Noise-agnostic searches

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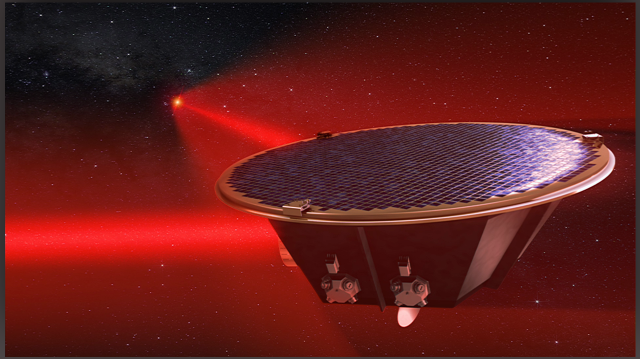
Towards a realistic forecast detection of Primordial Gravitational Wave Backgrounds
December 10, 2024

Layout

1. Data simulation and pre-processing
2. Likelihood
3. Spectrum models
4. Inference
5. Discussion



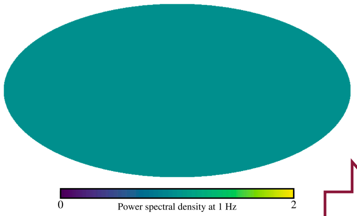
0. Preamble: our goals

- ✦ Encompass as many instrumental configurations as possible
- ✦ Be agnostic regarding the spectral shape of single-link noise sources
- ✦ Control the uncertainties
- ✦ Explore the detectable parameter space for various GW background models

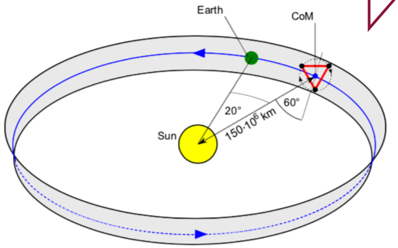
1. Data simulation and preprocessing

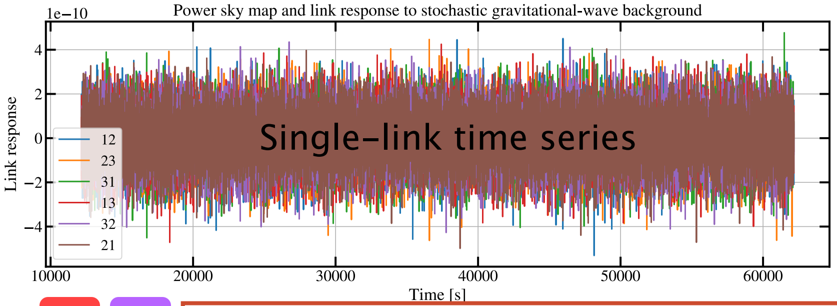
- We consider:
  - Unequal armlengths
  - Varying armlengths
  - Time-to-frequency transformations
- For that, we simulate time-domain interferometric measurements

Skymap



Orbits





Data produced in the context of a common project (see Nikos' talk)

[Bayle et al. 2023]

Spectra

TDI variables

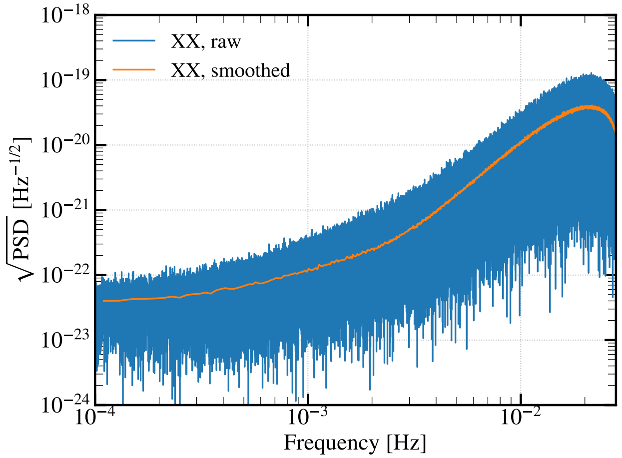
[Staab et al. 2023]

1. Data simulation and preprocessing

- We can build a « sufficient statistics » to lower the likelihood computation time

1. We compute the discrete Fourier transform (DFT) of the N x 3 TDI time series $\sim \sum_{n=0}^{N_x-1} w_n \mathbf{x}_n e^{-2\pi k n / N_x}$
2. We form the periodogram matrix $\mathbf{P}(f) = \tilde{\mathbf{d}}(f)\tilde{\mathbf{d}}(f)^{\dagger}$
3. We slice it into segments whose size increases with frequency
4. We compute the averaged periodogram matrix in each segment

$$\bar{\mathbf{P}}(f_j) \equiv \frac{1}{n_j} \sum_{k=j-\frac{n_j}{2}}^{j+\frac{n_j}{2}} \tilde{\mathbf{d}}(f_k)\tilde{\mathbf{d}}(f_k)^{\dagger}$$

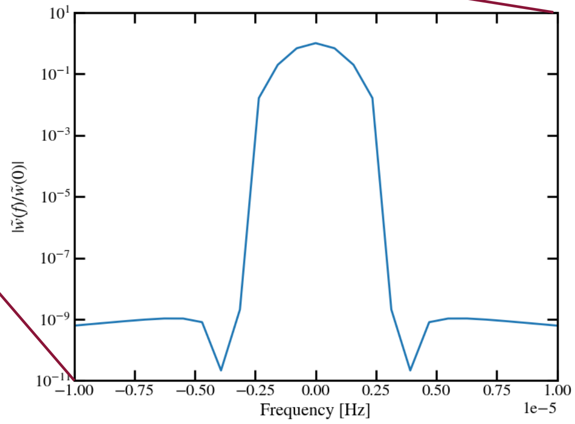


2. Likelihood

- To construct the likelihood, we need to caracterize the probability distribution of $\bar{\mathbf{P}}$
- Assume that the original time series $\mathbf{x}(t)$ is zero-mean stationary with 3 x 3 spectrum matrix $\mathbf{S}(f)$
- Then the DFT covariance is

$$\boldsymbol{\Sigma}(f_1, f_2) = \mathrm{E}\left[\tilde{\mathbf{d}}(f_1)\tilde{\mathbf{d}}^{\dagger}(f_2)\right] = \int_{-\infty}^{+\infty} \tilde{w}(f - f_1)\tilde{w}^*(f - f_2)\mathbf{S}(f)df$$

- Usually windowing reduces leakage (bias) but increases bin correlations
- The covariance is not exactly diagonal
- We need to account for these correlations when we average!



2. Likelihood

- If close frequency bins were not correlated → the average periodogram $\bar{\mathbf{P}}(f_j)$ would follow a Wishart distribution with a number of degrees of freedom (DoF) ν equal to the number of frequencies n_j in segment j

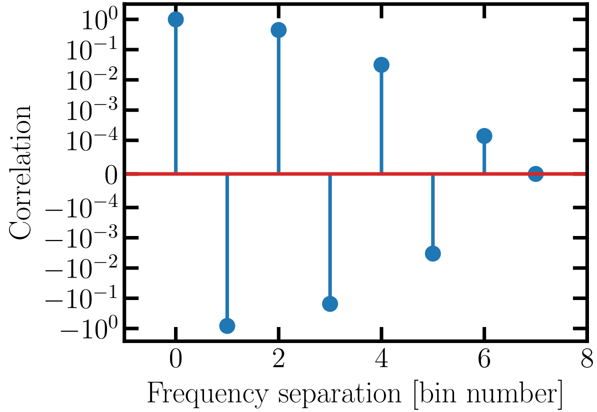
$$\log p\left(\bar{\mathbf{P}}(f_j)\right) = -\nu(f_j)\left[\mathrm{tr}(\mathbf{S}(f_j)^{-1}\bar{\mathbf{P}}(f_j)) + \log \mathbf{S}(f_j)\,|\right]$$

- Instead, we can approximate the distribution by a Wishart with effective number of DoF [see A. Ferrari, 2019]

$$\nu(f_j) = \frac{n_j}{1 + \frac{2}{n_j}\sum_{l<m}\rho_{lm}}$$

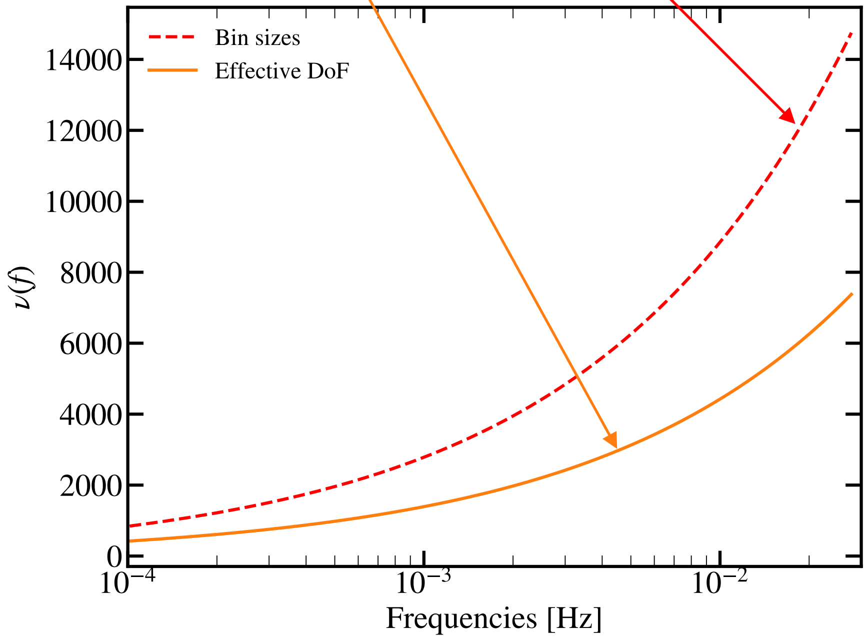
- It depends on the bin-to-bin correlation coefficients

$$\rho^{\alpha\beta}_{l,m} = \frac{|\tilde{\Sigma}_{\alpha\beta}(f_l, f_m)|^2}{S_{\alpha\beta}(f_l)S_{\alpha\beta}(f_m)}$$



2. Likelihood

$$\nu(f_j) = \frac{n_j}{1 + \frac{2}{n_j}\sum_{l<m} \rho_{lm}}$$



2. Likelihood

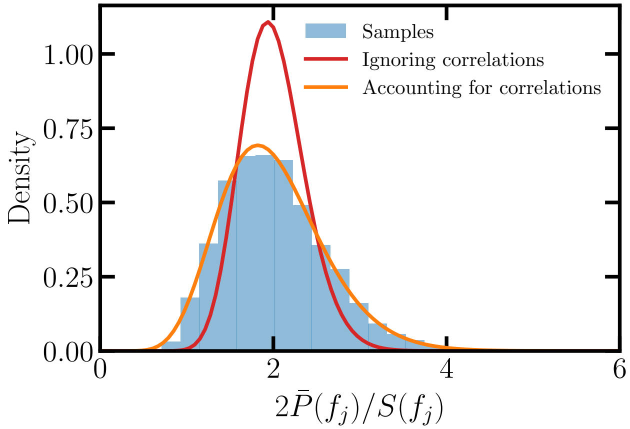
- The bin-to-bin correlation coefficients depends on the (unknown) spectrum in general:

$$\rho^{\alpha\beta}_{l,m} = \frac{\left| \int_{-\infty}^{+\infty} \tilde{w}(f - f_l)\tilde{w}^*(f - f_m)S_{\alpha\beta}(f)df \right|^2}{S_{\alpha\beta}(f_l)S_{\alpha\beta}(f_m)}$$

- But if the spectrum is smooth enough on the bandwidth of $\tilde{w}$, ρ_{lm} is independent of it (at 0th order):

$$\rho^{\alpha\beta}_{l,m} \approx \left| \int_{-\infty}^{+\infty} \tilde{w}(f - f_l)\tilde{w}^*(f - f_m)df \right|^2$$

- Accounting for correlations is important:



3. Spectrum models

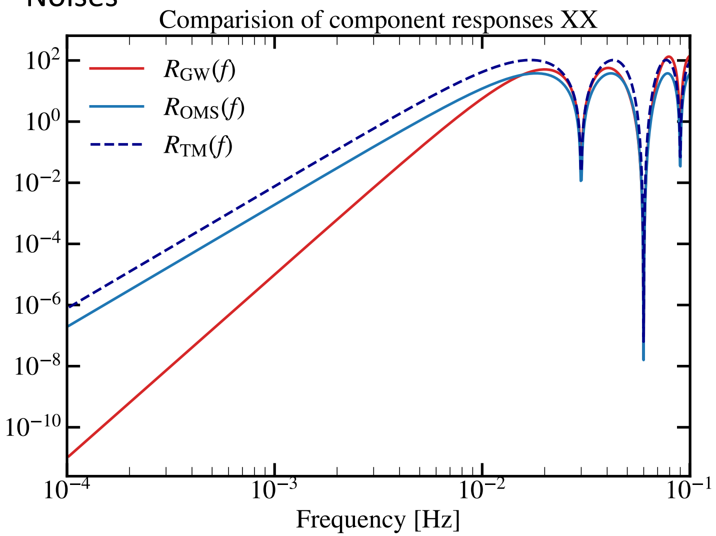
- The spectrum matrix is a sum of independent components:

Response matrices

Single-link PSDs = what we fit

$$\boldsymbol{S}(f) = \underbrace{\boldsymbol{R}_h(f)\, S_h(f, \theta_h)}_{\text{Signal}} + \underbrace{\sum_\alpha \boldsymbol{R}_\alpha(f)\, S_\alpha(f, \theta_\alpha)}_{\text{Noises}}$$

- With assumptions:
  - Orbits are prefeclty known, and so are the response functions
  - The noises affecting the links have the same PSDs
  - All components are stationary
  - Only 2 noise components for now $\alpha \in \{\mathrm{OMS}, \mathrm{TM}\}$



3. Spectrum models

✦ Signal models

- S_h (present-day SGWB PSD) is modeled by a parametrized template
- Related to the energy density as

$$S_h(f, \theta_h) = \frac{3H_0^2}{4\pi^2 f^3}\Omega_{\mathrm{GW}}(f, \theta_h)$$

✦ Noise models

- S_α for component α is modelled by Akima cubic splines with unknown knot locations

$$\log S_\alpha(x) = \sum_{p=0}^{3} a_{i,p}^{\alpha}(x - x_i^{\alpha})^p, \ \forall x \in [x_i^{\alpha}, x_{i+1}^{\alpha}], \ \ x = \log f$$

✦ Parameters

$$\theta = \left(\theta_h, (x_i^{\mathrm{TM}}), (\log a_{ip}^{\mathrm{TM}}), (x_i^{\mathrm{OMS}}), (\log a_{ip}^{\mathrm{OMS}})\right)$$

SGWB parameters → θ_h | Noise knot locations → (x_i^{TM}) | Noise knot log-amplitudes → $(\log a_{ip}^{\mathrm{TM}})$

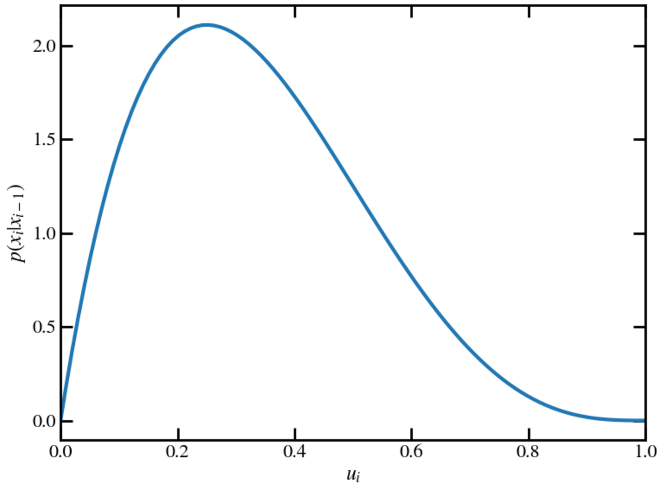
3. Spectrum models (priors)

- Signal priors
  - Energy density amplitude: Log-uniform
  - Other spectral parameters: uniform
- Noise priors
  - Spline PSD coefficients: log-uniform between 10 times above and below the injection
  - Spline log-frequency locations $x_i = \log f_i$: nested conditional beta distributions [QB+2023]

Probability of having the knot i at x_i given that the previous one was at x_{i-1} :

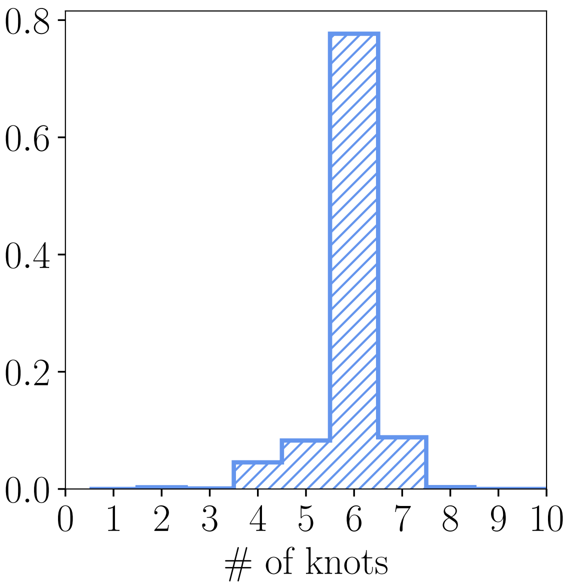
$$p\left(x_i | x_{i-1}\right) \propto u_i^{\alpha_i - 1}(1 - u_i)^{\beta_i - 1},$$

$$u_i \equiv \frac{x_i - x_{i-1}}{x_Q - x_{i-1}}$$



4. Inference

- Assume that the number k of spline knots is unknown
- We look for the optimal number of spline knots
- Bayesian inference using parallel-tempered reverse-jump MCMC with the `Eryn` sampler [Karnesis 2023]



Start with the model of dimension k, with parameter state θ_k

1. In-model step: usual Metropolis-Hastings
2. Birth-or-death step: propose a new dimension m and parameter θ_m

Acceptance probability $$\alpha = \min\left[1, \frac{p(y|\vec{\theta}_k)p(\vec{\theta}_k)q(\{k,\vec{\theta}_k\},\{m,\theta_m\})}{p(y,\vec{\theta}_m)p(\vec{\theta}_m)q(\{m,\vec{\theta}_m\},\{k,\theta_k\})}\right]$$

Likelihood Prior Proposal

4. Inference

✦ Example: injection of an isotropic power-law SGWB from stellar-mass black holes binaries

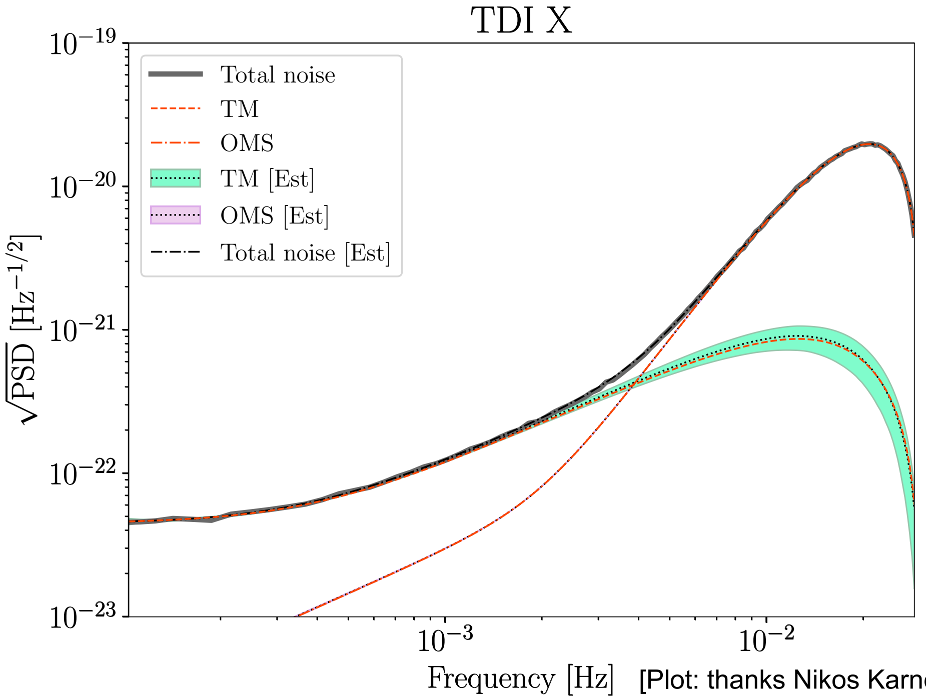
$$\Omega_{\mathrm{GW}}(f, \theta_h) = \Omega_0 \left(\frac{f}{f_0}\right)^n$$

$$\Omega_0 = 3.4 \times 10^{-13}$$

$$n = \frac{2}{3}$$

$$\theta_h = \left(\log \Omega_0, n\right)$$

Predictive posteriors: TDI PSDs

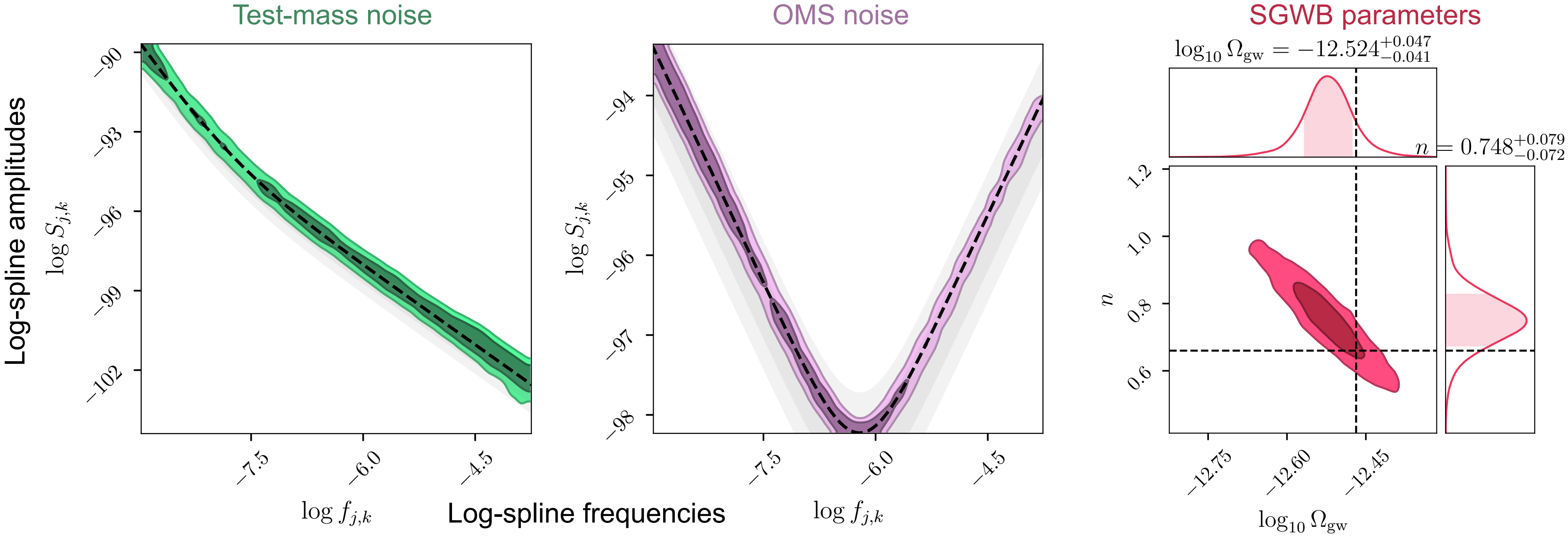


[Plot: thanks Nikos Karnesis]

4. Inference

- ✦ Example: injection of an isotropic power-law SGWB from stellar-mass black holes binaries

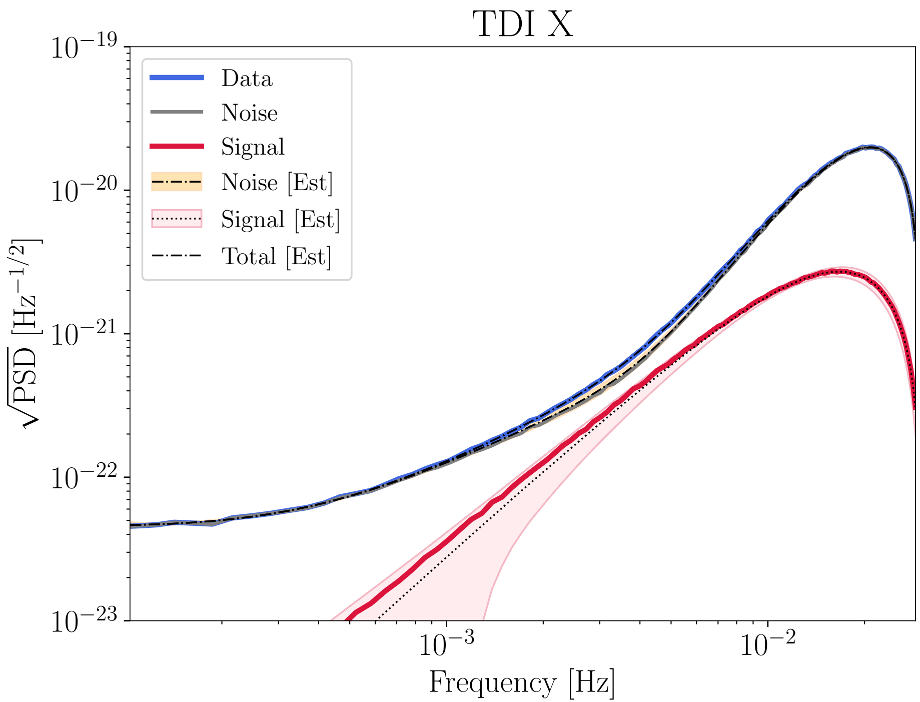
Predictive posteriors: Single-link noise PSDs



4. Inference

- ✦ Example: injection of an isotropic power-law SGWB from stellar-mass black holes binaries
- ✦ Investigation: **replace SGWB template by spline model**

$$\theta_h = \left((x_i^{\mathrm{GW}}), (a_{ip}^{\mathrm{GW}}) \right)$$



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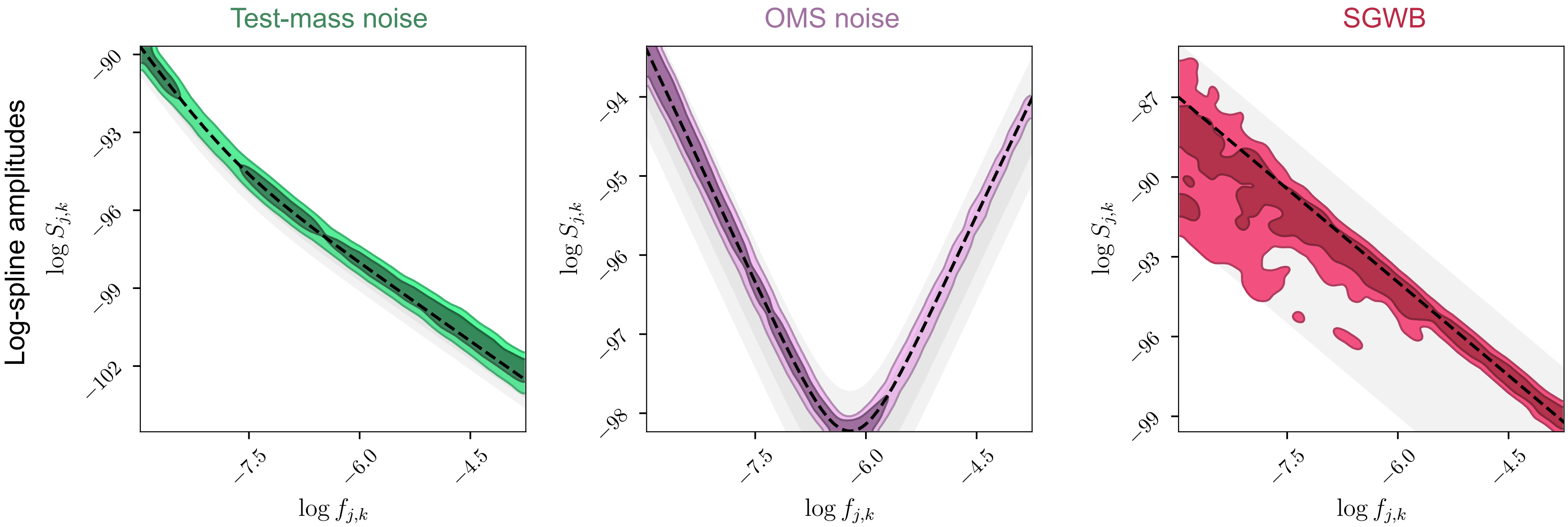
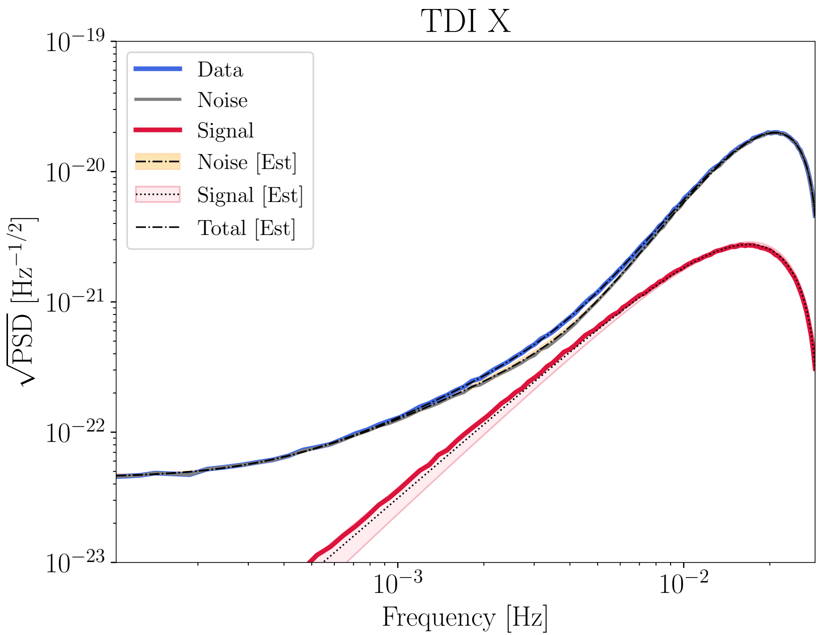


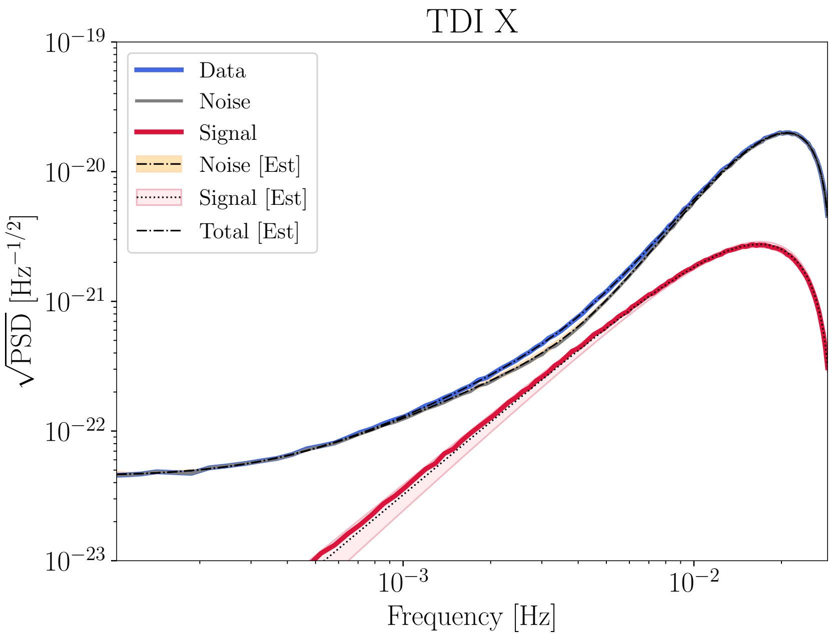
4. Inference

- ✦ Example: injection of an isotropic power-law SGWB from stellar-mass black holes binaries
- ✦ Investigation: testing the **influence of priors**

With uniform (flat) prior on noise

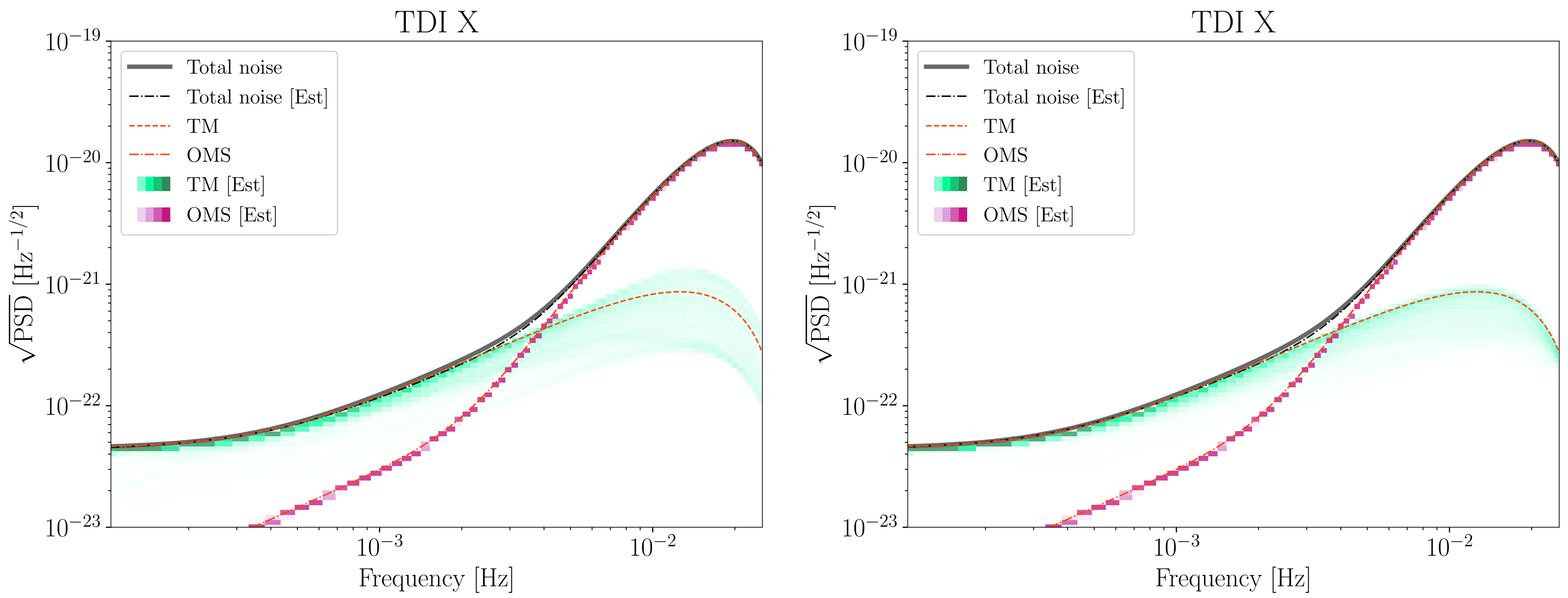


With informative (skewed) prior on noise



4. Inference

- Example: injection of an isotropic power-law SGWB from stellar-mass black holes binaries
- Investigation: testing the **influence of priors**



5. Discussion

- We are carrying out detectability study proping the (Ω_0, n) parameter space
  - →Based on Bayesian model comparison
- When increasning « agnosticism », we must be careful about the influence of priors
- We will have to go for non-equal single-link noise spectra
  - → is our modelling still adapted in this case?
- Armlenghts are more varying than in our simulations
  - → need to test the robustness of our pipelines
- We must model the non-stationarity of some components
  - → can be an advantage instead of an issue


Stochastic GW background generated with Midjourney

Thank you!