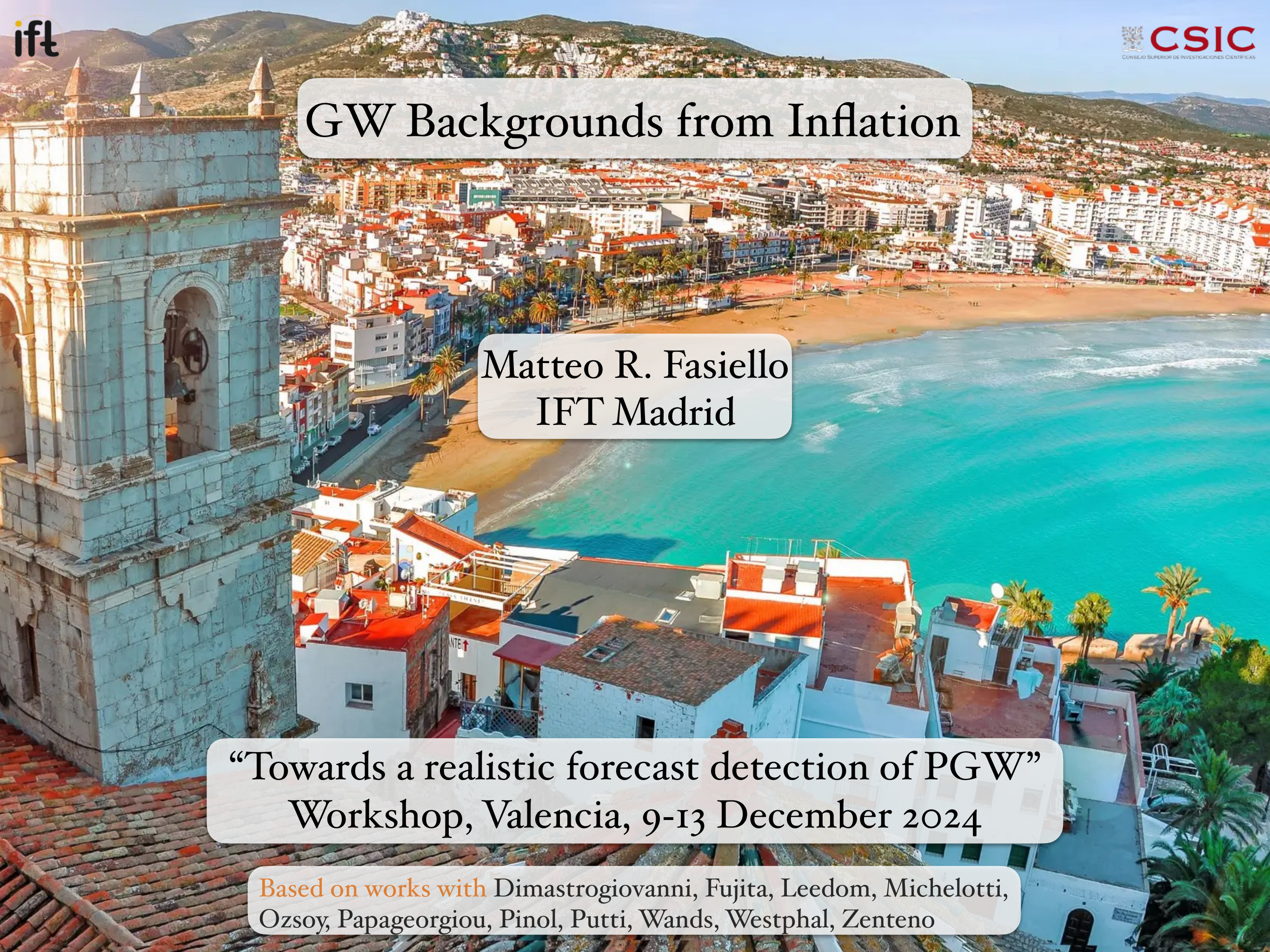




GW Backgrounds from Inflation

Matteo R. Fasiello
IFT Madrid

“Towards a realistic forecast detection of PGW”
Workshop, Valencia, 9-13 December 2024

Based on works with Dimastrogiovanni, Fujita, Leedom, Michelotti, Ozsoy, Papageorgiou, Pinol, Putti, Wands, Westphal, Zenteno

Inflation, the minimal paradigm, SFSR

Simplest realization: single-scalar field in slow-roll

- Scalar field :

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi) \approx -V(\phi)$$
$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi) \approx V(\phi)$$

$$\dot{\phi}^2 \ll V$$

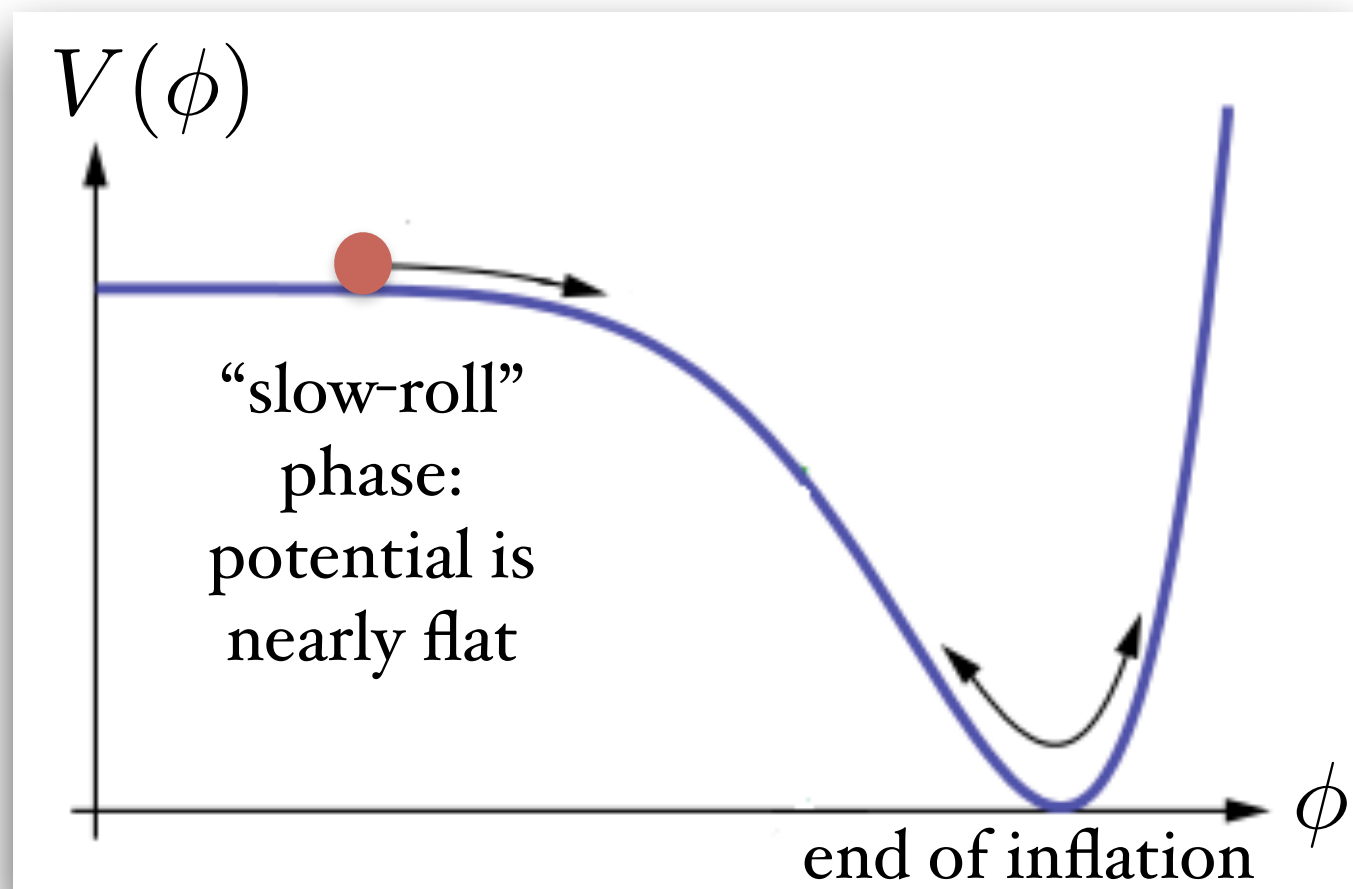
$$p_\phi \approx -\rho_\phi$$

start flat

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_{\text{P}}^2}{2} \left(\frac{V'}{V} \right)^2 \simeq \frac{3}{2} \frac{\dot{\phi}^2}{V} \ll 1$$

stay flat

$$|\eta| \equiv \frac{|\dot{\epsilon}|}{H\epsilon} \simeq -\frac{2}{3} \left(\frac{V''}{H^2} \right) + 4\epsilon \ll 1$$



Primordial Fluctuations

(minimal scenario)

$$ds^2 = (-dt^2 + a(t)^2 [e^{2\zeta} \delta_{ij} + \gamma_{ij}] dx^i dx^j)$$

scalar fluctuations

tensor perturbations

Primordial Fluctuations

(minimal scenario)

$$ds^2 = (-dt^2 + a(t)^2 [e^{2\zeta} \delta_{ij} + \gamma_{ij}] dx^i dx^j)$$

scalar fluctuations

$$\mathcal{P}_\zeta(k) = \frac{1}{8\pi^2} \frac{1}{\epsilon} \frac{H^2}{M_{\text{pl}}^2} \left(\frac{k}{k_*} \right)^{n_s-1}$$

$n_s - 1 \simeq -2\epsilon - \eta$

$$2.2 \times 10^{-9}$$

$$0.9649 \pm 0.0042$$

$$[k_* = 0.05 \text{ Mpc}^{-1}, 68\% \text{ C.L.}]$$

from Planck measurements
of CMB anisotropies

tensor perturbations

$$\mathcal{P}_\gamma^{\text{vacuum}}(k) = \frac{2}{\pi^2} \frac{H^2}{M_{\text{pl}}^2} \left(\frac{k}{k_*} \right)^{n_T}$$

energy scale of inflation

red tilt

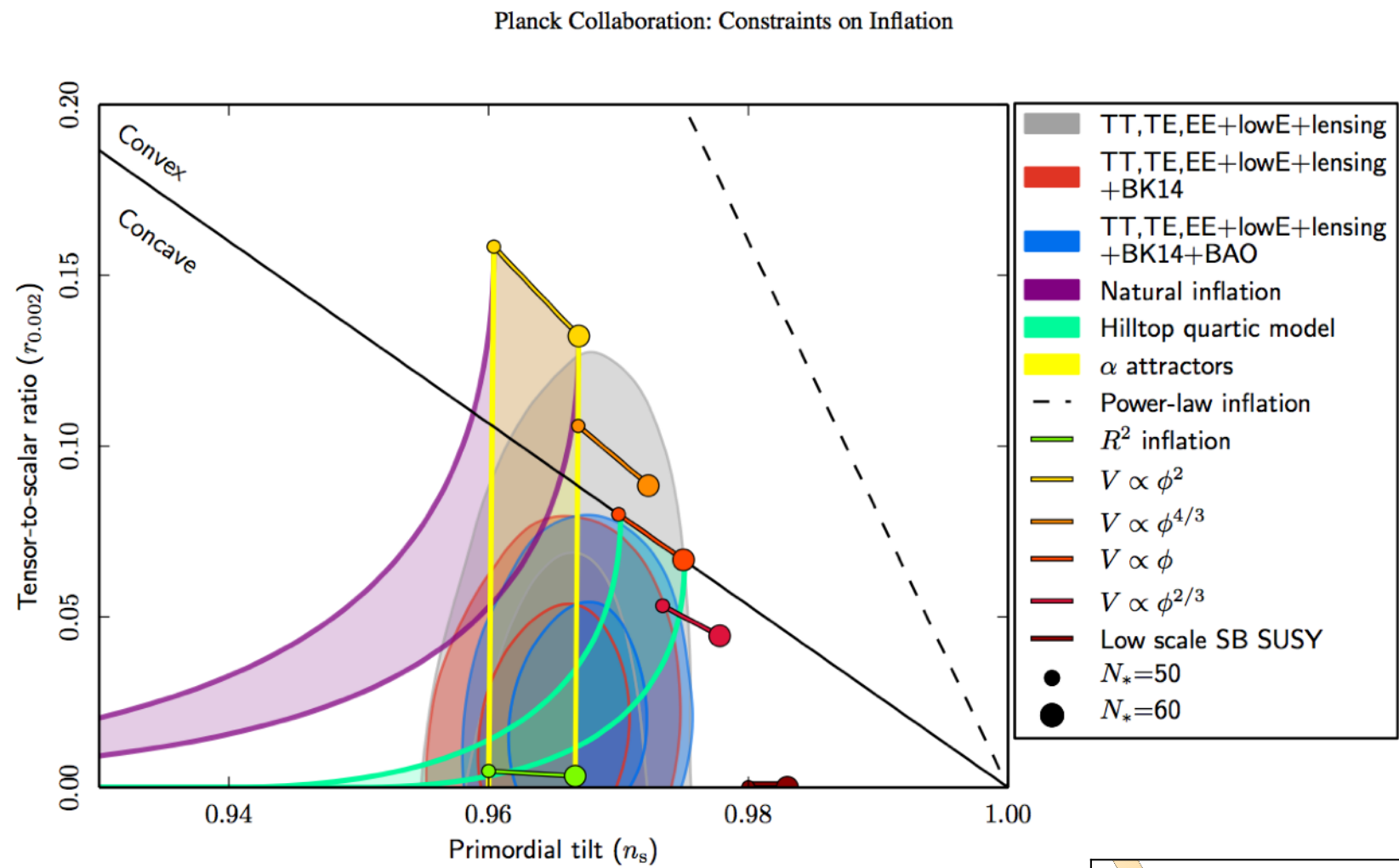
$$n_T \simeq -2\epsilon \simeq -r/8$$

tensor-to-scalar ratio $r \equiv \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta}$

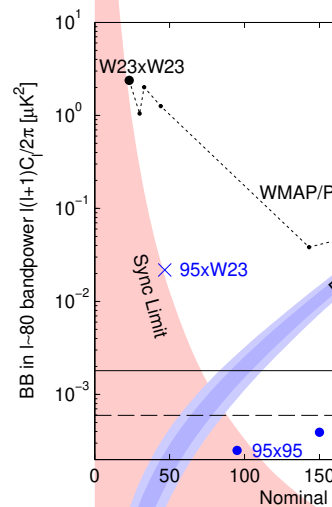
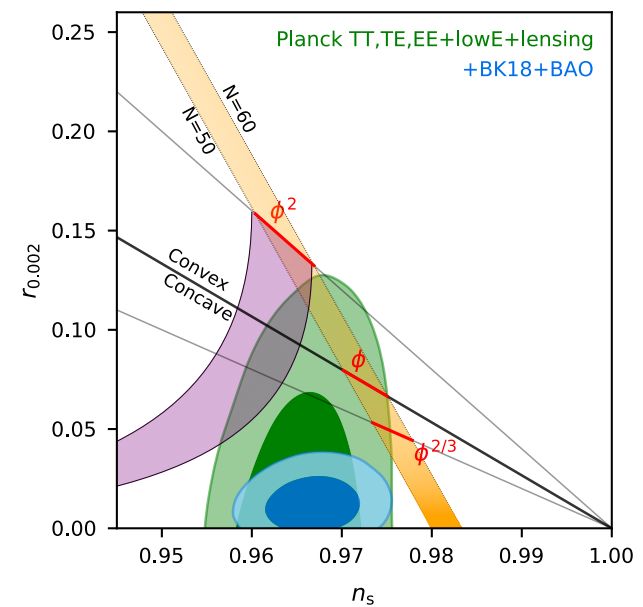
bounds $\left\{ \begin{array}{l} \text{current} \\ r < 0.032 \text{ (95\%CL, Planck}^+ \text{)} \\ \text{future} \\ r < 0.01 \text{ (CMB-S3); } r < 0.001 \text{ (-S4)} \end{array} \right.$

LiteBIRD

@ CMB scales



+BK update:



We may additionally ask for

technically natural light inflaton

give the theory a chance
at solving the eta
problem

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“embeddable” scenarios

a + if e.g. string theory
constructions can realise
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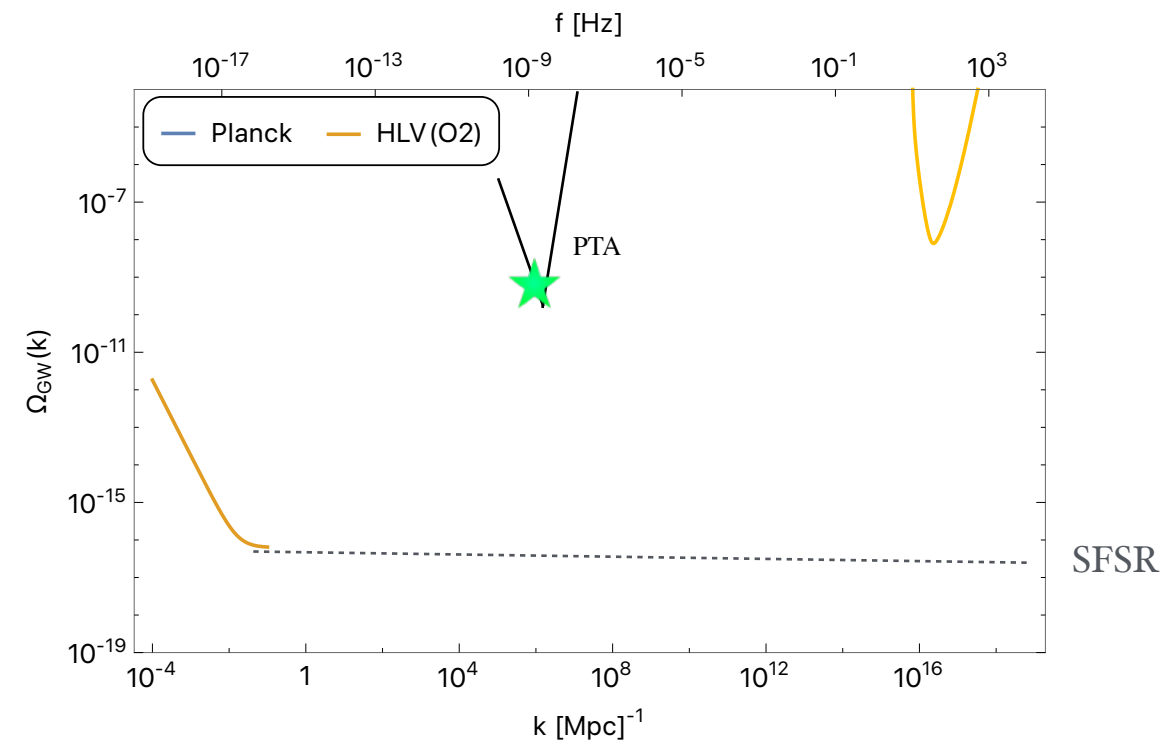
a + if e.g. string theory
constructions can realise
your Lagrangian

viable ones

CMB quite constraining,
many more bounds
coming
(20 decades in frequency)

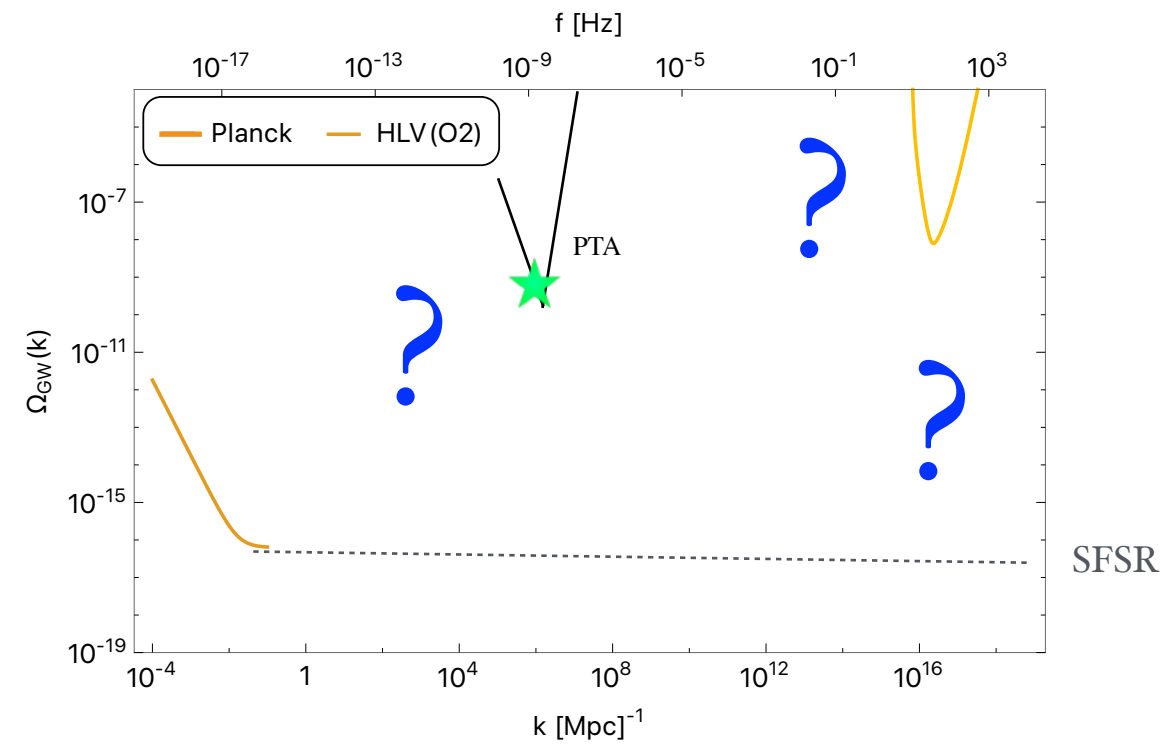
Hope for a little more (GW)

now



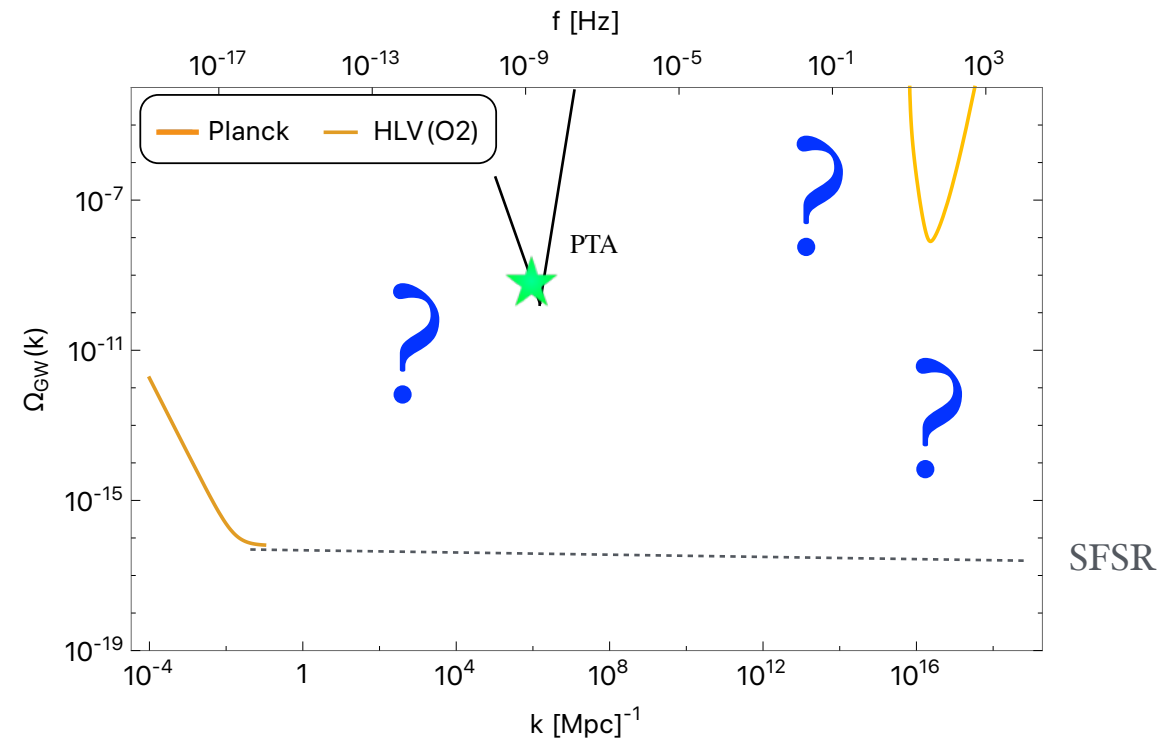
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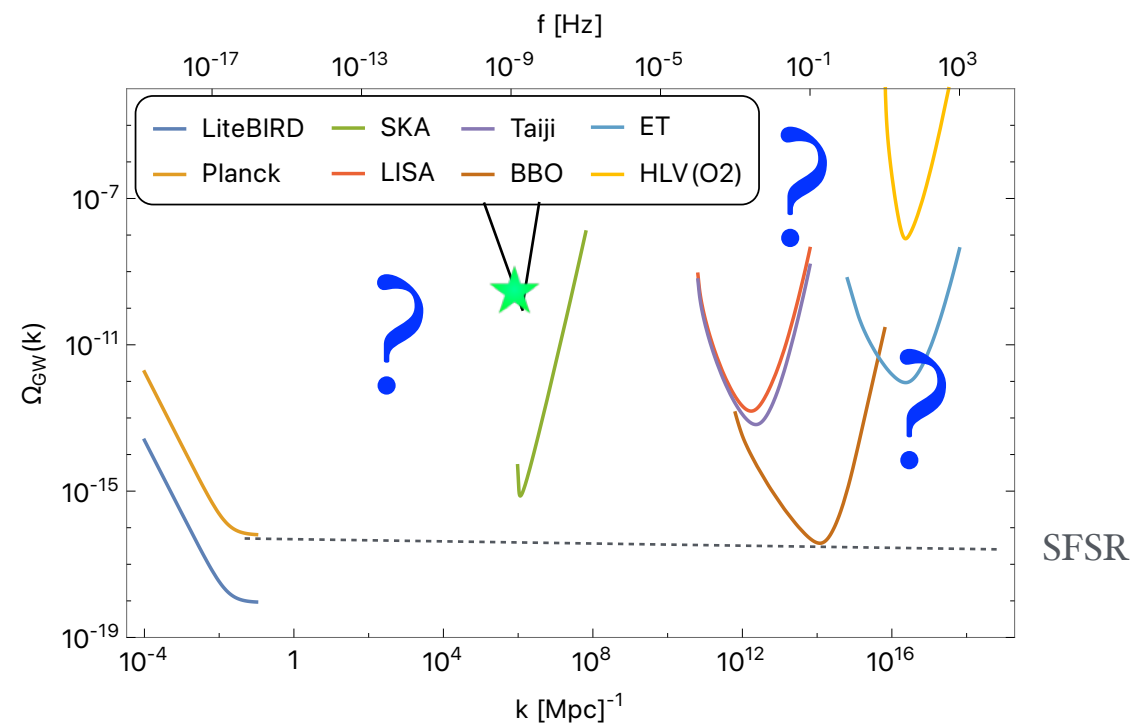


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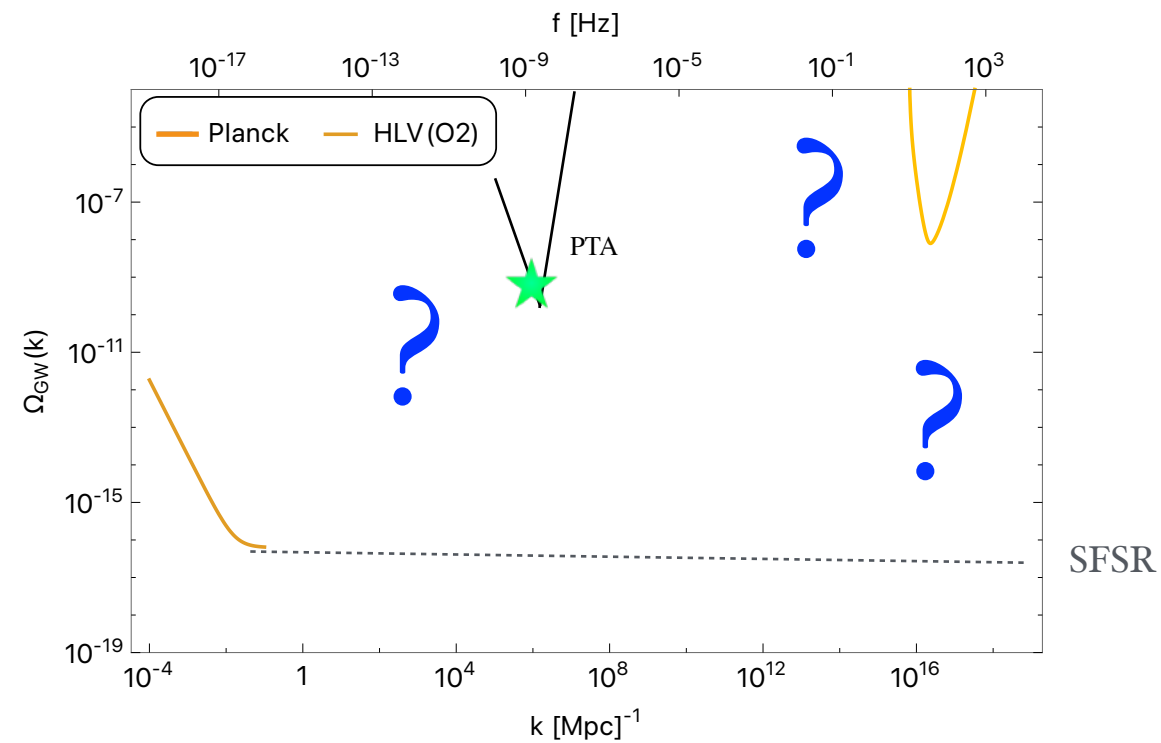


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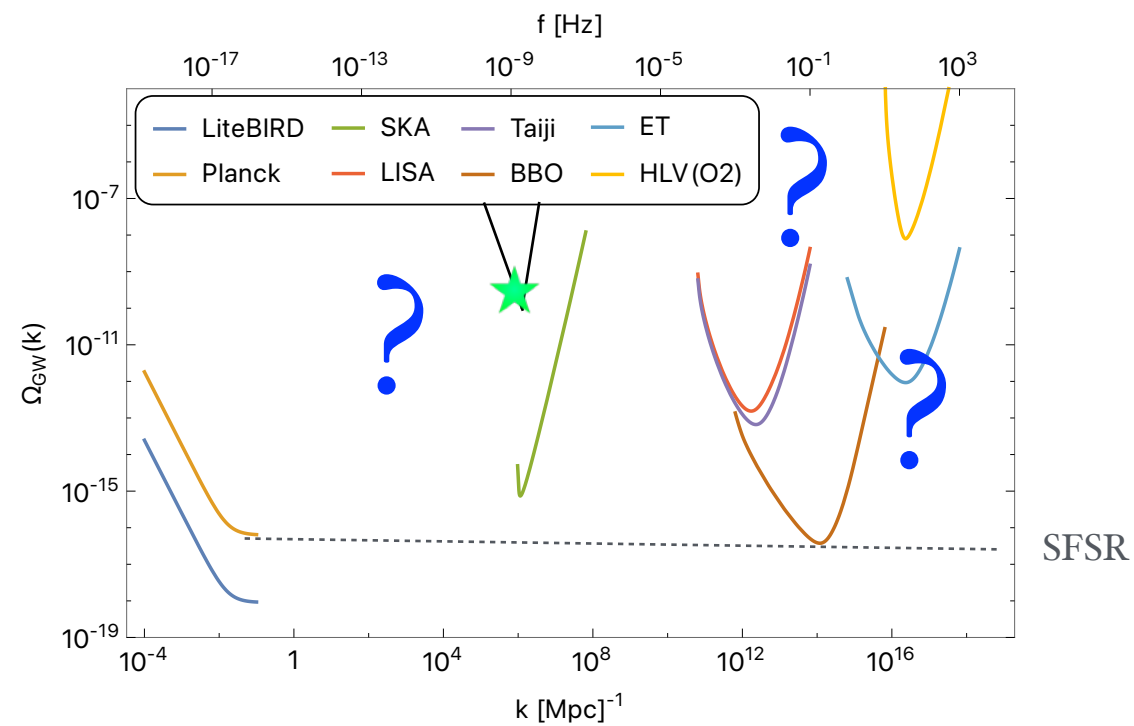


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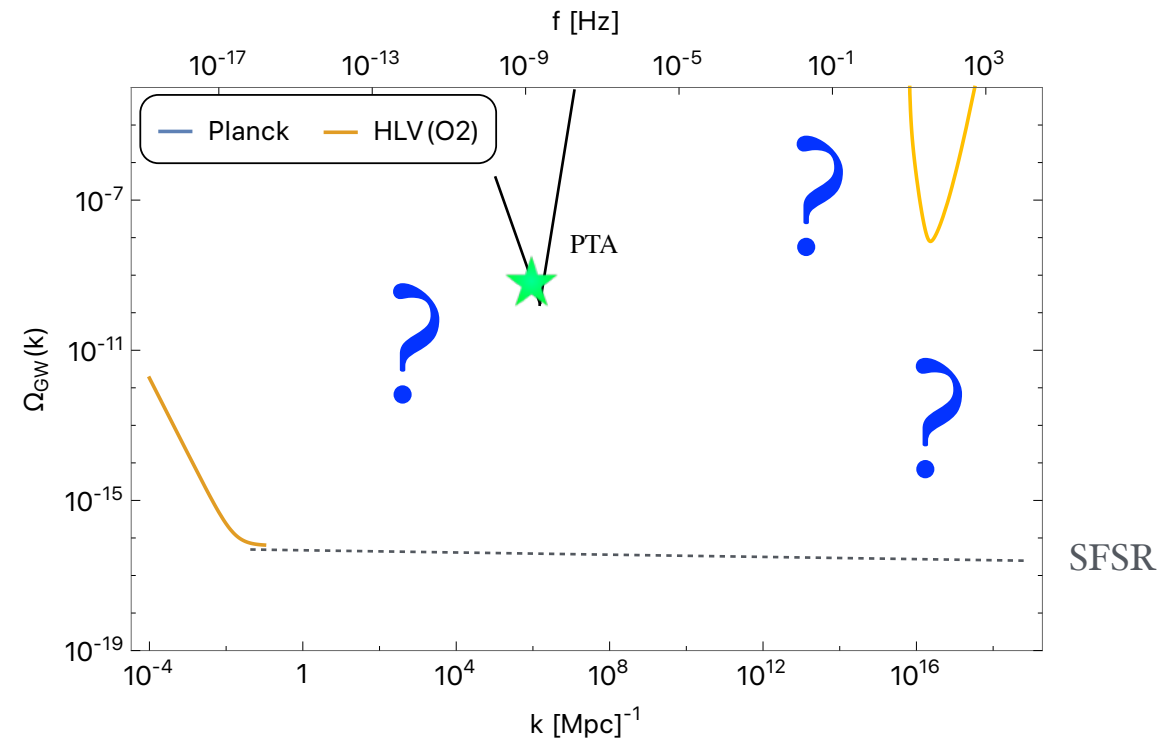
soon



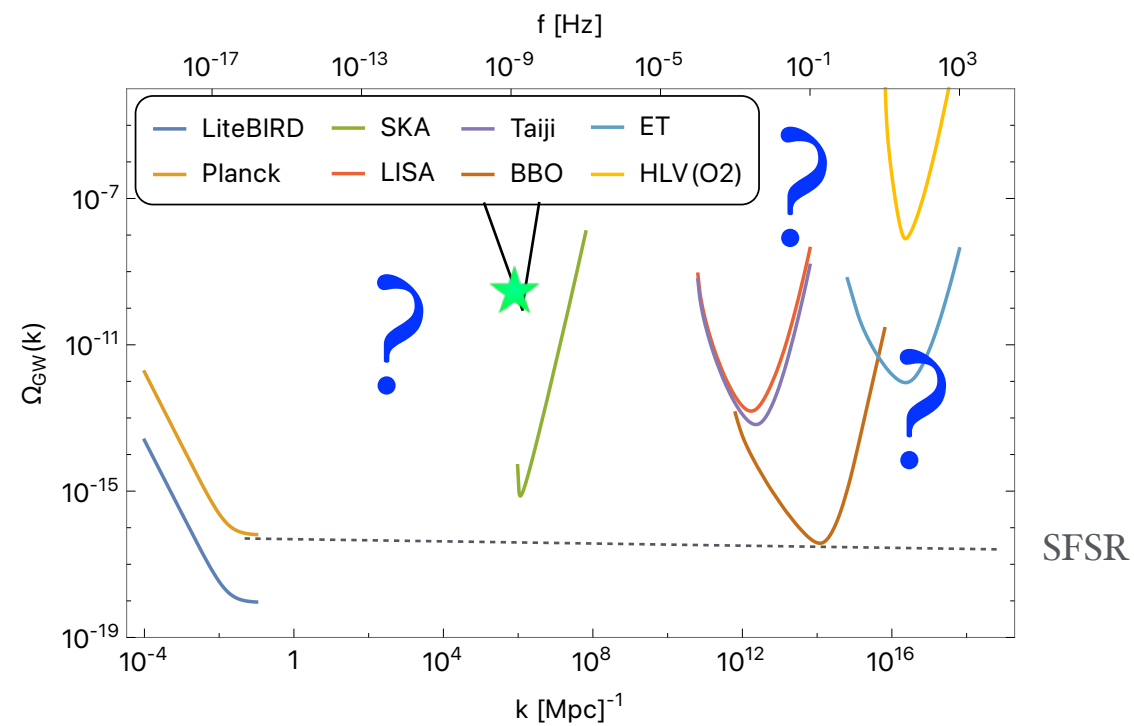
features

Hope for a little more (GW)

now



soon



blue or bump-like GW spectrum testable @ PTA or via laser interferometers

Typically multiple fields required

why go beyond single-field?

Likely

string theory

|

flux compactifications

|

4D EFT with many moduli fields

Testable

soon to cross key thresholds

$r < 0.001$ (CMB)

$f_{\text{NL}} < 1$ (LSS, 21cm)

GW signatures of new content:
PS: scale-dependence, chirality,
n-G: (amplitude, shape, angular)

Necessary

extraordinary claims
require extraordinary evidence

what to infer from
GW detection?
e.g. $r \longleftrightarrow H$ relation

This talk's focus on *Axion Inflation* models

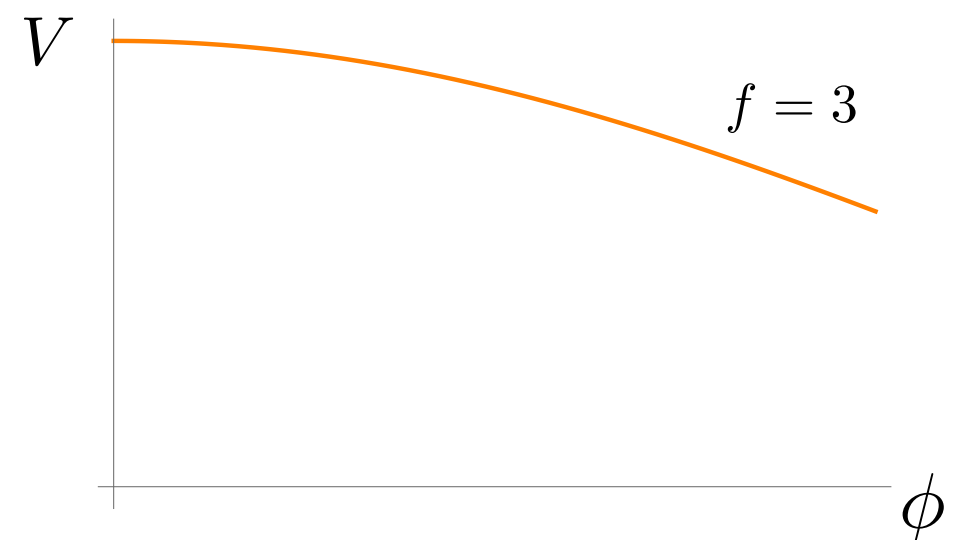
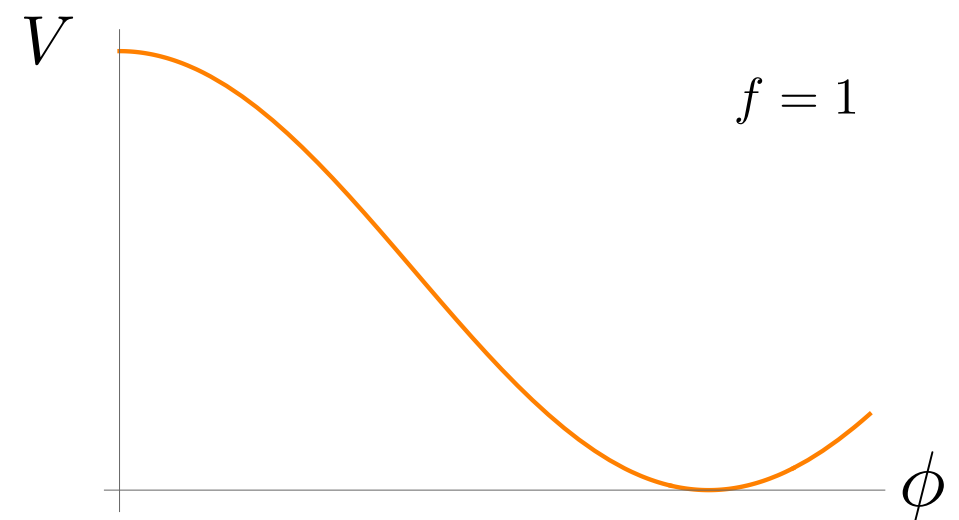
- Single field scenarios w/ e.g. inflection point & SIGW
- Non-attractor phase
- Multi-field inflation with an EFT perspective
- ...

Natural Inflation

$$\mathcal{L} = \sqrt{-g} \left[R[g] - (\partial\phi)^2 - \mu^4 (1 + \text{Cos}[\phi/f]) \right]$$

[Freese, Frieman, Olinto]

axion-like potential



simple

(technically) natural: shift symmetry

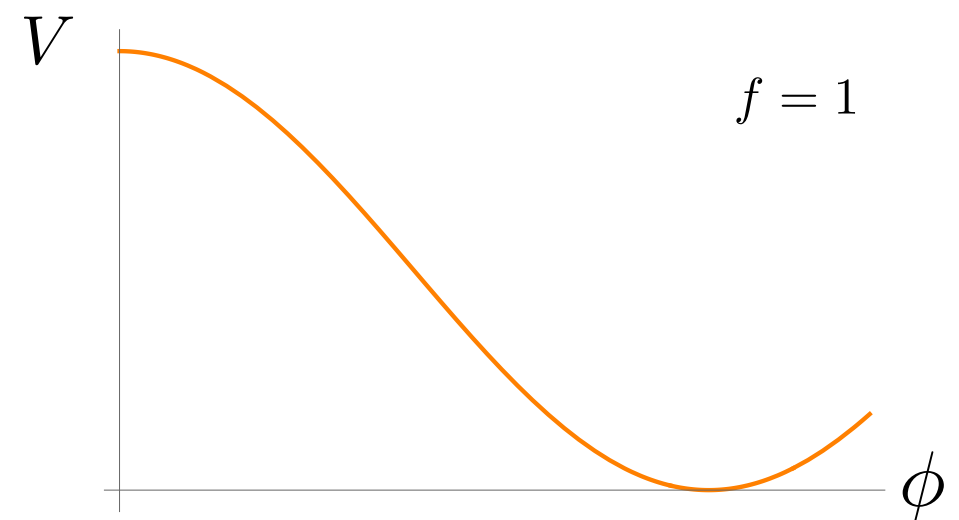
needs $f \gtrsim M_{\text{P}}$

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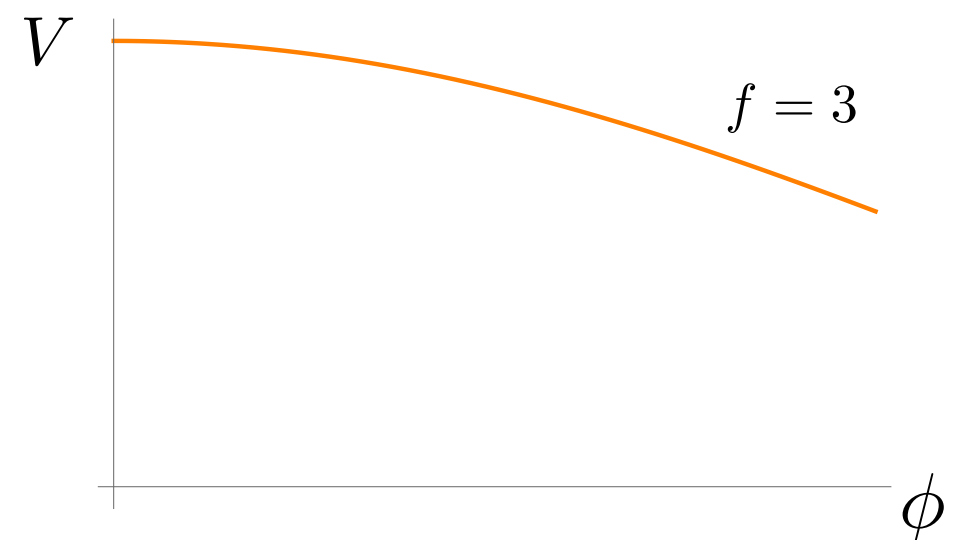
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needs $f \gtrsim M_{\text{P}}$



now ruled out

Natural Inflation + Gauge Sector

$$\mathcal{L} \supset \left[-\frac{1}{4}F^2 + \frac{\lambda\phi}{4f}F\tilde{F} \right] - (\partial\phi)^2 - U_{\text{axion}}(\phi)$$

$$\hookrightarrow U(\phi) \sim \mu^4[1 + \cos(\phi/f)]$$

- ◆ $\left\{ \begin{array}{l} \text{friction/dissipation slows the roll} \\ f \ll M_{\text{P}} \text{ realization} \\ \text{very interesting GW signatures !} \end{array} \right.$

Natural Inflation + Gauge Sector

general properties

$$\mathcal{L} \supset \left[-\frac{1}{4}F^2 + \frac{\lambda\phi}{4f}F\tilde{F} \right] - (\partial\phi)^2 - U_{\text{axion}}(\phi)$$

$\text{SU}(2)$
 $\text{U}(1)$

$$t''_{R,L} + \left[1 + \frac{2m_Q\xi}{x^2} \mp \frac{2}{x}(m_Q + \xi) \right] t_{R,L} = \tilde{\mathcal{O}}^{(1)}(\Psi_{R,L})$$

$$\left[\partial_\tau^2 + k^2 \pm \frac{2k\xi}{\tau} \right] A_\pm(\tau, k) = 0$$

Adshead & Wyman

Anber & Sorbo

Natural Inflation + Gauge Sector

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Gauge fields source GW, chiral signal if leading

Interferometer tests for chirality

{ cross-correlation @ different locations [Smith, Caldwell 2017]
 kinematically induced dipole [Seto 2006, Domcke et al 2019]

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$$\xi \equiv \frac{\lambda\dot{\phi}}{2fH}$$

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- ξ regulates the growth ==> possible blue spectrum

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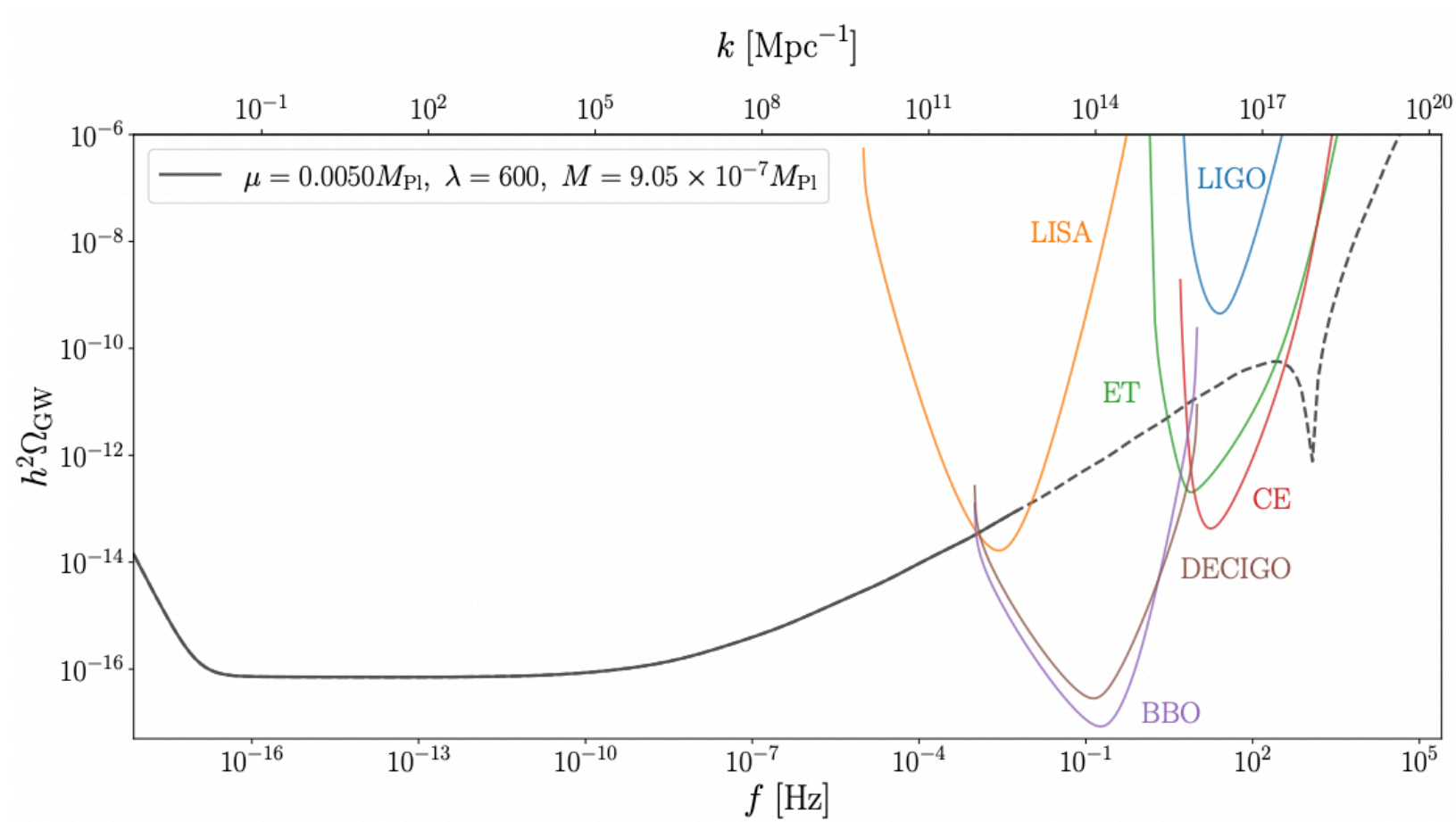
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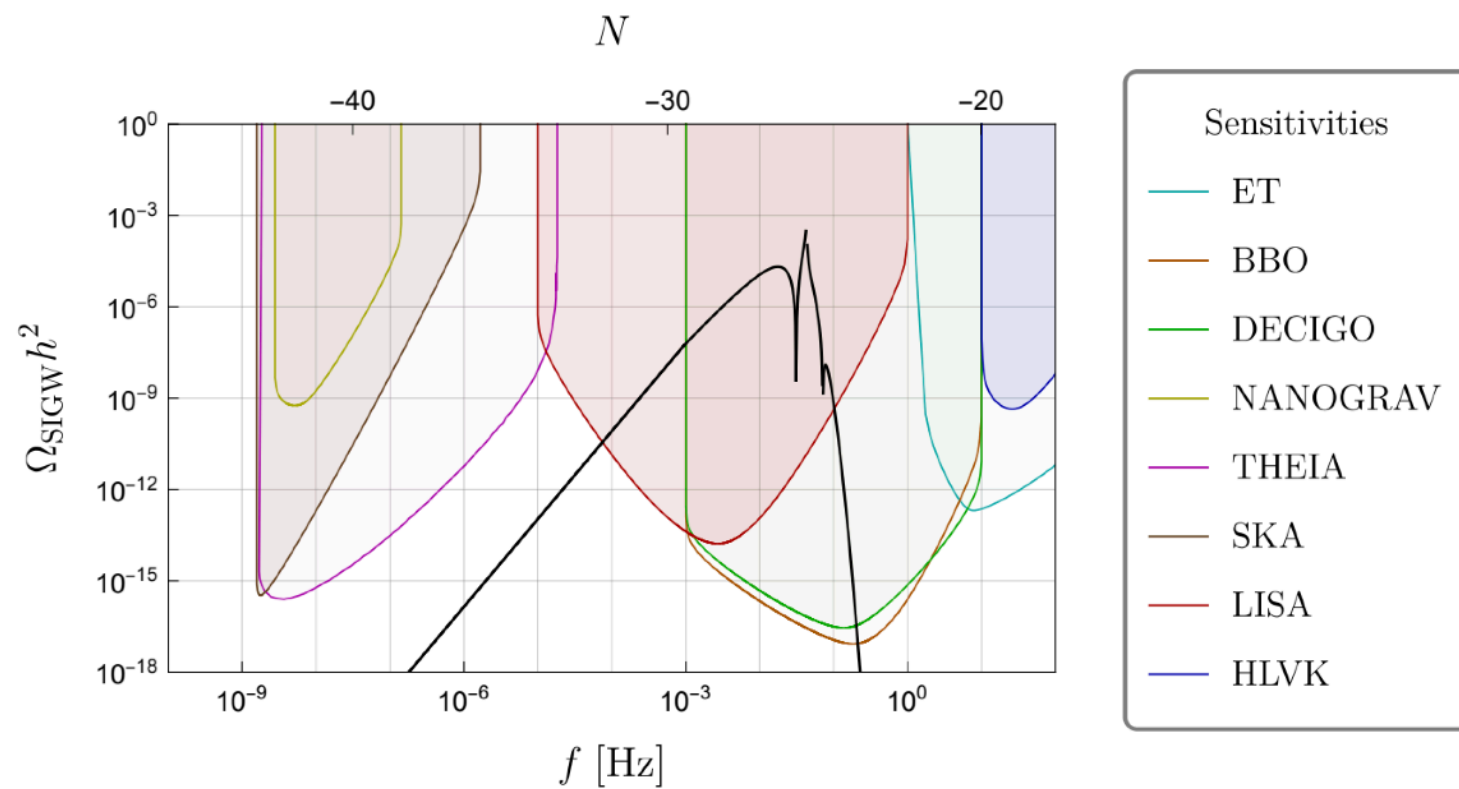
i.e. PTA & interferometers tests !

e.g.

blue



peaked



Not easy to keep things minimal/simple

Natural Inflation + U(1) sector

[Anber, Sorbo] , [...]

$$\mathcal{L} \supset \left[-\frac{1}{4}F^2 + \frac{\lambda\phi}{4f}F\tilde{F} \right] - (\partial\phi)^2 - U_{\text{axion}}(\phi)$$

$$\blacklozenge \left\{ f \ll M_{\text{P}} \text{ realization} \right.$$

Natural Inflation + U(1) sector

$\mathcal{N} \sim 10^5$
gauge fields

[Anber, Sorbo] , [...]

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many applications e.g. PBH context w/ spectator fields, [Peloso, Sorbo, Unal]
[Garcia-Bellido, Papageorgiou, Peloso, Sorbo]

$$\blacklozenge \left\{ f \ll M_{\text{P}} \text{ realization} \right.$$

Chromo Natural Inflation

SU(2)

[Adshead, Wyman]

$$\mathcal{L} \supset -\frac{1}{4}F^2 + \frac{\lambda\phi}{4f}F\tilde{F} - (\partial\phi)^2 - U_{\text{axion}}(\phi)$$

[Dimastrogiovanni, MF, Tolley]

[Dimastrogiovanni, Peloso],

[...]

◆ $\left\{ \begin{array}{l} f \ll M_{\text{P}} \text{ realization} \\ \text{very interesting GW signatures !} \end{array} \right.$

Chromo Natural Inflation

SU(2)

- attractor solution

[Wolfson, Maleknejad, Komatsu]

- emerging non zero VEV

[Domcke, Mares, Muia, Pieroni]

[Adshead, Wyman]

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[Dimastrogiovanni, Peloso],

[...]

◆ $\left\{ \begin{array}{l} f \ll M_{\text{P}} \text{ realization} \\ \text{very interesting GW signatures !} \\ \text{in tension with the data..} \end{array} \right.$

Spectator Chromo Natural Inflation

[Dimastrogiovanni, MF, Fujita]

[Obata, Soda]

$$\mathcal{L} \supset \mathcal{L}_{\text{inflaton}} - \frac{1}{4} F^2 + \frac{\lambda \chi}{4f} F \tilde{F} - (\partial \chi)^2 - U_{\text{axion}}(\chi)$$

SU(2)

◆ $\left\{ \begin{array}{l} f \ll M_{\text{P}} \text{ realization} \\ \text{same interesting GW spectrum} \\ \text{observationally viable} \end{array} \right.$

Primordial GW in SCNI

$$\mathcal{L} \supset \mathcal{L}_{\text{inflaton}} - \frac{1}{4}F^2 + \frac{\lambda\chi}{4f}F\tilde{F} - (\partial\chi)^2 - U_{\text{axion}}(\chi)$$

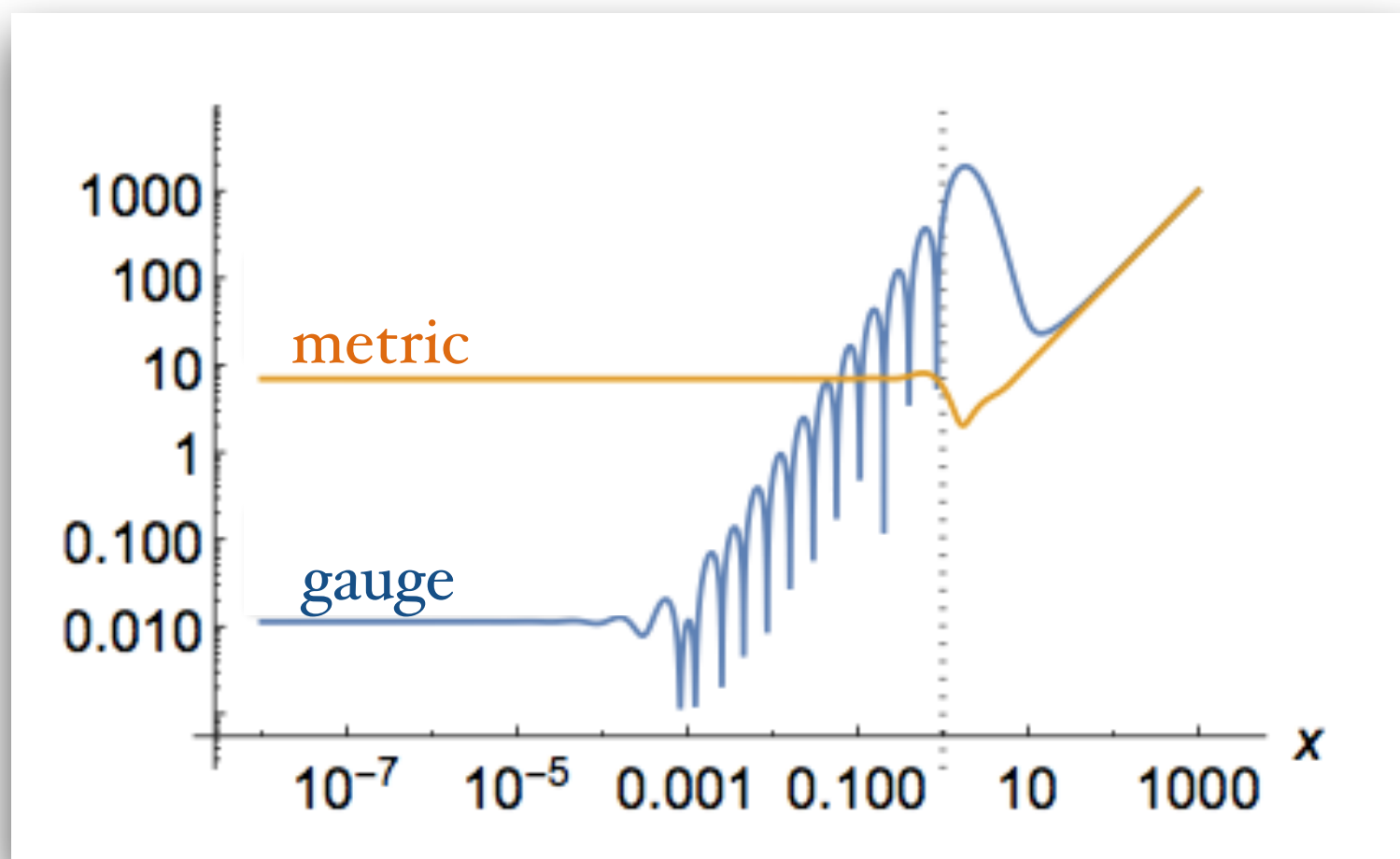
$$\text{SU}(2) \begin{cases} A_0^a = 0 \\ A_i^a = aQ\delta_i^a \\ \delta A_i^a = t_{ai} + \dots \end{cases}$$

$$\ddot{\gamma}_{ij}^\lambda + 3H\dot{\gamma}_{ij}^\lambda + k^2\gamma_{ij}^\lambda \propto t_{ij}^\lambda + \dots + \dots$$

[Dimastrogiovanni, MF, Fujita]

$$P_\lambda^{\text{sourced}} \gtrsim P_\lambda^{\text{vacuum}}$$

now possible!



SCNI

$$\mathcal{L} \supset \mathcal{L}_{\text{inflaton}} - \frac{1}{4}F^2 + \frac{\lambda\chi}{4f}F\tilde{F} - (\partial\chi)^2 - U_{\text{axion}}(\chi)$$

agnostic
on the potential

==>

SCNI?

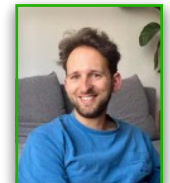
Chromo + non-Minimal Coupling

- Mechanisms to slow the rolling without abandoning naturalness

Einstein tensor

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2} \right) \partial_\mu \chi \partial_\nu \chi - V(\chi) - \frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a + \frac{\lambda \chi}{8f \sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a \right]$$

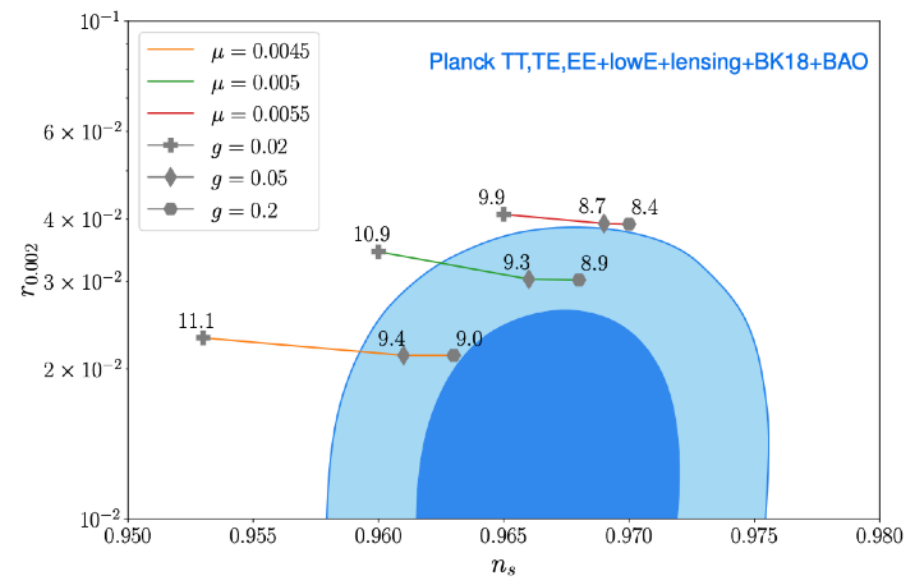
[Dimastrogiovanni, MF, Michelotti, Pinol 2023]



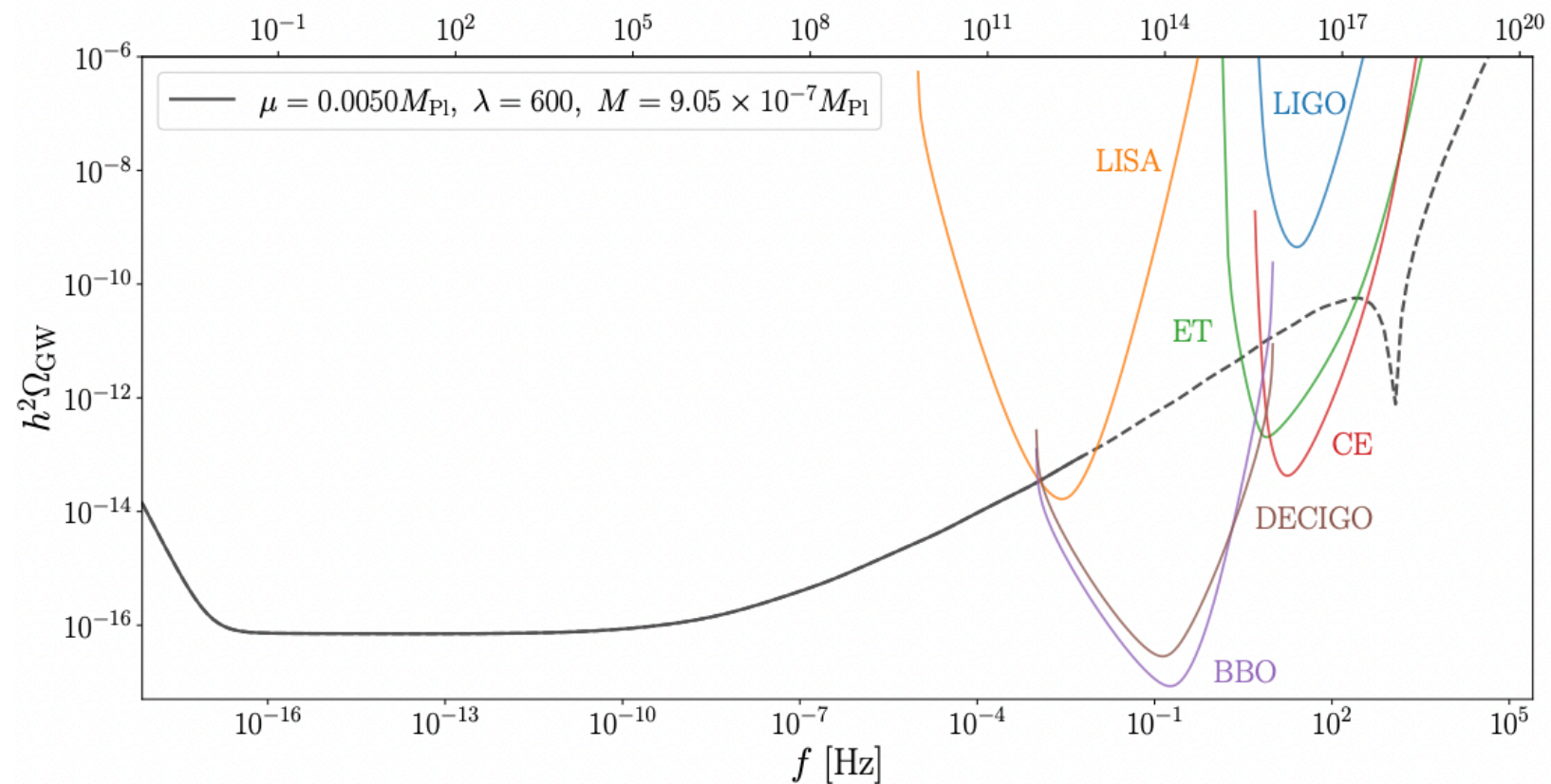
[see also Komatsu and Watanabe 2020]

more general coupling but w/o potential (kinetically driven)

Chromo + non-Minimal Coupling



[Dimastrogiovanni, MF, Michelotti, Pinol 2023]



Other natural axion-inflaton scenarios

Higgsed Chromo

another way to rescue the model

[Adshead, Martinec, Sfakianakis, Wyman]

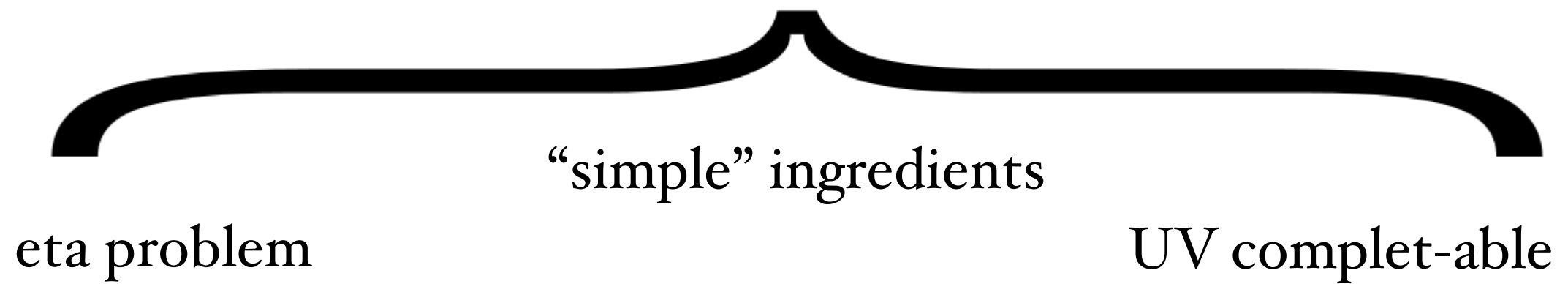
spontaneous breaking of the $SU(2)$ gauge symmetry

Goldstones modes provide additional d.o.f.s

these contribute more to scalars, decrease r

Appeal of axion-gauge field models

Model Building



scale-dependence non-Gaussianity



Testable

Challenges & Opportunities

[Anber, Sorbo]

[Iarygina, Sfakianakis, Sharma, Brandenburg 2023]

[Durrer, Sobol, Vilchinskii 2023]

[Dimastrogiovanni, MF, Papageorgiou 2024]

[Garcia-Bellido, Papageorgiou, Peloso, Sorbo 2023]...

(I) Gauge fields **backreaction** on the background

Tackled numerically under homogeneous assumption

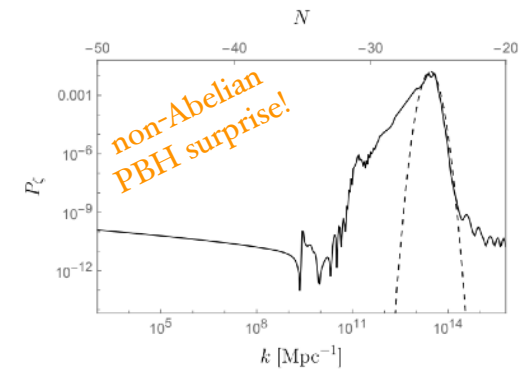
but see [Domcke, Ema, Sandner 2023]

Lattice the ultimate test

next talk!

[Figueroa, Lizarraga, Urio, Urrestilla 2023], [Caravano, Komatsu, Lozanov, Weller 2022]

[Caravano, Peloso 2024], [Figueroa, Lizarraga, Loayza, Urio, Urrestilla 2024]



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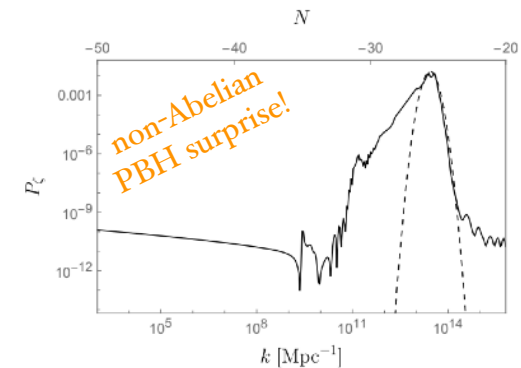
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(II) **Perturbativity** bounds



Explored, can be competitive with backreaction, need better numerical studies

[Ferreira, Ganc, Noreña, Sloth 2015]

[Papageorgiou, Peloso, Unal 2018]

[Dimastrogiovanni, MF, Michelotti, Ozsoy 2024]

Challenges & Opportunities

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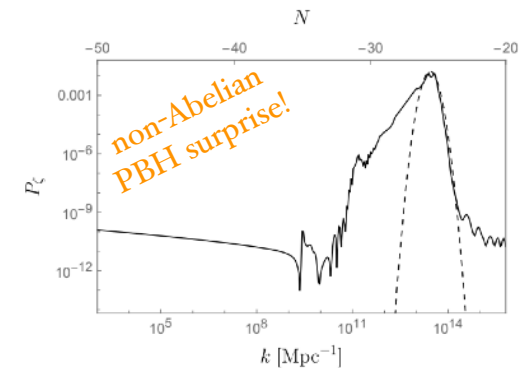
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[Figuerola, Lizarraga, Urio, Urrestilla 2023], [Caravano, Komatsu, Lozanov, Weller 2022]
 [Caravano, Peloso 2024], [Figuerola, Lizarraga, Loayza, Urio, Urrestilla 2024]



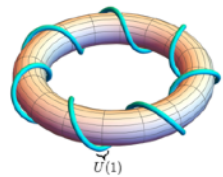
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 [Dimastrogiovanni, MF, Michelotti, Ozsoy 2024]

(III) **UV** completion



Abelian sector ok, non-Abelian ok but signatures only upon fine-tuning

...
 [Dall'Agata 2018]
 [Agrawal, Fan, Reece 2018]
 [Holland, Zavala, Tasinato 2020]
 [Bagherian, Reece, Xu 2022]
 [Cicoli et al 2023]
 [Dimastrogiovanni, MF, Leedom, Putti, Westphal 2023]...

Challenges & Opportunities

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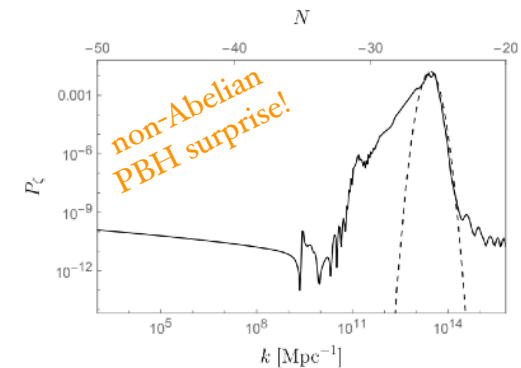
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Lattice the ultimate test
next talk!

[Figueroa, Lizarraga, Urio, Urrestilla 2023], [Caravano, Komatsu, Lozanov, Weller 2022]
 [Caravano, Peloso 2024], [Figueroa, Lizarraga, Loayza, Urio, Urrestilla 2024]



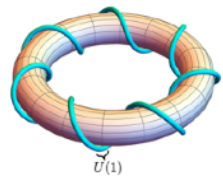
(II) **Perturbativity** bounds

Explored, can be competitive with backreaction, need better numerical studies



[Ferreira, Ganc, Noreña, Sloth 2015]
 [Papageorgiou, Peloso, Unal 2018]
 [Dimastrogiovanni, MF, Michelotti, Ozsoy 2024]

(III) **UV** completion



Abelian sector ok, non-Abelian ok but signatures only upon fine-tuning

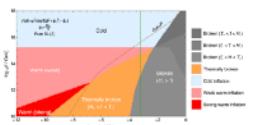
...
 [Dall'Agata 2018]
 [Agrawal, Fan, Reece 2018]
 [Holland, Zavala, Tasinato 2020]
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(IV) **Re-heating** with (dark) sectors

Lots of interesting work, past & future

...
 [Dufaux, Figueroa, Garcia-Bellido 2010]
 [Lozanov, Amin 2016]
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 [Adshead, Giblin, Pieroni, Weiner 2019]...

(VI) **Thermalization?**



...
 [Ferreira, Notari 2017 x 2]
 [Berghaus, Graham, Kaplan 2019]
 [DeRocco, Graham, Kalia 2021]
 [Mukuno, Soda 2024]...

(V) **Magneto-genesis, baryogenesis**

Not easy, but very intriguing

...
 [Fujita, Yokoyama 2014]
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 [Cado, Quirós 2022]
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Backreaction

(non-Abelian case)

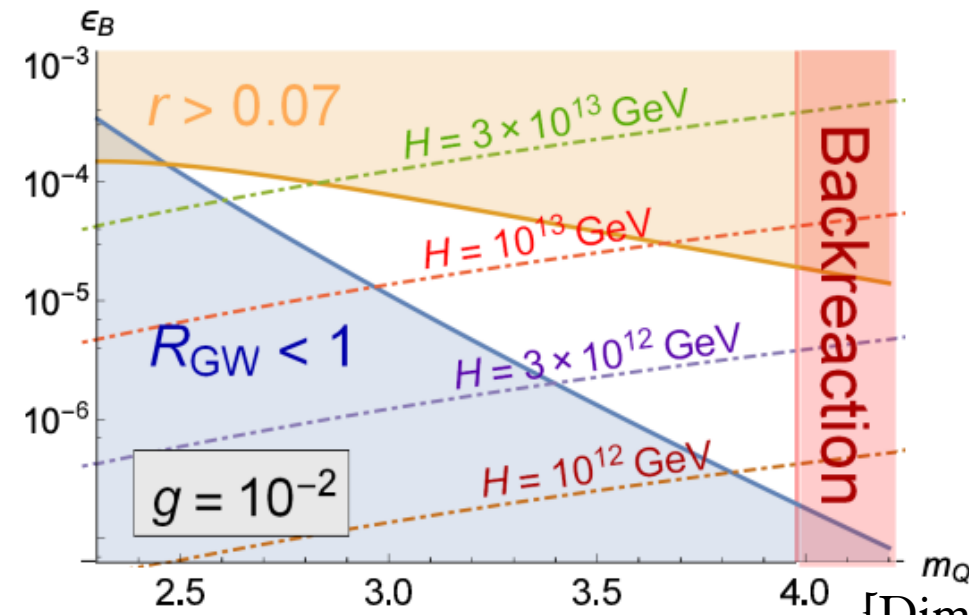
eom for the gauge field background

$$\ddot{Q} + 3H\dot{Q} + (\dot{H} + 2H^2)Q + 2g^2Q^3 = \frac{g\lambda}{f}\dot{\chi}Q^2 + \dots$$

(same goes for χ e.o.m.)

fluctuations backreact on
background ==> reduced regime of validity

$$\mathcal{T}_{BR}^Q \equiv \frac{g\xi}{3a^2}H \int \frac{d^3k}{(2\pi)^3} |t_R|^2 + \frac{g}{3a^2} \int \frac{d^3k}{(2\pi)^3} \frac{k}{a} |t_R|^2$$



[Dimastrogiovanni, MF, Fujita]



[Mirzagholi, Maleknejad, Lozanov 2020]

[Maleknejad, Komatsu 2018]

[Ishiwata, Komatsu, Obata 2021]

Backreaction* in (S)CNI

background

$$3H^2 M_p^2 = \frac{\dot{\chi}^2}{2} + U(\chi) + \frac{3}{2} \left[\left(\dot{Q} + HQ \right)^2 + g^2 Q^4 \right] + \rho_t$$

$$-2M_p^2 \dot{H} = \dot{\chi}^2 + 2 \left[\left(\dot{Q} + HQ \right)^2 + g^2 Q^4 \right] + Q \mathcal{T}_{BR}^Q + \frac{4}{3} \rho_t$$

$$\ddot{\chi} + 3H\dot{\chi} + U_\chi + \frac{3g\lambda}{f} Q^2 \left(\dot{Q} + HQ \right) + \mathcal{T}_{BR}^\chi = 0$$

$$\ddot{Q} + 3H\dot{Q} + \left(\dot{H} + 2H^2 \right) Q + gQ^2 \left(2gQ - \frac{\lambda\dot{\chi}}{f} \right) + \mathcal{T}_{BR}^Q = 0$$

gauge tensors
backreaction

$$\mathcal{T}_{BR}^\chi = -\frac{\lambda}{2a^2 f} \frac{d}{dt} \int \frac{d^3 k}{(2\pi)^3} \left(m_Q H - \frac{k}{a} \right) |t_L|^2$$

$$\mathcal{T}_{BR}^Q = \frac{g}{3a^2} \int \frac{d^3 k}{(2\pi)^3} \left(\xi H - \frac{k}{a} \right) |t_L|^2$$

$$\rho_t = \frac{1}{2a^2} \int \frac{d^3 k}{(2\pi)^3} \left[|\dot{t}_L|^2 + \left(\frac{k^2}{a^2} - 2gQ \frac{k}{a} \right) |t_L|^2 \right]$$

* Caveats

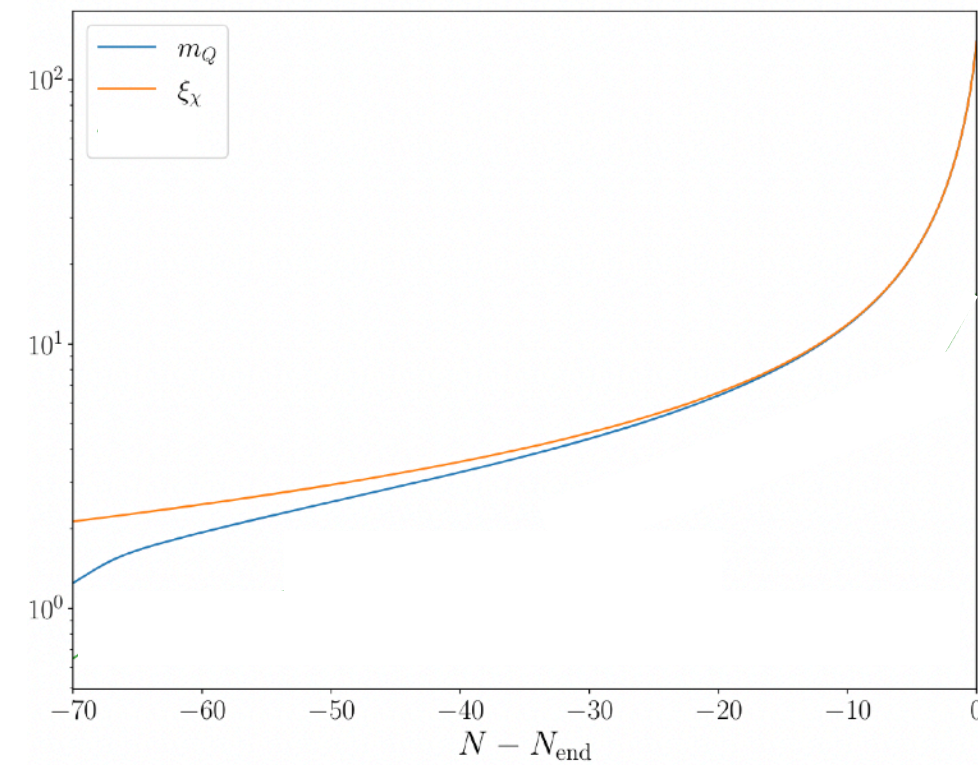
- lattice remains ultimate tool in this context
[Joanes' talk!]
- e.g. we assume homogeneous background
- scalar backreaction not considered
(small here but can be important for spectator models)
- ...

The effect of backreaction

$$m_Q \sim \xi$$



backreaction

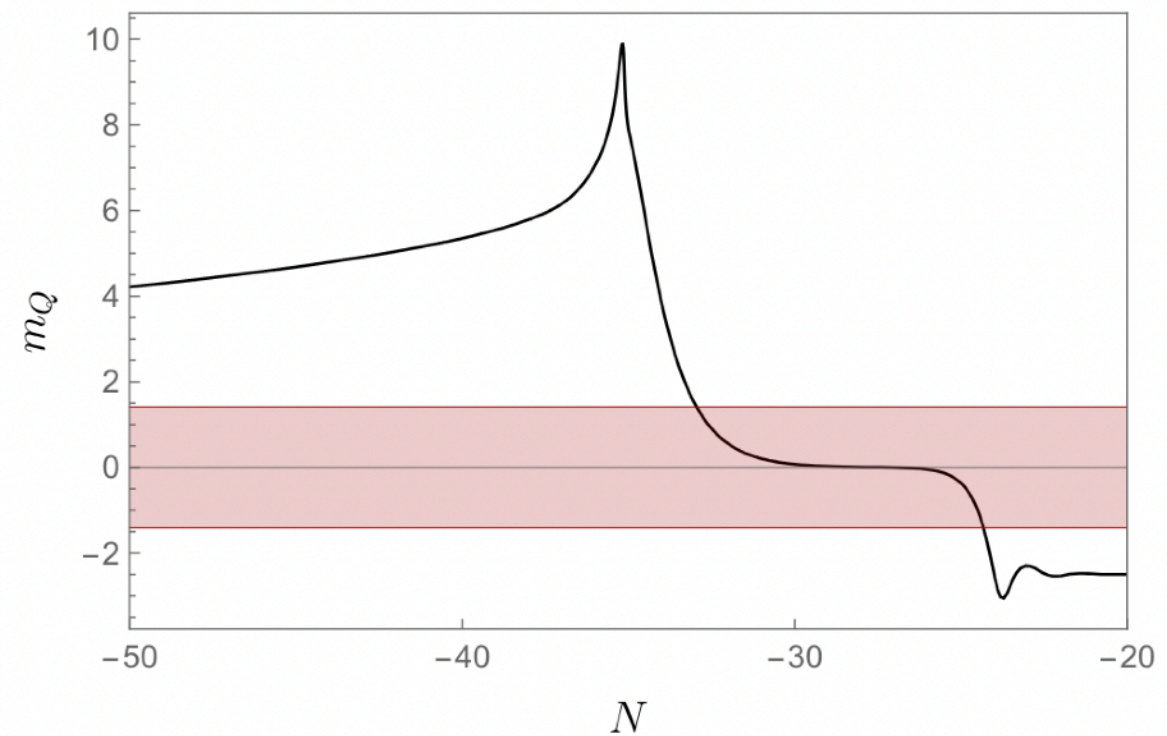
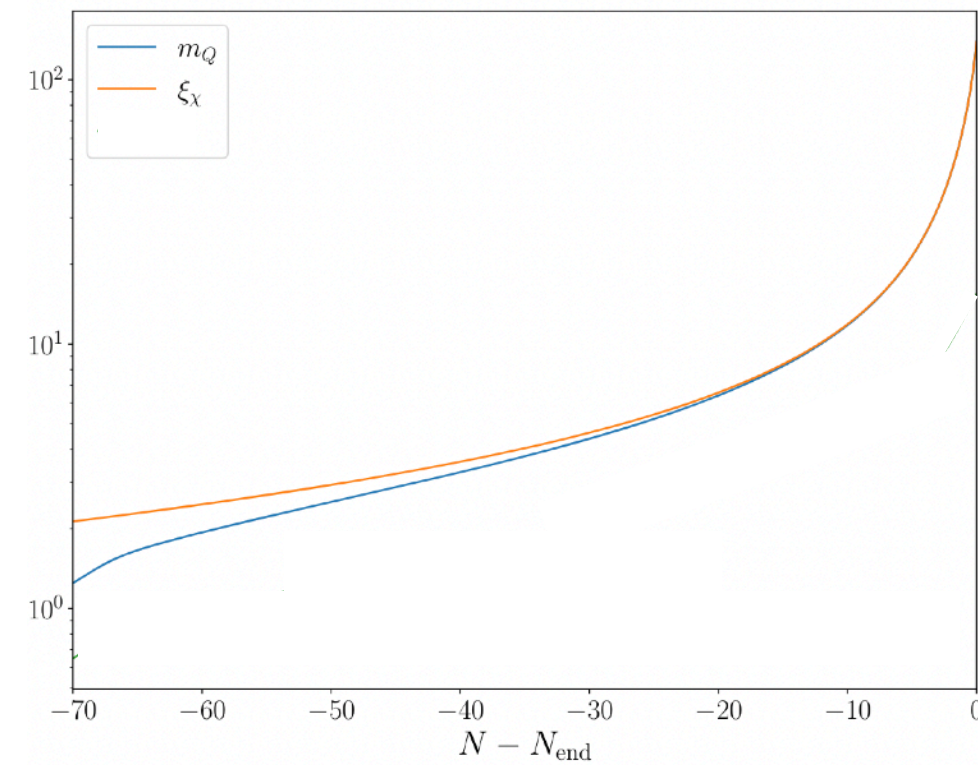


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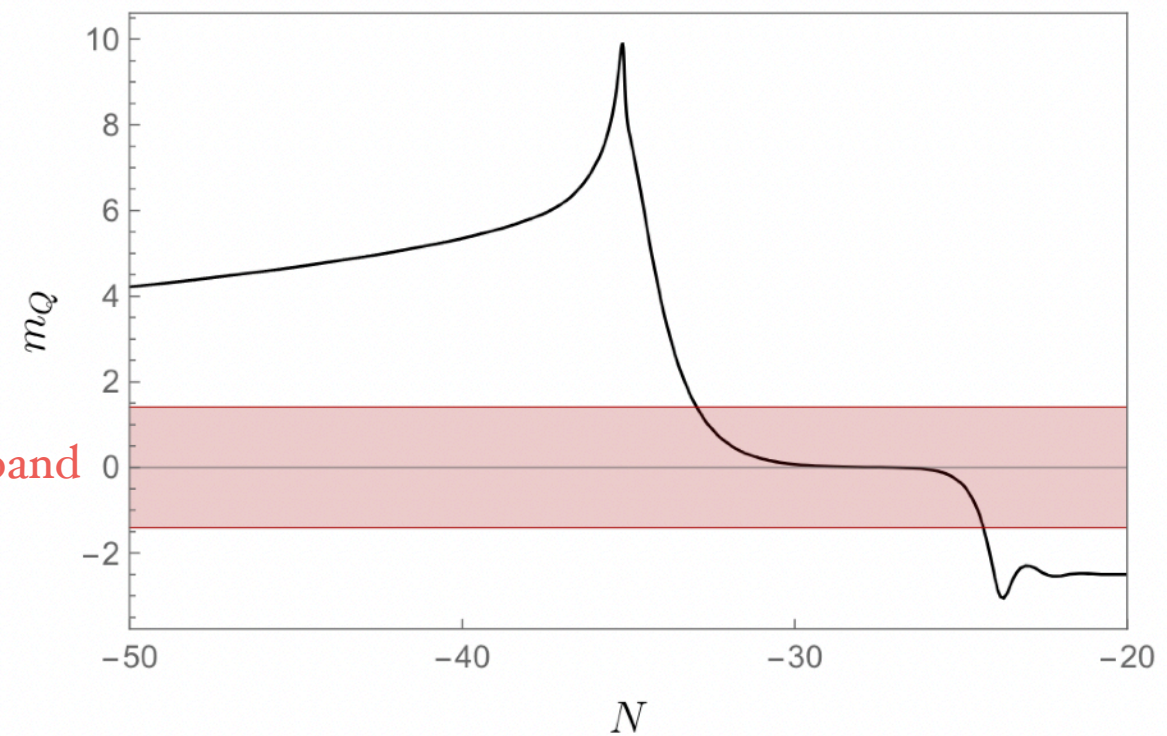
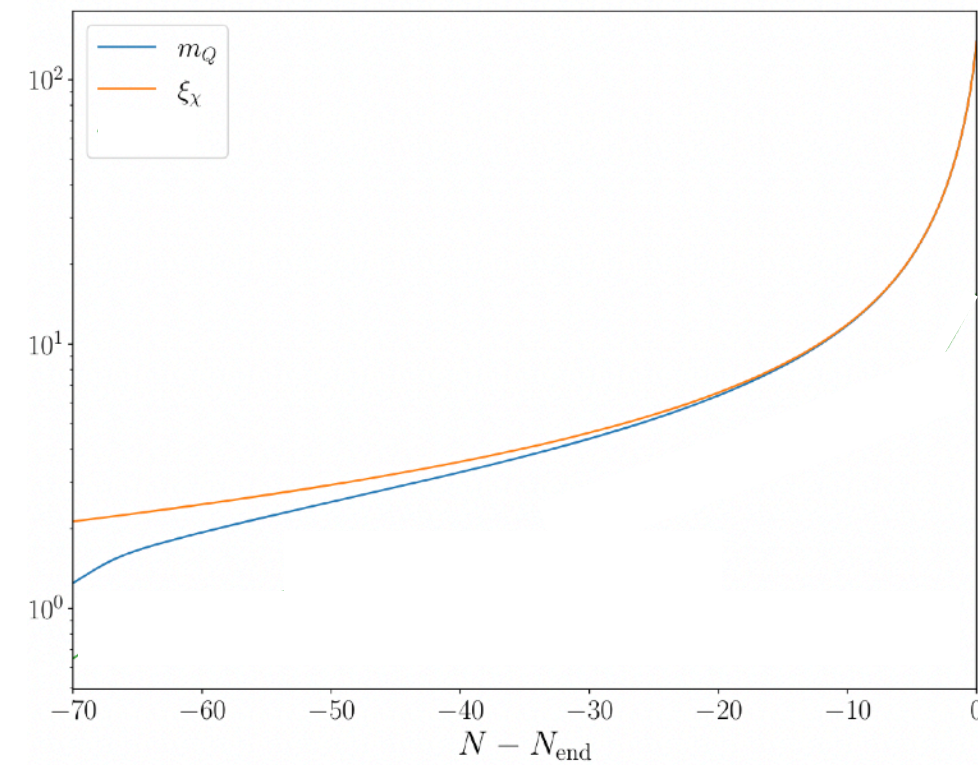


The effect of backreaction

$$m_Q \sim \xi$$



backreaction



Scalar Fluctuations



$$\hat{X}'' + \left(1 - \frac{2}{x^2} + \frac{\Lambda^2 m_Q^2}{2m_Q^2 + x^2}\right) \hat{X} - \frac{\Lambda m_Q^2}{x^2 \sqrt{\frac{1}{2} + \frac{m_Q^2}{x^2}}} \hat{\varphi}' + \frac{\sqrt{2}\Lambda (4m_Q^4 + 3m_Q^2 x^2 + x^4)}{x^2 (2m_Q^2 + x^2)^{3/2}} \hat{\varphi} + \frac{\sqrt{2}\Lambda m_Q}{x} \hat{Z}' - \frac{2\sqrt{2}\Lambda m_Q}{x^2} \hat{Z} = 0$$

axion - inflaton



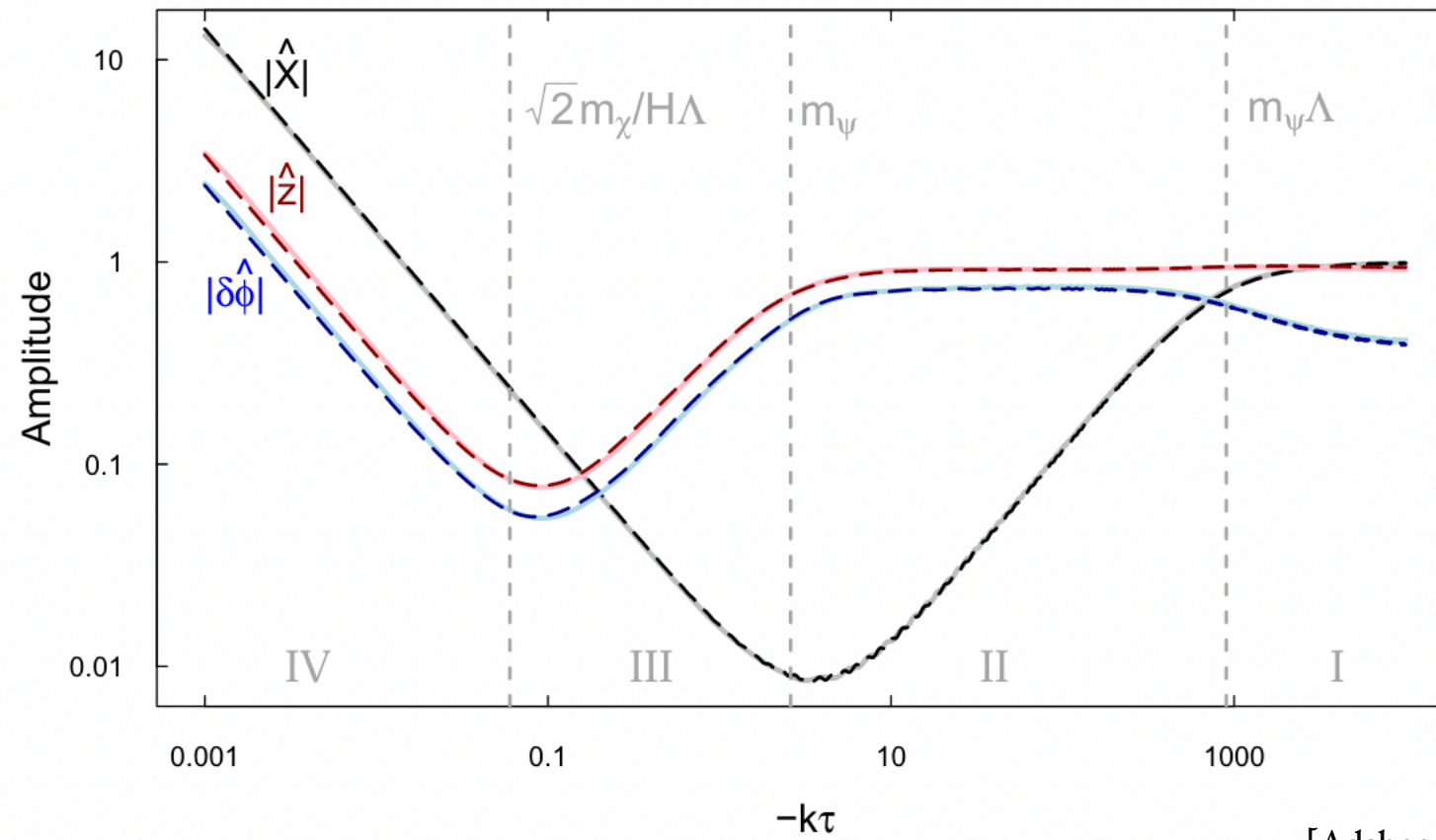
$$\hat{\varphi}'' + \frac{\left(1 + \frac{8m_Q^6}{x^6} + \frac{2m_Q^2(8m_Q^2 - 2m_Q\xi + 3)}{x^4} + \frac{2m_Q(4m_Q - \xi)}{x^2}\right)}{\left(1 + \frac{2m_Q^2}{x^2}\right)^2} \hat{\varphi} + \frac{\Lambda m_Q^2}{x^2 \sqrt{\frac{m_Q^2}{x^2} + \frac{1}{2}}} \hat{X}' +$$

$$+ \frac{\sqrt{2}\Lambda (2m_Q^4 + m_Q^2 x^2 + x^4)}{x^2 (2m_Q^2 + x^2)^{3/2}} \hat{X} - \frac{2(m_Q - \xi)\sqrt{2m_Q^2 + x^2}}{x^2} \hat{Z} = 0$$


gauge sector

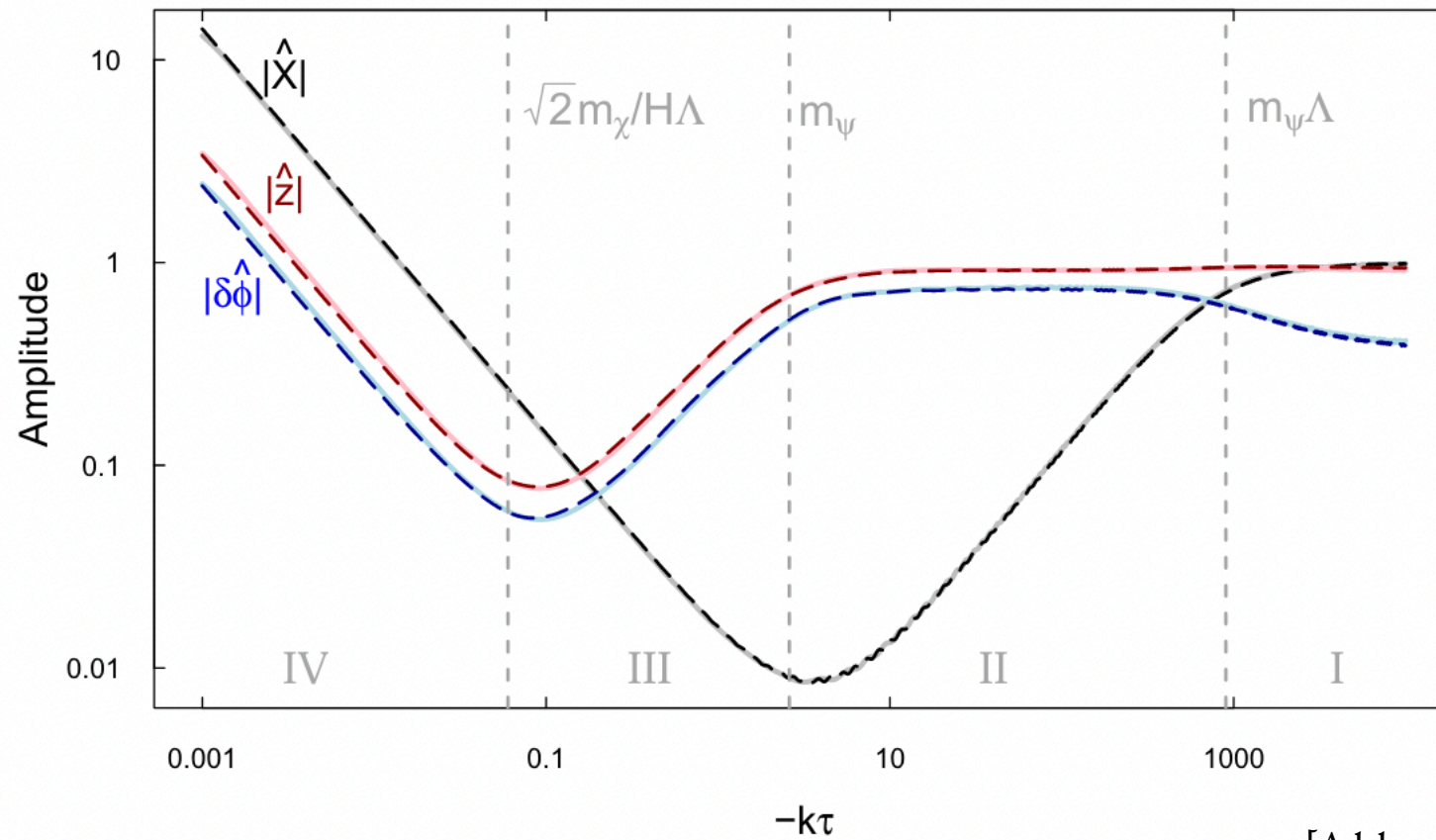
$$\hat{Z}'' + \frac{(4m_Q^2 - 2m_Q\xi + x^2)}{x^2} \hat{Z} - \frac{\sqrt{2}\Lambda m_Q}{x} \hat{X}' - \frac{\sqrt{2}\Lambda m_Q}{x^2} \hat{X} - \frac{2(m_Q - \xi)\sqrt{2m_Q^2 + x^2}}{x^2} \hat{\varphi} = 0$$

Scalar Fluctuations



[Adshead, Martinec, Wyman]

Scalar Fluctuations

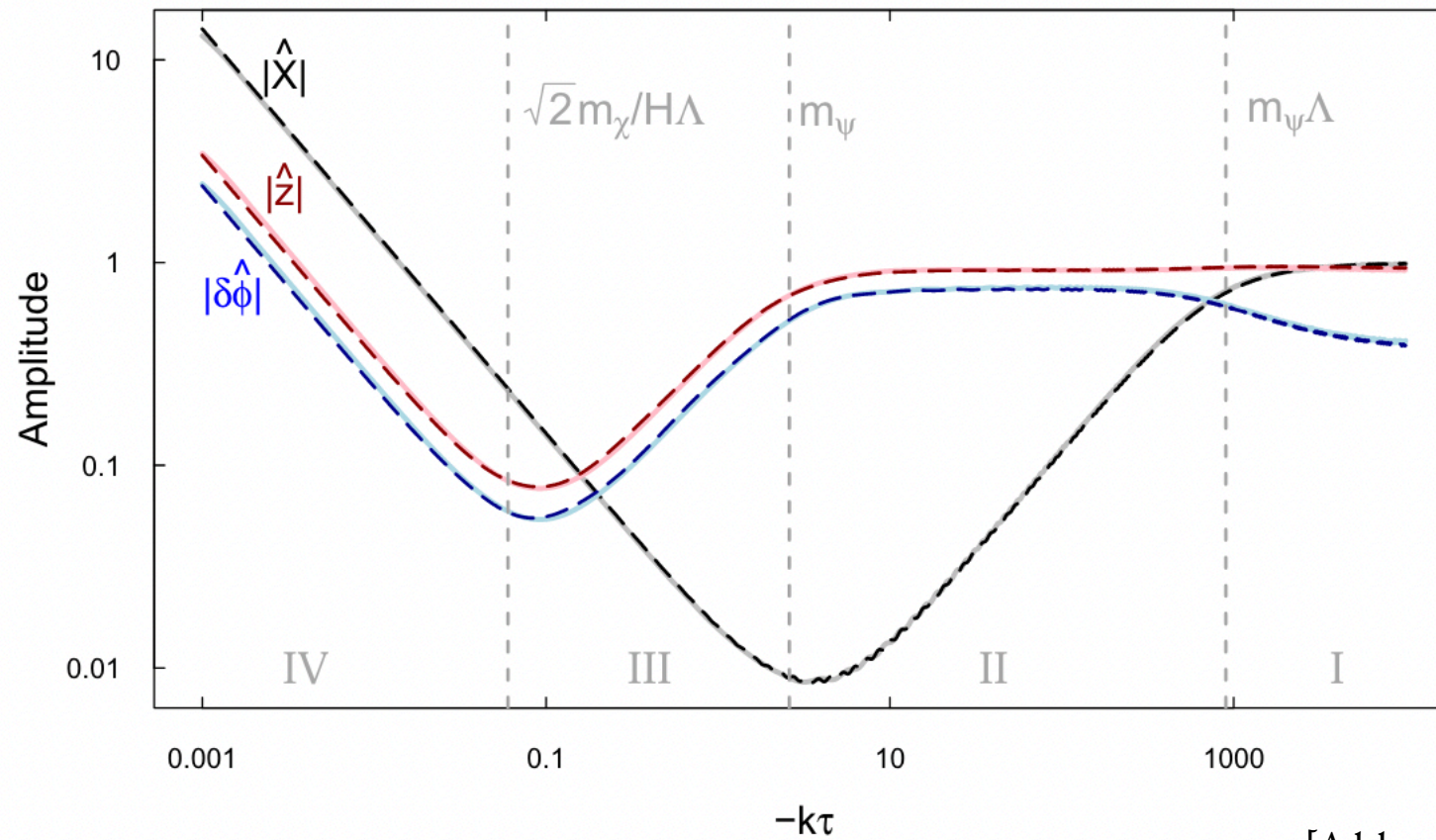


[Adshead, Martinec, Wyman]

- WKB approx in region I & II, to find

$$\omega^2 \sim 1 + \Lambda \left(3mQ^2\Lambda - 3mQ^2\Lambda \left(1 + \frac{8(1+mQ^2)x^2}{9mQ^4\Lambda^2} \right) \right) / (2x^2)$$

Scalar Fluctuations



[Adshead, Martinec, Wyman]

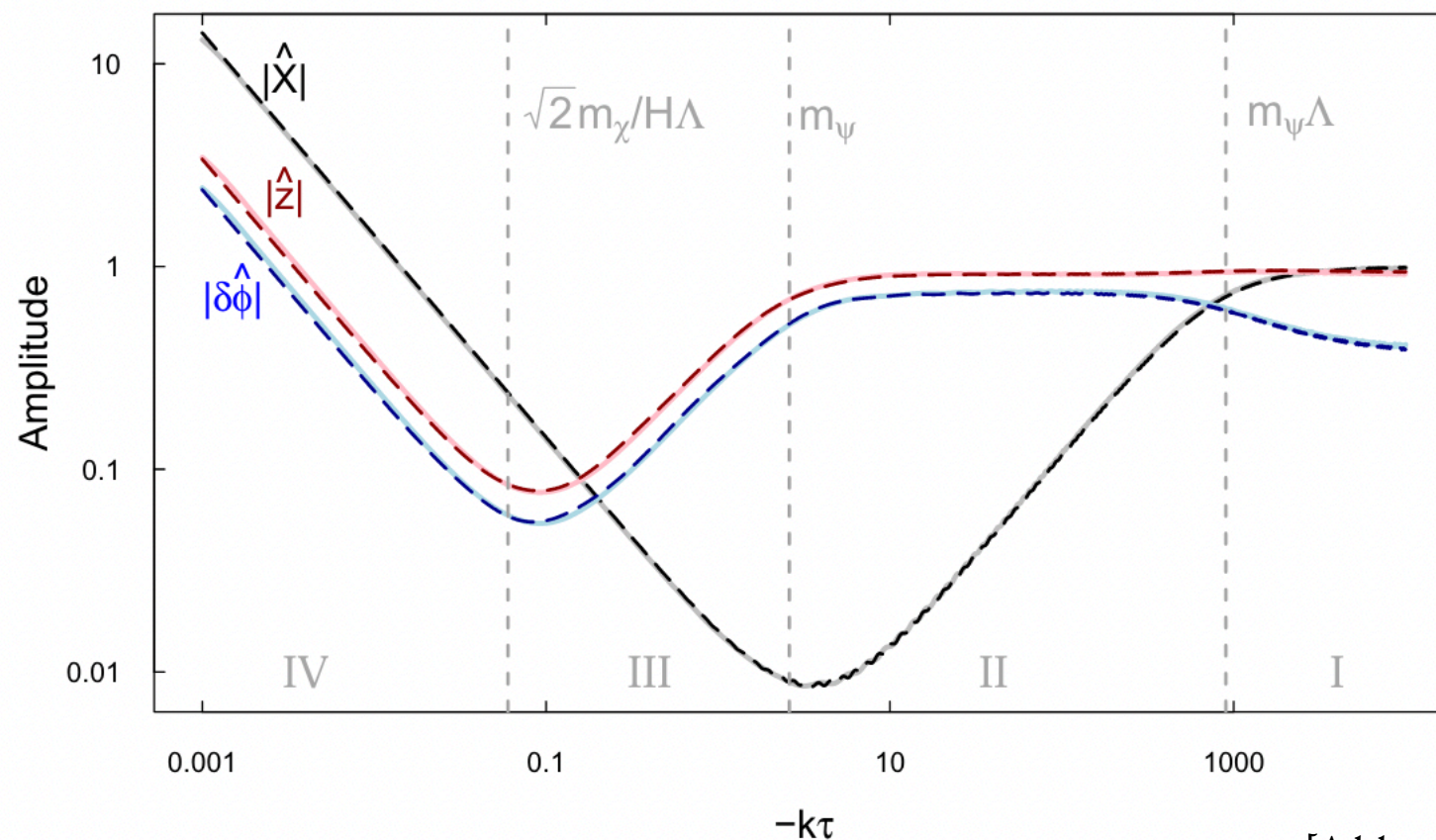
- WKB approx in region I & II, to find

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- Negative (i.e. instability in II for)

$$|m_Q|^2 < 2 \quad [\text{Dimastrogiovanni, Peloso}]$$

Scalar Fluctuations



[Adshead, Martinec, Wyman]

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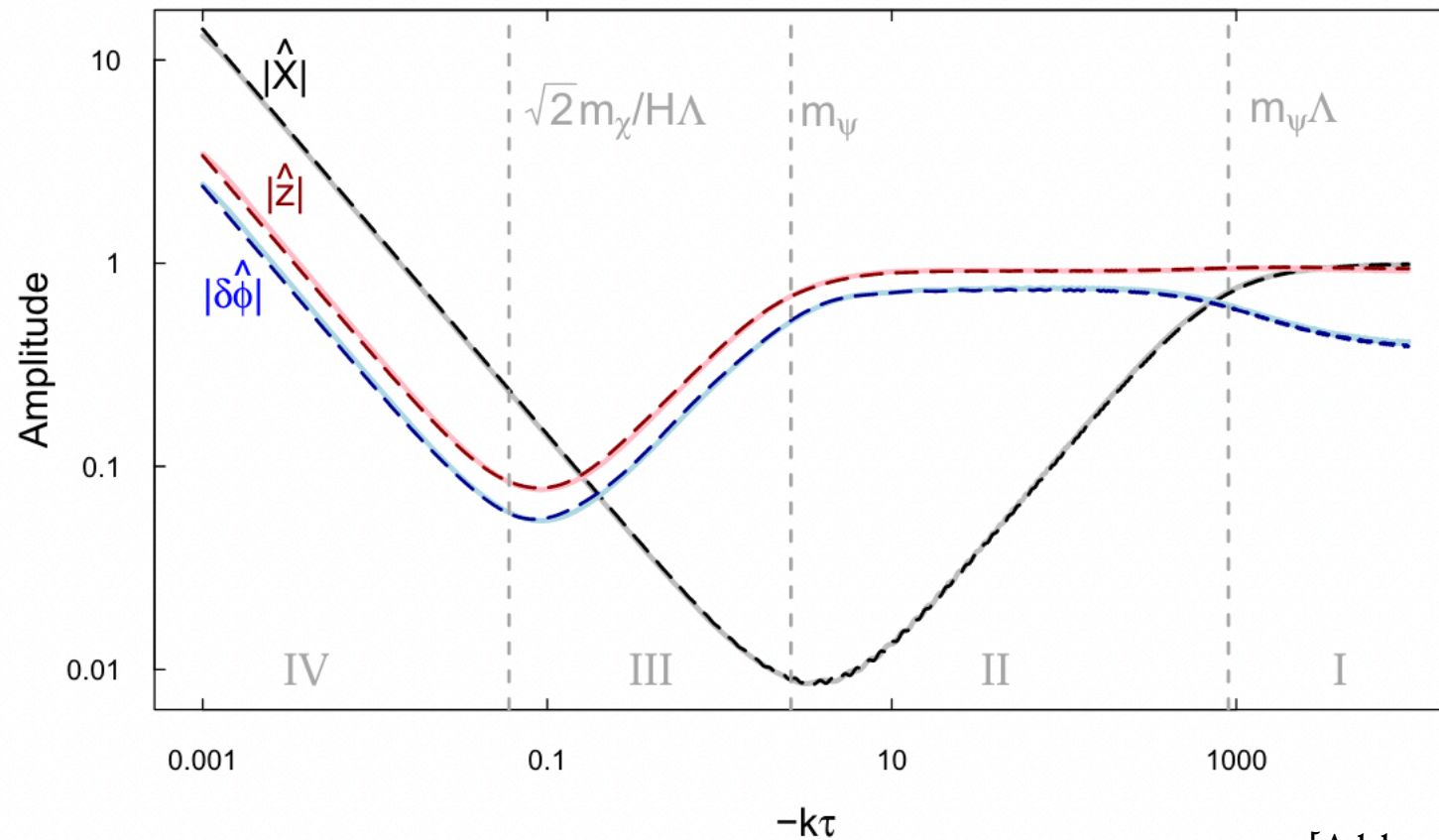
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$$|m_Q|^2 < 2 \quad [\text{Dimastrogiovanni, Peloso}]$$

- Serious issue given standard hierarchy

$$\frac{\lambda Q}{f} \equiv \Lambda \gg m_Q \equiv \frac{gQ}{H}$$

Scalar Fluctuations



[Adshead, Martinec, Wyman]

- WKB approx in region I & II, to find

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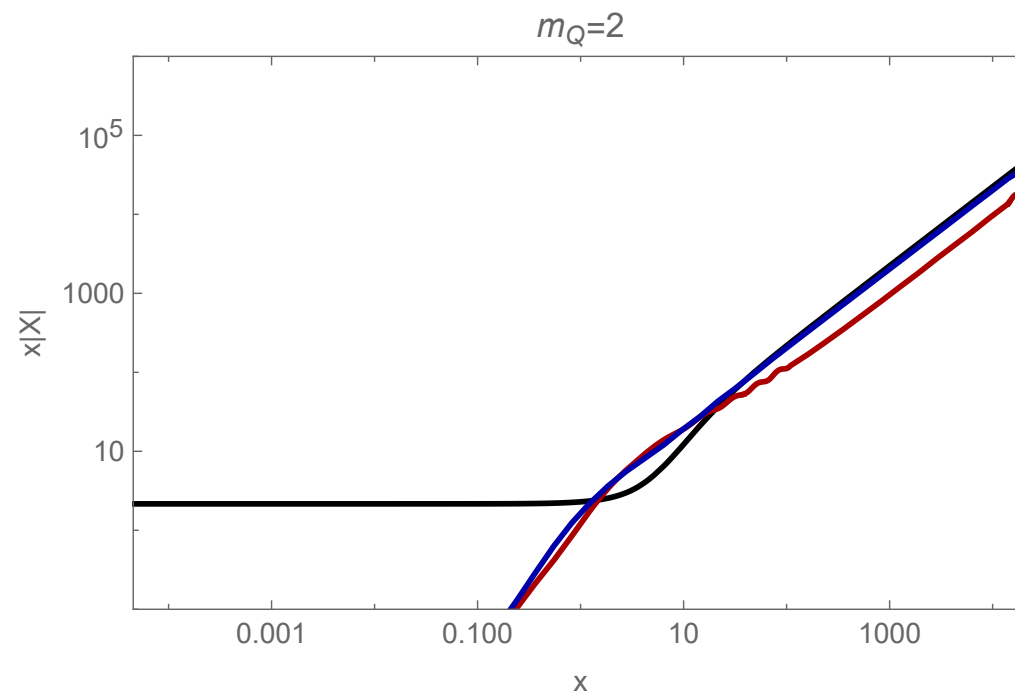
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AVOID

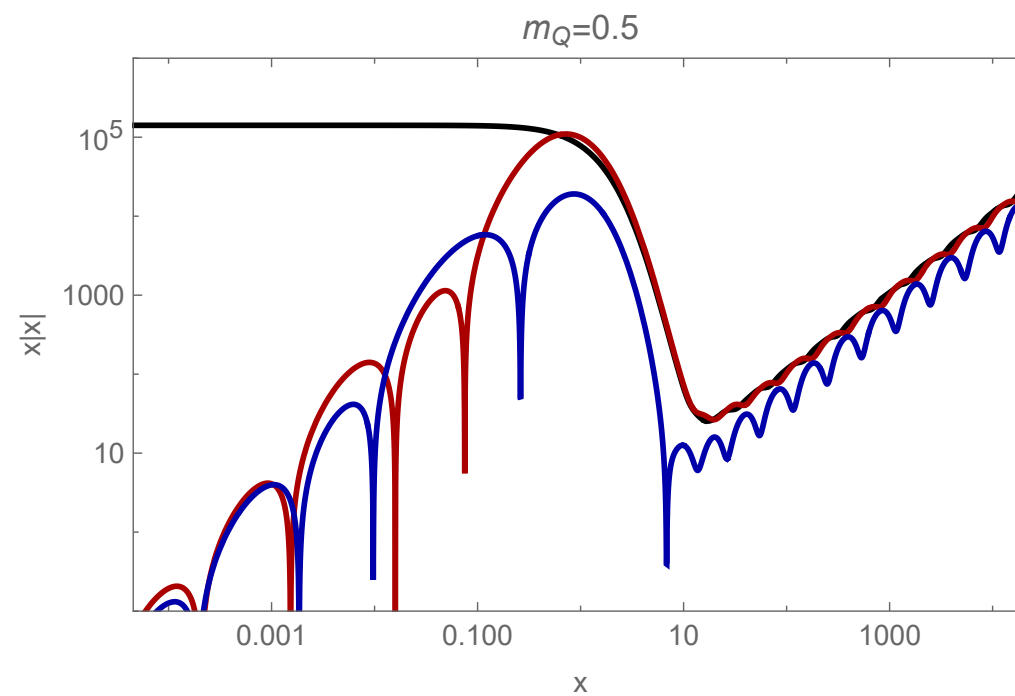
Effect of (finite) instability

standard

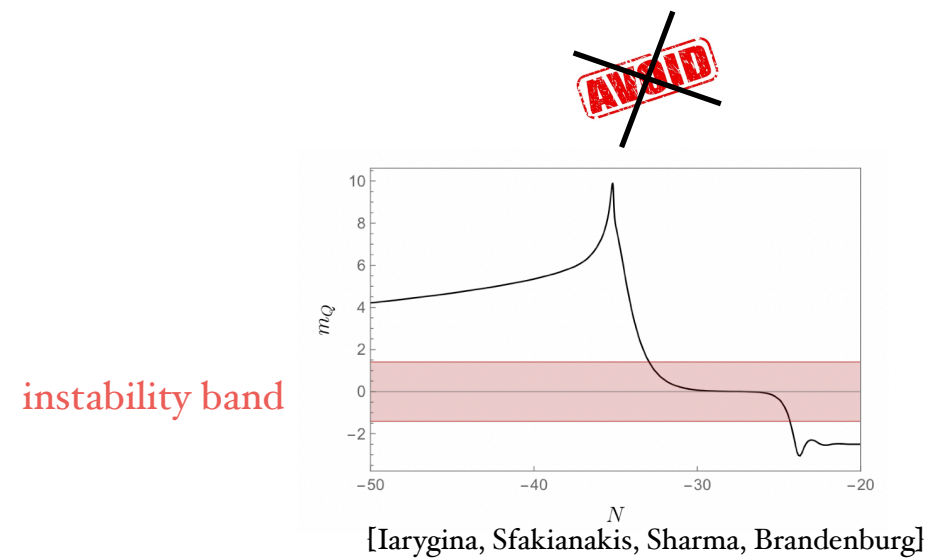


[Dimastrogiovanni, MF, Papageorgiou, 2024]

small m_Q larg-ish Λ



backreaction ==> cannot avoid



backreaction

==> extended inflation ==>

can inflate with smaller λ , smaller Λ/m_Q hierarchy, tamer instability

Instability Crossing is Generic

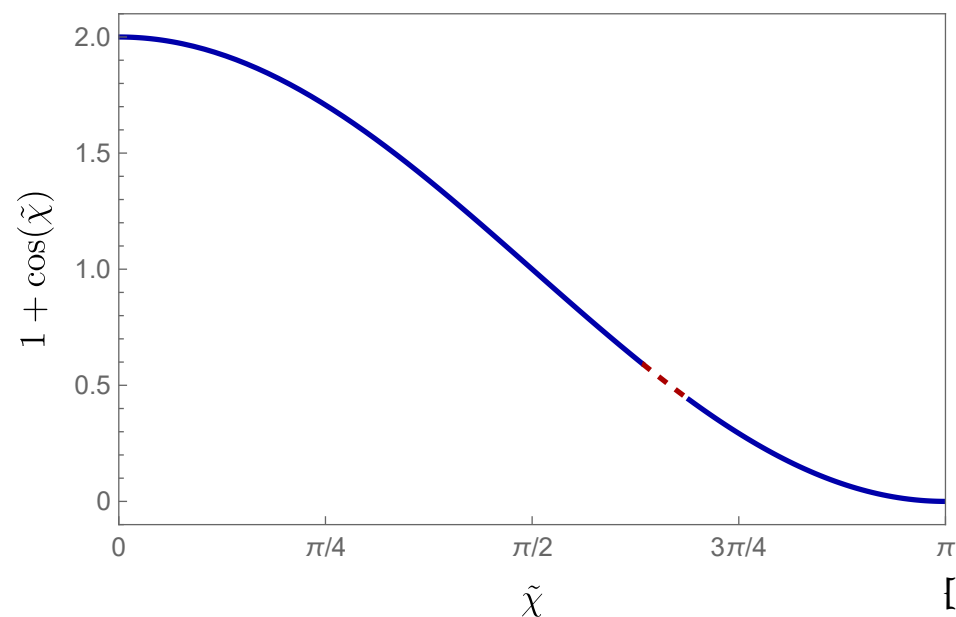
will quickly get to strong backreaction

onset_B = onset(m_Q, g)

$$m_Q \simeq \left(g \sqrt{\epsilon_H} \frac{M_p}{H} \right)^{1/2}$$

effect largely insensitive to global properties of the potential

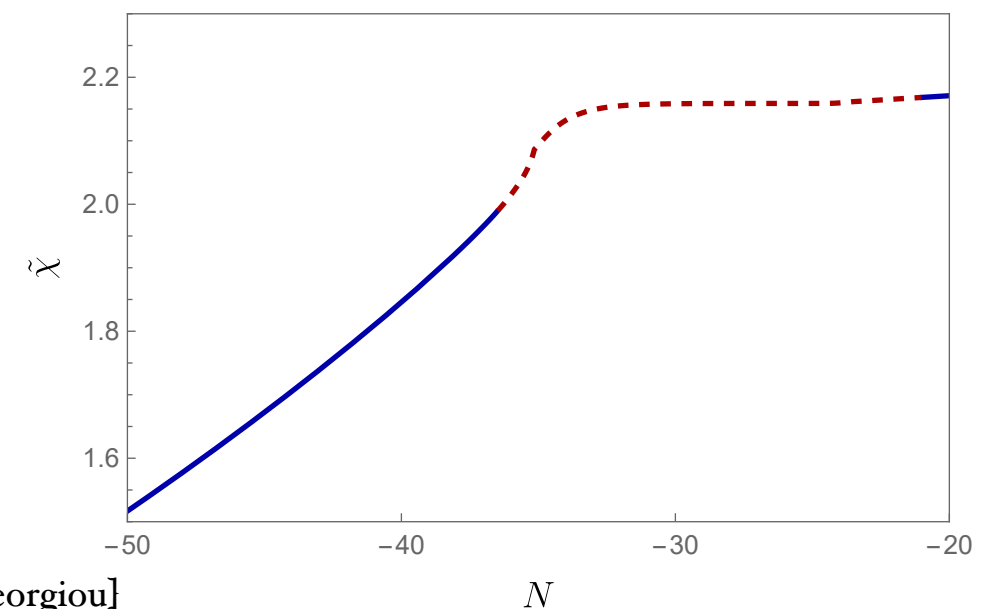
potential evol.



[Dimastrogiovanni, MF, Papageorgiou]

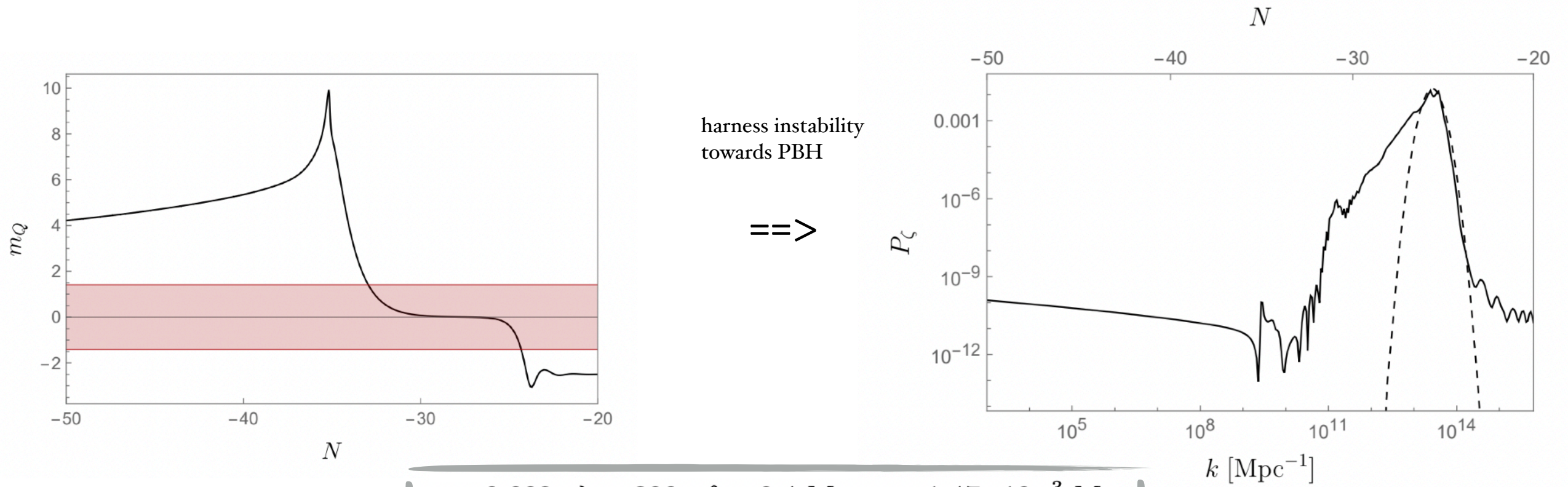
just need right local slope of the potential

field evolution

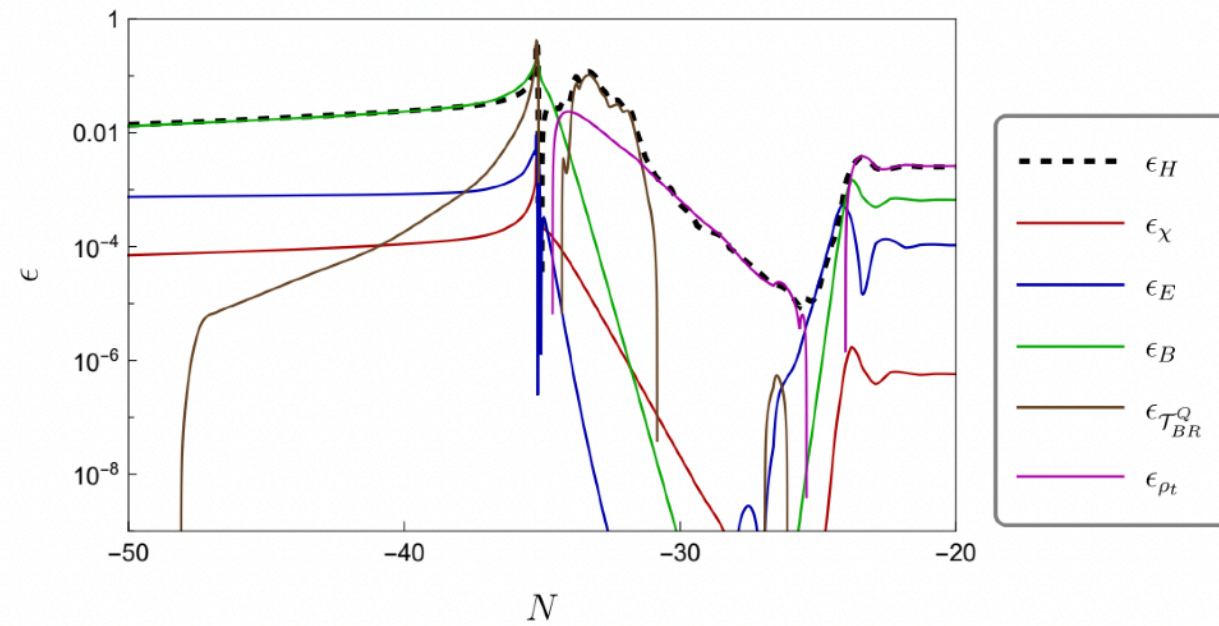


crossing spans very small field excursion

Power Spectrum



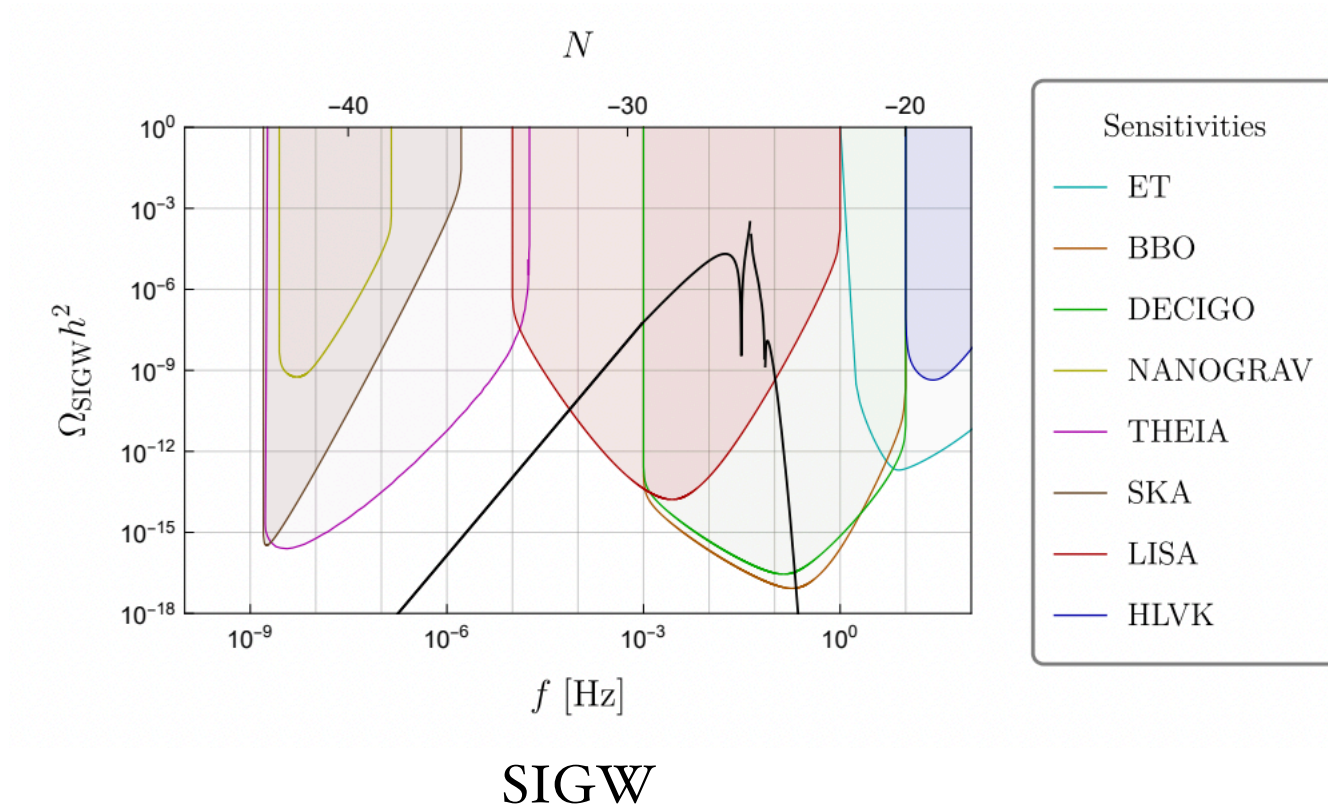
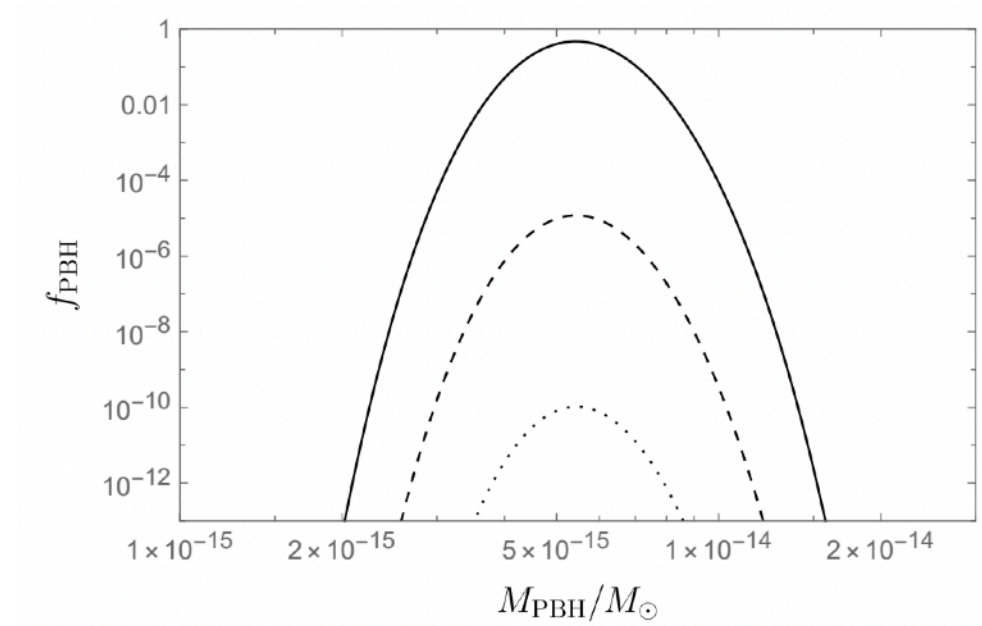
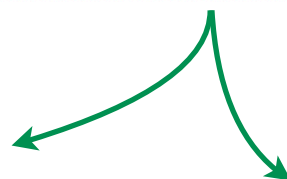
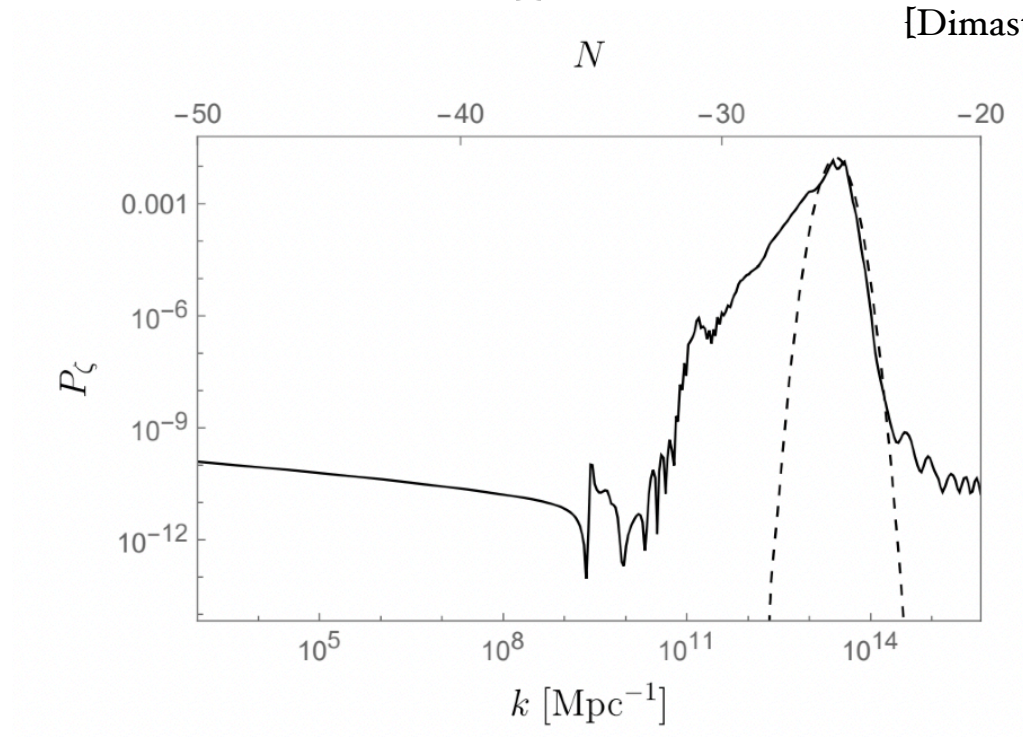
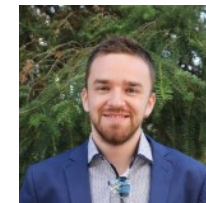
$$g = 0.002, \lambda = 300, f = 0.4 M_p, \mu = 1.47 \cdot 10^{-3} M_p$$



$$\epsilon_\chi \equiv \frac{\dot{\chi}^2}{2M_p^2 H^2}, \quad \epsilon_B \equiv \frac{g^2 Q^4}{M_p^2 H^2}, \quad \epsilon_E \equiv \frac{(\dot{Q} + HQ)^2}{M_p^2 H^2}, \quad \epsilon_{\mathcal{T}_{BR}^Q} \equiv \frac{Q}{2M_p^2 H^2} \mathcal{T}_{BR}^Q, \quad \epsilon_{\rho_t} \equiv \frac{2}{3M_p^2 H^2} \rho_t$$

Power Spectrum

[Dimastrogiovanni, MF, Papageorgiou, 2024]



SIGW

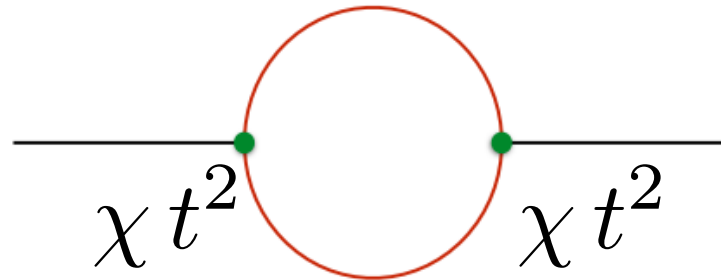
Mechanism largely insensitive to potential but



stay tuned (we found a viable & natural potential)!

Perturbativity

(SCNI case)



the same interaction enhancing sourced GW affects the 1-loop scalar PS

From $P_{\zeta}^{\text{tree}} \gg P_{\zeta}^{1-\text{loop}}$ or at least $P_{\zeta}^n \gg P_{\zeta}^{n+1}$ from a given n onwards

strong constraints on parameter space

====>

in SCNI sourced GW signal can be > vacuum but no more than 1 order of magnitude

[Dimastrogiovanni, MF, Hardwick, Assadullahi, Koyama, Wands 2018]

[Papageorgiou, Peloso, Unal 2018 & 2019]

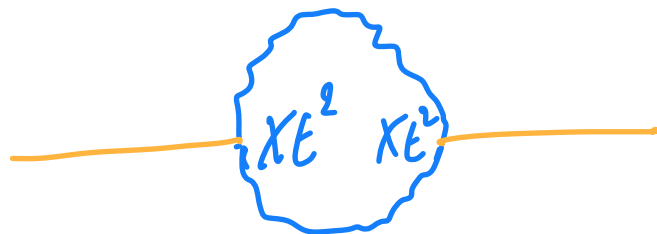
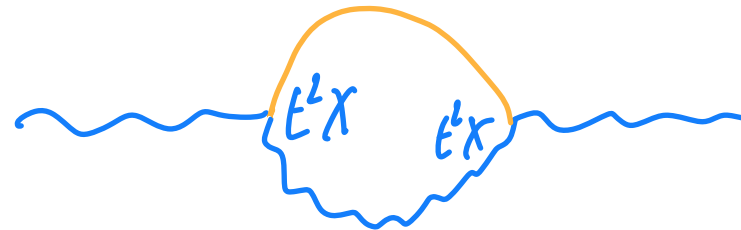
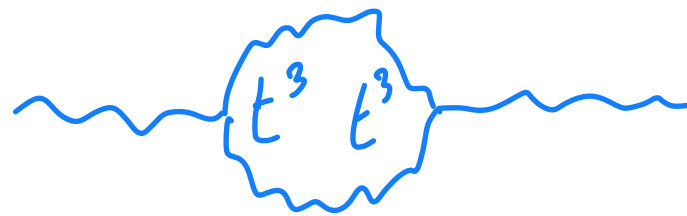
in similar context see also [Ferreira, Ganc, Noreña, Sloth 2015]

◆ (mostly CMB) bounds on non-Gaussianity play a similar role

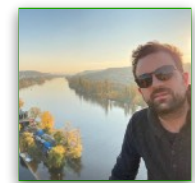
Perturbativity

(non-minimal CNI case)

Perturbativity and primordial non-Gaussianity bounds on all axion-gauge field models



[Dimastrogiovanni, MF, Michelotti, Ozsoy, **to appear**]

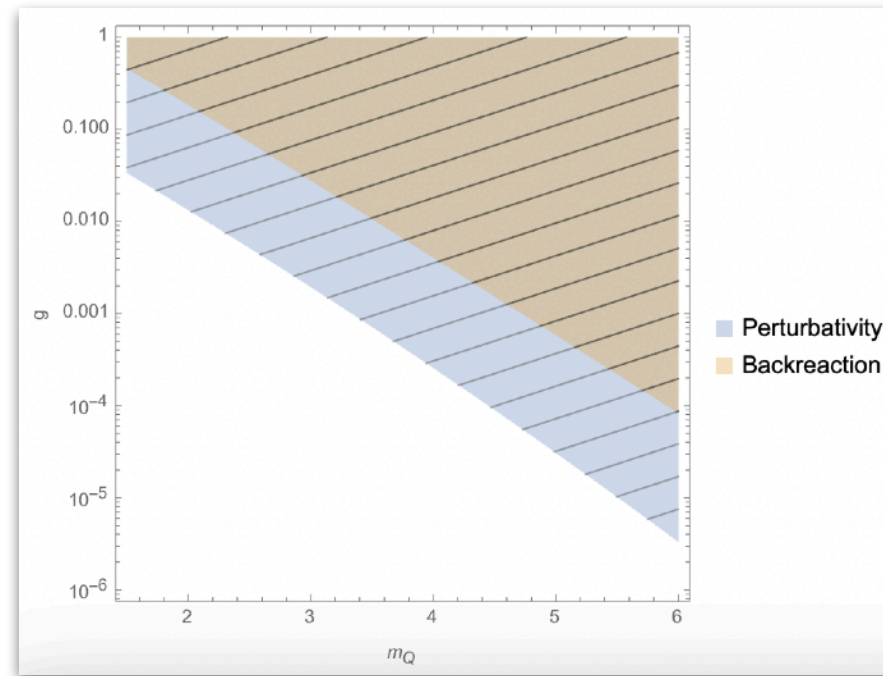


Perturbativity

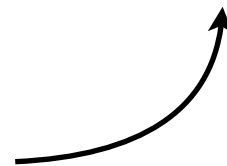
(non-Abelian case)



most stringent bound,
largely insensitive to potential,
indirectly dependent on λ

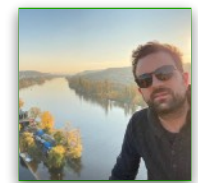


backreaction via
rough analytical estimate



universal behaviour in the non-Abelian case

[Dimastrogiovanni, MF, Michelotti, Ozsoy 2023]



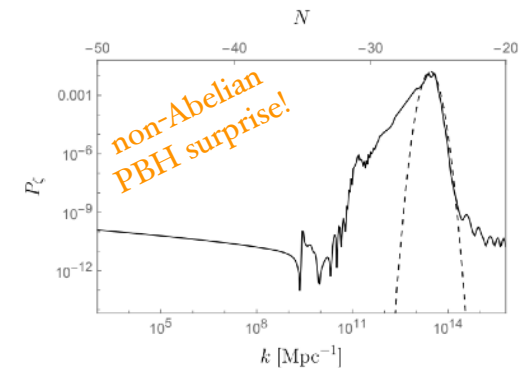
Challenges & Opportunities

[Anber, Sorbo]
 [Iarygina, Sfakianakis, Sharma, Brandenburg 2023]
 [Durrer, Sobol, Vilchinskii 2023]
 [Dimastrogiovanni, MF, Papageorgiou 2024]
 [Garcia-Bellido, Papageorgiou, Peloso, Sorbo 2023]...

(I) Gauge fields **backreaction** on the background

Tackled numerically under homogeneous assumption
 but see [Domcke, Ema, Sandner 2023]

Lattice the ultimate test [Figueroa, Lizarraga, Urio, Urrestilla 2023], [Caravano, Komatsu, Lozanov, Weller 2022]
 [Caravano, Peloso 2024], [Figueroa, Lizarraga, Loayza, Urio, Urrestilla 2024]



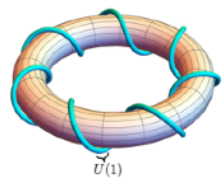
(II) **Perturbativity** bounds



Explored, can be competitive with backreaction, need better numerical studies

[Ferreira, Ganc, Noreña, Sloth 2015]
 [Papageorgiou, Peloso, Unal 2018]
 [Dimastrogiovanni, MF, Michelotti, Ozsoy 2024]

(III) **UV** completion



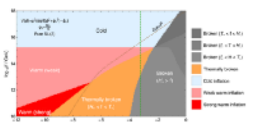
Abelian sector ok, non-Abelian ok but signatures only upon fine-tuning

...
 [Dall'Agata 2018]
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(IV) **Re-heating** with (dark) sectors

Lots of interesting work, past & future
 ...
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 [Lozanov, Amin 2016]
 [Adshead, Giblin, Weiner 2018]
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(VI) **Thermalization?**



(V) **Magneto-genesis, baryogenesis**

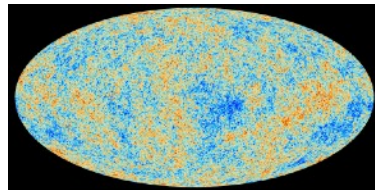
Not easy, but very intriguing

...
 [Fujita, Yokoyama 2014]
 [Caldwell, Devulder 2017]
 [Cado, Quirós 2022]
 [Brandenburg, Iarygina, Sfakianakis, Sharma 2024]

...
 [Ferreira, Notari 2017 x 2]
 [Berghaus, Graham, Kaplan 2019]
 [DeRocco, Graham, Kalia 2021]
 [Mukuno, Soda 2024]...

Thank you!

Chirality



CMB tests

single-field
slow-roll inflation

no chirality

$$\langle BT \rangle = 0 = \langle EB \rangle$$

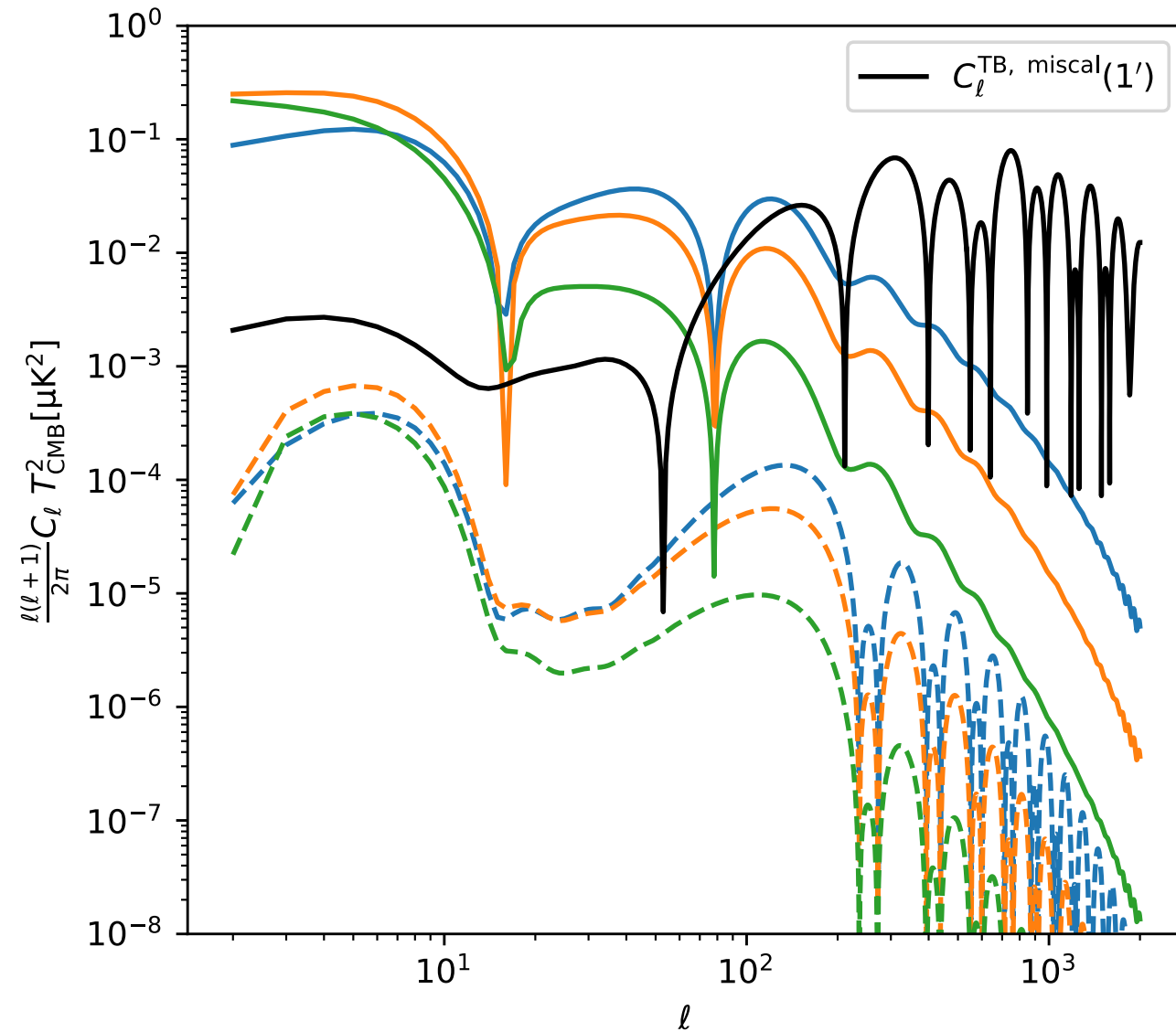
Chern-Simons
coupling

chirality

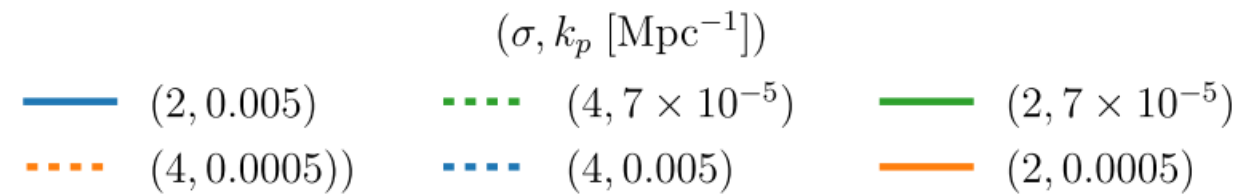
$$\langle BT \rangle \neq 0 \neq \langle EB \rangle$$

Chirality

LiteBIRD



[Thorne, Fujita, Hazumi, Katayama, Komatsu, Shiraishi, 2017]

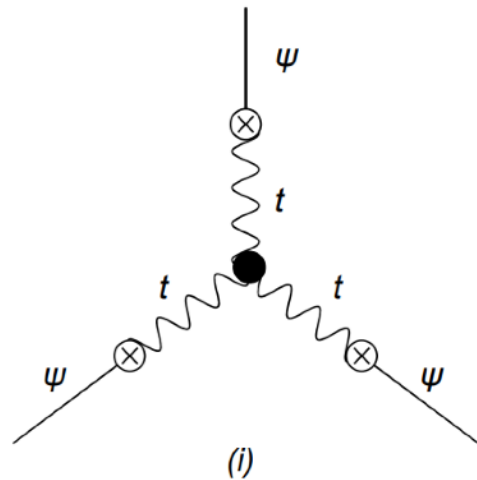


$$\mathcal{P}_h^{\text{sourced}} = r_* \mathcal{P}_\zeta \text{Exp} \left[- \frac{1}{2\sigma^2} \ln^2 \left(\frac{k}{k_p} \right) \right]$$

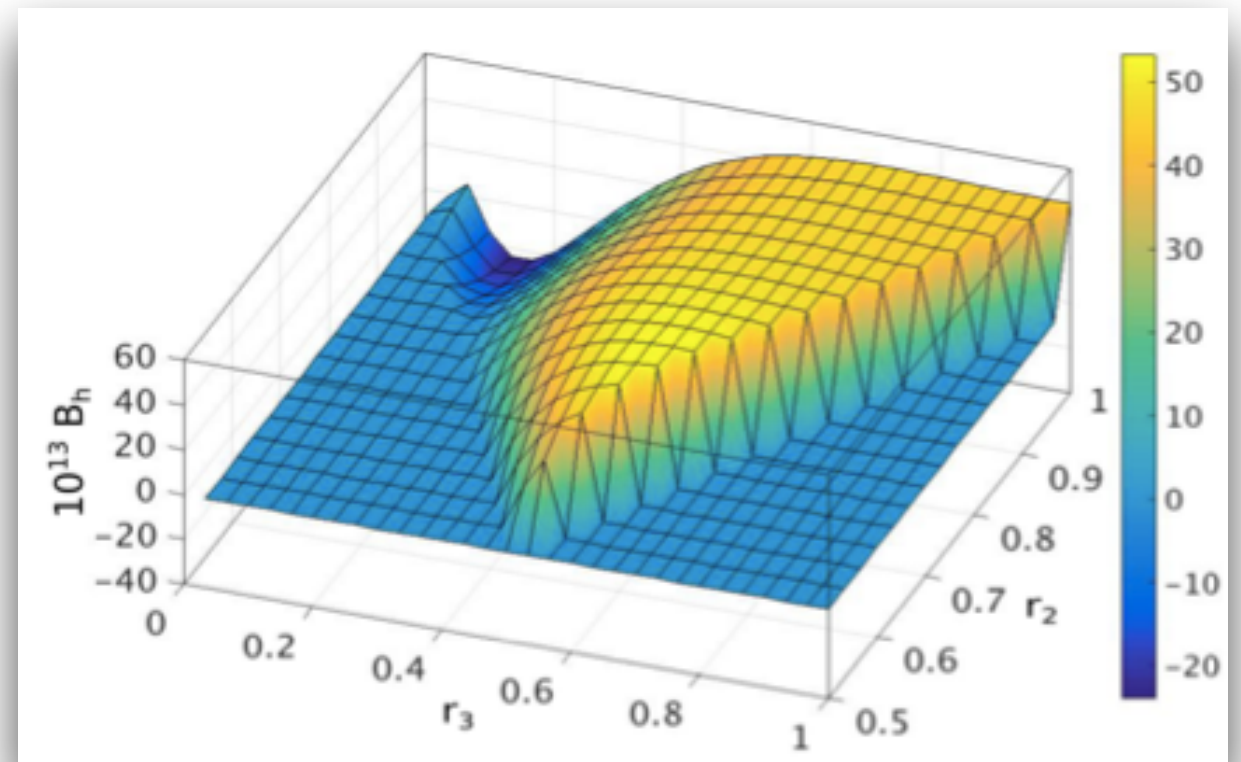
non-Gaussianity (TTT)

[Agrawal - Fujita - Komatsu 2017]

$$\text{n-G} \quad \langle h_R(\vec{k}_1) h_R(\vec{k}_2) h_R(\vec{k}_3) \rangle = (2\pi)^3 \delta^{(3)} \left(\sum_{i=1}^3 \vec{k}_i \right) B_h(k_1, k_2, k_3)$$



$\Psi = \text{GW}$
 $t = \text{tensor SU}(2)$



$$\frac{B_h}{P_\zeta^2} \lesssim r^2 10^6$$

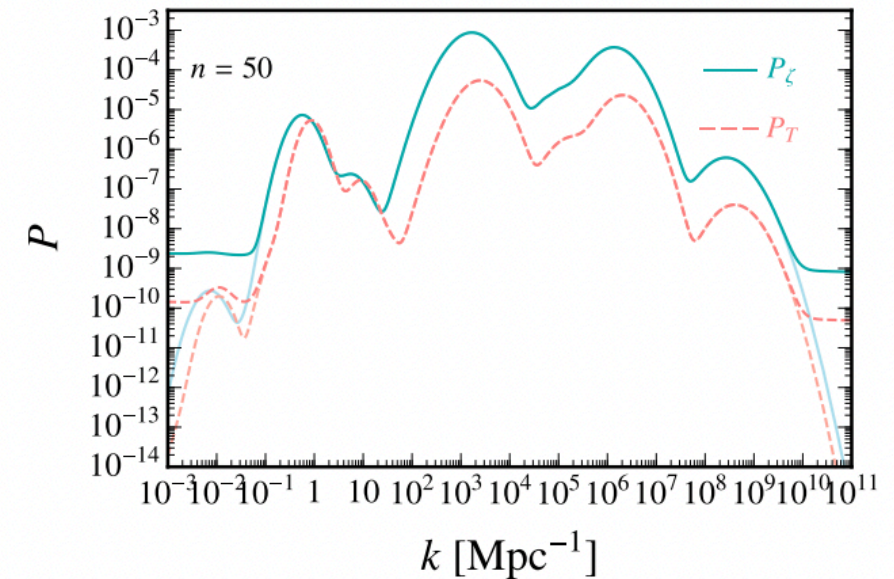
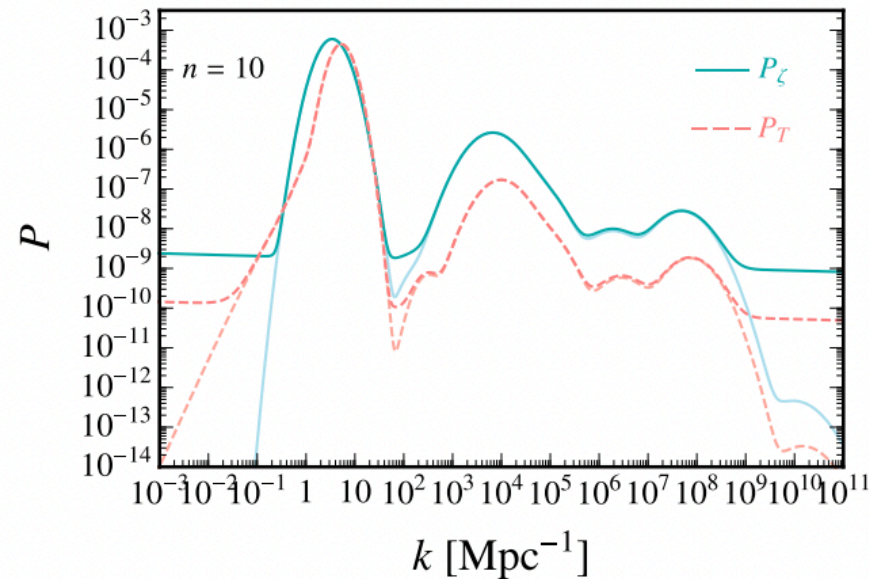
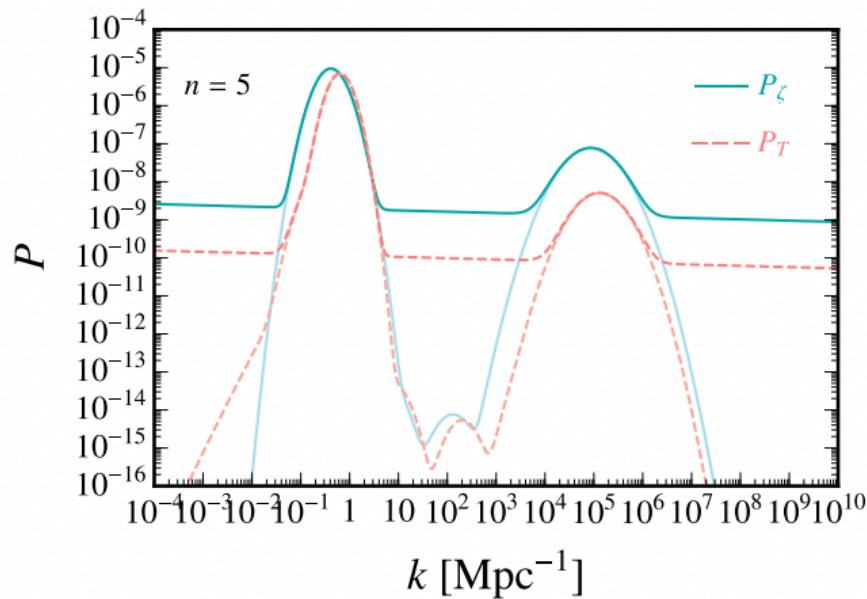
sourced nG tensors
 larger than in minimal SFSR

$m_Q = 3.45$
 $\epsilon_B \simeq 10^{-5}$
 $H \simeq 10^{13} \text{ GeV}$
 $r_{\text{vac}} \simeq 0.002$
 $r_{\text{sourced}} \simeq 0.04$

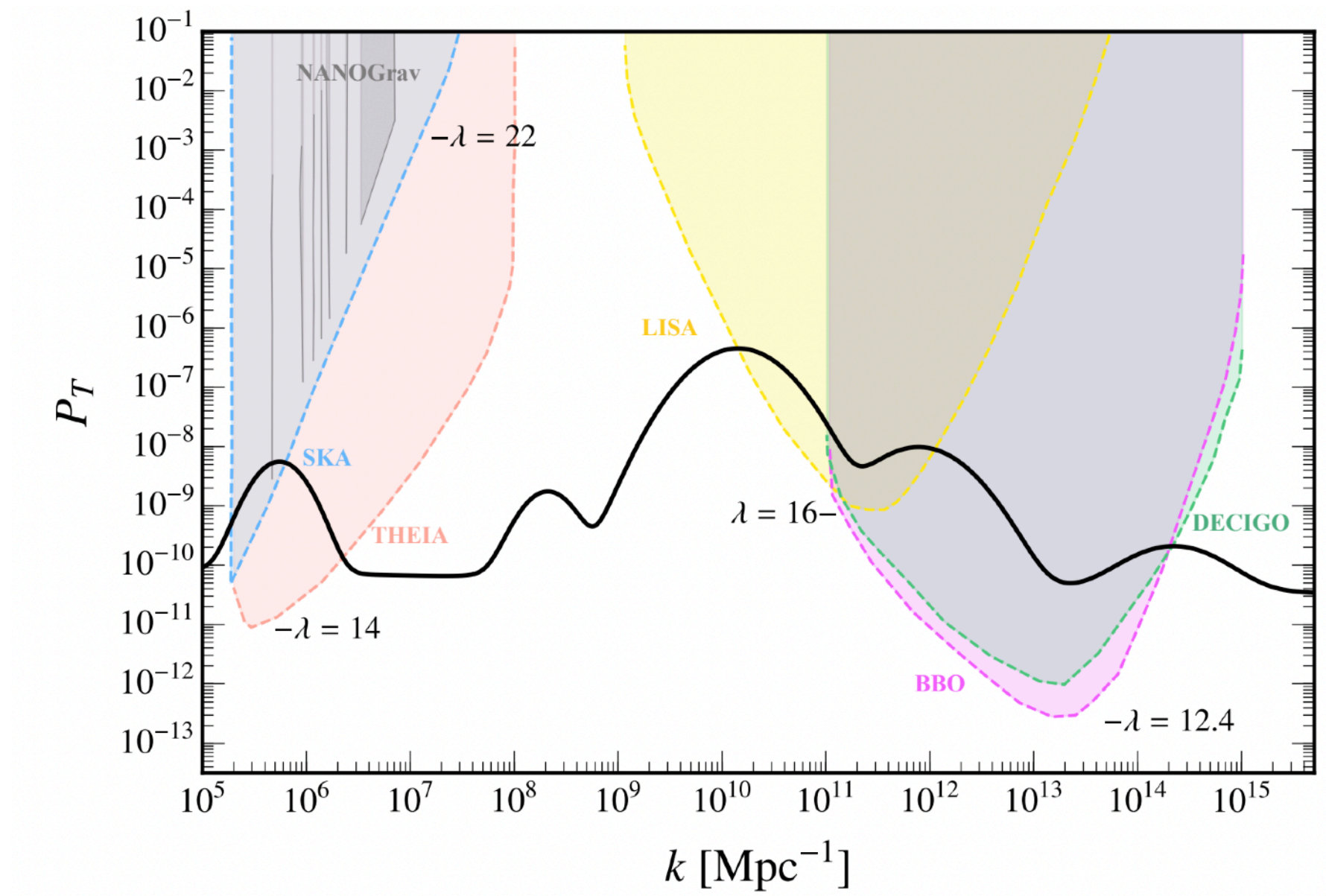
Multiple Abelian Spectator Sectors

$$-\frac{1}{2}(\partial\varphi)^2 - V_{inf}(\varphi) - \frac{1}{4}F_{ia\mu\nu}F_i^{a\mu\nu} - \frac{1}{2}(\partial\chi_i)^2 - V_{S_i}(\chi_i) - \frac{\lambda_i}{4f_i}\chi_i F_{ia\mu\nu}\tilde{F}_i^{a\mu\nu}$$

scalar (blue) & tensor (red) fluctuations



Multiple Abelian Spectator Sectors



10 fields, ad hoc initial conditions, Abelian case

Challenge III: UV Completion

(SCNI case)

field content easily obtained, key is strength of CS interaction, i.e. λ

interesting GW phenomenology requires $\lambda > 100$

very hard to obtain in
clockwork mechanisms

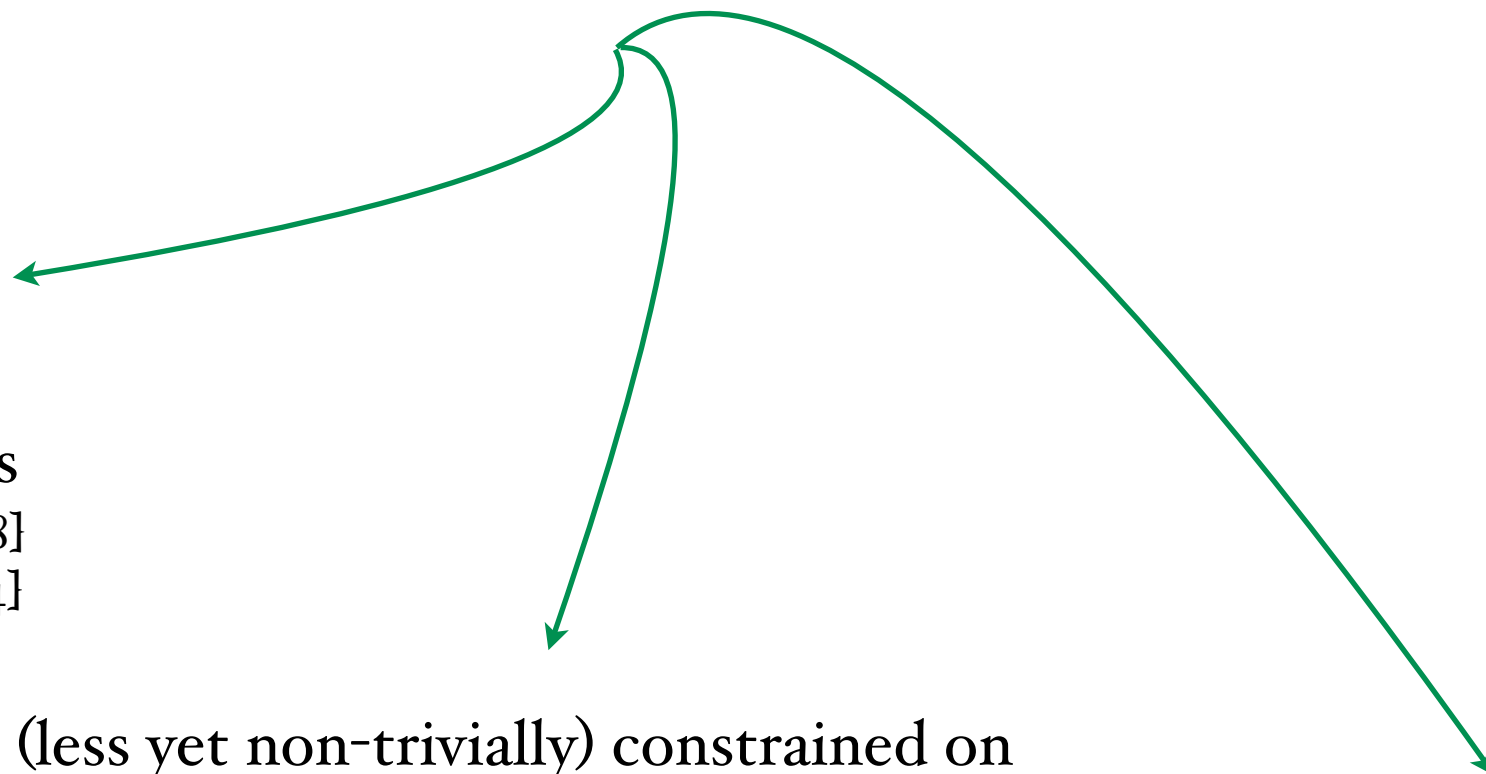
[Agrawal, Fan, Reece 2018]
[Kim, Nilles, Peloso 2004]

(less yet non-trivially) constrained on
more general unitarity grounds

[Agrawal, Fan, Reece 2018]
[Bagherian, Reece, Xu 2022]

nevertheless possible
in string theory

[Holland, Zavala, Tasinato 2020]



Natural Inflation + Gauge Sector

briefly on U(1)

$$\Phi'' + 2aH\Phi' - \nabla^2\Phi + a^2 \frac{dV(\Phi)}{d\Phi} = \left(\frac{\alpha}{f} a^2 \vec{E} \cdot \vec{B} \right)$$

E_{kin} in production of gauge quanta

[Anber, Sorbo]

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}^{\mu\nu} = -\frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

CS ==> parity breaking ==> different sols for polarisations

$$\left[\partial_\tau^2 + k^2 \pm \frac{2k\xi}{\tau} \right] A_\pm(\tau, k) = 0$$

enhanced polarisation $A_+(\tau, k) \propto e^{\pi\xi}$

$$\xi \equiv \frac{\lambda \dot{\phi}}{2fH}$$

[Barnaby, Peloso 2011, Barnaby, Namba, Peloso 2011, Bartolo et al 2014+...]

[Pajer, Peloso (2013)]

Natural Inflation + Gauge Sector

briefly on U(1)

non-linear sourcing w/ enhanced field



- observed scalar spectrum @ CMB & chiral GW
inflaton coupled to $\mathcal{N} \sim \mathcal{O}(10^5)$ gauge fields

- or for a different potential

- or introducing spectator axions

[Mukohyama, Namba, Peloso, Shiu 2014]

blue or bump-like, chiral, GW spectrum
possibly large GW non-Gaussianities

U(1) many many applications, e.g. in PBH context

By now a rich literature on the subject



...Anber - Sorbo 2009; Cook - Sorbo 2011; Barnaby - Peloso 2011;
Adshead- Wyman 2011; Maleknejad - Sheikh-Jabbari, 2011;
Dimastrogiovanni - MF - Tolley 2012; Dimastrogiovanni - Peloso 2012;
Adshead - Martinec -Wyman 2013; Garcia-Bellido - Peloso - Unal 2016;
Agrawal - Fujita - Komatsu 2017; Fujita - Namba - Obata 2018; Domcke -
Mukaida 2018; Iarygina - Sfakianakis 2021; ...

Supergravity embedding
[Dall'Agata]

Lots of research in this direction

+ gravitational Chern-Simons term
+ fermions production
+ back-reaction
[Komatsu et al, x 3]



+ gravitational leptogenesis
[Caldwell, Devulder]

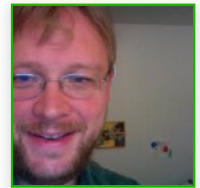
+ perturbativity bounds
[Papageorgiou, Peloso, Unal]

+ SCNI in string theory
[Holland, Zavala, Tasinato]

$$\epsilon_B \equiv g^2 Q^4 / (H M_{\text{Pl}})^2$$

Multiple Abelian Spectator Sectors

[Dimastrogiovanni, MF, Leedom, Putti, Westphal 2023]



Multiple Abelian Spectator Sectors

[Dimastrogiovanni, MF, Leedom, Putti, Westphal 2023]

crucial caveat

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how well my collaborators understand the string theory part

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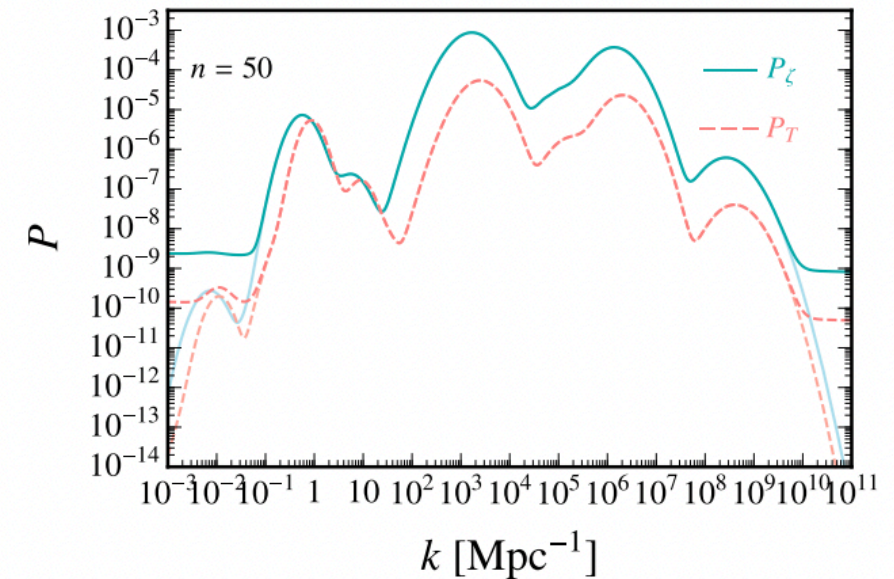
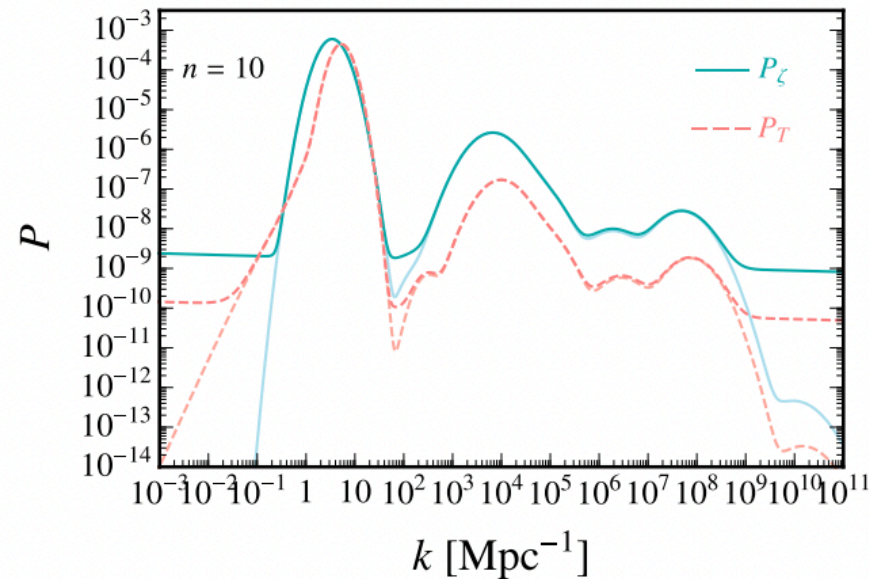
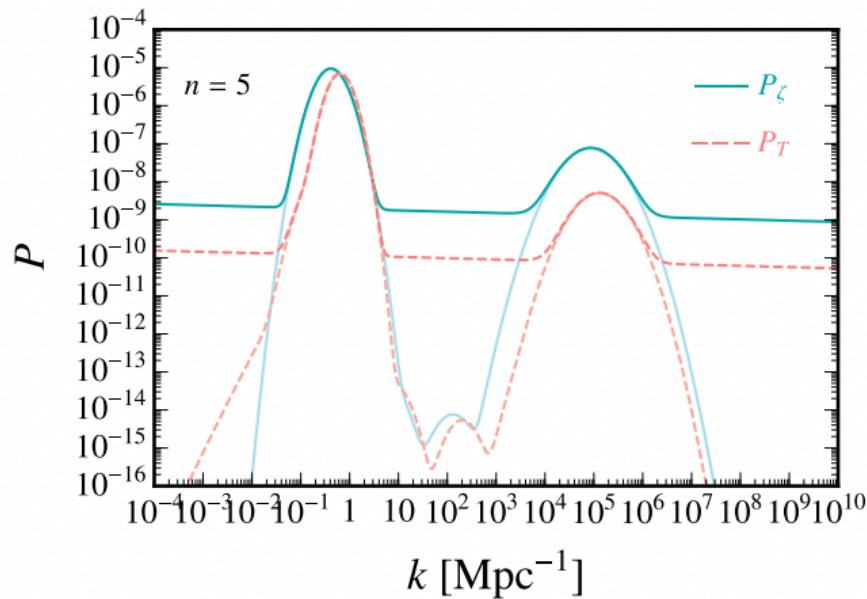
my level



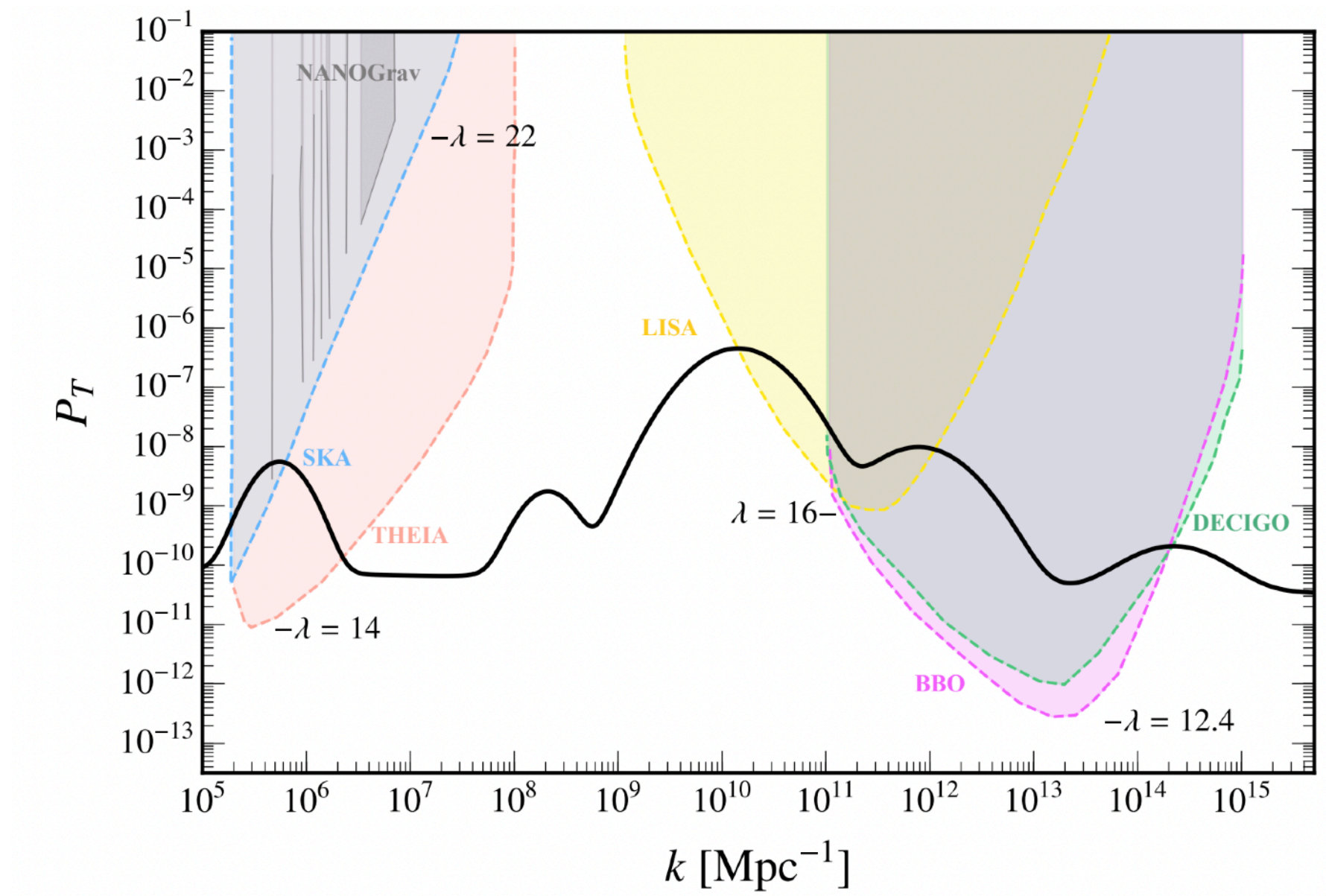
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scalar (blue) & tensor (red) fluctuations



Multiple Abelian Spectator Sectors



10 fields, ad hoc initial conditions, Abelian case

Naturalness of CS couplings is MASA models

phenomenological “needs”
(*caveat from backreaction!)

Abelian: $\lambda \sim 20$

non-Ab: $\lambda \sim \text{a few} \times \mathcal{O}(100)$

Abelian case

detectable GW from
orientifold-odd 2-form axion spectators

[Dimastrogiovanni, MF, Leedom, Putti, Westphal 2023]

non-Ab case

$N \sim 10^5$ D7-branes (i.e. fine tuning)

[Holland, Zavala, Tasinato 2020]

Why not clockwork in a Nutshell

[Agrawal, Fan, Reece 2018]
[Bagherian, Reece, Xu 2022]

$$2\pi f_\chi \sim k g^2 / 16\pi F \tilde{F}$$

From periodicity of axion field follows

$$\lambda = \frac{j \cdot k \cdot g^2}{8\pi^2}$$

Integer k from integrating out fermions carrying SU(2) gauge charge with χ -dependent masses

Validity of such EFT of fermions (and gauge) fields needs $k < 4\pi/g^2$

act on “j” instead via clockwork

$$\sum_{i=1}^{n-1} \mu_{i+1}^4 \cos \left[\frac{m_i \chi_i}{f_i} + \frac{\chi_{i+1}}{f_{i+1}} \right] + \mu_1^4 \cos \frac{\chi_1}{f_1}$$

Integrating out heaviest modes $\Rightarrow$ parametrically light χ with effective potential having $j = \Pi m_i$ ✓

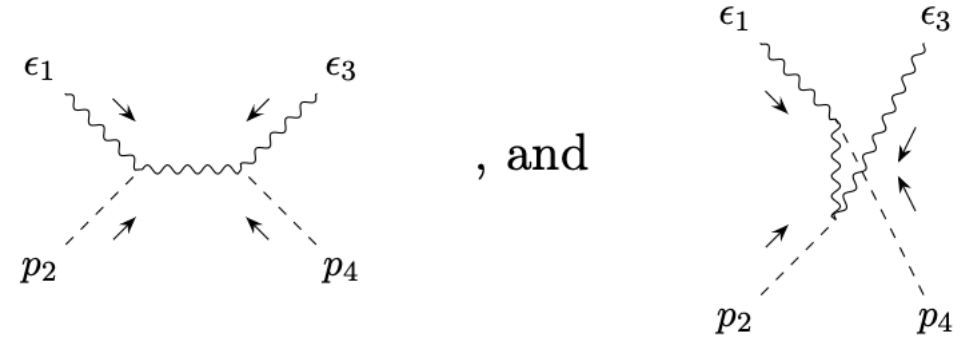
each cosine mediates axion scattering $\Rightarrow$ perturbative unitarity bound $\rightarrow$ upper bound on μ
 $\Rightarrow$ $j < f/\mu$
 $\lambda \leq \frac{f}{2\pi\mu}$

paired up with phenomenological constraints implies this mechanism does not work for the model

Unitarity Bounds

[Bagherian, Reece, Xu 2022]

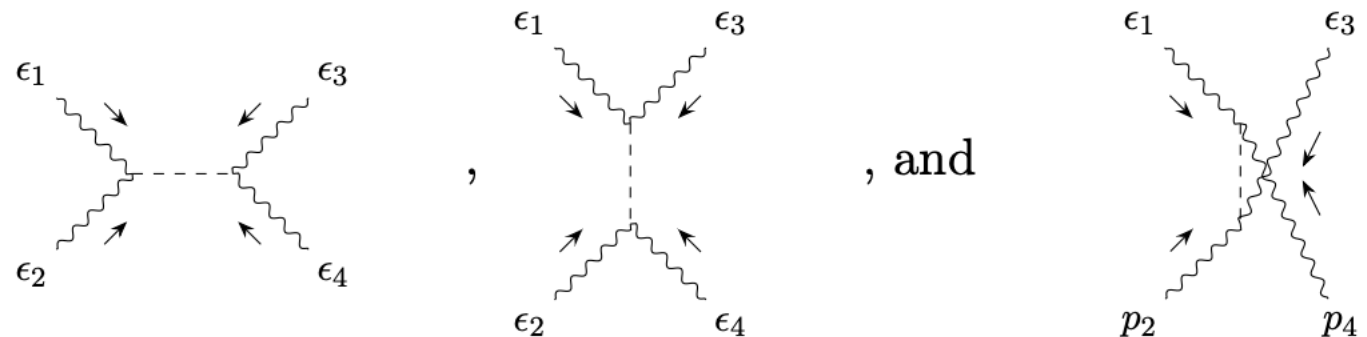
a: $2 \rightarrow 2$ axion gauge scattering amplitudes



$$\mathcal{M}_{+-}^{A\chi} = \delta^{ab} \frac{u+s}{su} \left(\frac{\lambda}{2f} \right)^2 \langle 14 \rangle^2 [34]^2 \propto \delta^{ac} \left(\frac{\lambda}{4f} \right)^2 2s(1 - \cos \theta),$$

$$\mathcal{M}_{++}^{A\chi} = -2\delta^{ac} \left(\frac{\lambda}{2f} \right)^2 \langle 13 \rangle^2 \propto \delta^{ab} \left(\frac{\lambda}{4f} \right)^2 4s(1 - \cos \theta).$$

b: gauge scattering mediated by axion



$$\mathcal{M}_{++++}^{AA} = - \left(\frac{\lambda}{4f} \right)^2 \left[\frac{\delta^{ab}\delta^{cd}}{s - m_\chi^2} [12]^2 [34]^2 + \frac{\delta^{ac}\delta^{bd}}{t - m_\chi^2} [34]^2 [13]^2 + \frac{\delta^{ad}\delta^{bc}}{u - m_\chi^2} [14]^2 [23]^2 \right]$$

a & b much weaker (than cw) bound $\mu \lesssim 8\sqrt{\pi} \frac{f}{\lambda}$ small SCNI region still viable

String Theory Embedding

[Holland, Zavala, Tasinato 2020]

Framework

Kahler inflation in type IIB large volume string compactifications

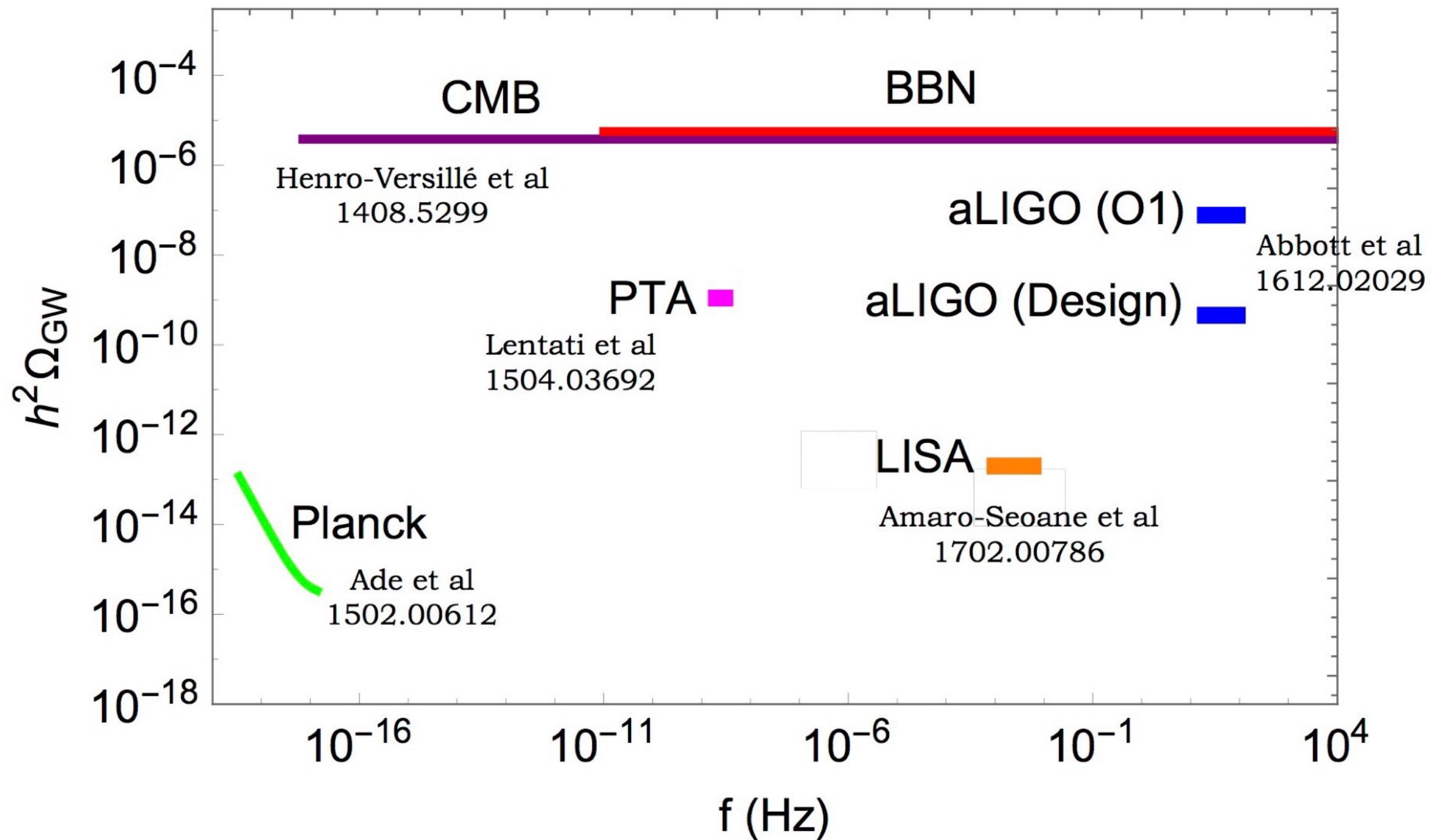
need spectator sector associated with gaugino condensation on multiply magnetised D7-branes

Successful inflation (+large GW enhancement) hinges on suitable values of three parameters:

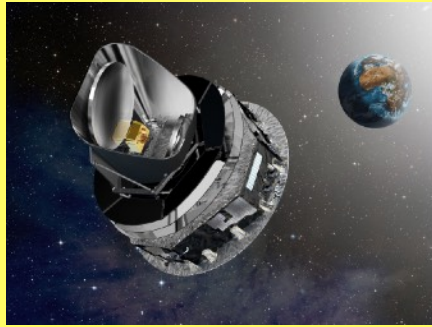
- magnetic flux
- degree of the condensing gauge group
- wrapping number of the D7-brane

gauge group degree ($\sim$ number of D7s) $N \sim 10^5$ not easy to realise, yet necessary for phenomenology!

Observational bounds/sensitivities for SGWB



Scalar bispectrum: current bounds



$$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \quad f_{\text{NL}}^{\text{equil}} = -26 \pm 47 \quad f_{\text{NL}}^{\text{ortho}} = -38 \pm 24$$

[68 % CL]

Scalar bispectrum: future bounds

- **LSST**
- **SKA**
- **SPHEREx**

$$\sigma(f_{\text{NL}}^{\text{local}}) \simeq 1$$

- **21-cm** $\sigma(f_{\text{NL}}^{\text{local}}) \lesssim 10^{-1}$
[Munoz, Ali-Haïmoud, Kamionkowski]

Tensor bispectrum

- **Planck**
 $f_{\text{NL}}^{\text{tens}} = (8 \pm 11) \times 10^2$
[68 % CL]

$$f_{\text{NL}}^{\text{tens}} \equiv \frac{B_{\gamma}^{+++}(k, k, k)}{(18/5)P_{\zeta}^2(k)}$$

(parity violating models / roughly equilateral)

- **LiteBIRD**
 $\sigma(f_{\text{NL}}^{\text{tens}}) = \text{a few}$
(possibly also with **PICO**)

The **axionic portion** of the spectator sector can arise **from dimensional reduction of p-forms in the 10d string theories**.

Gauge sector of spectator models depends greatly on which corner of the string landscape one works in. Largely focus on type IIB string theory compactified on orientifolded Calabi-Yau (CY) manifolds with quantized 3-form fluxes, D7-branes and O7-orientifold planes.

4d axions arise as KK zero modes of the 4-form C_4 and 2-form C_2 gauge fields.

The number of axions is governed by the **number of compact n-dimensional sub-manifolds** (n-cycles) of the 6d CY manifold chosen, **as well as the structure of the orientifold projection**: some number of 4-form axions are always present, while 2-form axions arise from a non-trivial ‘projection-odd’ sector of the orientifold action.

Gauge sectors are realized by the worldvolume theory of D7-branes wrapping 4-cycle submanifolds of the CY and permeating our 4d spacetime.

The two types of closed **string axions** differ in the way they couple to the D7-brane worldvolume gauge fields via Chern-Simons terms: the 4-form axions intrinsically couple to the worldvolume theory, while 2-form axions only acquire such a coupling in the presence of a particular type of quantized magnetic flux on the D7-brane. The “intrinsic” size of these CS couplings turns out to be too small to generate GW signals detectable with current or planned experiments.

Both CS couplings increase linearly with the number of times the D7-brane “wraps” a 4-cycle, and the 2-form axion CS coupling in addition increases with the amount of magnetic flux used on the D7-brane to generate it.

better definition of gauge invariant quantities

$$\sum_a (E_i^a)^2 \propto Q^2$$

$$\sum_a (B_i^a)^2 \propto Q^4$$