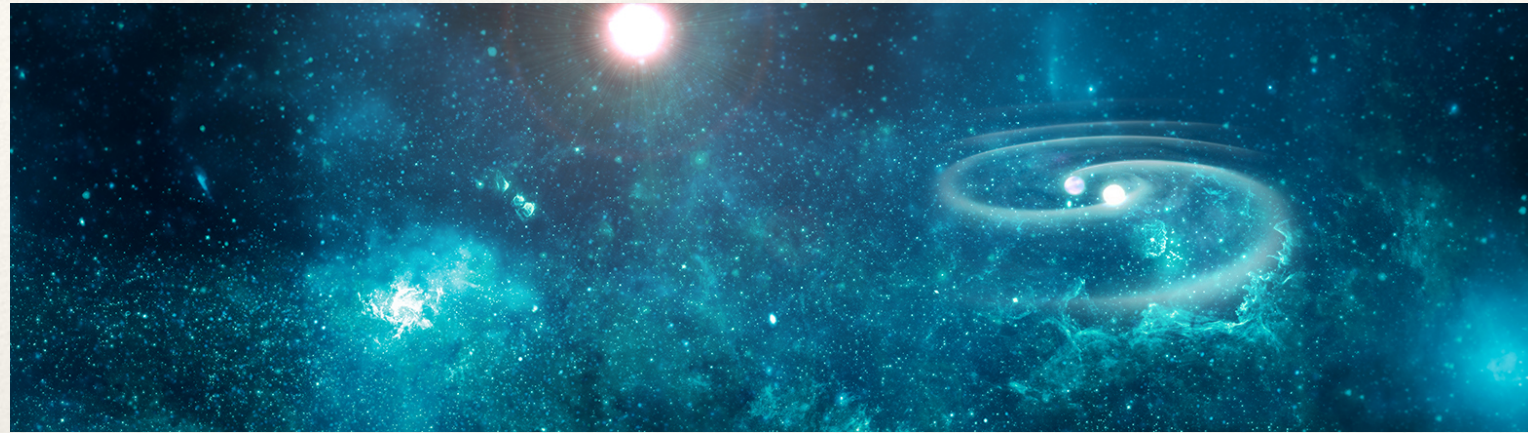


Stanislav (Stas) Babak.

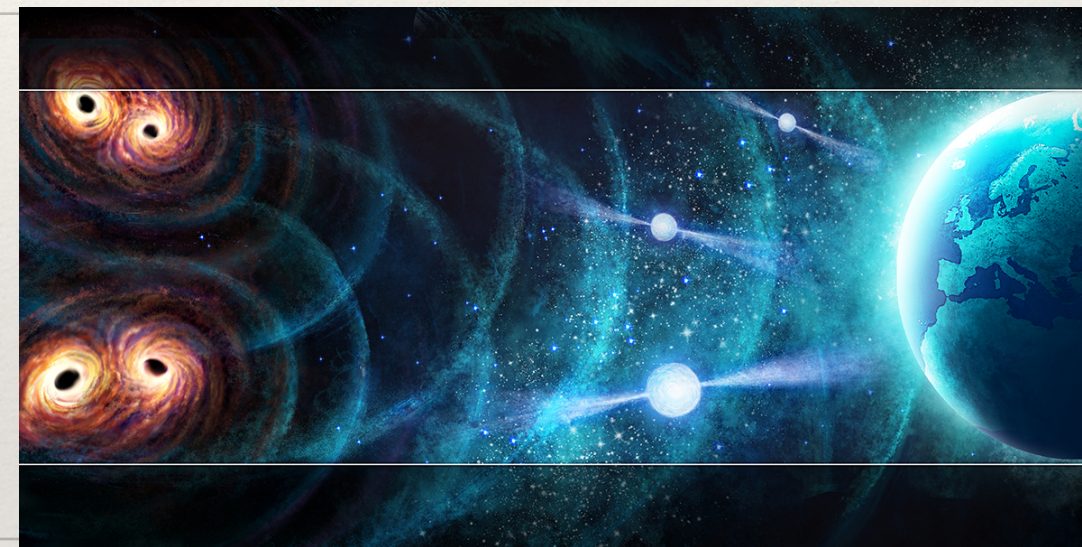
AstroParticule et Cosmologie, CNRS (Paris)



Université
Paris Cité



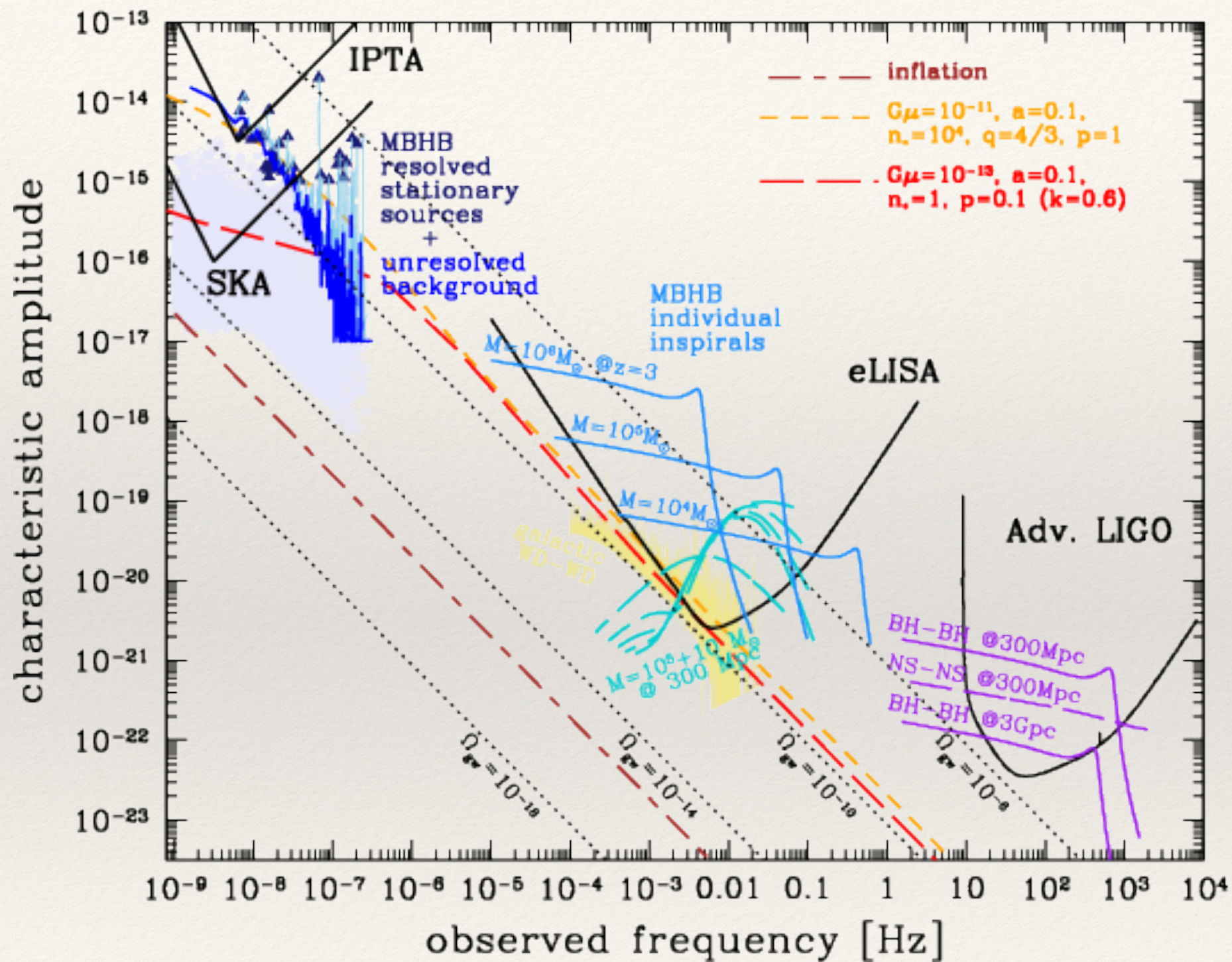
Pulsar Timing Array: what do you hear?



December 2024



GW landscape

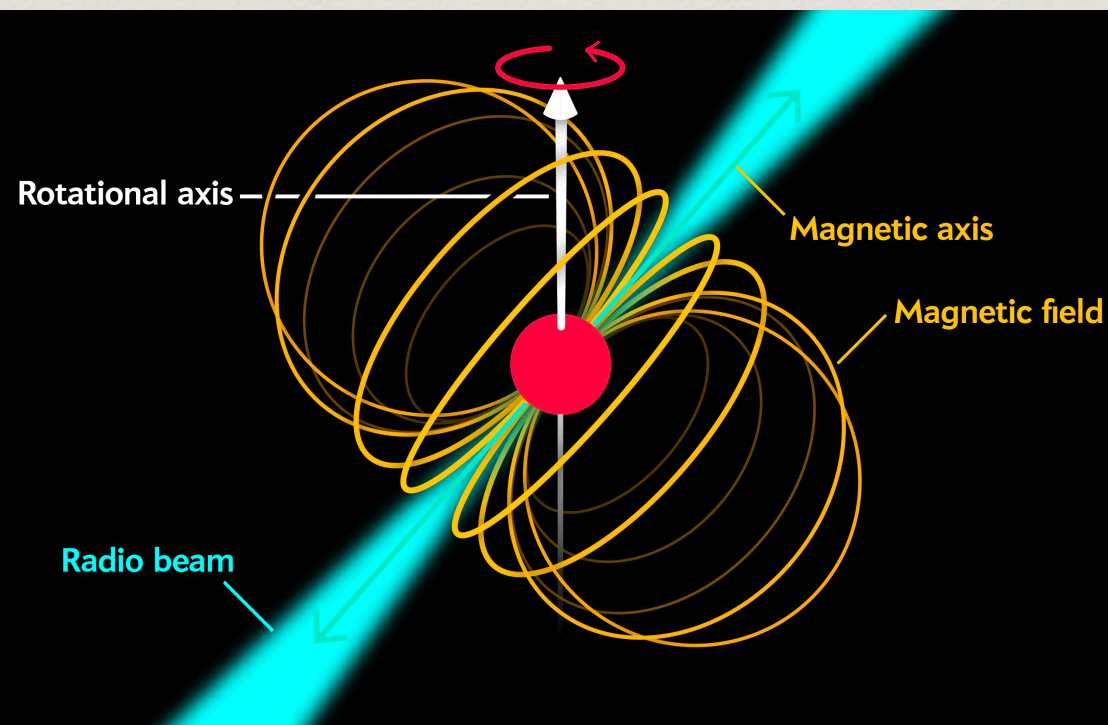
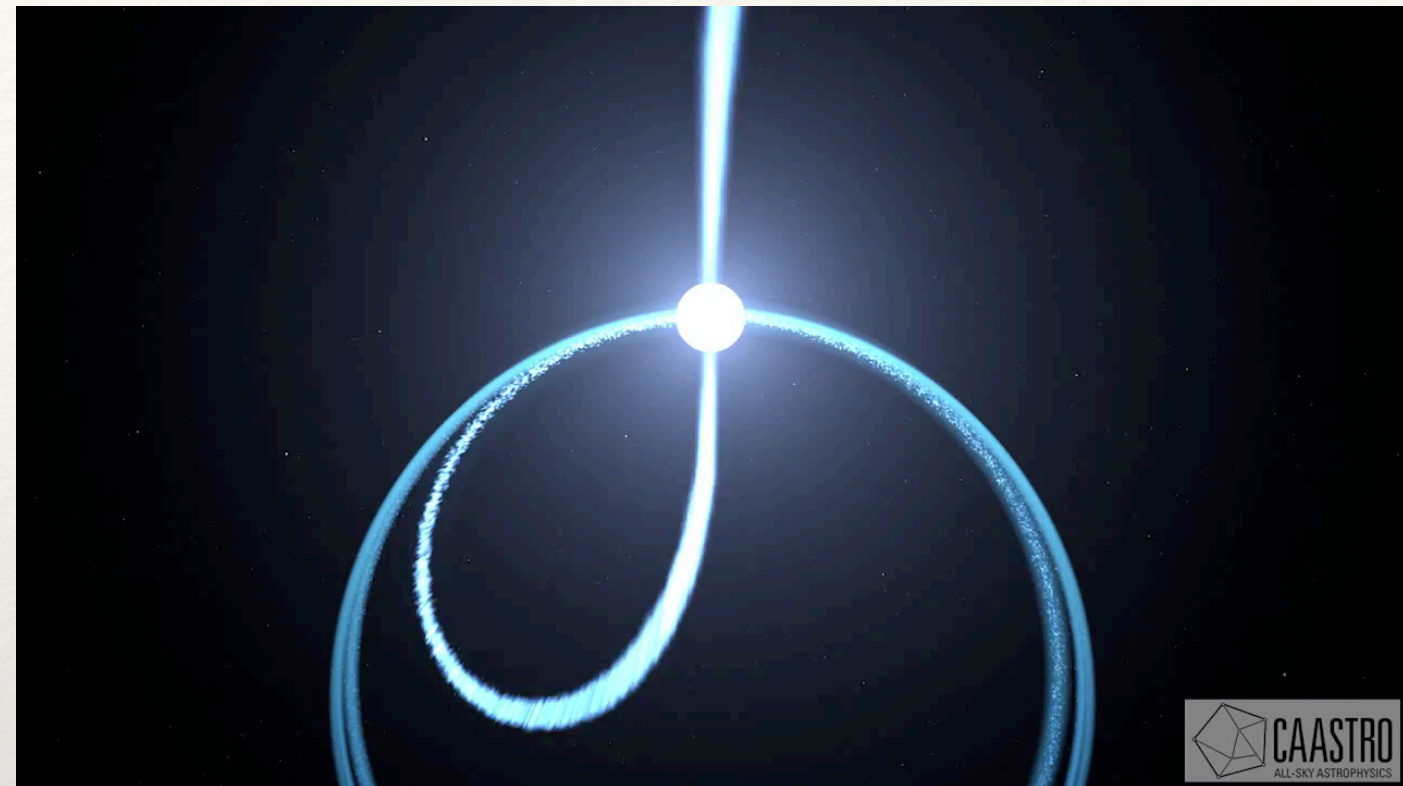


Millisecond pulsars

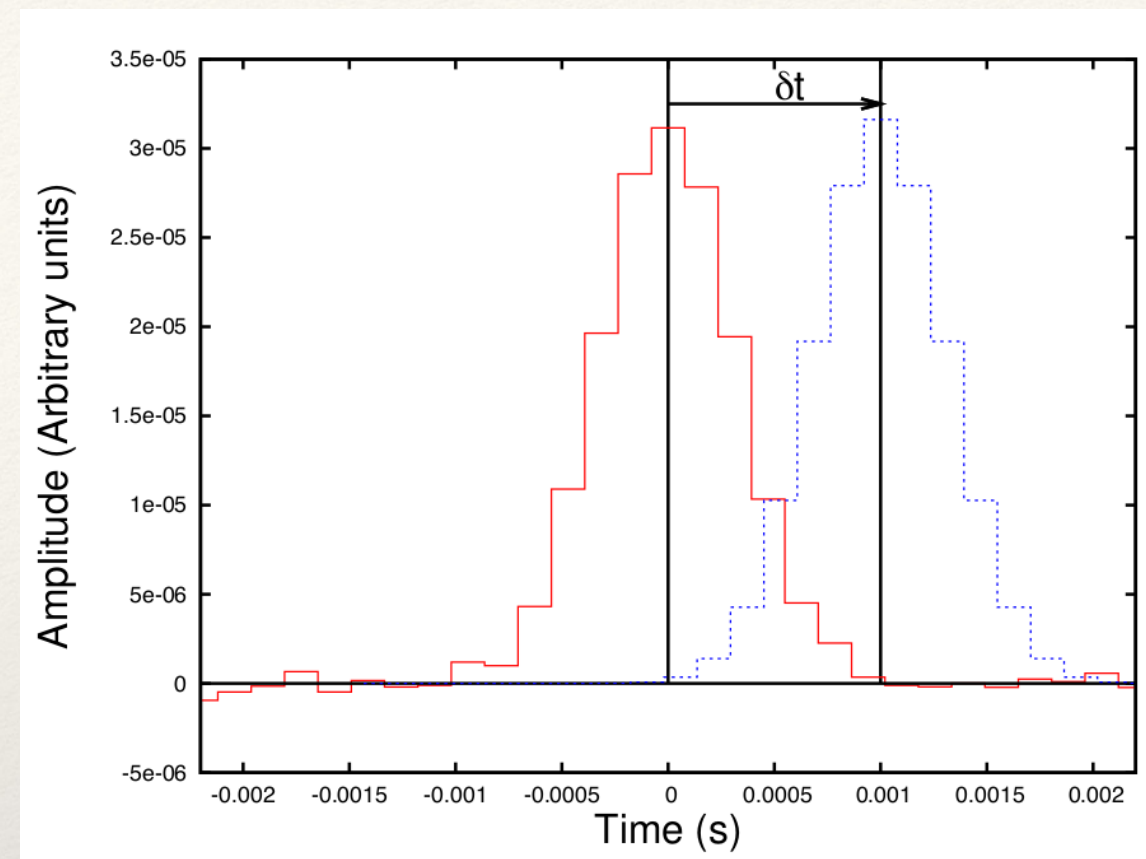
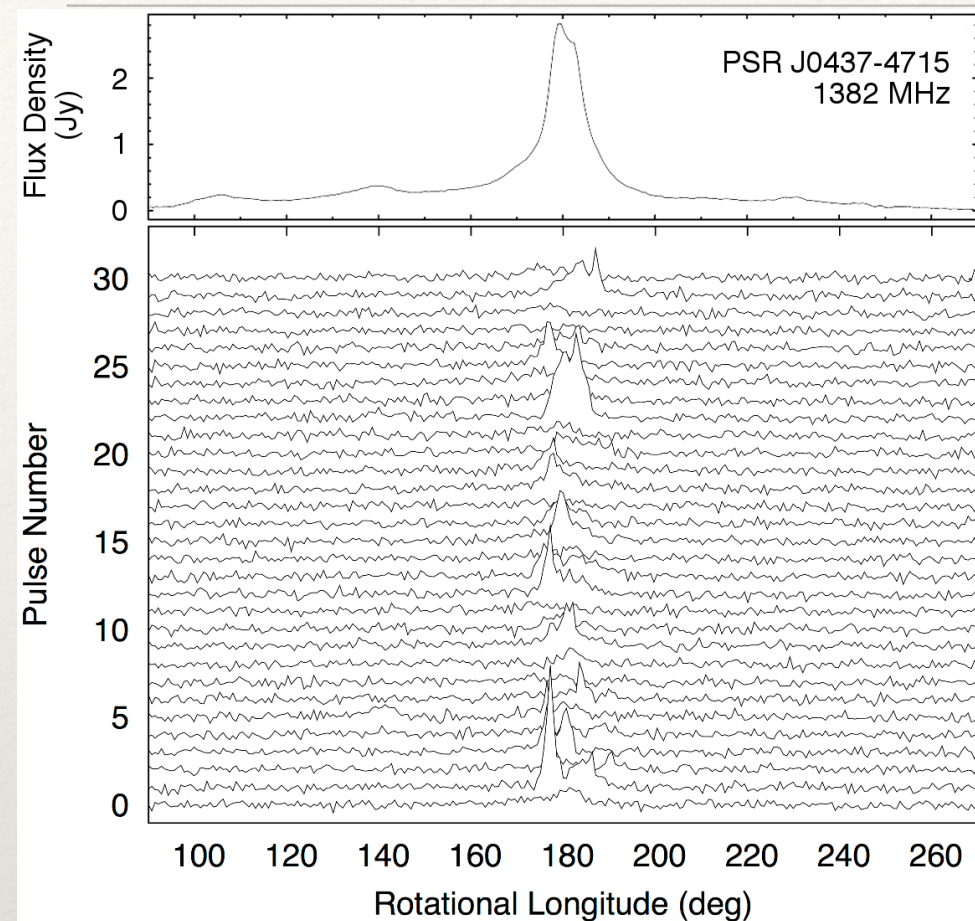


- Millisecond pulsars: period of rotation $\sim$ millisecc
- Often in binaries
- Very old NSs, very stable rotation
- The most accurate clock on the long time scale (decades)

Jen Christiansen



Pulsar timing



[Figs: credits
S. Burke-Spolar & L. Lentati]

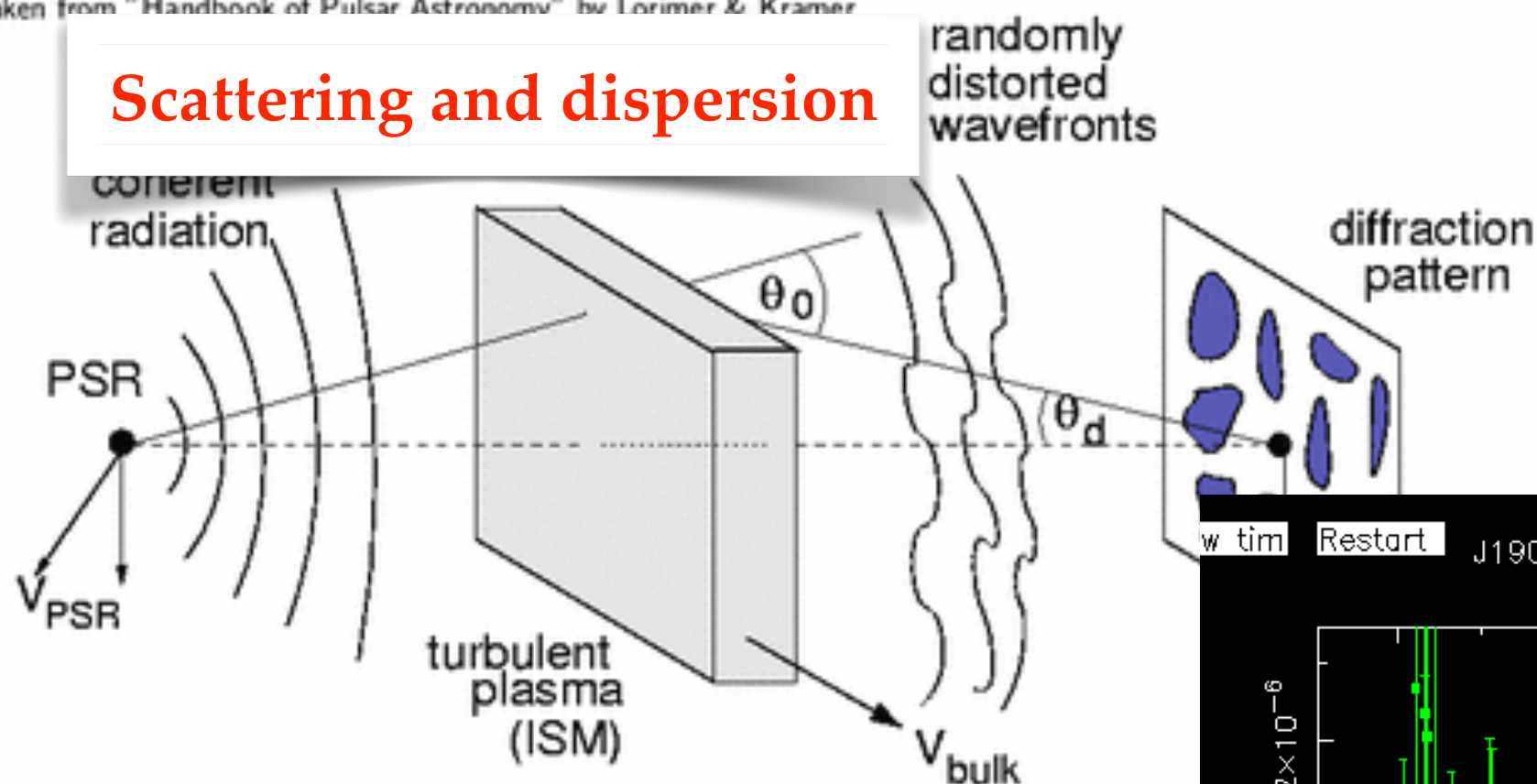
- Each observed radio pulse profile has a lot of micro-structure. If we average over ~hour the (average) profile is very stable
- We can use the average pulse profile to estimate the time-of-arrival (TOA) of the pulses.
- The idea is to measure the TOA, and compare to the expected TOA. We know the spin of the pulsars, so we can predict the TOA. The difference between measure and expected TOA: *residuals*



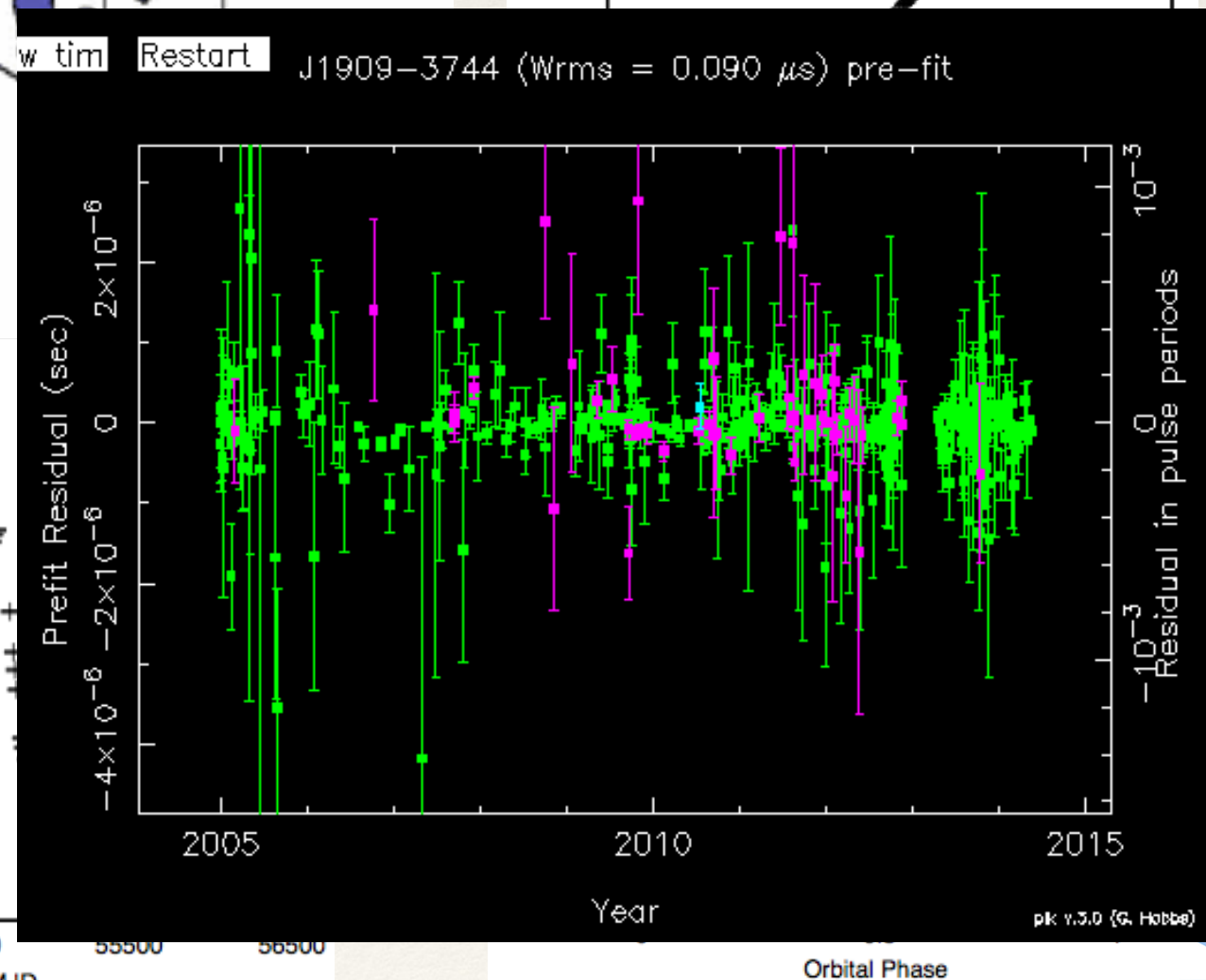
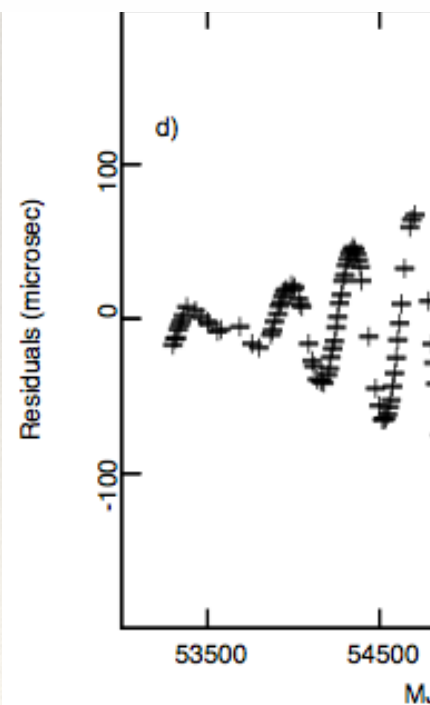
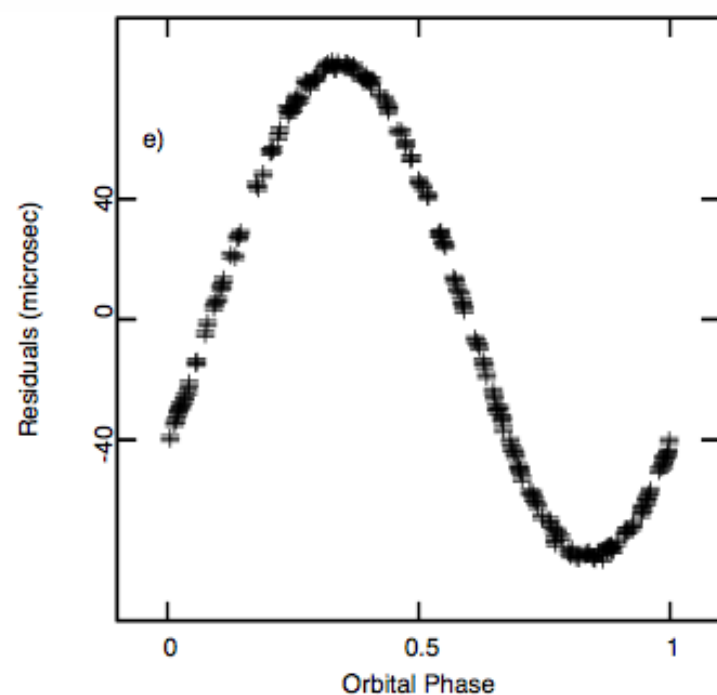
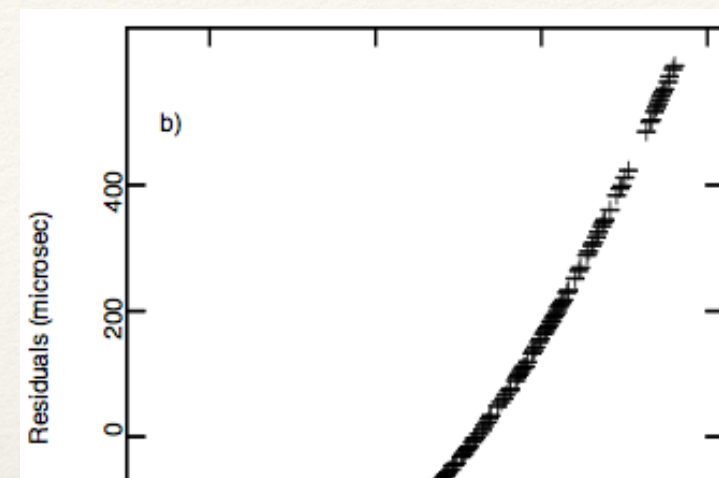
Predicting arrival time

Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Scattering and dispersion



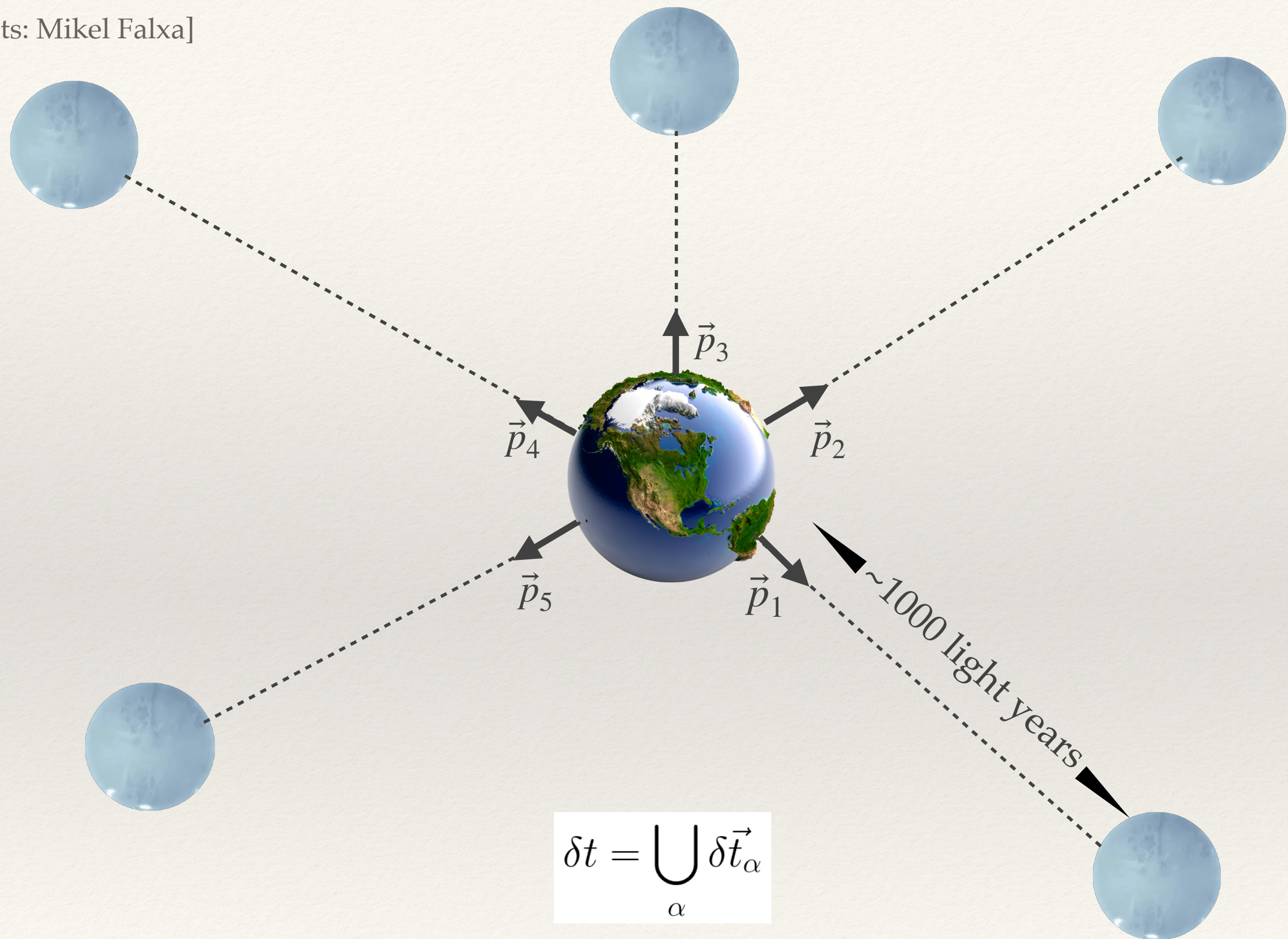
rate of change of spin



Pulsar Timing Array



[credits: Mikel Falxa]



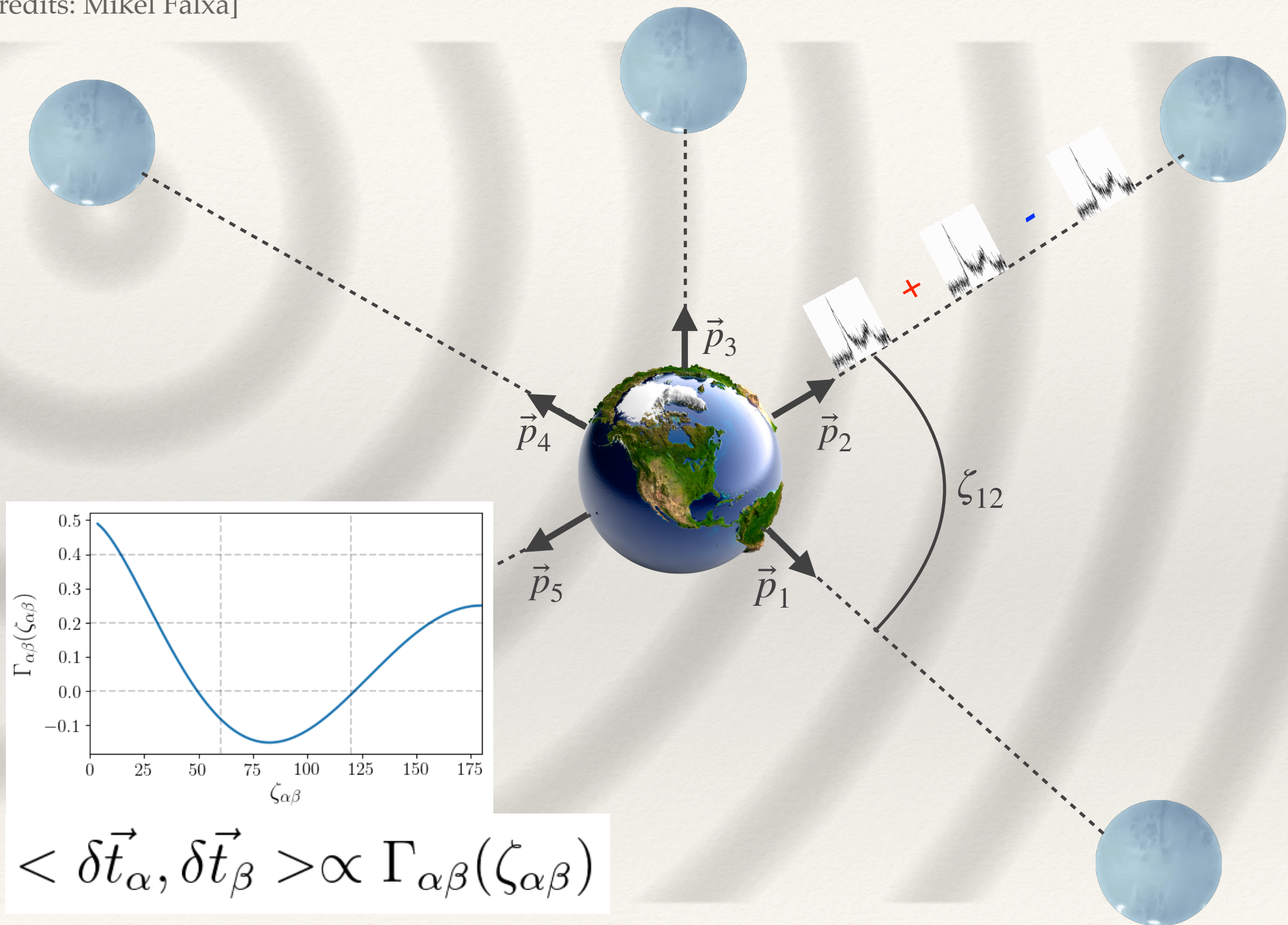
$$\delta t = \bigcup_{\alpha} \delta \vec{t}_{\alpha}$$



Pulsar Timing Array



[credits: Mikel Falxa]

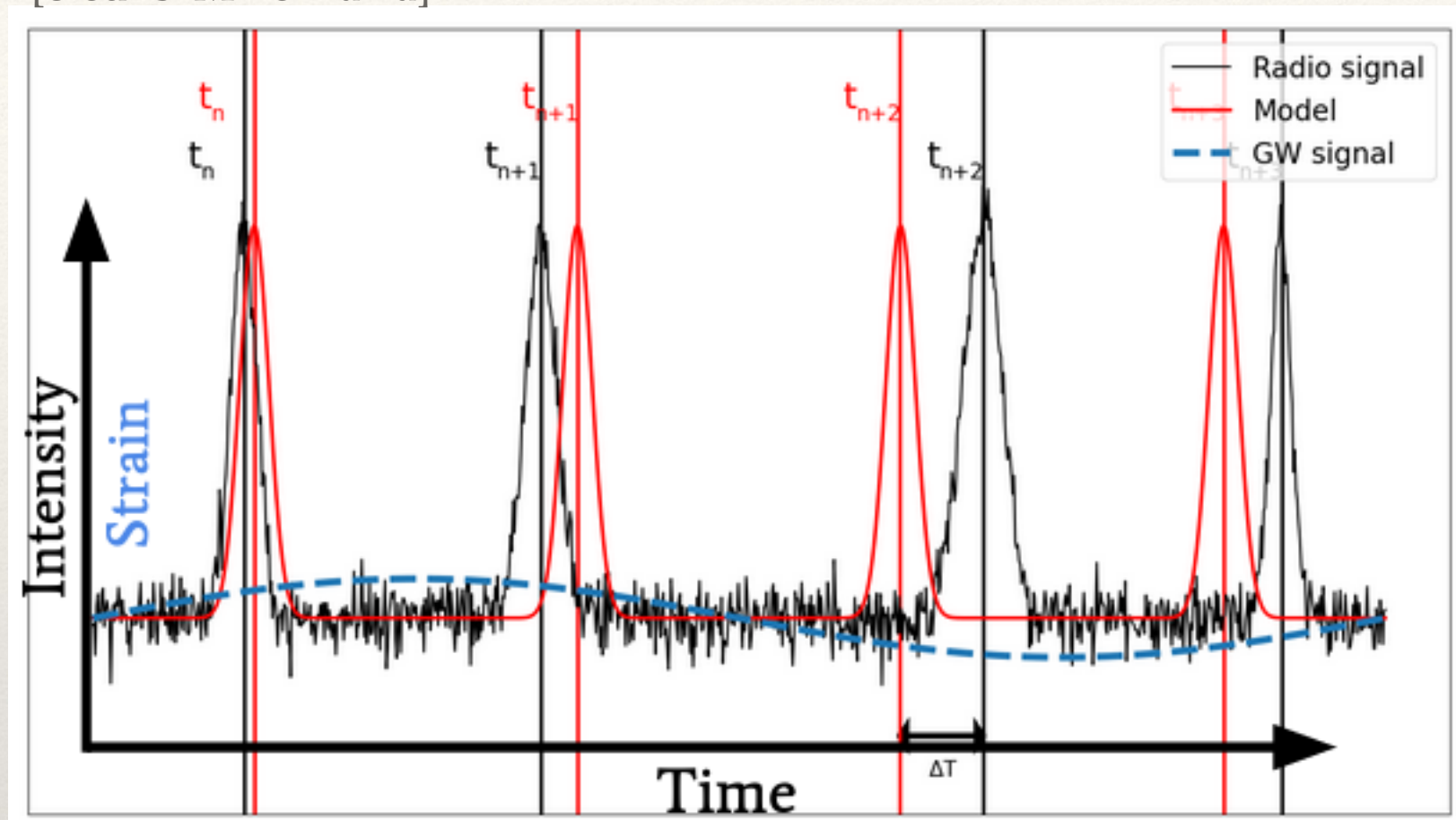


$$\langle \delta \vec{t}_\alpha, \delta \vec{t}_\beta \rangle \propto \Gamma_{\alpha\beta}(\zeta_{\alpha\beta})$$



Timing Residuals

[credits: Mikel Falxa]



$$dt = t_{toa}^p - t_{toa}^o = dt_{errors} + \delta\tau_{GW} + noise$$

Errors in fitting the model $\nearrow$ due to GWs

Detection statistic and search algorithm



- We assume that noise is Gaussian: the likelihood function (likelihood of the signal with given parameters) is

$$P(\vec{\delta t}, \vec{\theta}) = \frac{1}{\sqrt{(2\pi)^n \det(C)}} \exp \left(-\frac{1}{2} (\vec{\delta t} - \vec{s})^T C^{-1} (\vec{\delta t} - \vec{s}) \right),$$

- $\vec{\delta t}$ - concatenated residuals from all pulsars in the array: total size n
- $\vec{s}$ - is a model of deterministic signals (for example GW signals from individually resolvable SMBHBs)
- C is the noise variance-covariance matrix (size $n \times n$)

$$C_{\alpha i, \beta j} = C_{\alpha\beta}^{wn} \delta_{ij} + C_{ij}^{rn} \delta_{\alpha\beta} + C_{ij}^{dm} \delta_{\alpha\beta} + C_{\alpha i, \beta j}^{GW} + \dots$$

white
measurement
noise

red noise
spin
noise

dispersion
variation
noise

stochastic GW
signal



Noise modelling in PTA



- White noise — not very interesting. Two parameters per backend per pulsar: unaccounted noise.
- Red noise: very generic noise description in freq. domain

$$S(f) = A_{rn}^2 f^{-\gamma}$$

common, uncorrelated
red noise

$$S_{\alpha}(f) = A_{rn,\alpha}^2 f^{-\gamma_{\alpha}}$$

red noise in each pulsar

- DM (dispersion measurement variation) noise: depends on the radio-frequency of observation

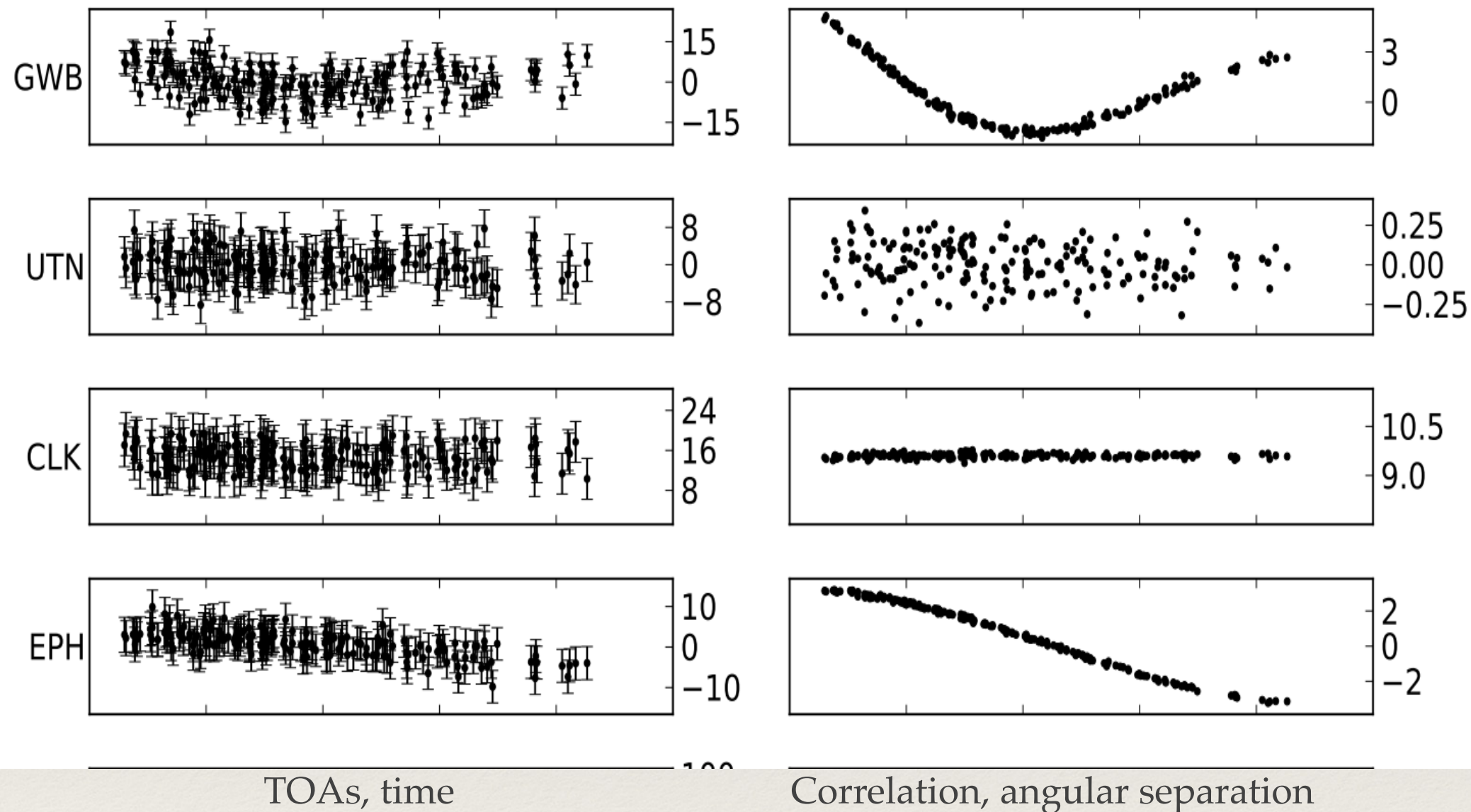
$$S_{DM}(f) \propto \frac{A_{dm}^2}{\nu^2} f^{-\gamma_{dm}}$$

- Correlated red noise processes

$$S_{\alpha\beta} = \Gamma_{\alpha\beta} A_{cor}^2 f^{-\gamma_{cor}} \quad \text{— includes also cross spectrum between each pair of pulsars: } \Gamma_{\alpha\beta} \text{ - spacial correlation coefficients}$$



Correlated noise



GWB (Quadrupole)
Hellings-Downs

Uncorrelated RN

Clock error (monopole)

Ephemeris (dipole)

stochastic GW from population of SMBHBs:

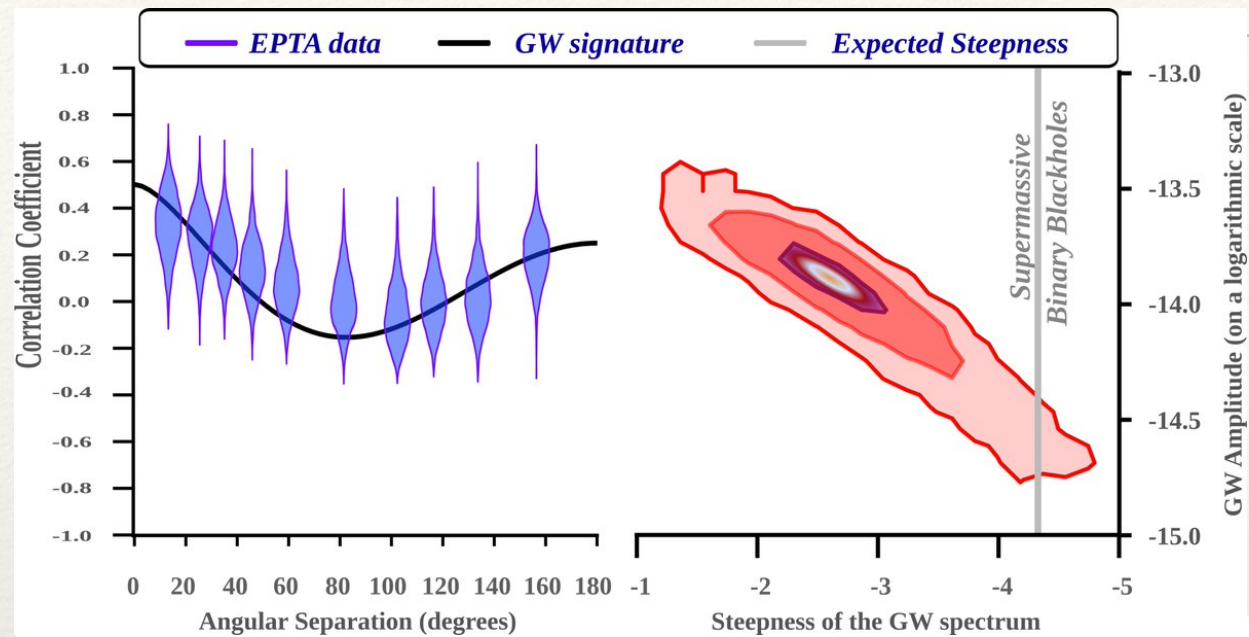
$$S_{\alpha\beta}^{SMBHB} = \Gamma_{\alpha\beta}^{H-D} A_{GW}^2 f^{-13/3}$$



IPTA



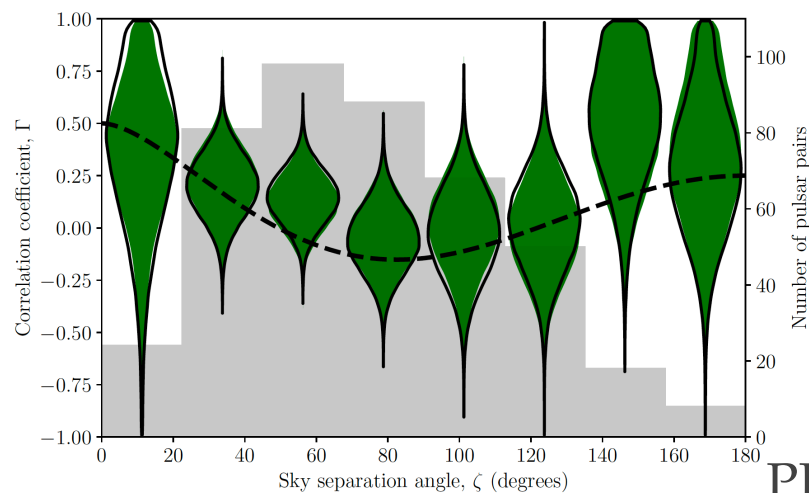
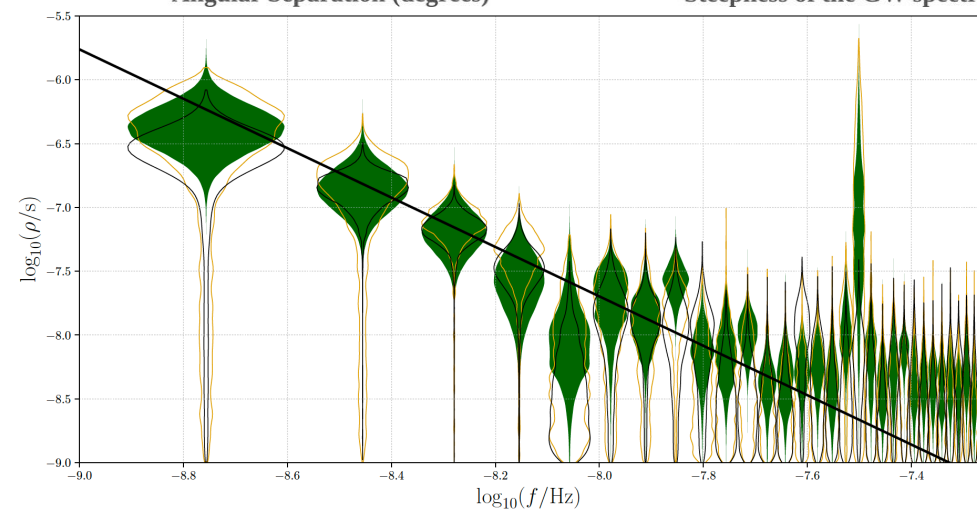
Stochastic Gravitational Waves? SGWB



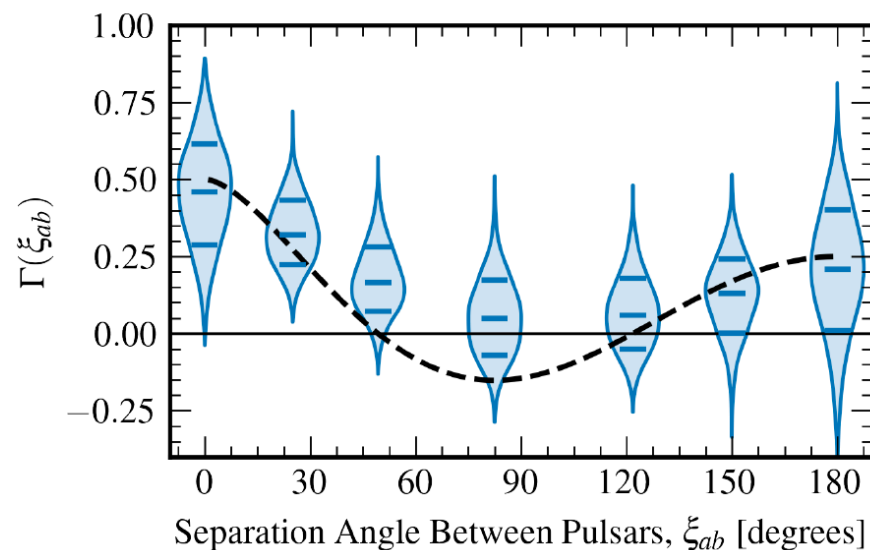
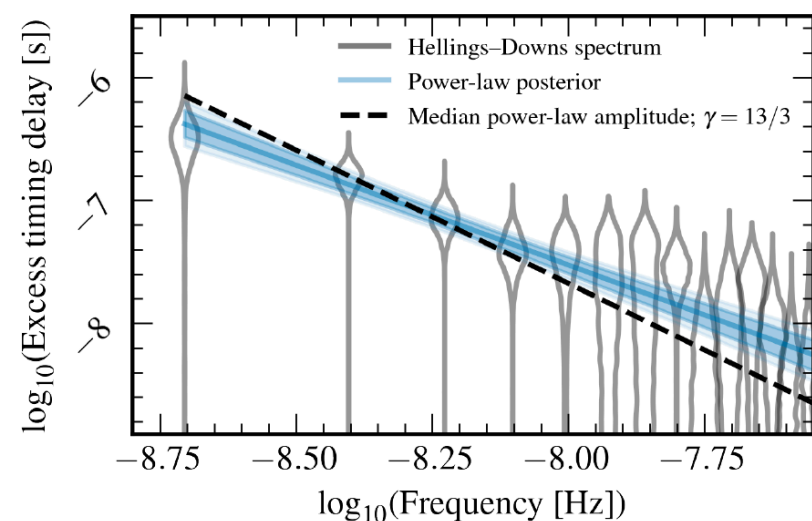
Indication of emerging GW signal:

- Violin plots: estimation of the pairwise correlation as f-n of angular separation
- SGWB modelled as power-law

EPTA+InPTA:
2306.16214



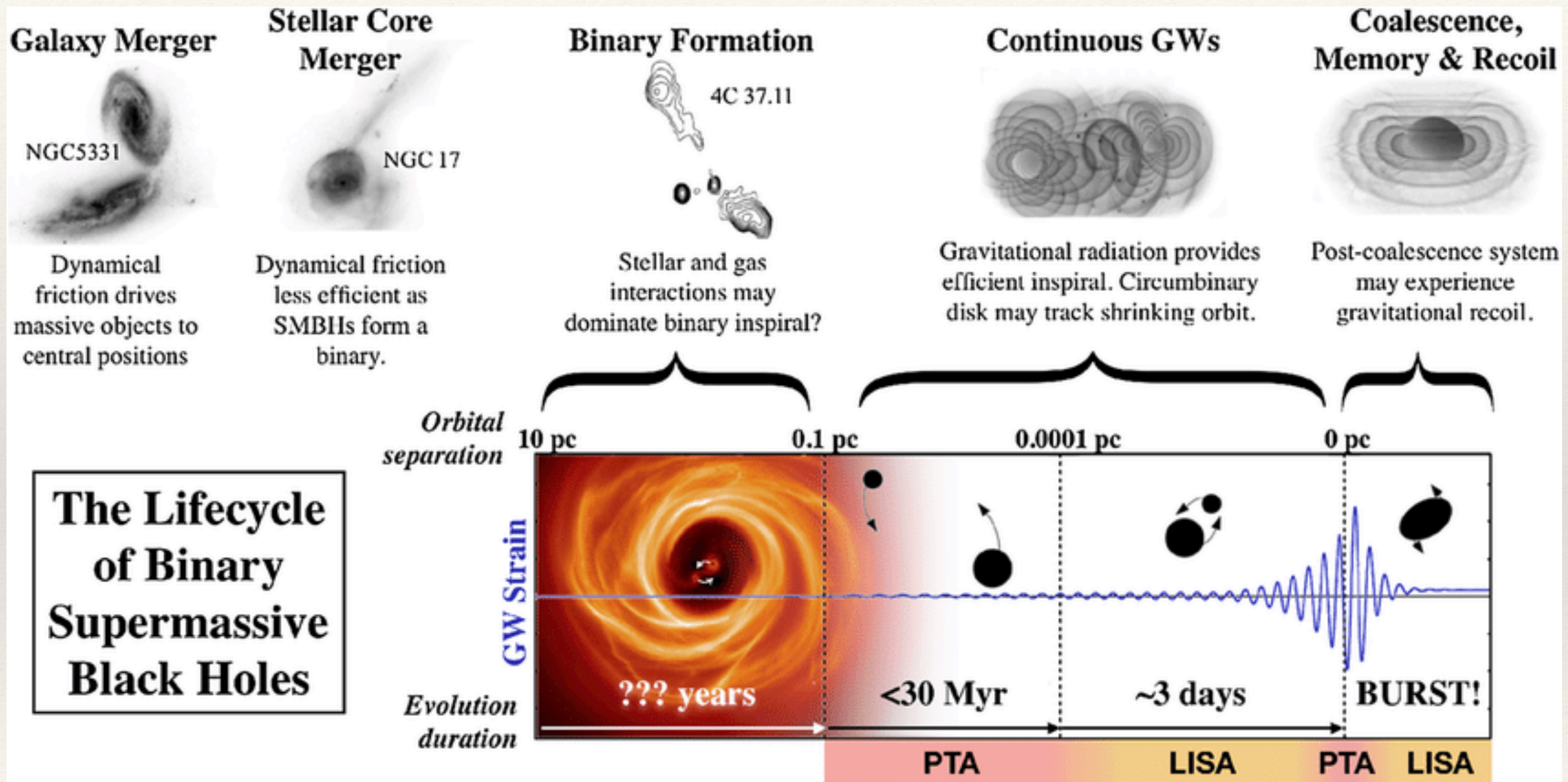
PPTA: 2306.16215



NG: 2306.16213



PTA: what we expect to see (SMBHBS)



[S. Burke-Splolaor A&A review (2019)]

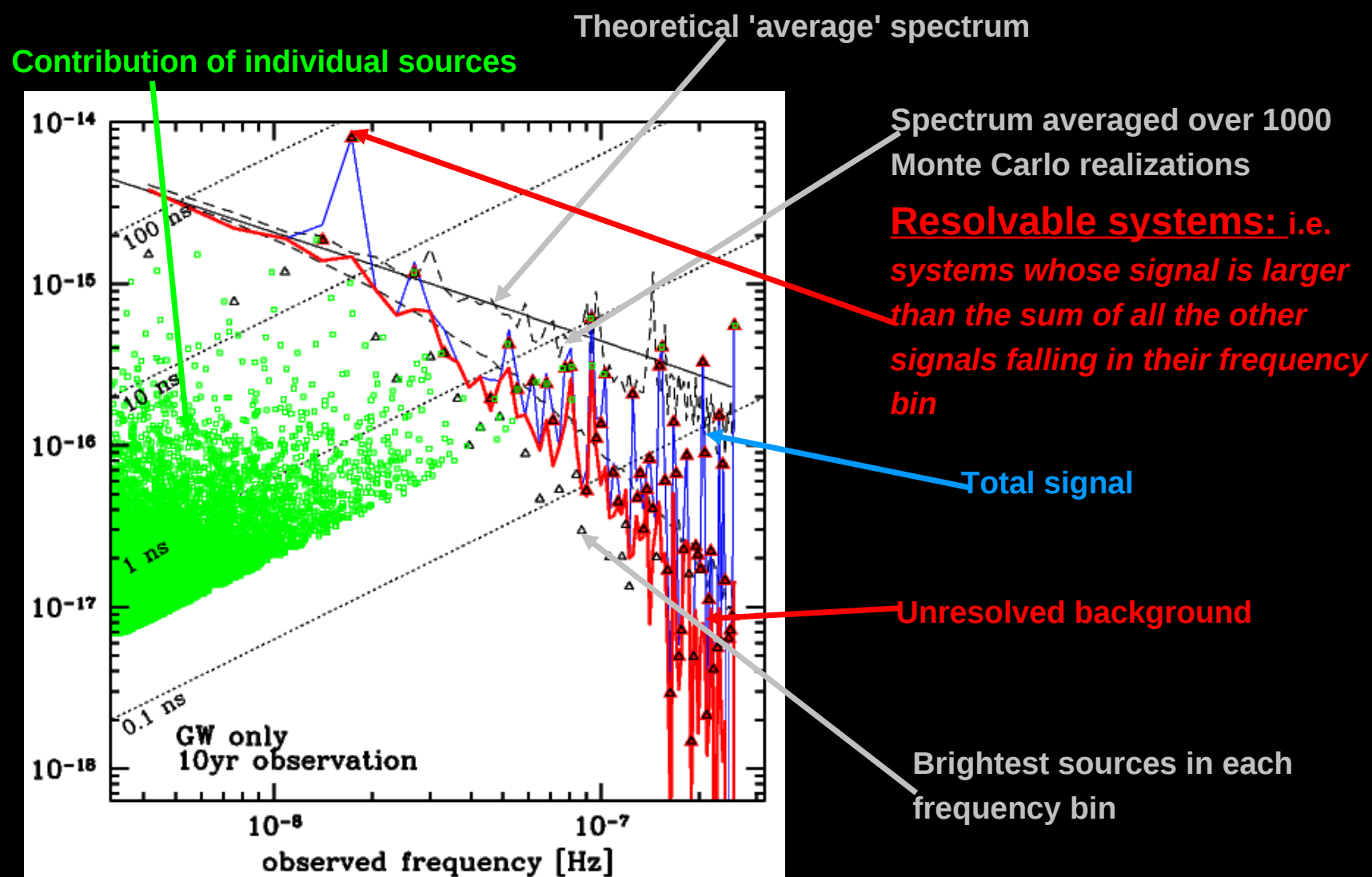


Supermassive black hole binaries



- Population of supermassive black hole binaries (mass $10^7 - 10^{10}$ solar) on very broad orbit (period $\sim$ year(s))
- The orbital evolution due to GW emission is very slow: $\frac{dE}{dt} \propto \eta(M/r)^5$
signal is (almost) monochromatic over period of observations

Signal from a MBHB population



GW signal from the population of SMBH binaries: forms a stochastic signal at low freqs. (similar to Galactic binaries in LISA)

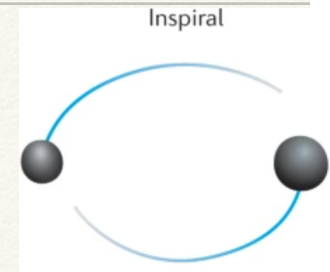
[Credits: A. Sesana]



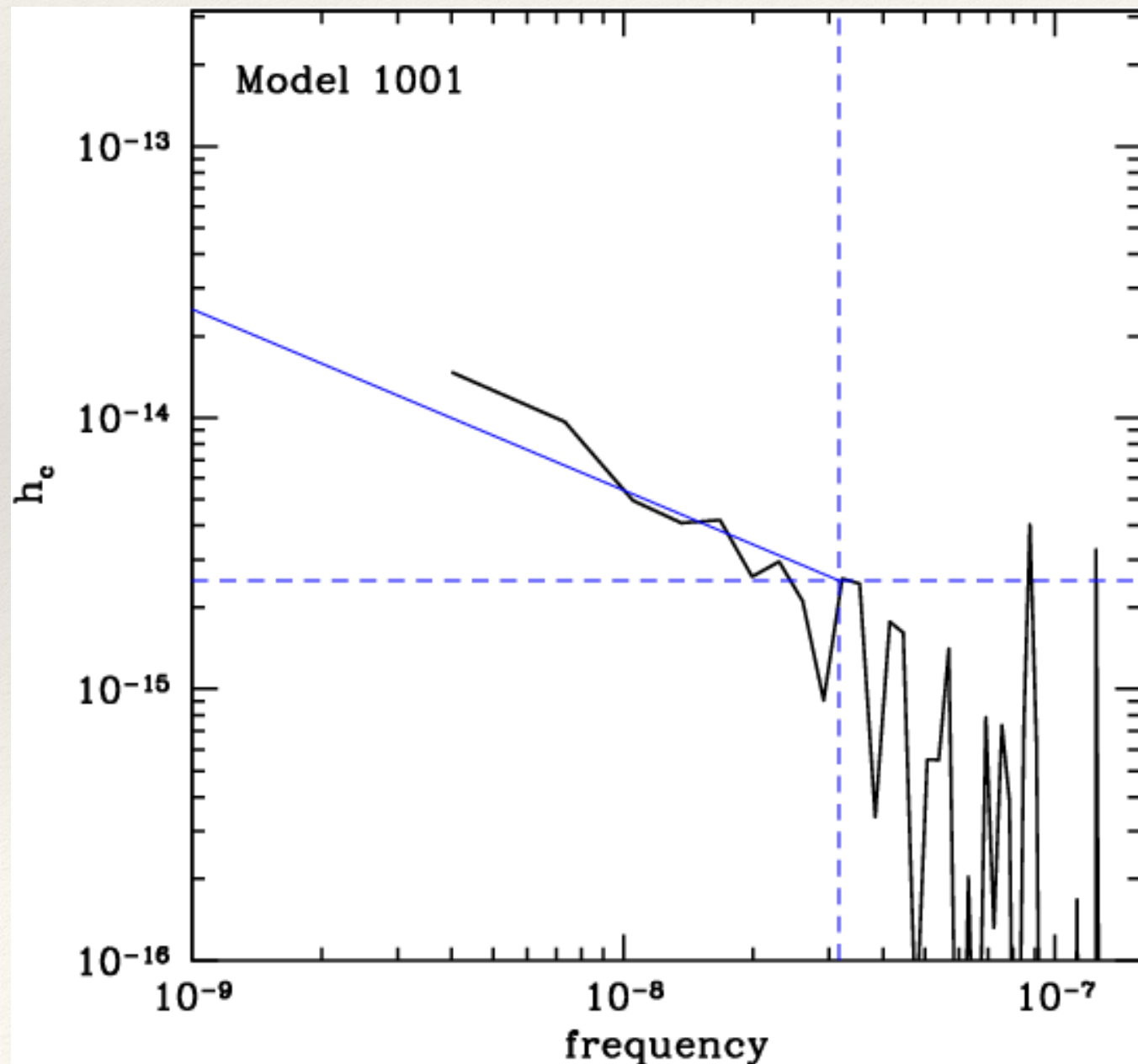
Continuous GW signal (CGW)



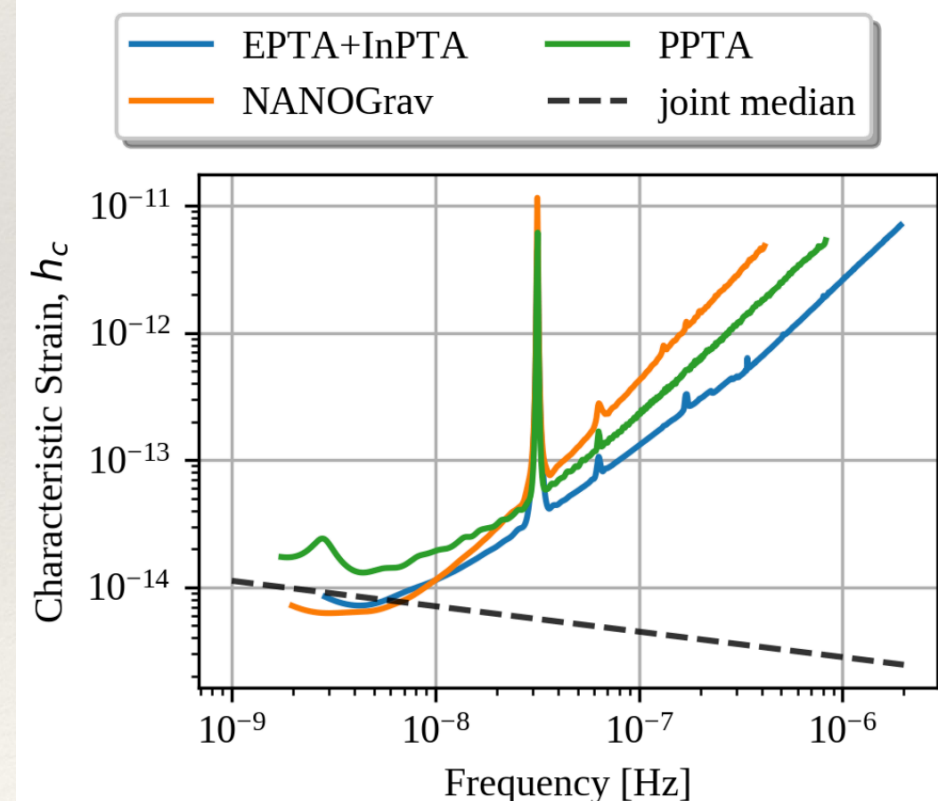
Searching for GW signal from individual SMBHB binary:



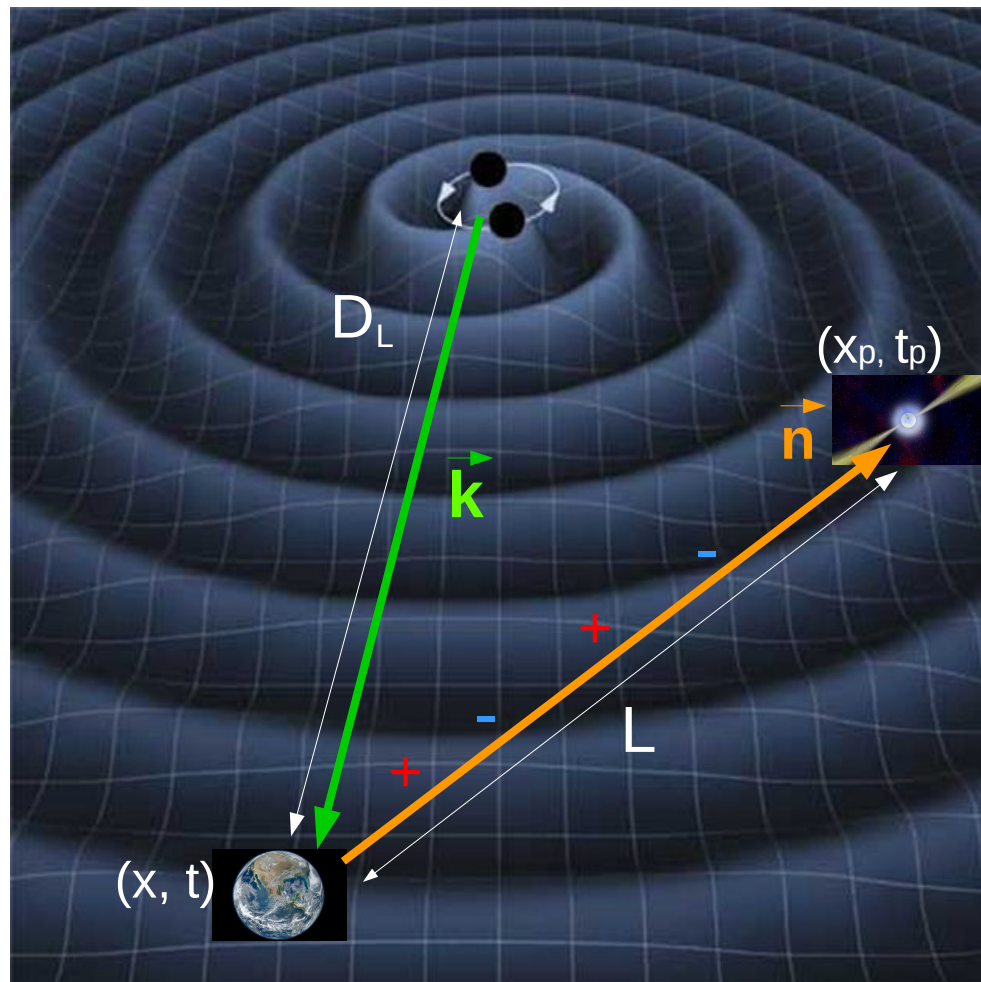
- Assume circular orbit
- Bayesian approach
- Strategy: all-sky search with simplistic model -> follow up candidates relaxing simplified assumptions on the reduced prior range



Sensitivity of PTAs [2309.00693]



Response to GW signal



$$dt = t_{toa}^p - t_{toa}^o = dt_{errors} + \delta\tau_{GW} + noise$$

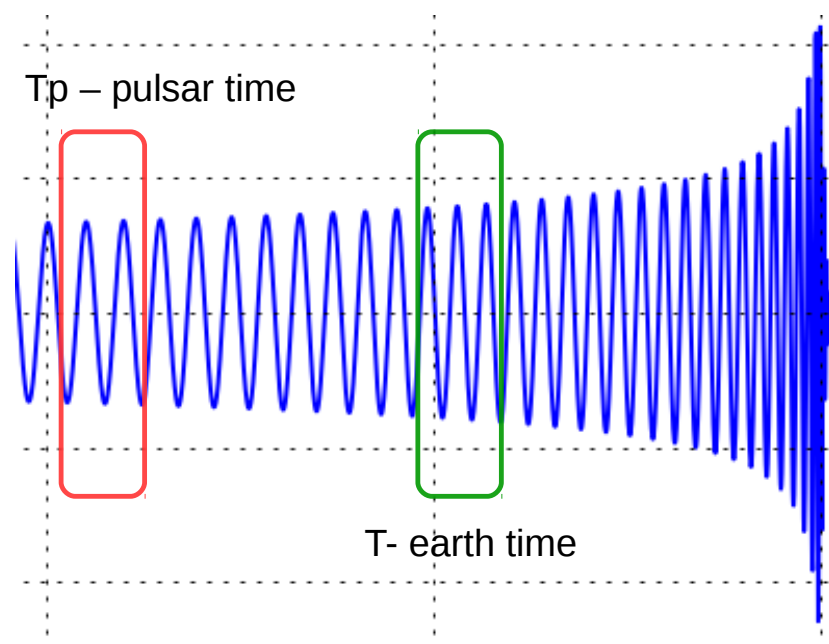
$$\delta\tau_{GW} = r(t) = \int_0^t \frac{\delta\nu}{\nu_0}(t') dt'; \quad \frac{\delta\nu}{\nu_0} = \frac{1}{2} \frac{\hat{n}^i \hat{n}^j \Delta h_{ij}}{1 + \hat{n} \cdot \hat{k}}$$

Familiar from LISA

$$\Delta h_{ij} = h_{ij}(t_p = t - L(1 + \hat{n} \cdot \hat{k})) - h_{ij}(t)$$

t_p — pulsar time, ~ time of emission of the radio pulse:

- depends on the relative position of a pulsar and GW source
- depends on the distance to the pulsar L
- $L \sim \text{few kpc} \sim 10^5 \text{ years}$ — “pulsar” term $h(t_p)$ contains info about the system 10^5 years in the past as compared to the “earth” term
- pulsar term depends on the pulsar.



CGW signal



Consider non-spinning SMBH binary in circular orbit

- pulsar and earth terms: each is monochromatic signal
- frequency. of pulsar term might or might not coincide with the earth term:
 $t_p = t - L(1 + \hat{n} \cdot \hat{k})$
- amplitude of the pulsar term is larger: $\sim \omega^{-1/3}$

$$s_\alpha = F_\alpha^+(\hat{k}, \hat{n}_\alpha) \left[\frac{h_+(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_+(t, \omega)}{2\pi f} \right] + \quad \alpha - \text{pulsar index}$$

$$F_\alpha^\times(\hat{k}, \hat{n}_\alpha) \left[\frac{h_\times(t_p^\alpha, \omega_\alpha)}{2\pi f_\alpha} - \frac{h_\times(t, \omega)}{2\pi f} \right]$$

relative position
pulsar and GW source

Pulsar term
 $\omega_\alpha = \omega(t - L_\alpha(1 + \hat{n}_\alpha \cdot \hat{k}))$

Earth term coherent
across pulsars

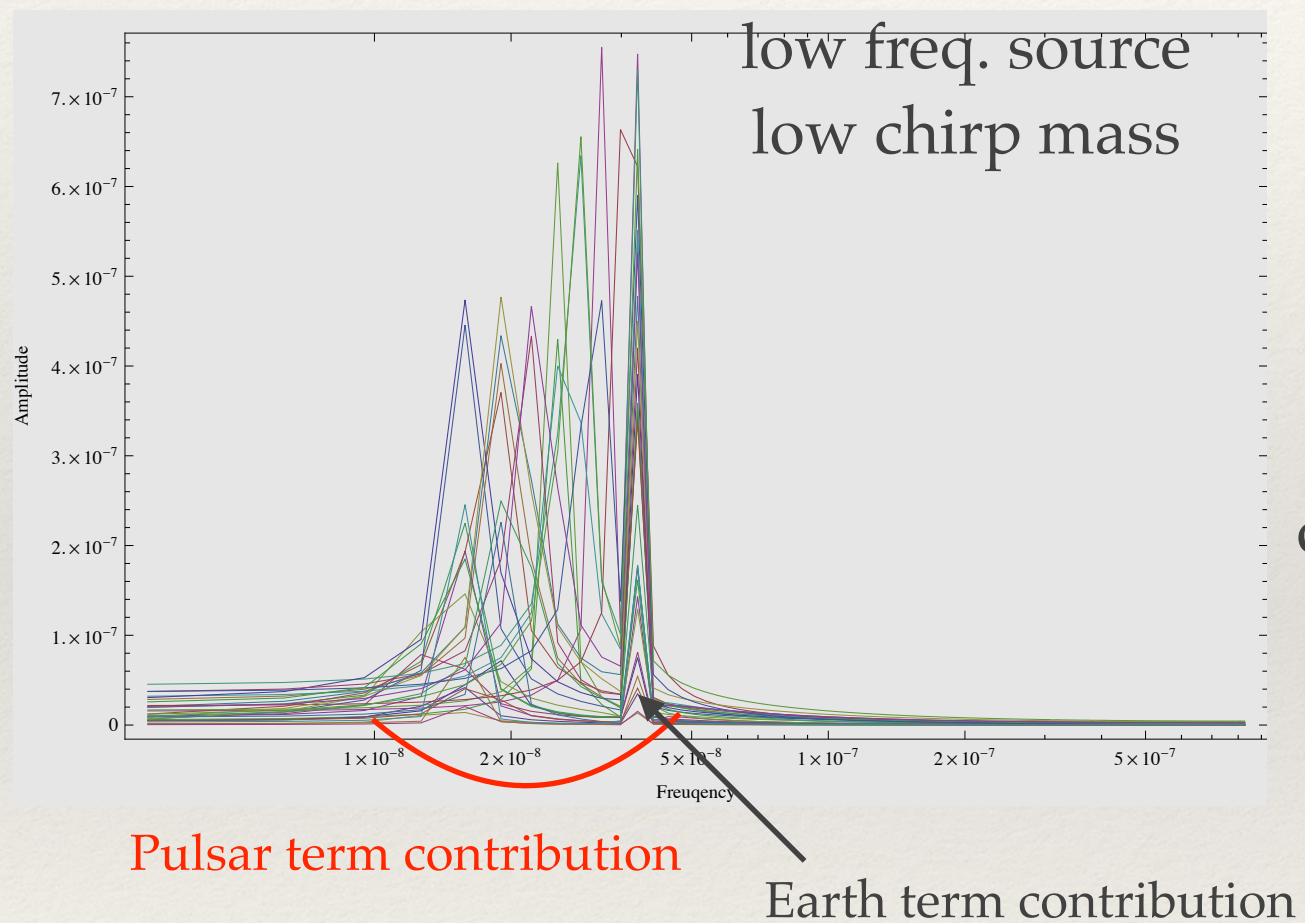


GW signal in PTA

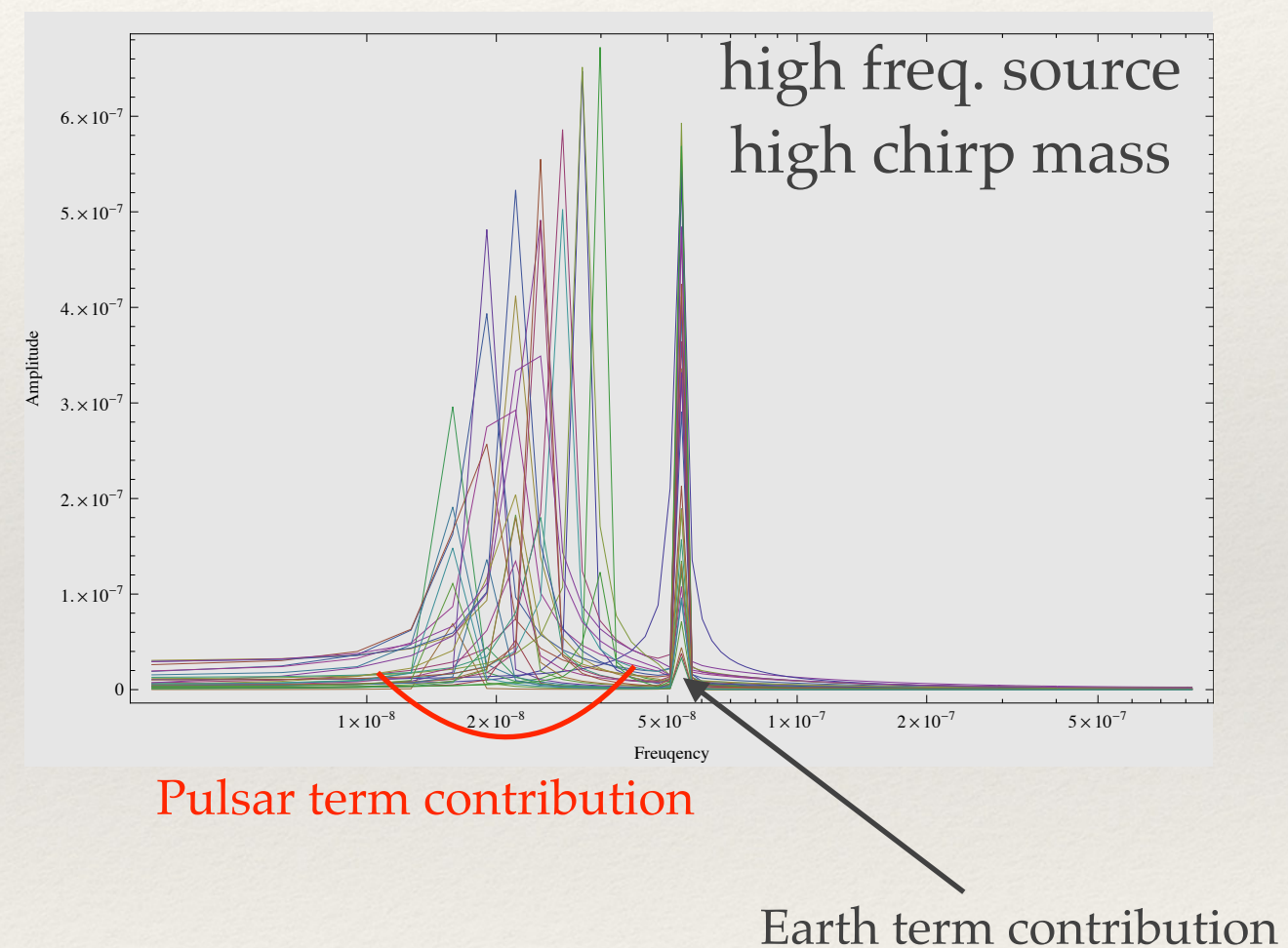


Response to GW signal of PTA in freq. domain

credits: A. Petiteau



or

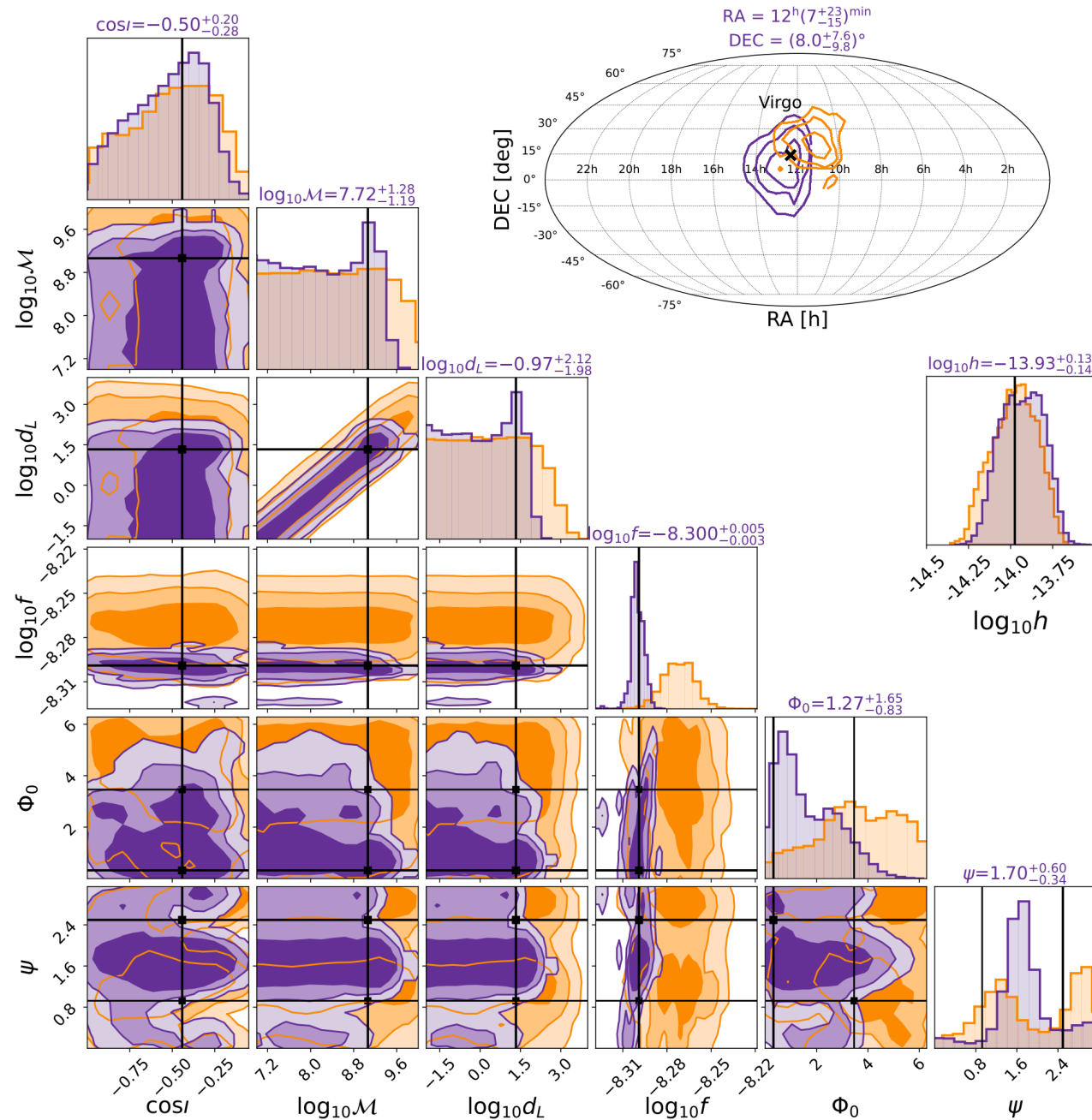


Do we need to include Pulsar term?

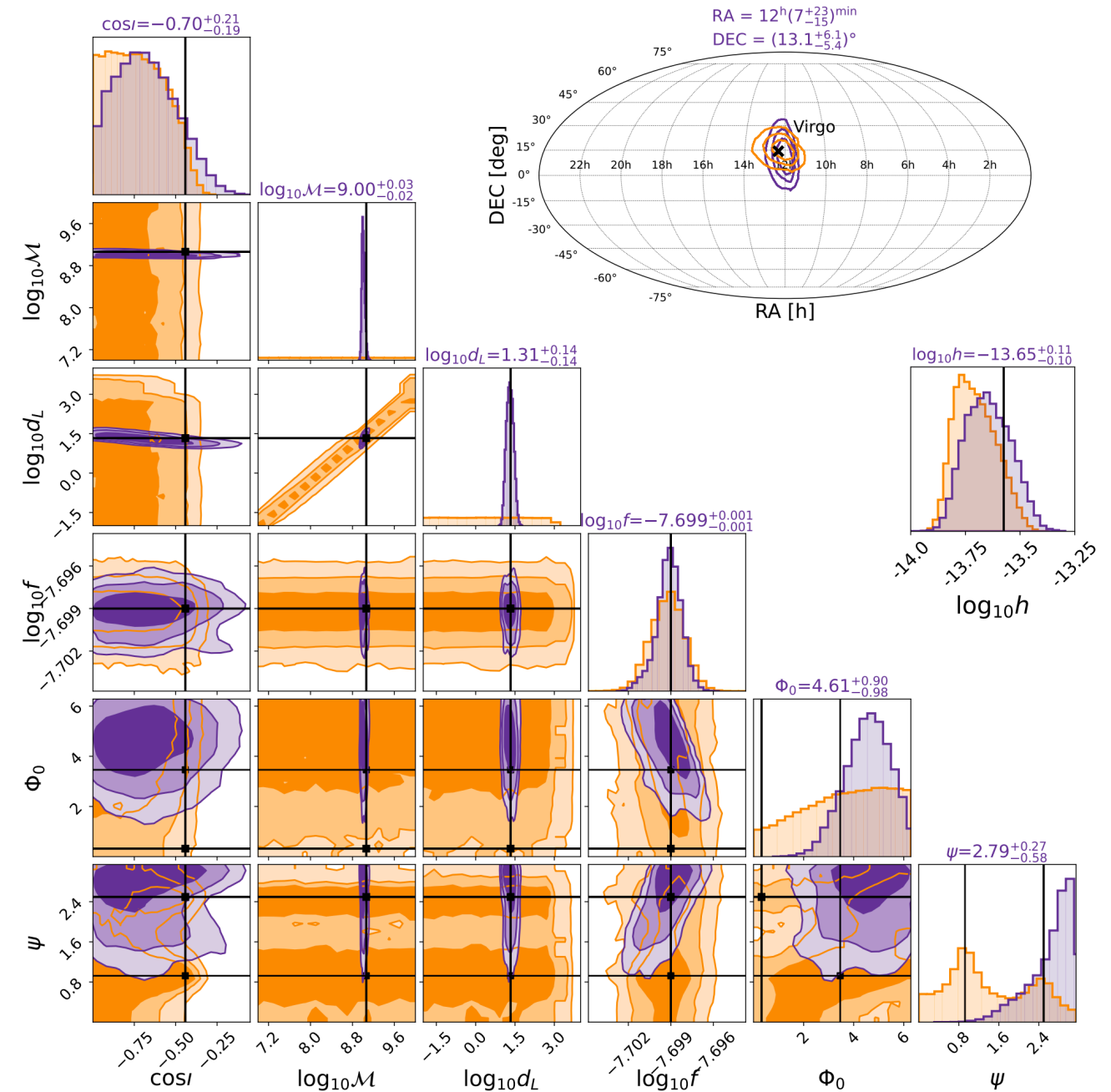


[Ferranti et al 2407.21105]

low freq. P-term corrupts E-term



high freq., P and E terms separated in freq.



Orange: with Earth term only
Purple: using both Earth and Pulsar terms



Do we need pulsar term?



- *Search:* We can search for CGW with Earth term only and then follow up with the E+P (full) template. E-term search contains only 8 parameter per circular source.
- *Parameter estimation:* E-term only might give you biased estimates.



Can we detect CGW with GWB



[Cornish&Sesana 2013, Allen 2023]

Main feature of GWB (stochastic GW background) is HD correlation

1. Consider GW propagating in the direction $\hat{\Omega}$. Compute correlations between residuals induced in two pulsars separated on the sky γ :

$$(r_\alpha | r_\beta) = \frac{1}{4\pi} \int (r_\alpha | r_\beta) d\Omega = |H_+^2 + H_\times^2| \mu_{\alpha\beta}$$

Sky averaging over direction of
GW propagation
(uniform distribution of sources)

$$\cos \gamma = \hat{p}_\alpha \cdot \hat{p}_\beta$$

$$\mu_{\alpha\beta} = \frac{1 - \cos \gamma}{2} \ln \left(\frac{1 - \cos \gamma}{2} \right) - \frac{1 - \cos \gamma}{12} + \frac{1}{3}(1 + \delta(\gamma))$$

2. *Alternatively*: we fix the source, and move a pair of pulsars over the sky keeping the angular separation (γ) fixed:

$$\mu_{\alpha\beta} = \frac{1}{4\pi^2} \int (r_\alpha | r_\beta) \delta(\cos \gamma - \hat{p}_\alpha \cdot \hat{p}_\beta) d\Omega_\alpha d\Omega_\beta$$

Hellings-Downs correlation

In reality pulsars are not uniform

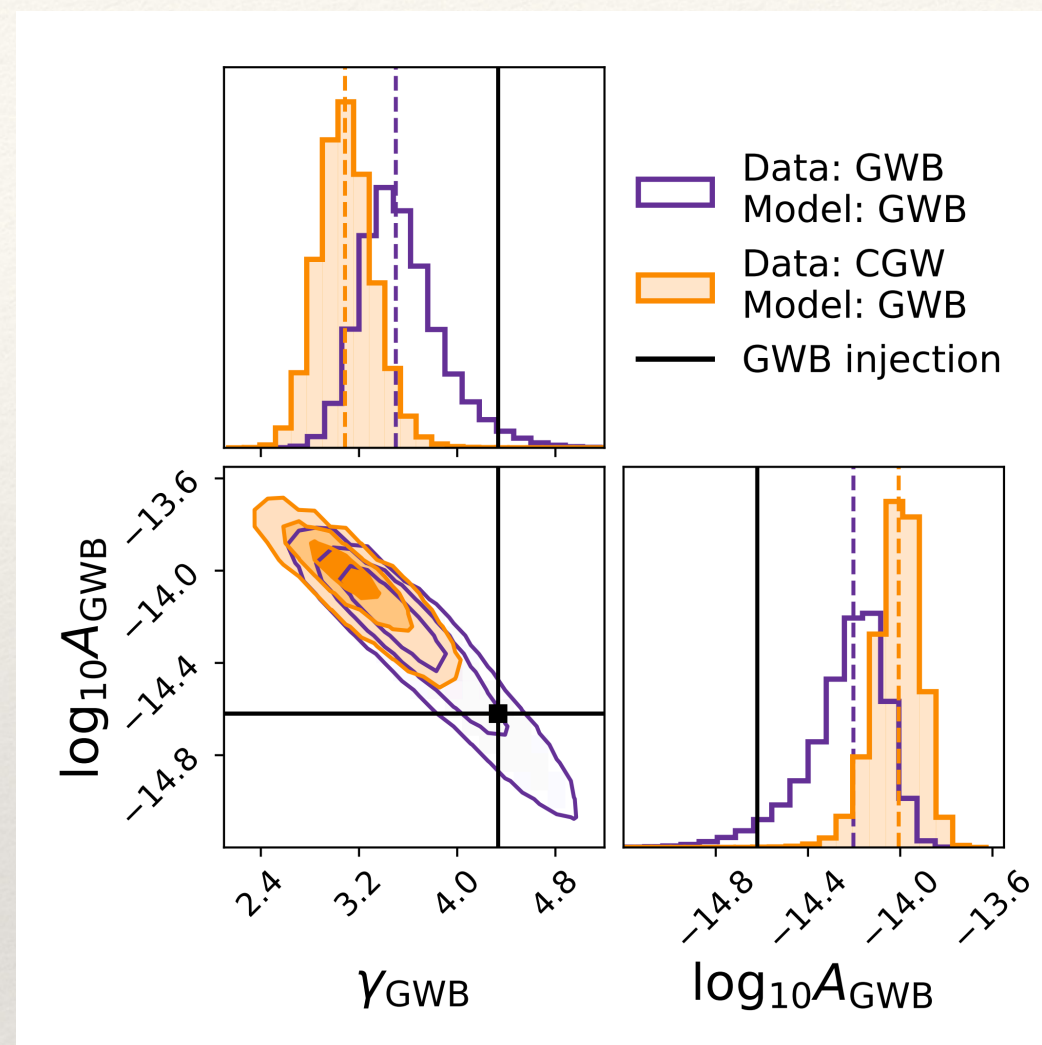
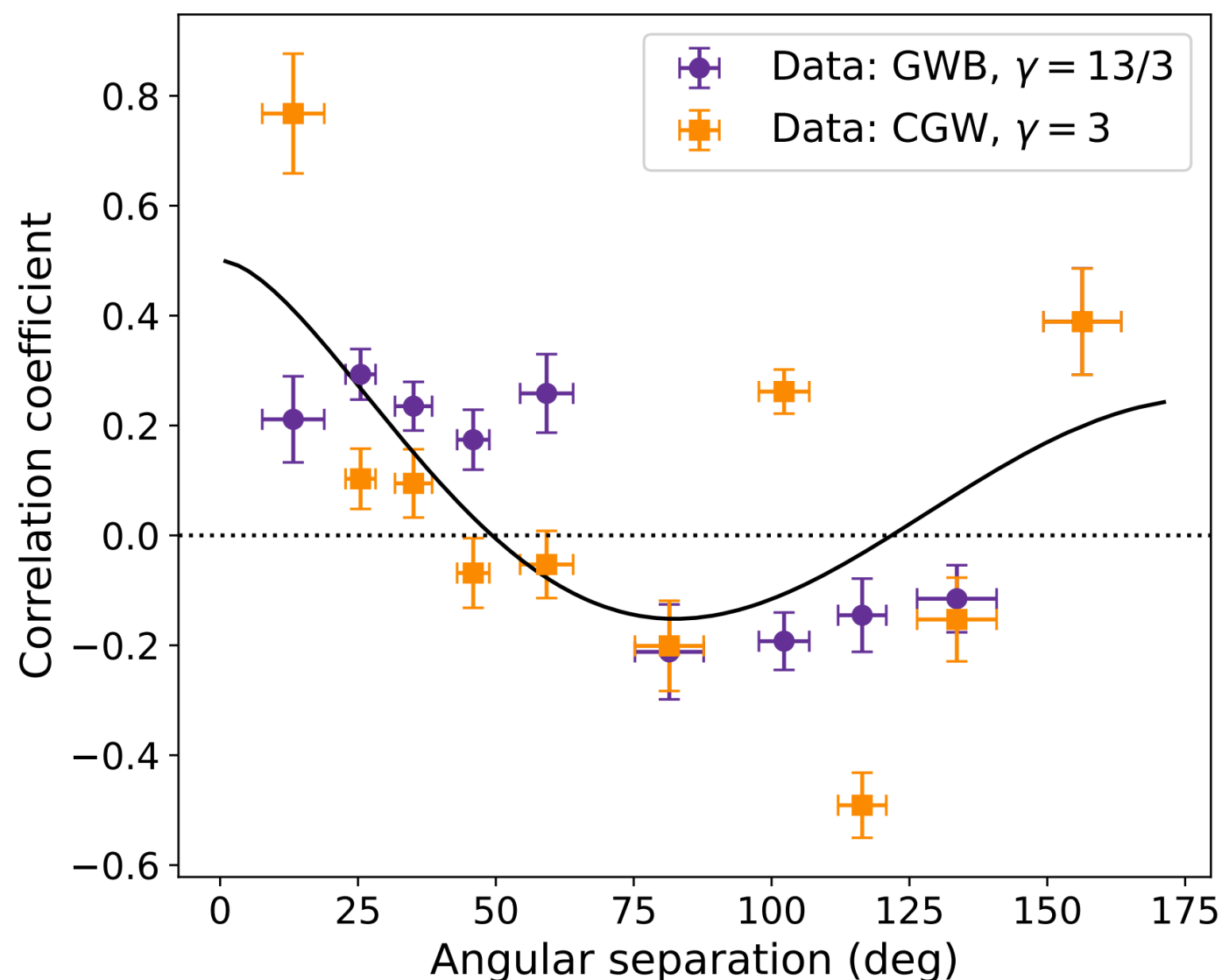


What do we hear?

[Ferranti et al 2407.21105]

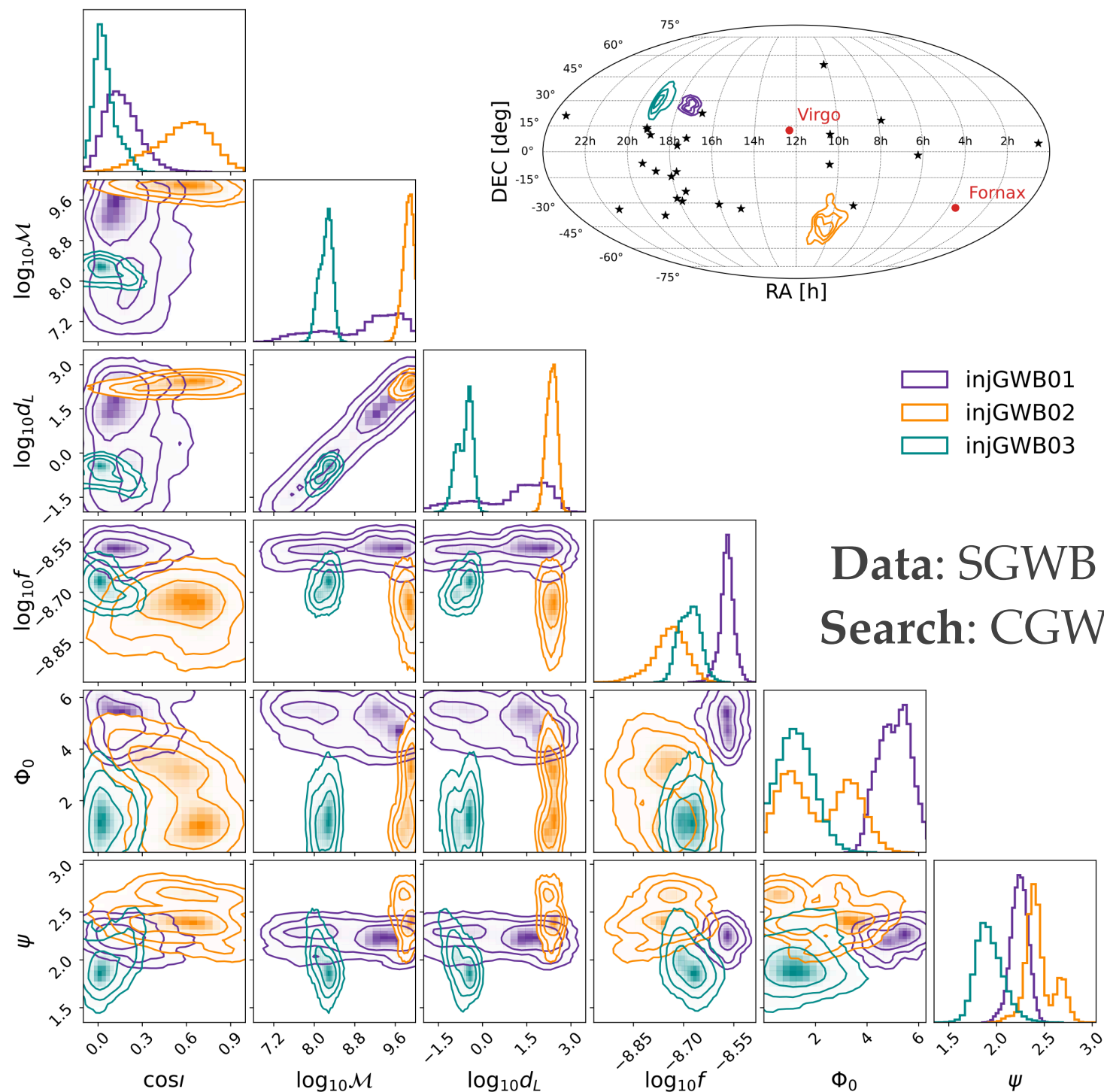
Simulated data

We recover H-D (quadrupolar) correlation analysing both SGWB and CGW



BF 10 for GWB

Do we see/hear a CGW?



We search for CGW in the simulated data containing only background (no bright sources in the population)

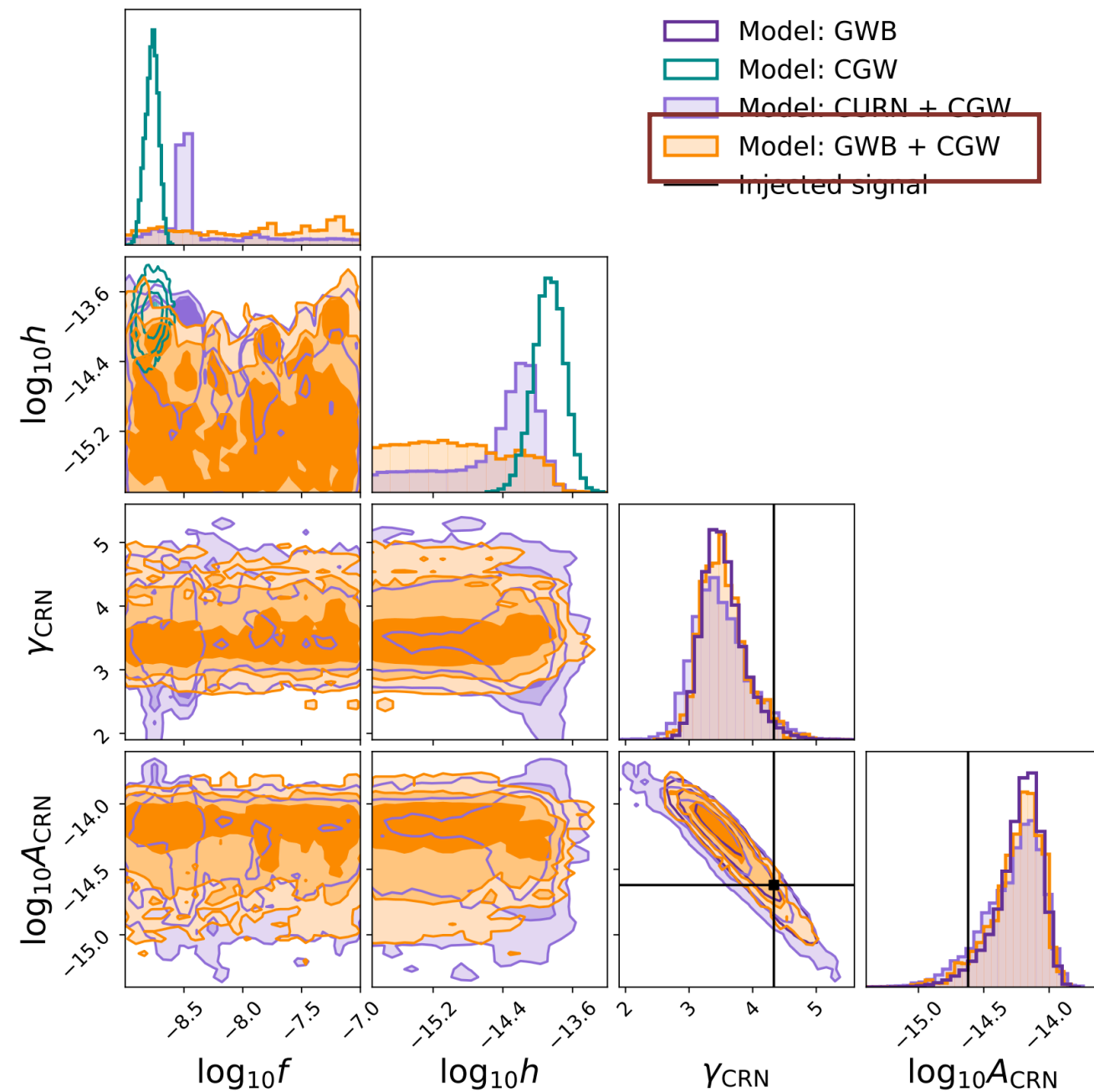
It is hard to find a black cat in the dark room especially if it is not there

Then what did we find?

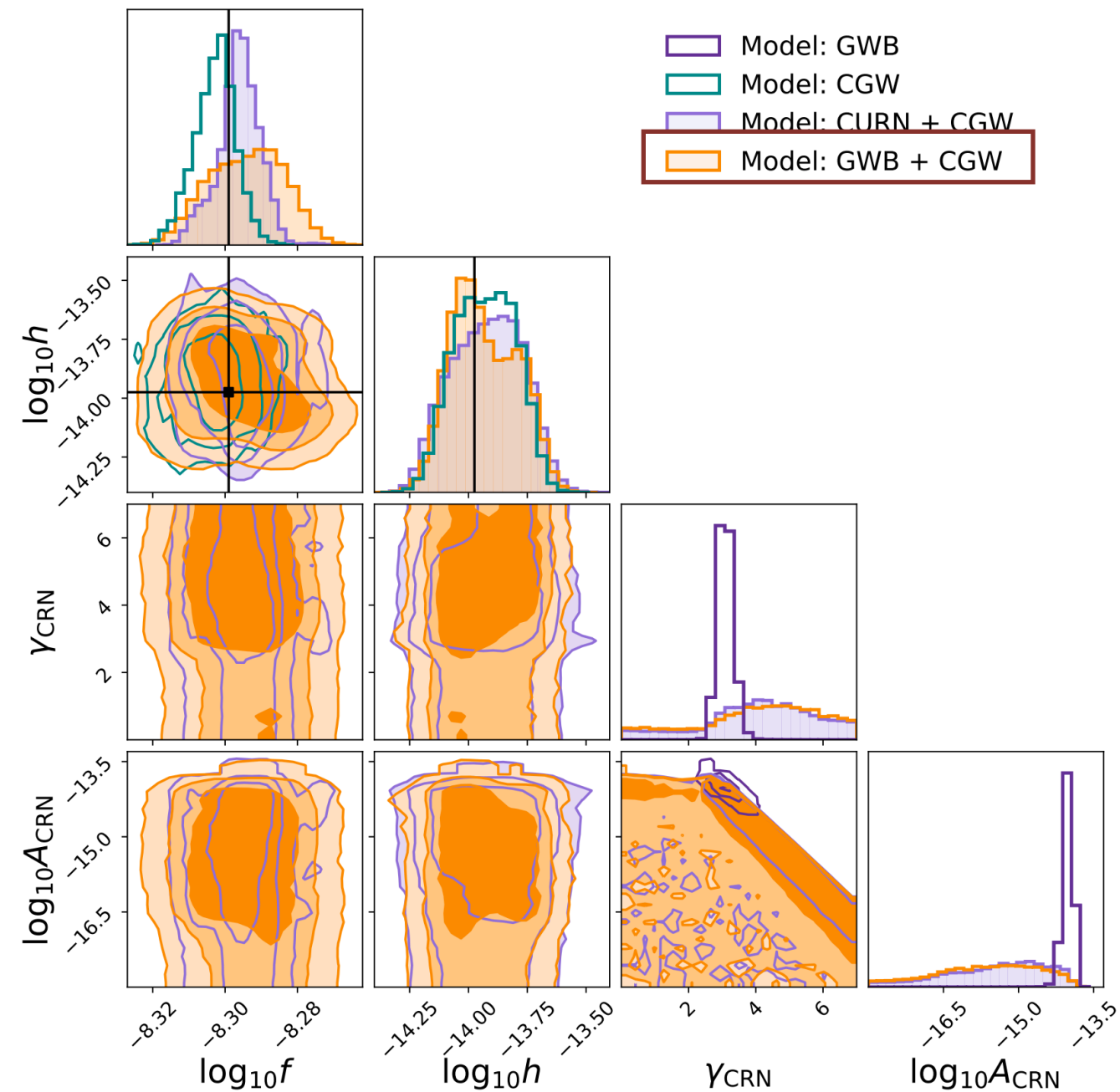
Breaking degeneracy

[Ferranti et al 2407.21105]

“strong” GW signal



Correctly recovering SGWB

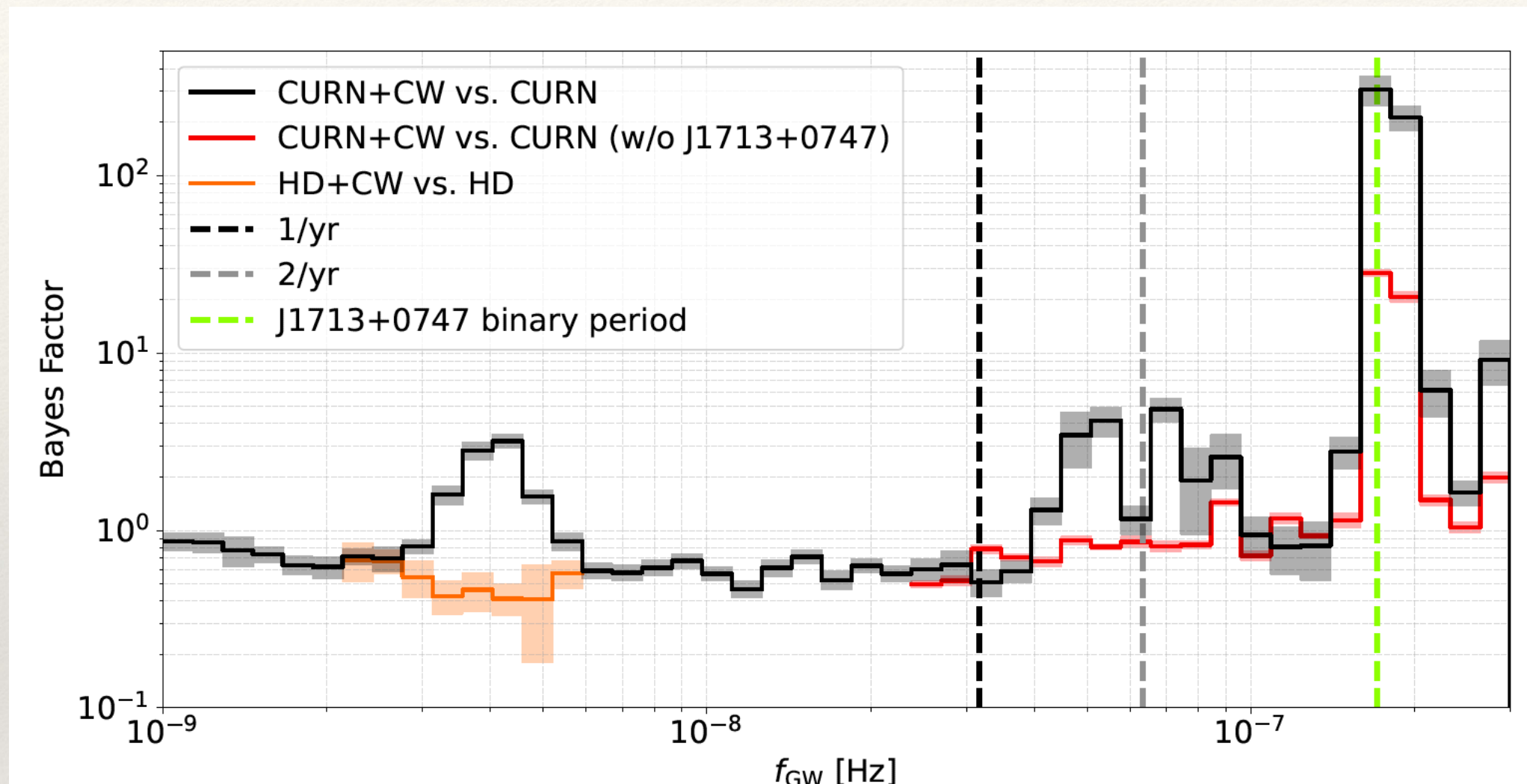


Correctly recovering circular CGW

CGW signal in NanoGrav



[NG: 2306.16222]



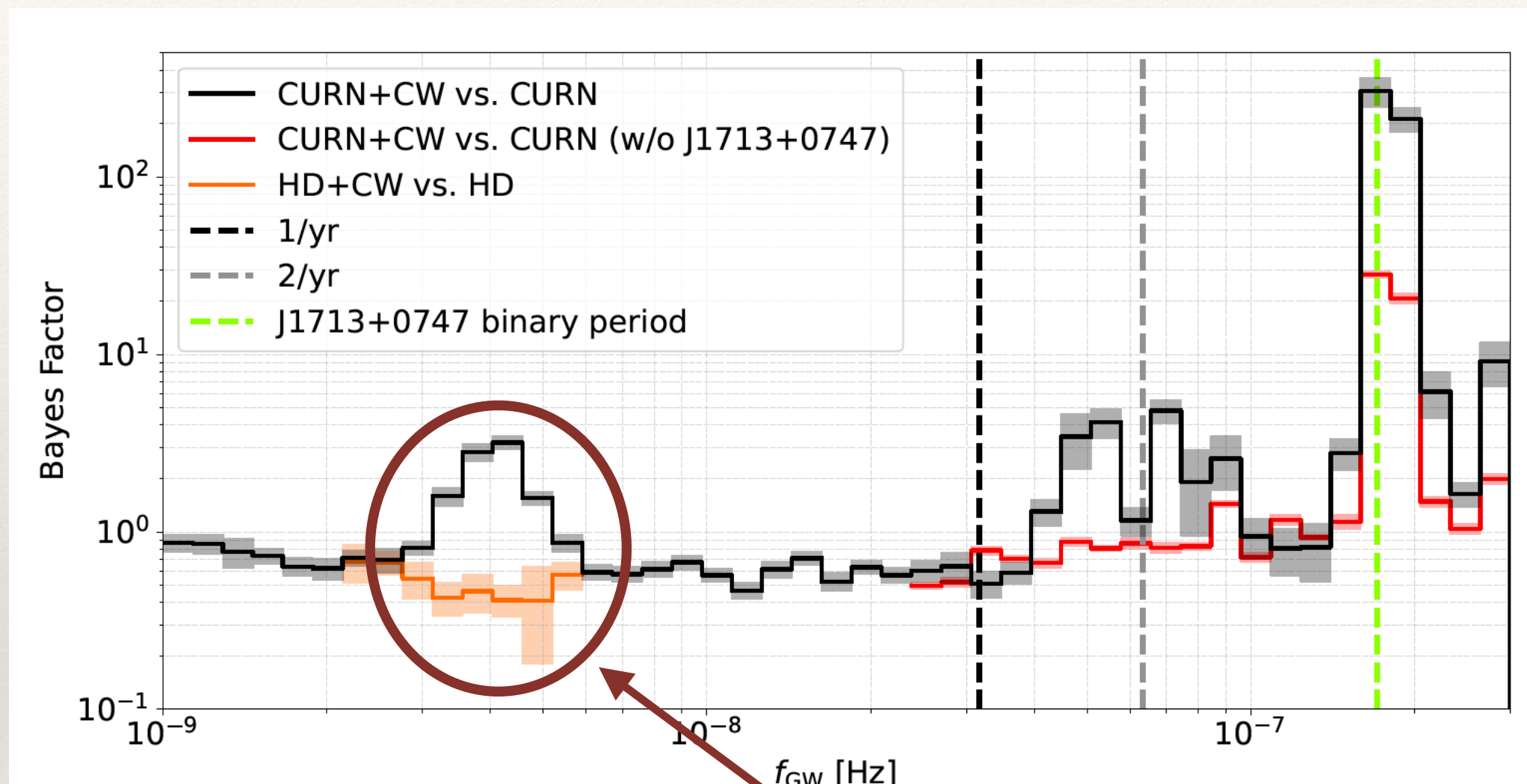
Bayesian all-sky search for a SMBHB in circular orbit: Bayes factor for presence of CGW



CGW signal



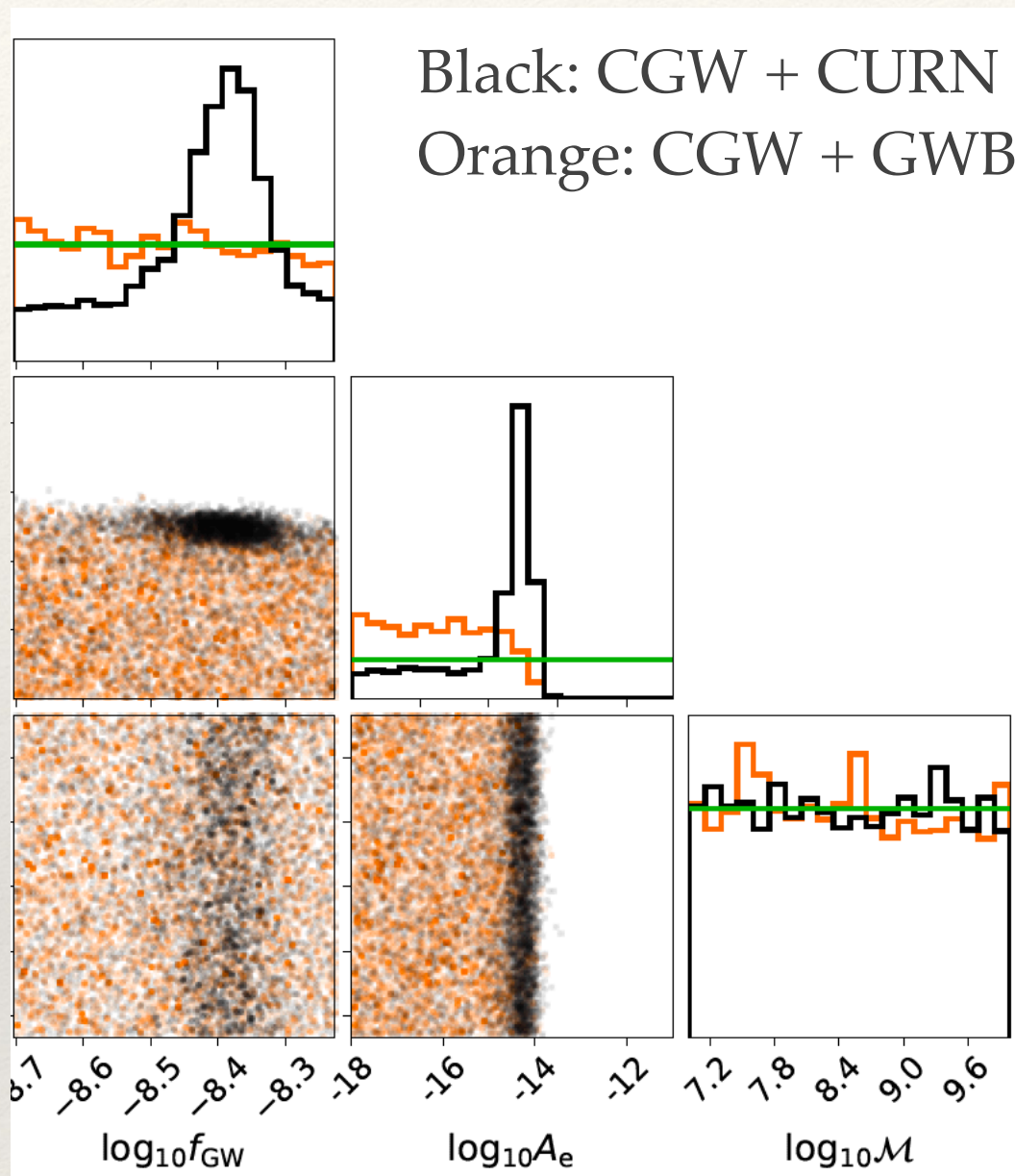
[NG: 2306.16222]



- Bayesian all-sky search for a SMBHB in circular orbit: Bayes factor for presence of CGW
- I concentrate on the low-frequency candidate

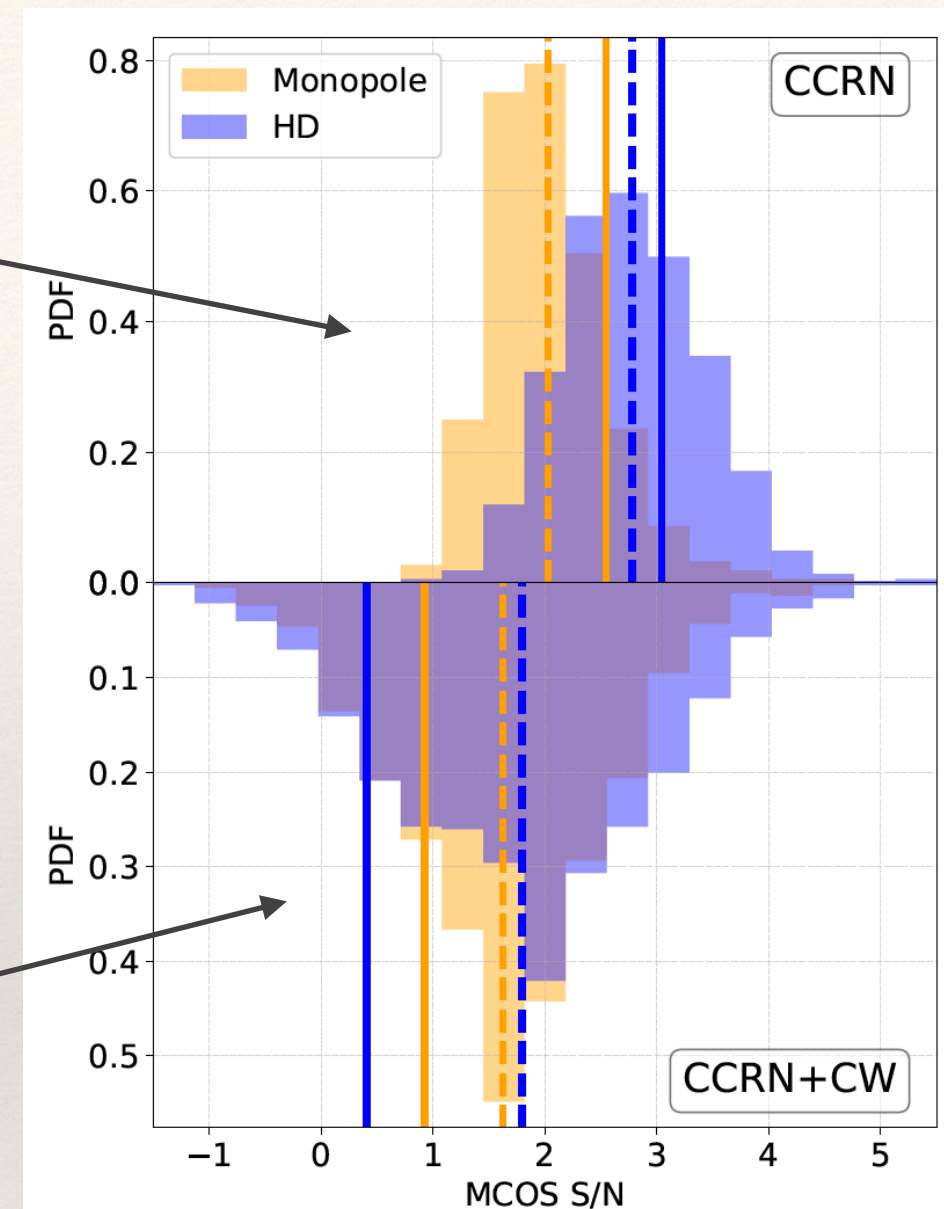


CGW in NG data

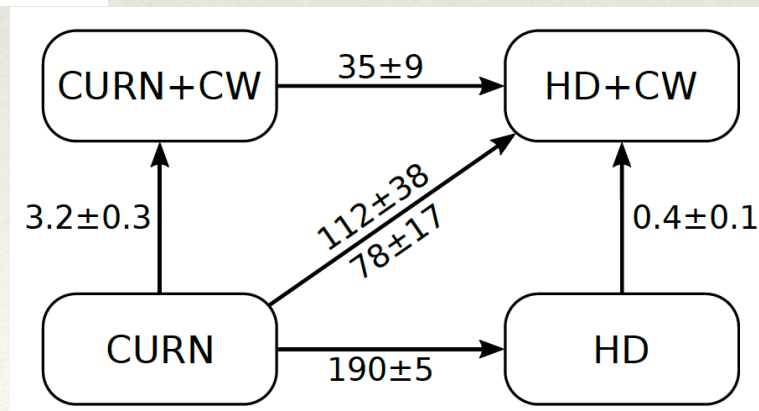


SNR of GWB and
monopole in NG data

SNR of GWB and
monopole in NG data
after removing CGW



$$f \in (3.2, 5.0) \text{ nHz}$$

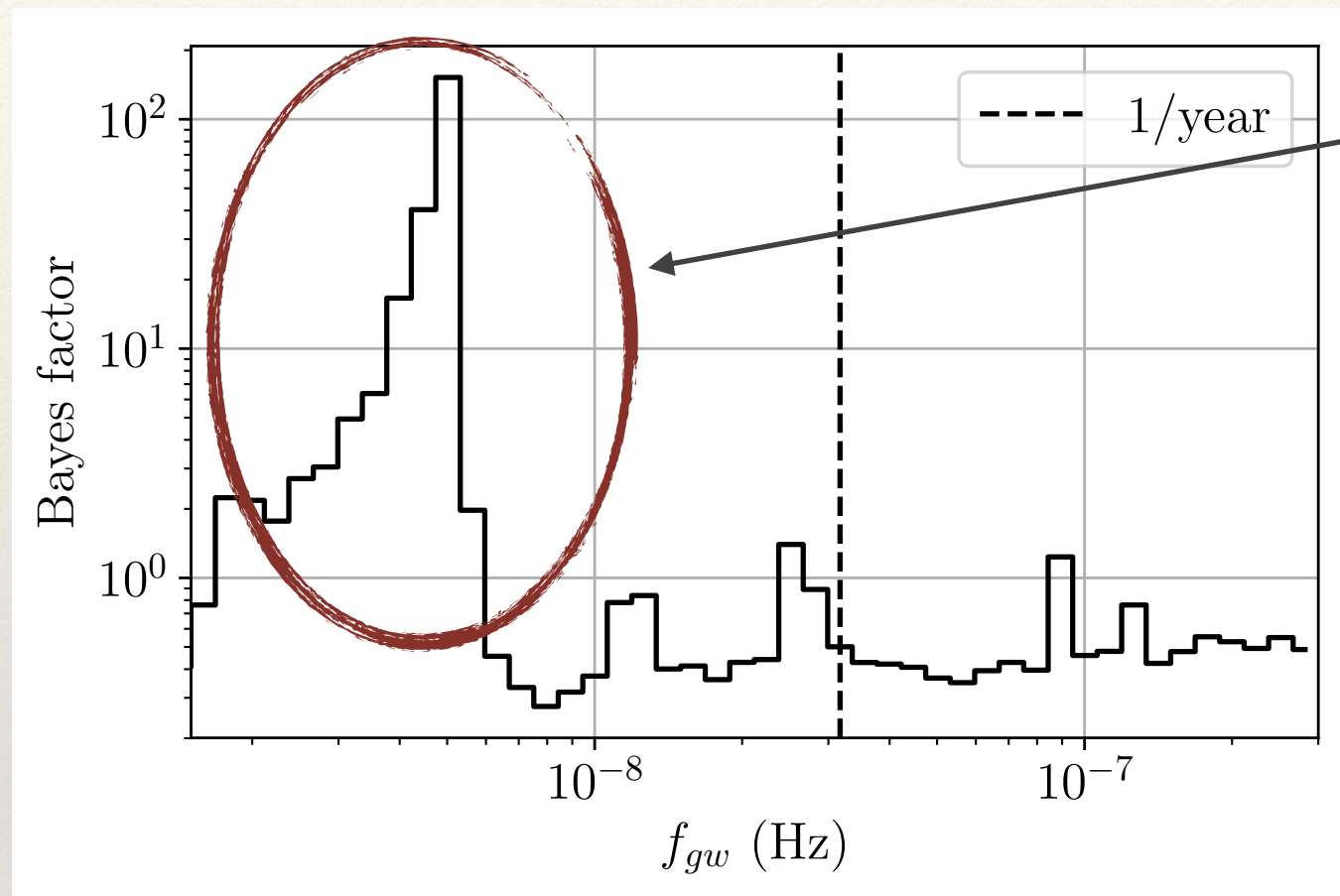


CGW signal in EPTA



BF: PSRN+CURN+CGW / PSRN+CURN

[EPTA+InPTA: 2306.16226]

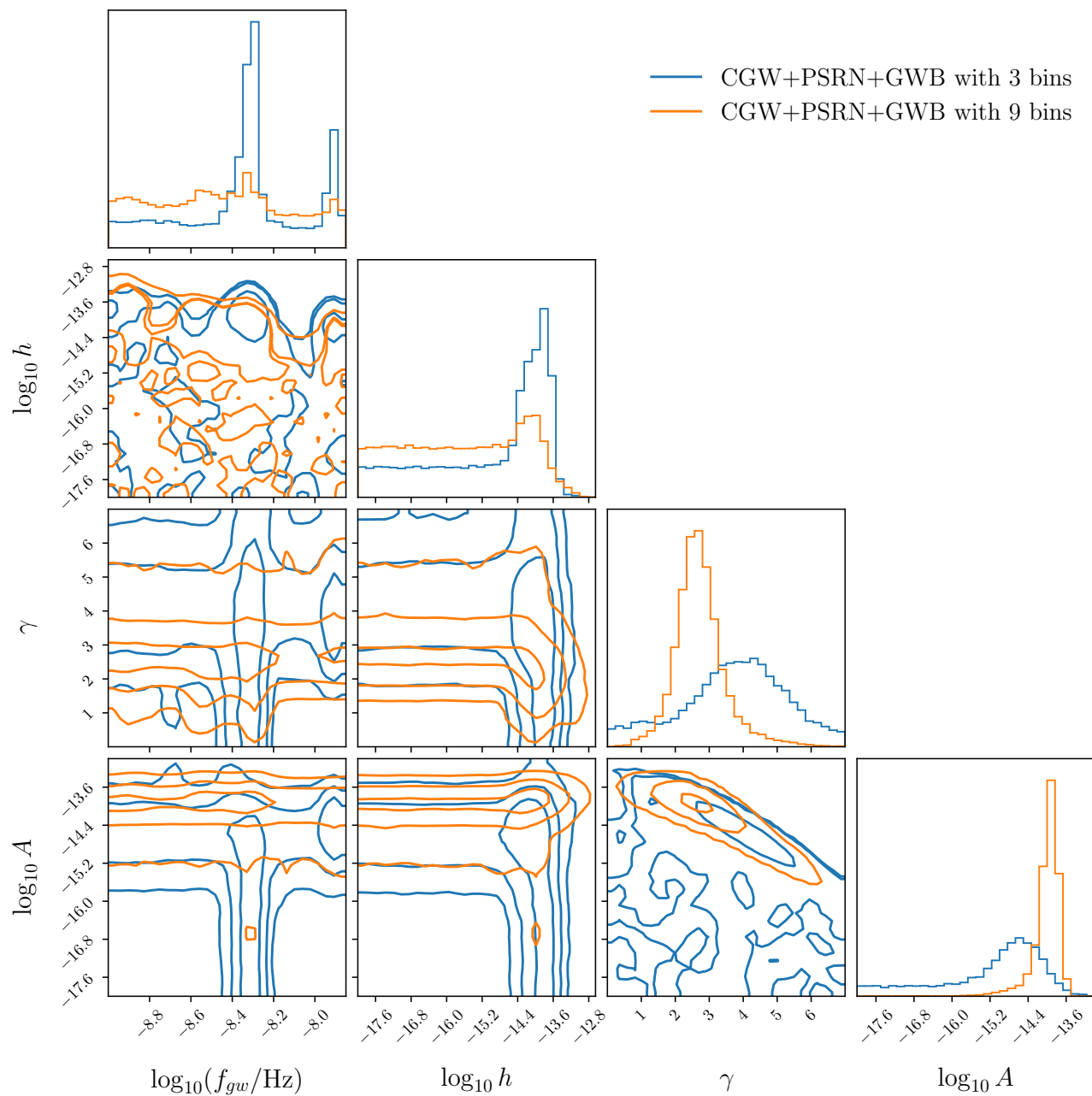


CGW candidate at low frequency

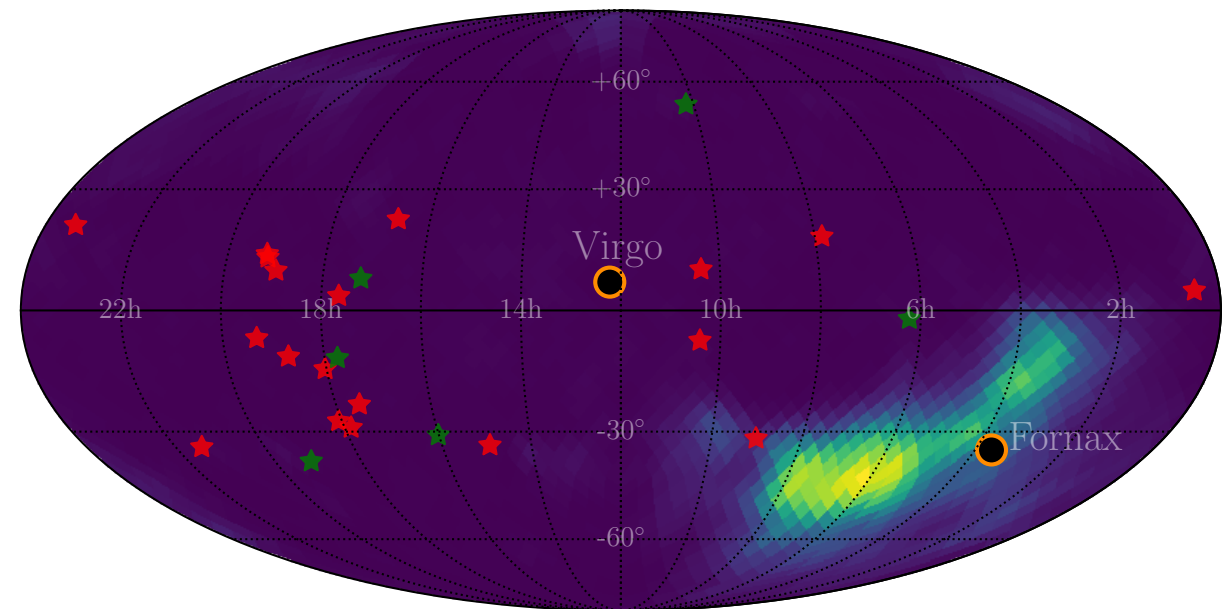
Bayesian search for CGW using Earth
term only



CGW signal in EPTA



$$f \in (3.2, 6.0) \text{ nHz}$$

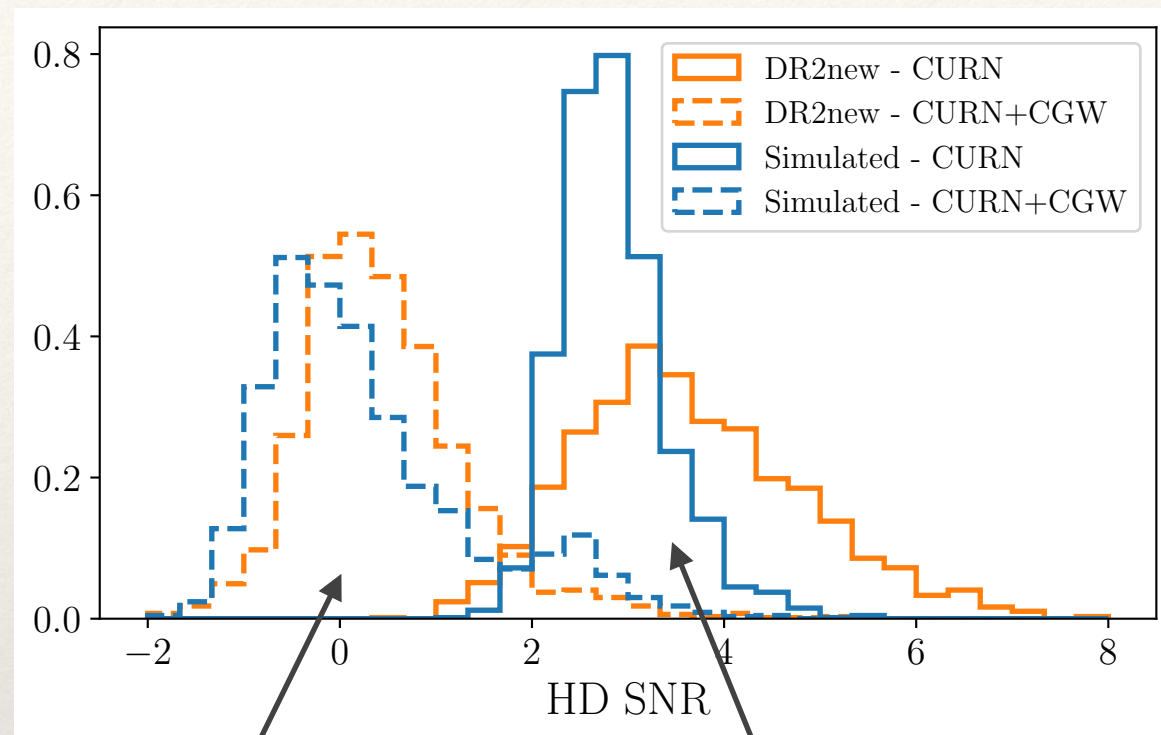


CGW: circular, Earth and Pulsar terms

Model comparison	Bayes factor
CGW+PSRN vs PSRN	4000
CGW+PSRN+CURN vs PSRN+CURN, 3 bins	12
CGW+PSRN+CURN vs PSRN+CURN, 9 bins	4
CGW+PSRN+GWB vs PSRN+GWB, 3 bins	1
CGW+PSRN+GWB vs PSRN+GWB, 9 bins	0.7



CGW signal in EPTA



GWB SNR after
removing CGW

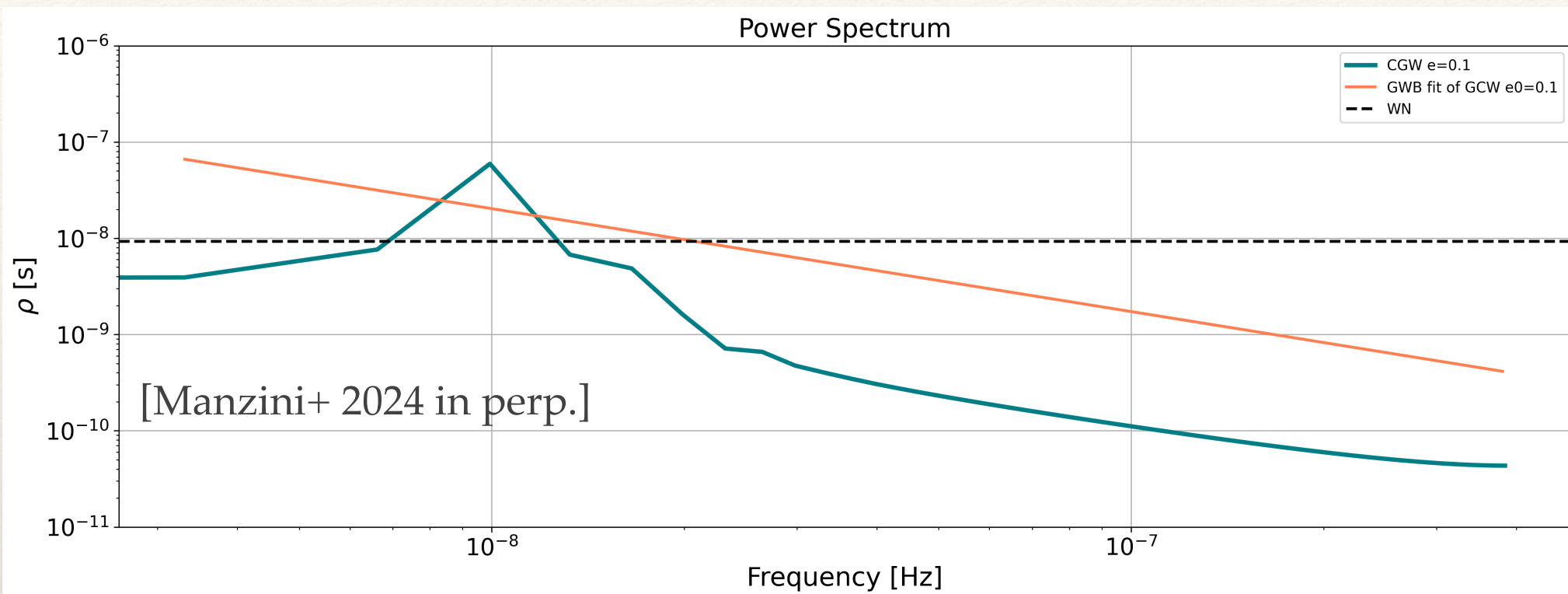
GWB SNR

Weak signal...
Need more data and more pulsars
for proper interpretation

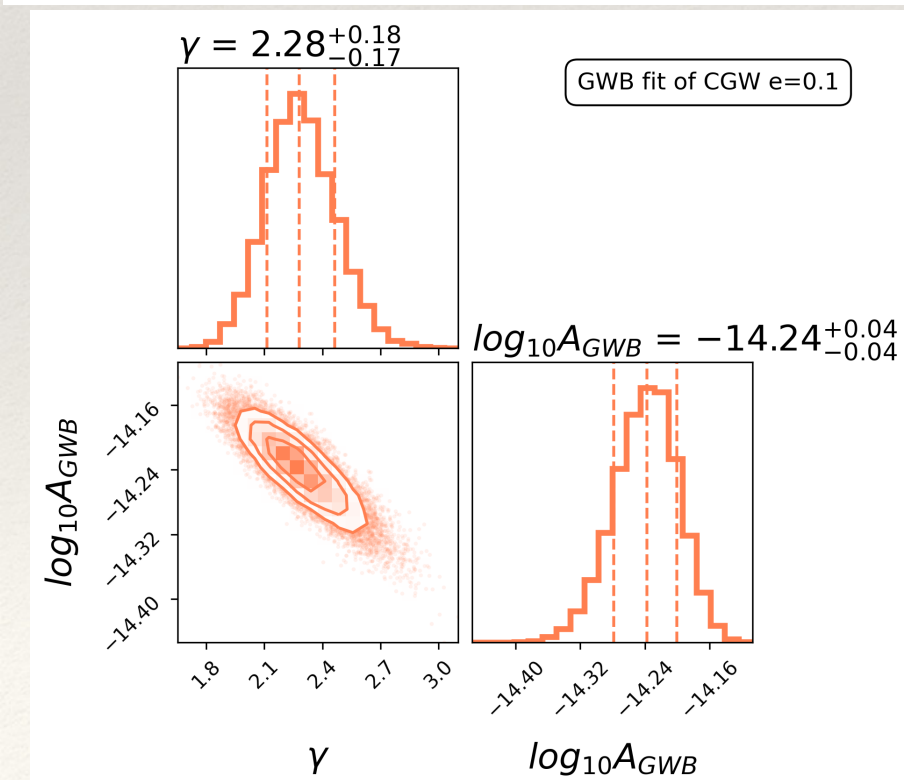


What about eccentric binaries?

Eccentricity spreads the power across harmonics $\rightarrow$ brings it closer to the stochastic signal



eccentricity=0.1



Simulated data

Recovered as
SGWB

Peters & Mathews

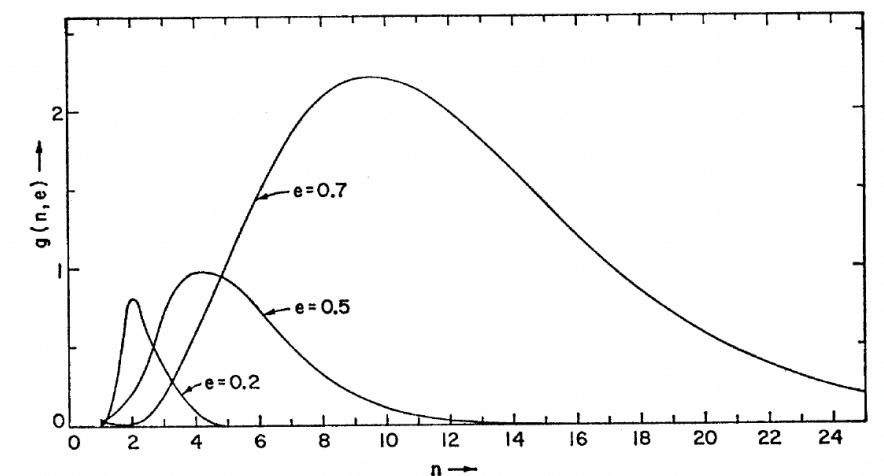
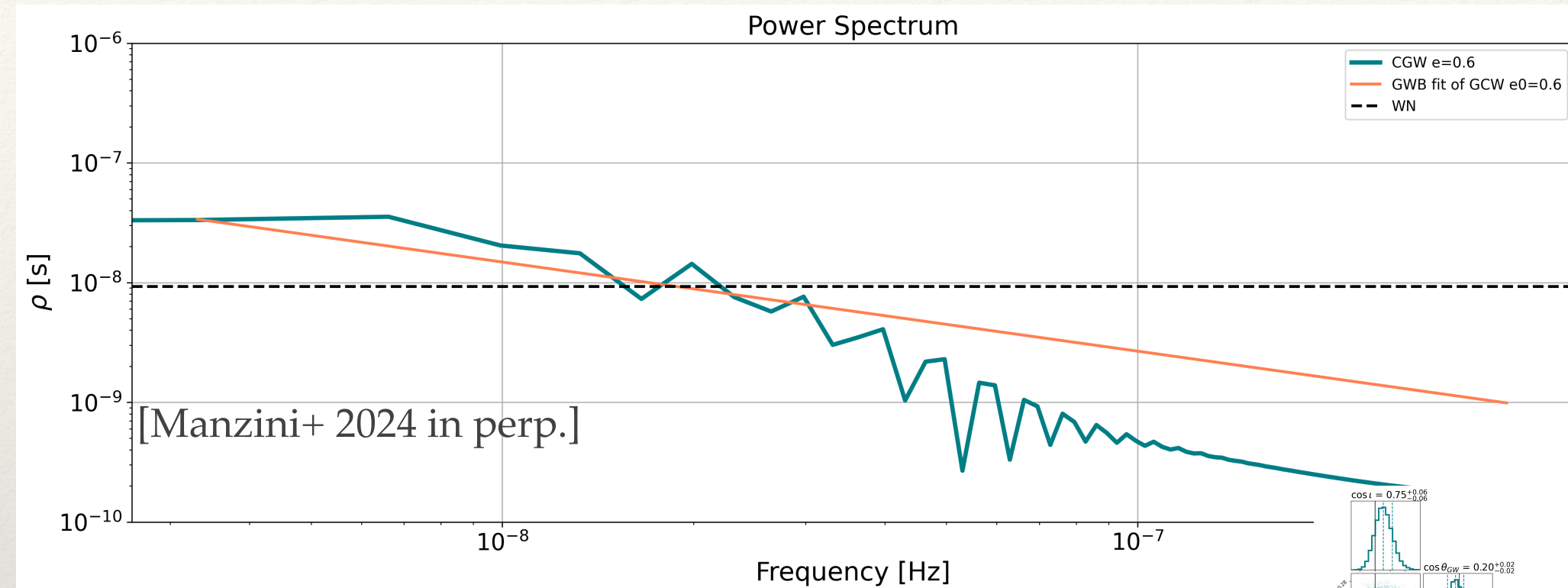


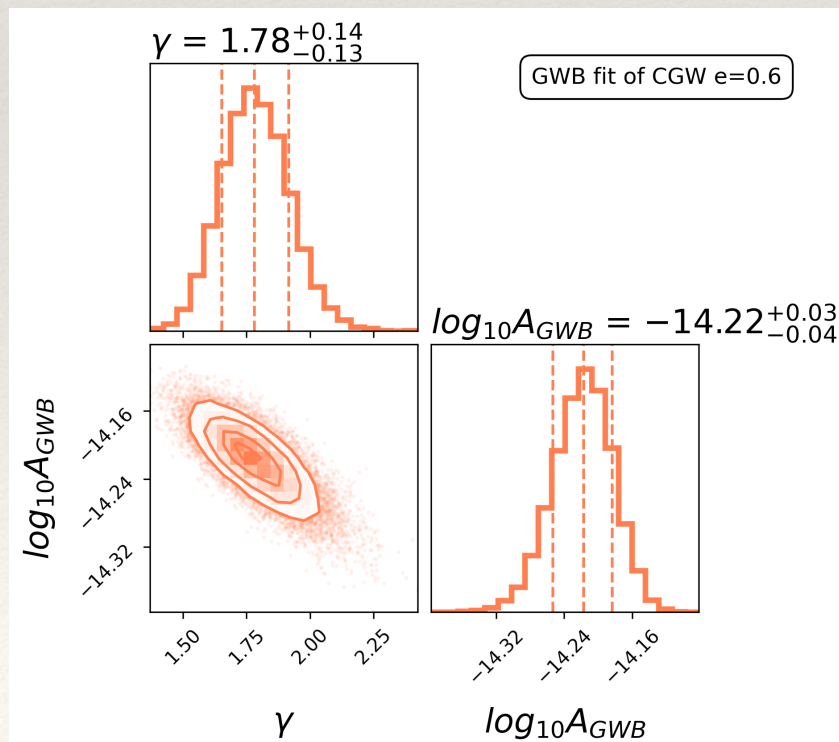
FIG. 3. $g(n,e)$, the relative power radiated into the n th harmonic for $e=0.2, 0.5$, and 0.7 .

What about eccentric binaries?

Eccentricity spreads the power across harmonics $\rightarrow$ brings it closer to the stochastic signal



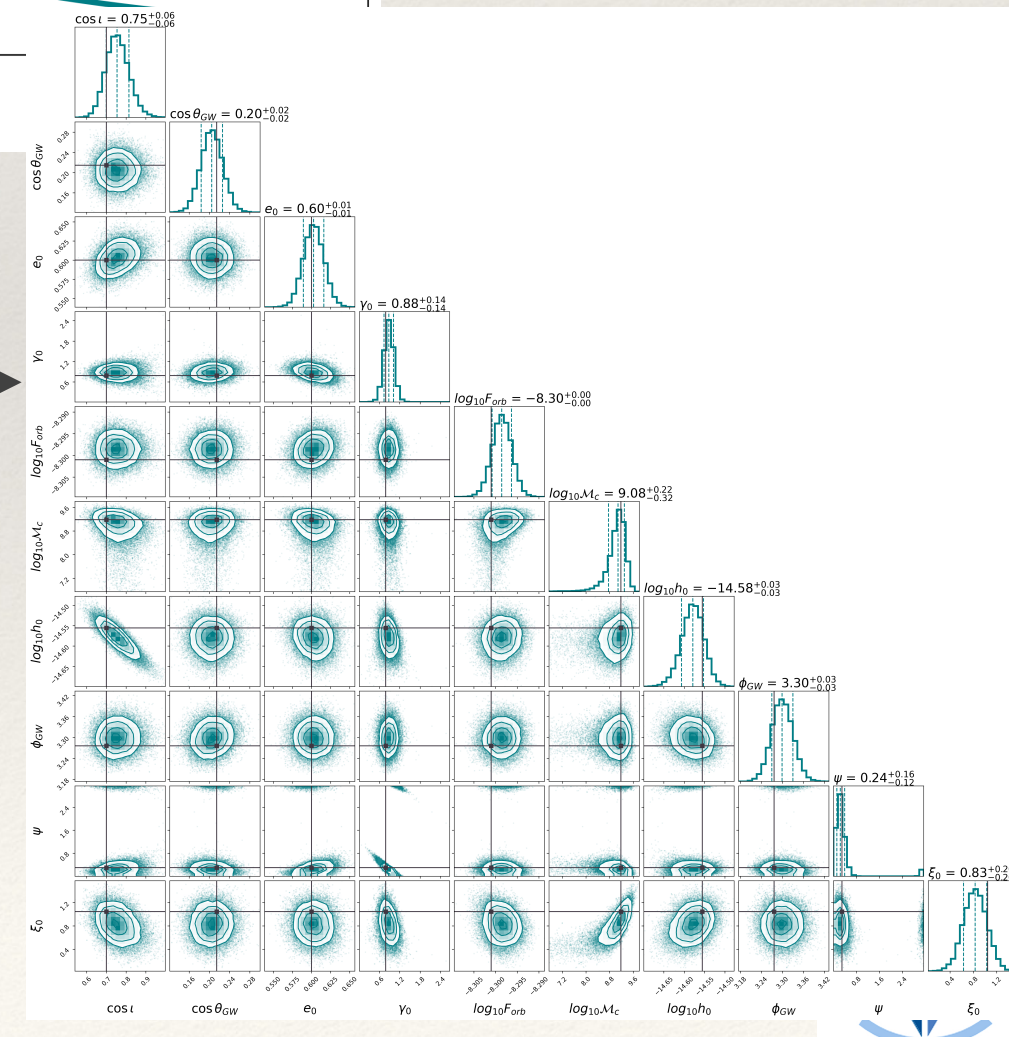
eccentricity=0.6



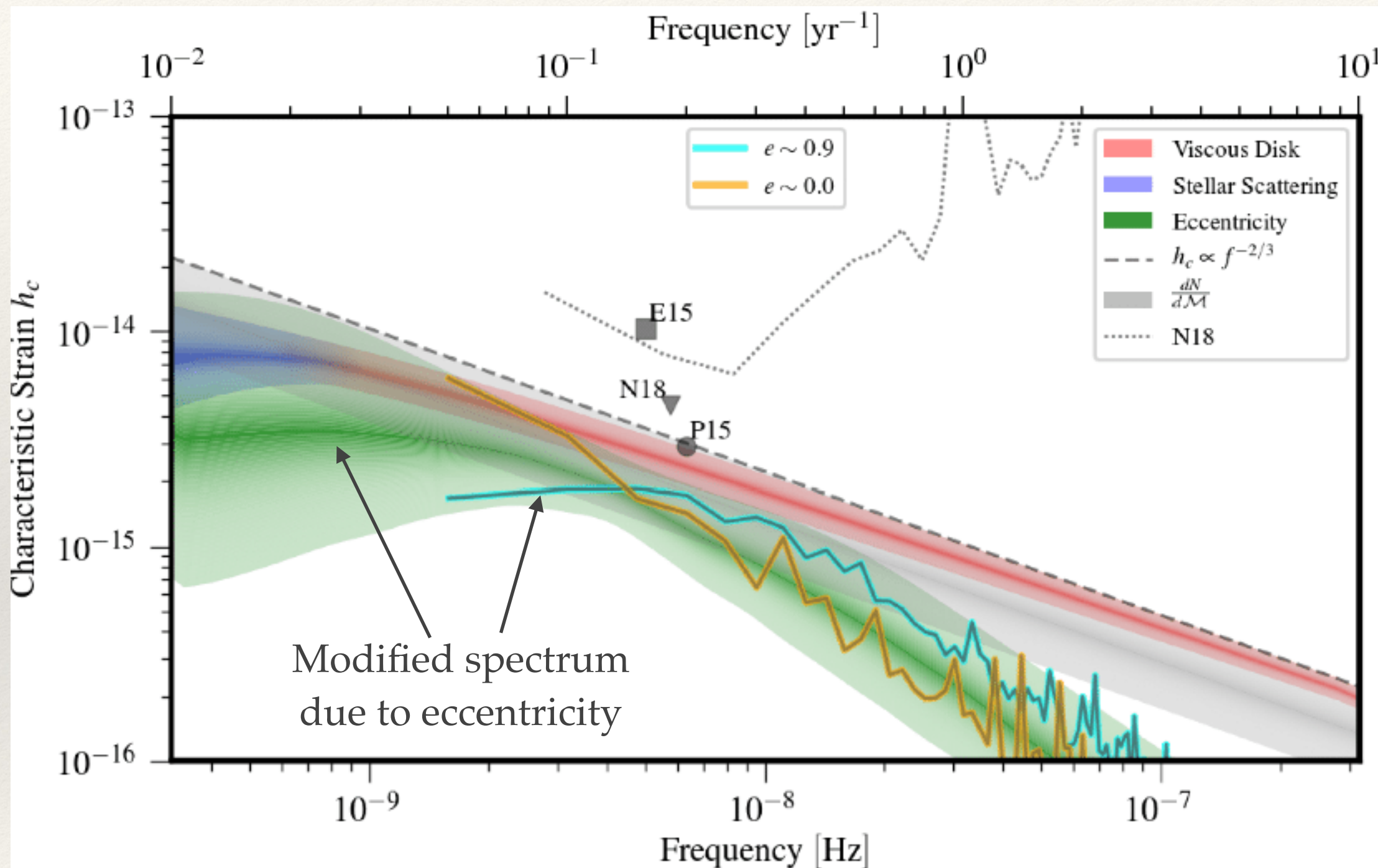
Simulated data

Recovered as
CGW (eccentric)

Recovered as
SGWB



Population of eccentric SMBHBs



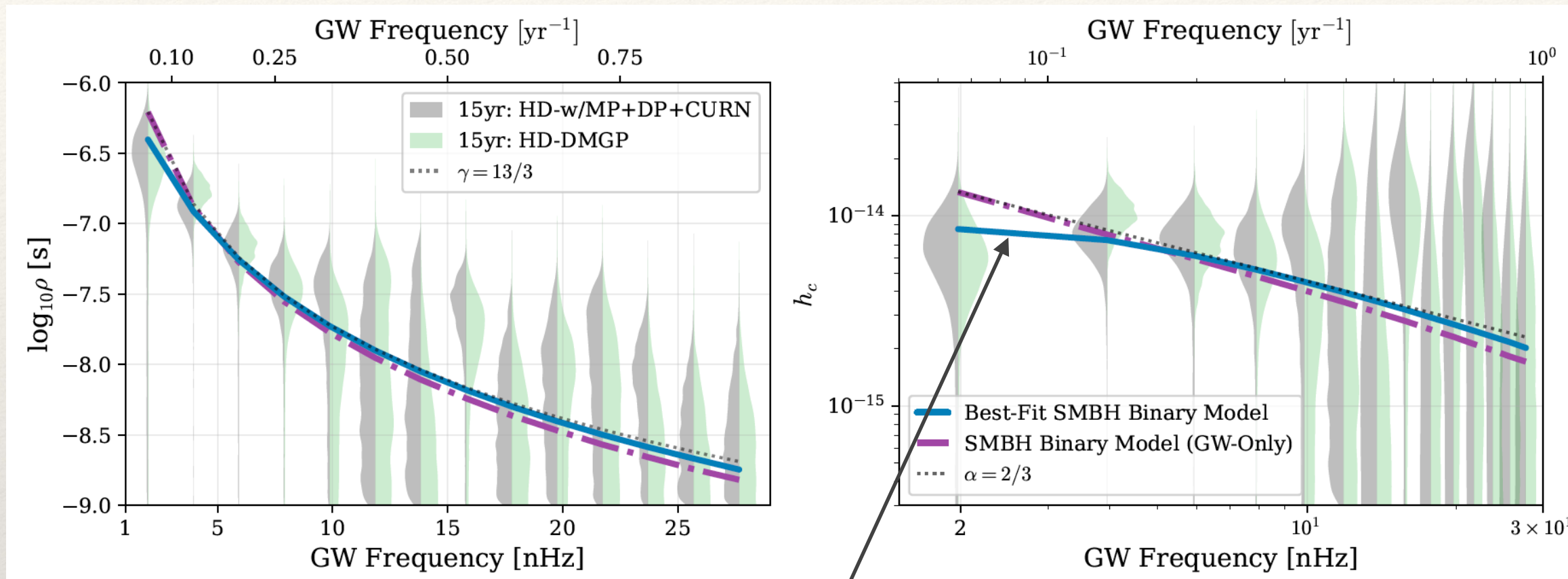
[S. Burke-Splolaor A&A review (2019)]



Population of eccentric SMBHBs



[NG: 2306.16220]



The model which allows eccentricity and interaction with stellar environment better describes the observed data (than circular GW-driven population)

[See Andrea's talk]

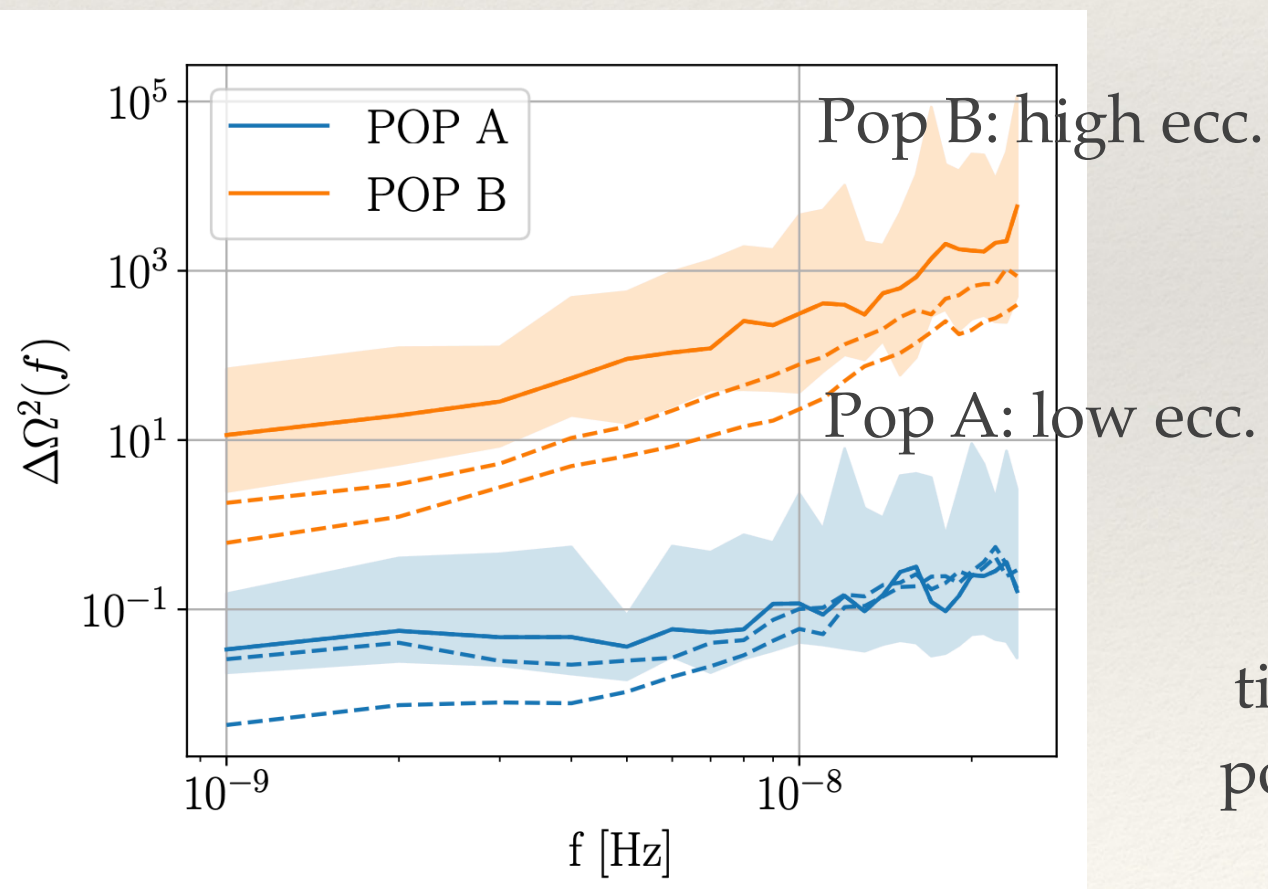


Non-stationarity?

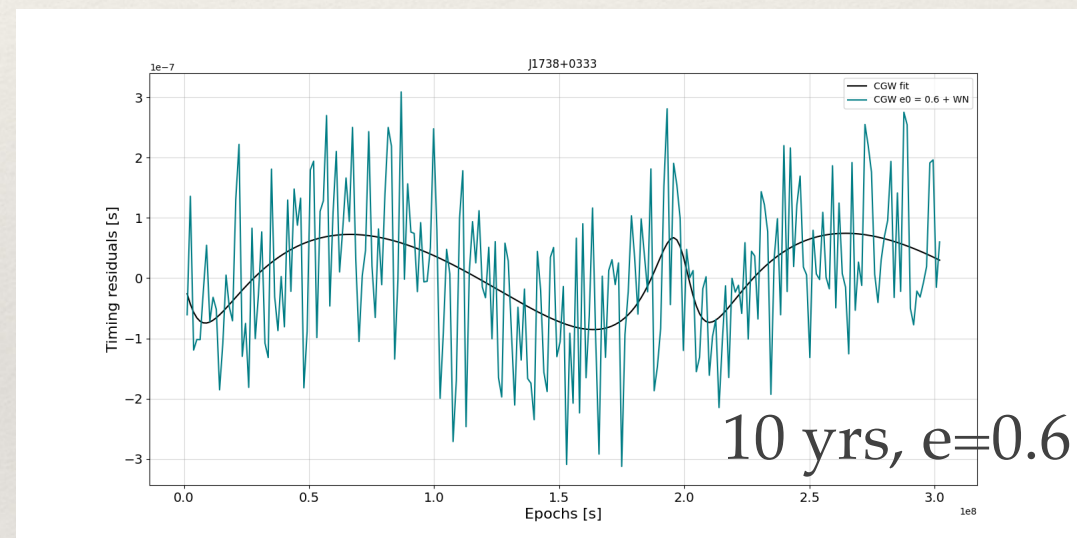


The population of eccentric SMBHB

- Dominated by massive binaries $> 10^8 M_\odot$ [Quelquejay+ 2024 in prep]
- Dominated by relatively close and bright binaries 100-1000
- If eccentricity is significant (>0.5): significant variations of the residuals amplitude over one period: significant for low frequencies
 - stochasticity is destroyed at high frequencies (small number statistic): “non-stationarity”
 - Non-stationarity at low freq.



[Falxa+ 2412.01899]



Even weak eccentric CGW signal $\text{SNR} \approx 1$ (w.r.t the SGWB) could induce a significant time-dependent fluctuations: non-stationarity possible way of separating CGW from SGWB



So... What remains in your brain after this talk?



So... What remains in your brain after this talk?

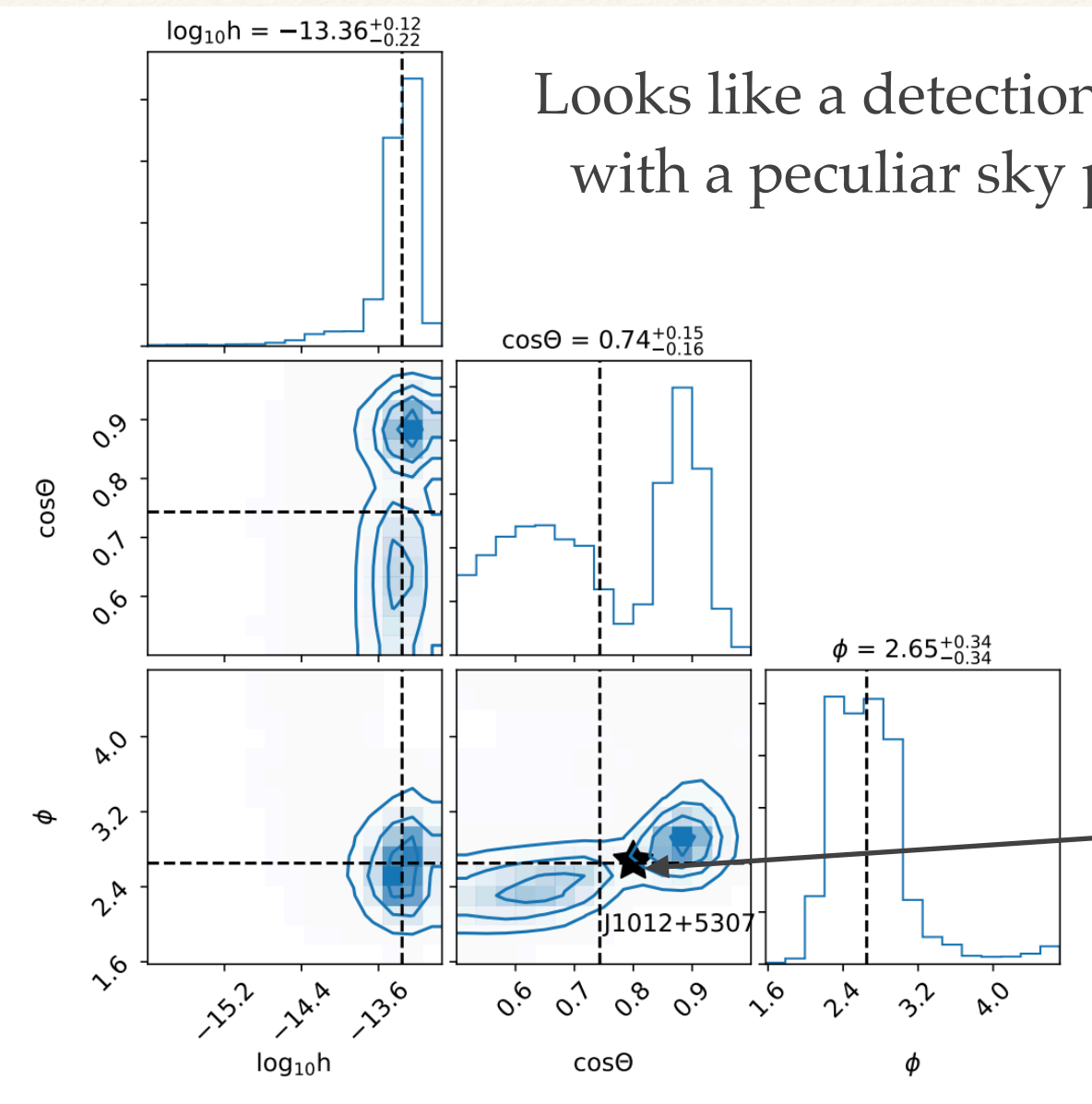


1. PTA is cool
2. Continuous GW sources much more interesting than stochastic signal
3. What do PTAs detect?
 - Probably SGWB but ... need a solid confirmation
4. Search for CGW
 - Ok to search using Earth term only (not optimal, possible biases)
 - Earth+Pulsar terms should be used to improve and interpret the preliminary results of E-term only search
5. Can we distinguish between CGW and SGWB? - Yes/probably
 - Use combined analysis: SGWB+CGW
 - CGW will produce anisotropy (one or several hot spots)
 - Non-stationarity in SGWB indicates that it dominates by a few eccentric binaries.

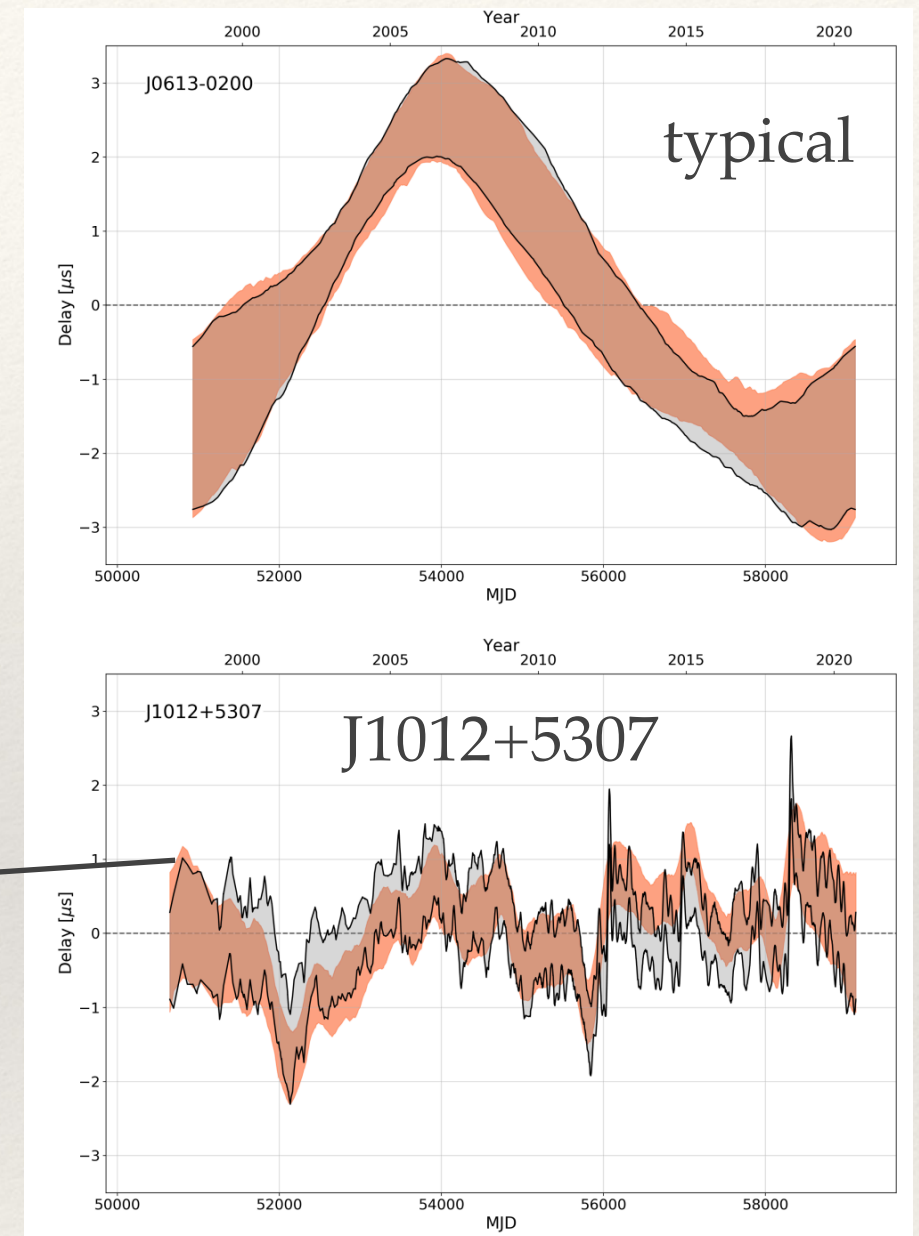


Effect of the noise model

[Falxa+ 2303.10767]



Red noise



Going from orange noise model to grey eliminates spurious CGW

New IPTA dataset (DR3)



Credits: Kuo Liu

IPTA DR3 dimensions

- In total **121** pulsars in full DR3;
 - The biggest / most sensitive PTA dataset ever made !!

Dataset	Number of pulsars	Time span	Frequency range
EPTA DR2	25	24.5 yr	283 – 5107 MHz
NANOGrav 15-yr	68	15.9 yr	302 – 3988 MHz
PPTA DR3	24	18.1 yr	704 – 4032 MHz
InPTA DR1	15	3.5 yr	300 – 1460 MHz
MeerKAT DR2	83	4.5 yr	856 – 1712 MHz
CHIME DR1	11	2.5 yr	400 – 800 MHz
LOFAR+NenoFar	17	9.6 yr	35 – 190 MHz
IPTA DR3	121	~25 yr	~30 – 5000 MHz

3

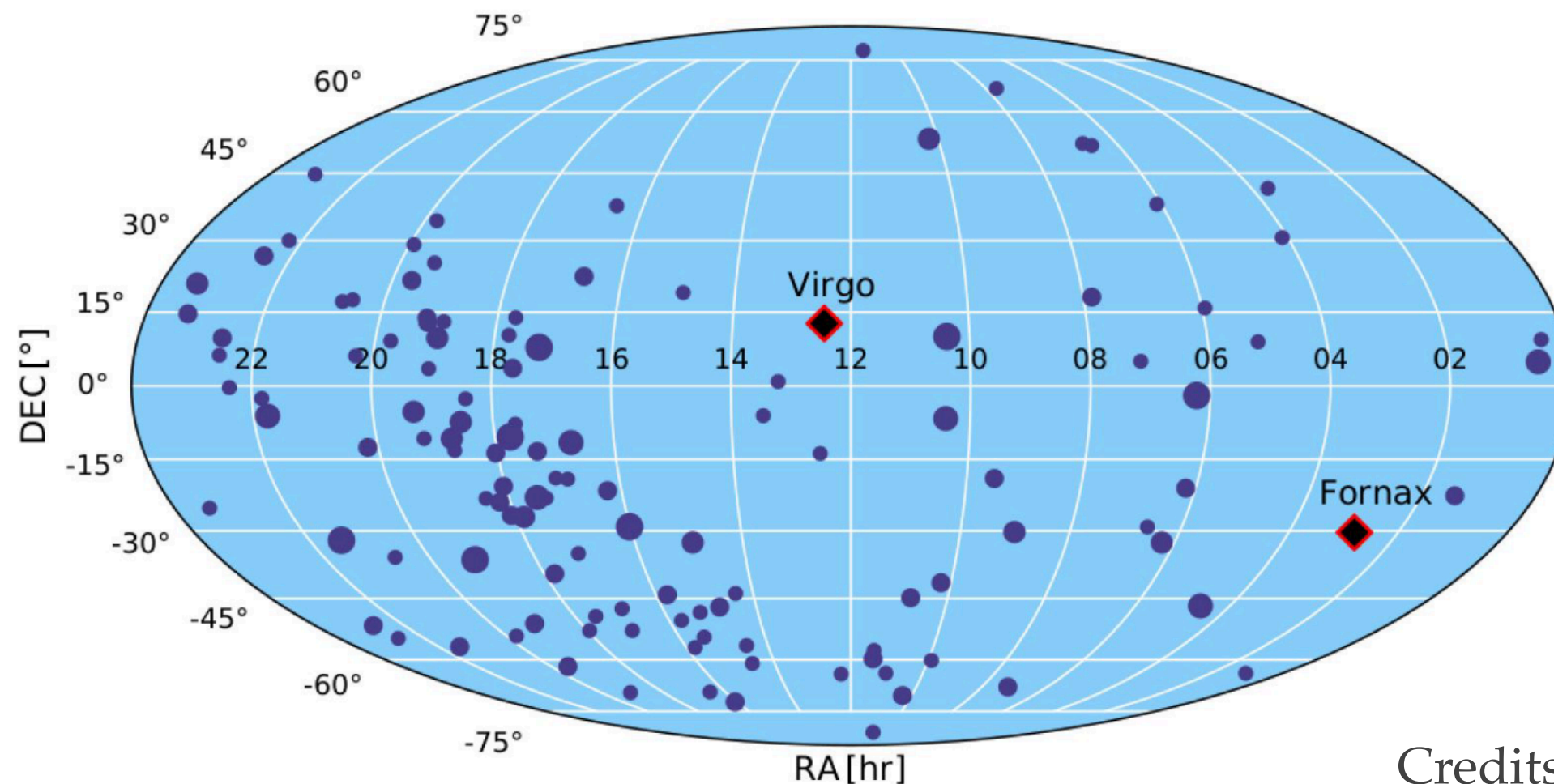


New IPTA dataset (DR3)



IPTA DR3 dimensions

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Credits Kuo Liu

4



What's next?



IPTA data combination:

- We combine the data from IPTA: EPTA, NG, InPTA, PPTA
- We use additional data (MeerKAT, Chime)
- Better coverage (dense) in time (smaller cadence)
- Better coverage in radio freq: DM and scattering variations
- Not dominated by a single radiotelescope: should see/handle systematics

Kind of summary...

- We are pretty sure that the observed signal is GW
- We are not sure about its nature
- We got so excited that made a big press release
- In relativity we need to look at IPTA data, we need longer high quality data. It is “GW detection in slow motion”

