Next-generation statistical inference tools

Simulation-based inference, marginal statistics & accelerated nested sampling

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Contents

The standard approach if you are fortunate enough to have a likelihood function $P(D|\theta)$:

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

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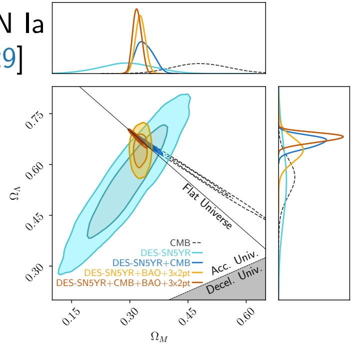
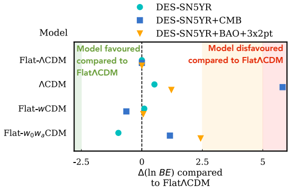
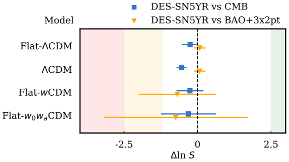
   - spend some time being philosophical
2. Sample posterior $\mathcal{P}(\theta|D)$
   - use out-of-the-box MCMC tools such as emcee or `MultiNest`
   - make some triangle plots
3. Optionally compute evidence $\mathcal{Z}(D)$
   - e.g. nested sampling or parallel tempering
   - do some model comparison (i.e. science)
   - talk about tensions

DES Y5 SN Ia [2401.02929]







LBI: Likelihood-based inference

The standard approach if you are fortunate enough to have a likelihood function $P(D|\theta)$:

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} \qquad \text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence}}$$

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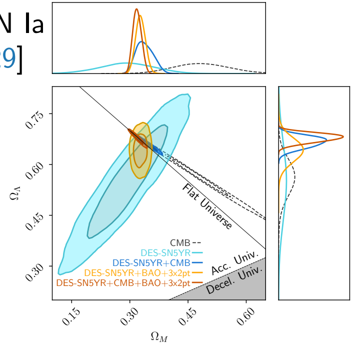

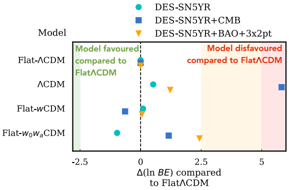

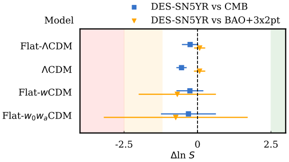

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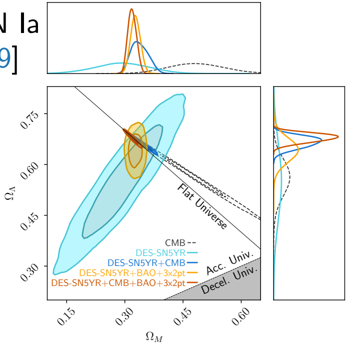

$$\mathcal{P}(\theta|D) = \frac{\mathcal{L}(D|\theta)\pi(\theta)}{\mathcal{Z}(D)} \qquad \text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence}}$$

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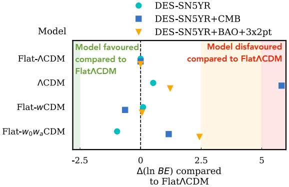

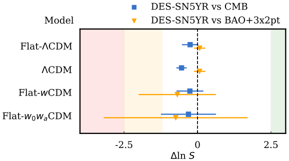

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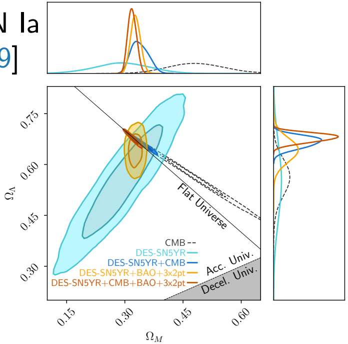

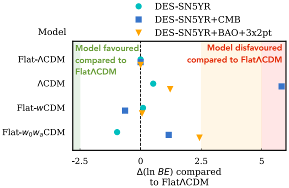

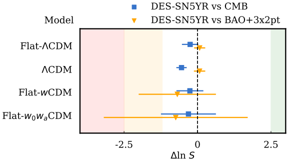

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$$\mathcal{P} \times \mathcal{Z} = \mathcal{J} = \mathcal{L} \times \pi, \qquad \text{Joint} = \mathcal{J} = P(\theta, D)$$

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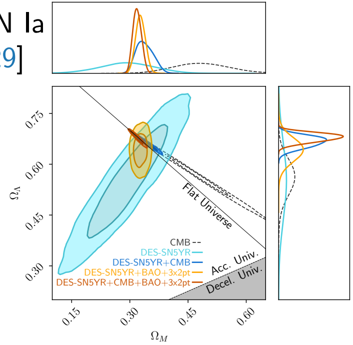

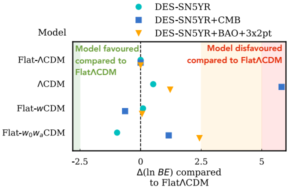

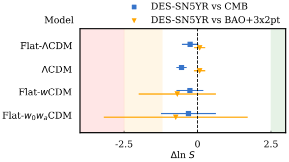

The three pillars of Bayesian inference

Parameter estimation

What do the data tell us about the parameters of a model? *e.g. the size or age of a ΛCDM universe*

$$P(\theta|D,M) = \frac{P(D|\theta,M)P(\theta|M)}{P(D|M)}$$

$$\mathcal{P} = \frac{\mathcal{L}\times\pi}{\mathcal{Z}}$$

$$\text{Posterior} = \frac{\text{Likelihood}\times\text{Prior}}{\text{Evidence}}$$

Model comparison

How much does the data support a particular model? *e.g. ΛCDM vs a dynamic dark energy cosmology*

$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

$$\frac{\mathcal{Z}_M\Pi_M}{\sum_m Z_m\Pi_m}$$

$$\text{Posterior} = \frac{\text{Evidence}\times\text{Prior}}{\text{Normalisation}}$$

Tension quantification

Do different datasets make consistent predictions from the same model? *e.g. CMB vs Type IA supernovae data*

$$\mathcal{R} = \frac{\mathcal{Z}_{AB}}{\mathcal{Z}_A\mathcal{Z}_B}$$

$$\log\mathcal{S} = \langle\log\mathcal{L}_{AB}\rangle_{\mathcal{P}_{AB}} - \langle\log\mathcal{L}_A\rangle_{\mathcal{P}_A} - \langle\log\mathcal{L}_B\rangle_{\mathcal{P}_B}$$

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What do the data tell us about the parameters of a model? *e.g. the masses and spins of a BBH collision*

$$P(\theta|D,M) = \frac{P(D|\theta,M)P(\theta|M)}{P(D|M)}$$

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How much does the data support a particular model? *e.g. IMRPhenom vs EOBNR waveform models*

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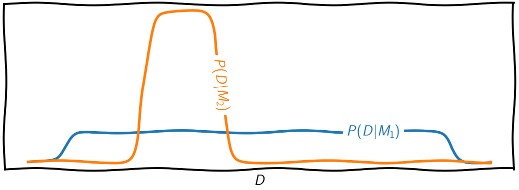
Model comparison $\mathcal{Z} = P(D|M)$

- Bayesian model comparison allows mathematical derivation of key philosophical principles.

Viewed from data-space D:

Popper's falsificationism

- Prefer models that make bold predictions.
- if proven true, model more likely correct.



- Falsificationism comes from normalisation

Viewed from parameter-space θ:

Occam's razor

- Models should be as simple as possible
- ... but no simpler

- Occam's razor equation:

$$\log \mathcal{Z} = \langle \log \mathcal{L} \rangle_{\mathcal{P}} - \mathcal{D}_{\mathrm{KL}}$$

- "Occam penalty": KL divergence between prior π and posterior $\mathcal{P}$.

$$\mathcal{D}_{\mathrm{KL}} \sim \log \frac{\text{Prior volume}}{\text{Posterior volume}}$$

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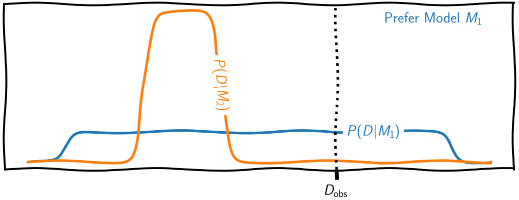

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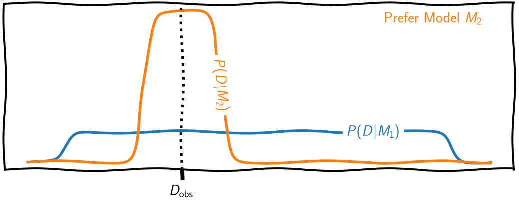

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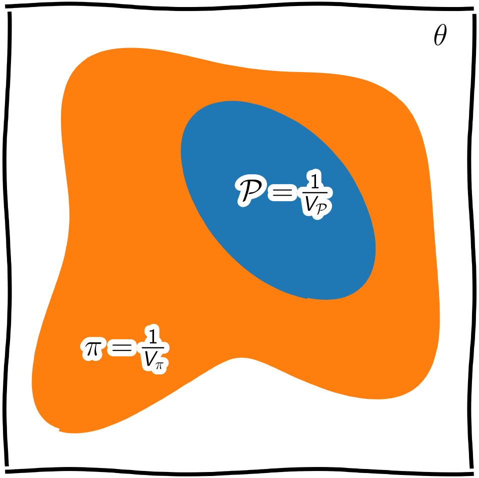
Why do sampling?

- The cornerstone of numerical Bayesian inference is working with **samples**.
- Generate a set of representative parameters drawn in proportion to the posterior $\theta \sim \mathcal{P}$.
- The magic of marginalisation $\Rightarrow$ perform usual analysis on each sample in turn.
- The golden rule is **stay in samples** until the last moment before computing summary statistics/triangle plots because

$$\boxed{f(\langle X \rangle) \ne \langle f(X) \rangle}$$

- Generally need $\sim \mathcal{O}(12)$ independent samples to compute a value and error bar.



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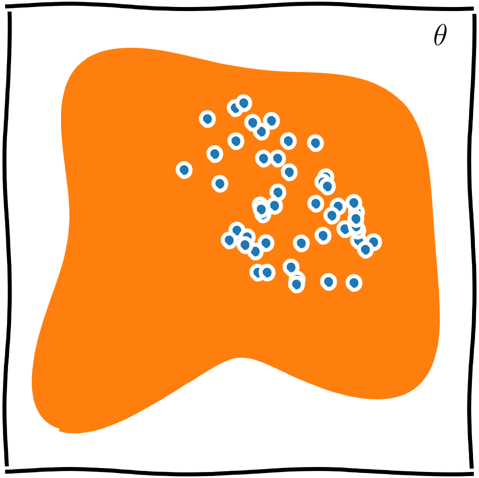
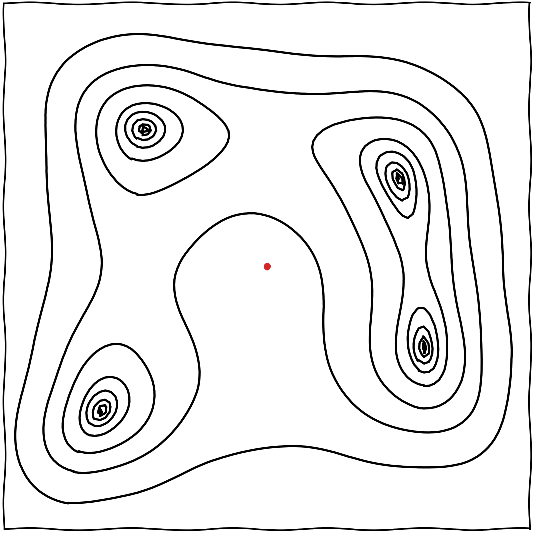


MCMC



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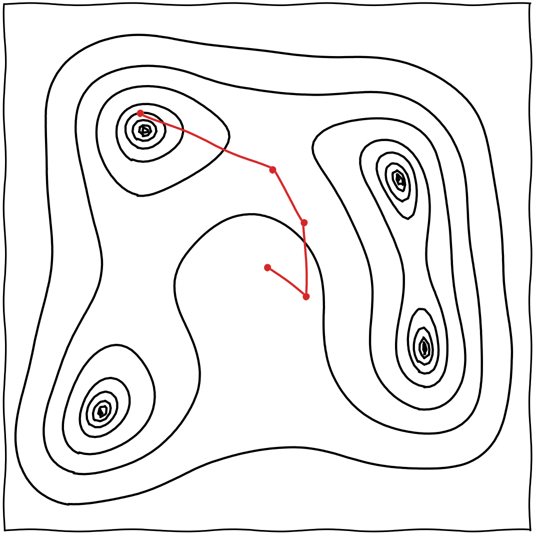

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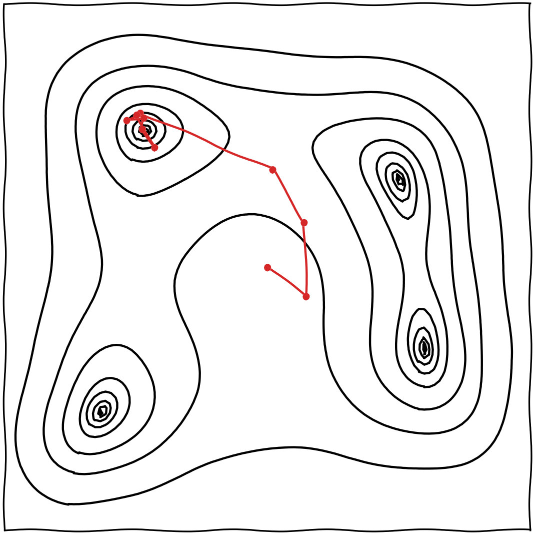

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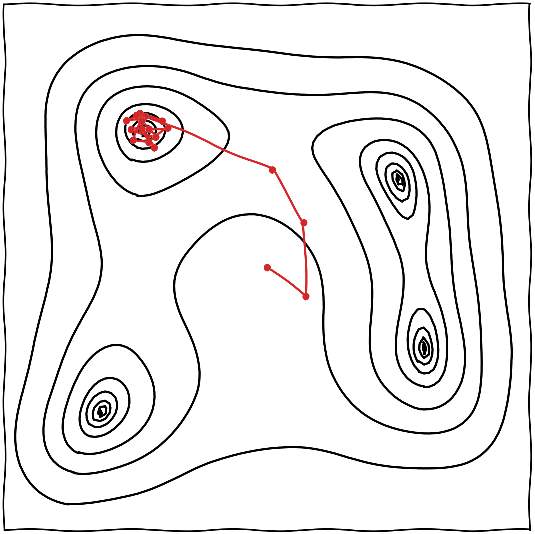

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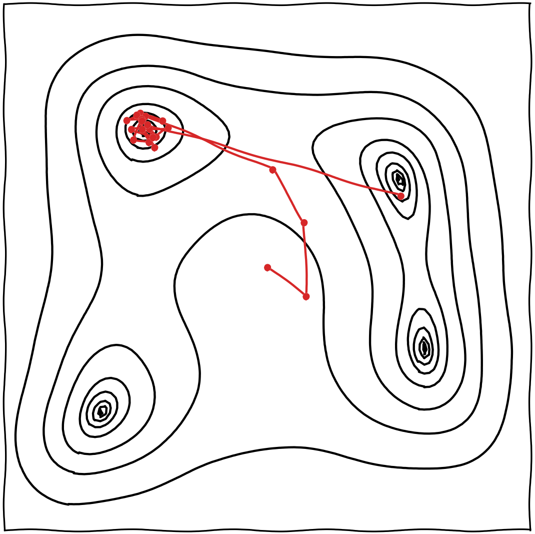

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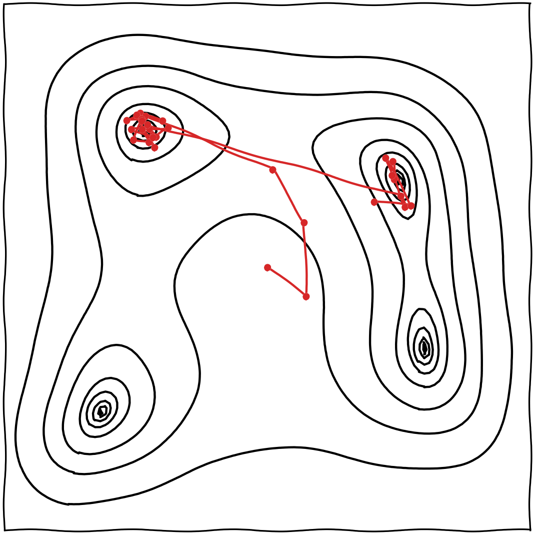

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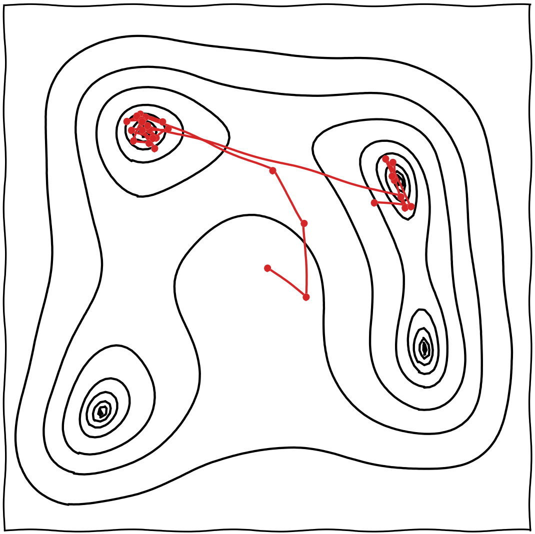

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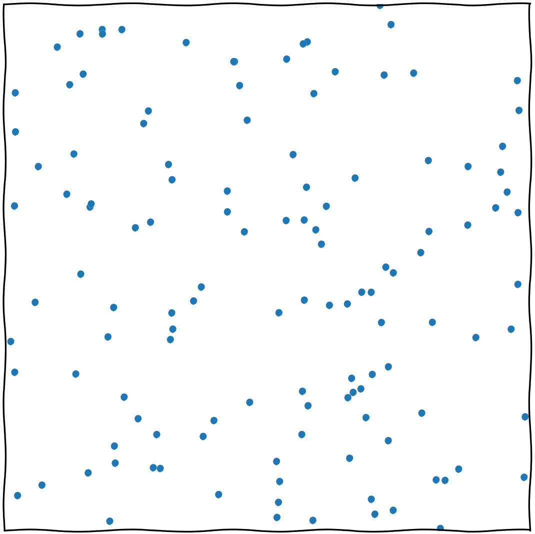

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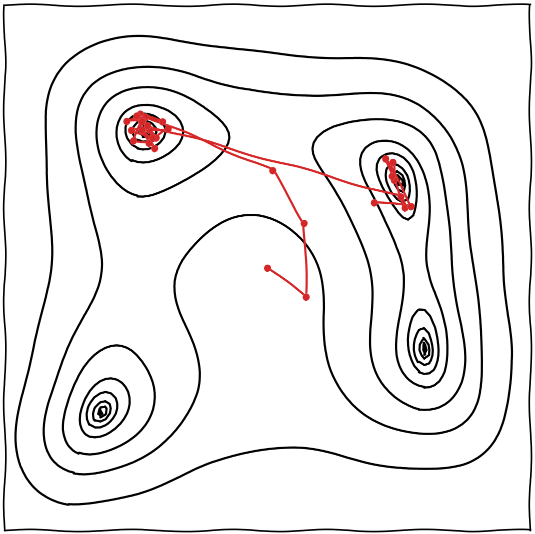

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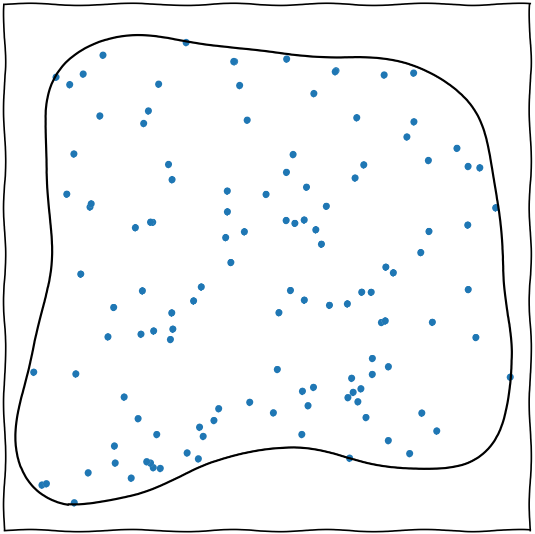

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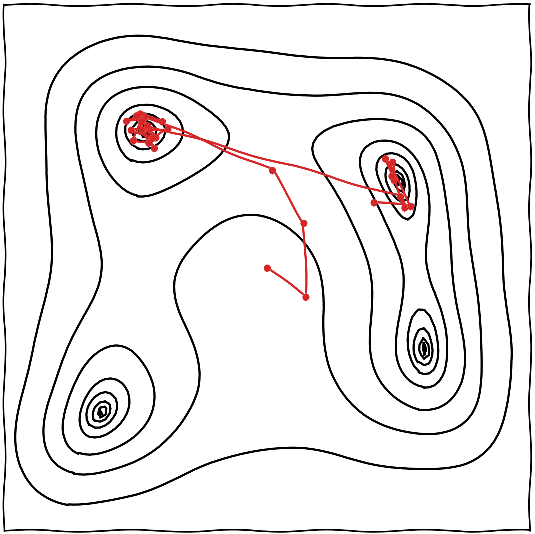

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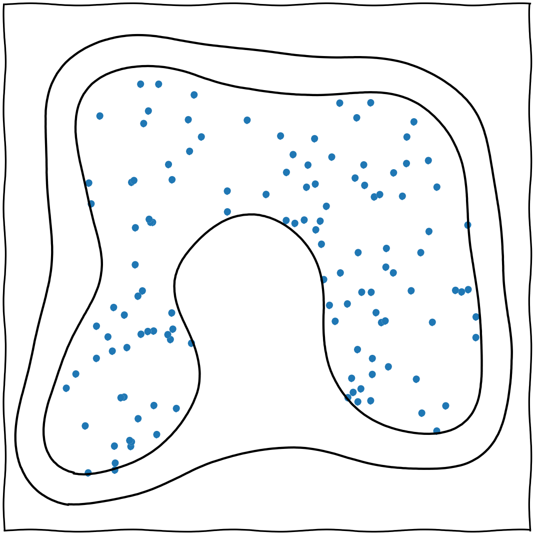

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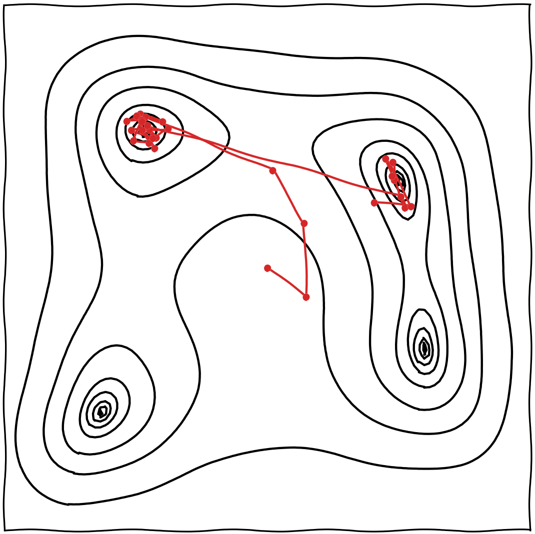

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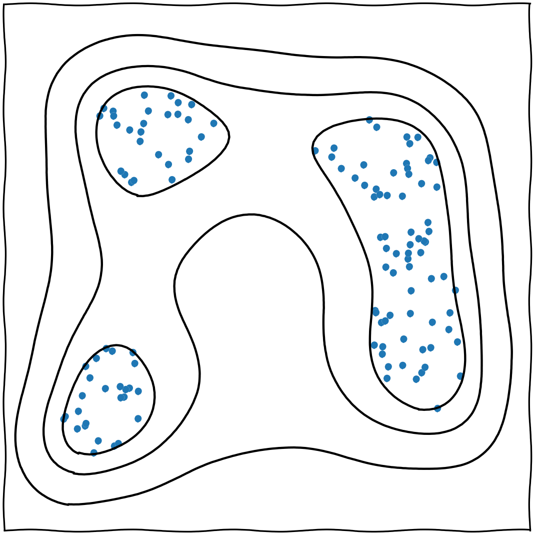

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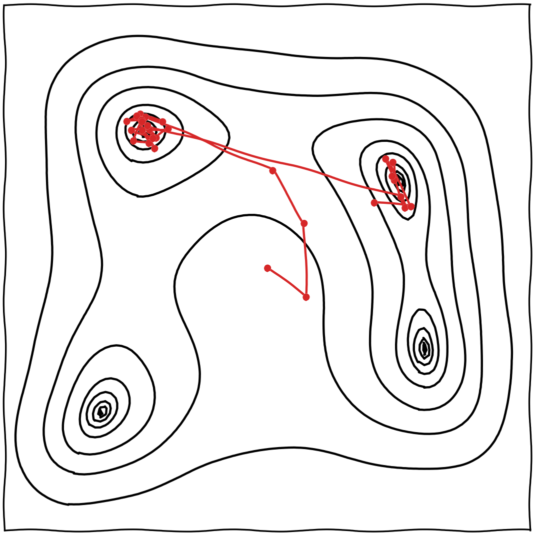

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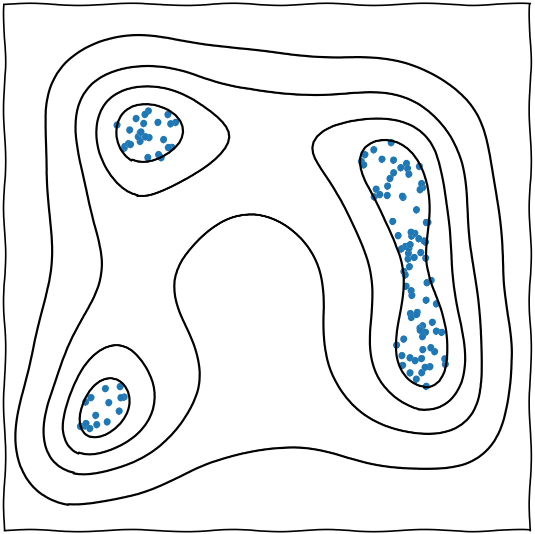

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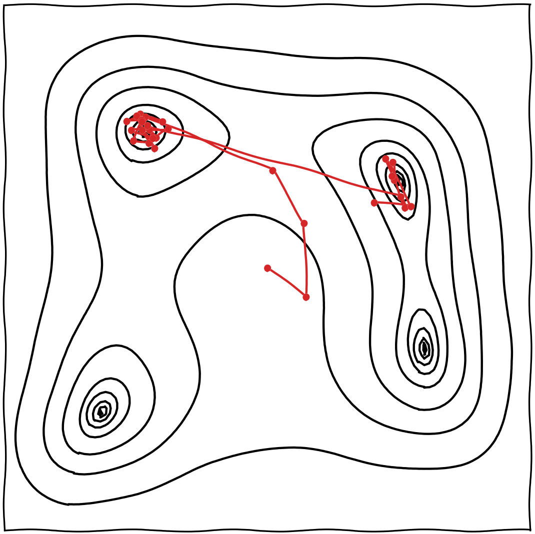

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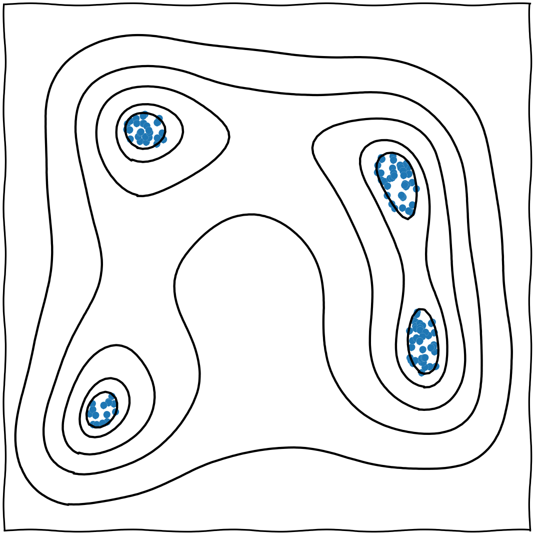

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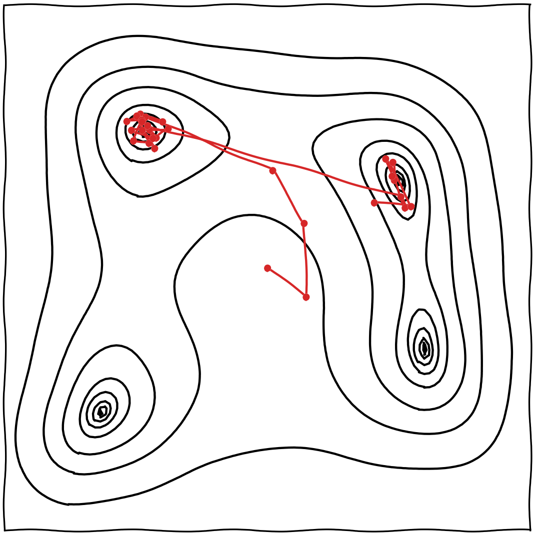

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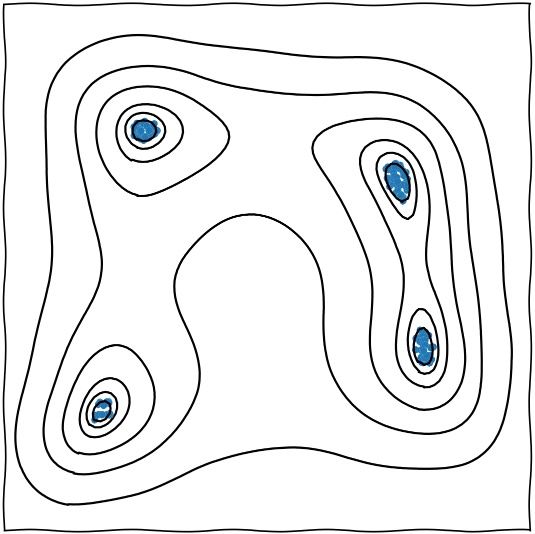

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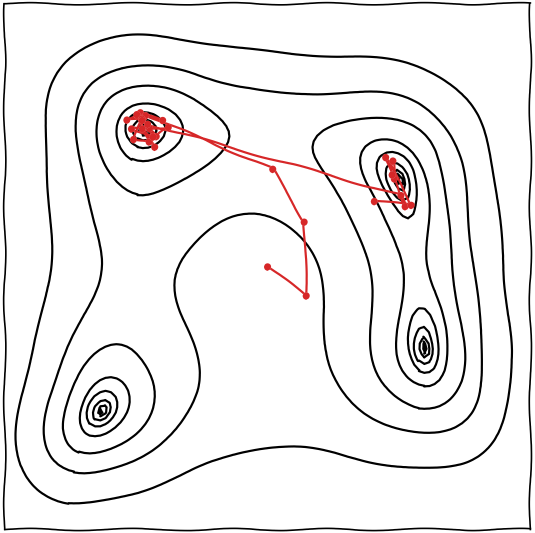

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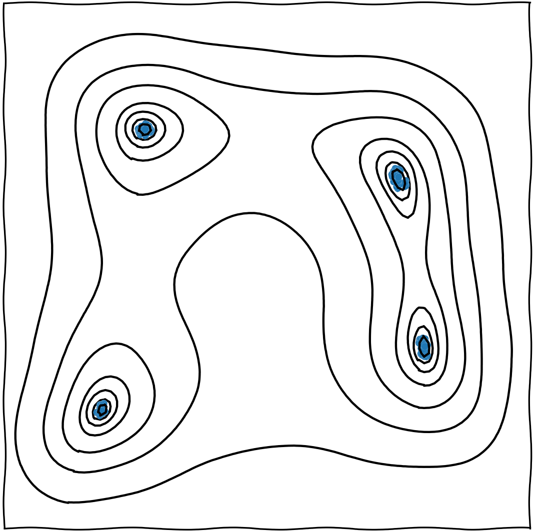

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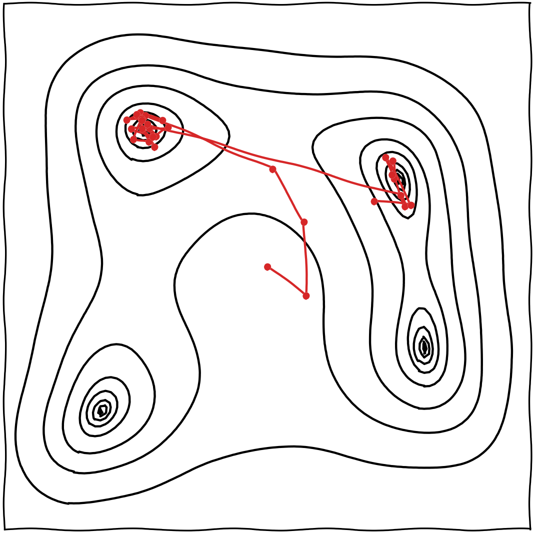

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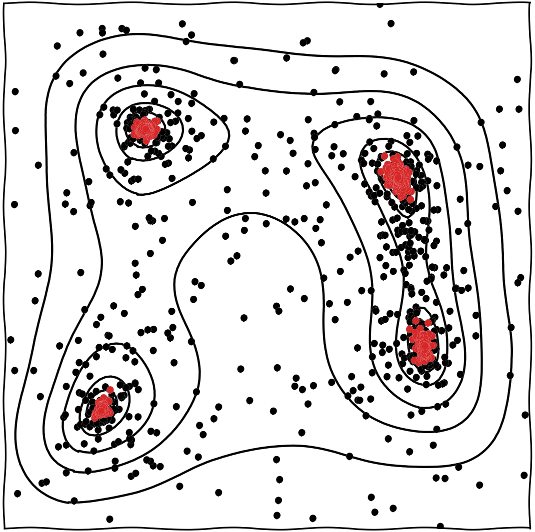
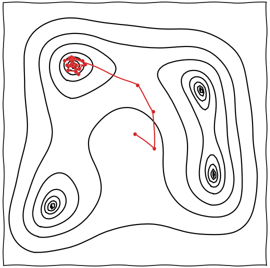


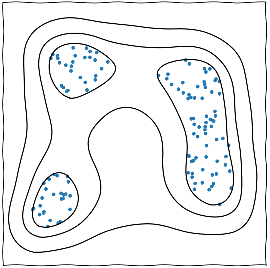
MCMC

- Single "walker"
- Explores posterior
- Fast, if proposal matrix is tuned
- Parameter estimation, suspiciousness calculation
- Channel capacity optimised for generating posterior samples



Nested sampling

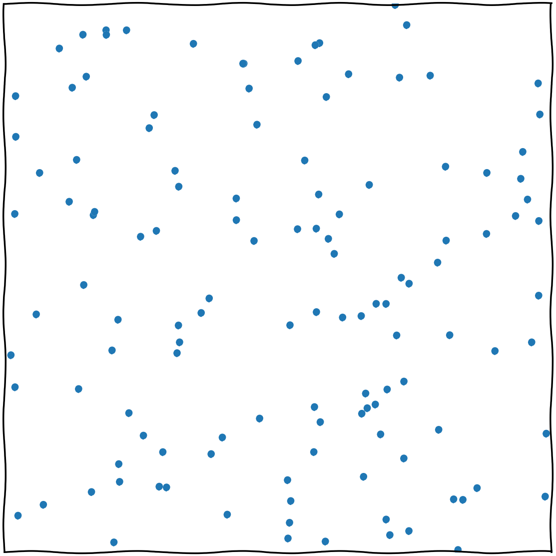
- Ensemble of "live points"
- Scans from prior to peak of likelihood
- Slower, no tuning required
- Parameter estimation, model comparison, tension quantification
- Channel capacity optimised for computing partition function



The nested sampling meta-algorithm: live points

- Start with n random samples over the space.
- Delete outermost sample, and replace with a new random one at higher integrand value.
- The "live points" steadily contract around the peak(s) of the function.
- We can use this evolution to estimate volume *probabilistically*.
- At each iteration, the contours contract by $\sim \frac{1}{n}$ of their volume.
- This is an exponential contraction, so

$$\int f(x)dV \approx \sum_i f(x_i)\Delta V_i, \qquad V_i = V_0 e^{-i/n}$$



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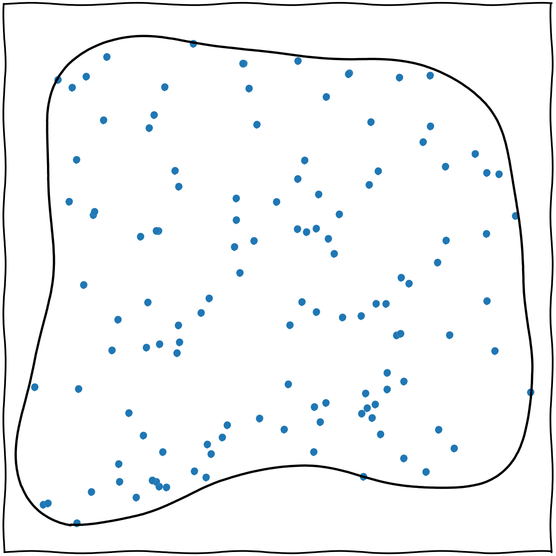

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- Start with n random samples over the space.
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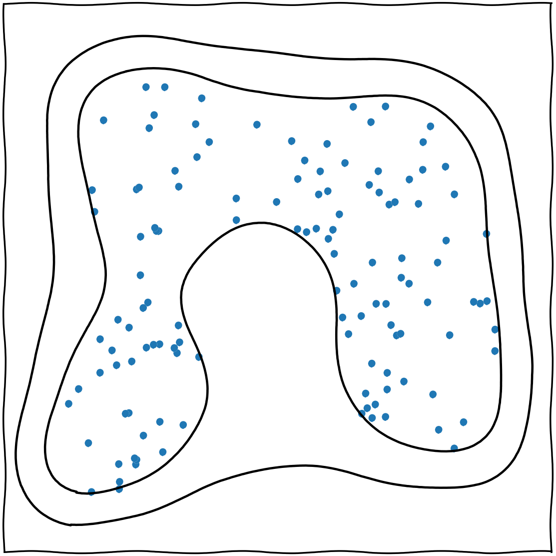

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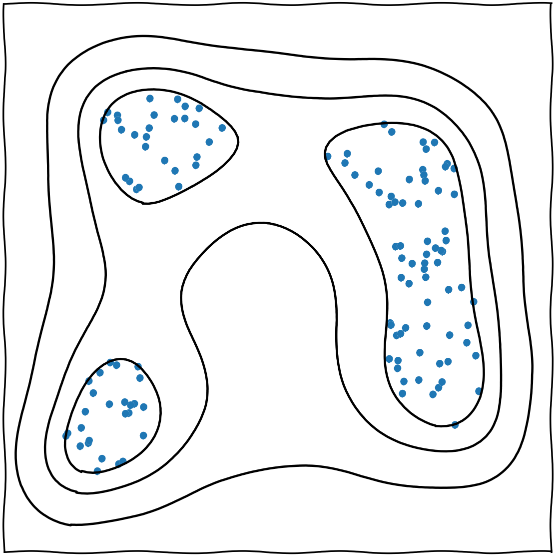

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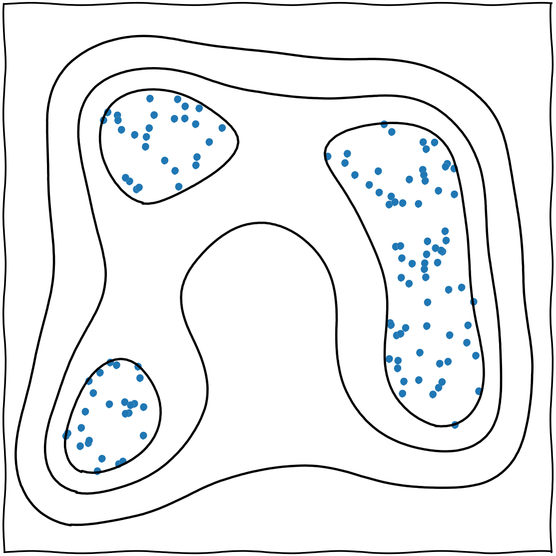
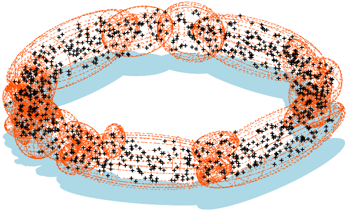
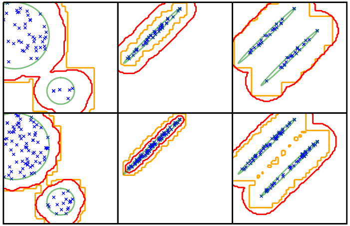


Implementations of Nested Sampling [2205.15570](NatReview)

MultiNest [0809.3437]

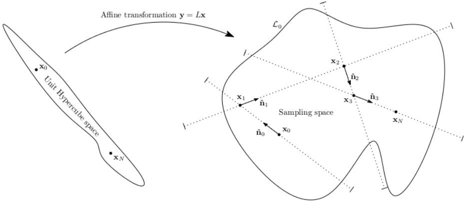


UltraNest [2101.09604]

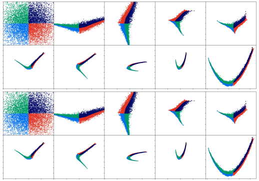


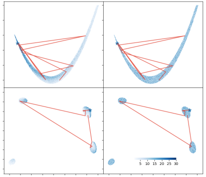
nautilus [2306.16923]

PolyChord [1506.00171]



NeuralNest [1903.10860]



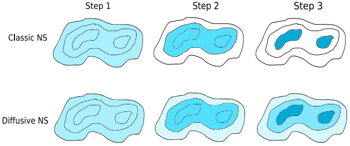


nessai [2102.11056] cpnest

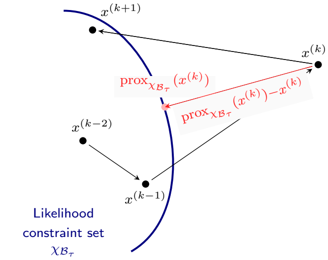
nora [2305.19267]

jaxns [2012.15286]

DNest [1606.03757]



ProxNest [2106.03646]



dynesty [1904.02180]

Types of nested sampler

- Broadly, most nested samplers can be split into how they create new live points.
- i.e. how they sample from the hard likelihood constraint $\{\theta \sim \pi : \mathcal{L}(\theta) > \mathcal{L}_*\}$.

Rejection samplers

- e.g. `MultiNest`, `UltraNest`.
- Constructs bounding region and draws many invalid points until $\mathcal{L}(\theta) > \mathcal{L}_*$.
- Efficient in low dimensions, exponentially inefficient $\sim \mathcal{O}(e^{d/d_0})$ in high $d > d_0 \sim 10$.

Chain-based samplers

- e.g. `PolyChord`, `ProxNest`.
- Run Markov chain starting at a live point, generating many valid (correlated) points.
- Linear $\sim \mathcal{O}(d)$ penalty in decorrelating new live point from the original seed point.

- Nested samplers usually come with:
  - *resolution* parameter n_{live} (which improve results as $\sim \mathcal{O}(n_{\text{live}}^{-1/2})$.
  - set of *reliability* parameters [2101.04525], which don't improve results if set arbitrarily high, but introduce systematic errors if set too low.
  - e.g. `Multinest` efficiency `eff` or `PolyChord` chain length n_{repeats}.

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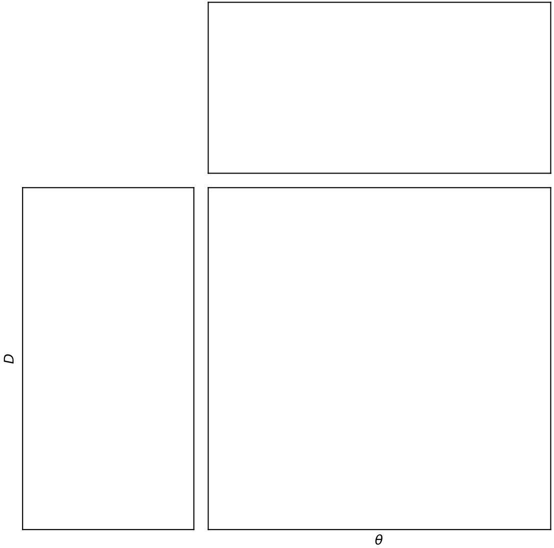

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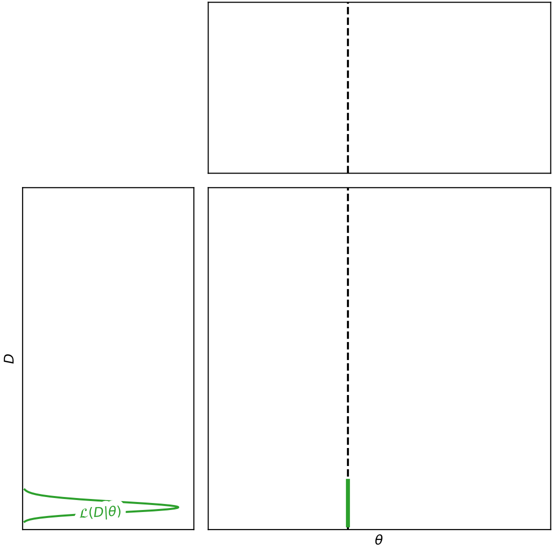

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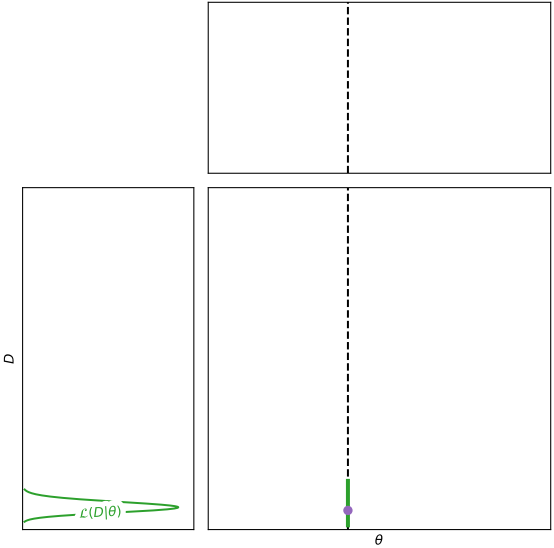

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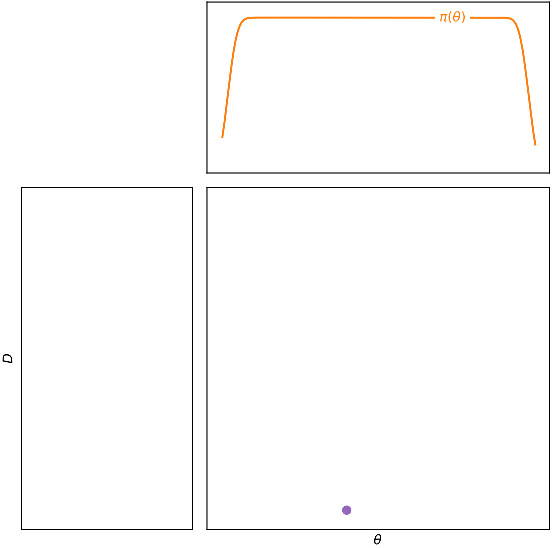

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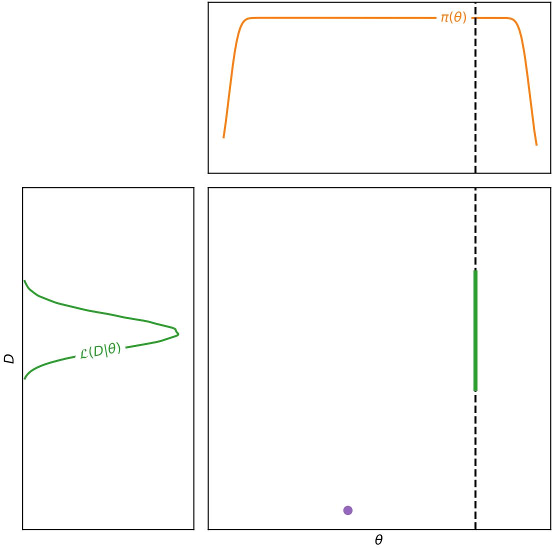

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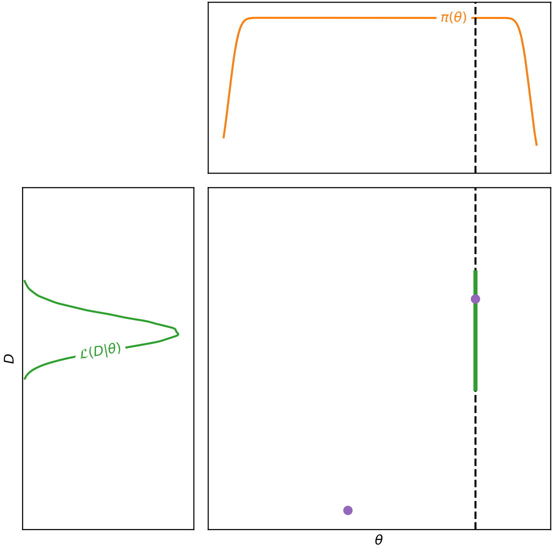

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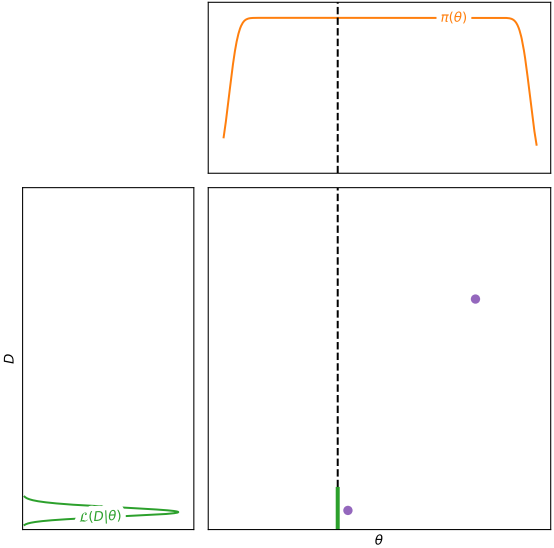

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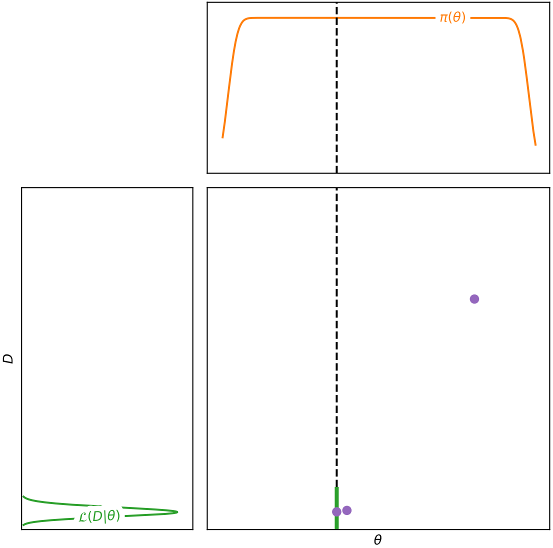

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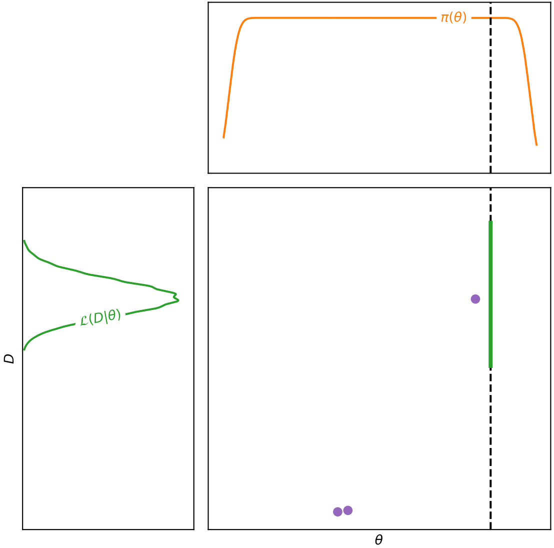

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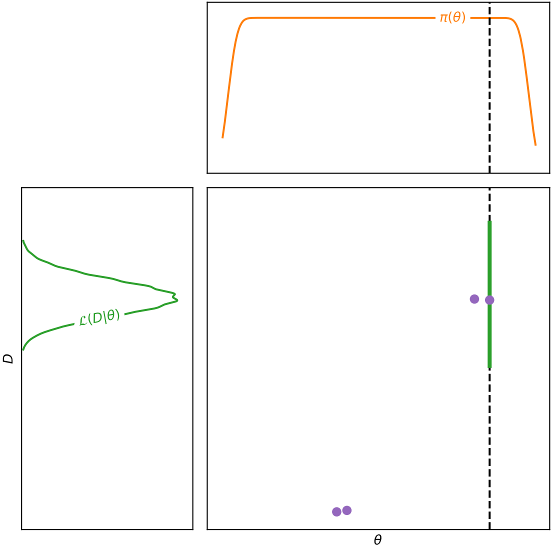

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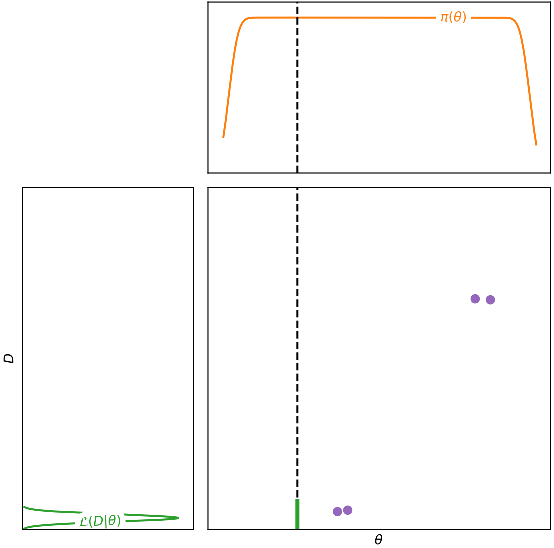

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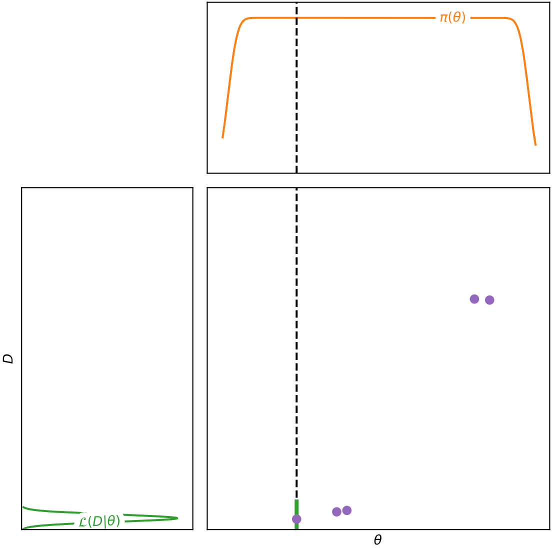

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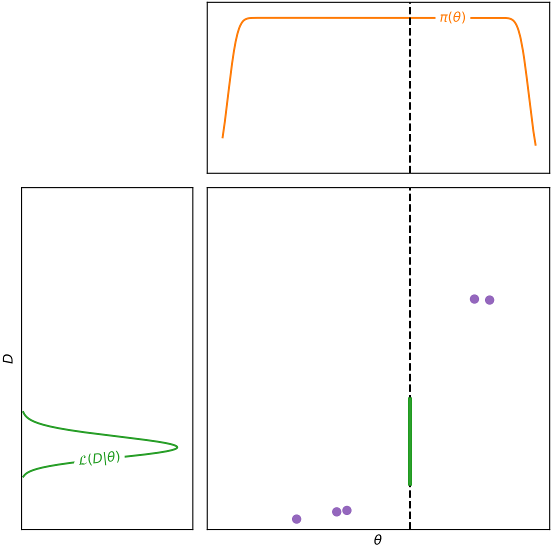

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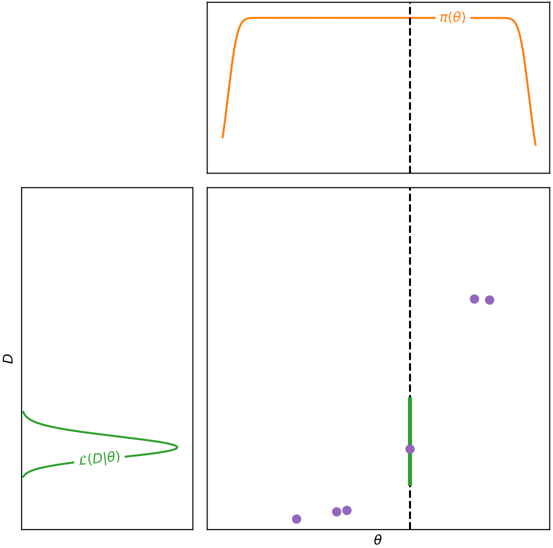

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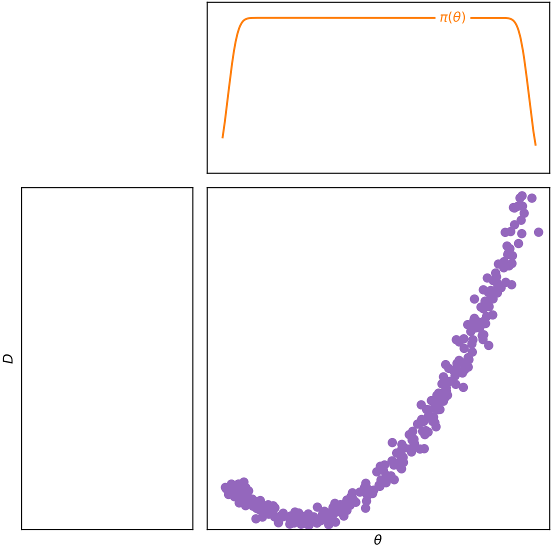

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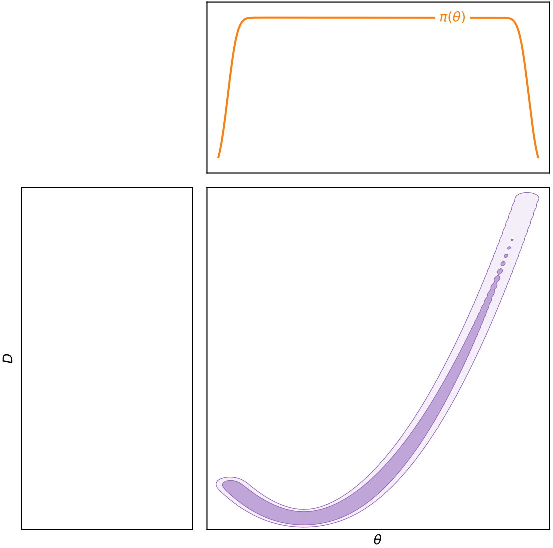

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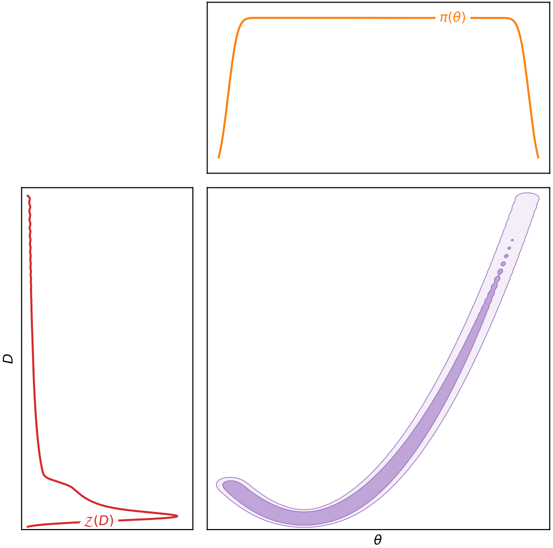

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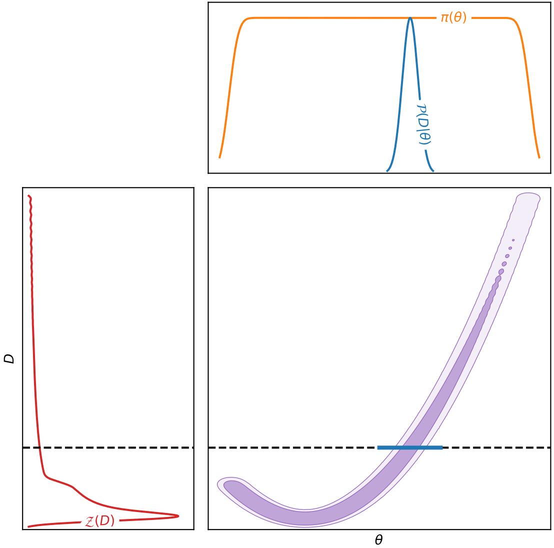

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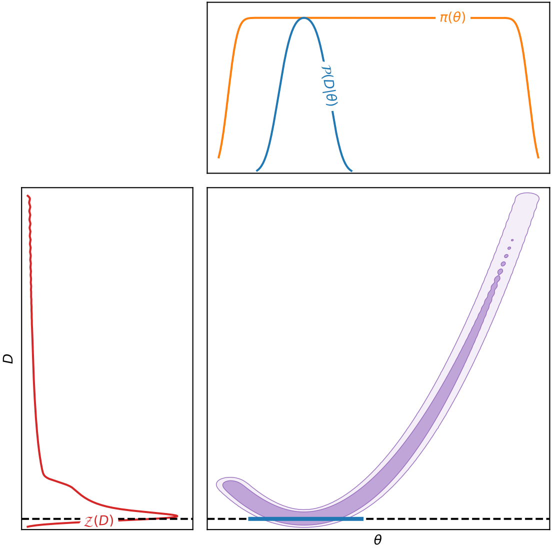
- With a prior $\pi(\theta)$ can generate samples from joint distribution $\mathcal{J}(\theta, D) = \mathcal{L}(D|\theta)\pi(\theta)$ *the "probability of everything"*.
- Task of SBI is take joint $\mathcal{J}$ samples and learn posterior $\mathcal{P}(\theta|D)$ and evidence $\mathcal{Z}(D)$ and possibly likelihood $\mathcal{L}(D|\theta)$.
- Present state of the art achieves this using *machine learning* (neural networks).
  - My group's research tries to removes machine learning `github.com/handley-lab/lsbi`.



SBI: Simulation-based inference

- What do you do if you don't know $\mathcal{L}(D|\theta)$?
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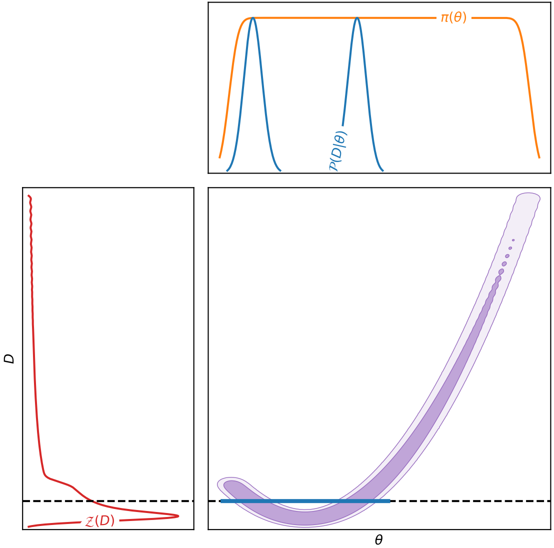

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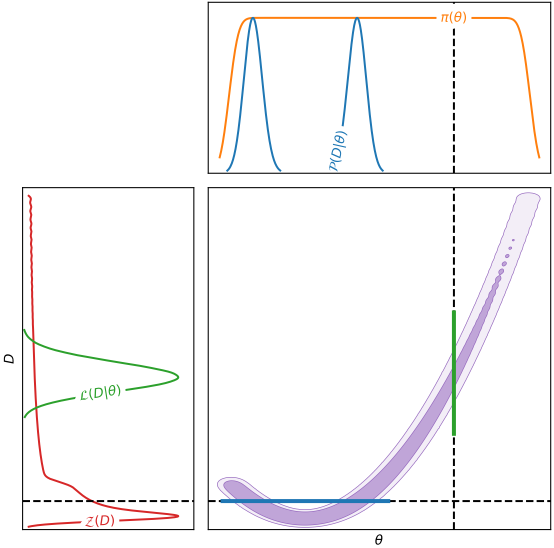
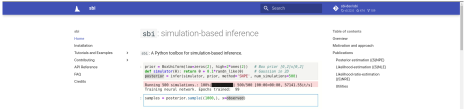


Why SBI?

SBI is useful because:

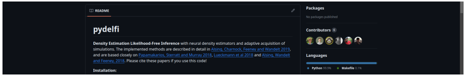
1. If you don't have a likelihood, you can still do inference
   - This is the usual case beyond CMB cosmology
2. Faster than LBI
   - emulation – also applies to LBI in principle
3. No need to pragmatically encode fiducial cosmologies
   - Covariance computation implicitly encoded in simulations
   - Highly relevant for disentangling tensions & systematics
4. Equips AI/ML with Bayesian interpretability
5. Lower barrier to entry than LBI
   - Much easier to forward model a systematic
   - Emerging set of plug-and-play packages
   - For this reason alone, it will come to dominate scientific inference



github.com/sbi-dev

github.com/undark-lab/swyft

github.com/florent-leclercq/pyselfi



github.com/justinalsing/pydelfi

SBI in astrophysics

- 2024 has been the year it has started to be applied to real data.
- Mostly for weak lensing
- However: SBI requires mock data generation code
- Most data analysis codes were built before the generative paradigm.
- It's still a lot of work to upgrade cosmological likelihoods to be able to do this (e.g. `plik` & `camspec`).

Investigating the turbulent hot gas in X-COP galaxy clusters

S. Dupourqué[1], N. Clerc[1], E. Pointecouteau[1], D. Eckert[2], S. Ettori[3], and F. Vazza[4,5,6]

Dark Energy Survey Year 3 results: simulation-based cosmological inference with wavelet harmonics, scattering transforms, and moments of weak lensing mass maps II. Cosmological results

M. Gatti,[1,*] G. Campailla,[2] N. Jeffrey,[3] L. Whiteway,[3] A. Porredon,[4] J. Prat,[5] J. Williamson,[3] M. Raveri,[2] B.

Neural Posterior Estimation with guaranteed exact coverage: the ringdown of GW150914

Marco Crisostomi[1,2], Kallol Dey[3], Enrico Barausse[1,2], Roberto Trotta[1,2,4,5]

Applying Simulation-Based Inference to Spectral and Spatial Information from the Galactic Center Gamma-Ray Excess

Katharena Christy,[a] Eric J. Baxter,[b] Jason Kumar[a]

KiDS-1000 and DES-Y1 combined: Cosmology from peak count statistics

Joachim Harnois-Déraps[1★], Sven Heydenreich[2], Benjamin Giblin[3], Nicolas Martinet[4], Tilman Tröster[5], Marika Asgari[1,6,7], Pierre Burger[8,9,10],Tiago Castro[11,12,13,14], Klaus Dolag[15], Catherine Heymans[3,16], Hendrik Hildebrandt[16], Benjamin Joachimi[17] & Angus H. Wright[16]

KiDS-SBI: Simulation-Based Inference Analysis of KiDS-1000 Cosmic Shear

Maximilian von Wietersheim-Kramsta[1,2,3], Kiyam Lin[1], Nicolas Tessore[1], Benjamin Joachimi[1], Arthur Loureiro[4,5], Robert Reischke[6,7], and Angus H. Wright[7]

Simulation-based inference of deep fields: galaxy population model and redshift distributions

Beatrice Moser,[a,1] Tomasz Kacprzak,[a,b] Silvan Fischbacher,[a] Alexandre Refregier,[a] Dominic Grimm,[a] Luca Tortorelli[c]

SimBIG: Cosmological Constraints using Simulation-Based Inference of Galaxy Clustering with Marked Power Spectra

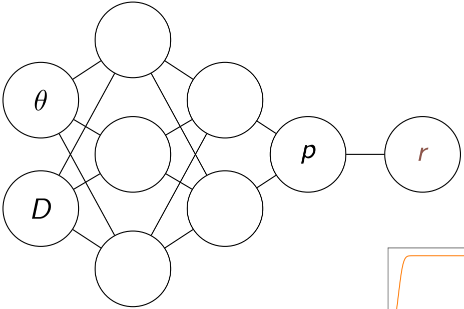
Elena Massara,[1,2,*] ChangHoon Hahn,[3] Michael Eickenberg,[4] Shirley Ho,[5] Jiamin Hou,[6,7] Pablo Lemos,[8,9,5] Chirag Modi,[4,5] Azadeh Moradinezhad Dizgah,[10,11] Liam Parker,[5,12] and Bruno Régaldo-Saint Blancard[4]

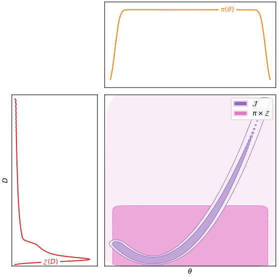
Neural Ratio Estimation

- SBI flavours: github.com/sbi-dev/sbi

NPE	Neural posterior estimation
NLE	Neural likelihood estimation
NJE	Neural joint estimation
NRE	Neural ratio estimation

- NRE recap:
  1. Generate joint samples $(\theta, D) \sim \mathcal{J}$
     - *straightforward if you have a simulator:* $\theta \sim \pi(\cdot)$, $D \sim \mathcal{L}(\cdot|\theta)$
  2. Generate separated samples $\theta \sim \pi$, $D \sim \mathcal{Z}$
     - *aside: can shortcut step 2 by scrambling the* (θ, D) *pairings from step 1*
  3. Train probabilistic classifier p to distinguish whether (θ, D) came from $\mathcal{J}$ or $\pi \times \mathcal{Z}$.
  4. $\frac{p}{1-p} = r = \frac{P(\theta,D)}{P(\theta)P(D)} = \frac{\mathcal{J}}{\pi\times\mathcal{Z}} = \frac{\mathcal{L}}{\mathcal{Z}} = \frac{\mathcal{P}}{\pi}$.
  5. Use ratio r for parameter estimation $\mathcal{P} = r \times \pi$





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Bayesian proof

- Let $M_\mathcal{J}$: $(\theta, D) \sim \mathcal{J}$, $M_{\pi\mathcal{Z}}$: $(\theta, D) \sim \pi \times \mathcal{Z}$
- Classifier gives
  $p(\theta, D) = P(M_\mathcal{J}|\theta, D) = 1 - P(M_{\pi\mathcal{Z}}|\theta, D)$
- Bayes theorem then shows
  $\frac{p}{1-p} = \frac{P(M_\mathcal{J}|\theta,D)}{P(M_{\pi\mathcal{Z}}|\theta,D)} = \frac{P(\theta,D|M_\mathcal{J})P(M_\mathcal{J})}{P(\theta,D|M_{\pi\mathcal{Z}})P(M_{\pi\mathcal{Z}})} = \frac{\mathcal{J}}{\pi\mathcal{Z}}$,
  where we have assumed
  - $P(M_\mathcal{J}) = P(M_{\pi\mathcal{Z}})$,

  and by definition
  - $\mathcal{J}(\theta, D) = P(\theta, D|M_\mathcal{J})$
  - $\pi(\theta)\mathcal{Z}(D) = P(\theta, D|M_{\pi\mathcal{Z}})$.

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Why I like NRE

- The link between classification and inference is profound.
- Density estimation is hard – Dimensionless r divides out the hard-to-calculate parts.

Why I don't like NRE

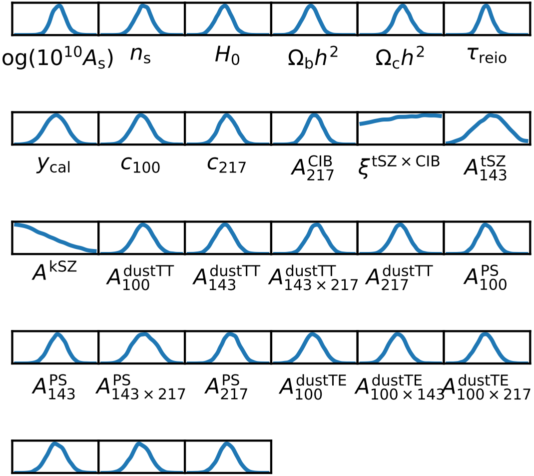
- Practical implementations require marginalisation [2107.01214], or autoregression [2308.08597].
- Model comparison and parameter estimation are separate [2305.11241].

Marginal inference

- Many cosmological likelihoods come with nuisance parameters that have limited relevance for onward inference.
- Notation: CMB cosmology

$\mathcal{L}$	Likelihood	(e.g. `plik`),
D	Data	(e.g. CMB),
θ	Cosmological parameters	(e.g. Ω_m, $H_0\ldots$),
α	Nuisance parameters	(e.g. $A_{\mathrm{planck}}\ldots$),
M	Model	(e.g. ΛCDM).

- Some marginal statistics (e.g. marginal means, posteriors. . .) are easy to compute.
- More machinery is needed for e.g. nuisance marginalised likelihoods and marginal KL divergences $\mathcal{D}_{\mathrm{KL}}$.



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- Notation: GW cosmology

$\mathcal{L}$	Likelihood	(e.g. LAL),
D	Data	(e.g. GW170817),
θ	Cosmological parameters	(e.g. , $H_0 \ldots$),
α	Nuisance parameters	(e.g. m_1, $m_2 \ldots$),
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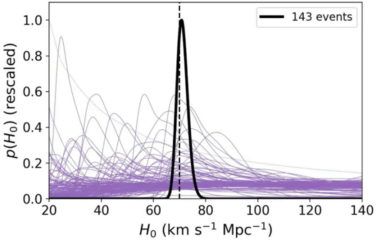

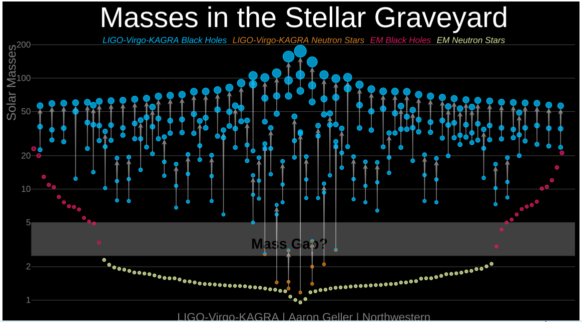

Nuisance marginalised likelihoods: Theory [2207.11457]

- Bayes theorem

$$\mathcal{L}(\theta,\alpha) \times \pi(\theta,\alpha) = \mathcal{P}(\theta,\alpha) \times \mathcal{Z} \qquad (1)$$

Likelihood × Prior = Posterior × Evidence

α: nuisance parameters, θ: cosmo parameters.

- Marginal Bayes theorem

$$\mathcal{L}(\theta) \times \pi(\theta) = \mathcal{P}(\theta) \times \mathcal{Z} \qquad (2)$$

- Non-trivially gives **nuisance-free likelihood**

$$\boxed{\mathcal{L}(\theta) = \frac{\mathcal{P}(\theta)\mathcal{Z}}{\pi(\theta)}} = \frac{\int \mathcal{L}(\theta,\alpha)\pi(\theta,\alpha)d\alpha}{\int \pi(\theta,\alpha)d\alpha} \qquad (3)$$

Key properties

- Given datasets A and B, each with own nuisance parameters α_A and α_B:
- If you use $\mathcal{L}_A(\theta)$, you get the same (marginal) posterior and evidence if you had run with nuisance parameters α_A (ditto B).
- If you run inference on $\mathcal{L}_A(\theta) \times \mathcal{L}_B(\theta)$, you get the same (marginal) posterior and evidence if you had run with all nuisance parameters α_A, α_B on.

  (weak marginal consistency requirements on joint $\pi(\theta,\alpha_A,\alpha_B)$ and marginal priors)

Harry Bevins

PhD→JRF

$$\boxed{\mathcal{L}(\theta) = \frac{\mathcal{P}(\theta)\mathcal{Z}}{\pi(\theta)}}$$

- To compute the nuisance marginalised likelihood, need:
  1. Bayesian evidence $\mathcal{Z}$
  2. Marginal prior and posterior **densities**

$\boxed{\mathcal{L}(\theta,\alpha)}$ $\boxed{\pi(\theta,\alpha)}$

1. Bayesian evidence $\mathcal{Z}$: g
   - Nested sampling
   - Parallel tempering (pocomc, ptmcmc)
   - Sequential Monte Carlo (SMC)
   - MCEvidence
2. Marginal prior $\pi(\theta)$ and posterior $\mathcal{P}(\theta)$ densities:
   - Histograms of samples
   - Kernel density estimation
   - Normalising flows / Diffusion models
   - ...

- Emulators usually much faster than original likelihoods
- `margarine`: PyPI, github.com/htjb/margarine

Nuisance marginalised likelihoods: Practice [2205.12841]

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↓ ↓

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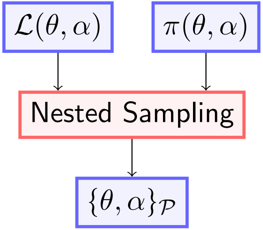

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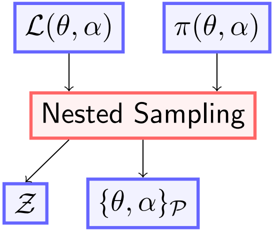

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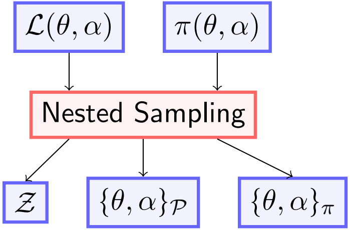

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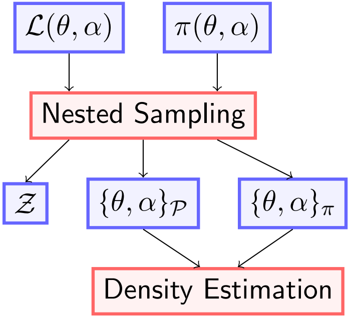

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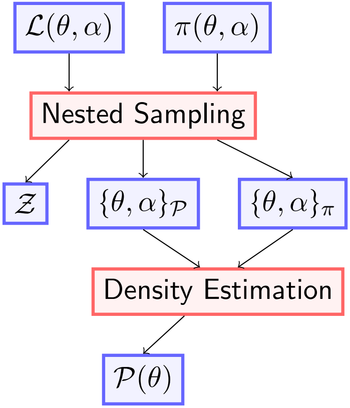

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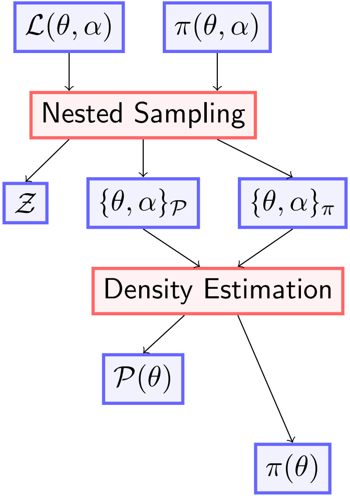

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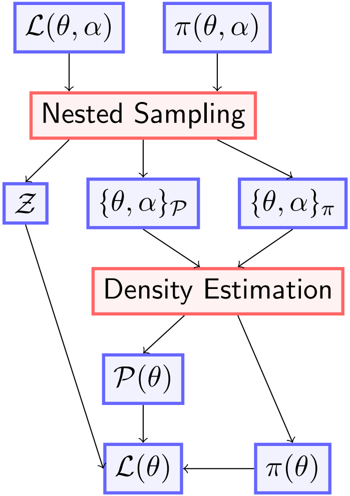

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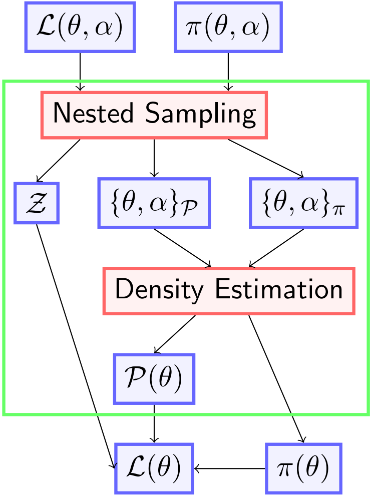

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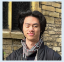
Nuisance marginalised likelihoods: Example uses

Ming Yang
MSci



- Library of pre-trained bijectors to be used as priors/emulators/nuisance marginalised likelihoods (DiRAC allocation `unimpeded`)
- e.g. easy to apply a *Planck*/DES/HERA/JWST prior or likelihood to your existing MCMC chains without needing to install the whole cosmology machinery.
- Hierarchical modelling:
  - Usually, have N objects, each with nuisance parameters α_i, and shared parameters of interest θ.
  - Likelihood $\mathcal{L}(\{D_i\}|\theta,\{\alpha_i\}) = \prod_i^N \mathcal{L}_i(D_i|\theta,\alpha_i)$ has $N\times \texttt{len}(\alpha_i)+\texttt{len}(\theta)$ parameters
  - Instead, break problem down into N runs on $\texttt{len}(\theta)+\texttt{len}(\alpha_i)$ parameters, and one final one on $\texttt{len}(\theta)$ parameters, using nuisance marginal likelihoods $\mathcal{L}_i(\theta)$.
  - In addition to computational tractability, also can perform model comparison with nuisance marginalised likelihoods.

The scaling frontier of nested sampling

Zixiao Hu

MSci 2023

How fast in nested sampling?

$$\boxed{T = T_{\mathcal{L}} \times n_\text{live} \times \mathcal{D}_\text{KL} \times f_\text{sampler}}$$

How accurate is nested sampling?

$$\boxed{\sigma(\log \mathcal{Z}) \approx \sqrt{\mathcal{D}_\text{KL}/n_\text{live}}}$$

in d dimensional parameter space:

- $T_{\mathcal{L}}$: likelihood eval time $\sim \mathcal{O}(d)$
- n_live: number of live points $\sim \mathcal{O}(d)$
- $\mathcal{D}_\text{KL}$: KL divergence from prior to posterior $\approx \log V_\pi/V_\mathcal{P}$ $\sim \mathcal{O}(d)$
- f_sampler: efficiency of point generation region$\sim \mathcal{O}(e^{d/d_0})$ or path$\sim \mathcal{O}(d)$

- Algorithmically improving f_sampler is only a fraction of the story!
- $\mathcal{D}_\text{KL}$ appears twice, so improvements here are quadratically important.
- Gradients give you d more information.

- $T \sim \mathcal{O}(d^4)$ whilst polynomial is far from ideal, athough progress can be made on all fronts.

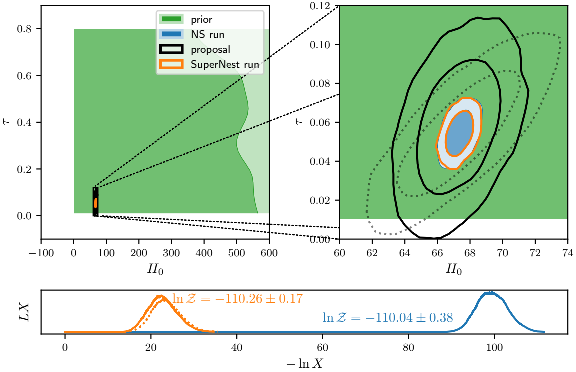
Accelerated nested sampling with beta flows

- For constant "run quality" σ,

$$T = T_{\mathcal{L}} \times n_{\mathrm{live}} \times \mathcal{D}_{\mathrm{KL}} \times f_{\mathrm{sampler}}, \qquad \sigma \approx \sqrt{\mathcal{D}_{\mathrm{KL}}/n_{\mathrm{live}}}$$

$$\Rightarrow \boxed{T = T_{\mathcal{L}} \times \sigma \times \mathcal{D}_{\mathrm{KL}}^2 \times f_{\mathrm{sampler}}}$$

  so if you can reduce the KL divergence, then quadratic gains to be made
- This can be crudely achieved by choosing a narrower prior π^* and then correcting the evidence $\mathcal{Z} = \mathcal{Z}^* \frac{V_\pi^*}{V_\pi}$ (REACH [2210.07409])
- This can be made more sophisticated with `SuperNest` [2212.01760] & posterior repartitioning
- Recent application to gravitational waves by Metha Prathaban Ongoing work



Jax-based nested samplers

David Yallup
PDRA

- **very** recent work over the past month
- Have implemented a nested slice sampler in `blackjax` [#755]

```
pip install git+https://github.com/handley-lab/blackjax@nested_sampling
import blackjax.ns.adaptive
```

- parallelised over `vmapped` likelihood & prior evaluations
- Plugs into `jim` [kazewong/jim] and `ripple` [2302.05329]
- interested in finding use-cases for such a sampler this week
- Also interested in understanding current limitations/strengths of `jax`/GPU GW programming

Conclusions

github.com/handley-lab

- **Next-generation inference:** Addressing challenges in astrophysical analysis, whether likelihoods are available or not.
- **Simulation-based inference (SBI):** Leveraging simulations and machine learning for posterior estimation and model comparison when only a forward model exists. Advantages include speed, implicit covariance handling, and ease of incorporating systematics.
- **Marginal statistics:** Efficient computation of nuisance-marginalised likelihoods using `margarine`. Applications to hierarchical modeling and building prior/likelihood libraries.
- **Accelerated nested sampling:** Improving the scaling of nested sampling with techniques like beta flows, posterior repartitioning, and `SuperNest`.
- **Jax-based nested samplers:** Introducing recent work on parallel nested sampling implementations in `Jax` for increased performance.

SGW relevant portions

- Marginal inference
  - Earth term/pulsar term from Stas Babak is ripe for a `margarine` based approach.
  - Linked to the SBI approach for GWB global fit that Bryan Zaldivar is working on.
- Nested sampling is an (underused) alternative for computing Bayes factors in the PTA community
  - current approaches are often Savage-Dicke density ratio/PTMCMC parallel tempering.
- Global fitting could benefit from accelerated approaches
  - `jax`-based GW codes are currently in development.
  - ML offers many opportunities for acceleration (emulation, proposals, . . .)