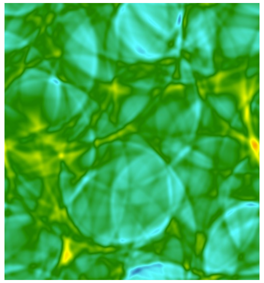

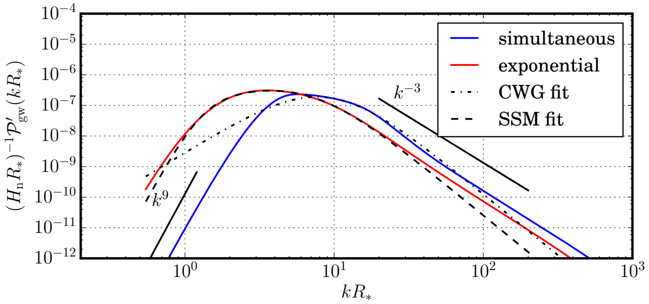


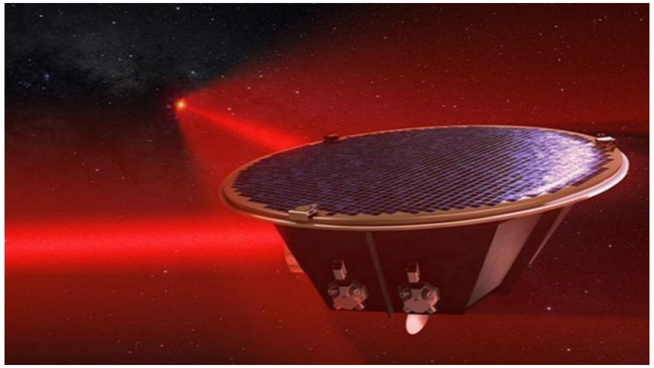

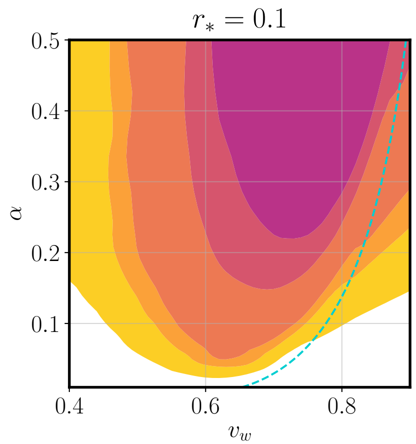


# Gravitational waves from first order phase transitions


**Mark Hindmarsh**

Helsinki Institute of Physics & Dept of Physics, University of Helsinki

and

Department of Physics & Astronomy, University of Sussex


Towards … Valencia
11. joulukuuta 2024

SUOMEN AKATEMIA
FINLANDS AKADEMI
ACADEMY OF FINLAND

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LISA CONSORTIUM

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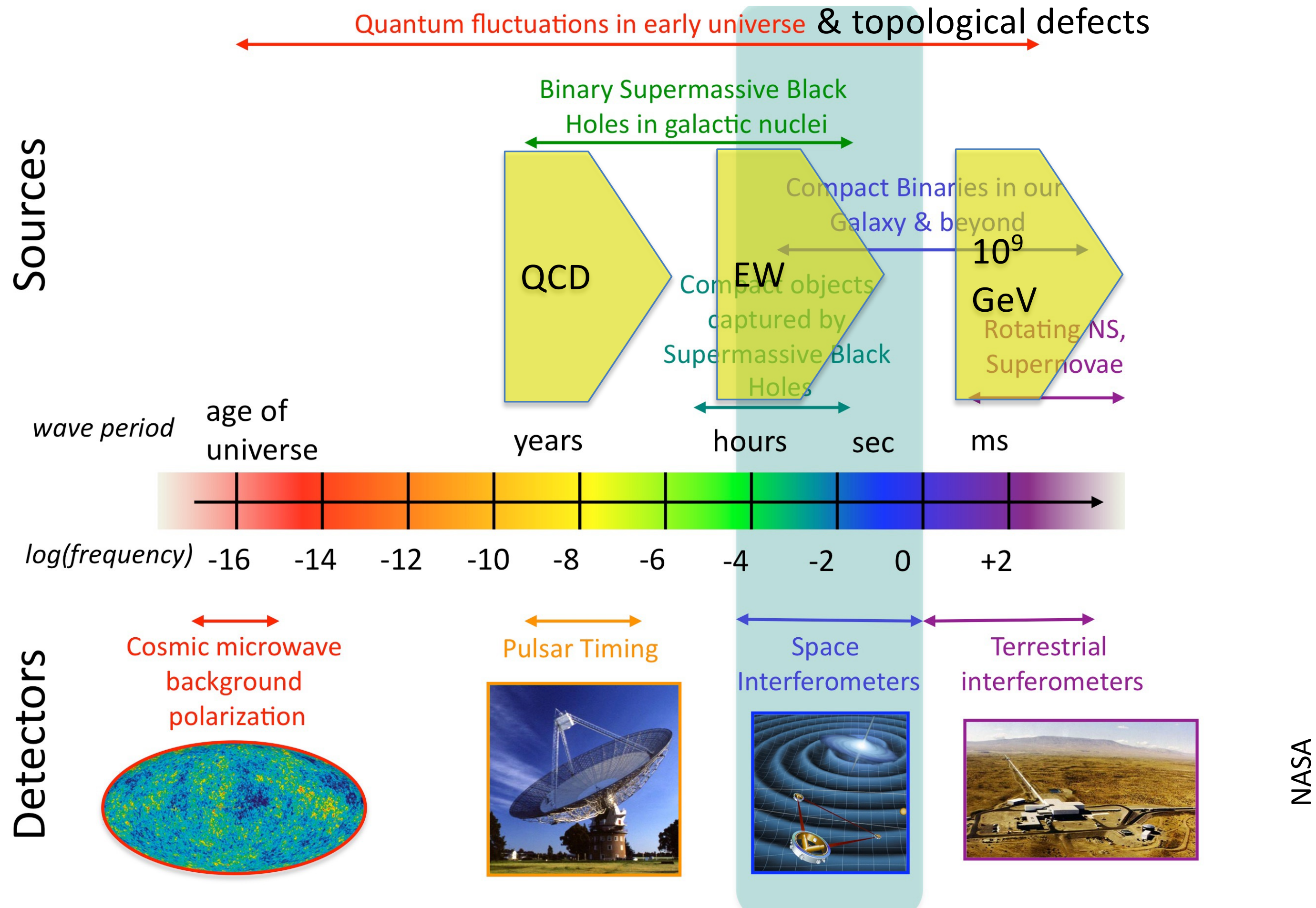
# Gravitational wave spectrum

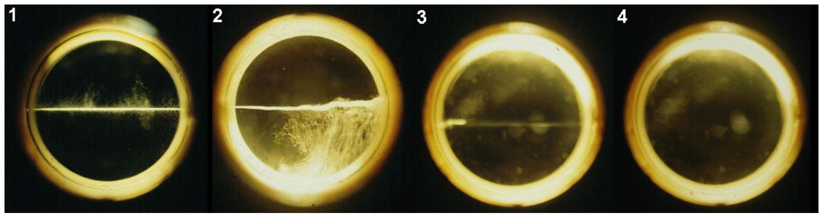


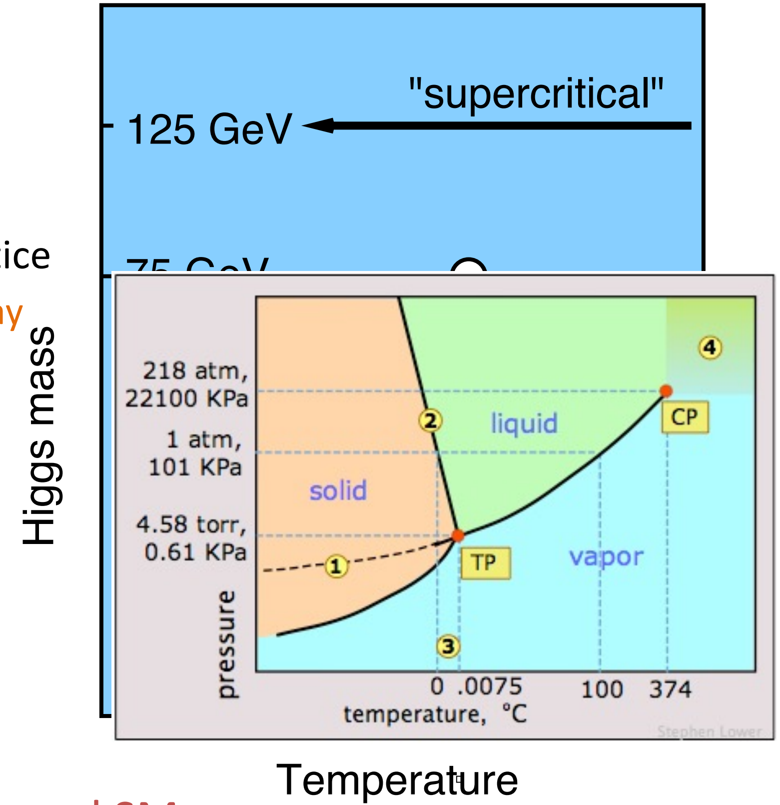
# Electroweak transition: 100 GeV, 10 ps

- Perturbative: weakly first order transition
  Kirzhnitz, Linde (1972,4)
- But: SM is not weakly coupled at high T
  Linde (1980)
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)

P Schicho talk, Friday

- SM transition at $m_h \approx 125$ GeV is a cross-over
  - a supercritical fluid

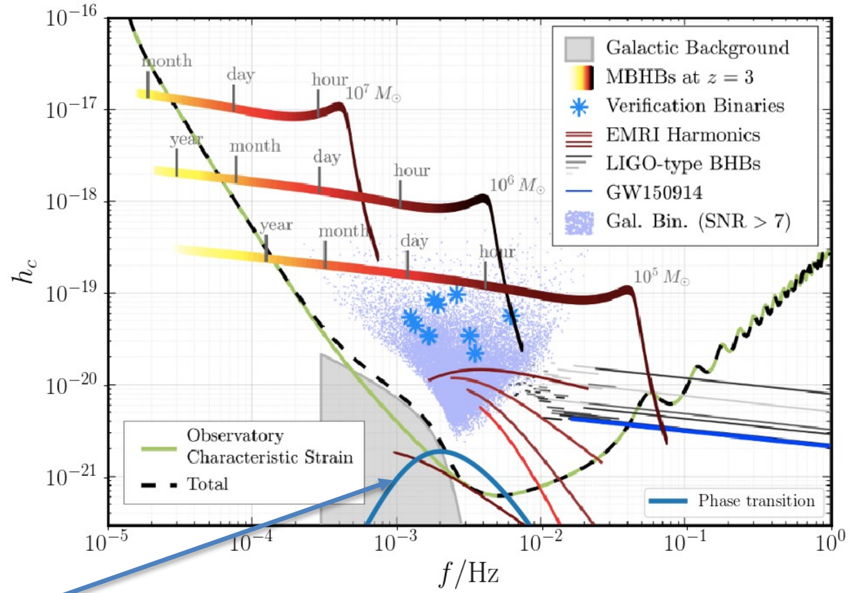


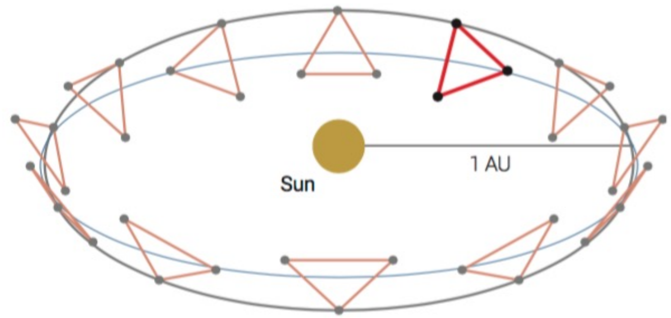

- Search for 1st order transition is a search for physics beyond SM

# Laser Interferometer Space Antenna

- Launch mid 2030s
- 4-year mission (up to 10 years)
- 2.5M km arms
- Science objectives:
  - White dwarves
  - Black holes
  - Galaxy mergers
  - Extreme gravity
  - TeV-scale early Universe
- Other missions: Taiji, TianQin
- Proposals: DECIGO, BBO




LISA sensitivity




LISA

# Little bangs in the Big Bang

- 1st order transition by nucleation of bubbles of low-*T* phase

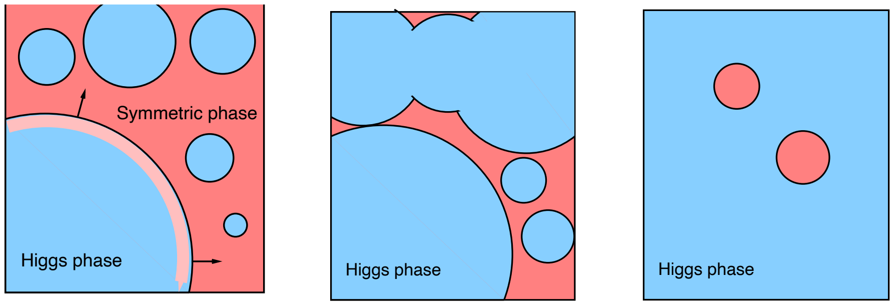
  Langer 1969, Coleman 1974, Linde 1983

- Nucleation rate/volume *p*(*t*) rapidly increases below $T_c$
- Expanding bubbles generate pressure waves in hot fluid
- Gravitational wave (GW) production
- GW spectrum has information about phase transition
- Departure from equilibrium: needed for baryogenesis

  Sakharov (1967)

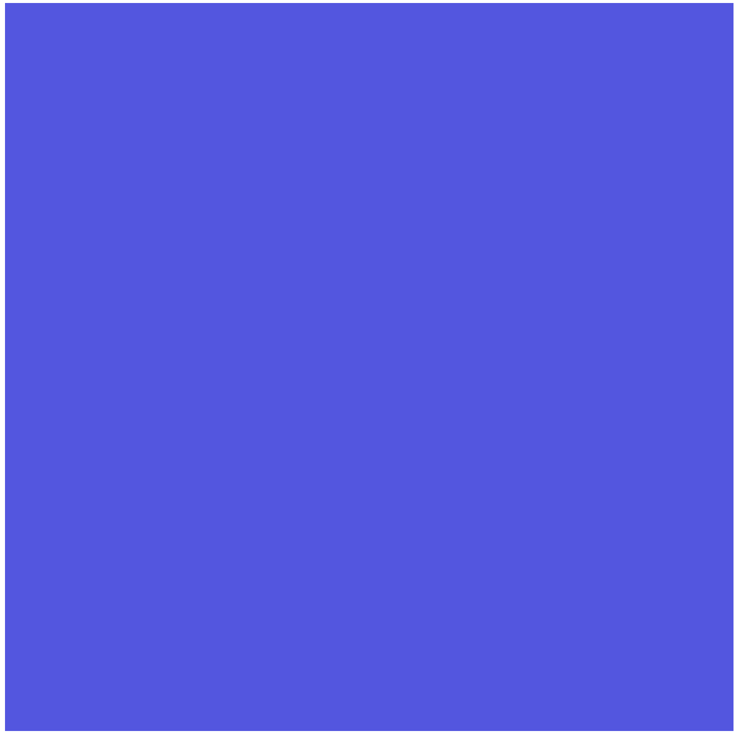
  Kuzmin, Rubakov, Shaposhnikov (1985)

M. Postma, Friday




Steinhardt (1982); Hogan (1983,86); Gyulassy et al (1984); Witten (1984)

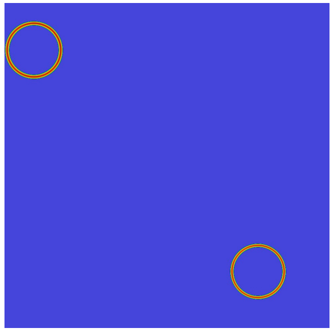
Fluid kinetic energy



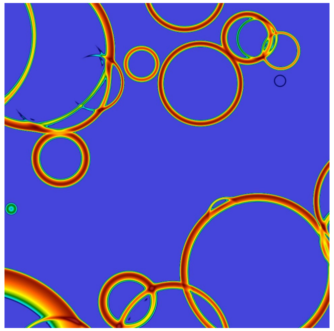
David Weir

MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

# Phases of a phase transition



1



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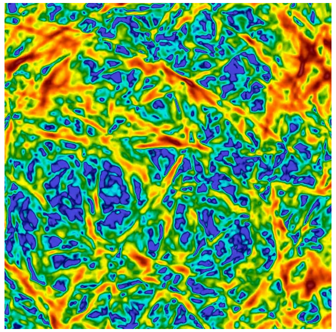


3


4

1. Bubble nucleation and expansion
2. Collision
3. Acoustic waves (vorticity)
4. Non-linear (shocks, turbulence)

$$\tau_{\mathrm{nl}} \sim L_f/\bar{U}_f$$

$L_{\mathrm{f}}$ – fluid flow length scale

$U_{\mathrm{f}}$ – RMS fluid velocity

'exponential' nucleation rate/volume p(t)

$$p(t) = p_n e^{\beta(t-t_n)}$$

$$\tau_{\mathrm{co}} = \beta^{-1}$$ Ekstedt, Gould, Hirvonen 2023

$\beta$ – transition rate parameter

$\beta > H$ for successful transition

Guth, Weinberg 1983; Enqvist et al 1992;
Turner, Weinberg, Widrow 1992

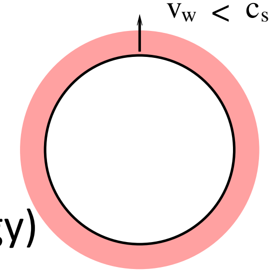
Review: MH, Lüben, Lumma, Pauly 2021

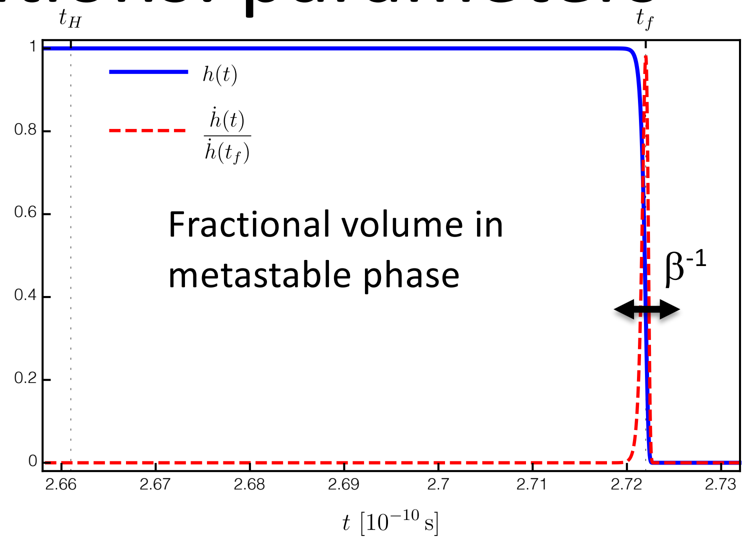
# GWs from first order phase transitions: parameters

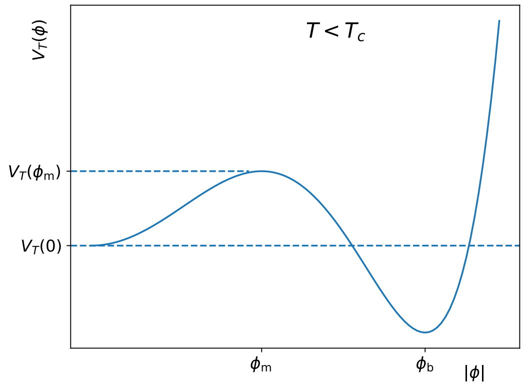
- Parameters of transition:
  - $T_n$ = Temperature at nucleation
  - $\beta$ = transition rate ( = - d log $p$ / d$t$)
  - $v_w$ = Bubble wall speed
  - $\alpha$ = (potential energy change*)/(heat energy)
  - $c_{s1}$, $c_{s2}$ = sound speeds   Giese et al 2020
- Useful derived parameters:
  - $r_*$ = (bubble centre spacing $R_*$)/Hubble length
  - $K$ = fluid kinetic energy fraction

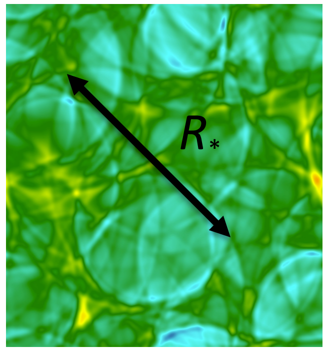
Steinhardt '84
Espinosa et al 2010

- Fluid kinetic energy makes GWs
  - Energy release via self-similar bubble solutions



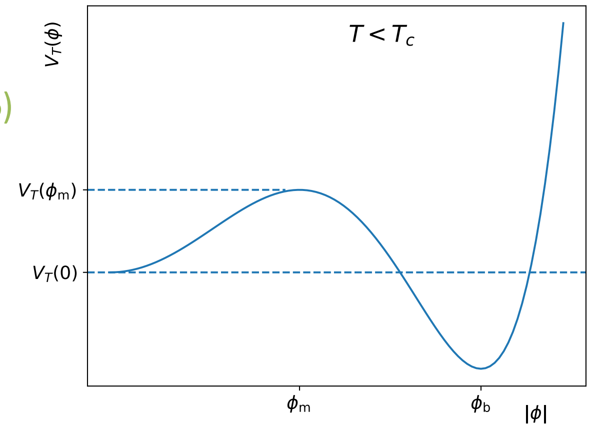




$$* \quad \frac{1}{4}\Delta(e - 3p)$$

# GWs from an early universe phase transition

Assume rapid transition, $\beta >> H$, neglect expansion of universe




- Effective theory: Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - **Higgs field** $\Box\phi - V_T'(\phi) = \eta_T(\phi) U \cdot \partial\phi$
    - $V_T(\phi)$ equation of state
    - $\eta_T(\phi)$ field-fluid coupling (models friction)
  - **Relativistic fluid (ideal limit)**

$$T_{\text{f}}^{\mu\nu} = (e + P)U^\mu U^\nu + P g^{\mu\nu}$$

$$\partial_\mu T_{\text{f}}^{\mu\nu} + \partial^\nu \phi V_T'(\phi) = \eta_T(\phi)(U \cdot \partial\phi)\partial^\nu\phi$$

  - **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \quad \Rightarrow \quad \tilde{h}_{ij}(\mathbf{k}) = \Lambda_{ij,kl}^{TT} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)

# GWs from an early universe phase transition

Assume rapid transition, $\beta >> H$, neglect expansion of universe

- Effective theory: Ignatius et al (1994), Kurki-Suonio, Laine (1996)
  - **Higgs field** $-\ddot{\phi} + \nabla^2\phi - V_T'(\phi) = \eta_T(\phi) W(\dot{\phi} + V^i\partial_i\phi)$
    - $V_T(\phi)$ equation of state
    - $\eta_T(\phi)$ field-fluid coupling ("friction", high T: $\eta_T(\phi) \propto \phi^2/T$ )
  - **Relativistic fluid**

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi} W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

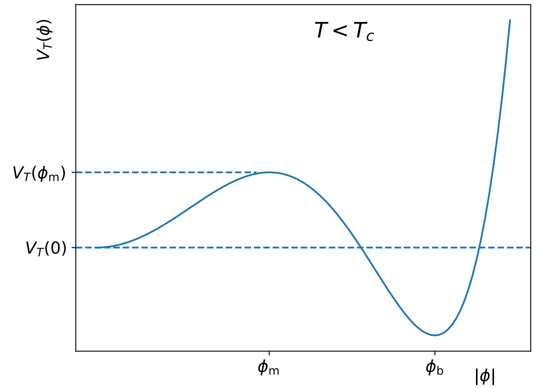
$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i P + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - $E$ = W*(energy density), $Z_i$ = momentum density, $V_i$ = 3-velocity, $W$ = Lorentz factor
  - Discretisation * Wilson & Matthews (2003)

    Also: Brandenburg, Enqvist, Olesen (1996); Giblin, Mertens (2013); Jinno, Konstandin, Rubira (2020)
  - **Metric perturbation** (GW strain)

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G T_{ij} \quad \Rightarrow \quad \tilde{h}_{ij}(\mathbf{k}) = \Lambda_{ij,kl}^{TT} u_{kl}(\mathbf{k})$$

Garcia-Bellido, Figueroa, Sastre (2008)



# Connection to fundamental theory

- Scalar hydrodynamics $-\ddot{\phi} + \nabla^2\phi - V_T'(\phi) = \eta_T(\phi) W(\dot{\phi} + V^i \partial_i \phi)$

- Scalar effective potential $V_T(\phi)$ → equilibrium, quasi-eqm. ($T_n$, $\alpha$, $\beta$, $c_s$, $g_{\rm eff}$)
- Scalar-fluid coupling $\eta_T(\phi)$ → non-equilibrium ($v_w$)

Ekstedt, Schicho, Tenkanen 2024
Ekstedt, Gould, Hirvonen 2023

Ekstedt et al 2024

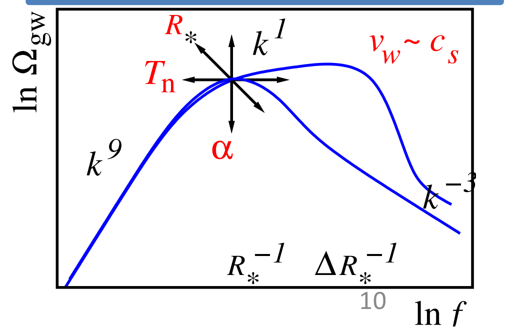
$$\Omega_{\rm gw}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

**Phase transition parameters :**
$T_n$ = nucleation temperature
$g_{\rm eff}$ = effective d.o.f. in plasma
$\alpha$ ~ (potential energy)/(thermal energy)
$c_{s1}$, $c_{s2}$ = sound speed(s)
$\beta$ = transition rate
$v_{\rm w}$ = bubble wall speed

→ *Simulations, Modelling* →

$H_{\rm n}(T_n, g_{\rm eff})$ (Hubble rate)
$K(v_{\rm w}, \alpha, c_s)$ (kinetic energy fraction)
$R_*(\beta, v_{\rm w})$ (mean bubble separation)

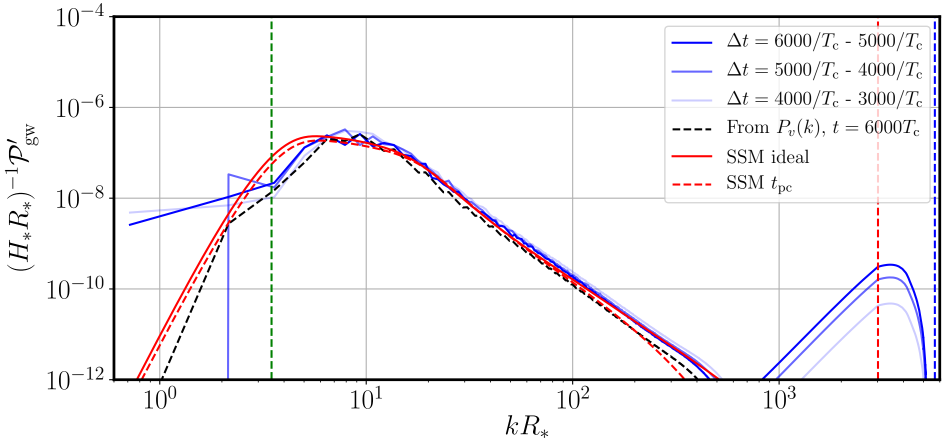
**GW spectrum**
$\Omega_{\rm p}$ = peak amplitude
$f_{\rm p}$ = peak frequency
$\sigma_{\rm i}$ = shape parameters

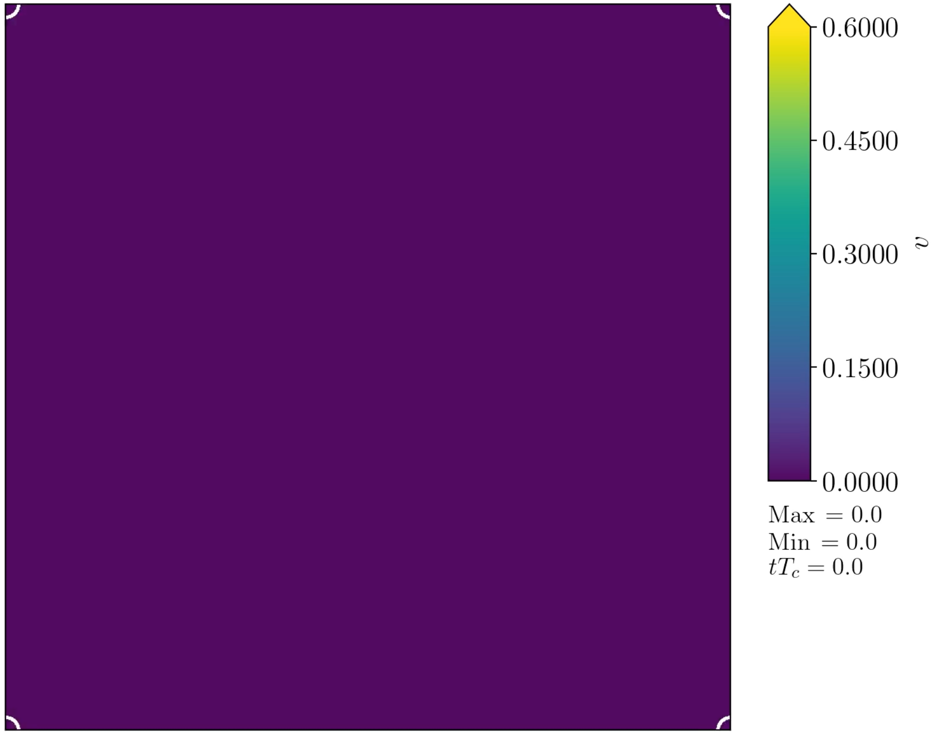


# 3D hydrodynamic simulations of phase transitions

Hindmarsh et al 2013, 2015,2017, 2019; Jinno et al 2022; Caprini et al 2024

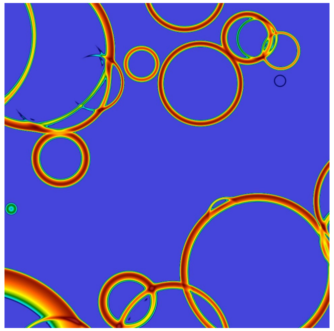
- Relativistic fluid + scalar order parameter ("Higgs")
- Linearised gravitational wave production
- Key output: GW power spectrum
  (fractional GW energy density per log wavenumber)

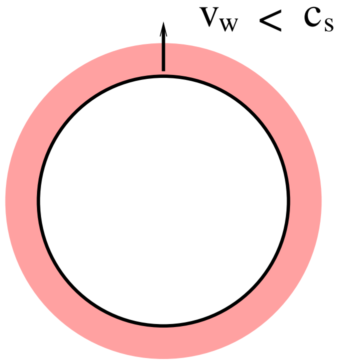


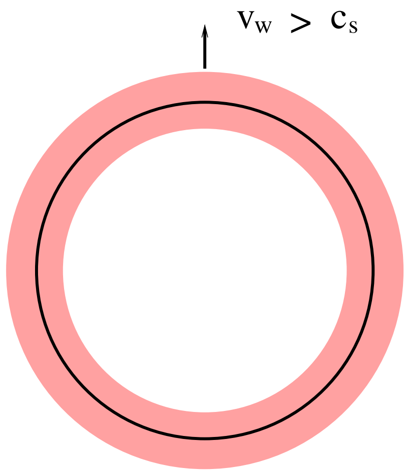

- While sound waves persist, GW power spectrum grows
- Plot: GW power spectrum **growth rate** (scaled by $H_n$)

# Towards a model: relativistic combustion

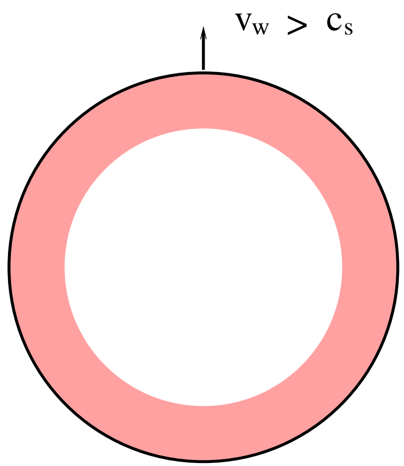





Deflagration



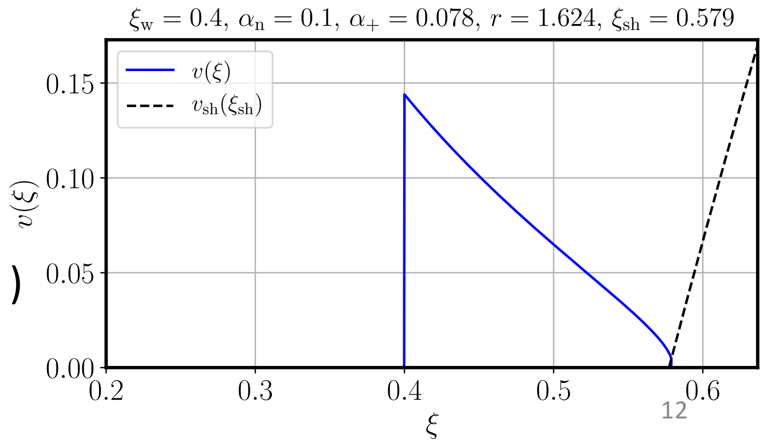

Supersonic deflagration
("hybrid")




Detonation

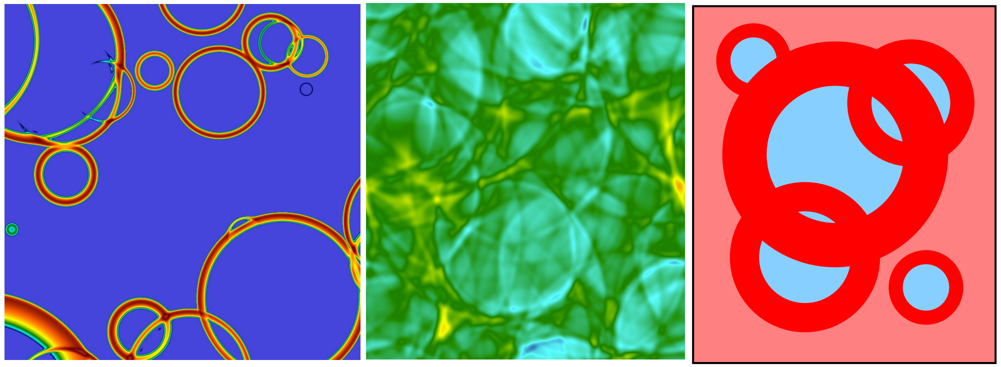
Landau & Lifshitz; Steinhardt (1984)
Kurki-Suonio, Laine (1991), Espinosa et al (2010)

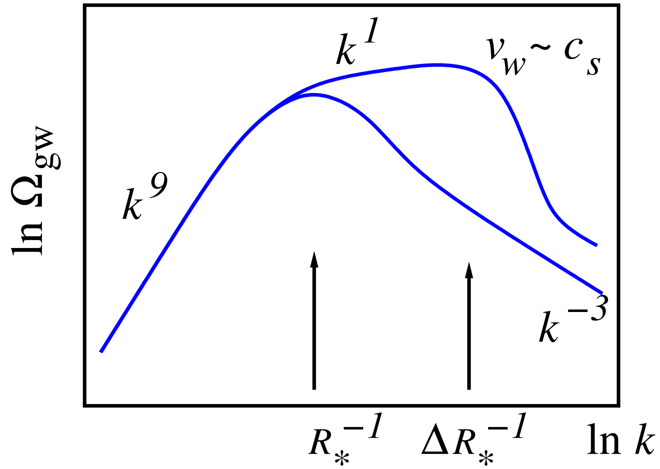
- Large scales: ideal relativistic hydrodynamics
- Microphysics: $e(T)$, $p(T)$, $v_w$
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
    - **Similarity solution $v(x)$, $w(x)$: $\xi = r/t$**
- Low-friction or ultra-strong transition: can be "runaway" ($v_w \to 1$ )
    - (not considered here) Bodeker Moore 2010, 2017



# GWs from phase transitions: Sound shell model

- GWs from Gaussian velocity field
  Caprini, Durrer, Servant (2007,2009)
- Velocity field: weighted addition of self-similar sound "shells" $\mathbf{v}_{\mathbf{q}}(t_i)$ from bubbles
  MH 2016, MH, Hijazi (2019)
- Two length scales:
  - Bubble spacing $R_*$
  - Shell width $R_*|v_w - c_s|/c_s$
- Double broken power law
  - $\mathcal{P}_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude proportional to:
  - Bubble spacing
  - Shear stress lifetime
  - (Kinetic energy)$^2$
- Similar: bulk flow model ($k^1$ at low $k$)
  Jinno, Takimoto 2017; Konstandin 2018





# Estimating GW power

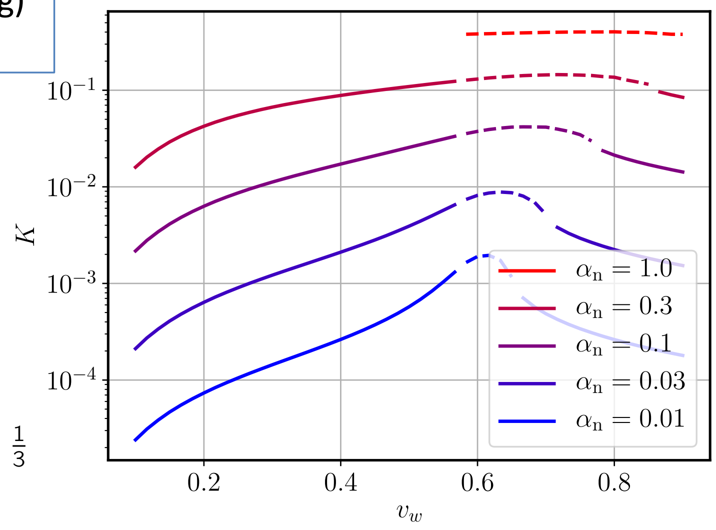
$$\Box h \sim T \longrightarrow P_{\dot{h}}(t,k) \sim \int^t dt_1 \int^t dt_2 \cos[k(t-t_1)]\cos[k(t-t_1)]\langle T_k(t_1)T_k^*(t_2)\rangle$$

- GW energy fraction: $\longrightarrow$ $\Omega_{\rm gw} \sim (H_{\rm n}\tau_{\rm v})(H_{\rm n}\tau_{\rm c})K^2$
  - $H_{\rm n}$ Hubble rate (max nucleation)
  - $\tau_{\rm v}$ duration of stresses
  - $\tau_{\rm c}$ coherence time
- Coherence time:
  - $\tau_{\rm c} \sim R_*$ (bubble spacing)
- Shear stress lifetime (shocks):
  - $\tau_{\rm v} \sim H_{\rm n}^{-1}/(1 + U_{\rm f}/R_* H_{\rm n})$
  - $U_{\rm f}$ (~ RMS velocity) ~ $\sqrt{K}$
  - or $K = (4/3)U_{\rm f}^2$

$$\Omega_{\rm gw,0} \simeq F_{\rm gw,0}\frac{r_*}{1+\sqrt{K}/r_*}K^2\tilde{\Omega}_{\rm gw}$$

$R_*(\beta, v_{\rm w}$, also $\alpha_{\rm n})$ (mean bubble spacing) from bubble growth dynamics

$K(v_{\rm w}, \alpha_{\rm n})$ (kinetic energy fraction) from self-similar hydro solution




Numerical simulations:

$$\tilde{\Omega}_{\rm gw} = O(10^{-2})$$

Standard cosmology:

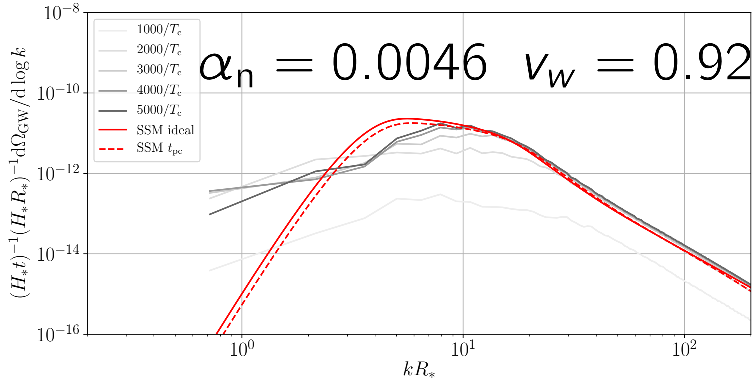
$$F_{\rm gw,0} = 3.6 \times 10^{-5}\left(\frac{100}{g_{\rm eff}}\right)^{\frac{1}{3}}$$

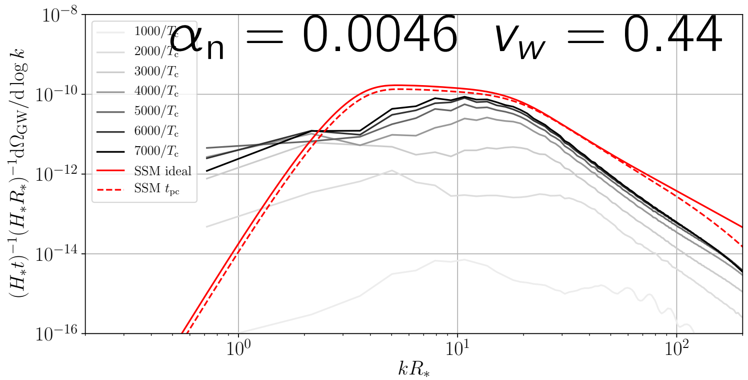
# Sound shell model vs. simulations $P_{gw}$

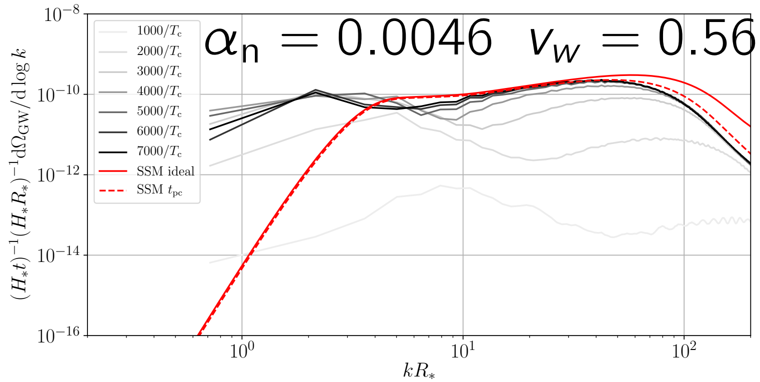
GW power growth rate

- Solid: ideal self-similar sound shell
- Dash: evolving sound shell at peak collision time in 1+1D scalar hydro
- Grey: simulations:
  - simultaneous nucleation of bubbles

Hindmarsh, Huber, Rummukainen, Weir 2017



# Sound shell model vs. simulations $P_{gw}$

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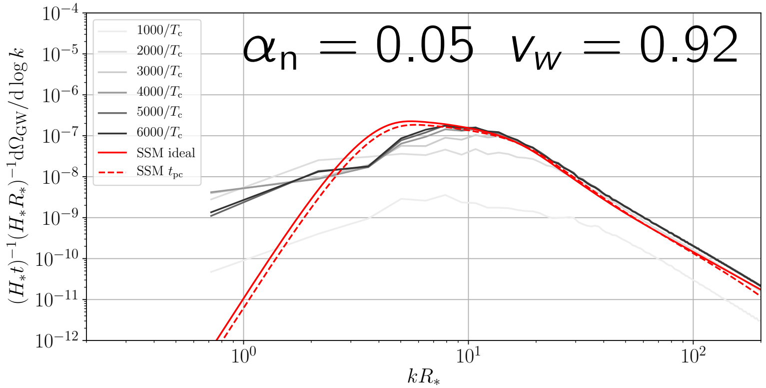
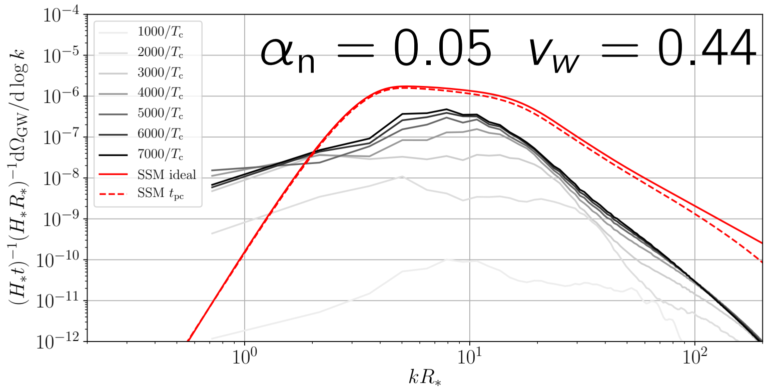
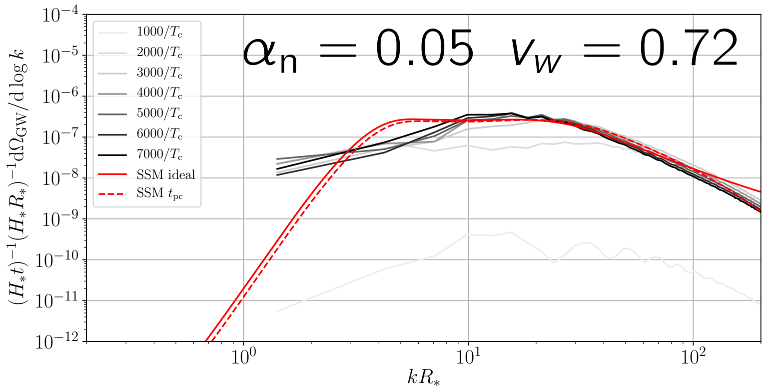
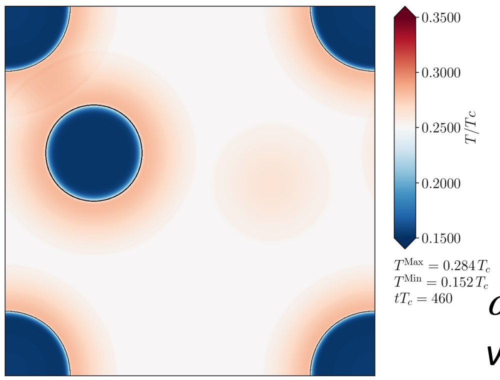


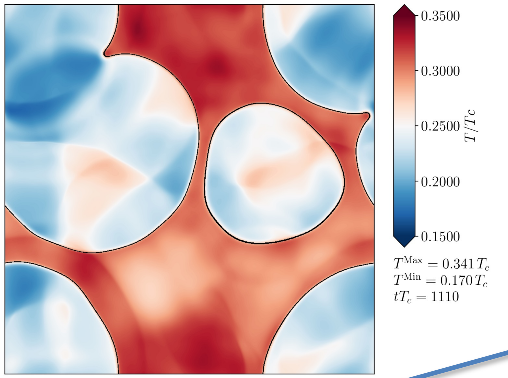
# Nonlinearities 1: Kinetic energy & GW suppression

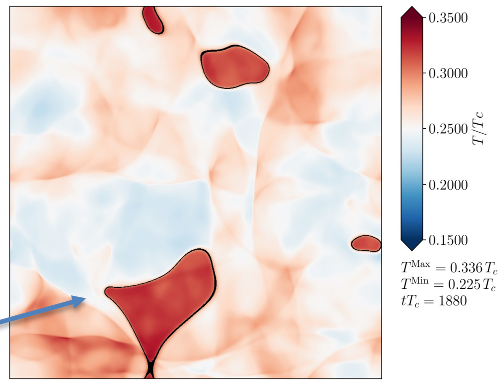



$T/T_c$

$\alpha$ = 0.5

$v_w$ = 0.44

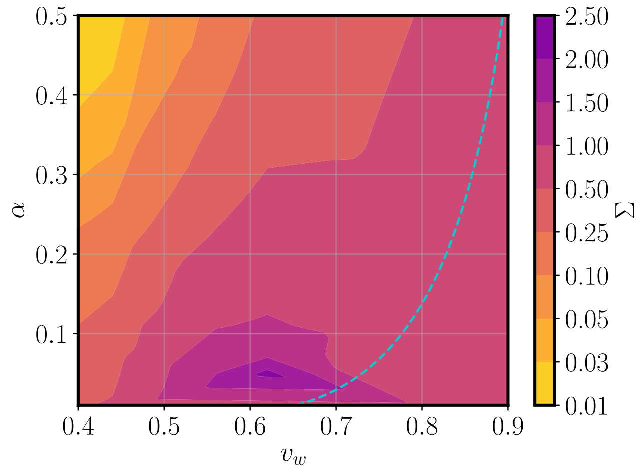



Cutting, MH, Weir, 2020

- **Deflagrations**: heat up fluid in front
- Pressure in front of wall increases, walls slow down
- Formation of hot droplets Cutting, Vilhonen, Weir (2022)
- Less transfer into kinetic energy, more into heat.
- Include **GW suppression factor** as a numerical parameter (**right**)
- Also: nucleation suppression
  - bigger bubbles, boosts signal

Al-Ajmi, MH (2023)

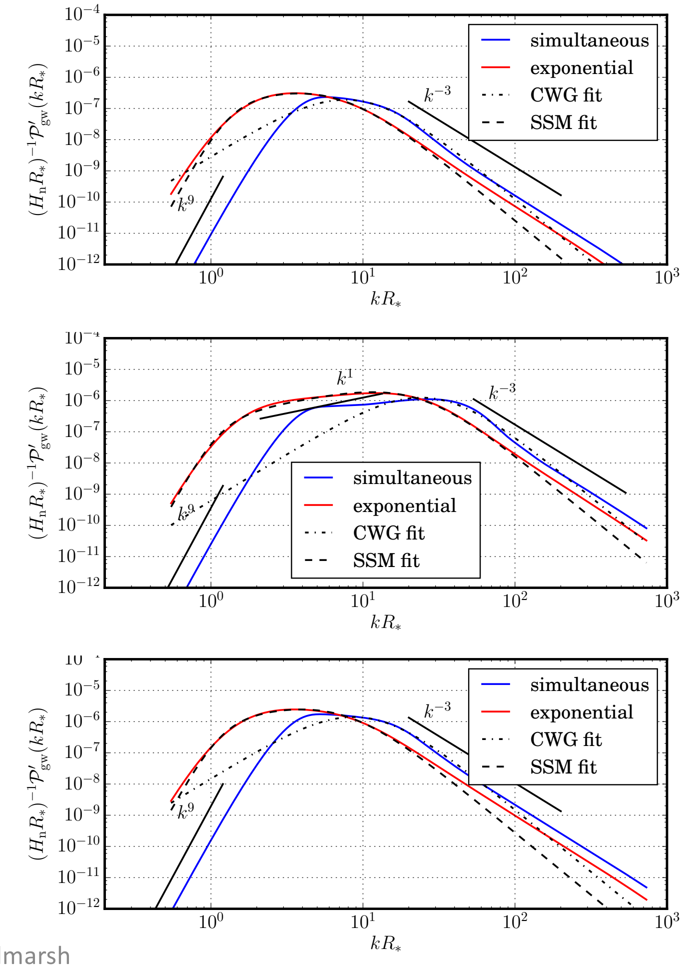



Gowling, MH (2021)

# SSM toolbox: PTtools

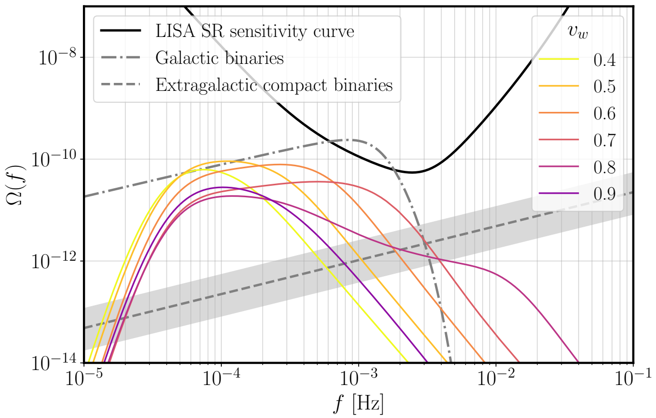
Cutting, Gowling, MH, Mäki

- Python package for GW spectra from 1st order phase transitions
- Currently: linearised approx with phenomenological corrections
  - intermediate strength ($\alpha \leq 0.3$)
  - Wall speeds $0.4 < v_\mathrm{w} < 0.9$
  - Sub-Hubble bubble separations ($r_* << 1$)
- Speed-up with numba
  - 50 ms for 100 frequencies
- Support for general eqn of state
- Public release early 2025

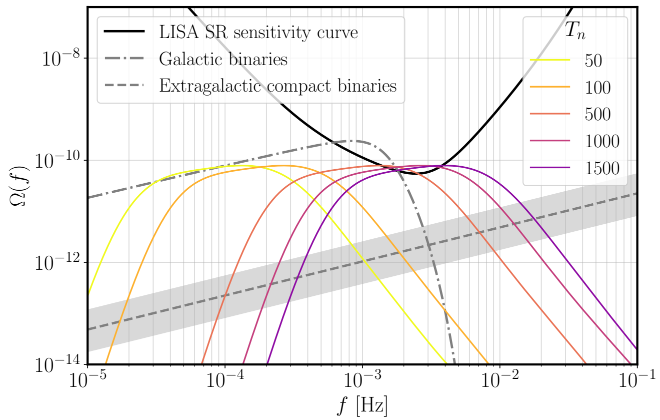


# GW power spectra in the SSM

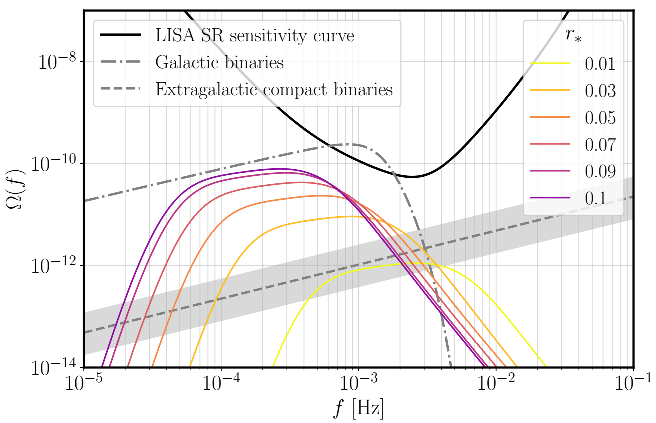
- **Sound shell model predictions: accuracy ~20% at peak for near-linear flows** Gowling, MH (2021)
  - strengths $\alpha \leq 0.3$; wall speeds $0.4 \leq v_w \leq 0.9$; sub-Hubble bubble separations $r_* \ll 1$



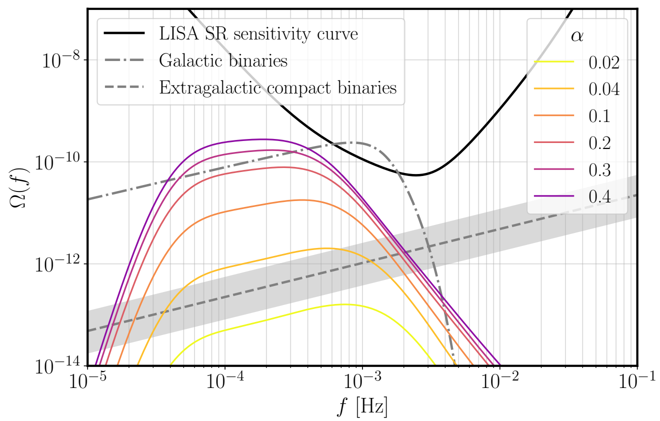

Vary wall speed




Vary transition temp

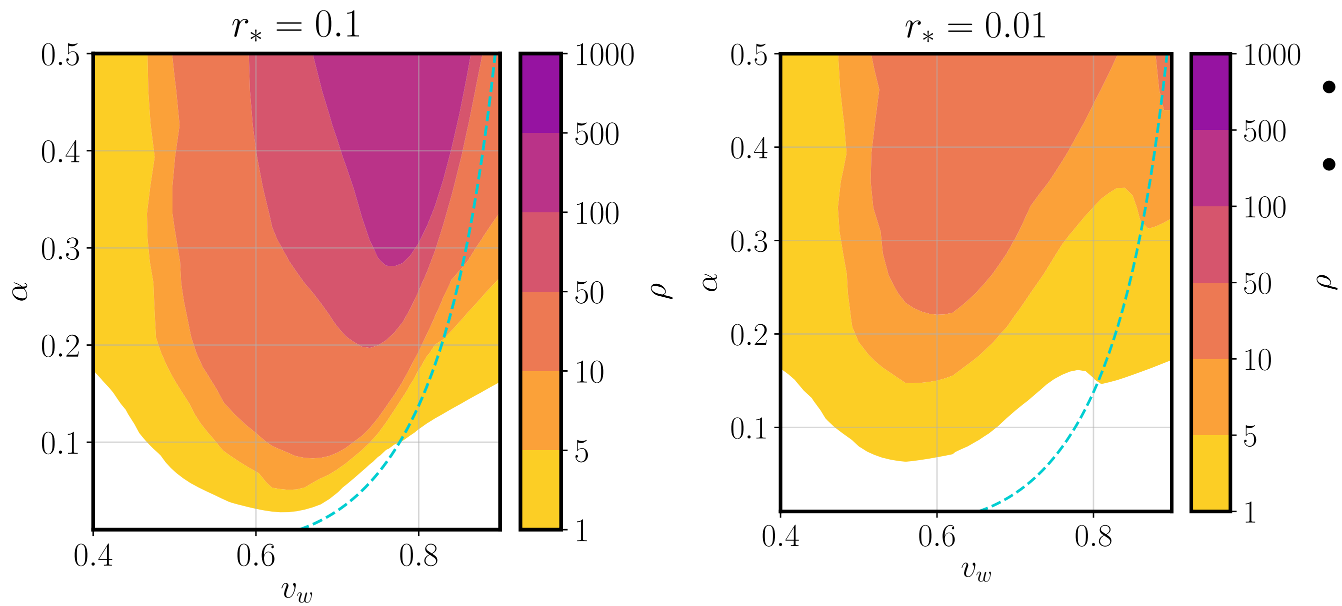



Vary bubble separation




Vary transition strength

# Signal-to-noise ratios (LISA)




- $T_n$ = 100 GeV
- $c_s = 1/\sqrt{3}$

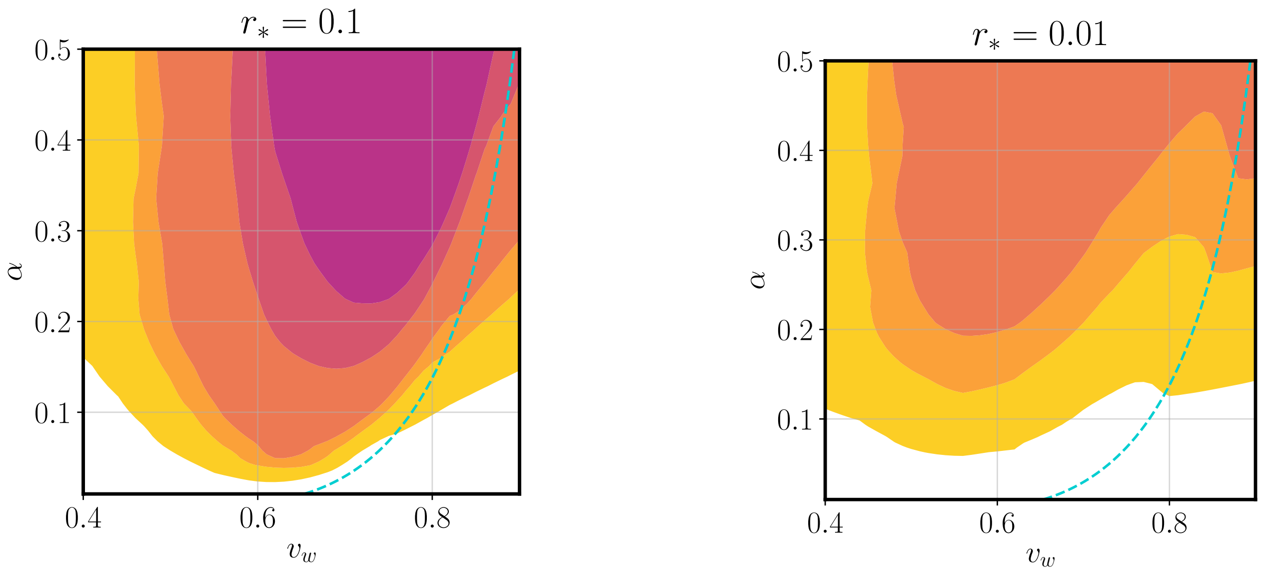
Gowling, MH (2021)

- Signal-to-noise ratio ρ ($t_{obs}$ = 4 years)
- "Worst case" galactic binary foreground
  - (NB annual variation aids removal)
- "LISA science requirements" instrument noise

MH, Hooper, Minkkinen, Weir (2024)

$$\rho^2(\vec{\theta}) = t_{obs} \int df \left( \frac{\Omega_{gw}(f;\vec{\theta})}{\Omega_{noise}(f)} \right)^2$$

# Signal-to-noise ratios (LISA)



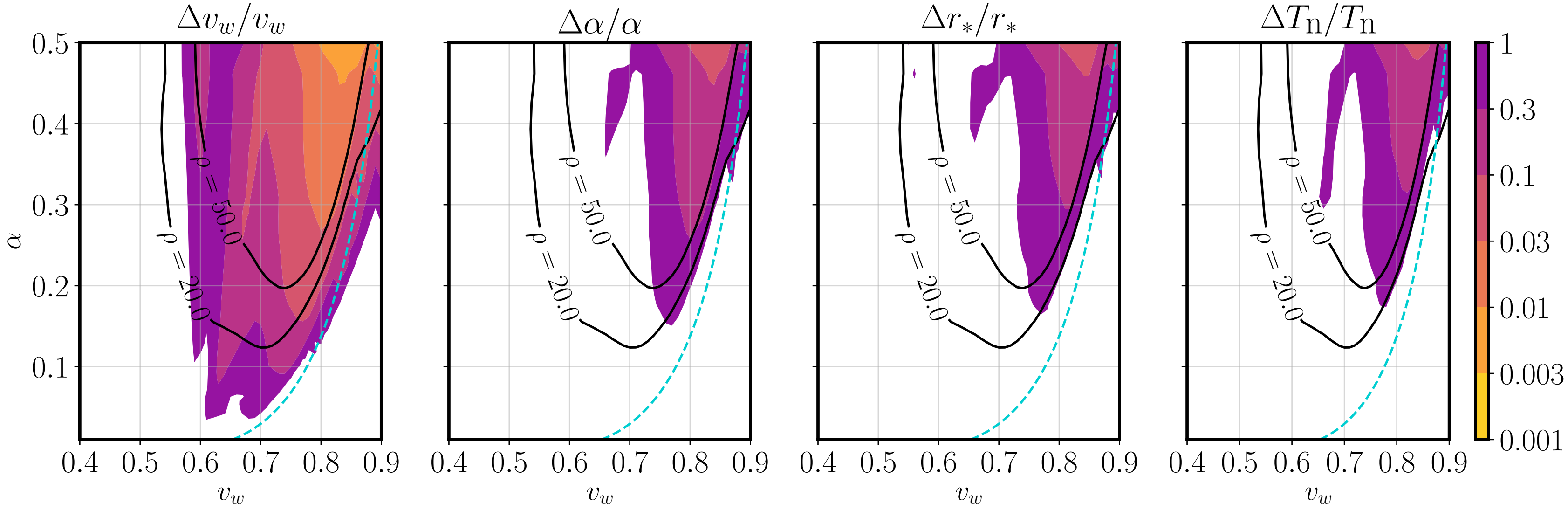

- $T_{\rm n}$ = 100 GeV
- $c_s = 1/\sqrt{3}$

Gowling, MH (2021)

- Signal-to-noise ratio r ($t_{\rm obs}$ = 4 years)
- Perfect removal of GB foreground
- "LISA science requirements" instrument noise

$$\rho^2(\vec{\theta}) = t_{\rm obs} \int df \left( \frac{\Omega_{\rm gw}(f;\vec{\theta})}{\Omega_{\rm noise}(f)} \right)^2$$

# Observability of PT parameters: Fisher analysis



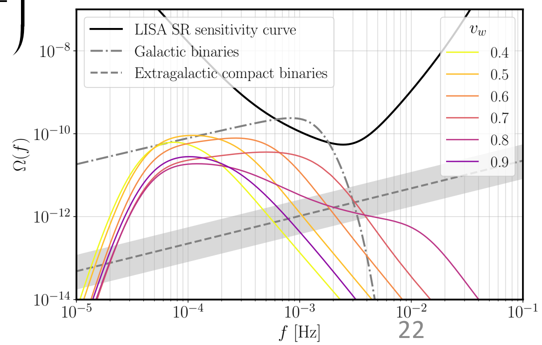

- $T_c$ = 100 GeV
- $c_s = 1/\sqrt{3}$
- $r_*$ = 0.1

Gowling, MH (2021)

- Covariance matrix ***C*** ($t_{obs}$ = 4 years)

$$C_{ij}^{-1}(\vec{\theta}) = 2t_{obs}\int df \frac{1}{\Omega^2_{noise}(f)}\left(\frac{\partial\Omega_{gw}(f;\vec{\theta})}{\partial\theta_i}\frac{\partial\Omega_{gw}(f;\vec{\theta})}{\partial\theta_j}\right)$$

- Uncertainty from eigenvalues of ***C***
  - "noise" = GB foreground + "LISA science requirements" instrument noise
- **Wall speed** is best determined parameter
  - shape dependence of power spectrum

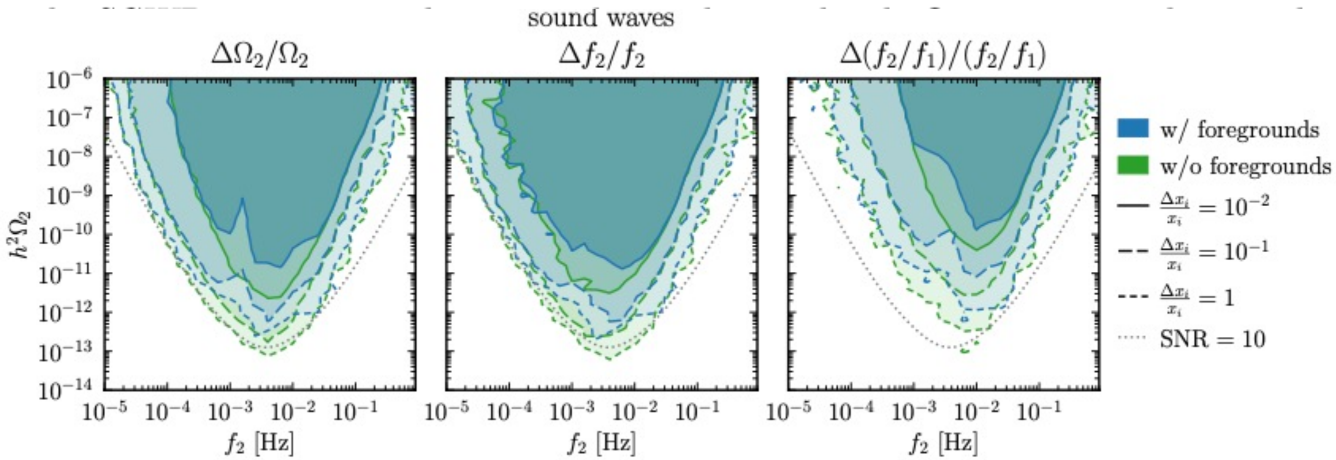


# Observability of parametrized spectra

Caprini et al (LISA CWG, 2024)

$$\Omega_{\rm GW}^{\rm DBPL}(f,\vec{\theta}_{\rm Cosmo}) = \Omega_{\rm int} \times S(f) = \Omega_2 \times S_2(f), \qquad (2.8)$$

$$S(f) = N\left(\frac{f}{f_1}\right)^{n_1}\left[1+\left(\frac{f}{f_1}\right)^{a_1}\right]^{\frac{-n_1+n_2}{a_1}}\left[1+\left(\frac{f}{f_2}\right)^{a_2}\right]^{\frac{-n_2+n_3}{a_2}},$$




- Marginalised posteriors from Polychord
  - “noise” = GB foreground + “LISA science requirements” instrument noise

# Recent advances in hydrodynamics

- Shape of GW spectrum at low *k: k*$^3$ and *k*$^1$

  Sharma, Dahl, Brandenburg, MH 2023; Roper-Pol, Procacci, Caprini 2023

- Effect of sound speed on GW spectrum

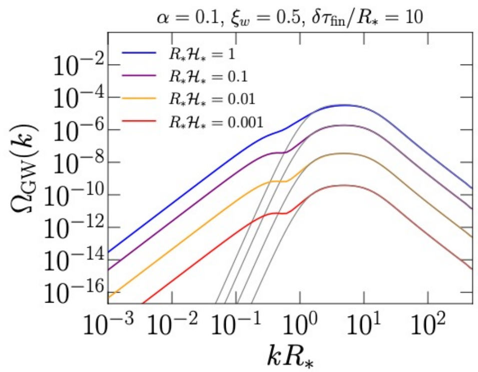
  Giese et al 2020, Giombi, Dahl, MH 2024

  Racco, Poletti 2022

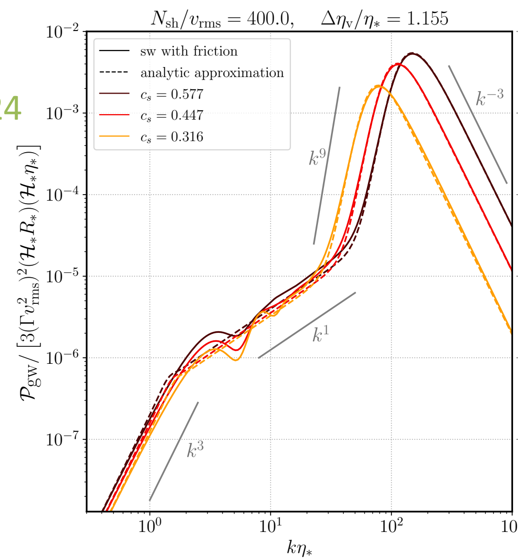
- GWs from strong transitions ($\alpha$ ~ 1)
  - $v_{rms}$ ~ 0.1 – 0.3, decaying flow (shock dissipation)
  - peak shape change from shock evolution
  - Decrease of GW growth rate, convergence

Dahl et al 2024

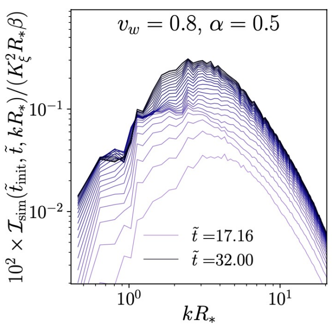
Caprini et al 2024



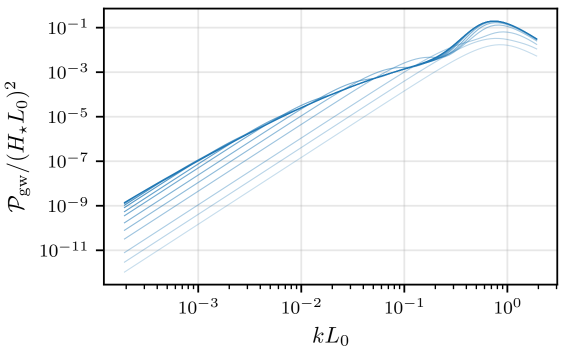

Roper Pol, Procacci, Caprini 2023




Giombi, Dahl, MH 2024




Caprini et al al 2024




Dahl et al al 2024

# Conclusions

- LISA and other missions will probe physics of electroweak-scale transitions from mid-2030s
  - Measure/constrain phase transition parameters
    - Wall speed likely to be best determined (if signal seen)
  - Parameters from underlying particle physics models
    - Recent progress with equilibrium parameters
    - Wall speed the hardest (non-equilibrium)
- Towards accurate calculations of GW power spectrum from parameters
  - Now: < 20% accuracy for strength $\alpha \leq 0.3$; speed $0.4 < v_w < 0.9$; bubble separation $r_* << 1$; sound speed $c_s = 1/\sqrt{3}$
  - Non-linear evolution ($\alpha \sim 1$) beginning to be investigated
  - Strongly supercooled transitions ($\alpha \gg 1$)?

Jinno et al 2019, Lewicki, Vaskonen 2022

- Ambition: make GWs as good a probe of the electroweak era as CMB is for the decoupling era