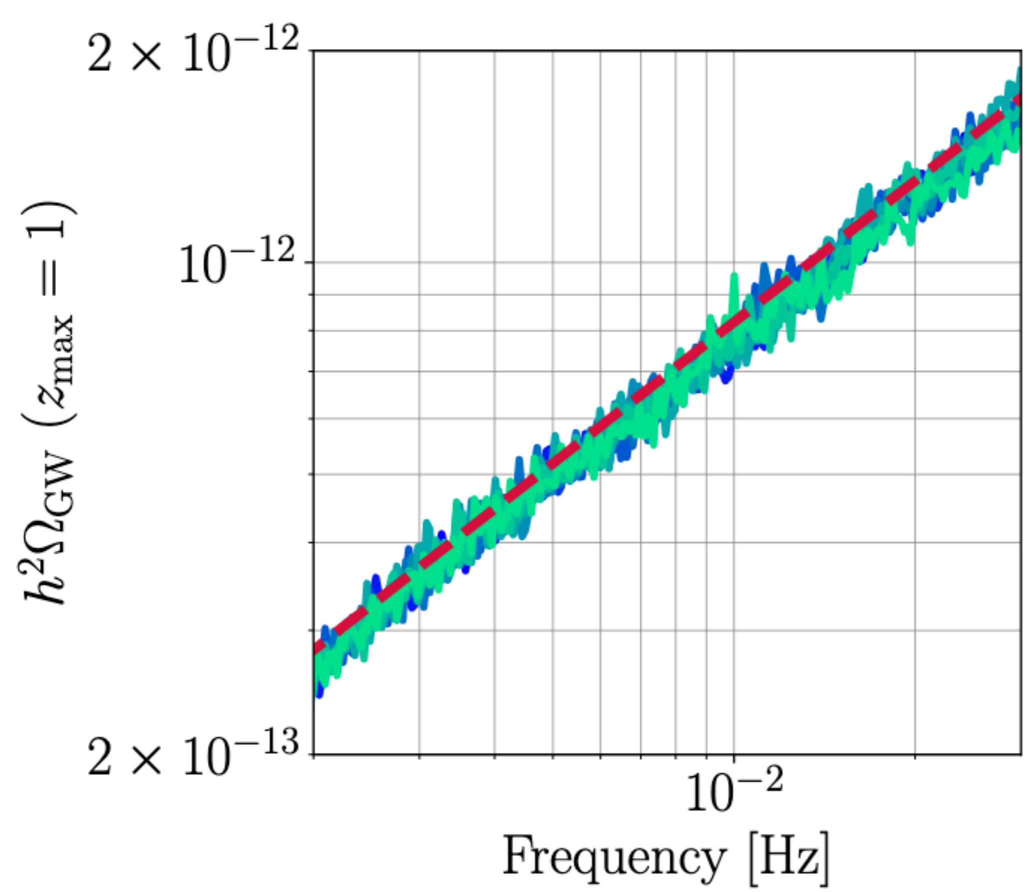
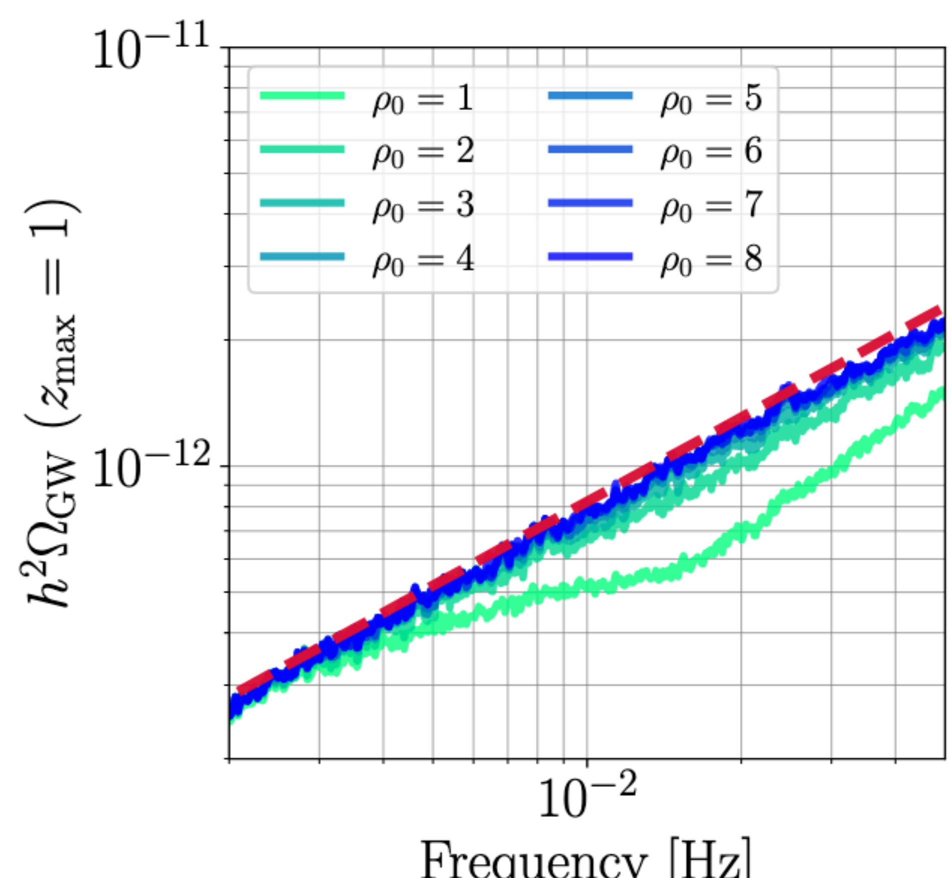
10 realisations, SNR>8, compatible with analytical prediction



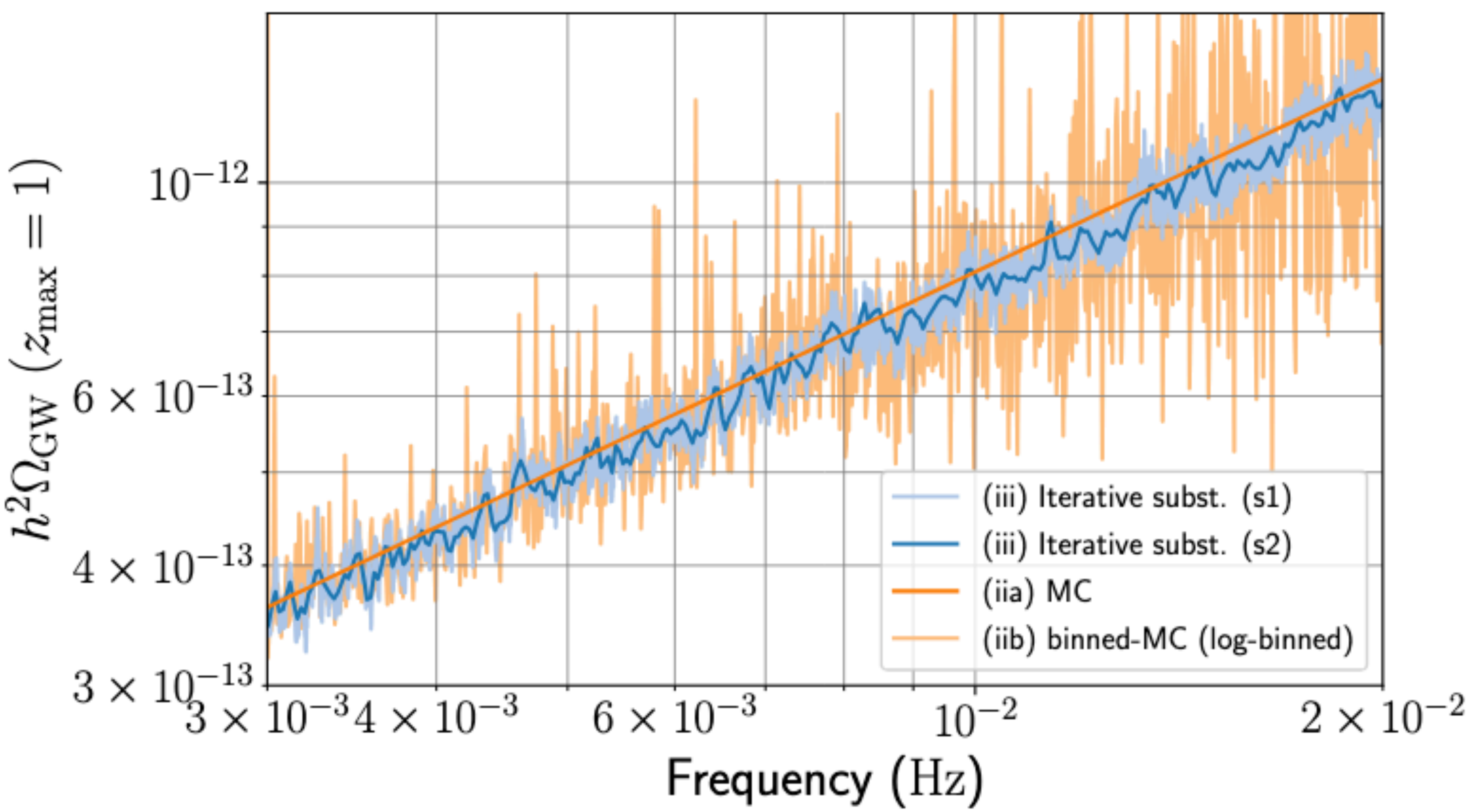
one realisation, variation of foreground shape with SNR threshold



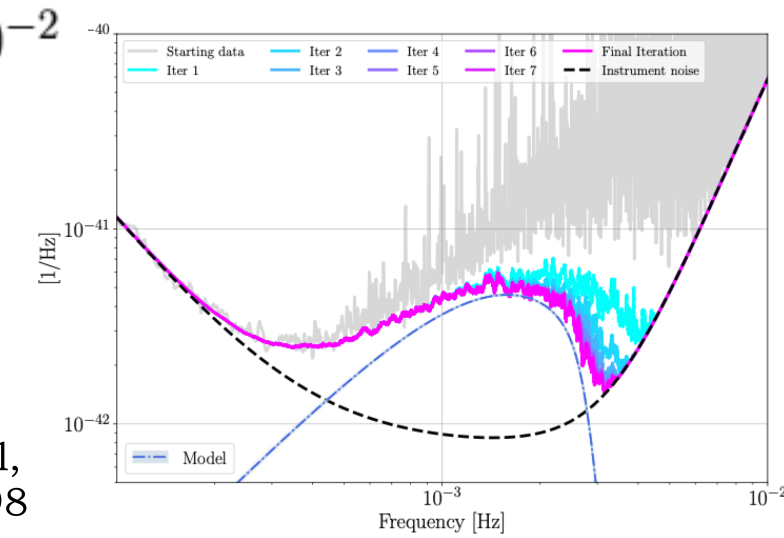
Computation of the foreground from the smBHBs

Comparison of the methods: all coherent

the instrument sensitivity is such that no deviation is observed form the analytical prediction of $f^{2/3}$ at $SNR_{thr} > 8$



Outcome could have been different, same method applied to Milky Way:

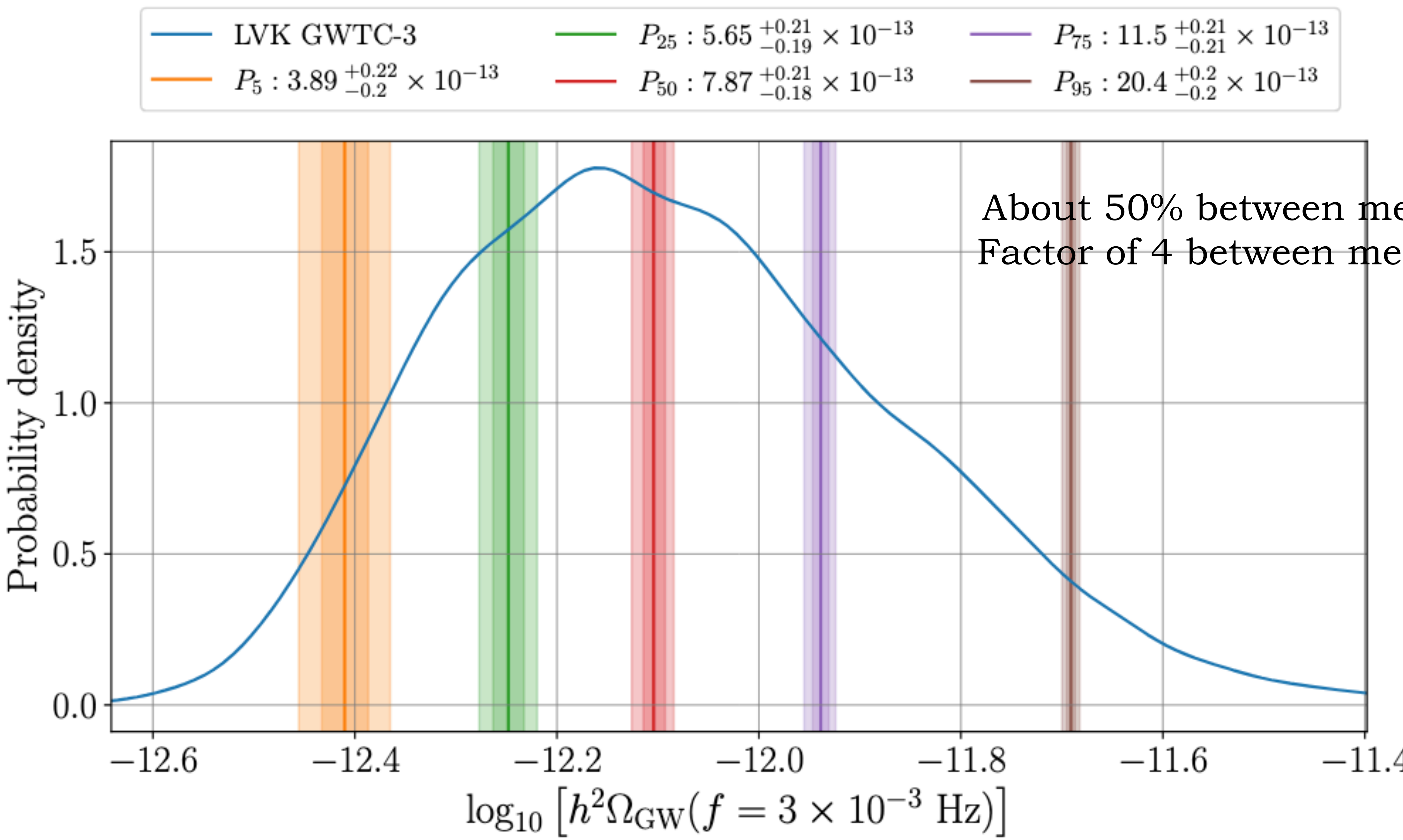


N. Karnesis et al, arXiv:2103.14598

Dependence of the foreground on population uncertainty

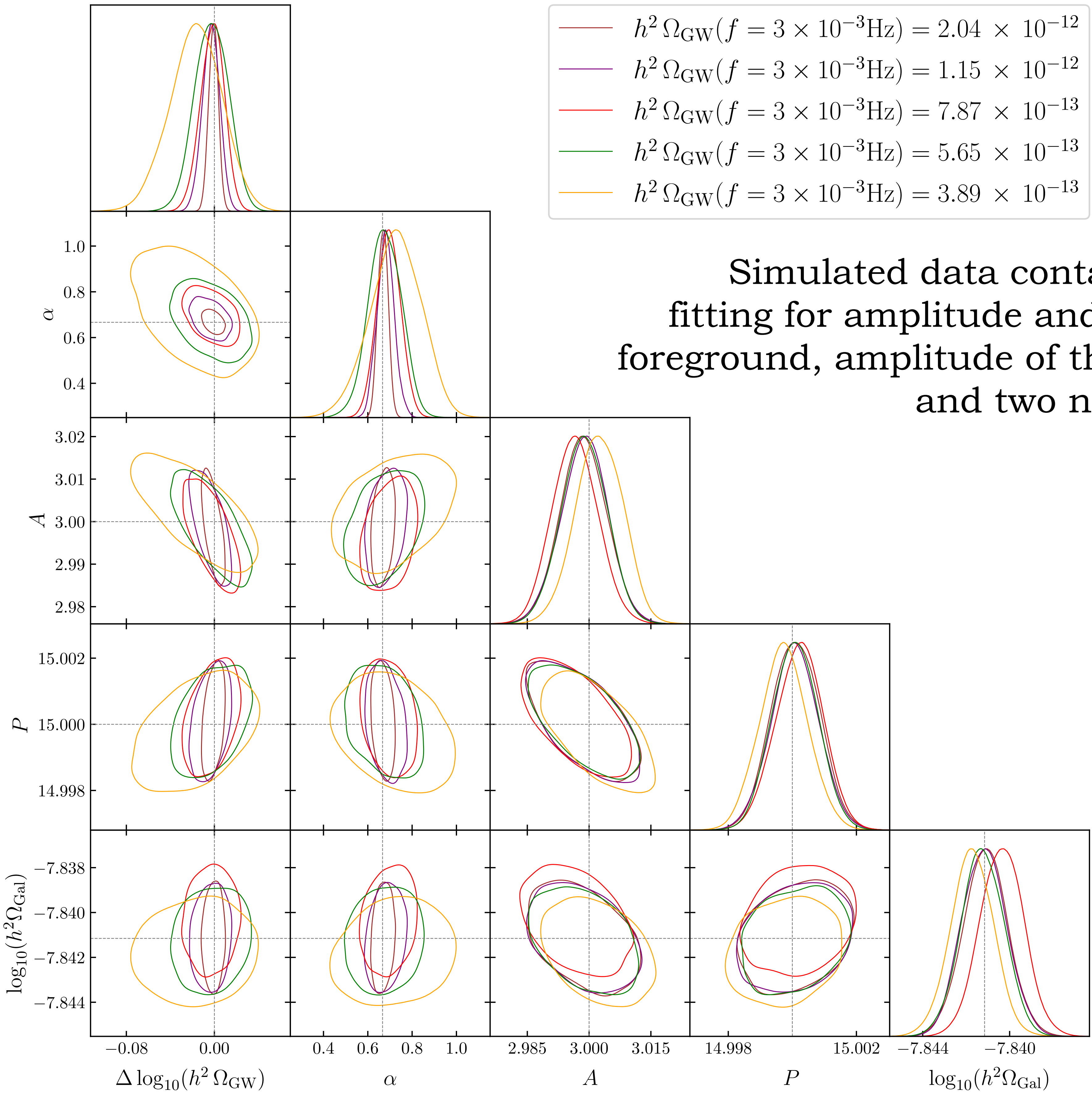
Calculation done using the analytical method

$$\frac{\rho_{\rm GW}^{\rm (tot)}}{\rho_c} = \frac{\pi^{2/3}}{3\,G\rho_c}\int\frac{{\rm d}f}{f}\,f^{2/3}\int{\rm d}\xi\int\frac{{\rm d}z}{H(z)(1+z)^{4/3}}(GM_c)^{10/3}\frac{{\rm d}^3N(z,\tau_c,\xi,\theta)}{{\rm d}\xi{\rm d}V_c{\rm d}\tau_c}$$



About 50% between median and P_5
Factor of 4 between median and P_{95}

Detection of the foreground



Simulated data containing galactic foreground, fitting for amplitude and spectral index of the smBHB foreground, amplitude of the galactic foreground (no shape) and two noise parameters

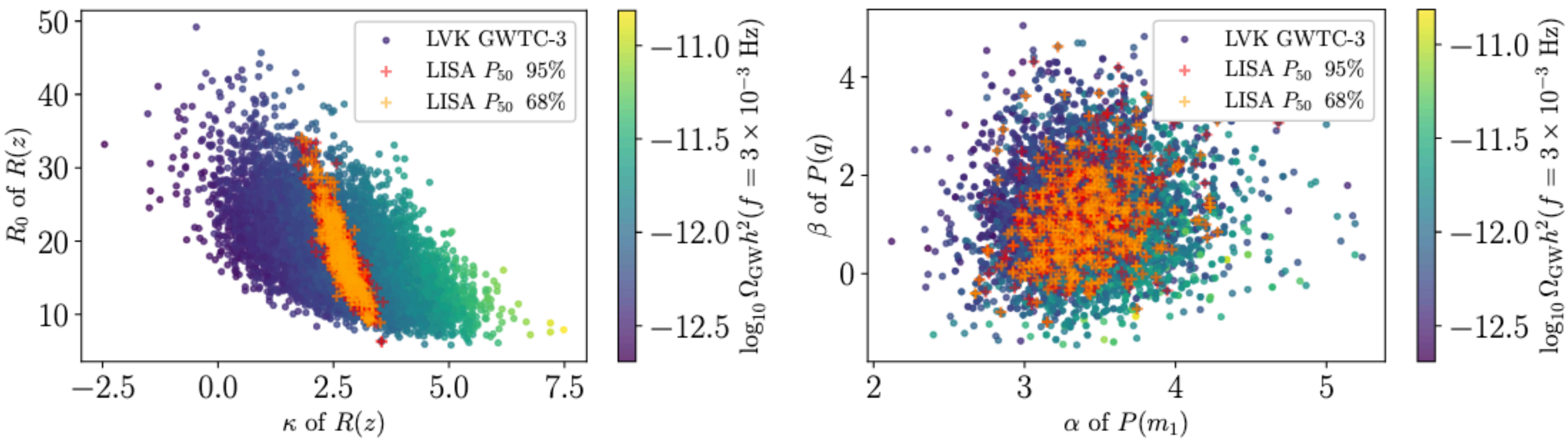
Relative uncertainty ranging from 5% (P_5) to 1% (P_{95})

Smaller than (present) uncertainty from population: LISA detection of the foreground can be informative about the population?

Detection of the foreground

Illustration of potential constraining power of a LISA detection on the population with respect to GWCT-3 population model:

Scatter plot of the GWTC-3 population parameter posterior sample and those compatible with 1 and 2 sigma LISA detection of P_{50}



By the time LISA flies, the population will be known much better.
However, LISA brings in information on high-redshift (3G will do as well?) and inspiralling phase

Effect of time delay

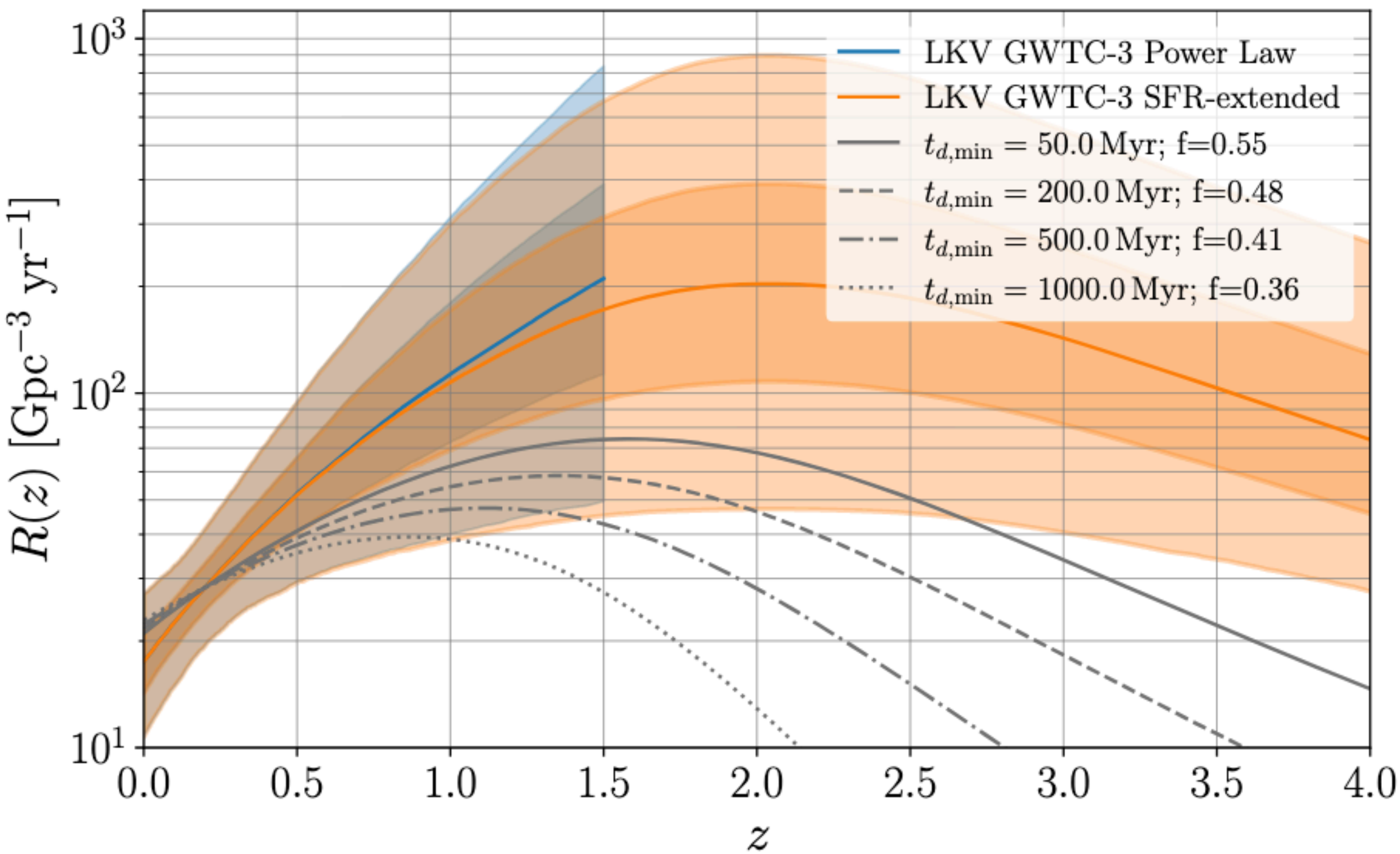
the merger rate is untested at higher redshift:
how much our assumption influences the foreground level?

Introduce a time delay between the formation of the star binary and its evolution into black hole binary

$$R(z) = \int_{t_{d,\min}}^{t_{d,\max}} R_{\rm SFR}\,(t(z) + t_d)\,p(t_d)\,\mathrm{d}t_d$$

$$p(t_d) \propto 1/t_d$$

f = foreground accounting for time delay / fiducial case



The effect remains within the GWTC-3 uncertainty: same effect of reducing P_{50} down to P_5

Foreground reduced to 36% for t_d = 1 Gyr but in tension with LVK

Results, caveats and things to improve

- The starting hypotheses can have an effect:
  - the assumption on the extension of the merger rate at high redshift *
  - the assumption that population parameters do not depend on redshift (but uniform τ_c should be good enough)
  - the assumption of circular binaries

- The foreground is robust against the universe's realisation but varies by a factor of 5 because of population uncertainty

- $SNR_{thr} < 8$ can be relevant in the case of archival searches, so it is important to keep in mind that the spectral shape can have slight variations

- The data are very noisy, and there will be residuals from other sources, so the 1-5% error on the amplitude is probably quite optimistic. But true searches can fold in the spectral index information

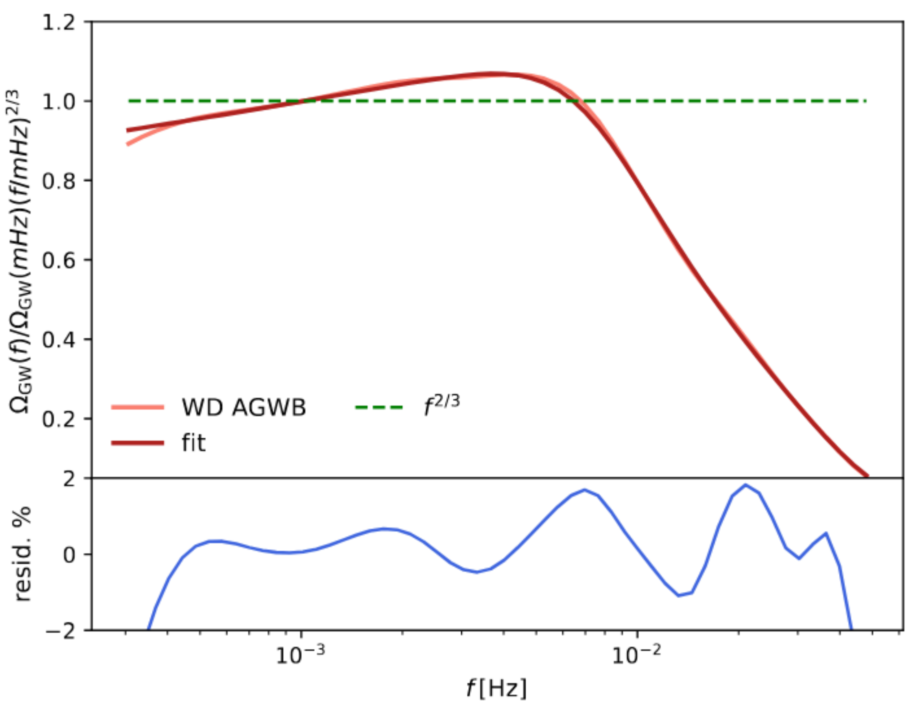
- The LISA measurement could still be promising to bring extra information on the population, possibly even in the case of 3G detectors being already operational… but unfortunately we probably don't care!

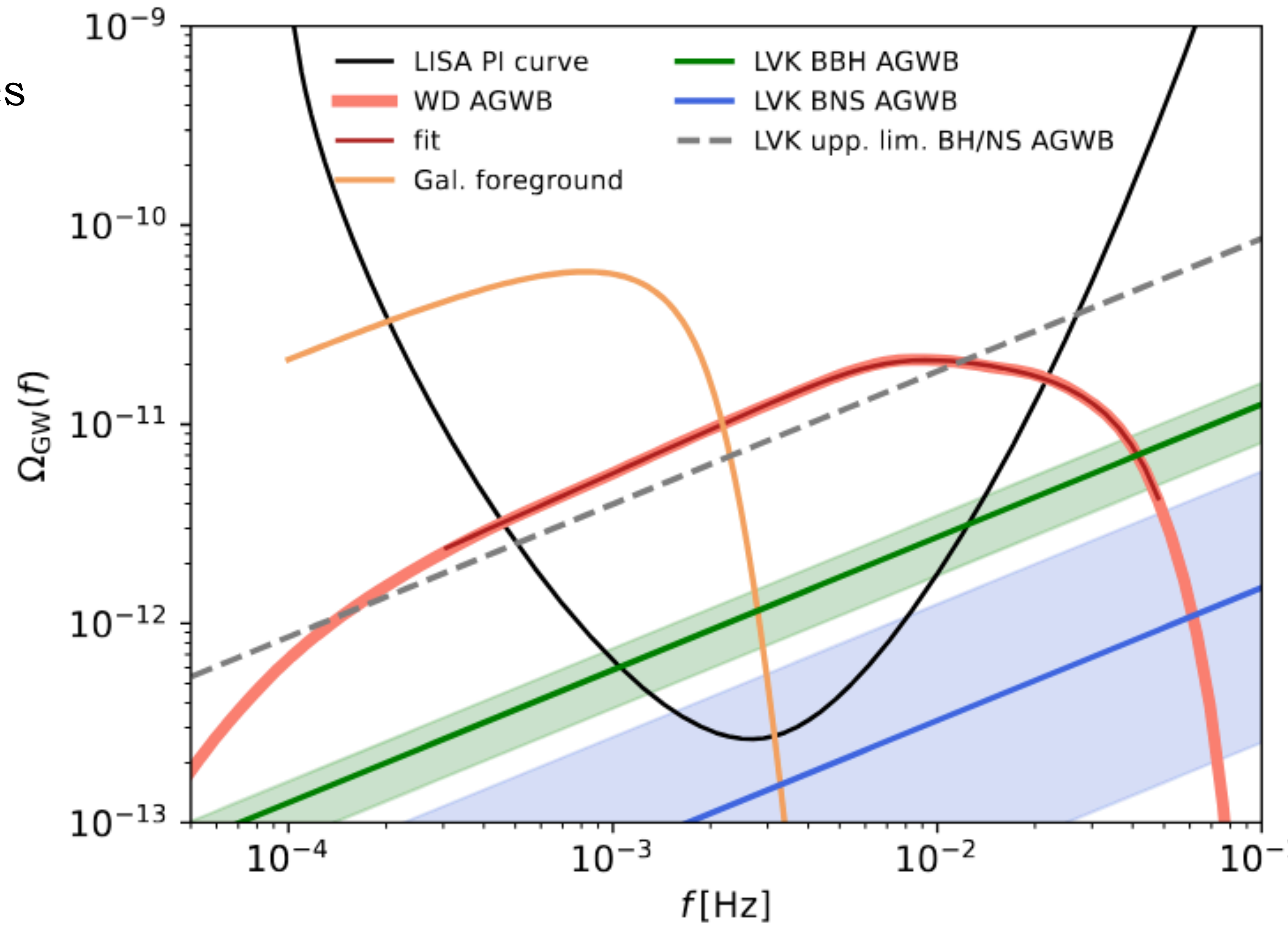
- Higher foreground from extra-galactic white dwarf binaries!

Results, caveats and things to improve

Staelens & Nelemans,
arXiv:2310.19448
60% higher than previous estimates
Factor of 10 more than P_{95}

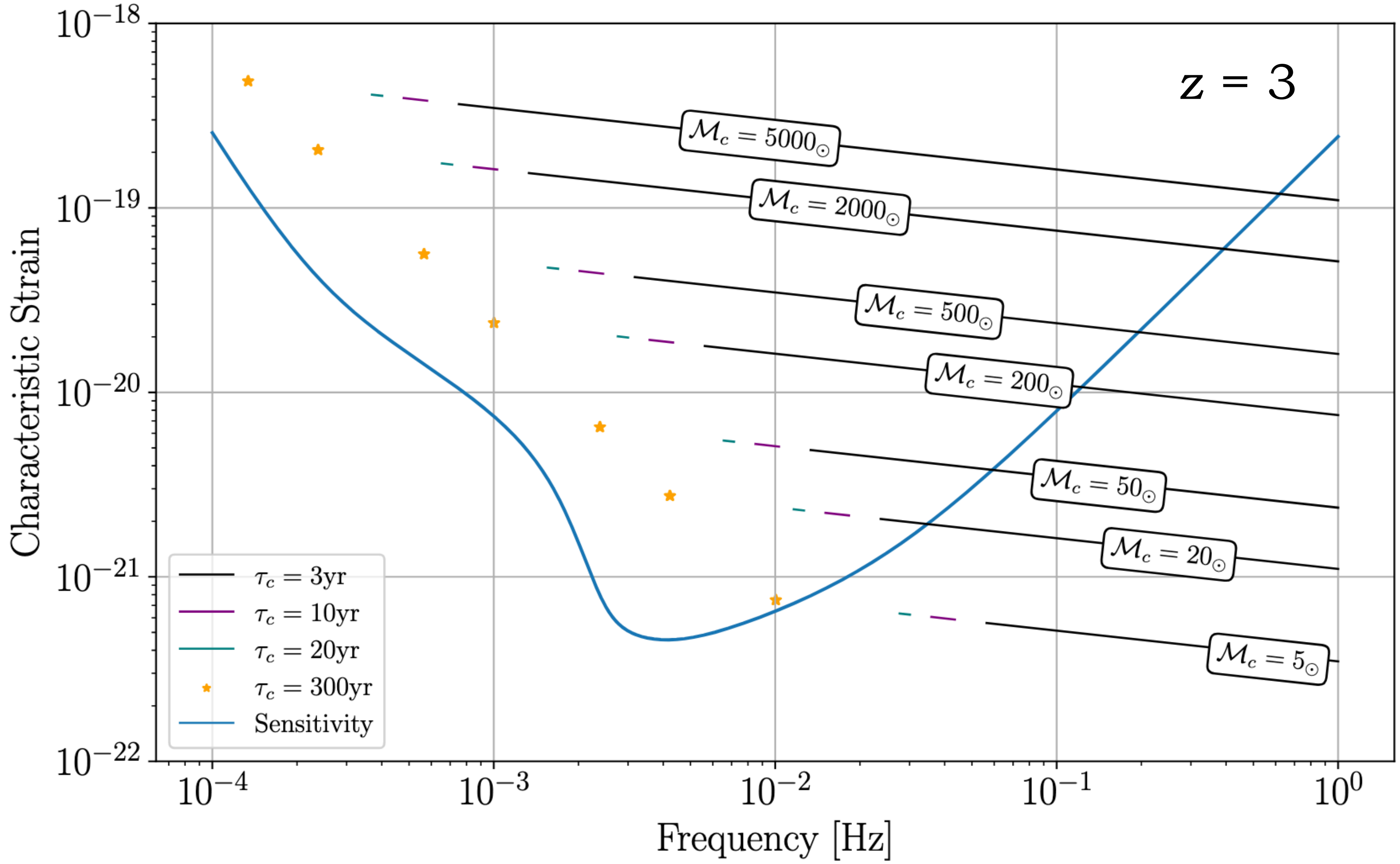
$$\Omega_{\rm WD}(3\,\mathrm{mHz}) \simeq 1.3 \cdot 10^{-11}$$





- Higher foreground from extra-galactic white dwarf binaries!

Resolved smBHBs in LISA: general behaviour

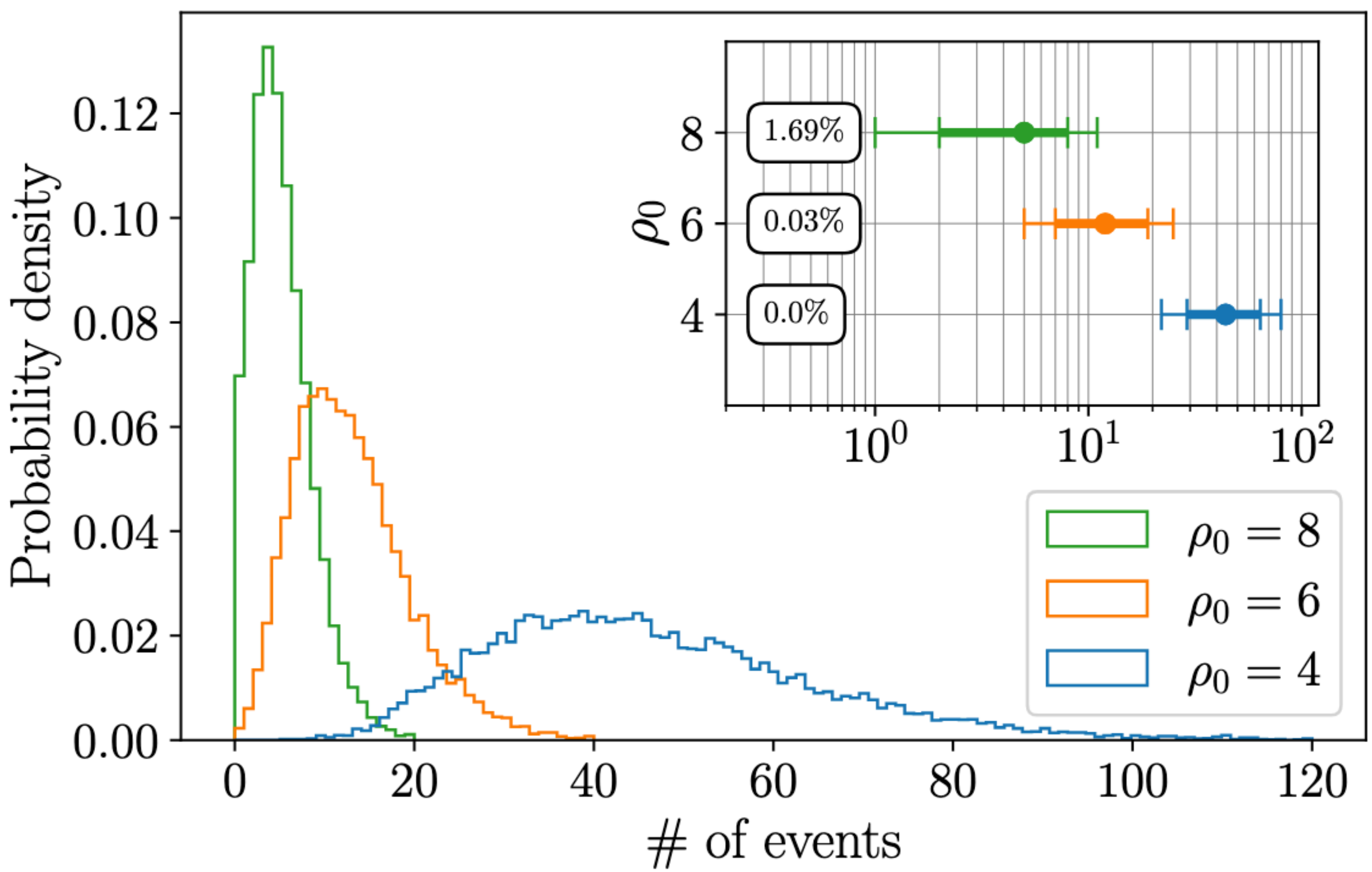


$$f(\tau) = \frac{1}{\pi}\left(\frac{G\,M_c}{c^3}\right)^{-5/8}\left(\frac{5}{256\,\tau}\right)^{3/8} \qquad \dot{f} = \frac{96}{5}\pi^{8/3}\left(\frac{G\,\mathcal{M}_c}{c^3}\right)^{5/3} f^{11/3}$$

- Sources with larger chips mass have higher SNR and appear in the LISA band at smaller frequency
- Sources with large τ_c appear in the LISA band at smaller frequency and drift less (quasi monochromatic)

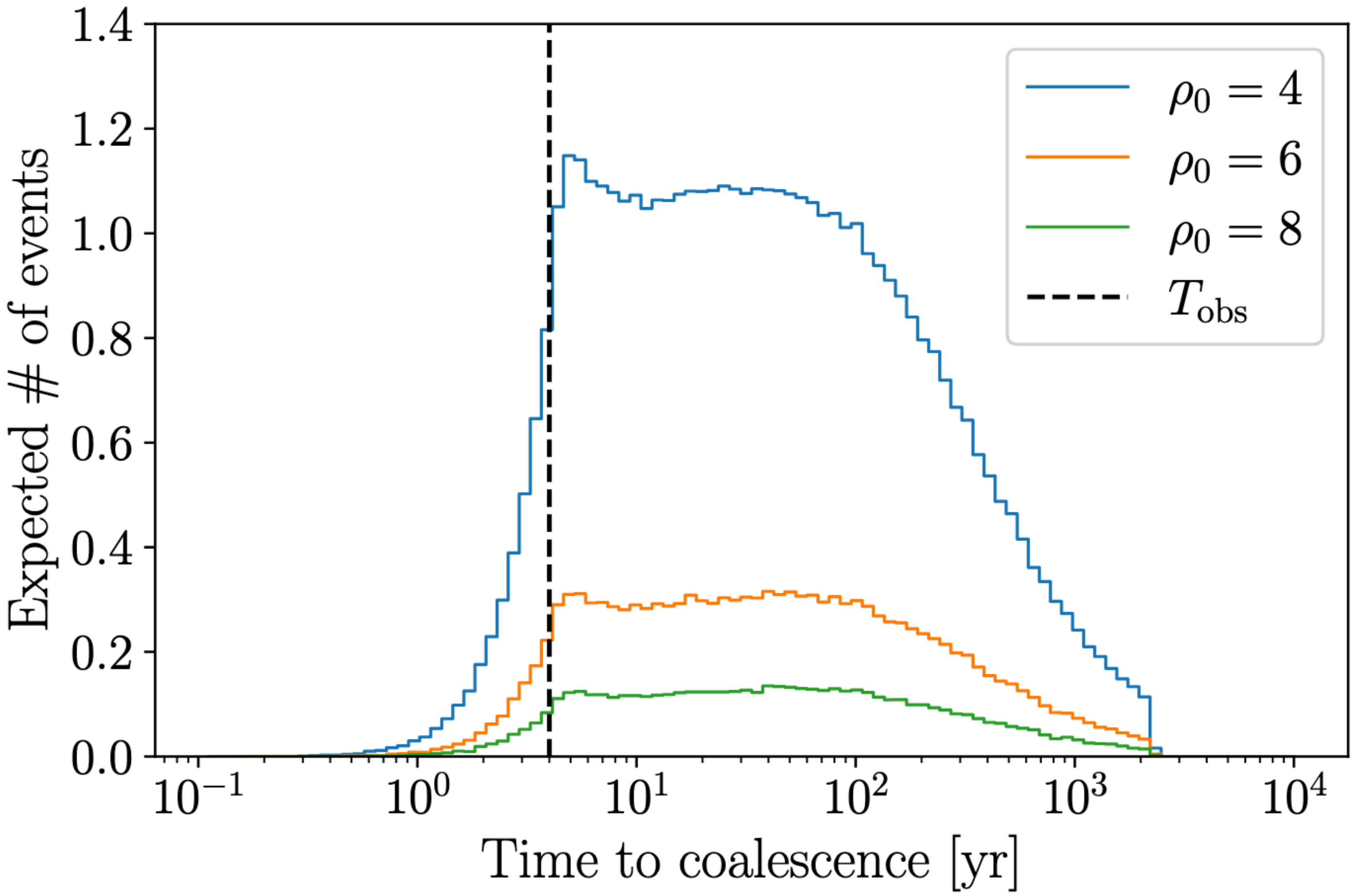
Resolved smBHBs in LISA: expected number in 4 yrs

- SNR_{thr} = 8: median value 5 sources, 2% probability of no sources being detected, 10% probability of more than 10 sources being detected
- SNR_{thr} = 6: median value 10 sources, practically zero probability of no sources being detected
- SNR_{thr} = 4: median value 50 sources, zero probability of no sources being detected



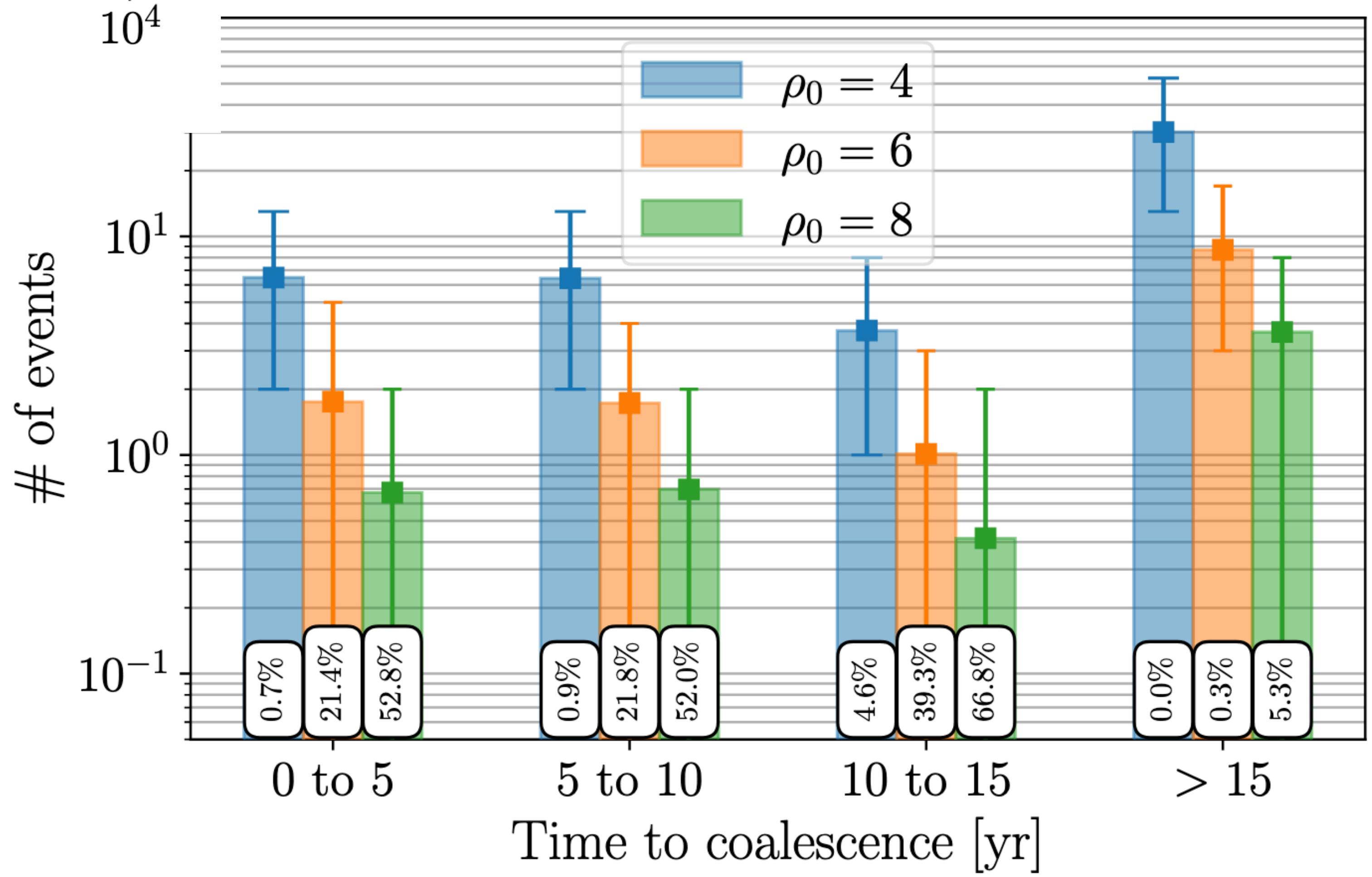
The quoted number of sources are marginalised over the population parameters

Resolved smBHBs in LISA as a function of time to coalescence

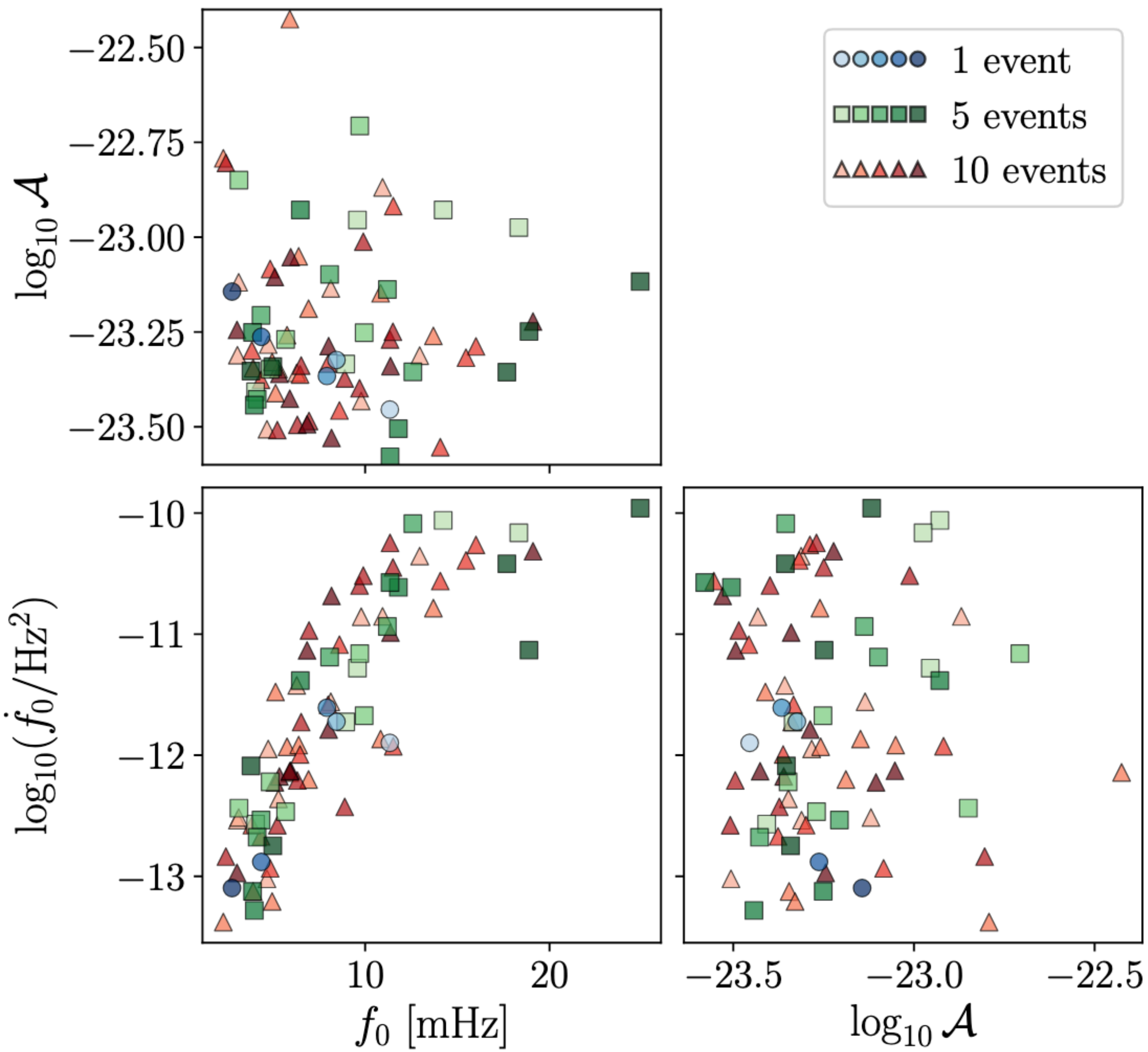


- Sources with small τ_c drift and have higher amplitude -> overall higher SNR
- But if they stay in band for too short, their SNR diminishes
- The equilibrium point is reached at $\tau_c \simeq 4$ yrs (LISA duration)
- The vast majority of sources have 10 yrs $\lesssim \tau_c \lesssim$ 100 yrs

- Multi-band sources must have $\tau_c \leq$ 15 yrs: they are about 1/3 of the total
- SNR_{thr} = 8: 2^{+3}_{-2} sources but 79% of the catalogues contain at least one multi-band source
- SNR_{thr} = 6: 4^{+6}_{-3} sources
- SNR_{thr} = 4: 16^{+14}_{-11} sources

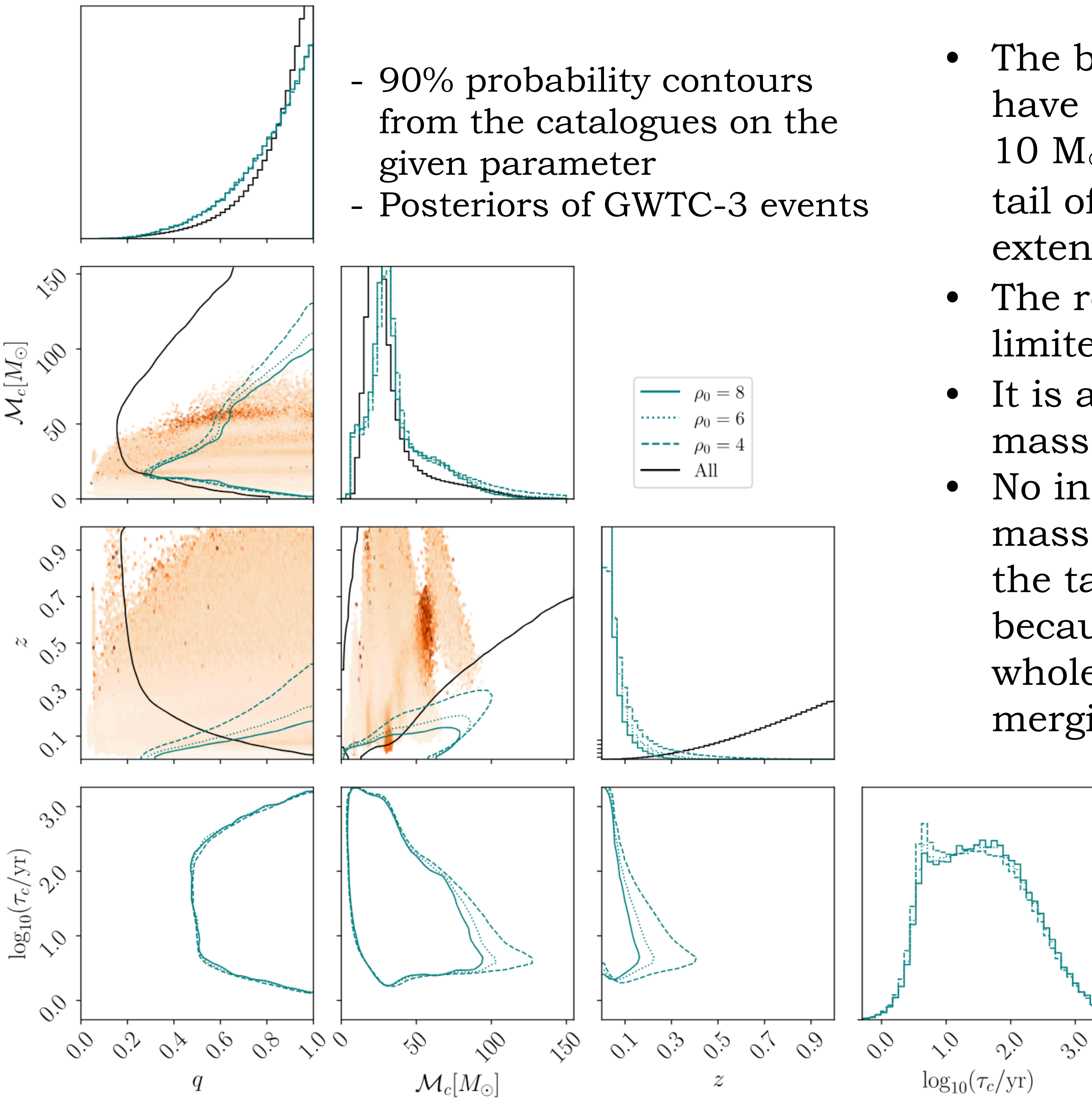


Resolved smBHBs in LISA: phenomenological parameters



- As the initial frequency increases the frequency drift increases
- Catalogues with less event tend to have smaller initial frequency (reflects the population)
- Sources with small initial frequency tend to have smaller amplitude (they stay in band longer and accumulate more SNR)
- Large variability across catalogues, representative of the population

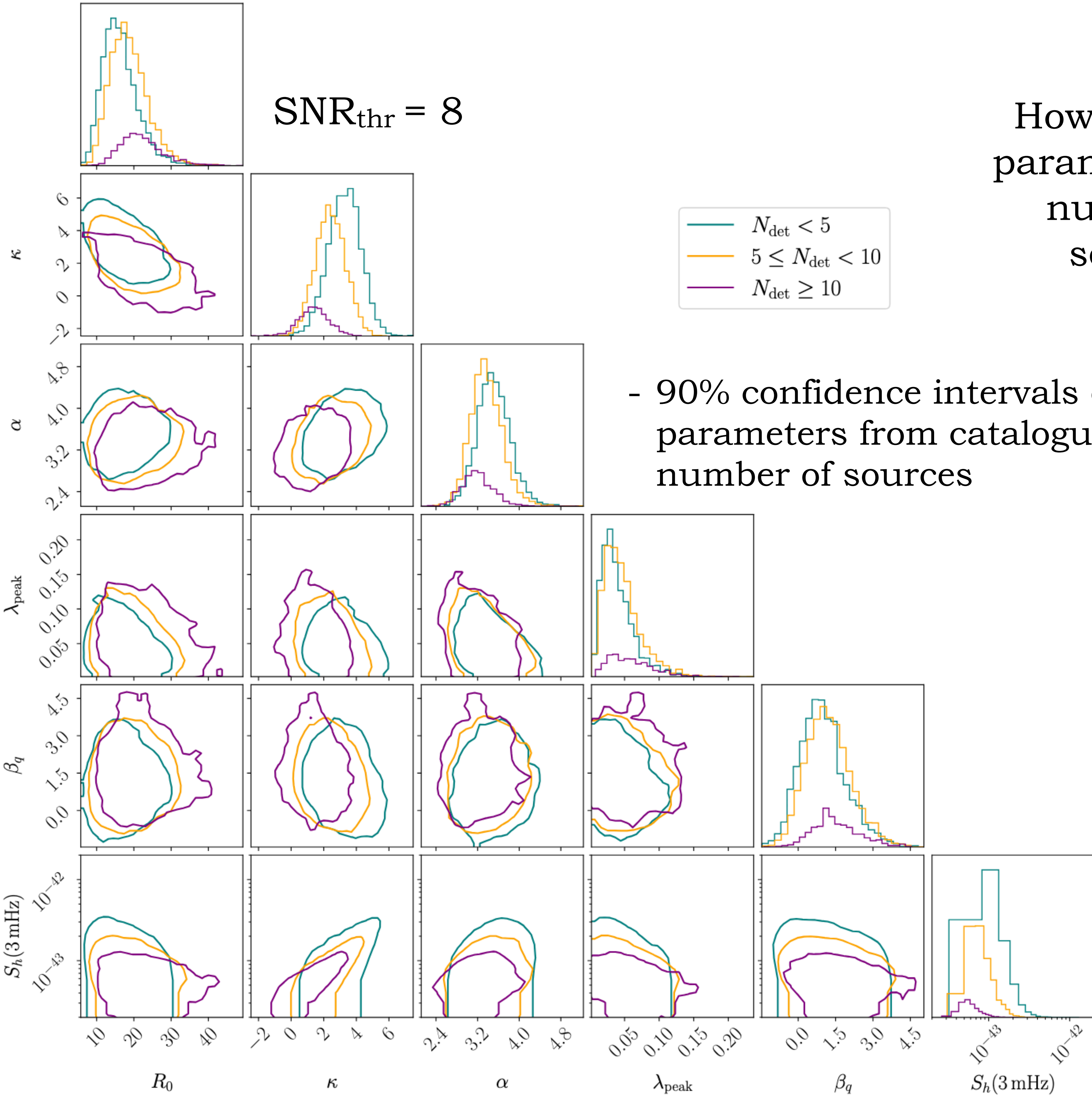
Resolved smBHBs in LISA: binary parameters



- 90% probability contours from the catalogues on the given parameter
- Posteriors of GWTC-3 events

- The bulk of the LISA sources have chirp mass $10\ M_\odot \lesssim M_c \lesssim 50\ M_\odot$ but the tail of the mass distribution extends to up to $M_c \sim 100\ M_\odot$
- The reach of LISA is strongly limited in redshift, $z \lesssim 0.1$
- It is also more limited in mass ratio compared to LVK
- No inconsistency at high mass - small z : LISA can see the tail of the distribution because it is sensitive to the whole population, LVK only to merging binaires

Resolved smBHBs in LISA: population parameters

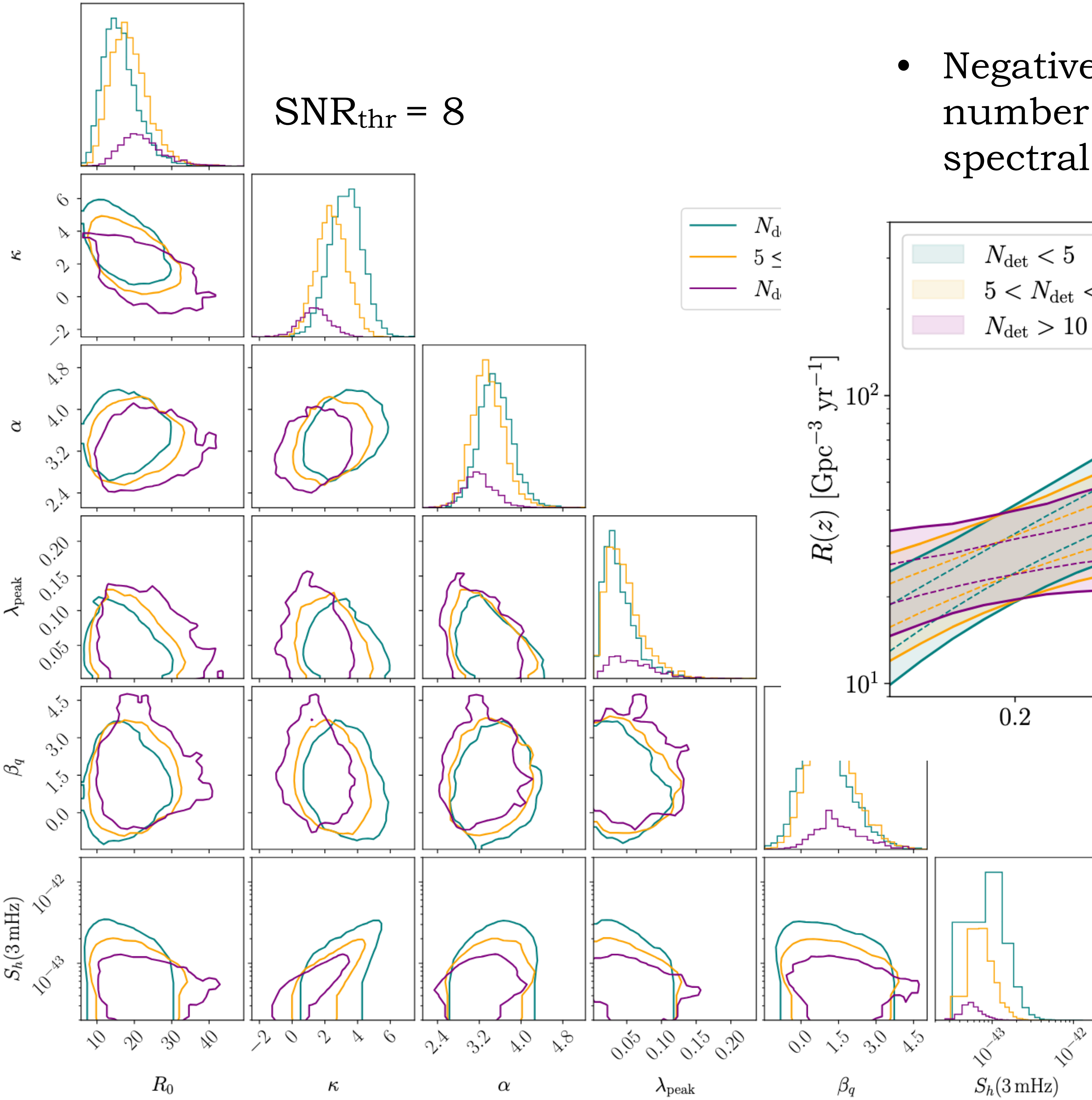


How do the population parameters influence the number of resolved sources in LISA?

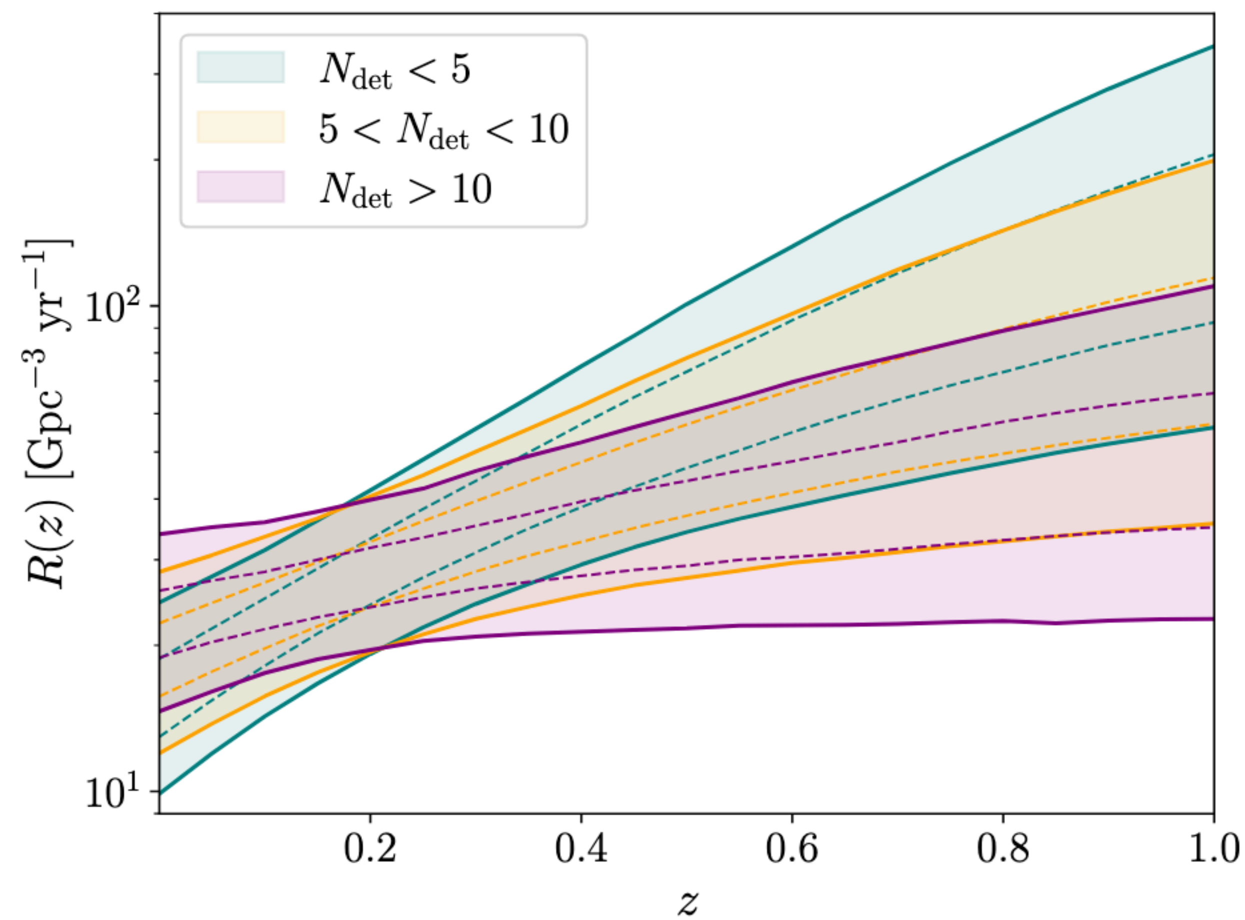
- 90% confidence intervals on population parameters from catalogues with given number of sources

Resolved smBHBs in LISA: population parameters

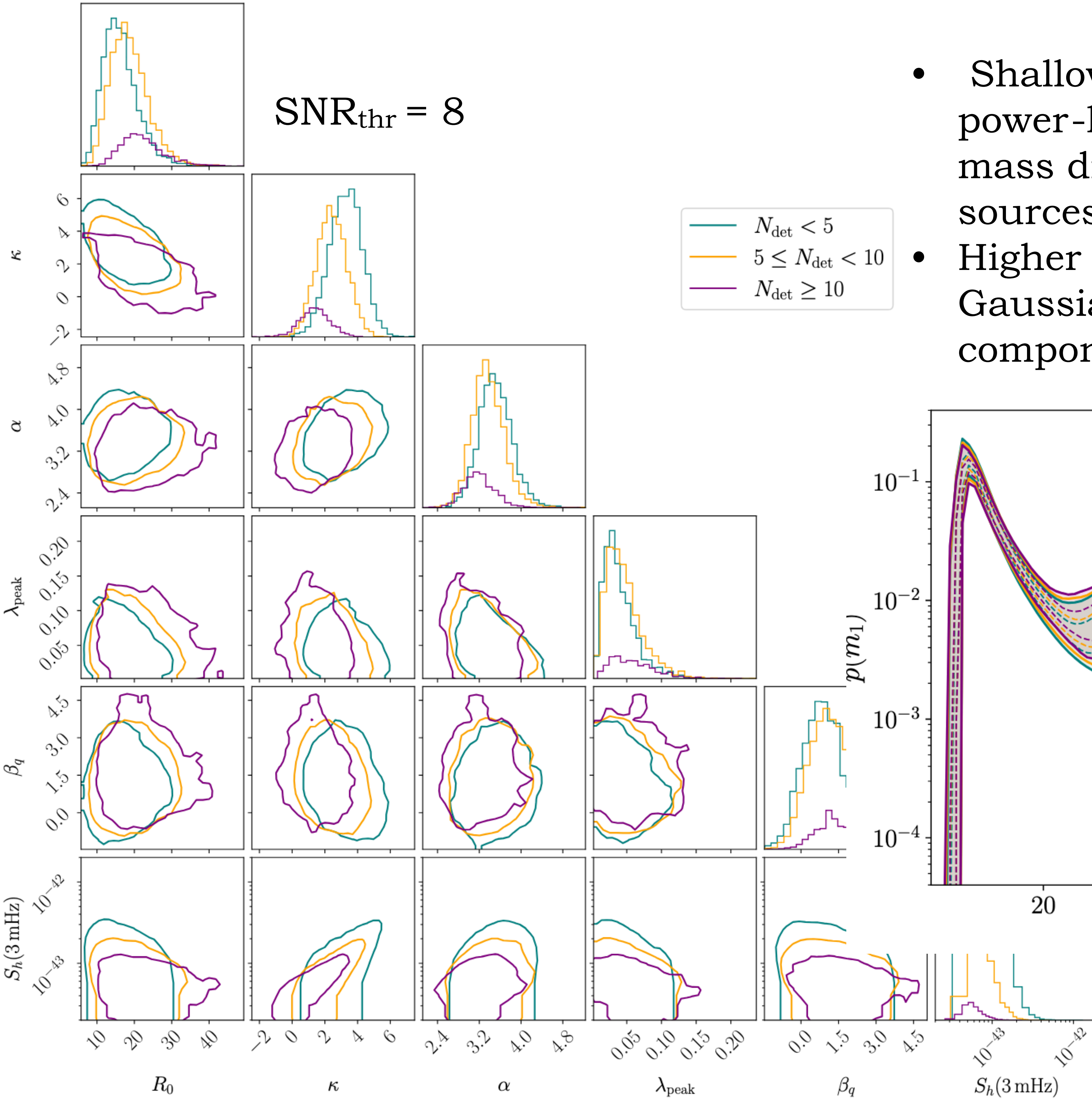
$\mathrm{SNR_{thr}} = 8$



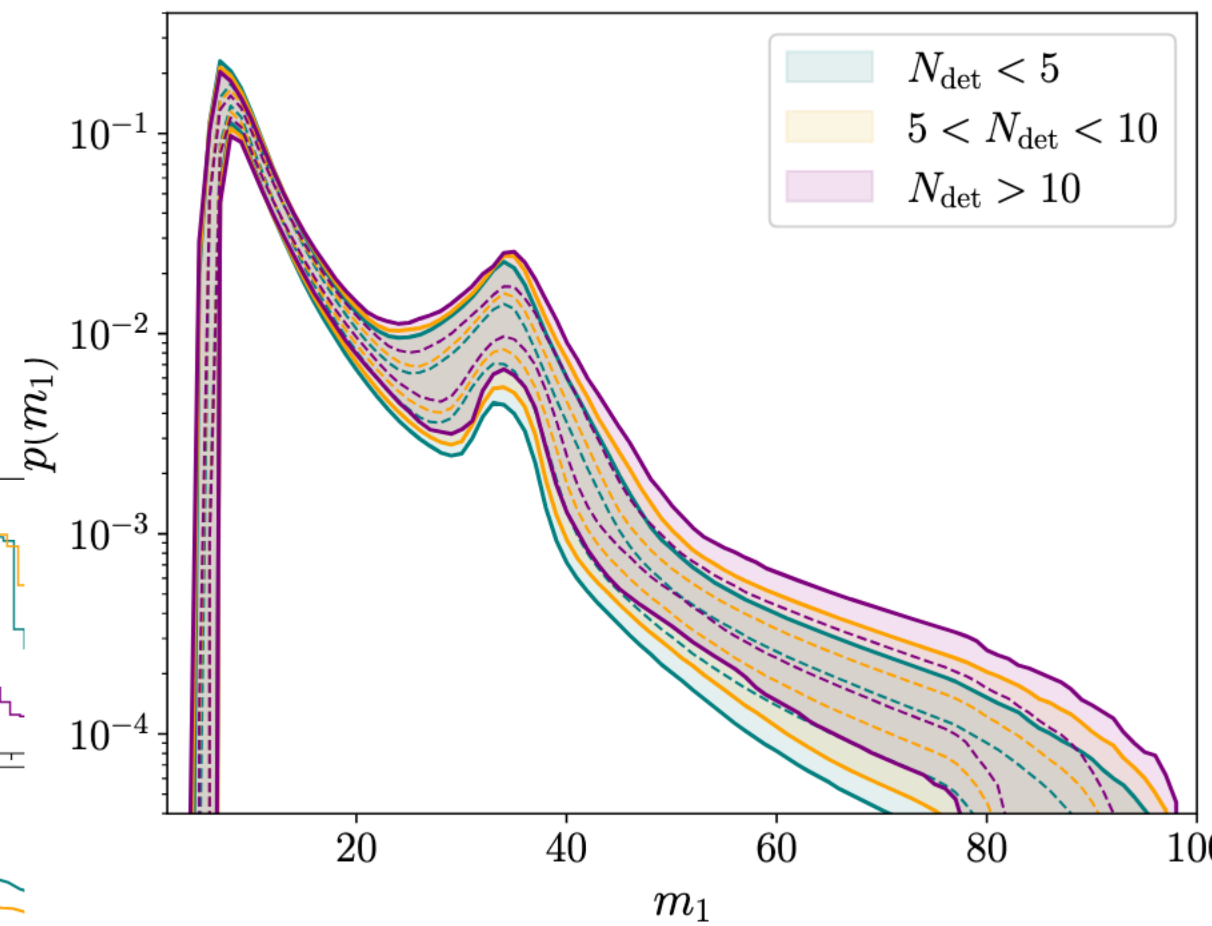
- Negative correlation between number of sources and the spectral index of the merger rate



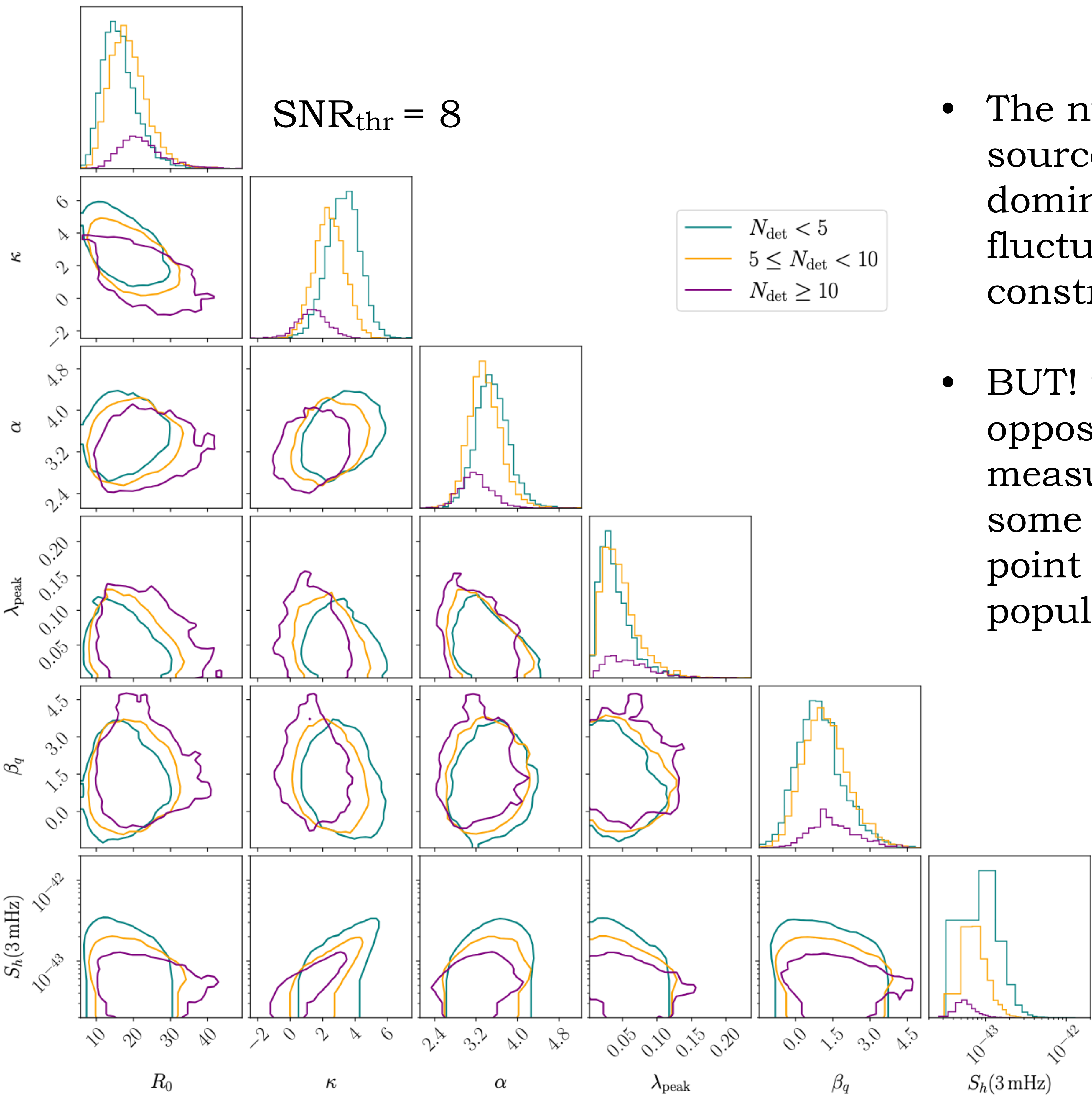
Resolved smBHBs in LISA: population parameters



- Shallower spectral index of the power-law component of the mass distribution gives more sources
- Higher contribution from the Gaussian mass distribution component gives more sources



Resolved smBHBs in LISA: population parameters

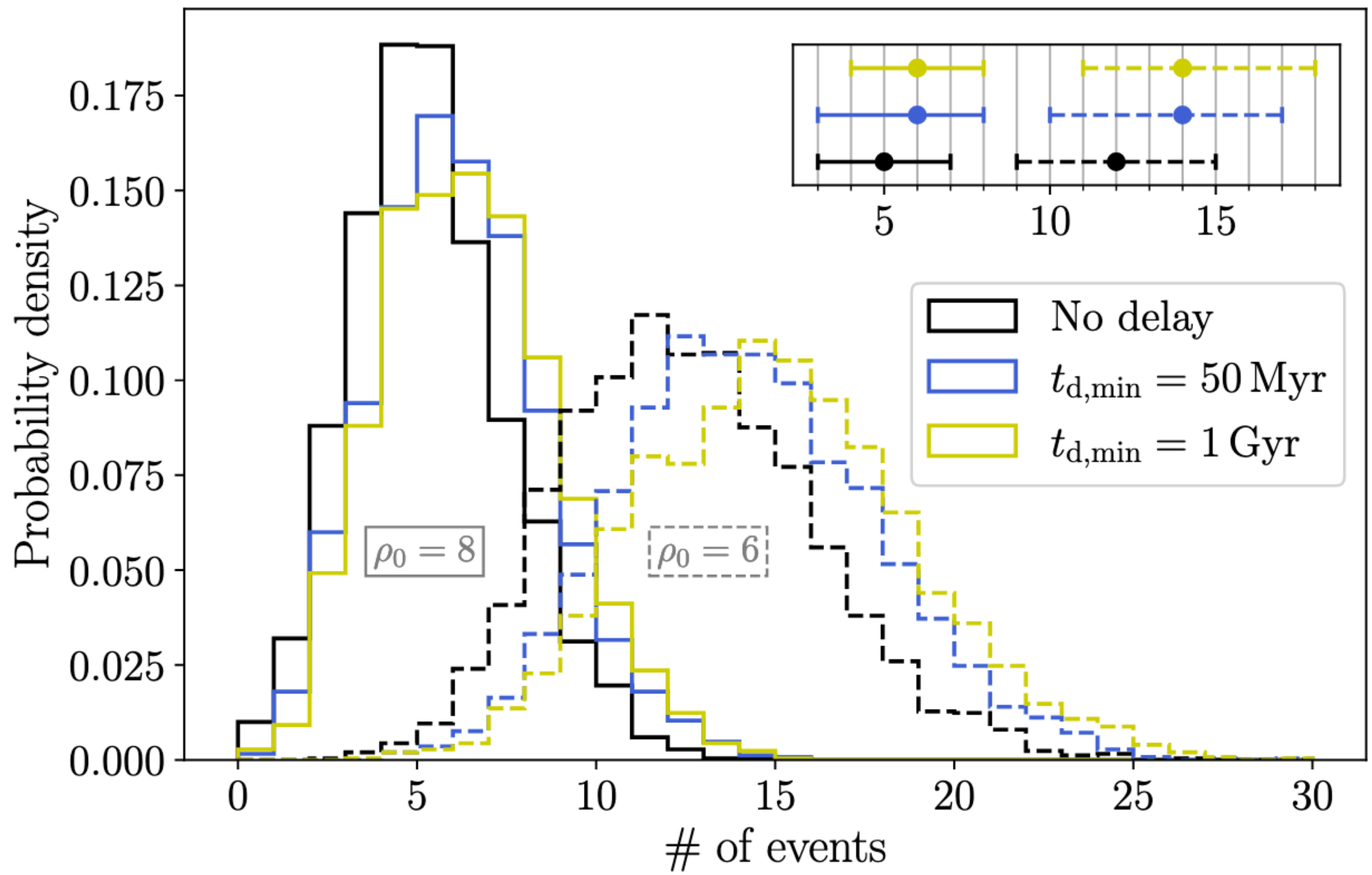


- The numbers of resolved sources are too small and dominated by statistical fluctuations to be exploited to constrain the population

- BUT! the foreground has the opposite correlations, so both measurements can provide some information, for example point to inconsistencies in the population model

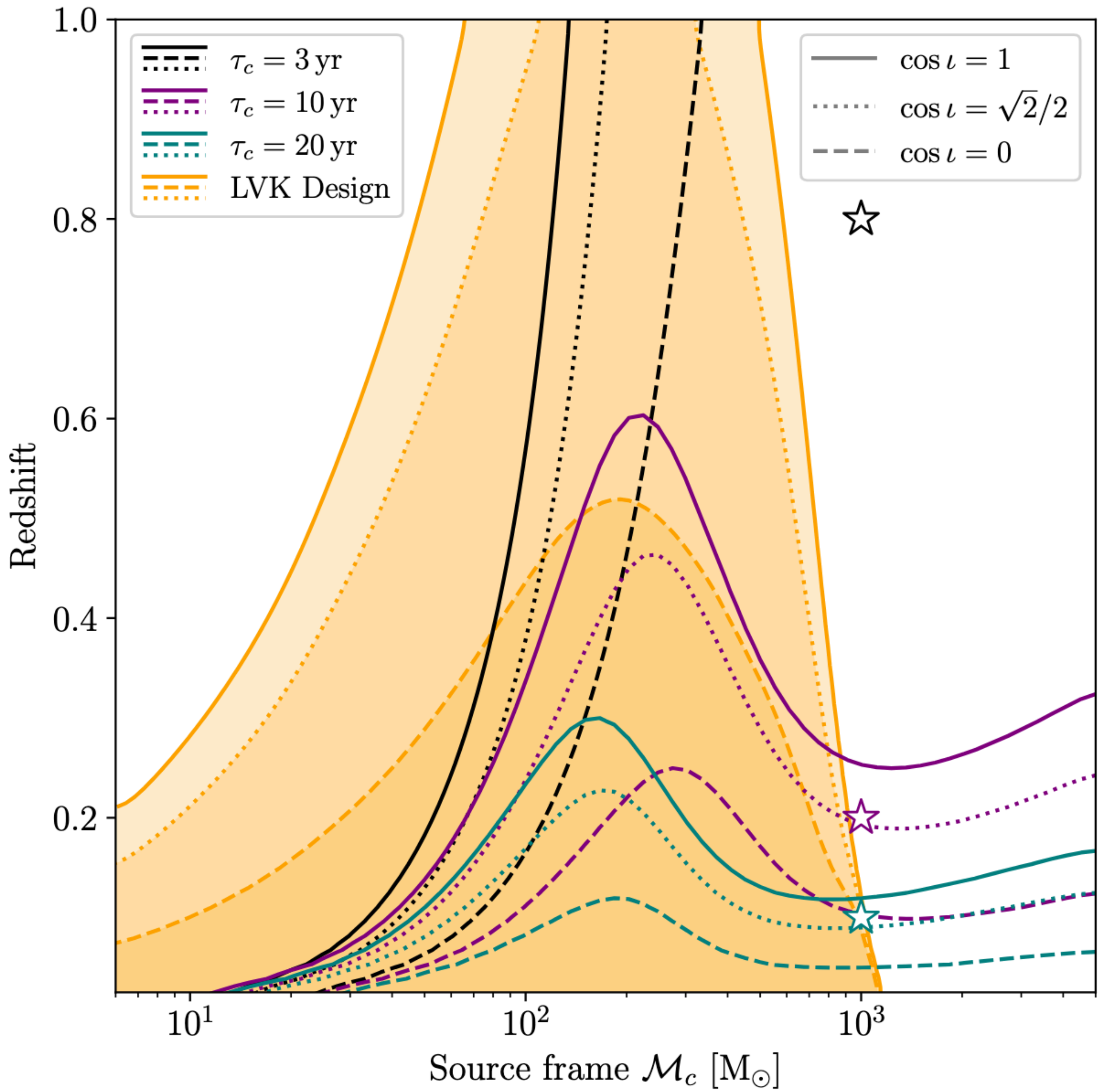
Resolved smBHBs in LISA: effect of time delay

- Compared to the foreground, the effect of time delay is even more negligible with respect to the statistical variability of the source numbers



Resolved smBHBs in LISA: “blind” comparison with LVK

What if the population of smBHB doesn’t fully obey the GWTC-3 inferred model?
Will LISA be sensitive to a population with different characteristics?



LVK waterfall plot at design sensitivity, network SNR>14
LISA waterfall plot at low mass

Resolved smBHBs in LISA: "blind" comparison with LVK

What if the population of smBHB doesn't fully obey the GWTC-3 inferred model?
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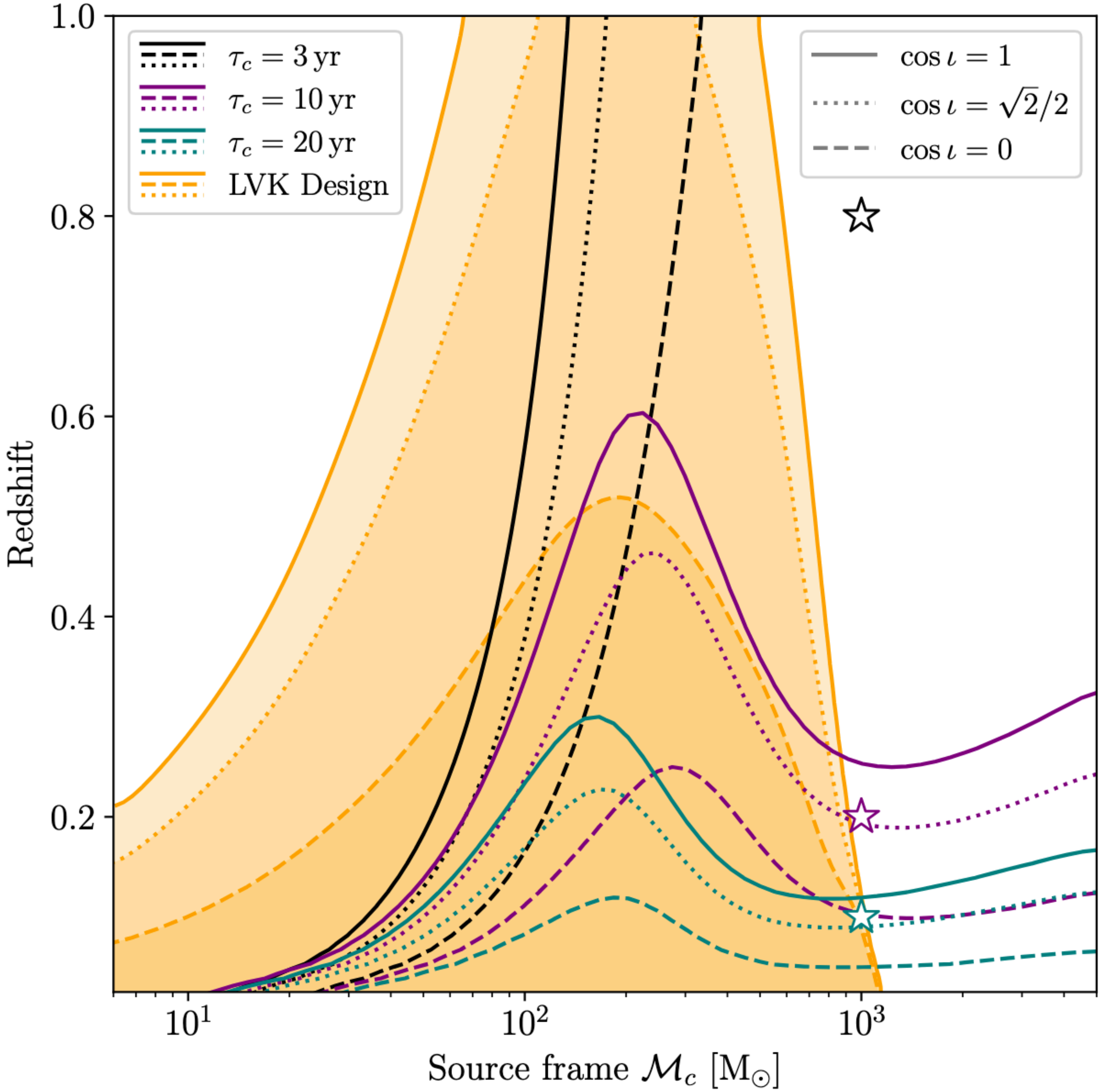
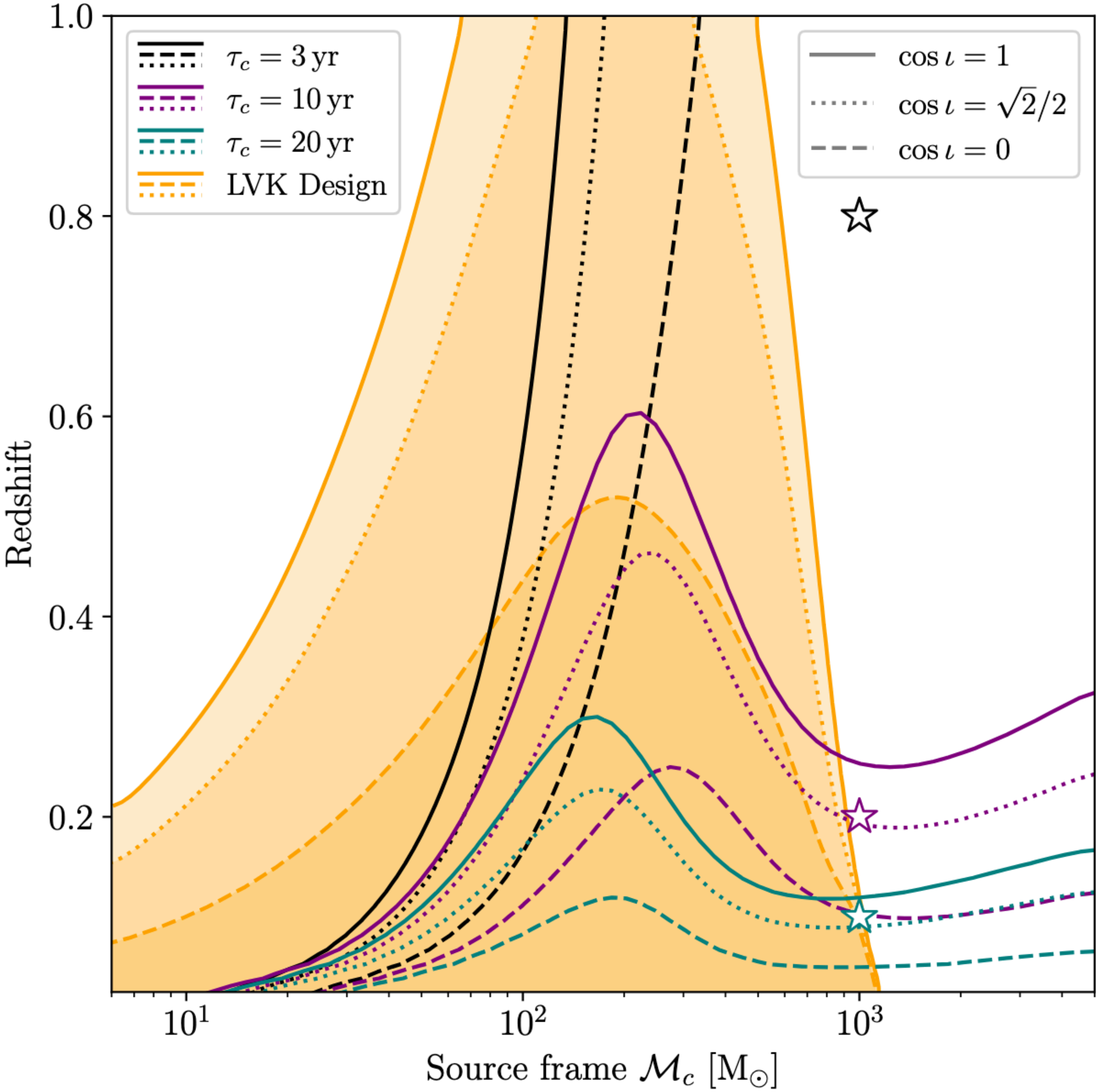


LVK waterfall plot at design sensitivity, network SNR>14
LISA waterfall plot at low mass

- Binary inclination w.r.t. the line of sight: face on improves SNR w.r.t. edge on
- Sources with small τ_c are visible up to large redshift when massive
- Sources with larger τ_c are visible only up to $z < 0.6$ because they have less SNR
- The peak structure is due to the presence of the galactic foreground

Resolved smBHBs in LISA: “blind” comparison with LVK

What if the population of smBHB doesn’t fully obey the GWTC-3 inferred model?
Will LISA be sensitive to a population with different characteristics?



LVK waterfall plot at design sensitivity, network SNR>14
LISA waterfall plot at low mass

- LISA’s reach is only really complementary to LVK if the smBHB population extends towards high masses
- However! The few sources that LISA will see will be characterised with extreme precision

Resolved smBHBs in LISA: parameter estimation on a few representative sources

Random selection of 12 sources with $8 < \mathrm{SNR} < 12$ and $20\ M_\odot \leq M_c \leq 40\ M_\odot$

Multiband: $\tau_c \leq 5$ yrs, 5 yrs $\leq \tau_c \leq 10$ yrs, 10 yrs $\leq \tau_c \leq 15$ yrs

Within τ_c ranges, two close to face on/off and two close to edge on

Run	$\mathcal{M}_c[M_\odot]$	$m_1[M_\odot]$	$m_2[M_\odot]$	q	χ_1	χ_2	$f^0_{\rm orb}$[mHz]	τ_c[yr]	z	d_L[Mpc]	$\sin b$	l	$\cos\iota$	ψ	$\phi_{\rm orb}$	SNR
High τ_c	43.10	53.48	45.87	0.9	0.4	0.3	4.4	11.44	0.08	392.599	0.4	4.1	1.0	3.5	0.4	10.01
	32.81	42.14	33.78	0.8	0.4	0.1	5.1	12.38	0.06	294.343	0.2	3.3	−0.9	2.7	0.7	9.18
	32.71	41.52	34.08	0.8	0.5	0.2	5.1	12.37	0.03	113.970	0.01	4.8	0.2	3.7	0.8	10.04
	21.42	31.62	19.37	0.6	0.4	0.3	7.1	10.59	0.01	57.04	0.06	2.9	0.2	0.4	3.6	12.08
Mid τ_c	34.60	42.76	36.97	0.9	0.2	0.5	5.4	9.7	0.07	344.103	0.1	5.3	−0.9	1.1	3.8	8.53
	32.31	49.86	28.08	0.6	0.1	0.4	5.8	9.0	0.06	301.756	0.2	0.4	−0.9	0.6	2.9	9.28
	26.07	33.97	26.48	0.8	0.1	0.5	6.6	9.3	0.02	82.63	−0.04	0.9	0.2	0.006	4.4	10.59
	23.00	26.44	26.40	1.0	0.1	0.1	7.2	9.1	0.01	64.86	−0.09	2.4	−0.03	3.7	4.6	11.17
Low τ_c	43.21	55.23	44.71	0.8	0.6	0.6	6.5	4.2	0.1	510.906	0.4	4.7	−1.0	6.0	4.7	11.12
	30.32	36.05	33.66	0.9	0.02	0.2	9.7	2.6	0.06	254.146	−0.4	4.5	0.9	2.9	3.0	10.55
	25.93	34.89	25.55	0.7	0.3	0.3	9.4	3.6	0.03	122.570	−0.3	1.8	0.3	0.6	5.6	10.59
	27.32	36.36	27.21	0.7	0.4	0.1	8.2	4.7	0.02	92.95	−0.4	4.4	−0.08	3.9	4.5	10.92
☆	1100.0	1263.6946	1263.4418	1.0	0.0	0.0	0.5	17.30	0.10	475.822	0.0	3.1	1	0.0	1	11.03
☆	1200	1378.5759	1378.3002	1.0	0.0	0.0	0.6	5.1	0.2	1012.2935	0.0	3.1	1	0.0	1	12.11
☆	1800.0	2067.8638	2067.4503	1.0	0.0	0.0	0.7	3.0	0.8	5162.1658	0.0	3.1	1	0.0	1	90.12

Three high mass sources with $M_c = 1000\ M_\odot$ and $z = 0.1, 0.2, 0.8$, face on

Resolved smBHBs in LISA: parameter estimation on a few representative sources

- The relative errors on chirp mass, initial frequency, τ_c are less than per mille (1 day or less for τ_c : multi-band OK)
- Parameters of sources with low τ_c are better determined (they chirp more)
- Counterintuitive: less-massive sources are slightly better determined, but not for the most massive ones)

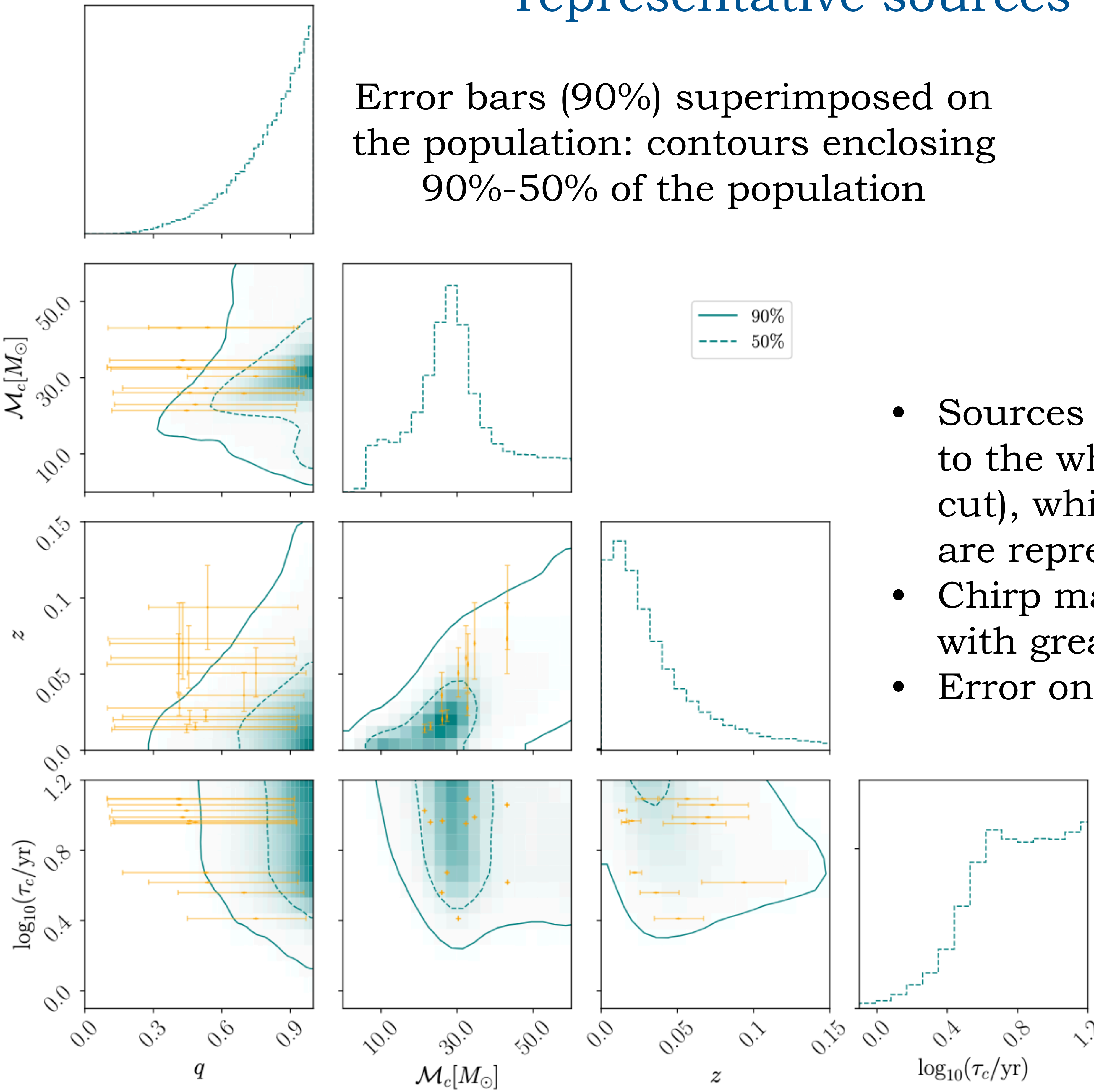
Run	$\Delta\mathcal{M}_c/\mathcal{M}_c$	$m_1[M_\odot]$	$m_2[M_\odot]$	q	χ_1	χ_2	$\Delta f^0_{\rm orb}/f^0_{\rm orb}$	$\Delta\tau_c/\tau_c$
High τ_c	2×10^{-4}	78^{+94}_{-27}	32^{+15}_{-15}	$0.4^{+0.5}_{-0.3}$	$-0.03^{+0.91}_{-0.85}$	$0.007^{+0.89}_{-0.90}$	7×10^{-7}	4×10^{-4}
	2×10^{-4}	60^{+75}_{-21}	25^{+11}_{-11}	$0.4^{+0.5}_{-0.3}$	$-0.04^{+0.92}_{-0.85}$	$0.004^{+0.89}_{-0.90}$	7×10^{-7}	4×10^{-4}
	2×10^{-4}	60^{+73}_{-20}	25^{+11}_{-11}	$0.4^{+0.5}_{-0.3}$	$-0.05^{+0.93}_{-0.84}$	$-0.004^{+0.90}_{-0.89}$	7×10^{-7}	4×10^{-4}
	1×10^{-4}	37^{+41}_{-12}	17^{+7}_{-7}	$0.4^{+0.5}_{-0.3}$	$-0.02^{+0.89}_{-0.84}$	$0.05^{+0.86}_{-0.93}$	5×10^{-7}	2×10^{-4}
Mid τ_c	2×10^{-4}	62^{+72}_{-20}	26^{+12}_{-12}	$0.4^{+0.5}_{-0.3}$	$-0.02^{+0.89}_{-0.85}$	$0.04^{+0.86}_{-0.91}$	9×10^{-7}	3×10^{-4}
	2×10^{-4}	56^{+65}_{-17}	25^{+10}_{-12}	$0.5^{+0.5}_{-0.3}$	$-0.04^{+0.89}_{-0.81}$	$0.04^{+0.86}_{-0.92}$	8×10^{-7}	3×10^{-4}
	1×10^{-4}	45^{+49}_{-14}	21^{+8}_{-9}	$0.5^{+0.5}_{-0.3}$	$0.002^{+0.86}_{-0.83}$	$0.05^{+0.85}_{-0.93}$	6×10^{-7}	2×10^{-4}
	1×10^{-4}	38^{+42}_{-11}	19^{+7}_{-8}	$0.5^{+0.4}_{-0.4}$	$-0.2^{+0.9}_{-0.7}$	$0.01^{+0.87}_{-0.90}$	6×10^{-7}	2×10^{-4}
Low τ_c	4×10^{-5}	68^{+29}_{-17}	37^{+11}_{-9}	$0.5^{+0.4}_{-0.3}$	$0.6^{+0.4}_{-0.3}$	$0.3^{+0.6}_{-1.0}$	5×10^{-7}	7×10^{-5}
	2×10^{-5}	40^{+12}_{-5}	30^{+4}_{-6}	$0.7^{+0.2}_{-0.3}$	$0.1^{+0.7}_{-0.6}$	$0.05^{+0.85}_{-0.90}$	4×10^{-7}	3×10^{-5}
	2×10^{-5}	35^{+12}_{-5}	25^{+4}_{-5}	$0.7^{+0.3}_{-0.3}$	$0.4^{+0.5}_{-0.5}$	$0.09^{+0.80}_{-0.87}$	3×10^{-7}	3×10^{-5}
	8×10^{-5}	44^{+39}_{-11}	23^{+7}_{-9}	$0.5^{+0.4}_{-0.4}$	$0.1^{+0.7}_{-0.5}$	$0.08^{+0.83}_{-0.92}$	5×10^{-7}	1×10^{-4}
☆	7×10^{-4}	2128^{+3500}_{-800}	787^{+413}_{-407}	$0.5^{+0.4}_{-0.4}$	$-0.02^{+0.91}_{-0.89}$	$0.004^{+0.89}_{-0.90}$	5×10^{-6}	1×10^{-3}
☆	7×10^{-4}	2250^{+3500}_{-800}	880^{+430}_{-450}	$0.4^{+0.4}_{-0.4}$	$-0.05^{+0.92}_{-0.84}$	$0.0001^{+0.89}_{-0.89}$	5×10^{-6}	2×10^{-3}
☆	5×10^{-6}	2157^{+126}_{-79}	1982^{+75}_{-110}	$0.4^{+0.6}_{-0.4}$	$0.05^{+0.75}_{-0.84}$	$-0.05^{+0.93}_{-0.85}$	9×10^{-7}	6×10^{-6}

Resolved smBHBs in LISA: parameter estimation on a few representative sources

- The relative errors on redshift are big (20-50%) but go down to 7% for the most massive source
- The inclination doesn't influence parameter estimation

Run	z	d_L[Mpc]	$\sin b$	l	$\cos\iota$	ψ	$\phi_{\rm orb}$
High τ_c	$0.07^{+0.02}_{-0.02}$	341^{+116}_{-110}	$0.38^{+0.02}_{-0.02}$	$4.15^{+0.01}_{-0.01}$	$0.8^{+0.2}_{-0.3}$	$-0.4^{+2.0}_{-2.0}$	$2.7^{+2.0}_{-2.0}$
	$0.06^{+0.02}_{-0.02}$	261^{+97}_{-87}	$0.23^{+0.03}_{-0.03}$	$3.30^{+0.01}_{-0.01}$	$-0.8^{+0.3}_{-0.2}$	$0.2^{+2.0}_{-2.0}$	$2.9^{+2.0}_{-2.0}$
	$0.028^{+0.009}_{-0.005}$	125^{+44}_{-22}	$0.004^{+0.089}_{-0.093}$	$4.850^{+0.009}_{-0.009}$	$0.2^{+0.2}_{-0.1}$	$0.5^{+1.6}_{-1.7}$	$2.4^{+1.8}_{-1.7}$
	$0.014^{+0.003}_{-0.002}$	61^{+15}_{-9}	$0.04^{+0.06}_{-0.11}$	$2.879^{+0.005}_{-0.005}$	$0.23^{+0.13}_{-0.09}$	$0.3^{+1.6}_{-3.0}$	$2.2^{+2.8}_{-1.7}$
Mid τ_c	$0.07^{+0.03}_{-0.02}$	327^{+133}_{-113}	$0.08^{+0.07}_{-0.19}$	$5.260^{+0.010}_{-0.010}$	$-0.8^{+0.3}_{-0.2}$	$-0.5^{+2.0}_{-2.0}$	$3.7^{+2.0}_{-2.0}$
	$0.06^{+0.02}_{-0.02}$	281^{+104}_{-95}	$0.22^{+0.02}_{-0.03}$	$0.431^{+0.008}_{-0.008}$	$-0.8^{+0.3}_{-0.2}$	$-0.4^{+2.0}_{-2.0}$	$3.5^{+2.0}_{-2.0}$
	$0.020^{+0.006}_{-0.003}$	90^{+28}_{-16}	$-0.01^{+0.08}_{-0.07}$	$0.897^{+0.007}_{-0.007}$	$0.21^{+0.14}_{-0.10}$	$0.005^{+1.6}_{-1.6}$	$2.8^{+1.7}_{-1.7}$
	$0.015^{+0.003}_{-0.002}$	69^{+13}_{-9}	$-0.07^{+0.16}_{-0.05}$	$2.354^{+0.005}_{-0.005}$	$-0.03^{+0.09}_{-0.09}$	$0.6^{+1.6}_{-1.6}$	$3.0^{+1.7}_{-1.7}$
Low τ_c	$0.09^{+0.03}_{-0.03}$	444^{+141}_{-138}	$0.371^{+0.009}_{-0.010}$	$4.694^{+0.005}_{-0.005}$	$-0.8^{+0.3}_{-0.2}$	$-0.1^{+2.0}_{-2.0}$	$3.3^{+2.0}_{-2.0}$
	$0.05^{+0.02}_{-0.02}$	233^{+80}_{-74}	$-0.358^{+0.007}_{-0.007}$	$4.513^{+0.004}_{-0.004}$	$0.8^{+0.2}_{-0.3}$	$0.6^{+2.0}_{-2.0}$	$3.7^{+2.0}_{-2.0}$
	$0.04^{+0.02}_{-0.01}$	164^{+71}_{-48}	$-0.280^{+0.010}_{-0.010}$	$1.753^{+0.004}_{-0.004}$	$0.5^{+0.4}_{-0.2}$	$0.3^{+1.9}_{-2.8}$	$3.5^{+2.1}_{-2.5}$
	$0.022^{+0.005}_{-0.003}$	99^{+21}_{-14}	$-0.382^{+0.008}_{-0.008}$	$4.426^{+0.005}_{-0.005}$	$-0.09^{+0.09}_{-0.10}$	$0.7^{+1.6}_{-1.6}$	$2.9^{+1.7}_{-1.7}$
☆	$0.08^{+0.02}_{-0.02}$	396^{+122}_{-117}	$-0.00003^{+0.15218}_{-0.15409}$	$3.14^{+0.08}_{-0.08}$	$0.8^{+0.2}_{-0.3}$	$-0.3^{+2.1}_{-2.0}$	$2.9^{+2.0}_{-2.0}$
☆	$0.17^{+0.04}_{-0.05}$	842^{+246}_{-246}	$-0.001^{+0.138}_{-0.142}$	$3.14^{+0.06}_{-0.06}$	$0.8^{+0.2}_{-0.3}$	$-0.3^{+2.0}_{-2.0}$	$2.9^{+2.0}_{-2.0}$
☆	$0.74^{+0.05}_{-0.08}$	4684^{+428}_{-584}	$-0.0009^{+0.0248}_{-0.0254}$	$3.139^{+0.006}_{-0.006}$	$0.90^{+0.08}_{-0.12}$	$-0.3^{+2.0}_{-2.0}$	$2.9^{+2.0}_{-2.0}$

Resolved smBHBs in LISA: parameter estimation on a few representative sources



Error bars (90%) superimposed on the population: contours enclosing 90%-50% of the population

- Sources have low q with respect to the whole population (SNR cut), while the other parameters are representative
- Chirp mass and τ_c determined with great precision
- Error on redshift increases with it

Resolved smBHBs in LISA: results

- If we keep SNR=8 as the relevant threshold, there are few sources! Especially multi-band (1/3 of the total, with 21% probability of no sources), and the numbers fluctuate due to population uncertainty: we need to be lucky!

- Even though the source number correlates with the population parameters, its high statistical variability makes it difficult for LISA to provide relevant information on the population if not combined with a foreground detection (however probably covered by extragalactic WD)

- The visible sources have very small redshift, and LISA can observe a tail at high masses allowed by the population model but not observed by LVK because of statistics

- LISA is still complementary to LVK because the parameter estimation is in general very good: it can characterise a few, low-redshift sources with great precision. Archival and multi-band searches are possible since sources parameters are very well determined (especially τ_c)

- This can have important consequences: tests of GR, exploration of the smBHB environment, binary formation channels…

- if a population of higher mass sources O(1000 $M_\odot$) exists, LISA will be able to fully characterise it