Towards a realistic forecast detection for Primordial Gravitational Wave Backgrounds

2024/12/12 - IFIC, Valencia

GW background signals from Inflation: lattice calculation

2303.17436 (*Phys.Rev.Lett. 131 (2023) 15, 151003*)
2411.16368
And ongoing work

Joanes Lizarraga (UPV/EHU)
in collaboration with D.G. Figueroa, Nicolás Loayza, Ander Urio and Jon Urrestilla

Introduction

- The model: $\boxed{\text{Axion-inflation} \quad + \quad \frac{\phi}{\Lambda} F\tilde{F}}$
  Shift symmetry $\phi \to \phi + c$

[K. Freese, J. A. Frieman, A. V. Olinto (PRL 65,3233 1990)]
...

- Why so interesting?

  - Very efficient energy transport Axion -> Gauge fields

  - Chiral instability $|A^+| \gg |A^-|$

[M. M. Anber, L. Sorbo (0908.4089)]
[J. Cook, L. Sorbo (1109.0022)]
[N. Barnaby, E. Pajer, M. Peloso (1110.3327)]

Very rich phenomenology:

- PBH
- GWs
- Non Gaussianities
- ...

[N. Barnaby, M. Peloso (1011.1500)]
[A. Linde, S. Mooji, E. Pajer (1212.1693)]
[E. Bugaev, P. Klimai (1312.7435)]
[N. Bartolo et al (LISA) (1610.06481)]
[J. G. Bellido, M. Peloso, C. Unal (1610.03763)]
.....

The model

$$V = \frac{1}{2}m^2\phi^2$$

$$m/m_{\rm p} = 6.16 \cdot 10^{-6}$$

$$\alpha_\Lambda/m_p = 1/\Lambda$$

Dynamical equations: @ FLRW

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B}$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \vec{\nabla}\phi\times\vec{E}\right)$$

$$\ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM}\right)$$

Constraint equations:

$$\vec{\nabla}\cdot\vec{E} = -\frac{\alpha_\Lambda}{a m_p}\vec{\nabla}\phi\cdot\vec{B}$$

$$H^2 = \frac{1}{3m_p^2}\left(\rho_{\rm K} + \rho_{\rm G} + \rho_{\rm V} + \rho_{\rm EM}\right)$$

$\phi \rightarrow$ axion-like scalar field

$A_\mu \rightarrow$ dark photon

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

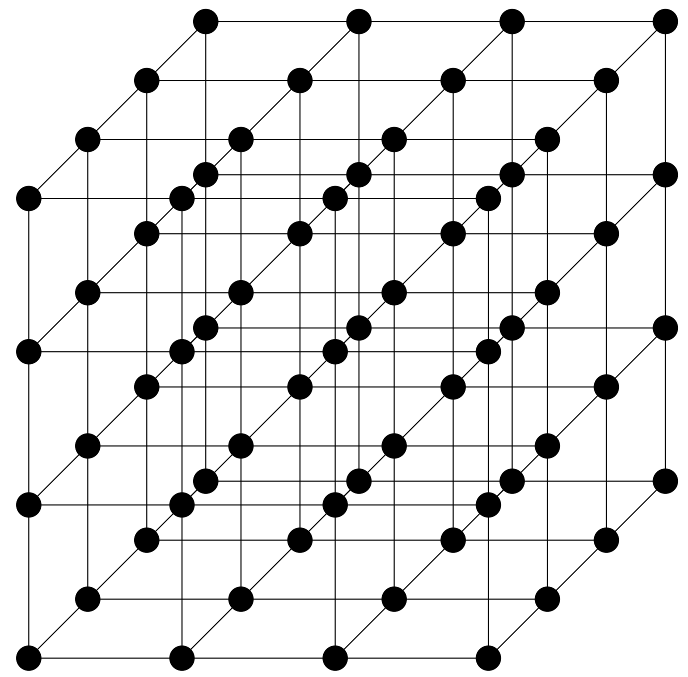
$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$

Our lattice study

[D.G. Figueroa, JLi, A. Urio, J. Urrestilla (2303.17436)]
[D.G. Figueroa, JLi, N. Loayza, A. Urio, J. Urrestilla (2411.16368)]

1- Full equations of motion: inhomogeneities allowed!

$$\vec{E}\cdot\vec{B} \rightarrow \vec{\nabla}^2\phi, \quad \vec{E}\cdot\vec{B} \rightarrow \vec{\nabla}\phi\times\vec{E}$$



2- Lattice gauge-invariant and shift symmetric.

[D. G. Figueroa, M. Shaposhnikov (1705.09629)]
[J. R. C. Cuissa , D. G. Figueroa (1812.03132)]

CosmoLattice

[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2006.15122)]
[D. G. Figueroa, A. Florio, F. Torrenti & W. Valkenburg (2102.01031)]

3- Strong backreaction during and until the end of inflation

From the linear regime $\rightarrow$ $\epsilon_H = 1$ $\rightarrow$

Discretisation procedure

[D. G. Figueroa, M. Shaposhnikov (1705.09629)]

Continuum

$$\frac{\phi}{\Lambda}\vec{E}\cdot\vec{B}$$

The only one that satisfies all conditions

Lattice

$$\boxed{\sum_i \frac{\phi}{\Lambda} E_i^{(2)} B_i^{(4)}}$$

Want to preserve:

1- Gauge transformations

$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

2- Bianchi identities

$$\vec{\nabla}\times\vec{E} = \dot{\vec{B}} \qquad \vec{\nabla}\cdot \ldots 0$$

3- Topological nature ... coupling

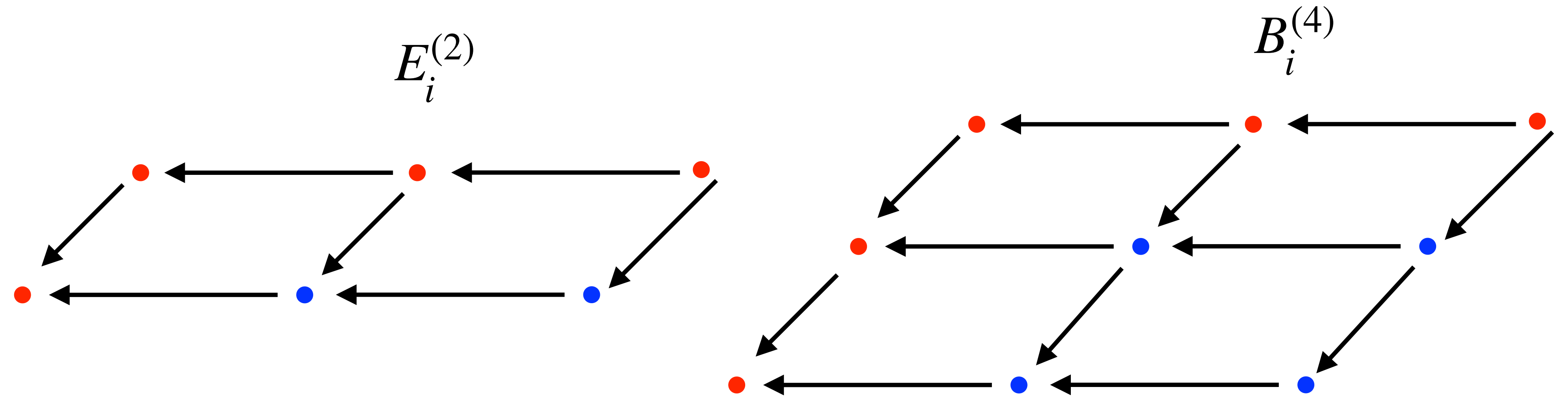
$$F_{\mu\nu}\tilde{F}^{\mu\nu} = \ldots$$

Exact lattice shift symmetry

4- Continuum limit to $\mathcal{O}(dx^2)$

$$E_i^{(2)} \equiv \frac{1}{2}(E_i + E_{i,-i}) \quad @ \quad n$$

$$B_i^{(4)} \equiv \frac{1}{4}(B_i + B_{i,-j} + B_{i,-k} + B_{i,-j-k}) \quad @ \quad n$$



Discretisation procedure

$$\dot{\pi}_\phi = -3H\pi_\phi + \frac{1}{a^2}\sum_i \Delta_i^- \Delta_i^+ \phi - \frac{dV(\phi)}{d\phi} + \frac{\alpha_\Lambda}{a^3 m_\mathrm{p}} \sum_i E_i^{(2)} B_i^{(4)}$$

$$\dot{E}_i = -HE_i - \frac{1}{a^2}\sum_{j,k}\epsilon_{ijk}\nabla_j^- B_k - \frac{\alpha_\Lambda}{2am_\mathrm{p}}\left(\pi_\phi B_i^{(4)} + \pi_{\phi,+i} B_{i,+i}^{(4)}\right)$$

$$+\frac{\alpha_\Lambda}{4am_\mathrm{p}}\sum_\pm \sum_{j,k} \epsilon_{ijk}\left\{\left[(\Delta_j^\pm \phi) E_{k,\pm j}^{(2)}\right]_{+i} + \left[(\Delta_j^\pm \phi) E_{k,\pm j}^{(2)}\right]\right\}$$

$$\sum_i \Delta_i^- E_i = -\frac{\alpha_\Lambda}{2am_\mathrm{p}} \sum_\pm \sum_i (\Delta_i^\pm \phi) B_{i,\pm i}^{(4)}$$

Conjugate momenta in kernels

$$\dot{\tilde{E}}_i = \mathcal{K}_{i,A}^L\left[a, b, \phi, A_i, \tilde{E}_i\right]$$

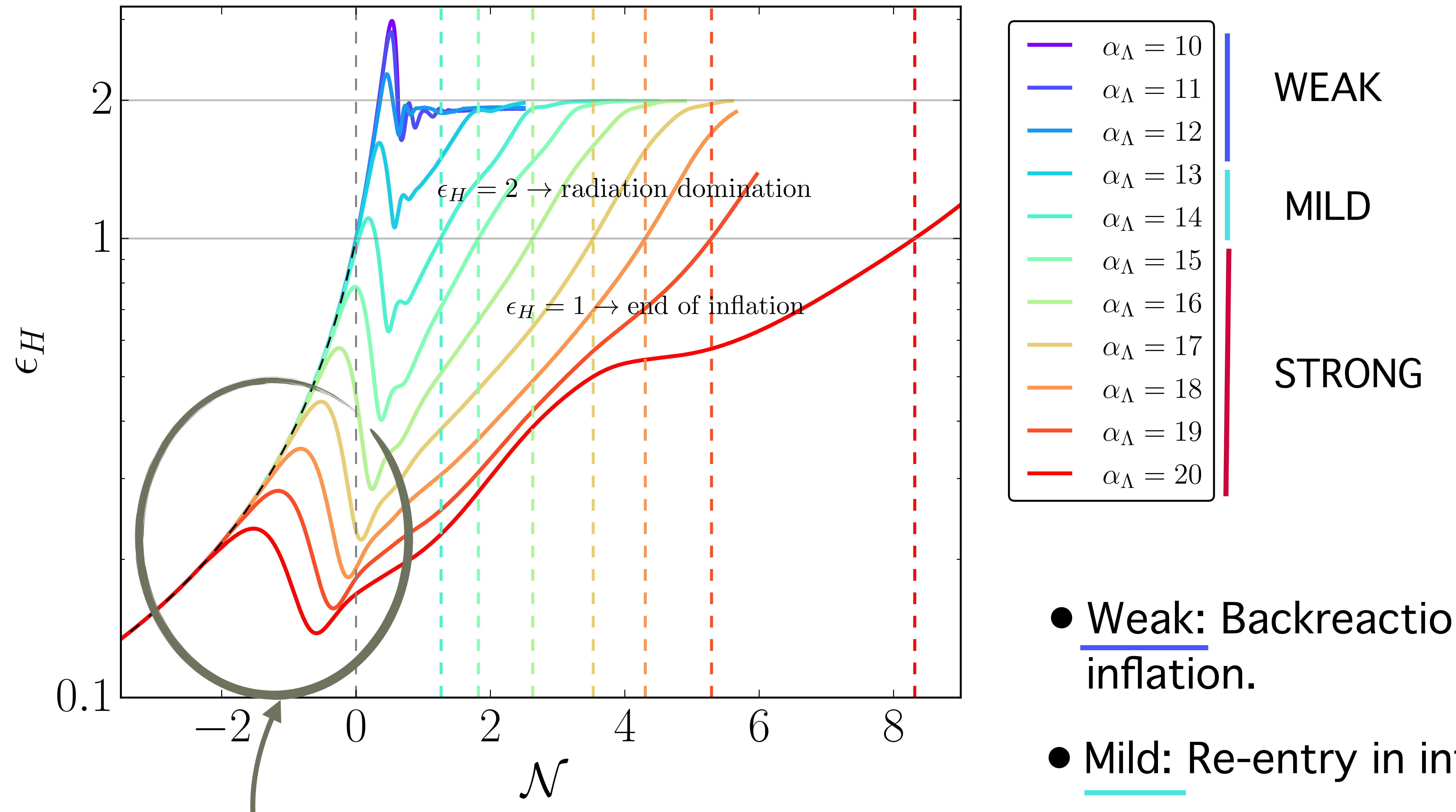
Not valid for usual symplectic time integrators: LF or VV

Need for a non-symplectic integrator:
Runge-Kutta

Outline

1. Uncovering the strong backreaction regime
    1. General features
    2. (Electro)magnetic slow-roll
    3. UV sensitivity
2. GW generation and detectability prospects
    1. Inflation
    2. Preheating

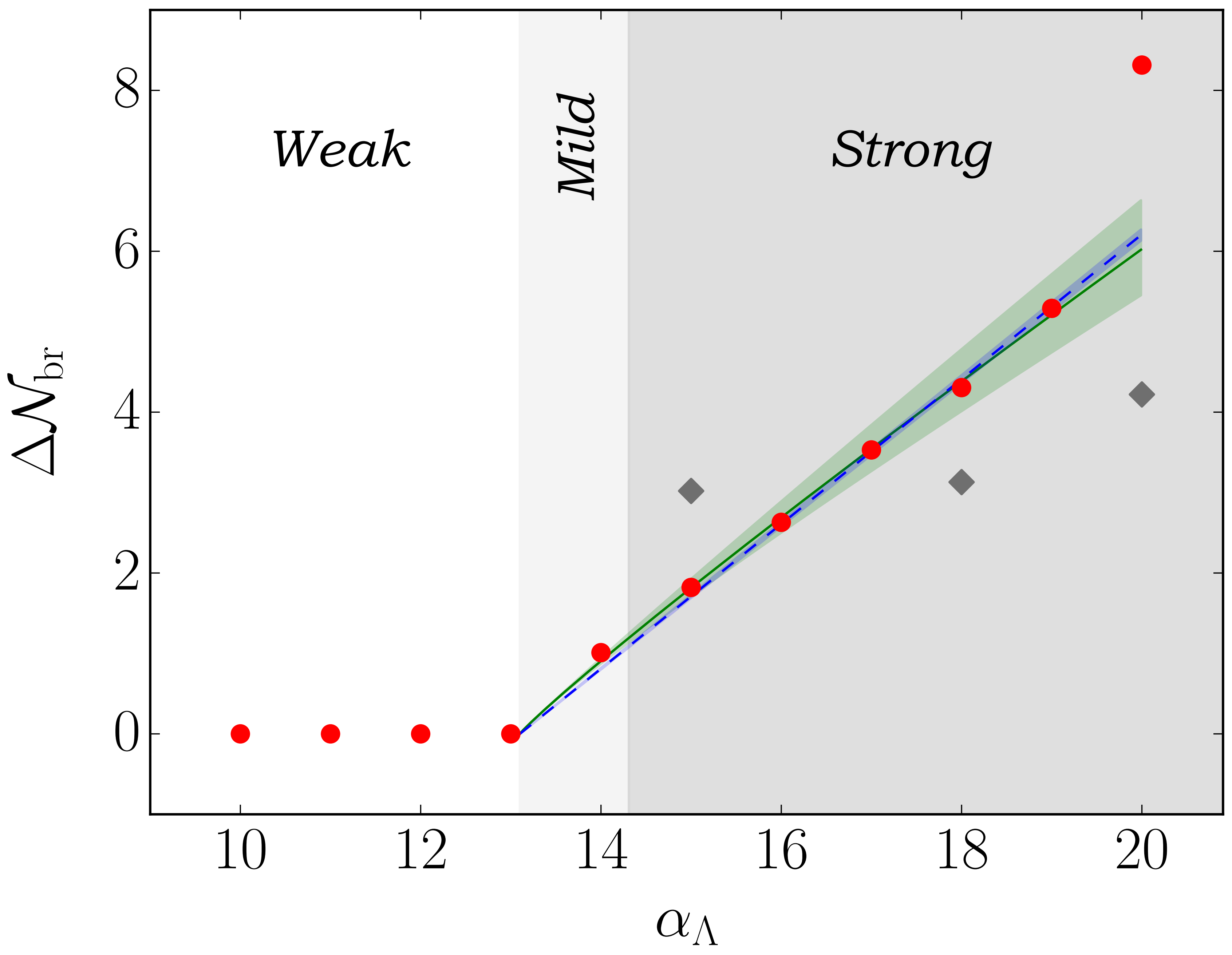
1. Uncovering the strong backreaction regime

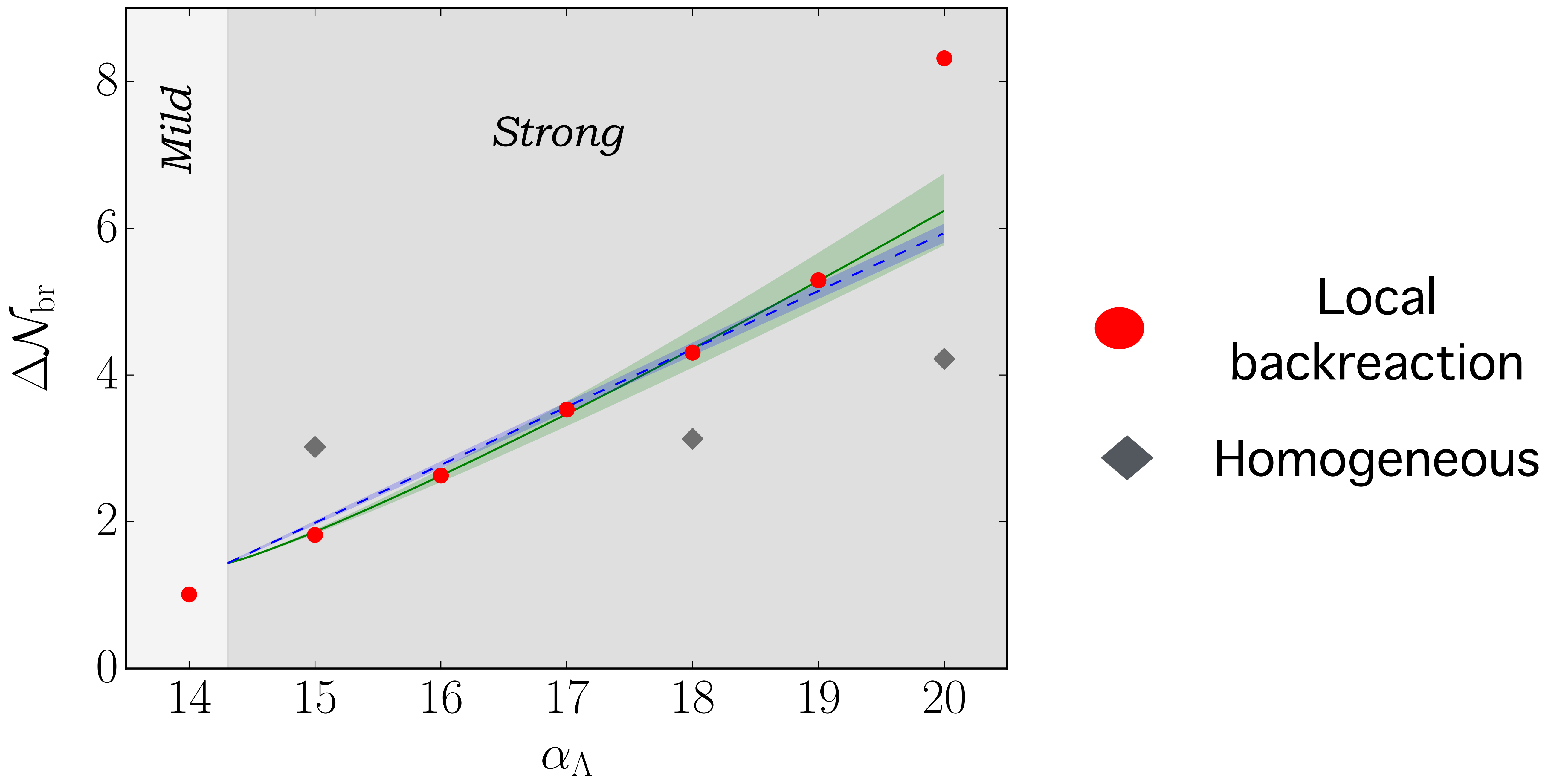


Bump before standard slow-roll end, earlier for higher couplings

- Weak: Backreaction only after inflation, no extension of inflation.
- Mild: Re-entry in inflation.
- Strong: Inflationary period exponentially grows with the coupling value!

1.1 General features



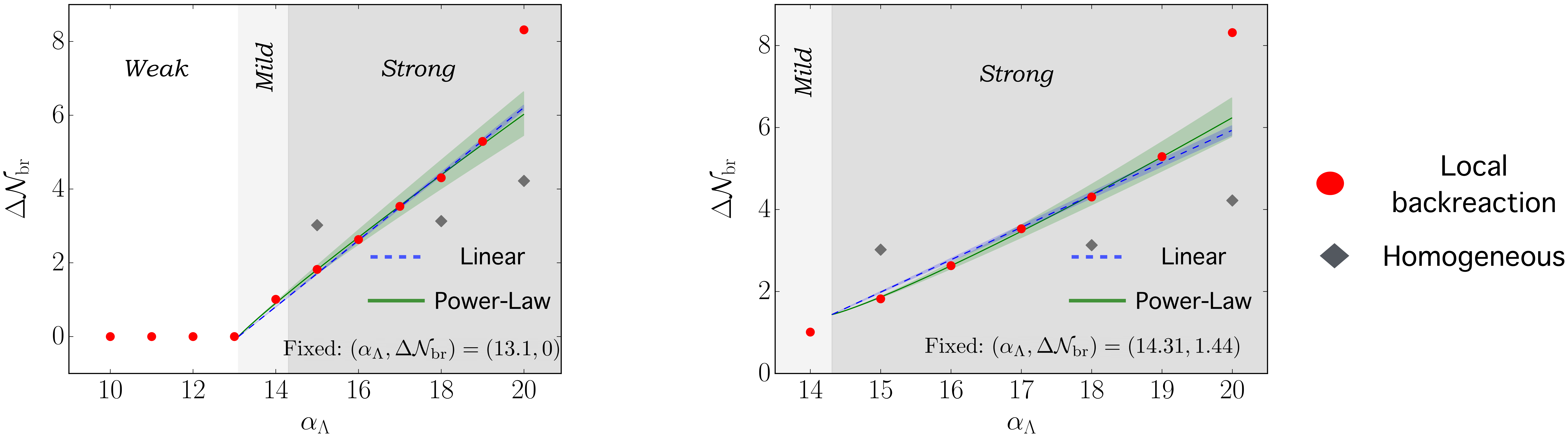


Local backreaction:
Inflationary period extends
~ linearly respect to the
coupling

VS

Homogeneous backreaction:
Inflationary period is clustered
~ 3-4 e-folds

1.1 General features



Linear fits approximately supported, but preferred mild curvature

Extrapolations

$\alpha_\Lambda = 25 \to \Delta\mathcal{N}_{\rm br} \sim 10 \; - \; 12$

$\alpha_\Lambda = 30 \to \Delta\mathcal{N}_{\rm br} \sim 15 \; - \; 18$

$\alpha_\Lambda = 35 \to \Delta\mathcal{N}_{\rm br} \sim 18 \; - \; 25$

1.1 General features

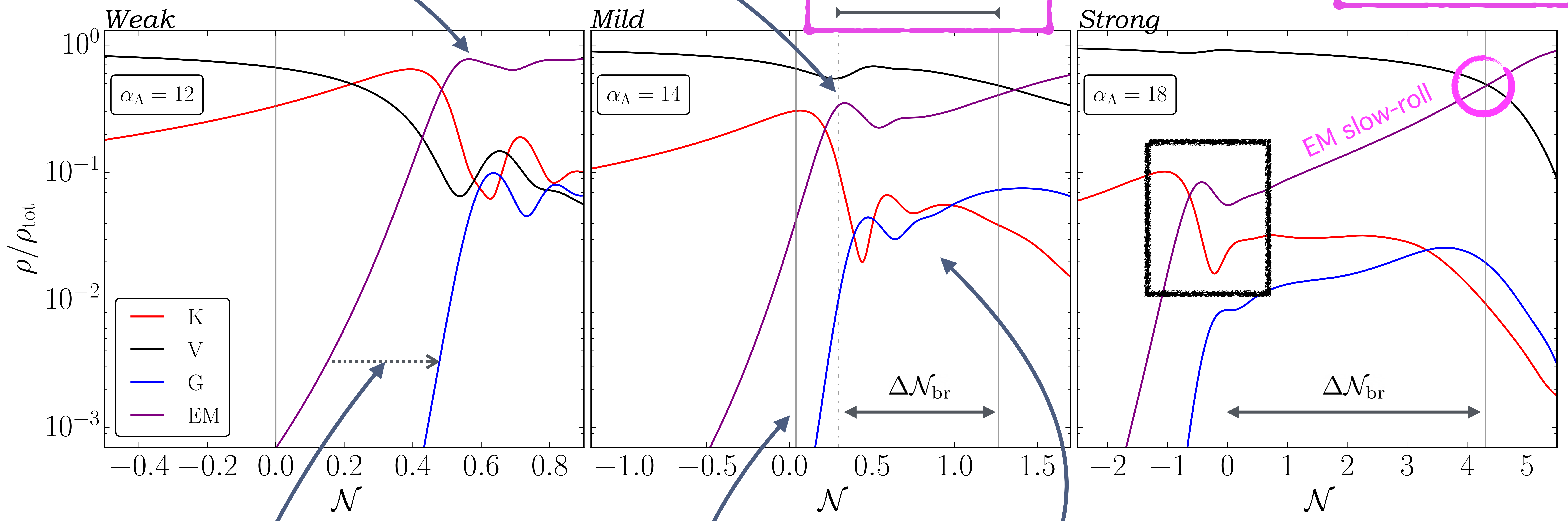
$$\epsilon_H = 1 + (2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM})/\rho_{\rm tot}$$

EM backreaction quite after standard slow-roll

Re-entry due to the EM backreaction

Interplay between V and EM in the re-entry

End of inflation when V=EM



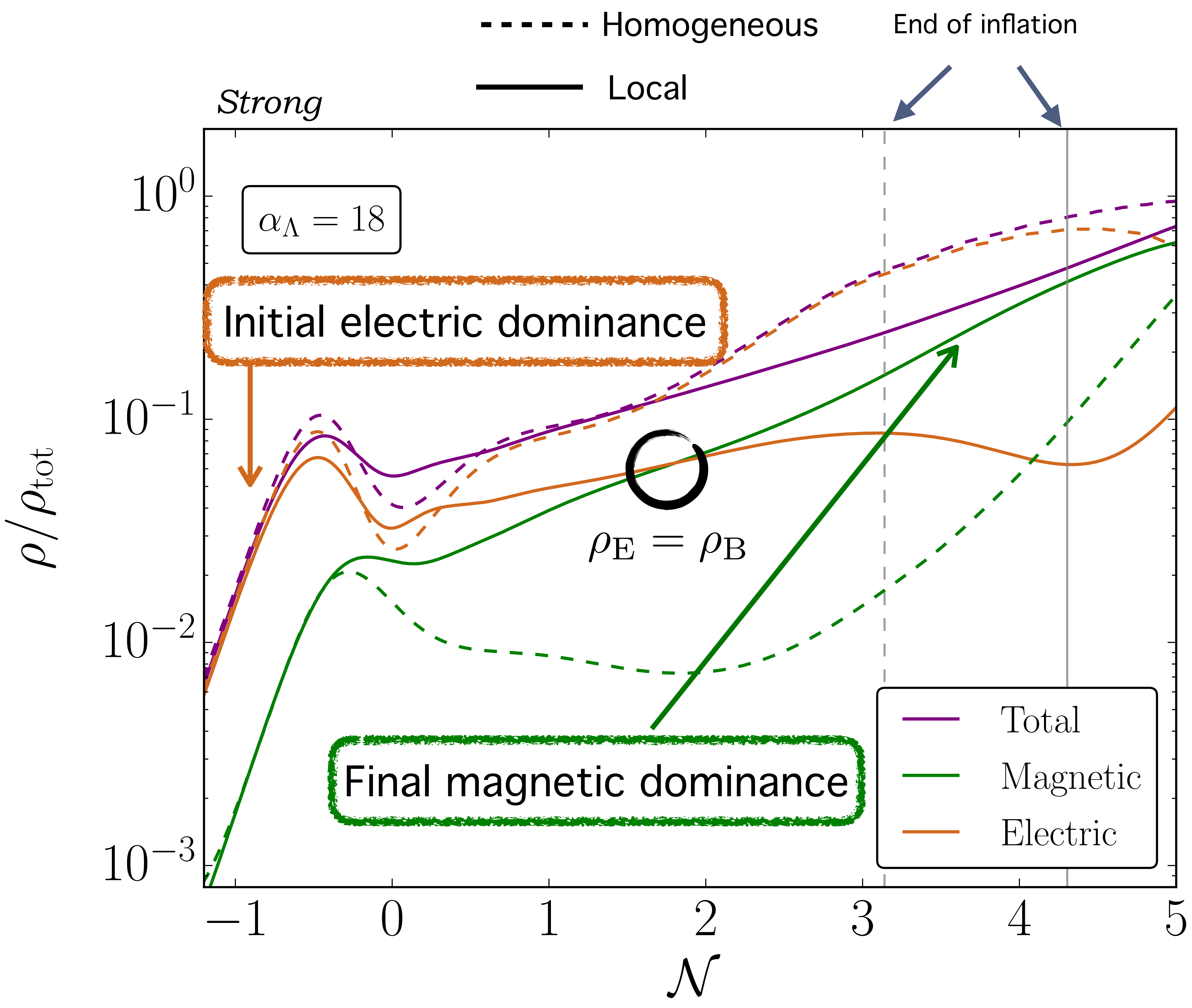
Local backreaction of the electromagnetic term feeds the gradients

Still first crossing almost at 0 efold

Comparable kinetic and gradient contribution during inflation

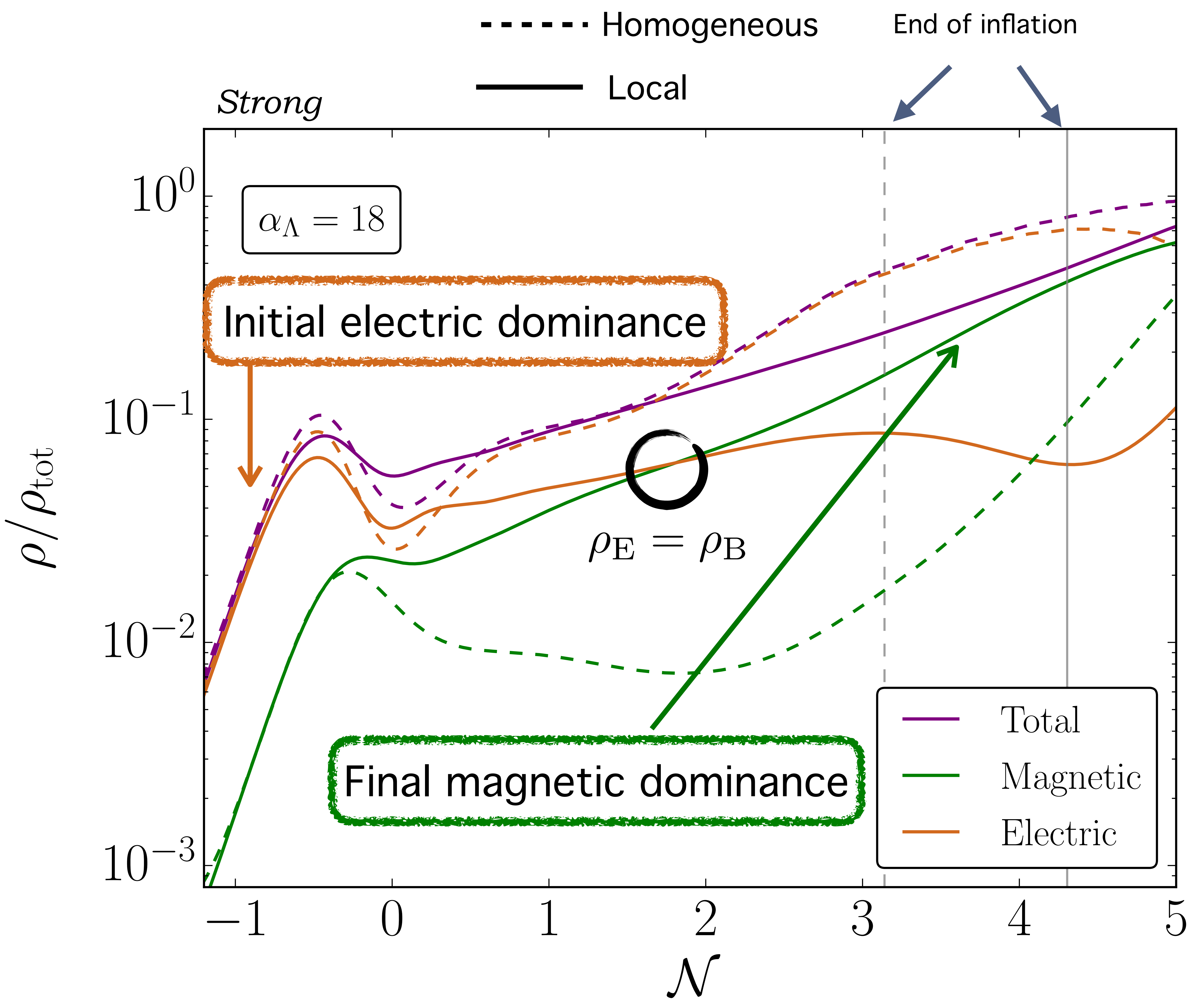
EM>K before 0 efold prevents the end of inflation

1.2 (Electro)magnetic slow roll

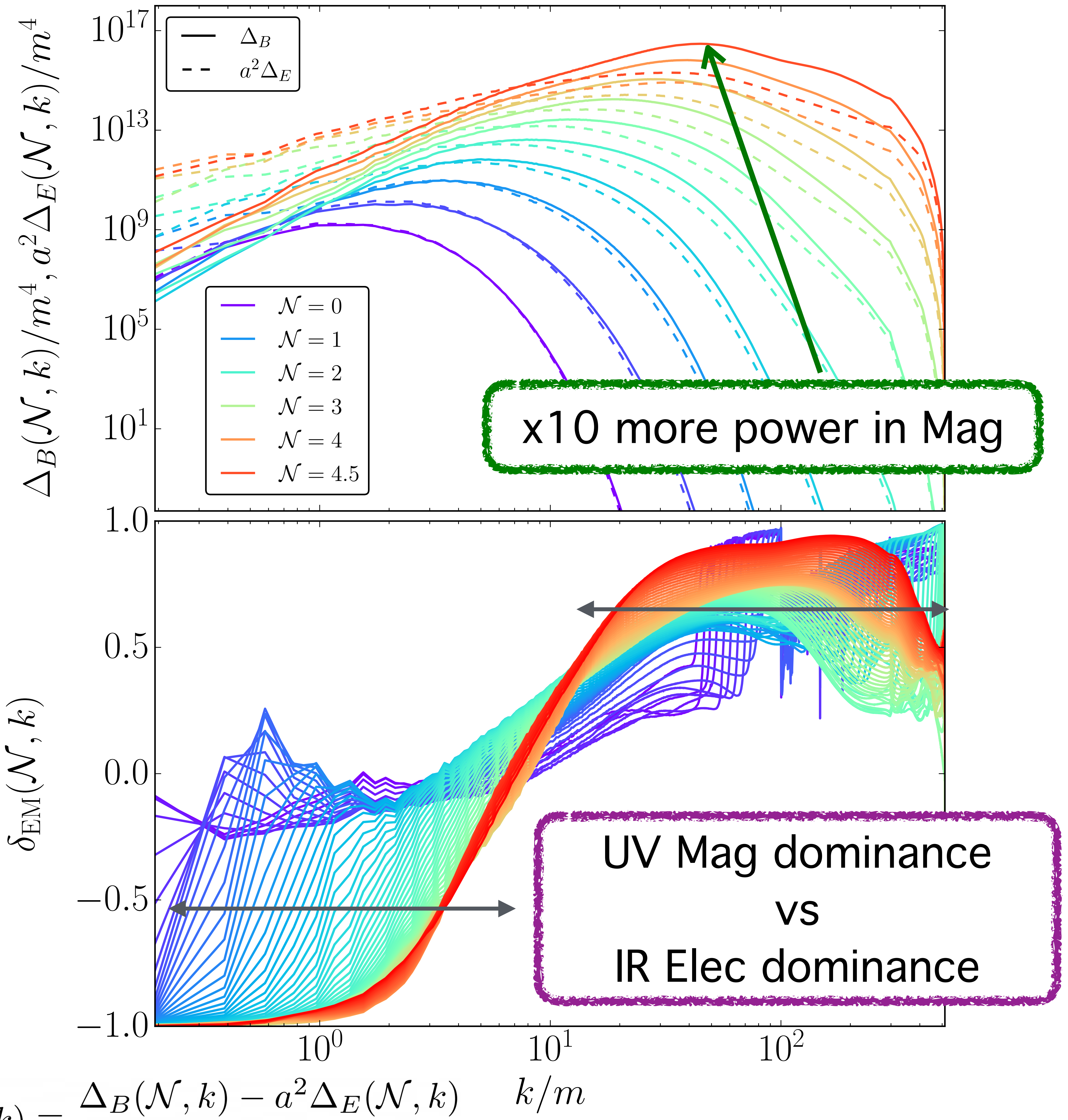


Homogeneous: complete electric dominance

1.2 (Electro)magnetic slow roll



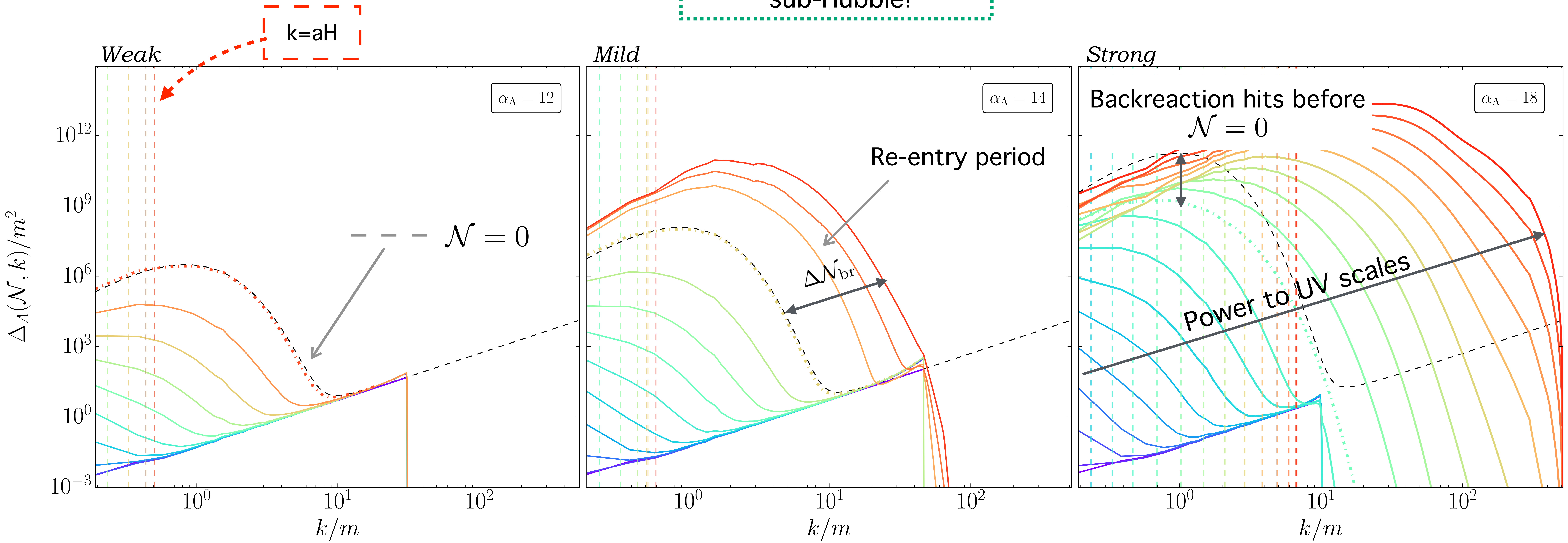
Homogeneous: complete electric dominance



$$\delta_{\mathrm{EM}}(\mathcal{N},k) \equiv \frac{\Delta_B(\mathcal{N},k) - a^2\Delta_E(\mathcal{N},k)}{\Delta_B(\mathcal{N},k) + a^2\Delta_E(\mathcal{N},k)}$$

1.3 UV sensitivity

Peak always remains sub-Hubble!



No noticeable changes during inflation

During the re-entry period, excitation moves towards UV scales

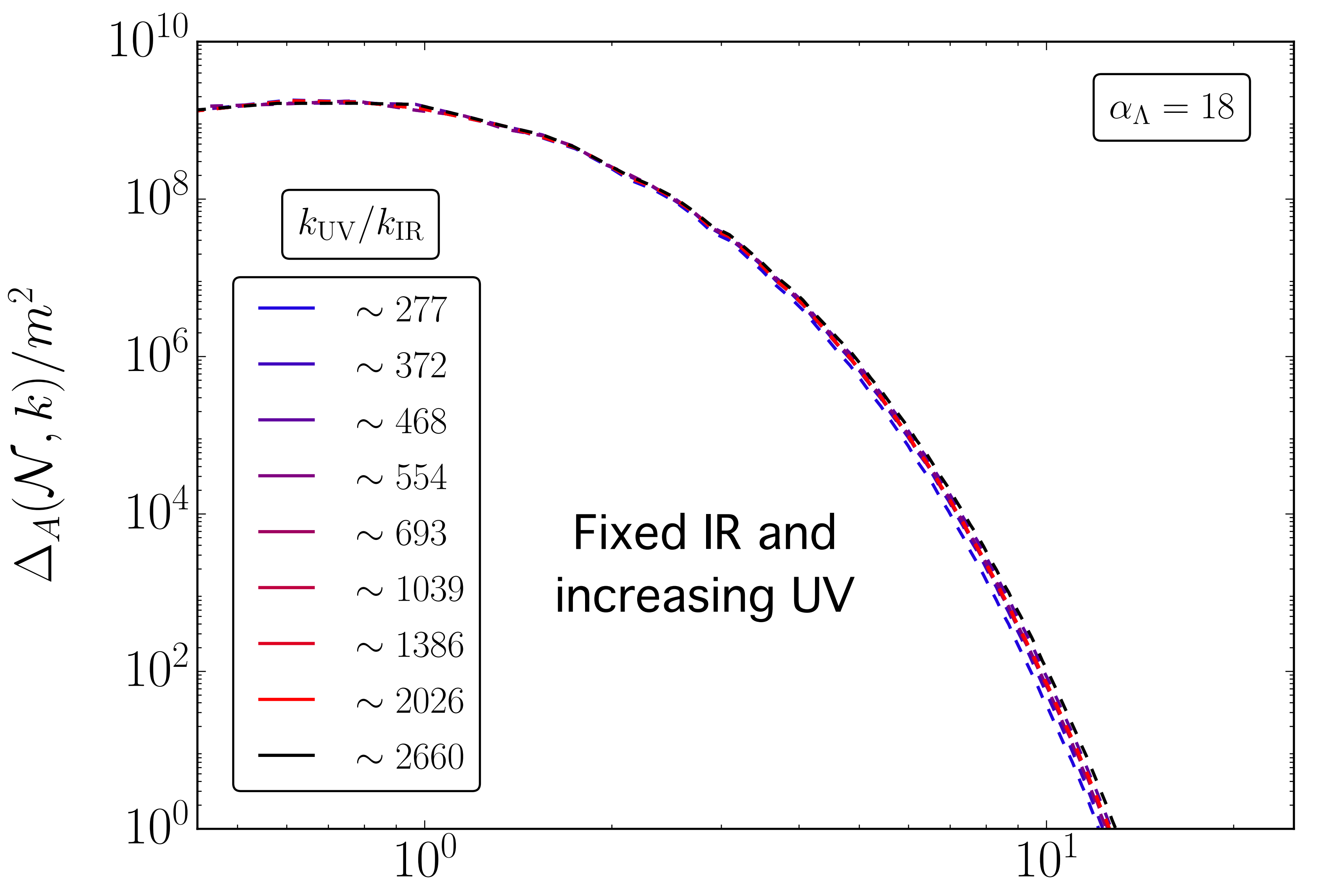
Considerable amount of extra e-folds: extreme UV sensitivity

$N = 320, \quad k_{\rm UV}/k_{\rm IR} \sim 280$

$N = 480, \quad k_{\rm UV}/k_{\rm IR} \sim 415$

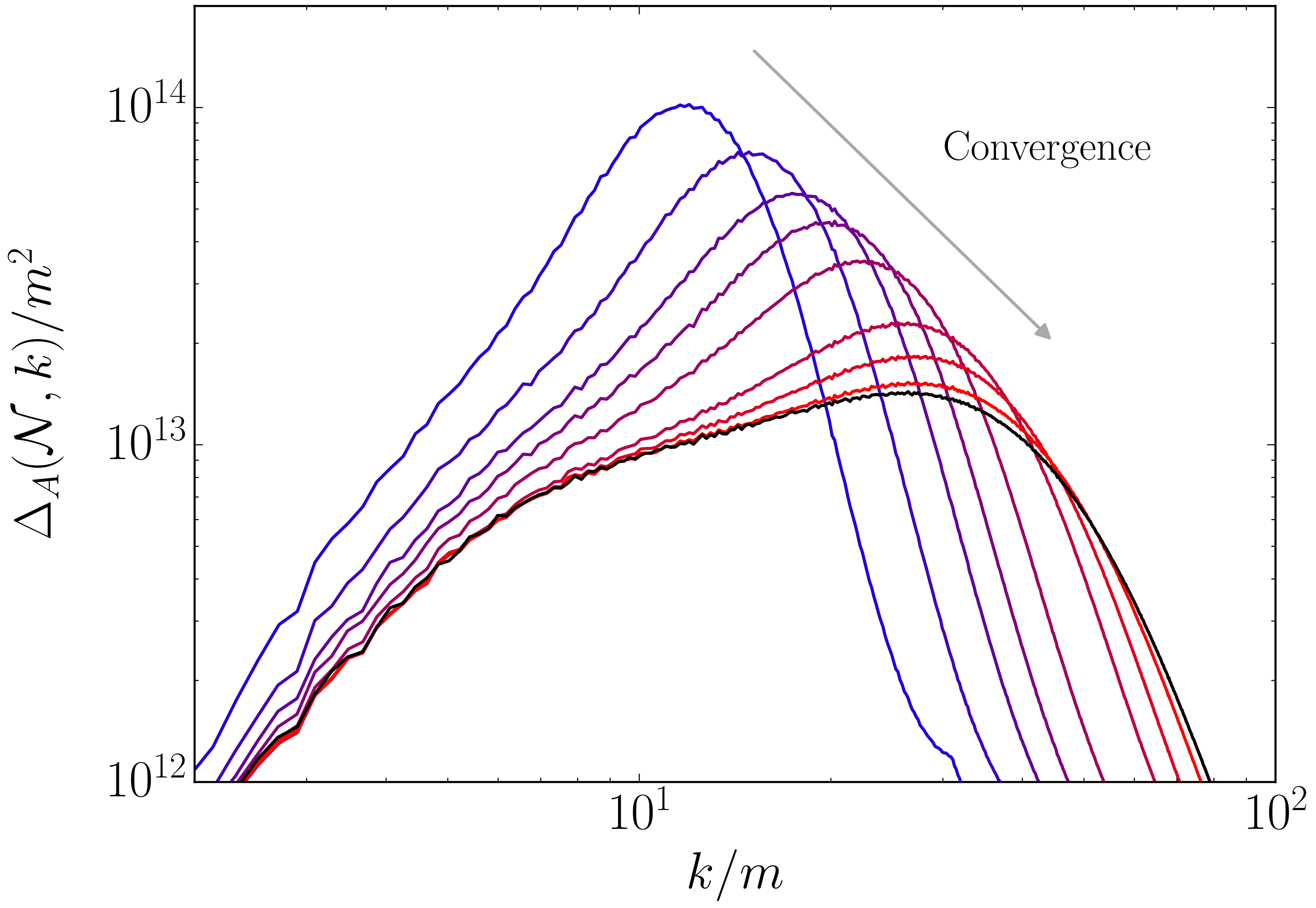
$N = 3072, \quad k_{\rm UV}/k_{\rm IR} \sim 2660$

1.3 UV sensitivity: convergence?



$\mathcal{N} = 0$

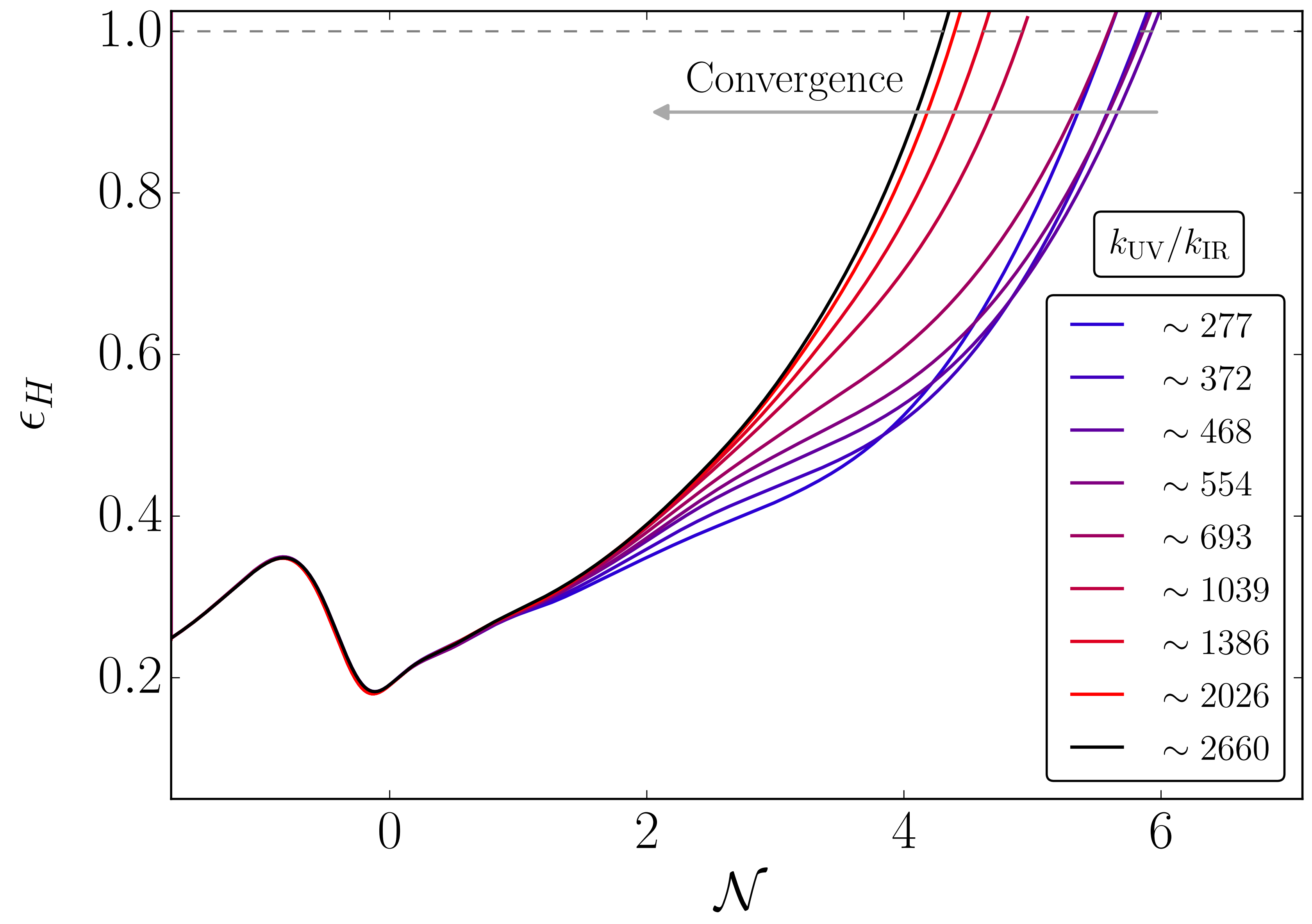
Common evolution



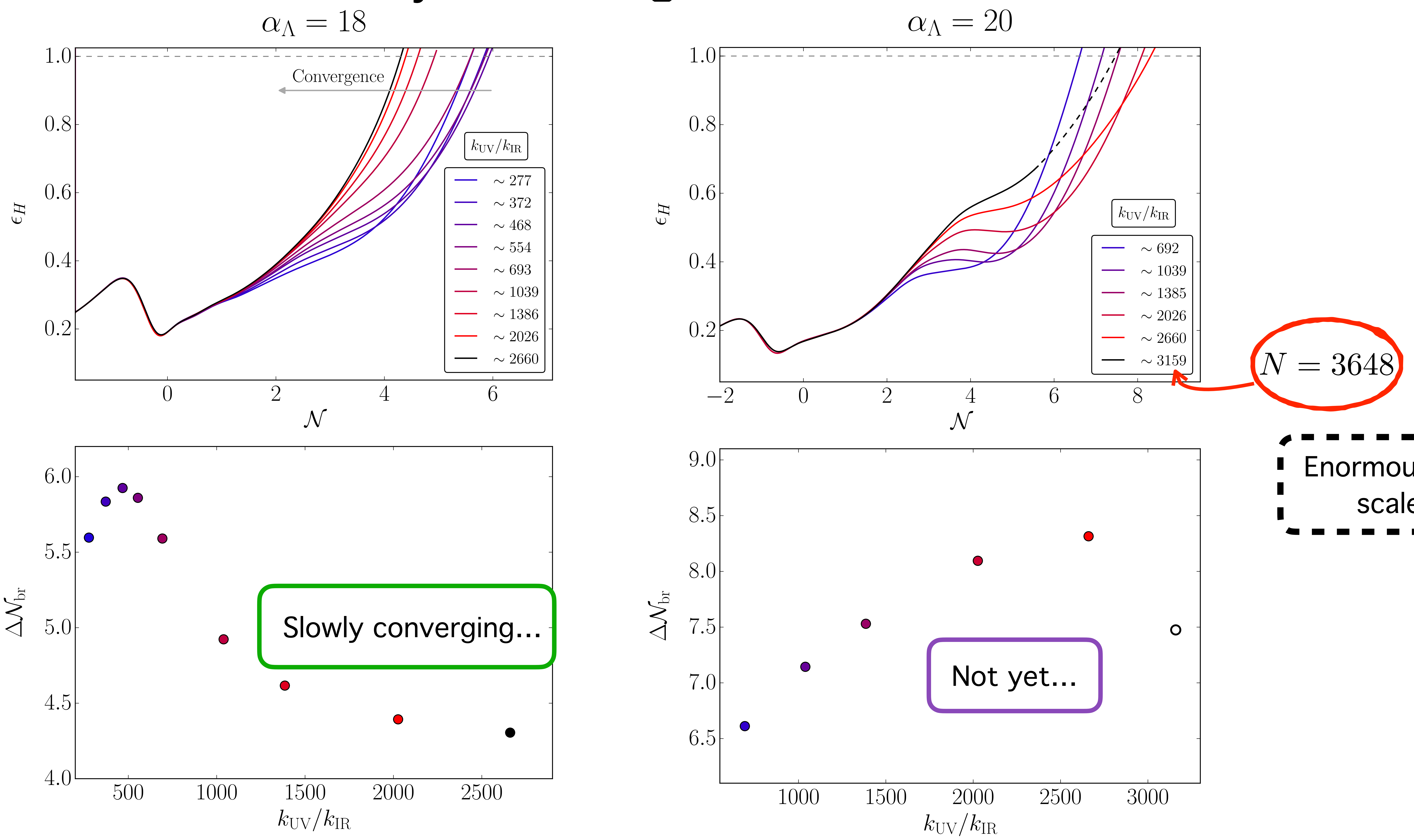
$\mathcal{N} = \mathcal{N}_{\mathrm{end}}$

During backreaction, different evolution depending on UV coverage

Lack of UV resolution distorts the power spectrum, and the self-consistent background evolution



1.3 UV sensitivity: convergence?



Enormous separation of scales required!

Conclusions 1

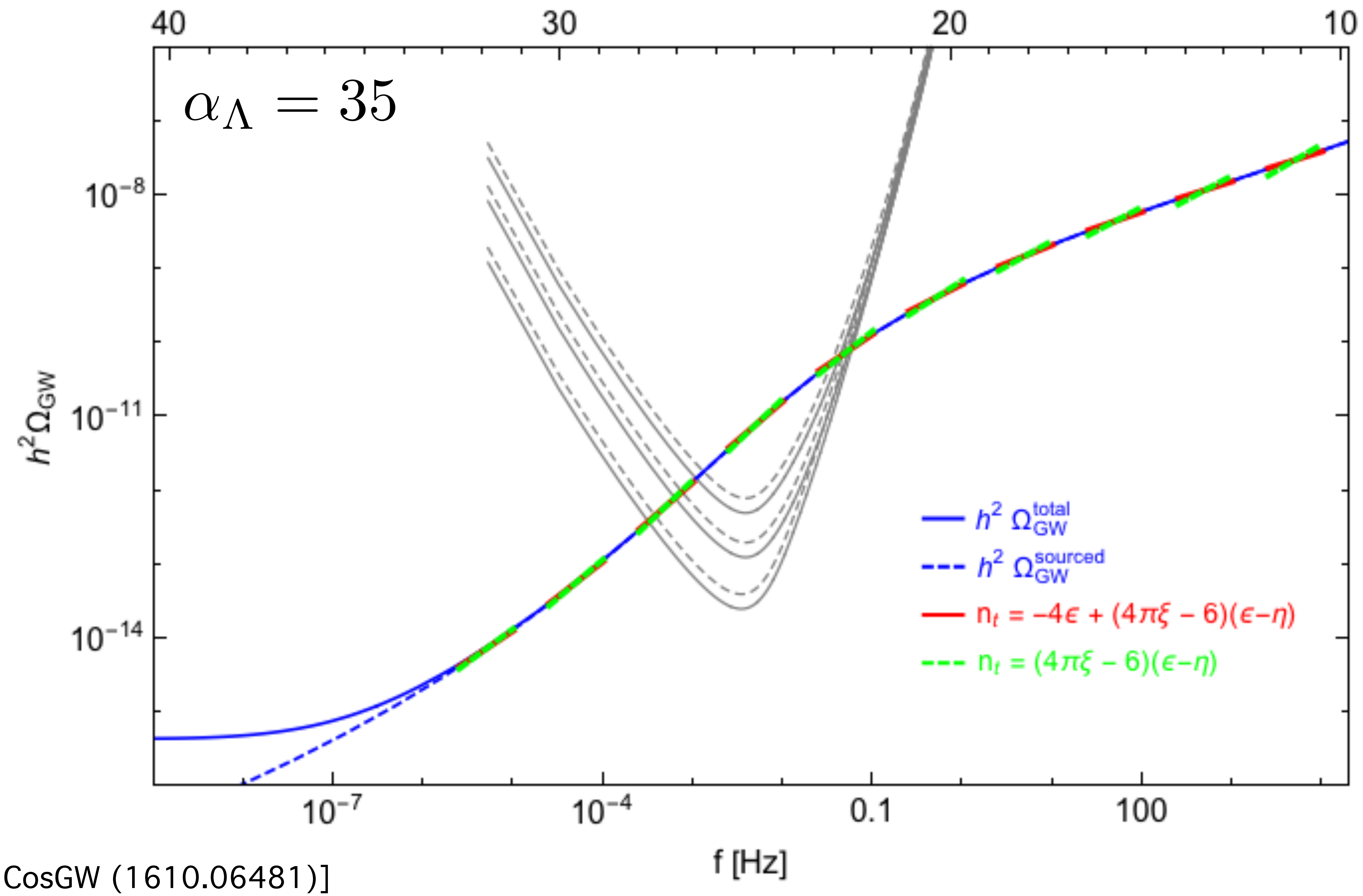
$$\boxed{\frac{\phi}{\Lambda} F\tilde{F}}$$

- Successful implementation of axion-inflation in lattice.
  - Correct reproduction of linear and homogeneous approximations

- Inhomogeneities affect crucially dynamics

- Many interesting features:
  - Extension of inflationary period
  - (Electro)magnetic slow roll
  - Relevance of UV scales and convergence

2. GW generation @ inflation: Asses possible observability



[LISA CosGW (1610.06481)]

GW signal from particle production during axion inflation

- Relevant parameters?
- Any insight from local backreaction procedure? (Lattice)
- Would the signal be detectable?

2. GW generation

Continuum

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = \frac{2}{m_p^2}\Pi^{\rm TT}_{ij}(\mathbf{x},t)$$

$$\Pi^{\rm TT}_{ij} = (\partial_i\phi\partial_j\phi - E_iE_j - \frac{1}{a^2}B_iB_j)^{\rm TT}$$

Lattice

$$\ddot{u}_{ij}(\mathbf{x},t) + 3H\dot{u}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}u_{ij}(\mathbf{x},t) = \frac{2}{m_p^2}\Pi^{\rm eff}_{ij}(\mathbf{x},t)$$

[J. Garcia-Bellido, D.G. Figueroa, A. Sastre (0707.0839)]

$$h_{ij}(\mathbf{k},t) = \Lambda^{\rm L}_{ij,kl}(\hat{\mathbf{k}})u_{kl}(\mathbf{k},t)$$

In Fourier
Recover physical dof

$$\Lambda^{\rm L}_{ij,kl}(\hat{\mathbf{k}}) = P^{\rm L}_{ik}(\hat{\mathbf{k}})P^{\rm L}_{jl}(\hat{\mathbf{k}}) - \frac{1}{2}P^{\rm L}_{ij}(\hat{\mathbf{k}})P^{\rm L}_{kl}(\hat{\mathbf{k}})$$

TT projector


CosmoLattice
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

–Technical Note II: Gravitational Waves–

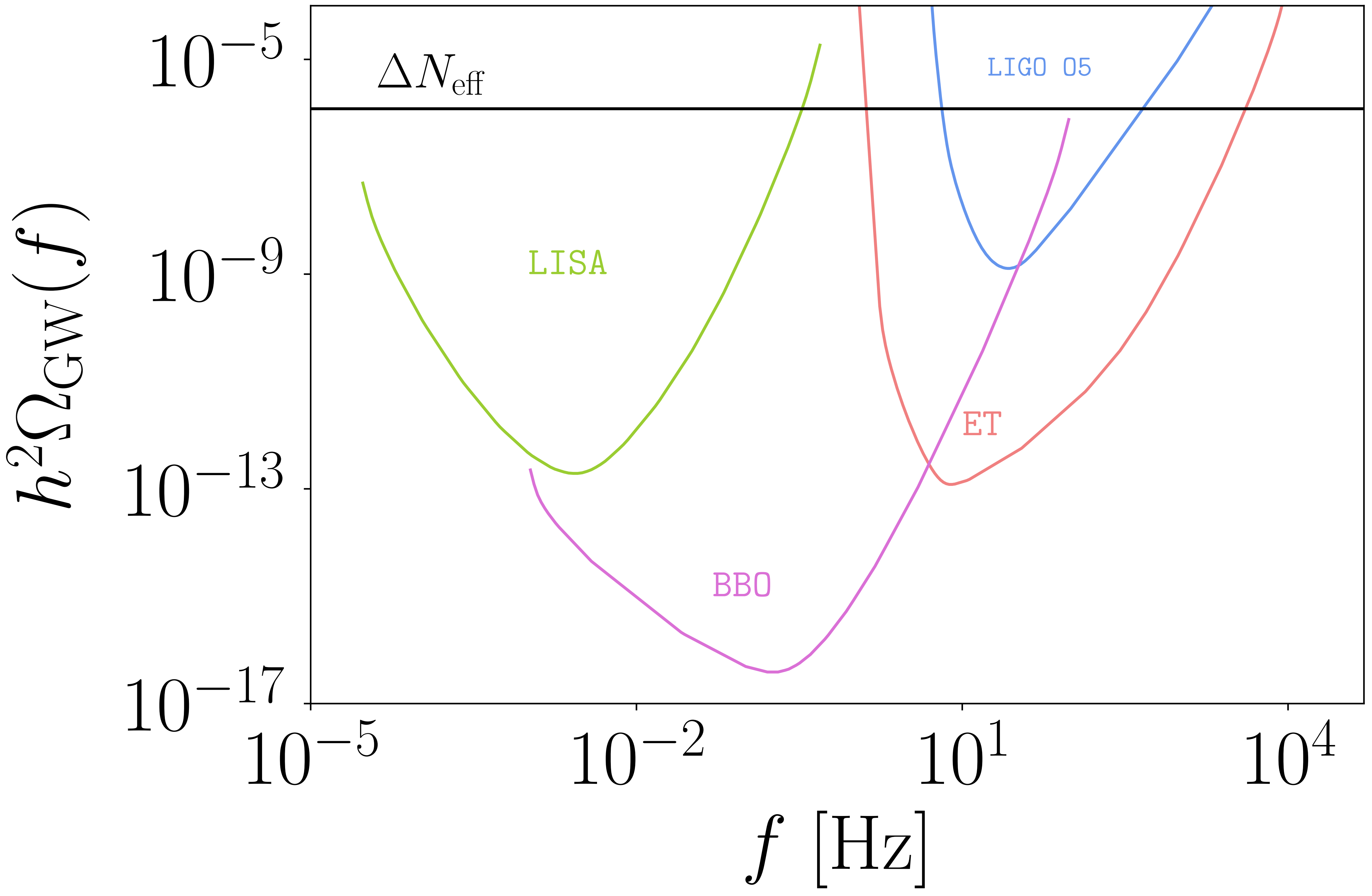
Written on May 6, 2022
(Corrected on June 20, 2023)

CosmoLattice
A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

– Technical Note III –
Gravitational Waves from U(1) Gauge Theories

Written on June 16, 2023

2. GW generation @ inflation: procedure



Possible observability at low frecuencies
deep inside inflation

Strategy:

1- Consider a set of strong couplings

$$\alpha_\Lambda = 15,\ 20,\ 25,\ 30,\ 35$$

2- Divide spectral range into ... lly superimposed pat...

3- Every patch ... sub-Hubble to very ... Hubble

4- Extract GW power spectrum

5- Concatenate all patches: whole cosmic evolution

Work in progress

2. GW generation @ inflation: procedure

Some notes on the procedure:

1- Preheating bounds $\alpha_\Lambda \lesssim 14$

[Adshead et al (1909.12842) & (1909.12843)]

Might be affected by different phenomena at the end on inflation (PBH dominance...)
We keep open the parameter window

2- Well inside inflation, negligible backreation.
Backreaction less solution valid until onset of non-linearities

$$\ddot{\phi} = -3H\dot{\phi} + \frac{1}{a^2}\vec{\nabla}^2\phi - m^2\phi + \frac{\alpha_\Lambda}{a^3 m_p}\vec{E}\cdot\vec{B}$$

$$\dot{\vec{E}} = -H\vec{E} - \frac{1}{a^2}\vec{\nabla}\times\vec{B} - \frac{\alpha_\Lambda}{a m_p}\left(\dot{\phi}\vec{B} - \vec{\nabla}\phi\times\vec{E}\right)$$

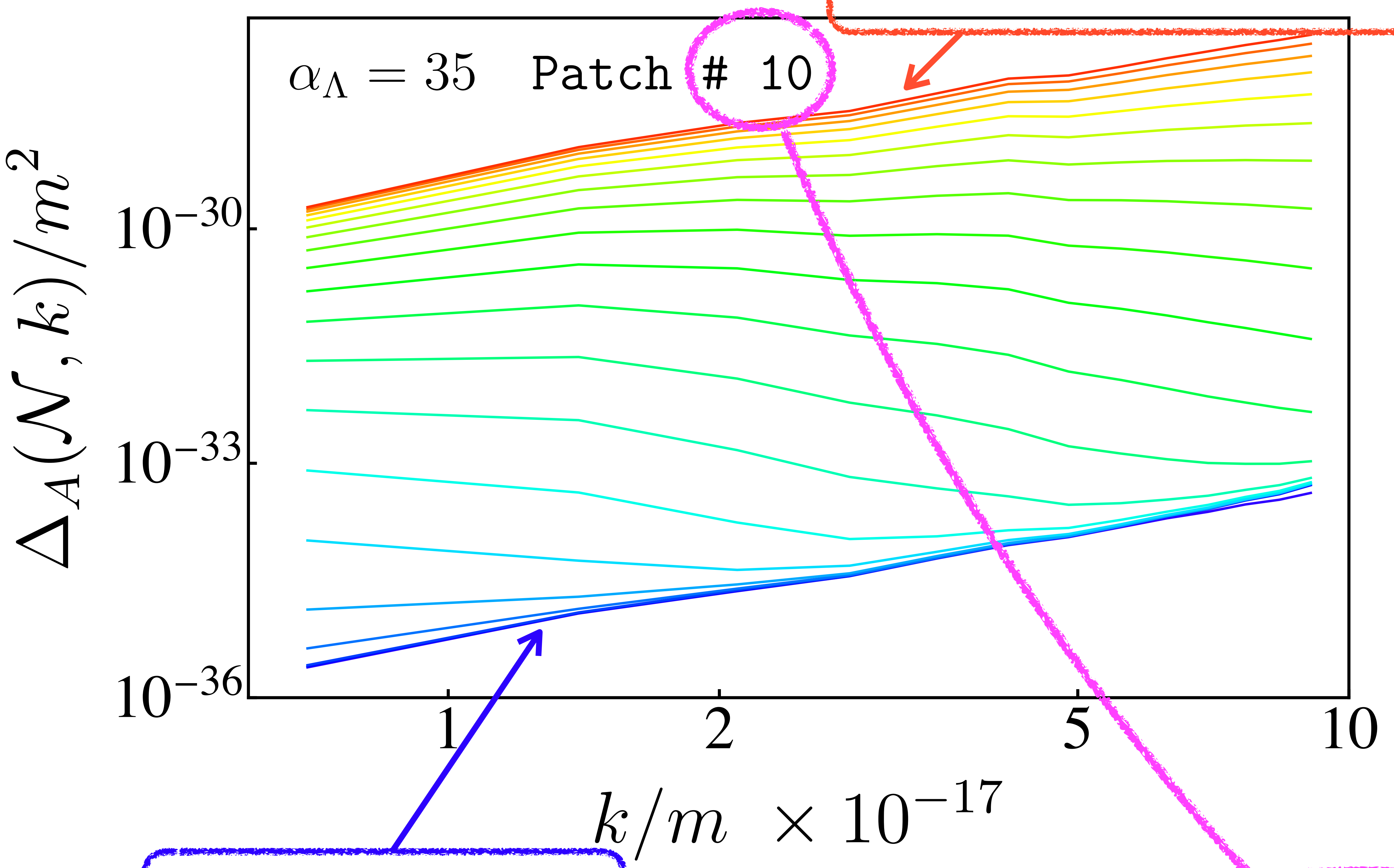
$$\ddot{a} = -\frac{a}{3m_p^2}\left(2\rho_{\rm K} - \rho_{\rm V} + \rho_{\rm EM}\right)$$

3- GW generated only through the tachyonic excitation of the gauge modes

$$\Pi^{\rm eff}_{ij} \approx E_i E_j - \frac{1}{a^2}B_i B_j$$

2. GW generation @ inflation: procedure

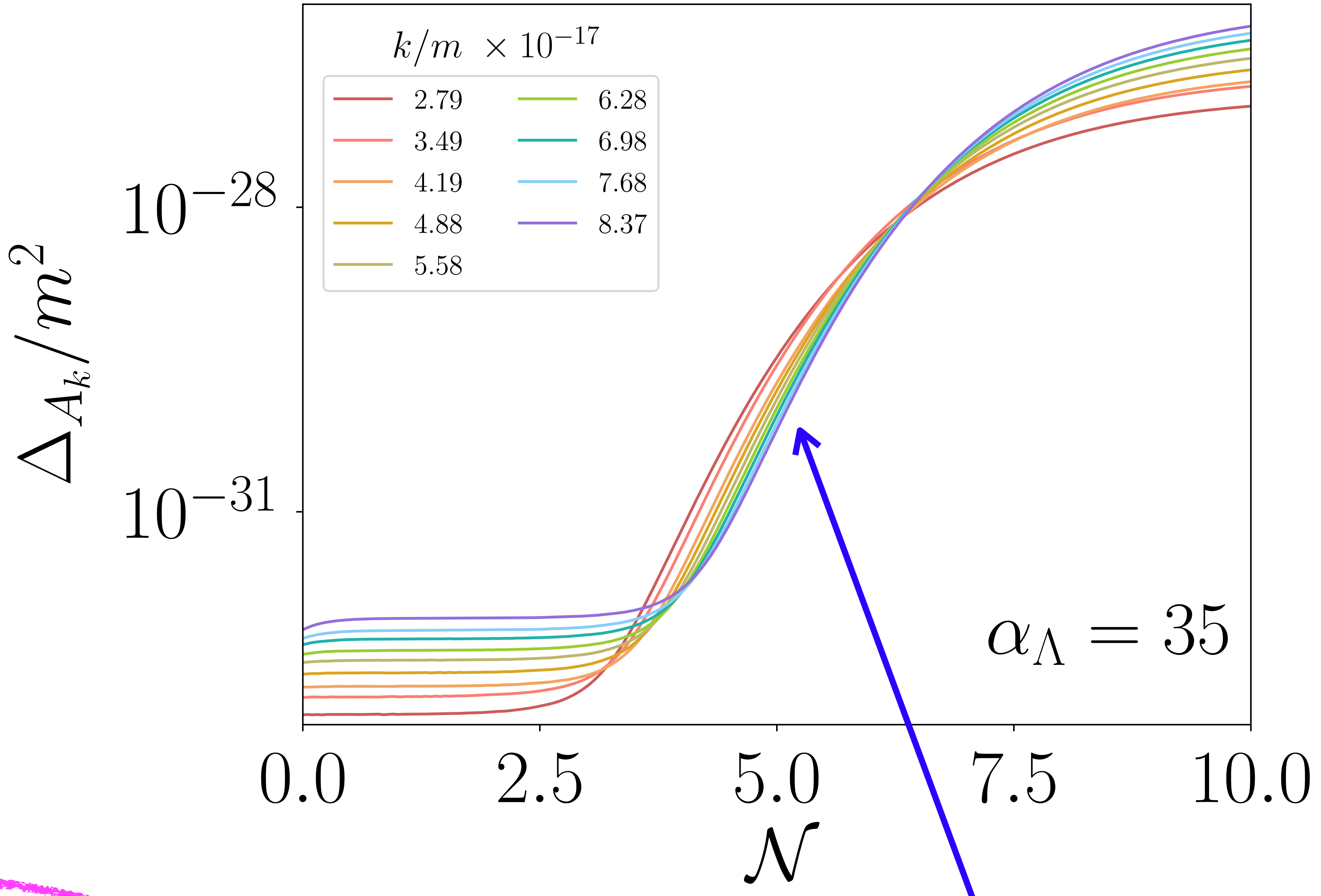
super-Hubble excitation
$\Delta\mathcal{N} = 10$



IC: vacuum solution
$\propto k^2$

~30-40 patches of
N~200 in total until end of
slow roll inflation

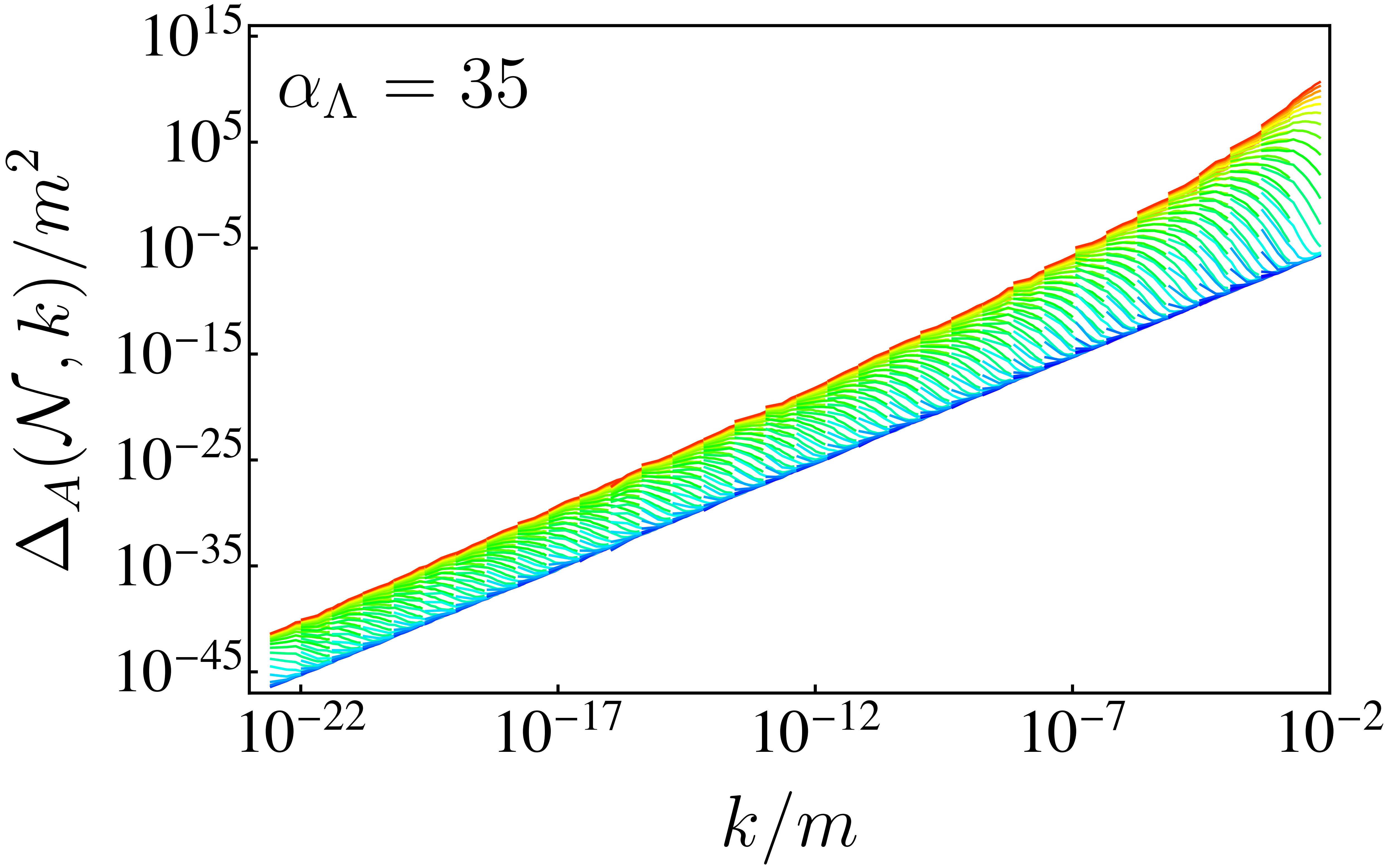
Mode by mode evolution

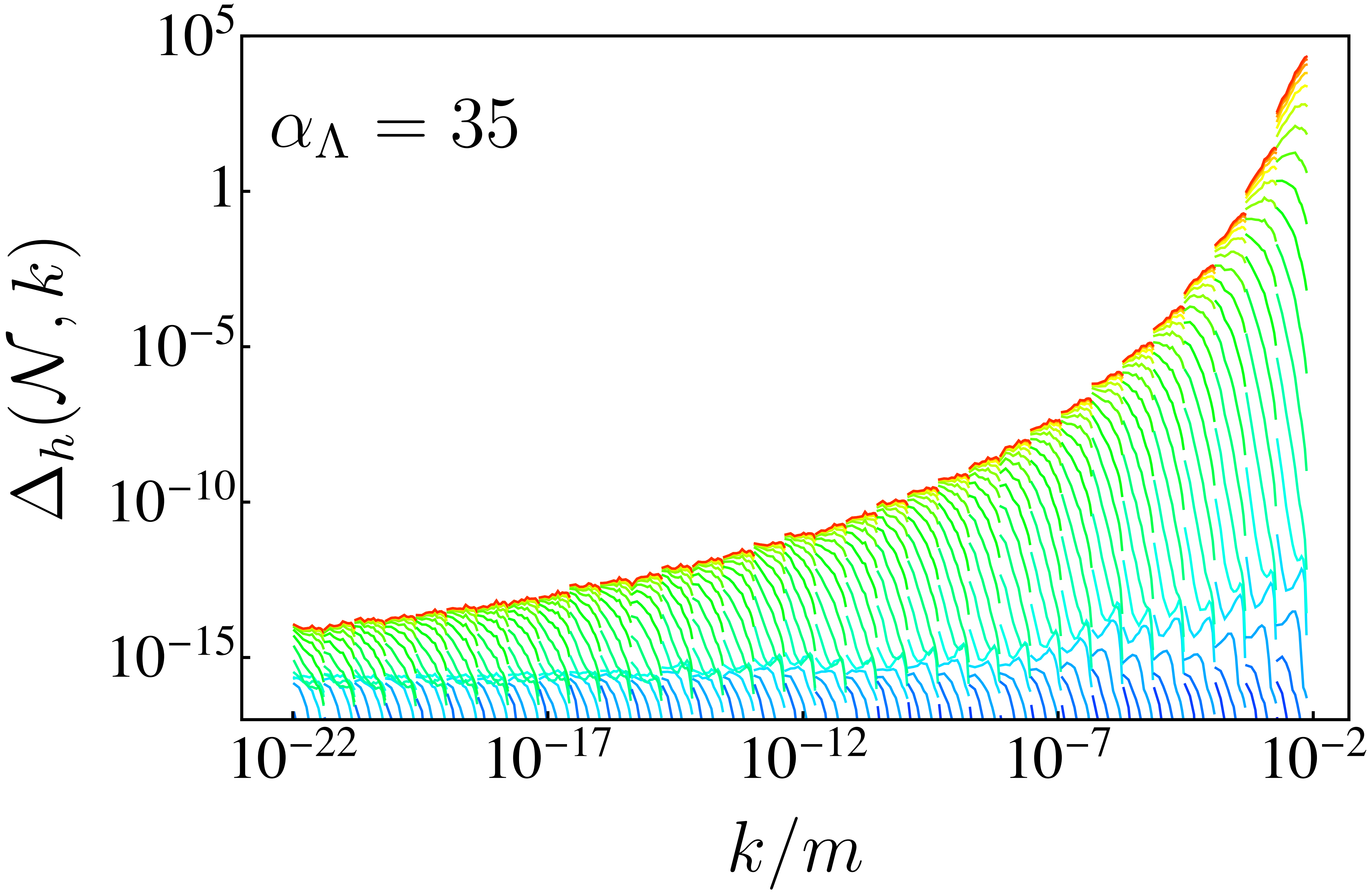


Tachyonic growth

2. GW generation @ inflation: procedure

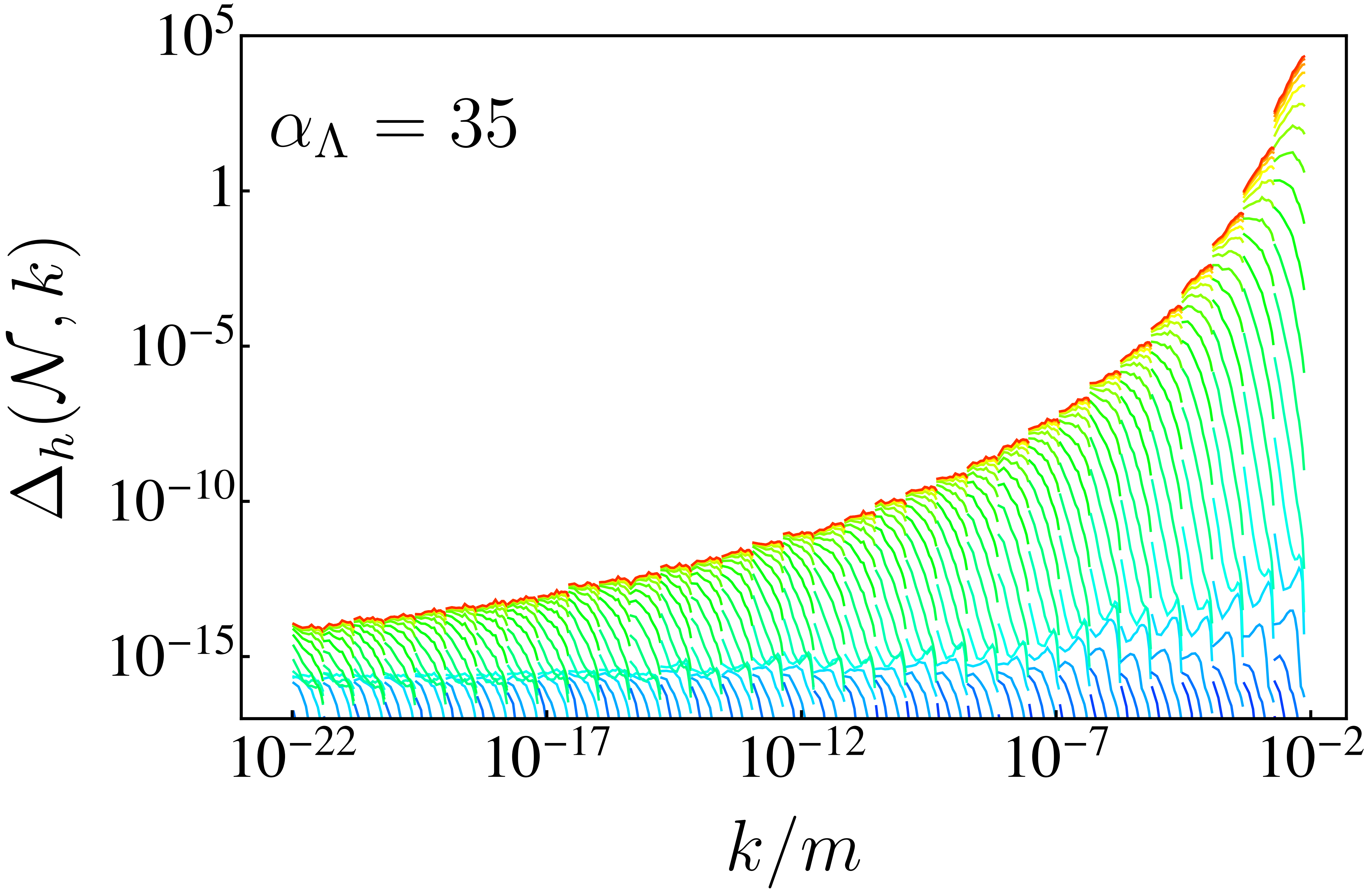
2. GW generation @ inflation: procedure

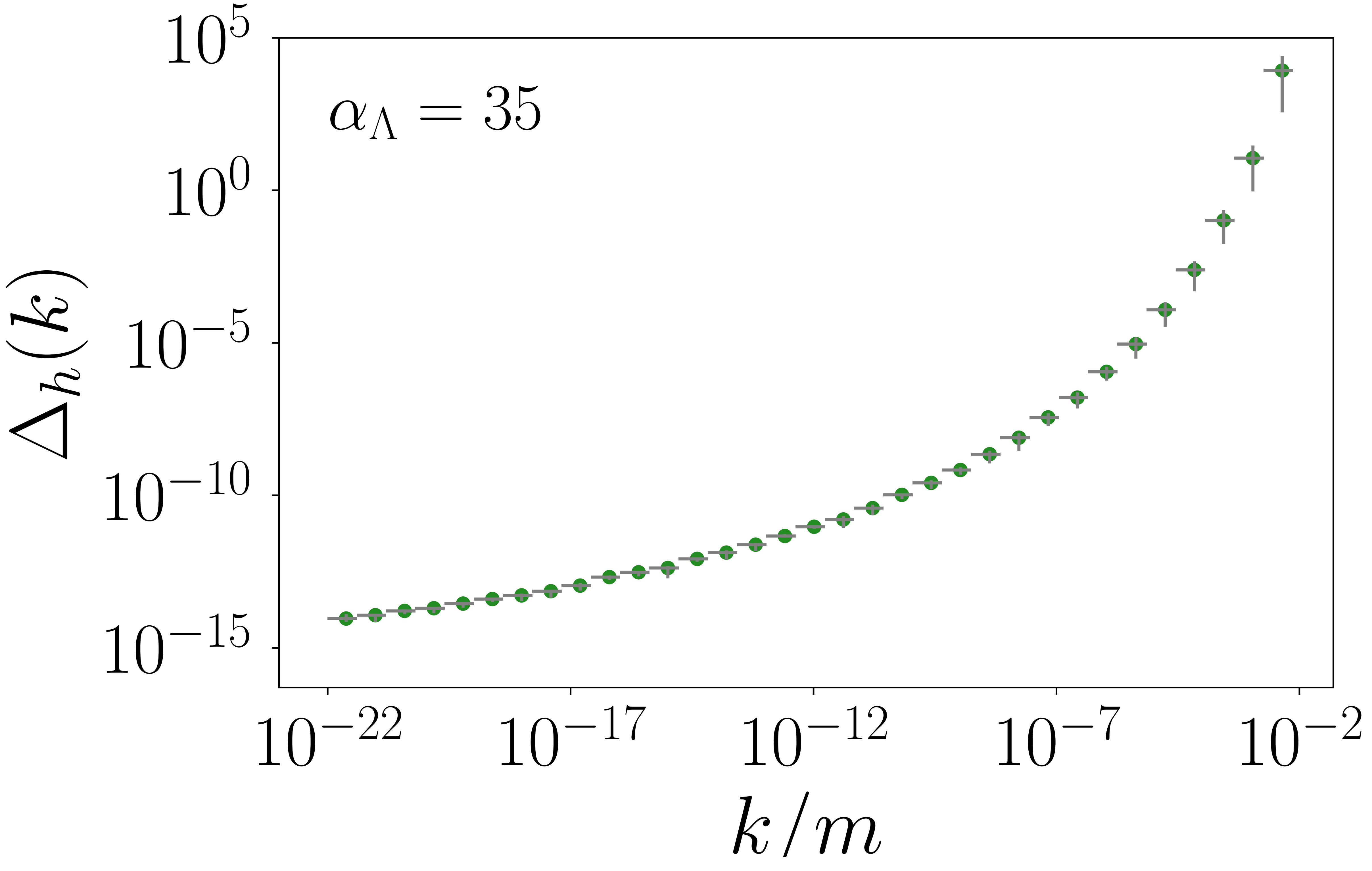




2. GW generation @ inflation: procedure

Re-cast information:
saturated super-Hubble value
with “error” bands for every patch

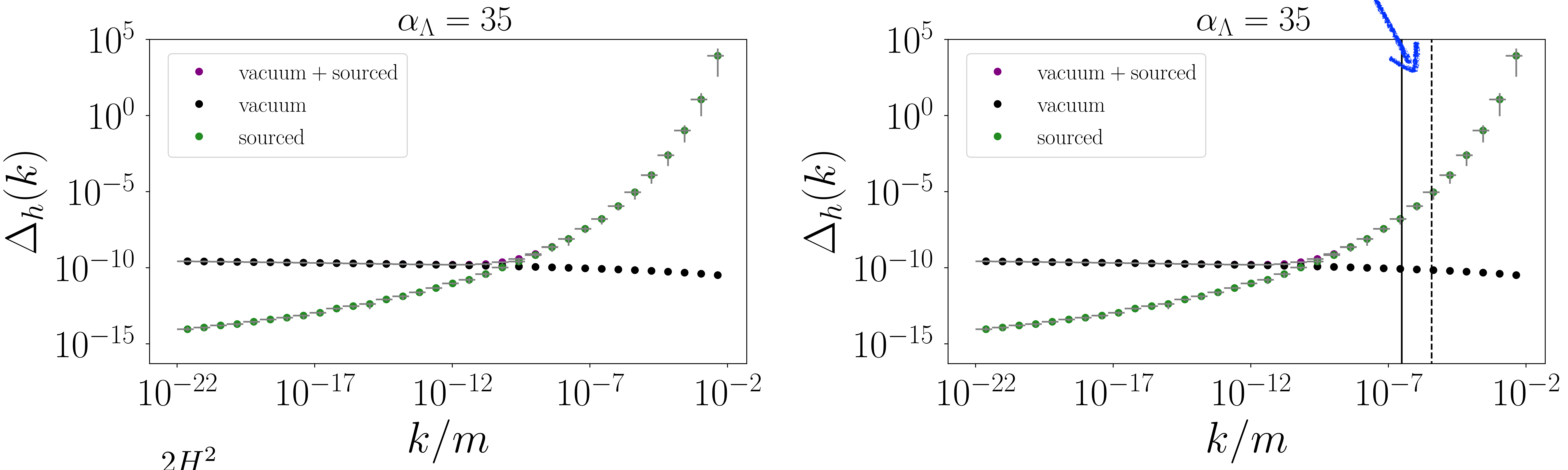




2. GW generation @ inflation: procedure

How does this compare with the vacuum prediction?

Onset of non-linearities
EM ~1%



$$\Delta_h^{\mathrm{vac}} = \frac{2H^2}{\pi^2 m_{\mathrm{pl}}^2}$$

All the way up to end of slow roll inflation

2. GW generation @ inflation: today

From comoving wavenumber to frecuency domain

Redshift affected by:

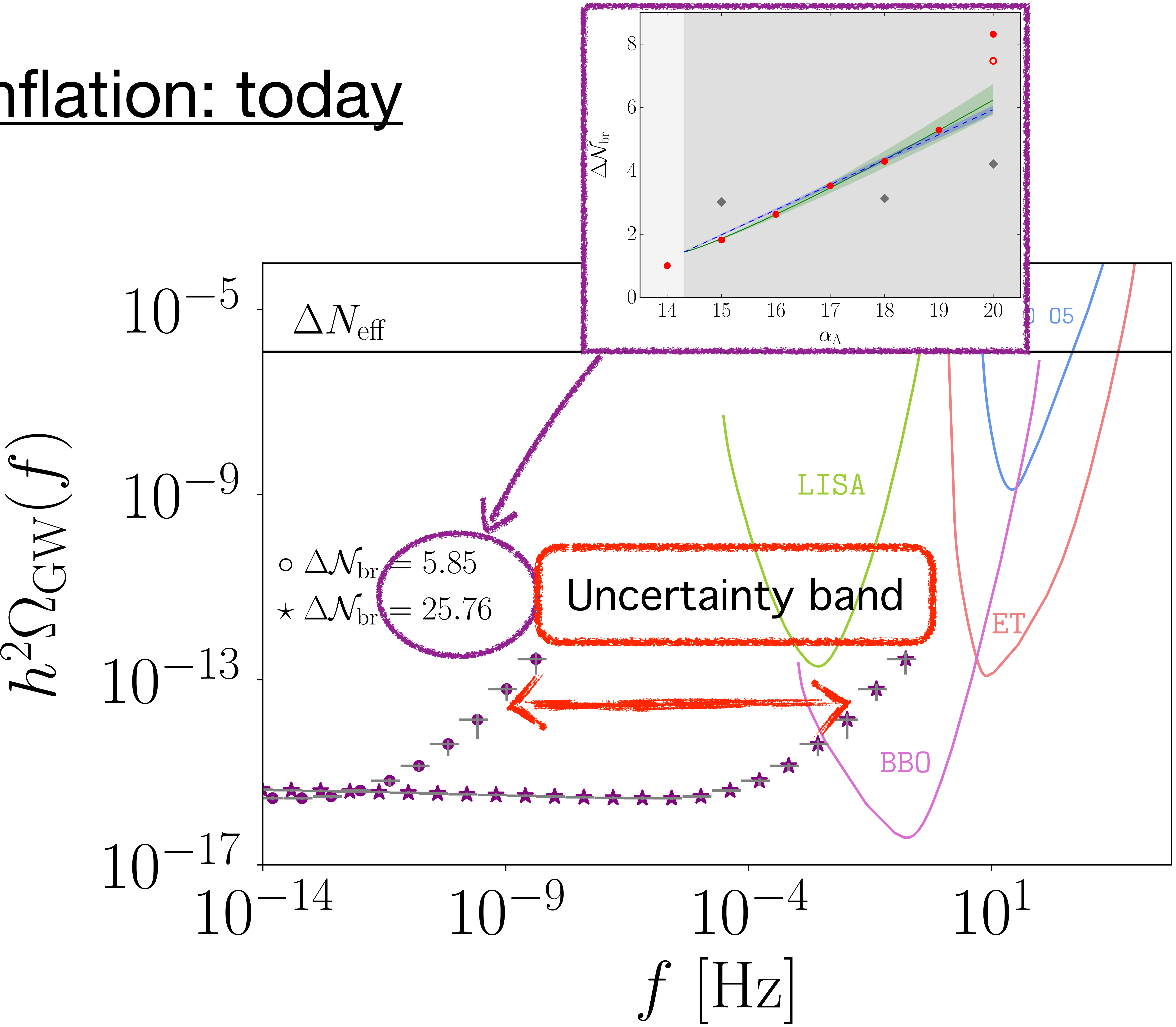
1- Number of extra e-folds in inflation

$$\Delta\mathcal{N}_{\rm br}$$

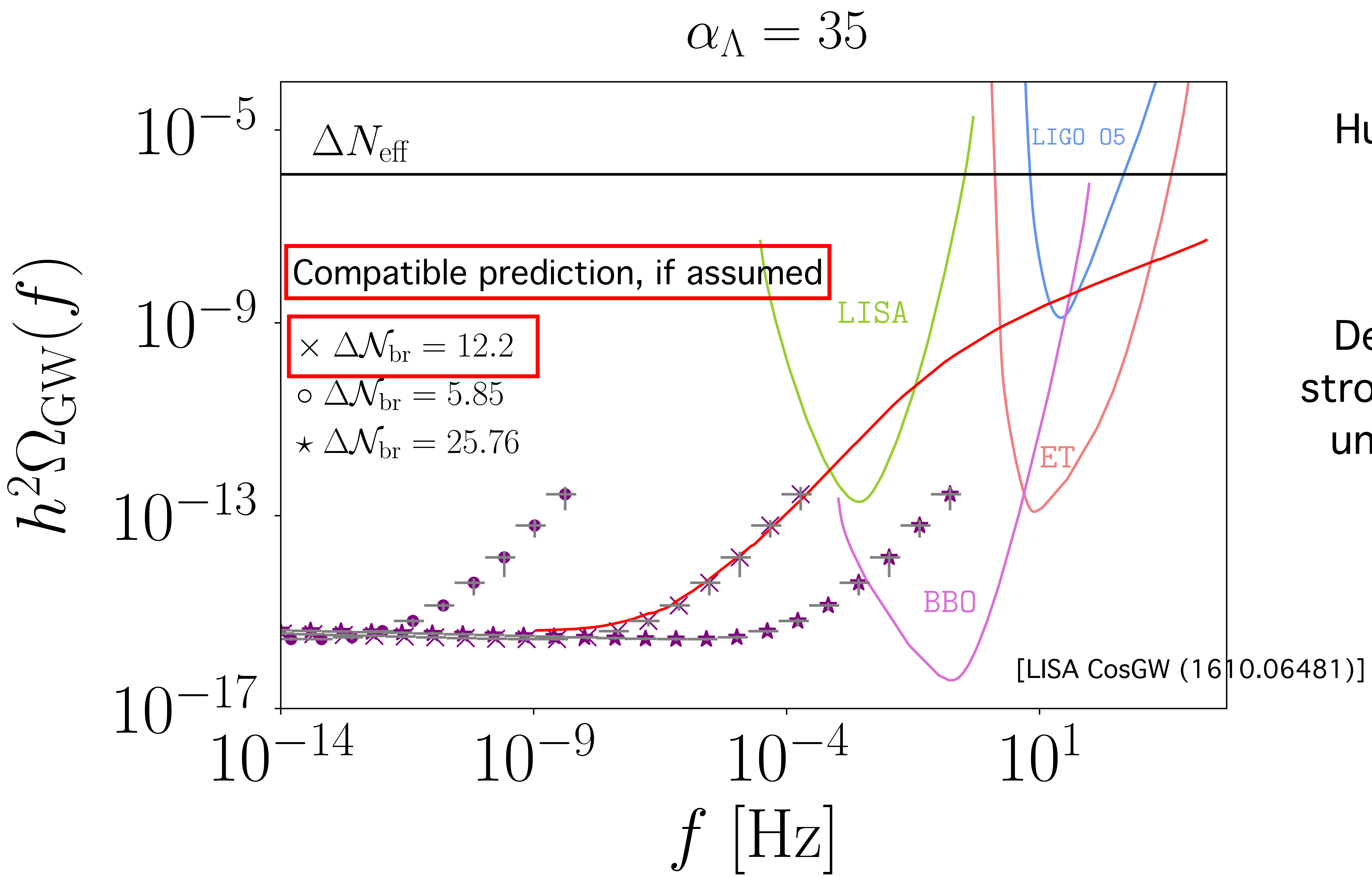
2- Details of evolution from the end of inflation to RD

$$\Delta\mathcal{N}_{\rm RD} \sim 2$$

Exact numbers only from lattice simulations



2. GW generation @ inflation: today



Huge uncertainty in the prediction

Detectability subject to strong backreaction details until the end of inflation

Dedicated lattice studies needed

2. GW generation @ inflation: today

Smaller couplings:

2. GW generation @ inflation: chirality?

$$\langle h_+(\mathbf{k})\, h_+(\mathbf{k}')\rangle \simeq 8.6 \times 10^{-7}\, \frac{H^4}{M_P^4}\, \frac{e^{4\pi\xi}}{\xi^6}\, \frac{\delta(\mathbf{k}+\mathbf{k}')}{k^3},$$

$$\langle h_-(\mathbf{k})\, h_-(\mathbf{k}')\rangle \simeq 1.8 \times 10^{-9}\, \frac{H^4}{M_P^4}\, \frac{e^{4\pi\xi}}{\xi^6}\, \frac{\delta(\mathbf{k}+\mathbf{k}')}{k^3}.$$

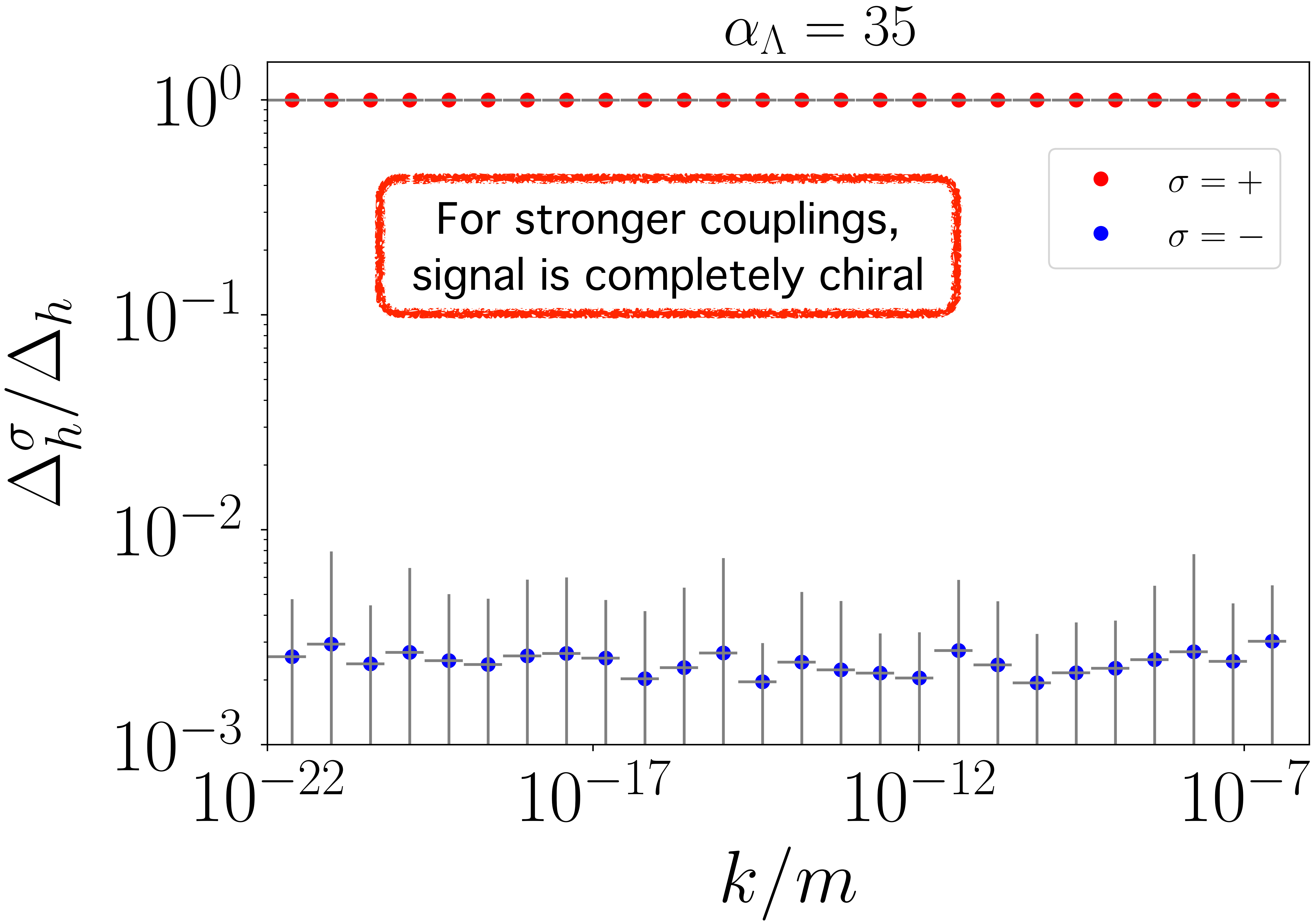
[L. Sorbo (1101.1525)]

High chirality imbalance expected

Power spectra

$$\Delta_h^\sigma = \frac{k^3}{2\pi^2}|h_{ij}^\sigma (h_{ij}^\sigma)^*|^2$$

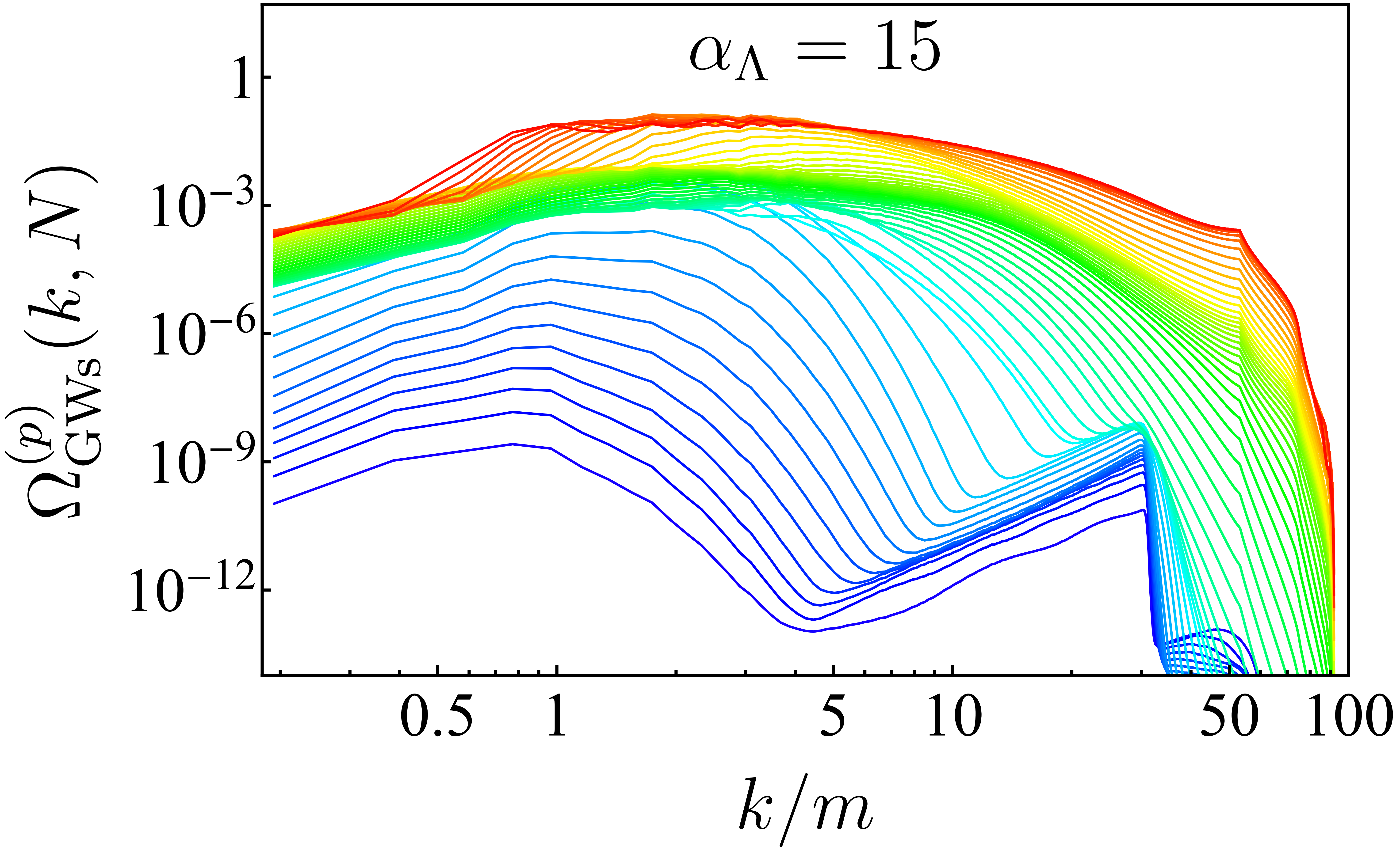
$$\Delta_h = \sum_{\sigma=\pm} \Delta_h^\sigma$$



2. GW generation @ Preheating

From fully local lattice simulations

Until RD $\epsilon_H = 2$



2. GW generation @ Preheating

From fully local lattice simulations

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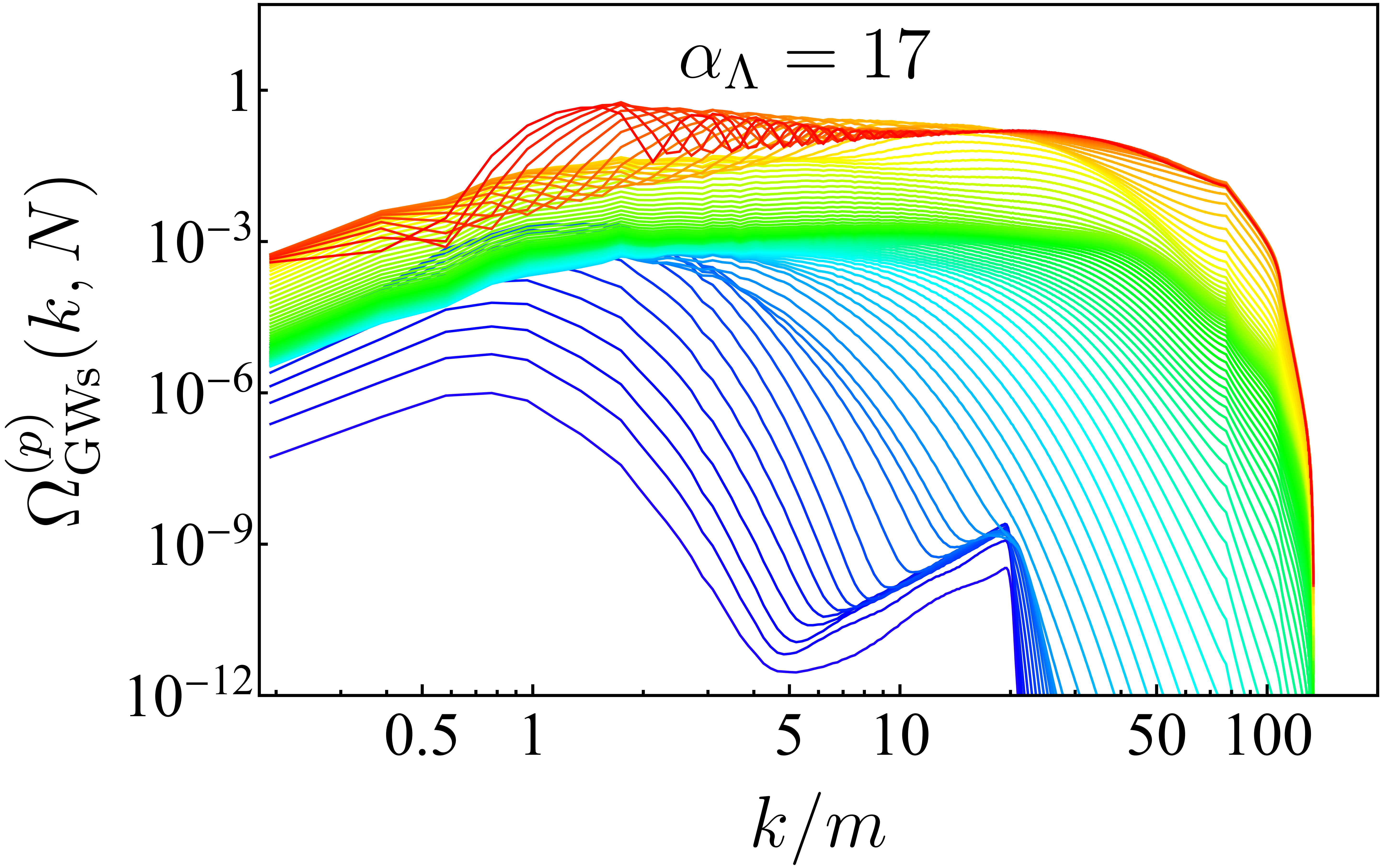
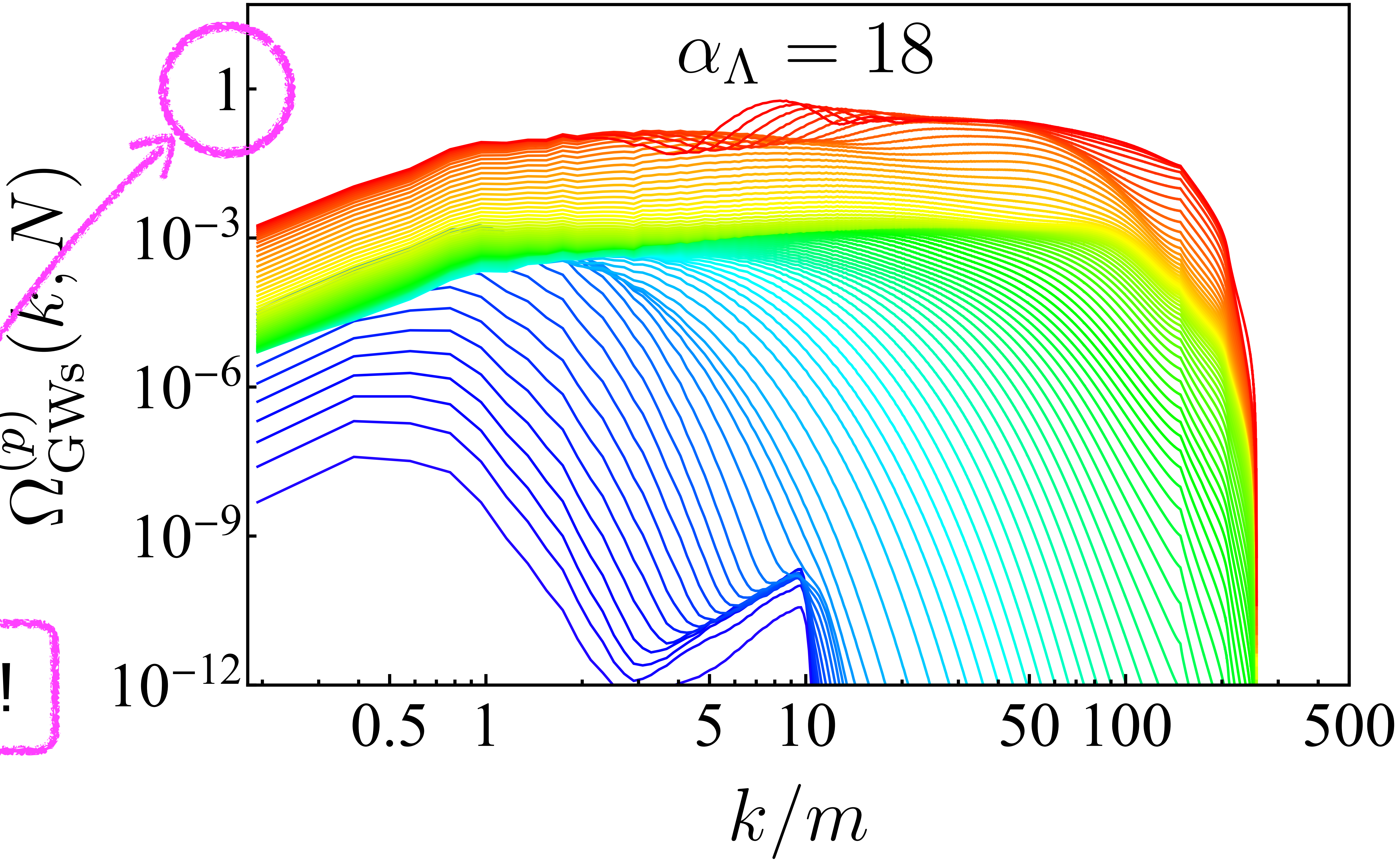


2. GW generation @ Preheating

From fully local lattice simulations

Until RD $\epsilon_H = 2$

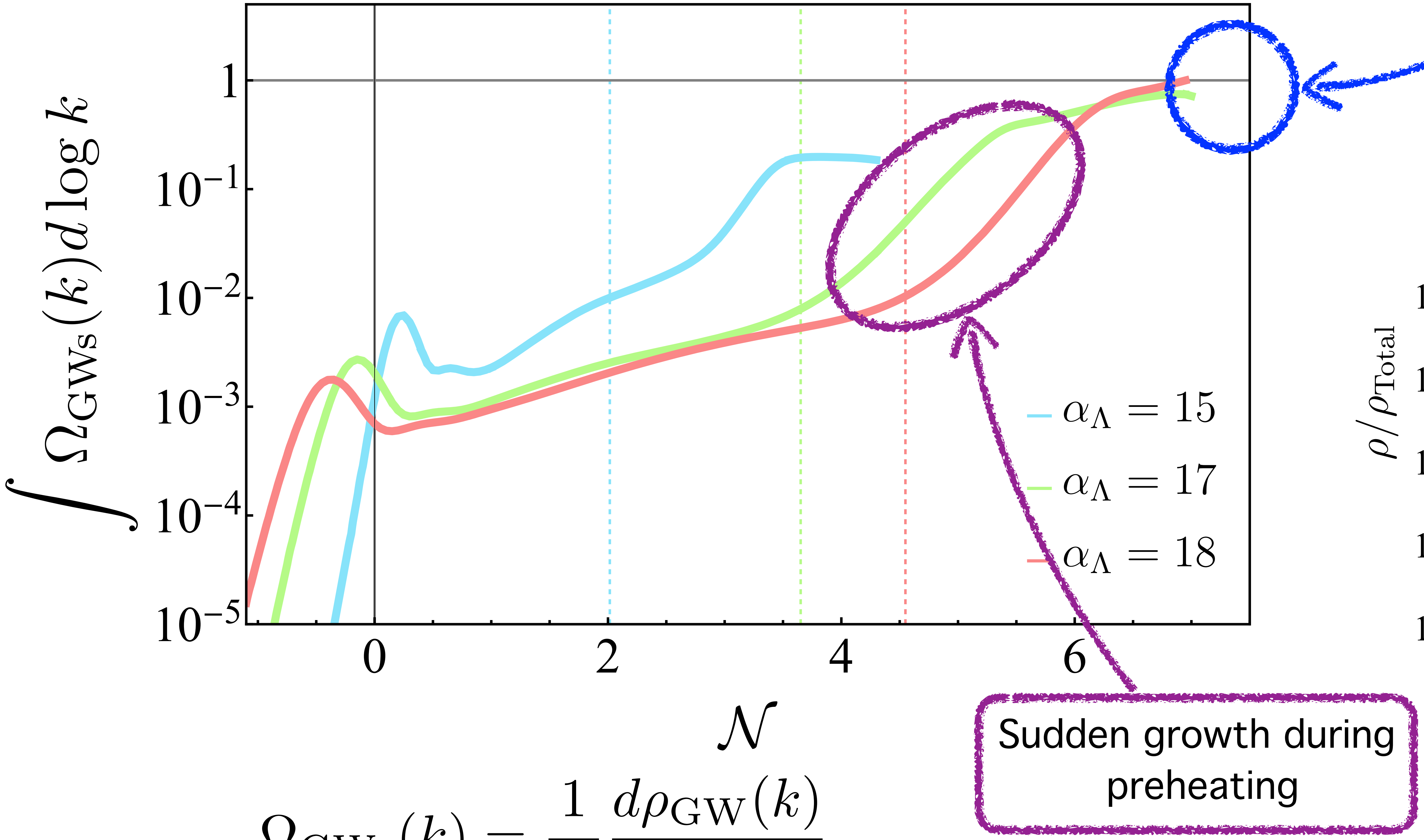


Huge signal!

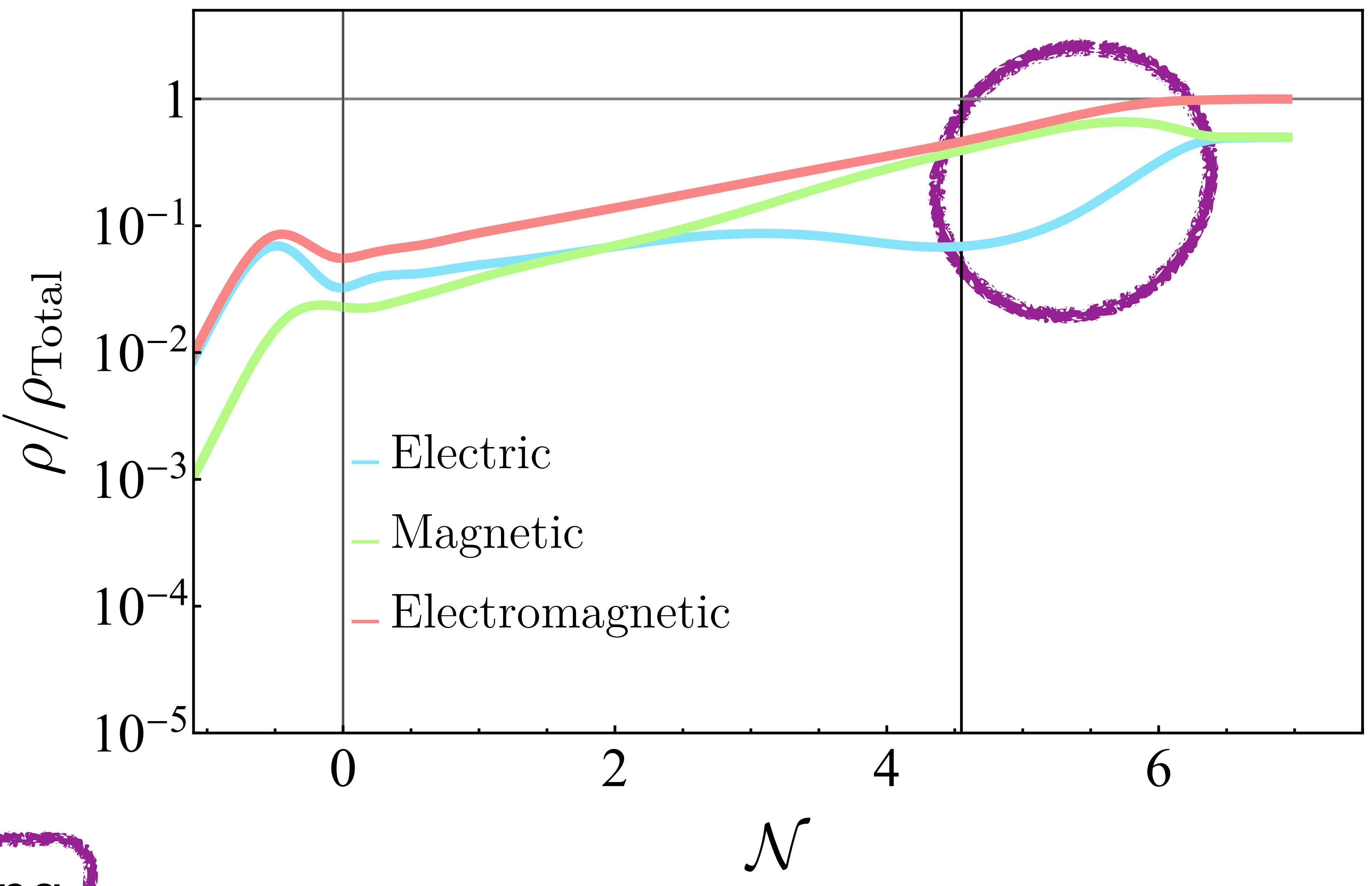
2. GW generation @ Preheating

From fully local lattice simulations

Until RD $\epsilon_H = 2$



$$\Omega_{\mathrm{GWs}}(k) = \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}(k)}{d\log k}$$



From (electro)magnetic slow roll to equipartition

Conclusions 2: GWs in axion inflation

Realistic detectability prospects are subject to:

- Accurate gravitational wave (GW) production modeling: analytical and lattice predictions seem to coincide.

- Accurate understanding of cosmic history:
    - Non-trivial effects during strong backreaction not only affect high frecuencies, but the whole signal.
    - Dedicated full lattice simulation for strong couplings are required.

- Preheating: preheating models suggest that gravitational backreaction must be included for an precise description.

Thank you!