IMPERIAL

Cosmic Twinkling & Shimmering

New windows on low-frequency GWs

Carlo Contaldi with Giorgio Mentasti
Valencia 12/12/2024

Outline

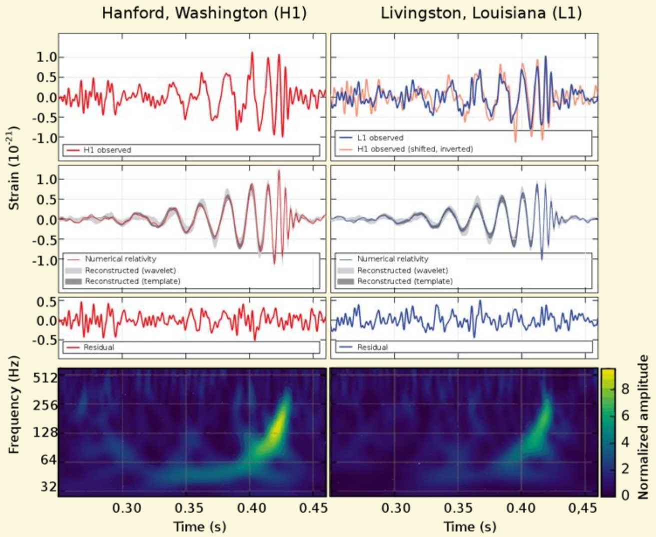
- Gravitational waves and detectors
- Stochastic Gravitational Waves Backgrounds (SGWB).
- The low-frequency SGWB
  - Pulsar Timing Arrays
  - Astrometry
  - Cosmic Shimmering
- Prospects

Gravitational Wave Interferometry

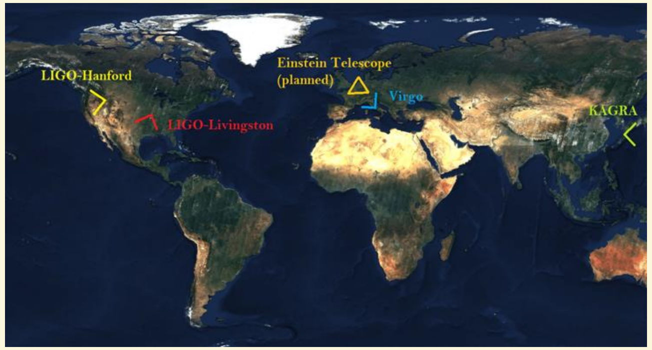
- 2014: First direct detection of a binary BH merger by LIGO-Virgo (m ~ 30 M_{sun}, f ~ 100Hz, d ~ 400 Mpc)

- 2024: A network of terrestrial gravitational wave interferometers

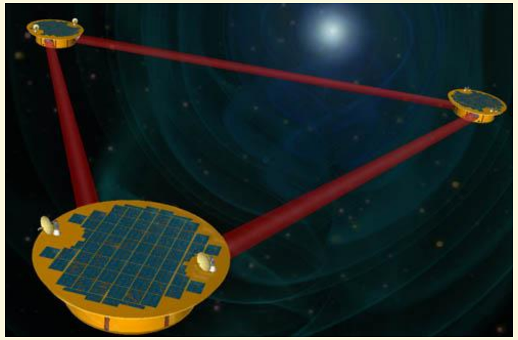
- ~ 2035: Space based (LISA) and future ground based instruments (ET, CE…)



LIGO GW150914 discovery event (2014)



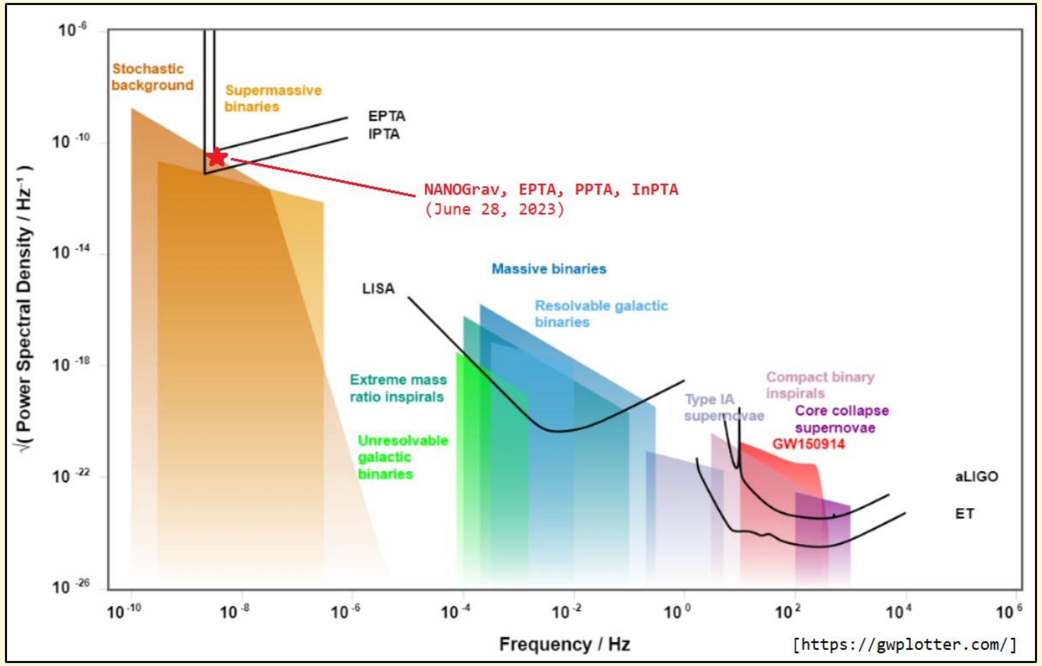
The network of ground-based detectors



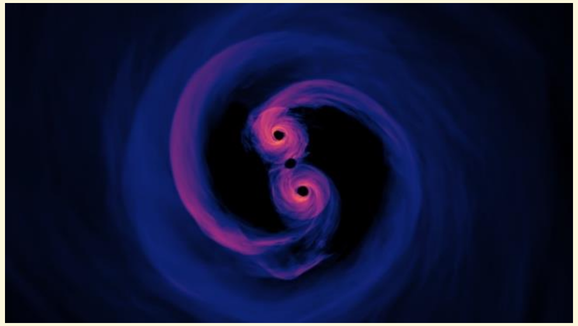
LISA, the planned space-based interferometer

Low-Frequency Gravitational Waves

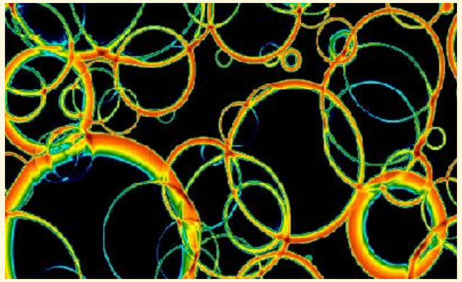
- GW interferometry: f ≥ uHz
- Pulsar Timing Array and astrometry probe the nHz band
- Many expected sources of nHz gravitational waves (supermassive BHs, phase transitions, ultralight DM…)



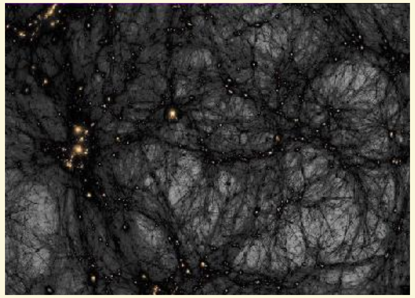
Frequency band of the gravitational wave sources and detectors [GWPlotter]



SMBH binary inspiral

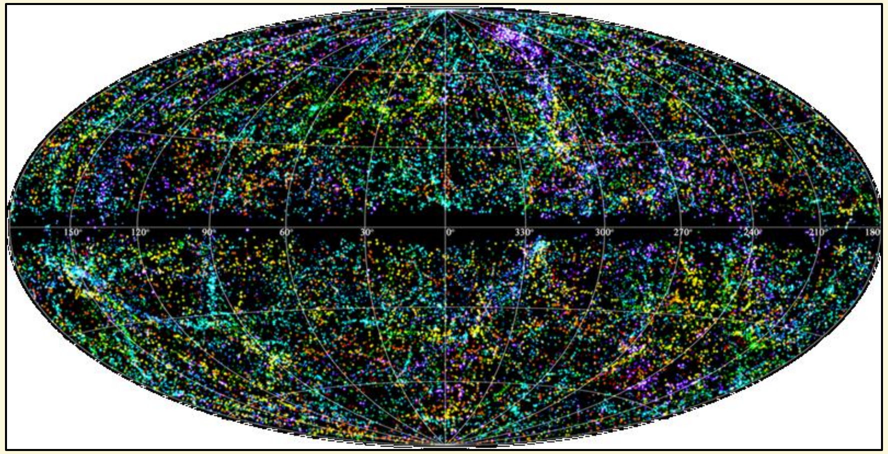


1st order phase transitions

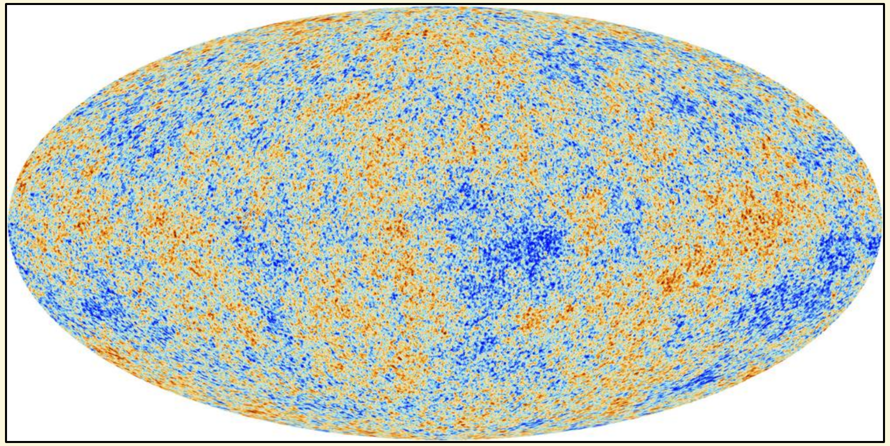


Ultralight Dark Matter

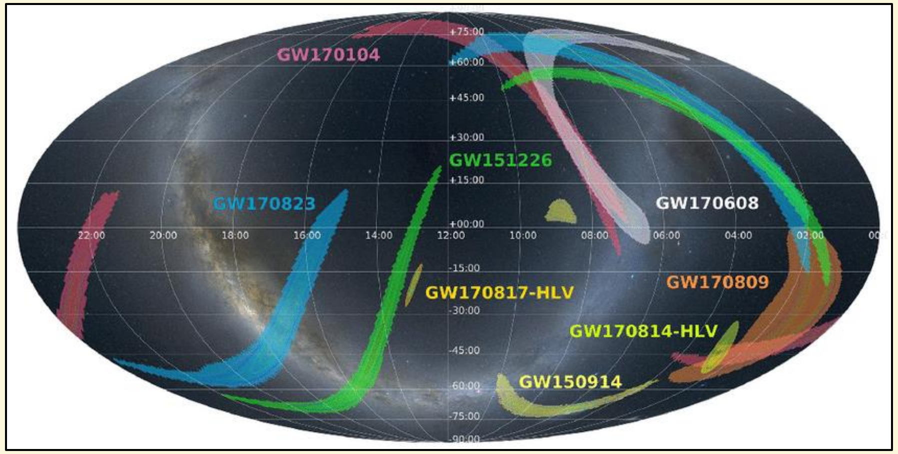
Coherent and Stochastic Signals



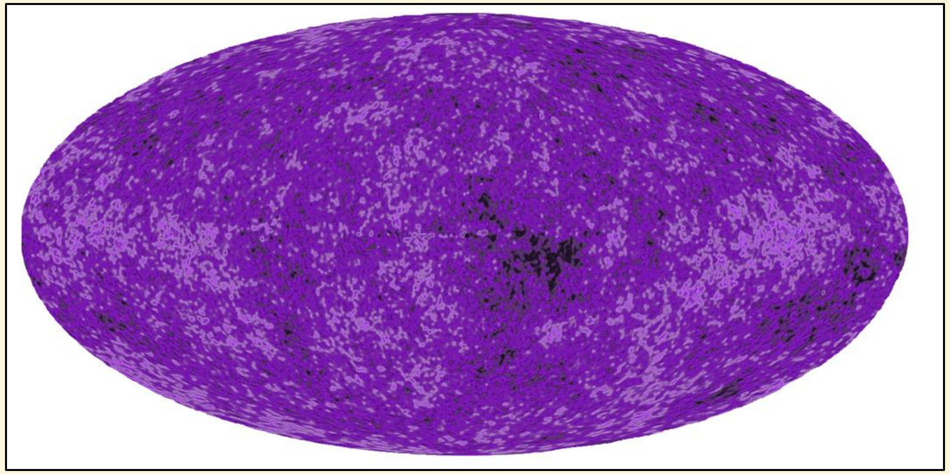
Resolved microwave sources



Unresolved microwave sources (CMB)

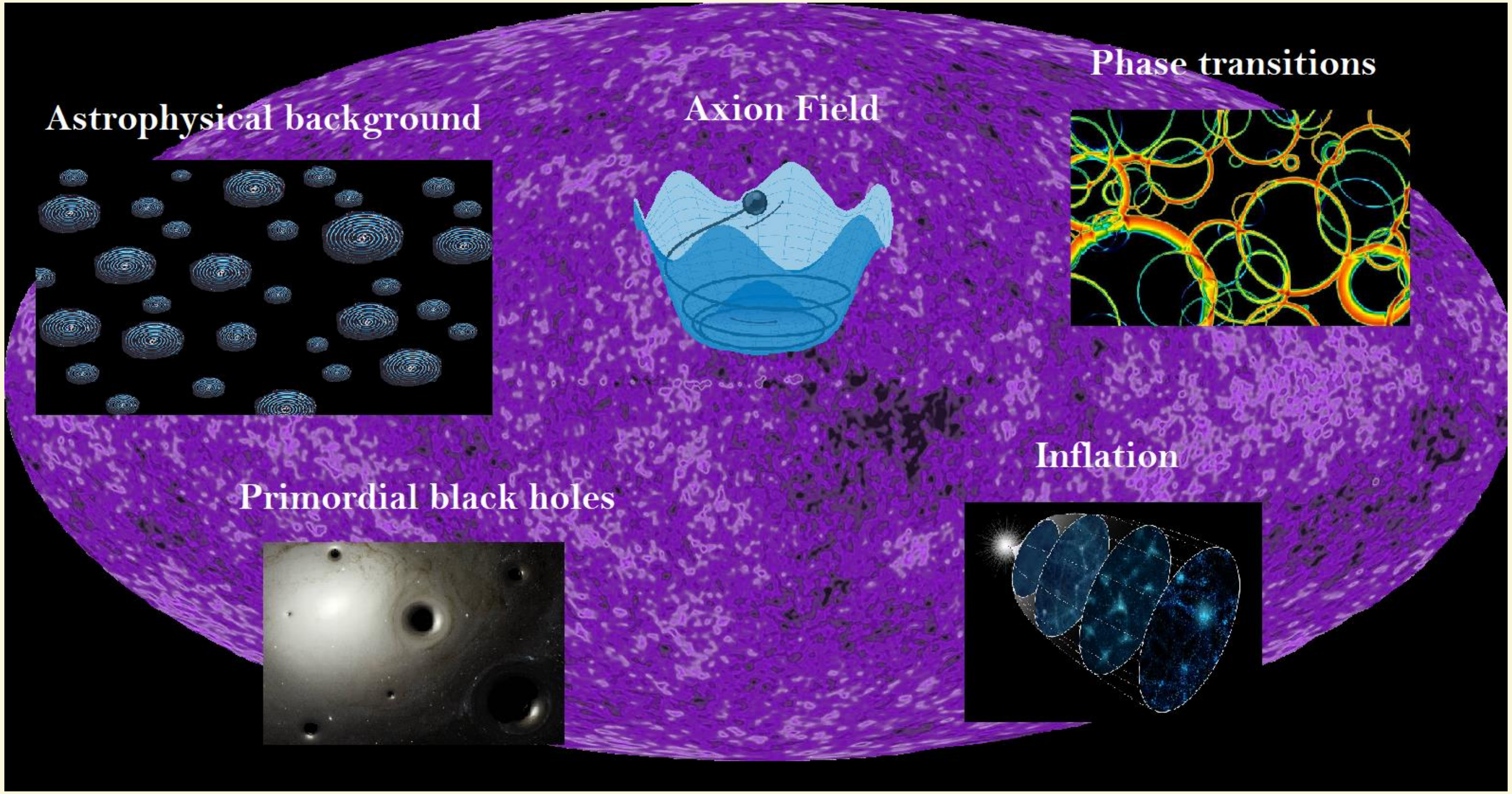


Resolved gravitational wave events



Unresolved gravitational wave background (SGWB)

Stochastic Background Sources



- Astrophysical contribution: (unresolved events)
- Cosmological contribution (inflation, cosmological defects, ...)

Coherent and Stochastic Searches

$$g_{ab}(t,\vec{x}) = \eta_{ab} + h_{ab}(t,\vec{x})$$

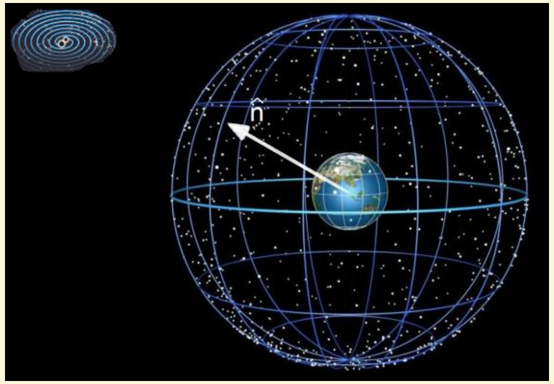
- **Coherent** search: a deterministic template for the GW signal

$$h_{ab}(t,\vec{x}) = \underbrace{e^{2\pi i f(t-\hat{n}\cdot\vec{x})}}_{\text{Planar wave}} \sum_\lambda \underbrace{h_\lambda(f,\hat{n})}_{\text{Amplitude}} \underbrace{e^\lambda_{ab}(\hat{n})}_{\text{Polarization tensors}}$$

- **Stochastic** search: *superposition* of many weak independent signals

$$h_{ab}(t,\vec{x}) = \int_{-\infty}^{+\infty} df \int d^2\hat{n}\, \underbrace{e^{2\pi i f(t-\hat{n}\cdot\vec{x})}}_{\text{Planar wave}} \sum_\lambda \underbrace{h_\lambda(f,\hat{n})}_{\text{Amplitude}} \underbrace{e^\lambda_{ab}(\hat{n})}_{\text{Polarization tensors}}$$

- GW amplitude promoted to a **stochastic** Gaussian variable



- Power spectrum

$$\langle h^*_\lambda(f,\hat{n})\, h_{\lambda'}(f,\hat{n})\rangle = \delta_{\lambda\lambda'}\delta(f-f')\delta(\hat{n}-\hat{n}')\mathcal{H}_\lambda(|f|,\hat{n})$$

Stochastic Power Spectrum

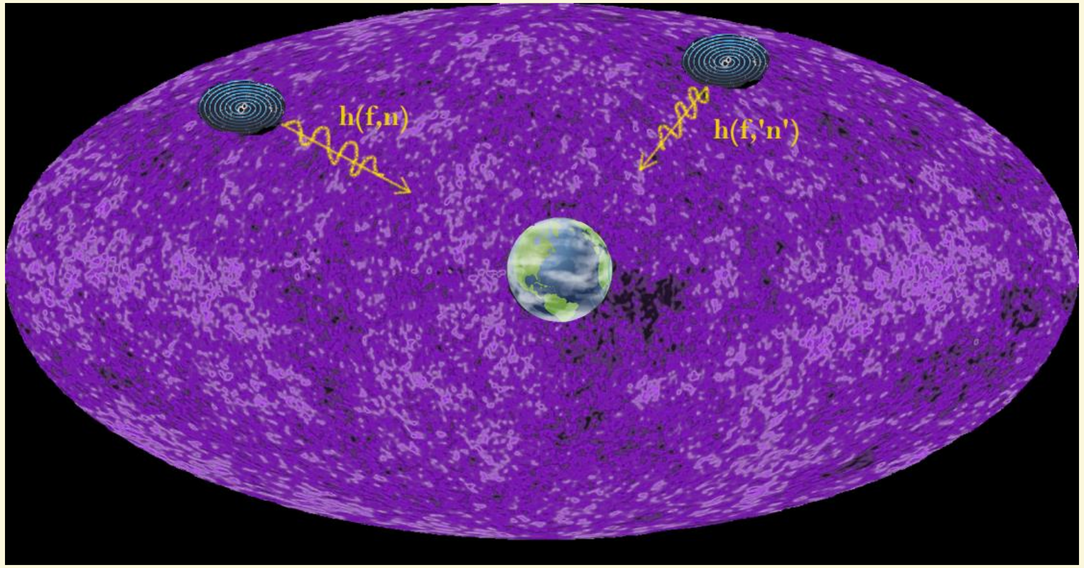
$$\langle h^*_\lambda(f,\hat{n})\, h_{\lambda'}(f,\hat{n})\rangle = \delta_{\lambda\lambda'}\delta(f-f')\delta^{(2)}(\hat{n}-\hat{n}')\mathcal{H}_\lambda(|f|,\hat{n})$$

- Uncorrelated GWs (no phase correlation between frequencies, polarisations, and directions) – *statistical isotropy*.

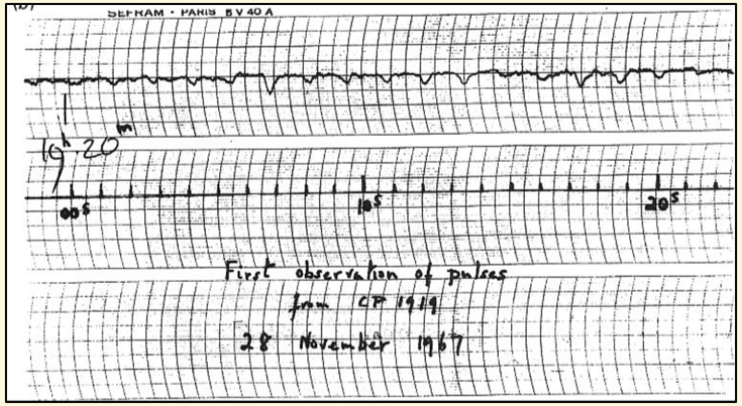
- …but intensity (power) can be anisotropic

- Stationarity

$$\langle h(t)h(t')\rangle \propto \mathcal{H}(t-t')$$



Pulsar Timing Arrays



- Pulsar's proper rotational frequency f_s
- Redshift measurement over timescales $\gg \frac{1}{f_s}$

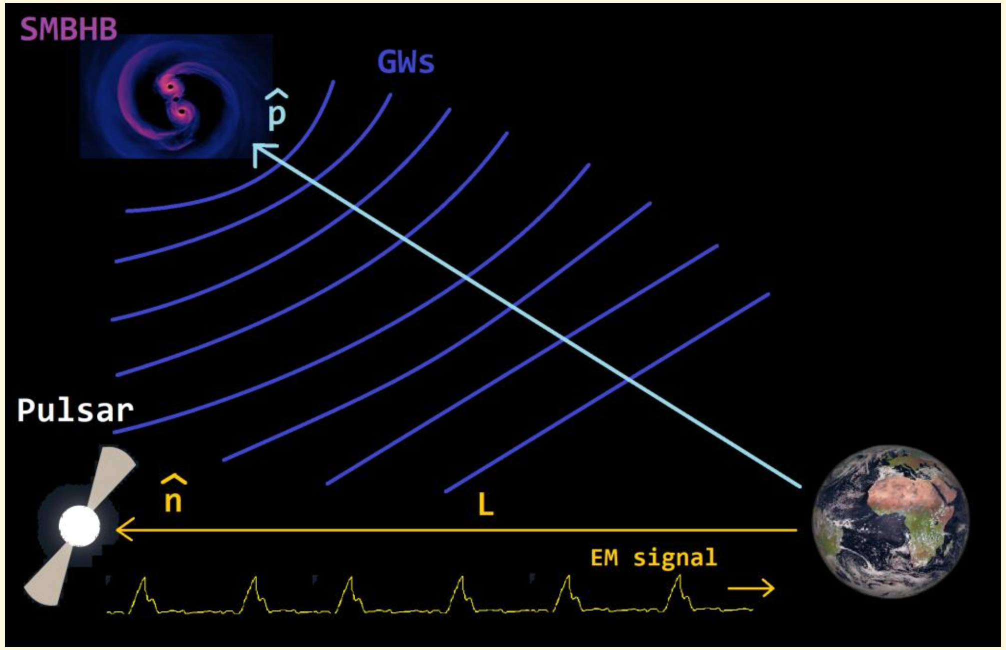
$$z(t) \equiv \frac{f_s - f_o(t)}{f_s} = \frac{1}{2}\frac{\hat{n}^i\hat{n}^j}{1+\hat{n}\cdot\hat{p}}\left[h_{ij}(t) - h^p_{ij}(t,\hat{n})\right]$$

"Earth term"

"pulsar term"

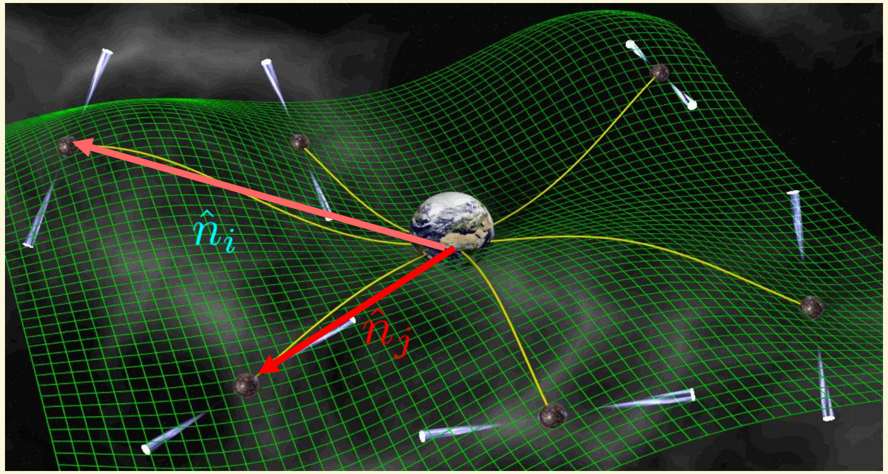
(dropped for large distances)

$$L \gg \lambda_{GW} \geq 10\, ly$$



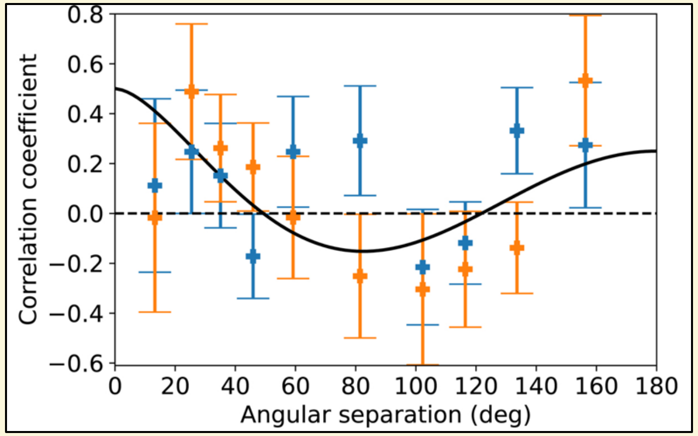
Hellings Downs Curve

- Redshifts are angularly correlated due to "Earth" term GW metric distortion.

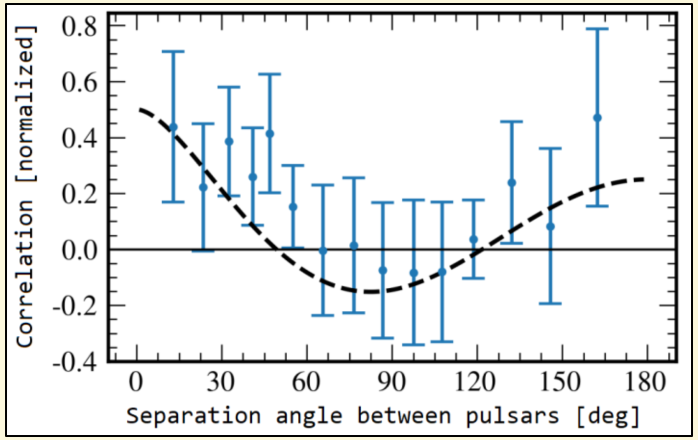


[David Champion, MPIRA]

$$\langle z_i(t) z_j(t) \rangle \propto \underbrace{\chi(\zeta_{ij})}_{\text{HD curve}} \int df \quad \underbrace{H(f)}_{\text{Power spectrum}} \quad e^{2\pi i f_{GW} t}$$



ePTA, 2023



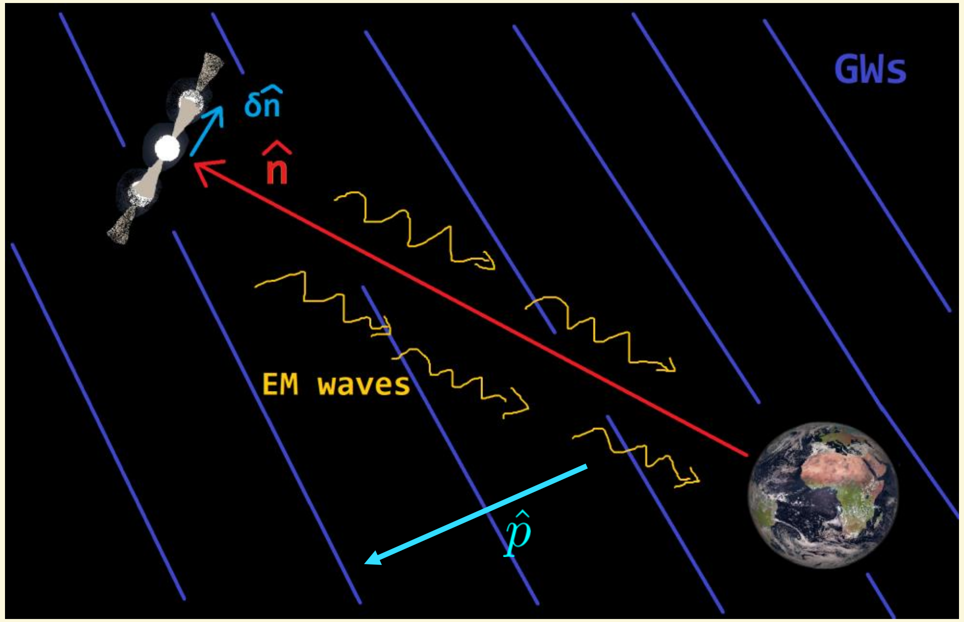
NANOGrav, 2023

$$\zeta_{ij} = \arccos(\hat{n}_i \cdot \hat{n}_j)$$

- Tests on non-Einstenian polarisations, anisotropies, spectral shape…

GW Astrometry

- Geodesics distorted by GWs along the line of sight.
- Apparent position and proper motion of emitters vary with time.



$$\delta n^i = \mathrm{Re}\left[\left(\left\{1 + \frac{i(2+p\cdot n)}{2\pi f\omega_0\lambda_s(1+p\cdot n)}\left[1 - e^{-2\pi i f\omega_0(1+p\cdot n)\lambda_s}\right]\right\} n^i + \left\{1 + \frac{i}{2\pi f\omega_0\lambda_s(1+p\cdot n)}\left[1 - e^{-2\pi i f\omega_0(1+p\cdot n)\lambda_s}\right]\right\} p^i\right)\frac{n^j n^k h_{jk} e^{-2\pi i f t_0}}{2(1+p\cdot n)} - \left\{\frac{1}{2} + \frac{i}{2\pi f\omega_0\lambda_s(1+p\cdot n)}\left[1 - e^{-2\pi i f\omega_0(1+p\cdot n)\lambda_s}\right]\right\} n^j h^i_j e^{-2\pi i f t_0}\right]$$

[Book & Flanagan 2011]

Distant Source Limit

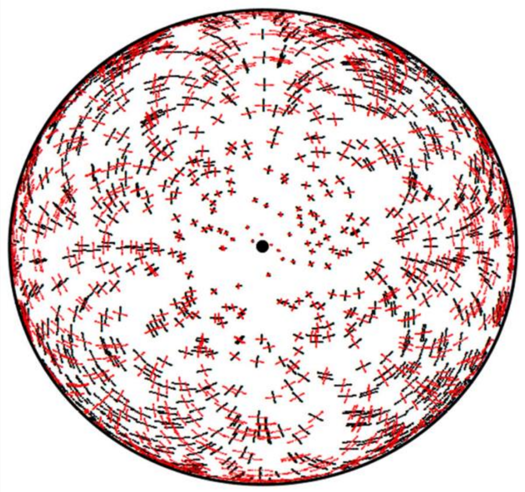
- Distant source limit: $L \gg \lambda_{\mathrm{GW}} \geq 10\mathrm{ly}$

$$\delta n^i(t) = \frac{1}{2}\left[\frac{n^i - p^i}{1 - p \cdot n} n^j n^k - n^j \delta^{ik}\right] h_{jk}(t)$$

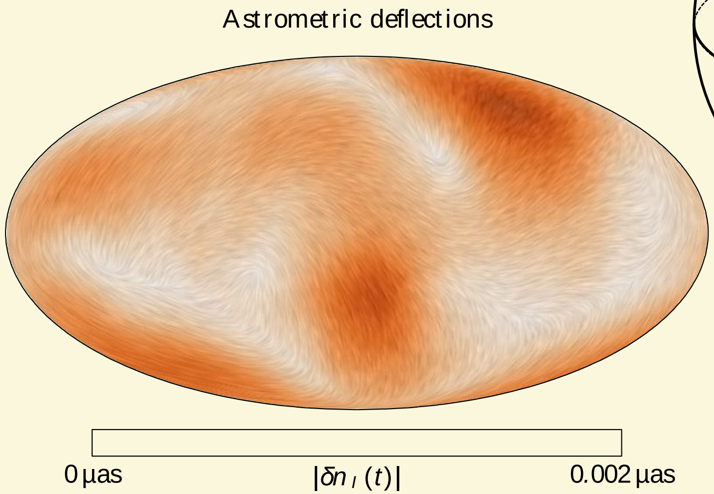
- **Resloved** source searches: e.g. monochromatic GW. Time dependence + angular response.

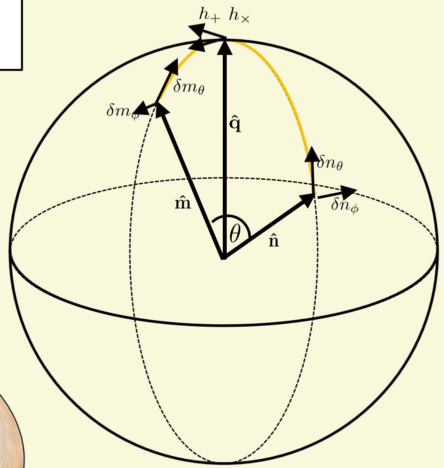
- **Stochastic** GWB searches: angular cross-correlating astrometric deflections + frequency spectrum.

$$\langle \delta m_\theta(t) \delta n_\theta(t) \rangle \propto \underbrace{\chi(\hat{m} \cdot \hat{n})}_{\text{Angular pattern}} \int df \underbrace{H(f)}_{\text{Power spectrum}} e^{2\pi i f_{GW} t}$$



[C.J. Moore et.al, 2017]

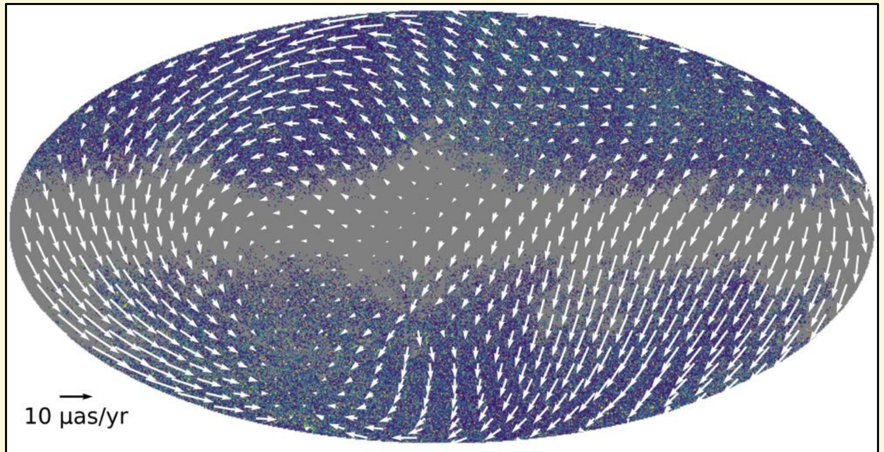




[Golat & Contaldi 2022]

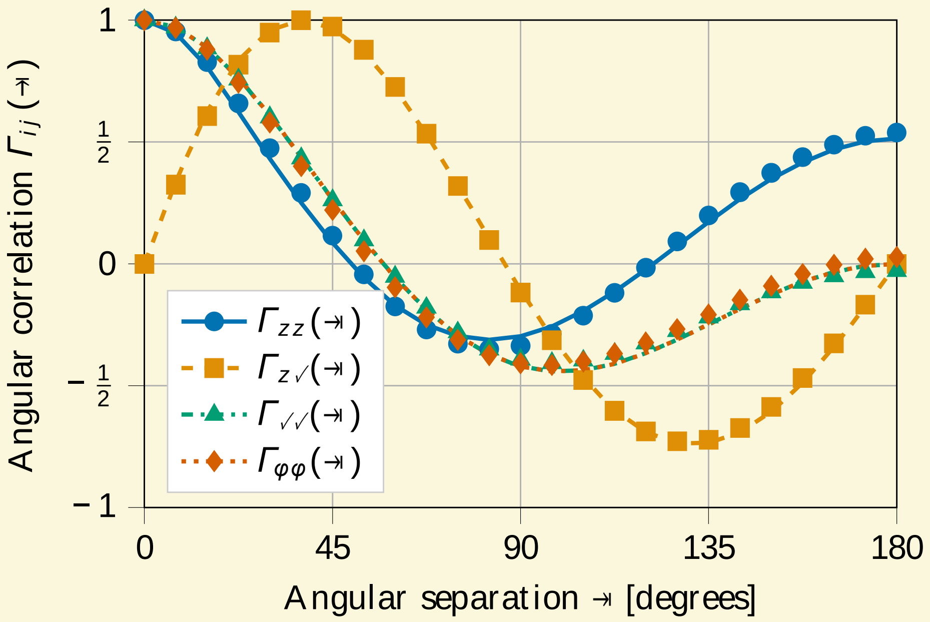
Astrometry with GAIA

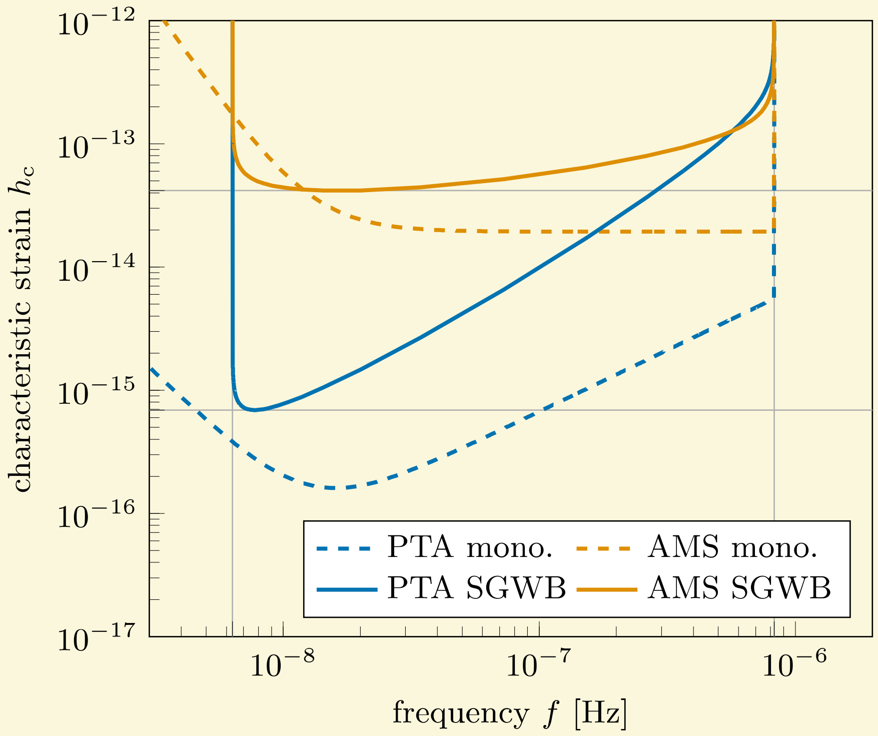
- Launched in 2013.
- Observation of 10^9 sources with astrometric precision of 10-100 µas.
- Each source is observed 80 times (5-year nominal mission) —10^{-9}-10^{-7} Hz window.
- Extension to 8-10 years.
- $\Omega_{GW} < 10^{-2}$ constraint on the stochastic GW background (cf. $\Omega_{GW} \sim 10^{-8}$ from PTA)



[S.Jaraba et al., 2023]

Astrometry





[Golat & Contaldi 2022]

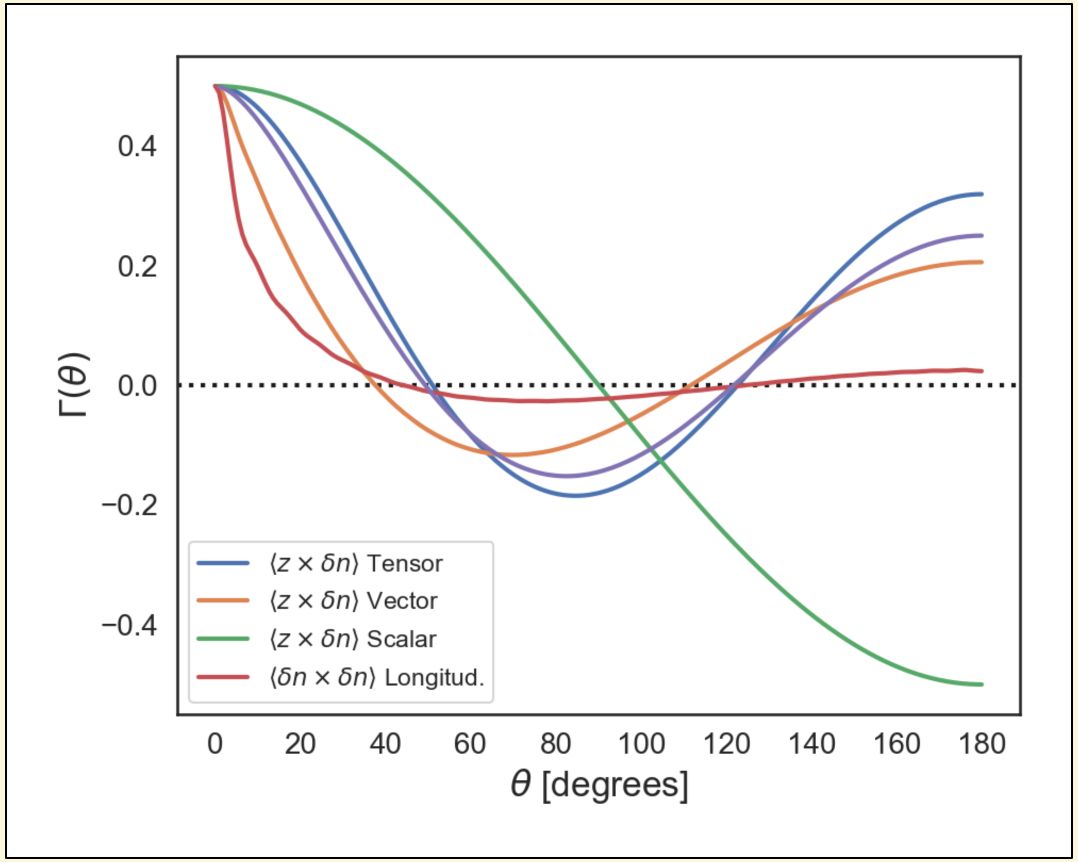
Astrometry x Redshift

- Can extend to cross-correlations of deflections and redshifts.
- + Non-Einsteinian polarisations
  - Metric travelling waves that are not TT.

$$\text{Tensor} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Vector} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\text{Scalar} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

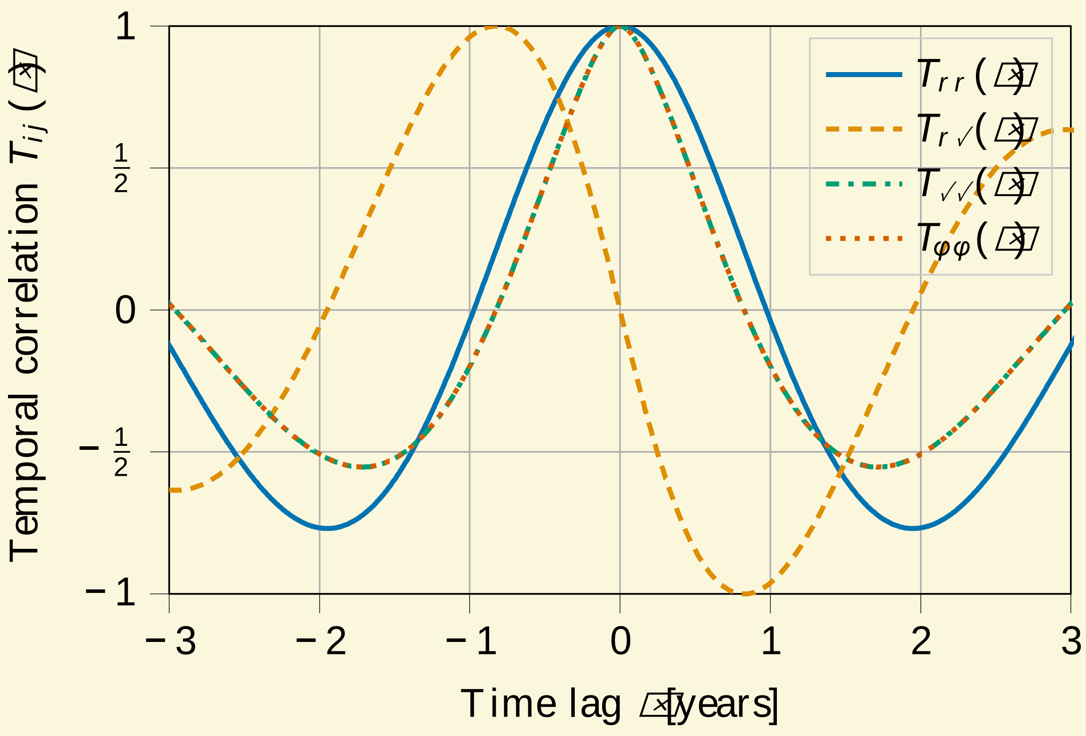


[Golat & Contaldi 2022]

Astrometry x Redshift

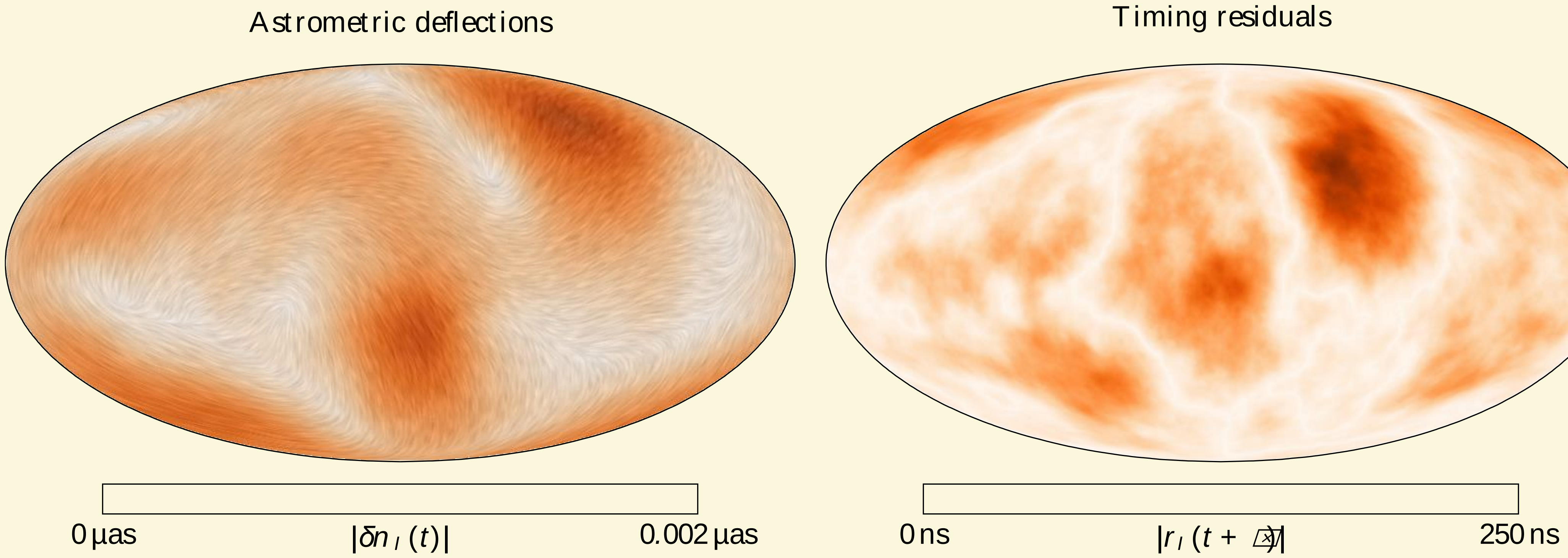
- Sparse vs Dense sampling of sky.
- Independent systematics.
- Verification of angular correlations.
- The feature of cross-correlation is an odd temporal component (lagged correlation).

$$\langle z(t)\delta n(t+\tau)\rangle = T(\tau)\Gamma(\theta)$$



[Golat & Contaldi 2022]

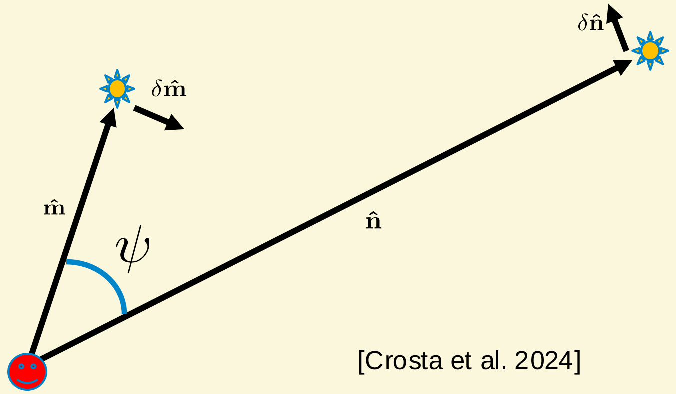
Astrometry x Redshift



Differential Astrometry

- Measuring absolute angles is difficult → differential angular measurements.

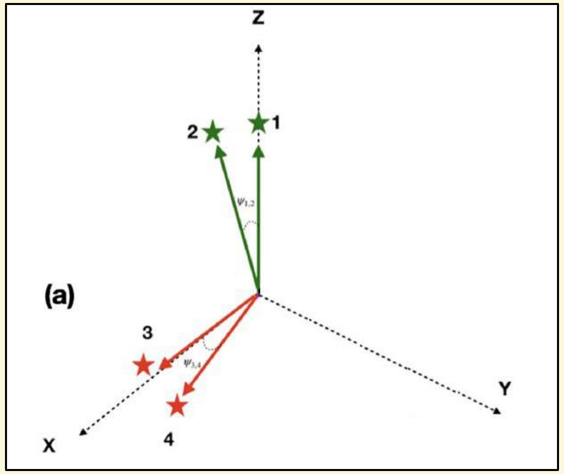
$$\delta\psi = -\frac{1}{\psi_0}\left[\frac{\hat{n}\cdot\hat{m}+\hat{p}\cdot\hat{m}}{2\,(1+\hat{n}\cdot\hat{p})}h_{jk}\hat{n}^j\hat{n}^k + \frac{\hat{n}\cdot\hat{m}+\hat{p}\cdot\hat{n}}{2\,(1+\hat{m}\cdot\hat{p})}h_{jk}\hat{m}^j\hat{m}^k - h_{ij}\hat{n}^i\hat{m}^j\right]$$



[Crosta et al. 2024]

- Cross-correlating **pairs** of close-by stars.

$$\langle\delta\psi_{12}(\hat{n}_1,\hat{n}_2)\,\delta\psi_{34}(\hat{n}_3,\hat{n}_4)\rangle$$



- Cross-correlating differential measurements and PTA

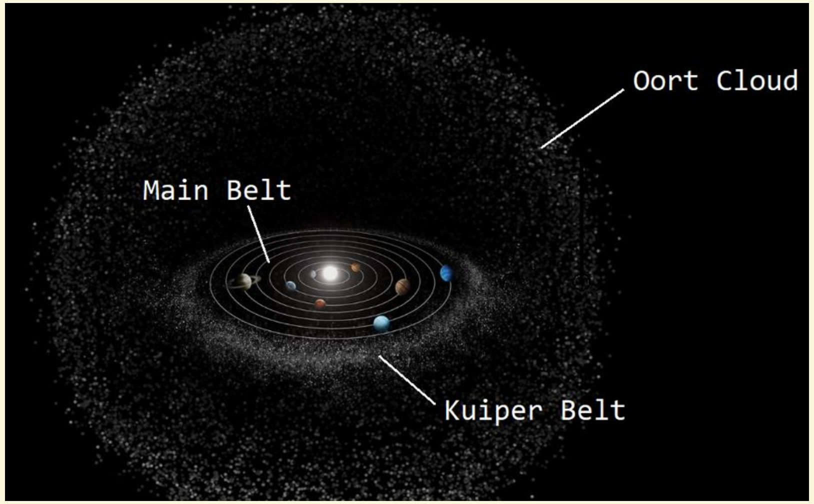
$$\langle\delta\psi_{12}(\hat{n})\,z(\hat{n}')\rangle$$

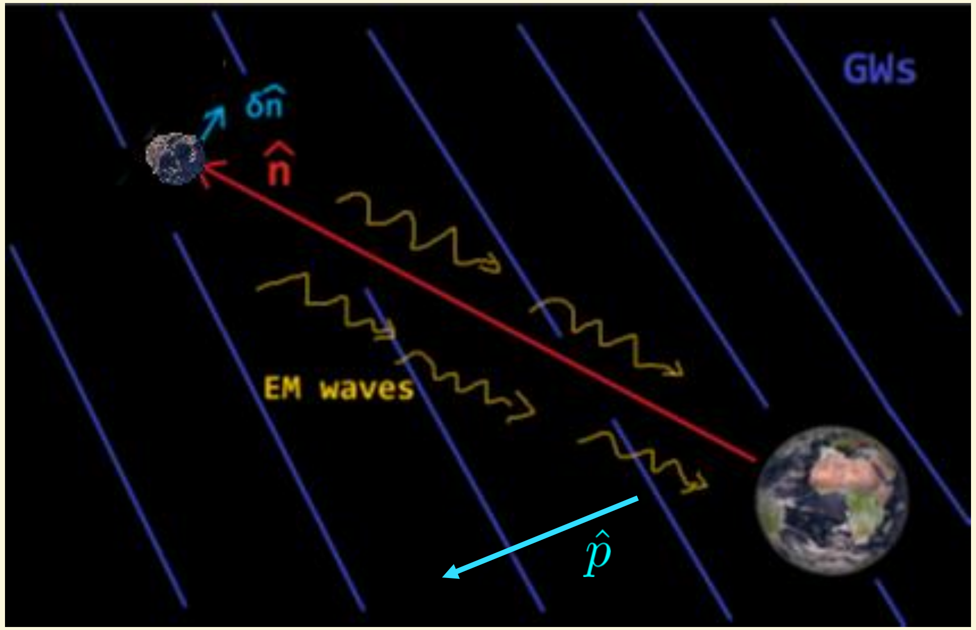
Solar System GW Astrometry

- $\sim 10^9$ small-sized objects in the solar system $\sim 10^6$ already known.
- Closer (L<< λ_{GW}) but fainter (apparent magnitude m > 9).
- Accelerating (but on long timescales).
- Short-distance limit of astrometry – distinct response function with simple agular structure.

$$\delta n^i(\hat{n}, t) = \frac{1}{2}\left(\delta^{ij} - \hat{n}^i \hat{n}^j\right) \hat{n}^k \, h_{jk}(t)$$

[Mentasti & Contaldi 2024]



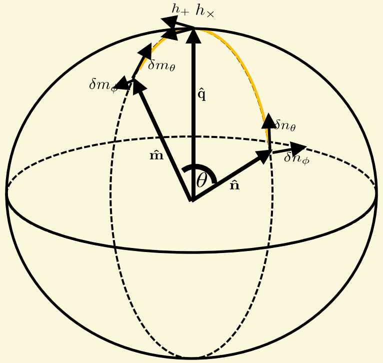


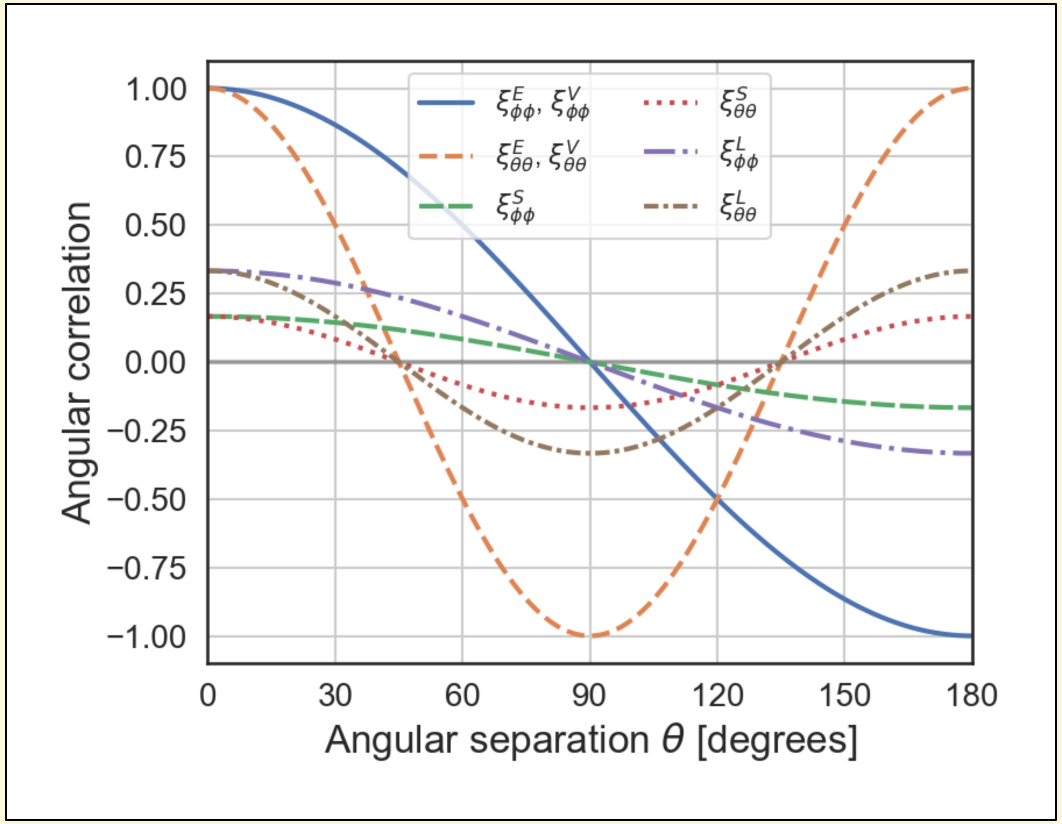
Solar System GW Astrometry

- Much simpler angular correlation structure

$$\langle \delta m_\theta(t) \delta n_\theta(t) \rangle \propto \underbrace{\chi(\hat{m}\cdot\hat{n})}_{\text{Angular pattern}} \int df \underbrace{H(f)}_{\text{Power spectrum}} e^{2\pi i f_{GW} t}$$

- How could this be done?





[Mentasti & Contaldi 2024]

Solar System GW Astrometry

- LSST: Operational from 2024 (now?).
- High astrometric accuracy, widefield, high sensitivity, high cadence.
- Also GAIA…

Vera C. Rubin (LSST)

	Currently Known	LSST Discoveries	Median number of observations	Observational arc length
Near Earth Objects (NEOs)	~14,500	100,000	(D>250m) 60	6.0 years
Main Belt Asteroids (MBAs)	~650,000	5,500,000	(D>500m) 200	8.5 years
Jupiter Trojans	~6,000	280,000	(D>2km) 300	8.7 years
TransNeptunian + Scattered Disk Objects (TNOs + SDOS)	~2,000	40,000	(D>200km) 450	8.5 years
Interstellar Objects (ISOs)	1	10	?	?

Effective Mirror Diameter	*6.7 m*
Field of view	*9.6 sq deg*
Survey length	*10 years*
Sky coverage	*~18,000 sq deg*
Site	*Cerro Pachon*
Filters	*ugrizy*
Typical seeing	*0.7"*
Exposure ('Visit') Time	*2x15 s /visit*
Data rate	*~15 TB/night*
Photometric accuracy	*10 mmag*
Astrometric accuracy	*50 mas*

[Vera C. Rubin Observatory / LSST]

Gaia (for solar system)

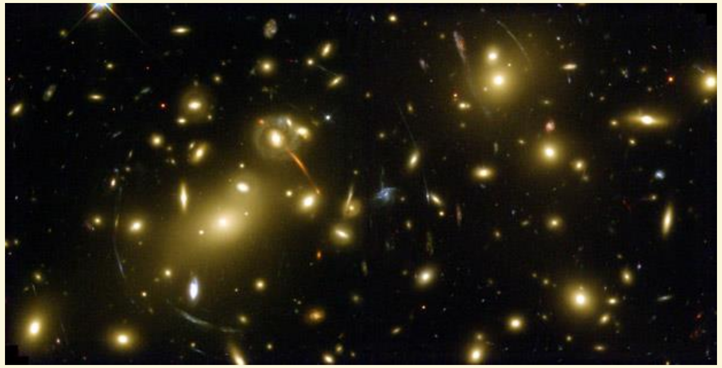
LSST

σ [mas]	γ	$N = 1 \times 10^5$	$N = 5 \times 10^6$
50.0	0	9.9×10^{-1}	2.0×10^{-2}
0.1	0	3.9×10^{-6}	7.9×10^{-8}
0.01	0	3.9×10^{-8}	7.9×10^{-10}
50.0	13/3	9.6×10^{-5}	1.9×10^{-6}
0.1	13/3	3.8×10^{-10}	7.7×10^{-12}
0.01	13/3	3.8×10^{-12}	7.7×10^{-14}

Detection limits for Ω_0

Gaia at LSST magnitudes

Cosmic Shear



- Metric perturbations due to dark matter induce distortions: magnification and shearing (ellipticity)

$$I_{\rm obs}(\hat{n}) = I_{\rm true}(\hat{n} + \delta\hat{n})$$

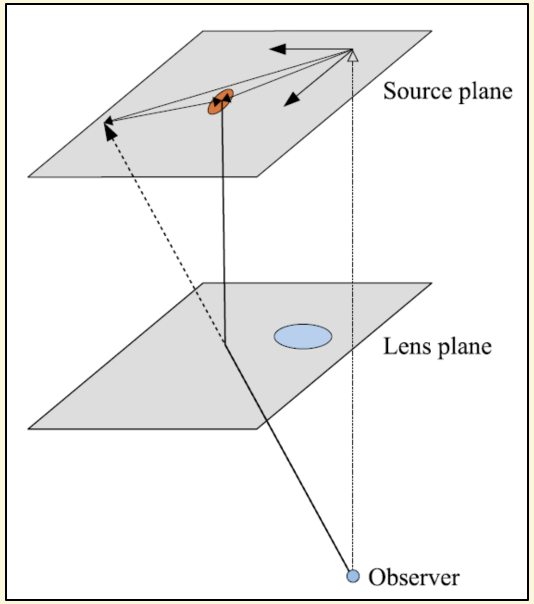
- Obtained from galaxy image second moments

$$q_{ij} = \langle \theta_i \theta_j \rangle_{I_{\rm obs}} = \frac{1}{F}\int d^2\theta I_{\rm obs}(\theta)\theta_i\theta_j$$

- Linear distortion tensor

$$A_{ij} \equiv \frac{\partial(\hat{n}_i + \delta\hat{n}_i)}{\partial\hat{n}^j} = \delta_{ij} + \frac{\partial(\delta\hat{n}_i)}{\partial\hat{n}^j}$$

ψ_{ij}



[Dodelson]

Cosmic Shimmering

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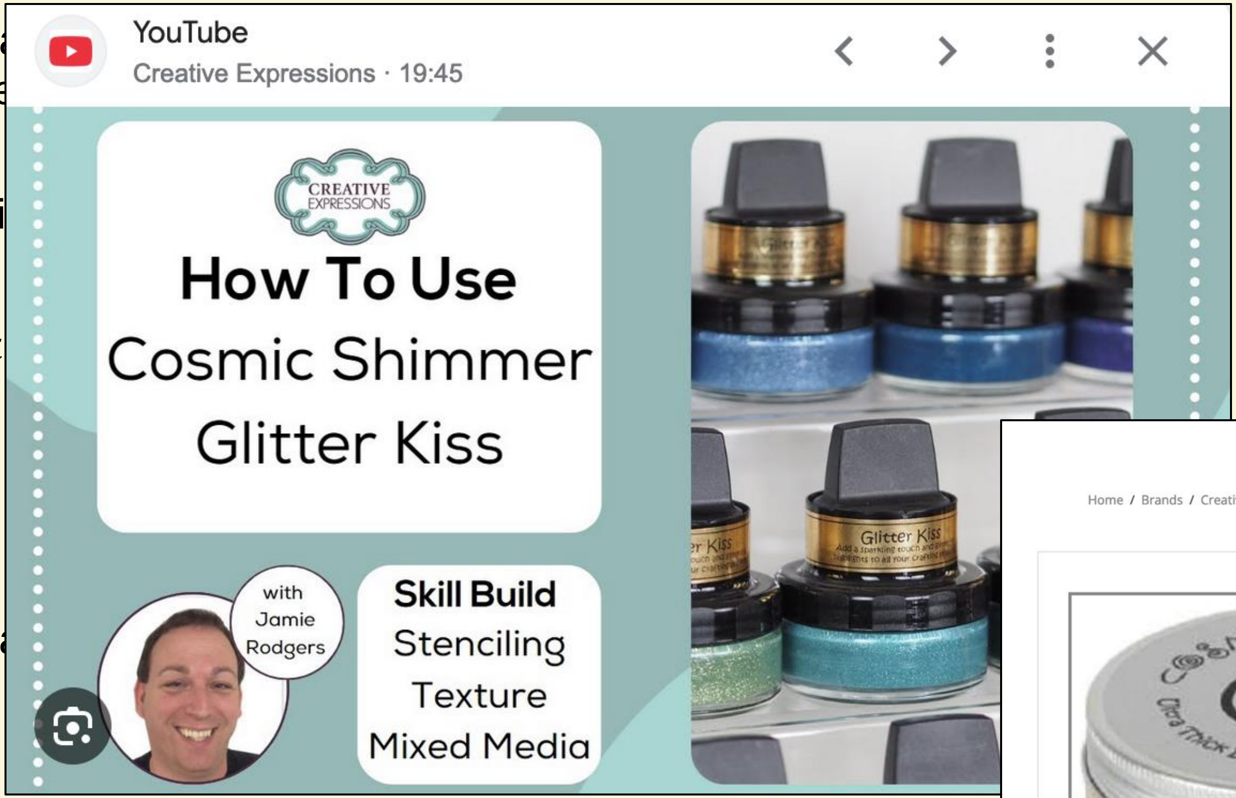


time

Cosmic Shimmering

- Time-varying galaxy image observed second momenta over nHz scale

$$\psi_{ij}(\hat{n},t) = \frac{1}{2}\left[\delta_{ij} - \frac{\hat{n}_i + \hat{q}_i}{1+\hat{q}_l\,\hat{n}^l}\hat{q}_j\right]\frac{h_{rs}(t)\hat{n}^r\hat{n}^s}{1+\hat{q}_l\,\hat{n}^l} + \frac{\hat{n}_i + \hat{q}_i}{1+\hat{q}_l\,\hat{n}^l}h_{jr}(t)\hat{n}^r - \frac{1}{2}h_{ij}(t)$$

- Decomposition of the distortion matrix

$$\psi_{ij} = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa \end{pmatrix} + \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} + \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

spin-0

spin-2

[Mentasti & Contaldi 2024]

Cosmic Shimmering

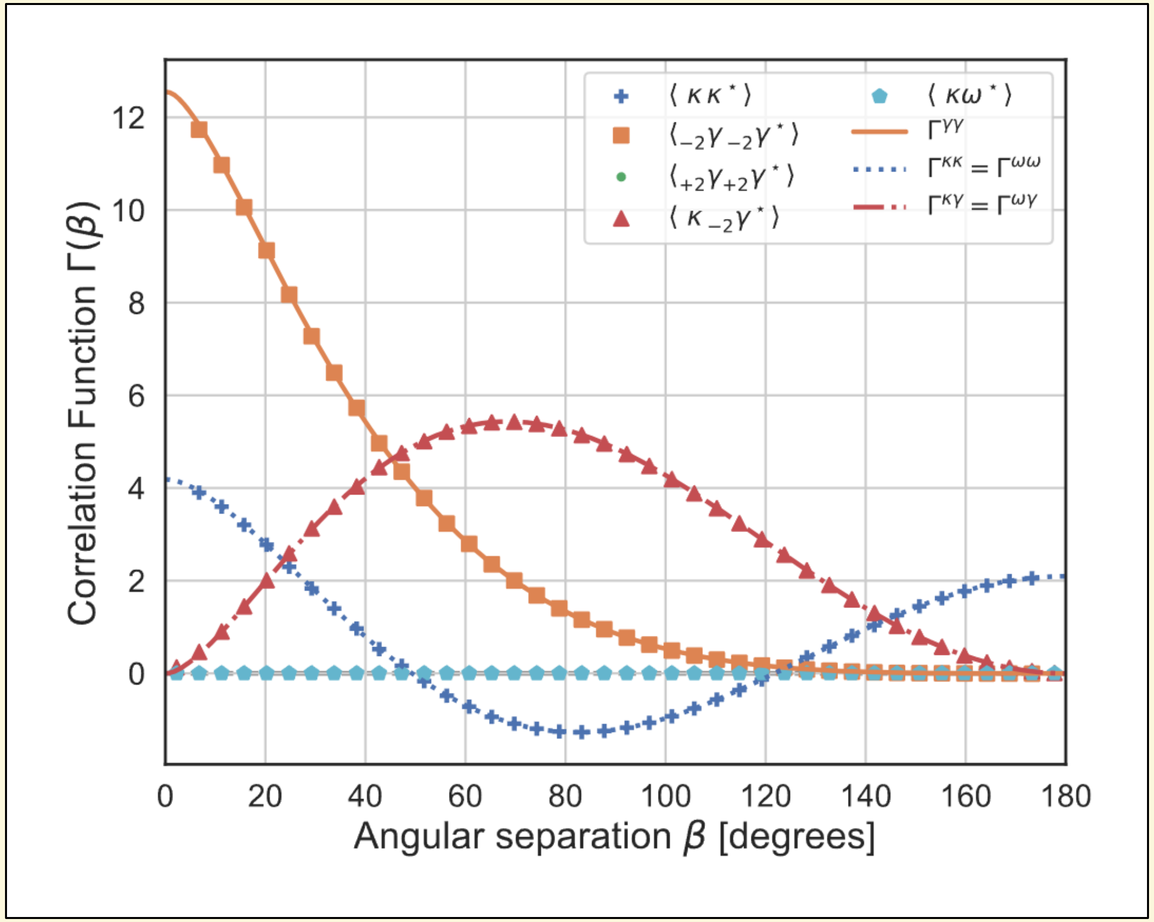
- Correlation functions for couples of observed galaxy ellipticities.
  - Generalised spin-weighted formalism.
  - Sensitive to polarised backgrounds (and non-Einstenian polarisations).

- State-of-the-art mission has ~100 mas angular precision and bad SNR.

- Possible detection for future missions

- Angular correlations:
  - Magnification → HD
  - Shearing → New correlation.
  - Shearing x Mag. → New correlation.
  - Mag. X Rotation → Uncorrelated.

Hellings Downs Analogues



[Mentasti & Contaldi 2024]

Cosmic Shimmering: Observability

- Signal and noise in astrometric measurements $$d_i(t) = h_i(t) + n_i(t) \quad i = 1...N$$
- Cross correlating datastreams $$\mathcal{C}_{ij}(f) = \tilde{d}_i(f)\tilde{d}_j^*(f)$$

$$\langle \mathcal{C}_{ij}(f) \rangle = \langle \tilde{d}_i(f)\tilde{d}_j^*(f) \rangle = \langle \tilde{h}_i(f)\tilde{h}_j^*(f) \rangle$$

- Gaussian noise $$\langle \tilde{n}^i(f)\tilde{n}^{j\star}(f') \rangle = \frac{1}{2}\delta_{ij}\delta(f - f')N(f)$$

- What noise?
  - PTA: white + red + other chromatic noise
  - Astrometry…we don't know!
  - White noise as working assumption

$$N(f) = N^{\mathrm{WN}} + N^{\mathrm{RN}}\left(\frac{f}{f_r}\right)^{\gamma_T^{\mathrm{RN}}} + N^{\mathrm{DM,SV}}(f)$$

$$N^{\mathrm{WN}} = 2\sigma^2\Delta t$$

cadence of samples

single measurement error

Cosmic Shimmering: Observability

- SNR forecasts.
  - T = 15 year survey.
  - Cadence = 3 days.

$$\mathrm{SNR} \simeq 12 \times 10^{-7} \left(\frac{0.1\mathrm{as}}{\sigma_\theta}\right)^4 \left(\frac{\Theta}{1\,\mathrm{arcmin}}\right)^4 \left(\frac{N}{10^9}\right)$$

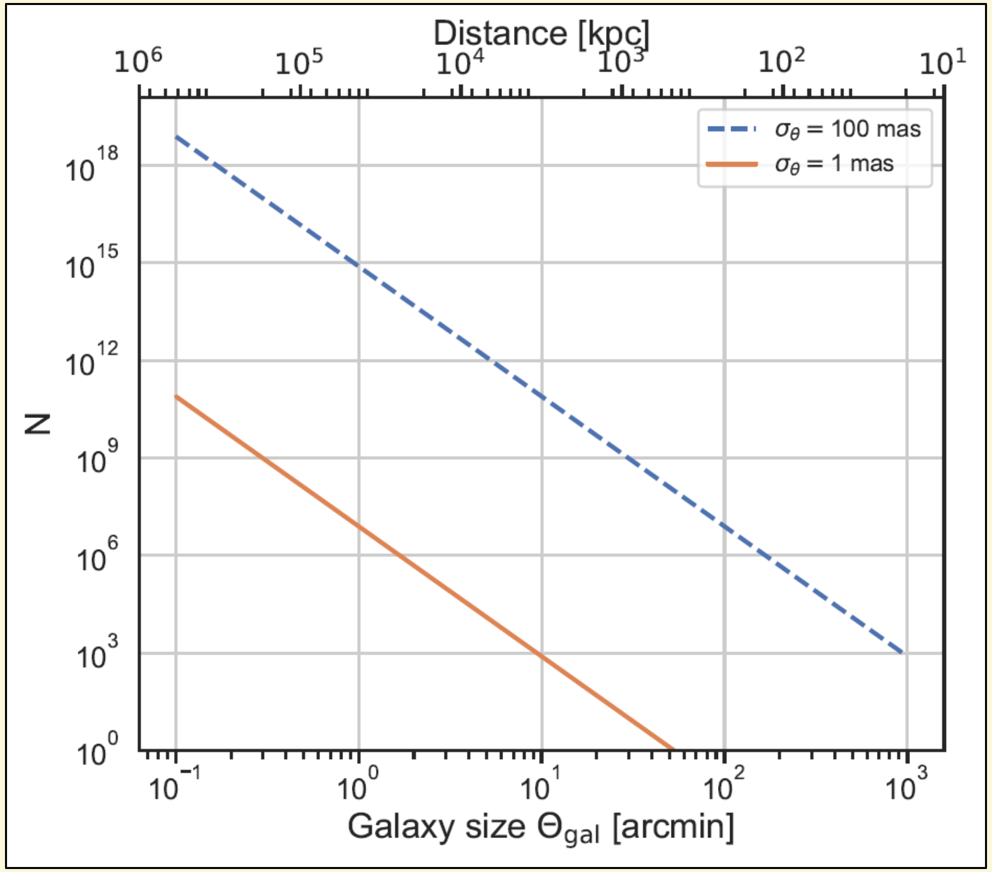
Resolution

Angular size of object

Number of objects

- Cross-correlation with PTA and Astrometry at nHz frequencies?

Number of galaxies of physical size 10 Kpc resolvable at different distances



Summary

- Evidence for nHz GW signal from PTA.

- Astrometry as a probe of GWs is maturing.
    - Difficult to assess the systematics

- Multiple channels (solar & extrasolar astrometry, *shimmering).*

- Cross-correlating PTA and astrometry: reduction of systematics…

- Data (optical surveys) are there, so use them – never say never!