

Towards a consistent picture of in-medium parton showering

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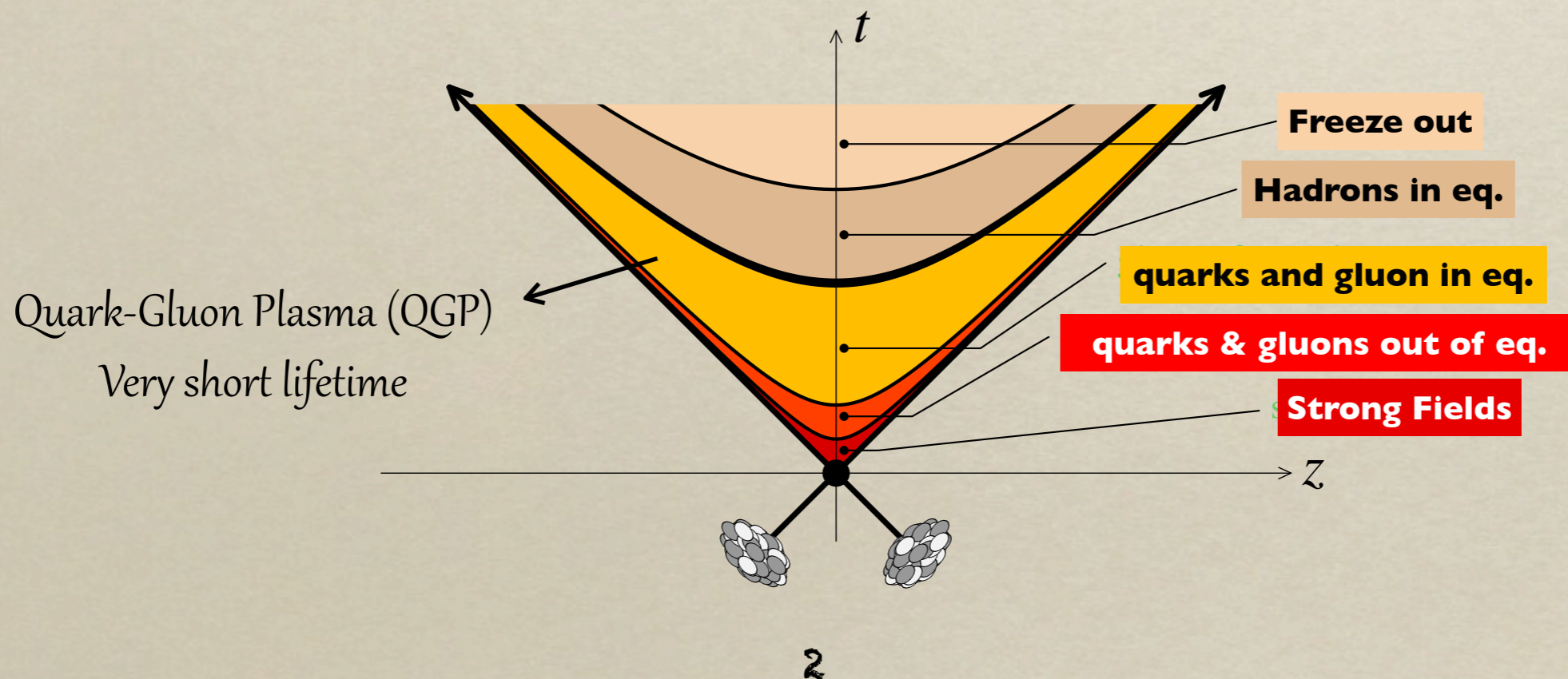
(Preprint in preparation.....)

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V CPAN days, Santiago de Compostela, Spain

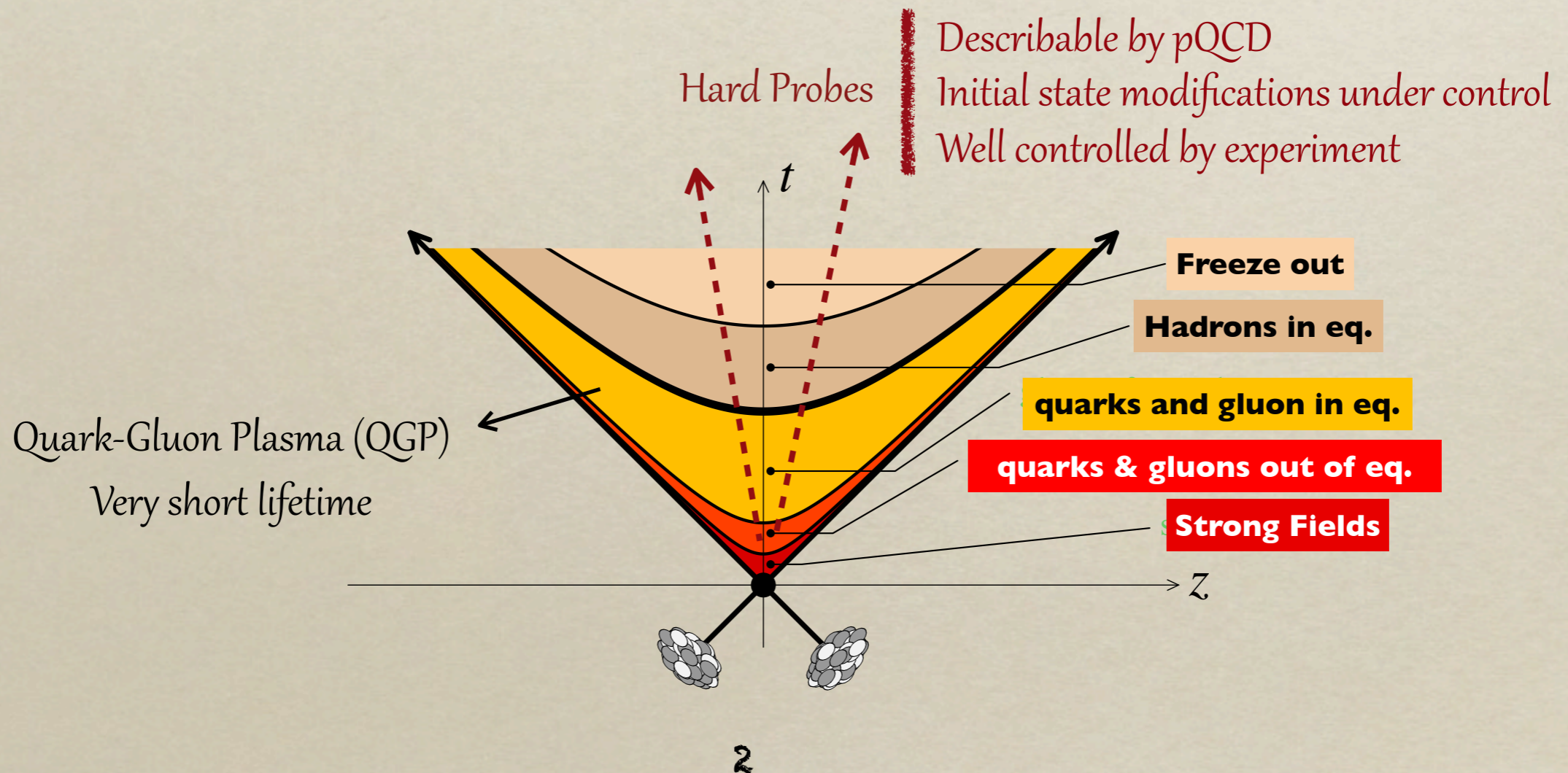
Introduction

- Heavy-ion collisions:
 - Production of the Quark-Gluon Plasma \Rightarrow Opportunity to study QCD in a regime that is usually not accessible perturbatively;
 - \Rightarrow Lifetime very short \Rightarrow Use of self generated probes!



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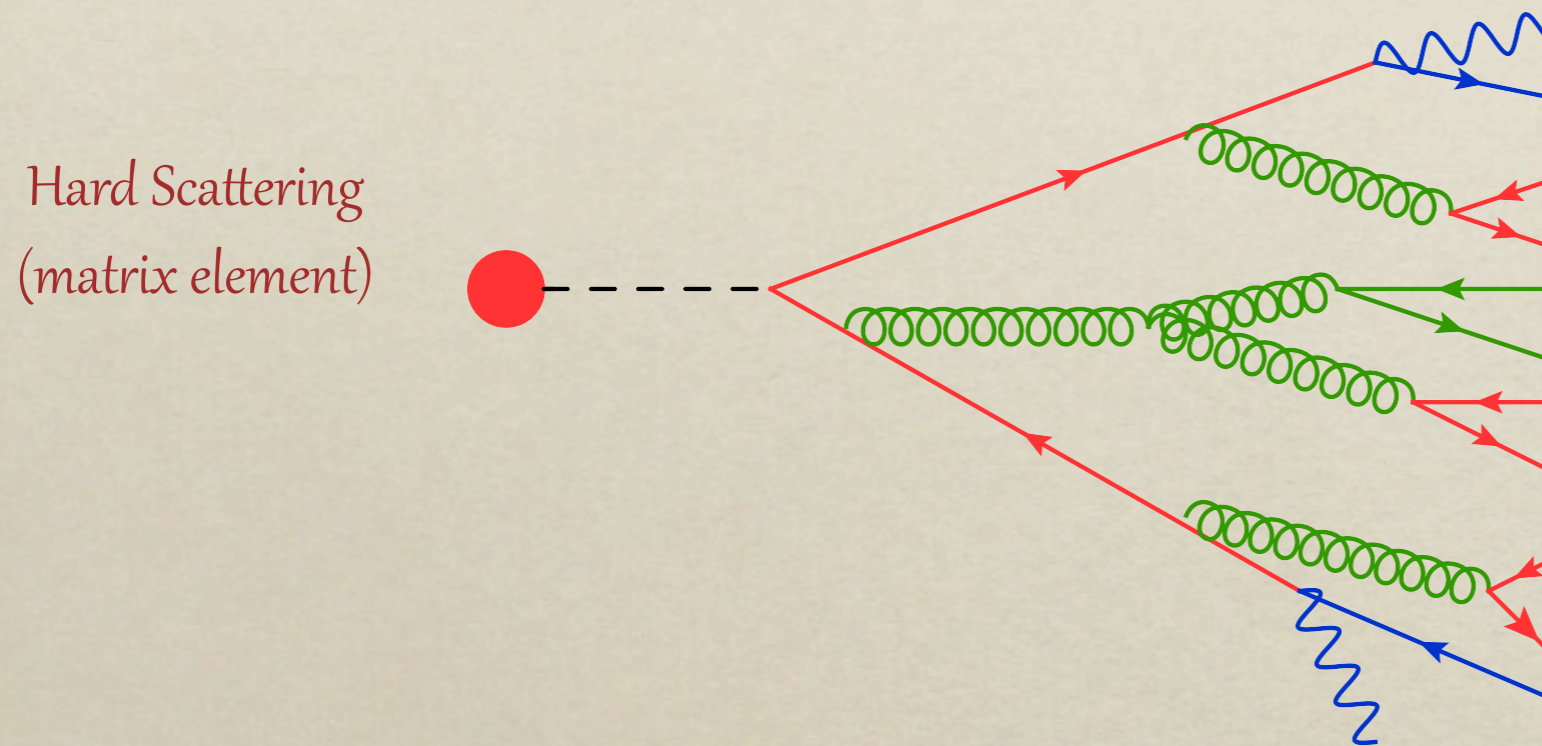
- Production of hard probes:

Hard Scattering
(matrix element)



Introduction

- Production of hard probes:

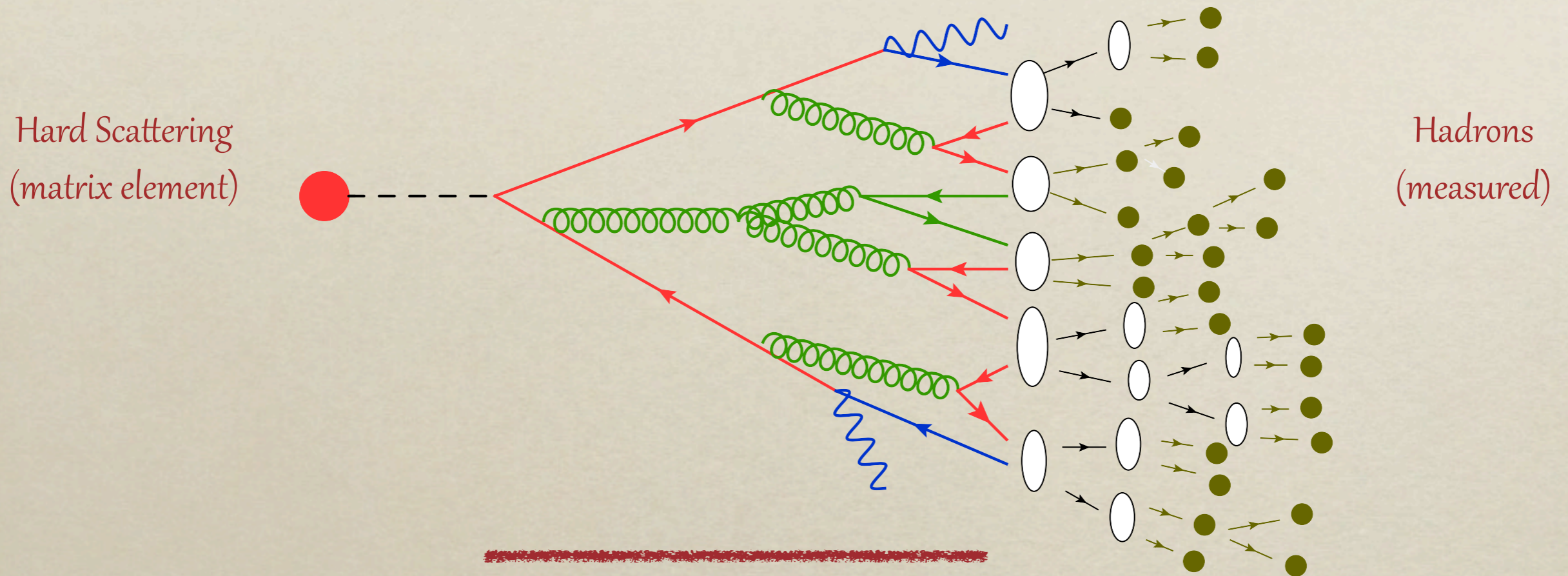


Parton showering:

- Calculable through pQCD
- Known in vacuum

Introduction

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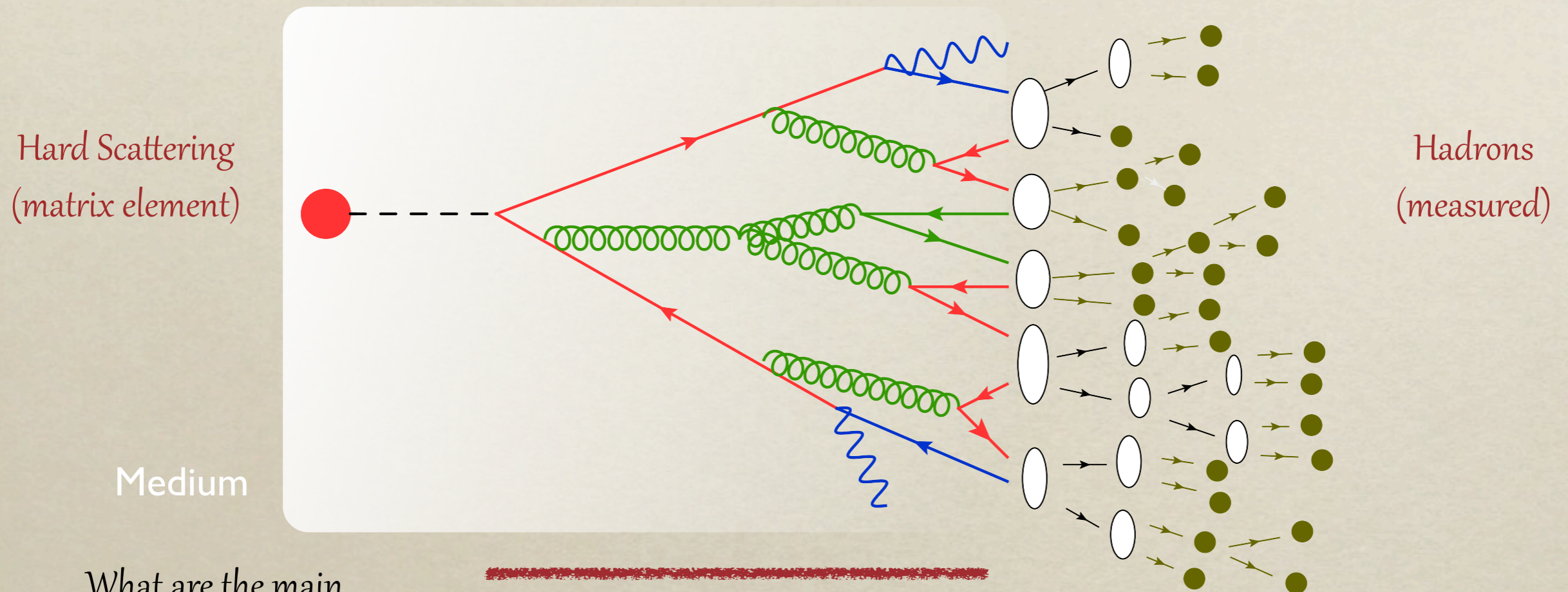
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Hadronization

- Universal/process independent
- Not described by pQCD

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What are the main mechanisms of in-medium energy loss (Jet Quenching)?

Parton showering:

- Calculable through pQCD
- Known in vacuum
- Medium?

Hadronization

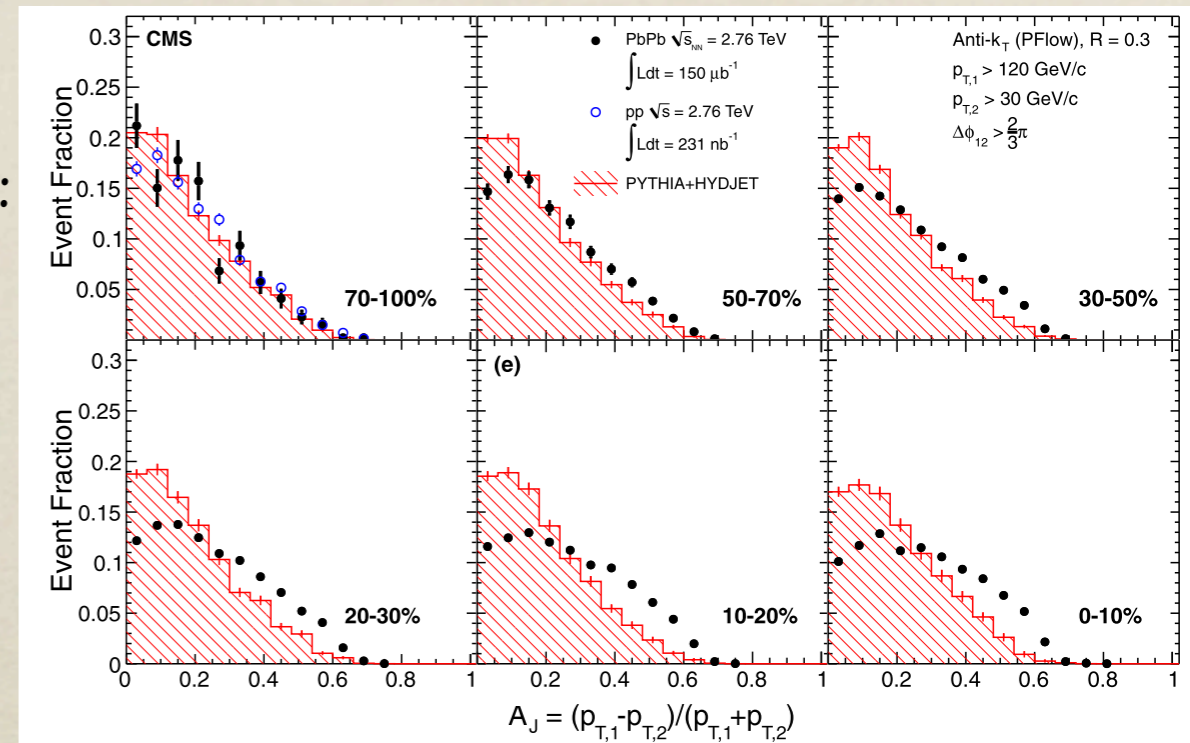
- Universal/process independent
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- Assumed to be outside the medium

Motivation

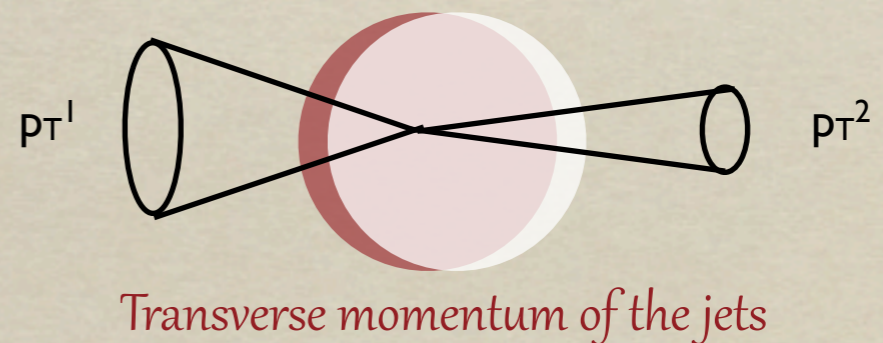
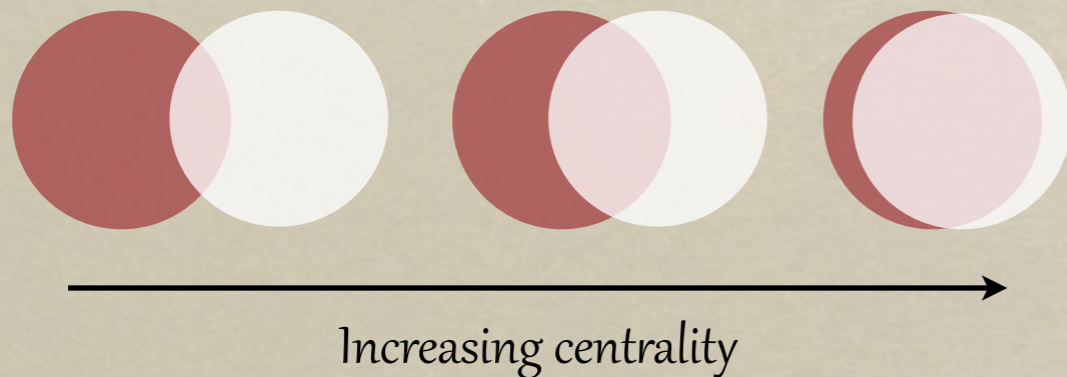
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 - Suppression of the particle yield, strong dijet asymmetry, modified jet fragmentation functions,...

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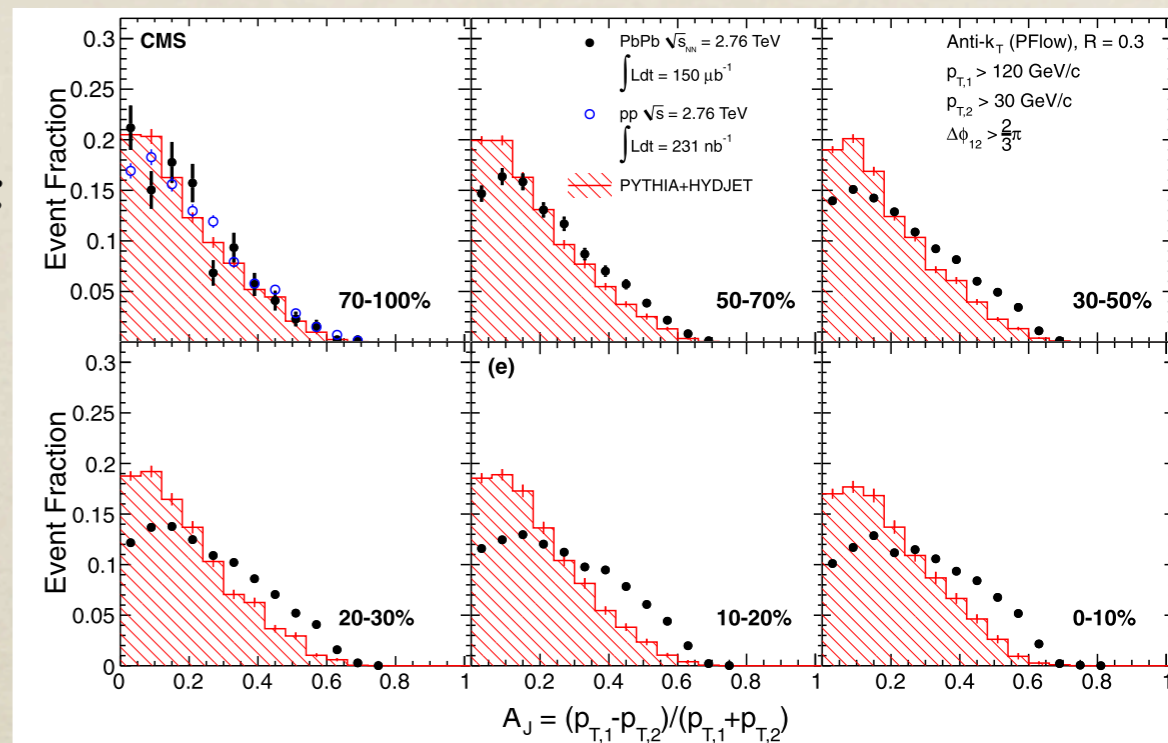


$$A_J = \frac{p_T^1 - p_T^2}{p_T^1 + p_T^2}$$



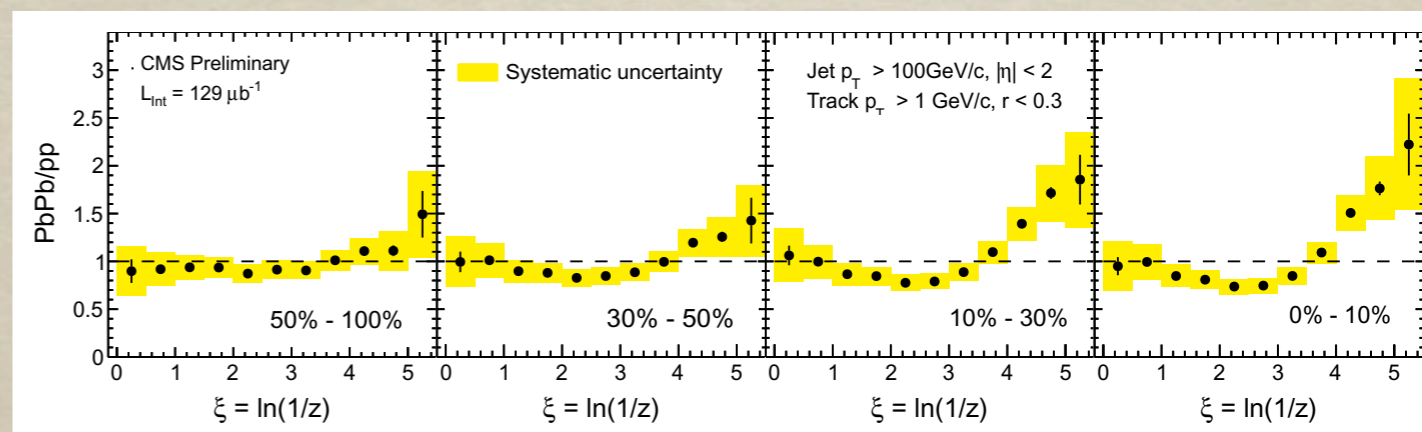
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$$A_J = \frac{p_T^1 - p_T^2}{p_T^1 + p_T^2}$$

$$\xi = \ln\left(\frac{1}{z}\right), \quad z = \frac{p_{||}^{tr}}{p^{jet}}$$



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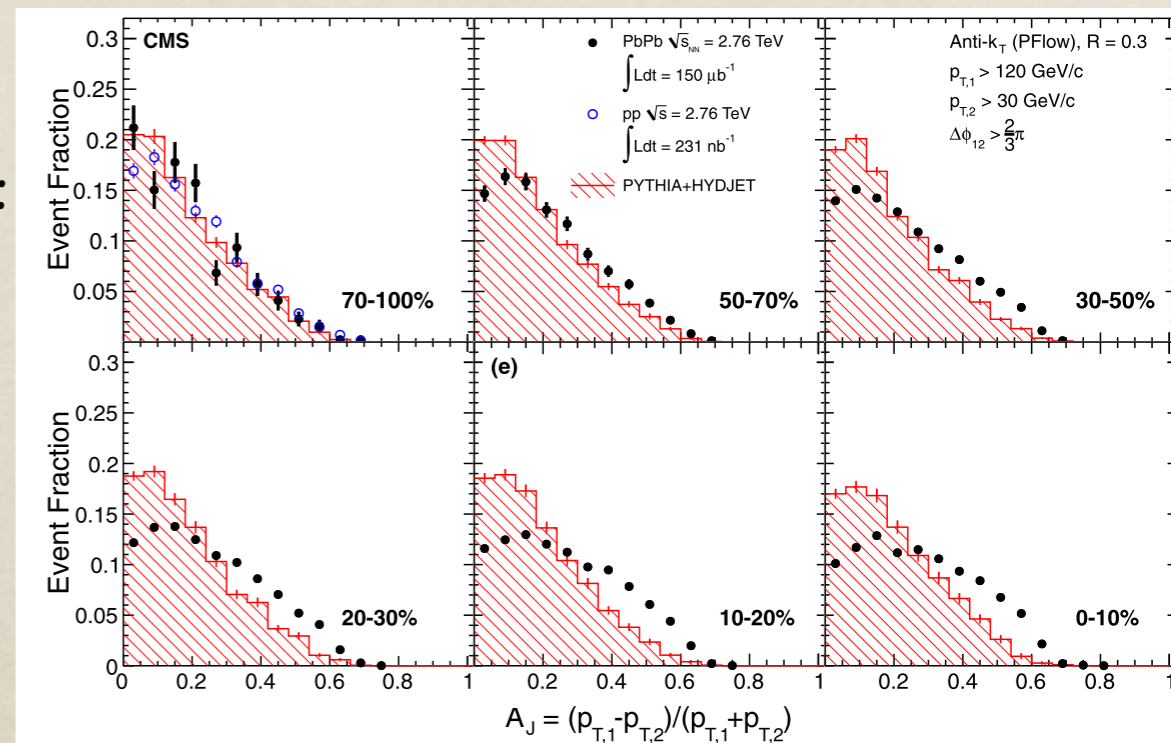
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⇒ Important to have a good description of energy loss mechanisms

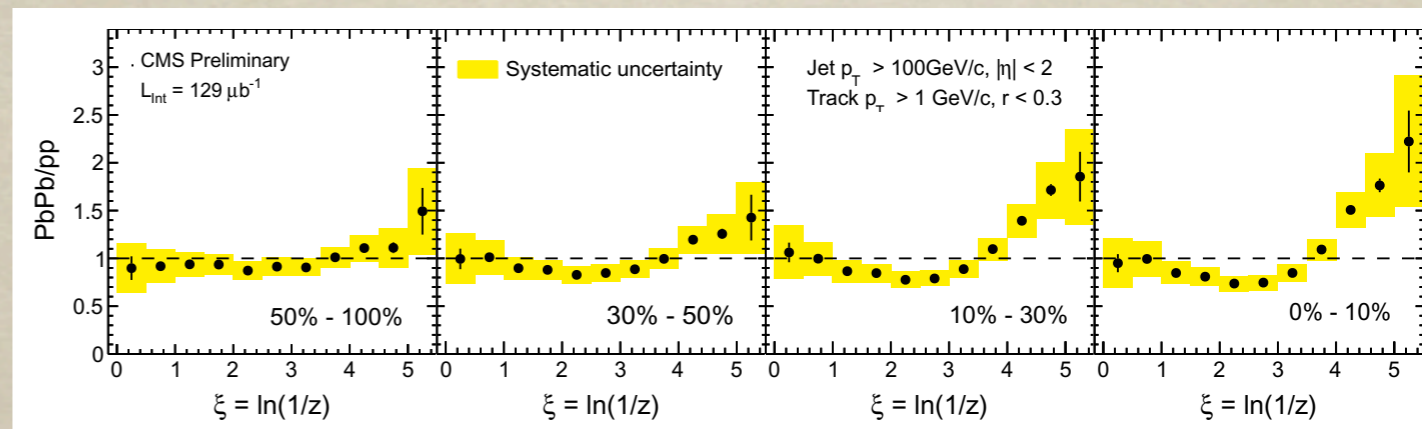


Particularly relevant for Monte Carlo codes
(usually employ phenomenological assumptions without a firm theoretical basis)

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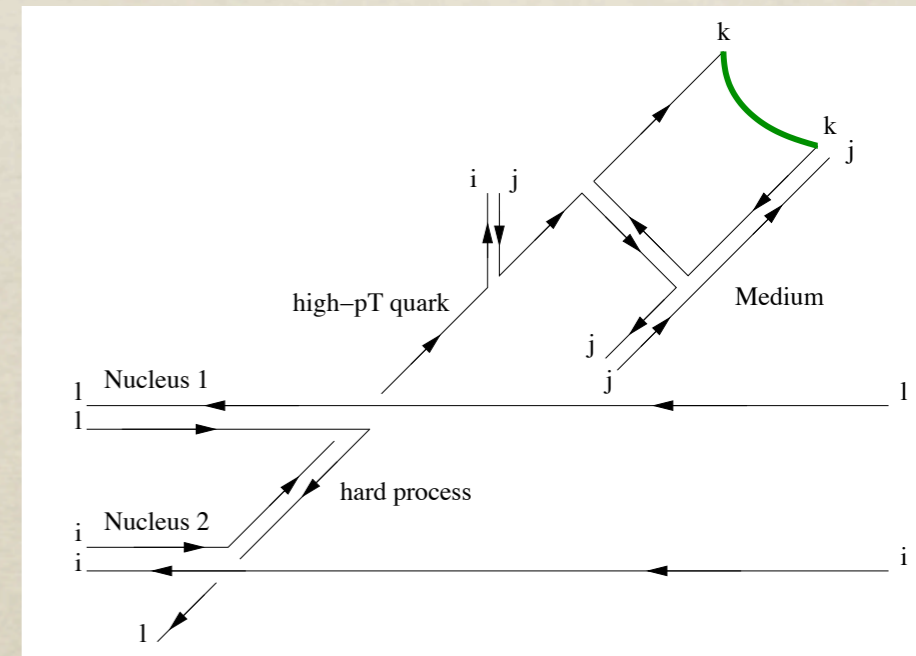
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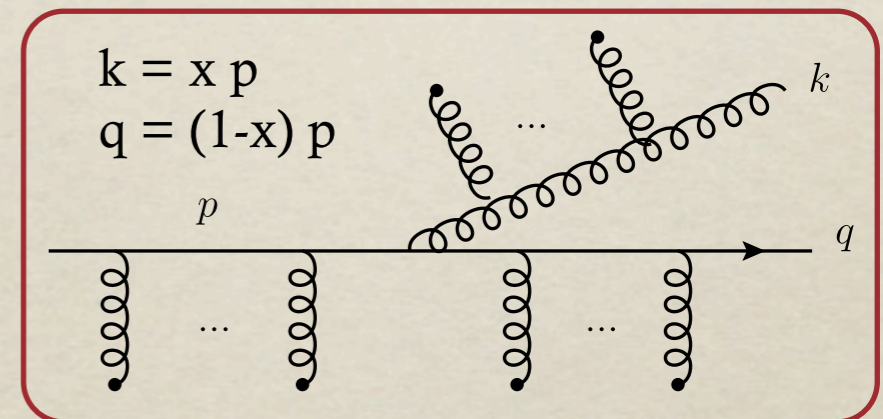
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- Previous works: [arXiv: 1204.2929 , 1209.4585 ^{Path-integral formalism} 1109.5619 ^{SCET}]

Interpolation function between soft and hard gluon radiation limits

Most energetic particle still constrained in the transverse plane

Gluon branching beyond eikonal approximation but small formation times
Approximation may not hold for the partons radiated near the edge of the medium

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- This work: gluon radiation without constrains on the formation time (finite medium)

Formalism

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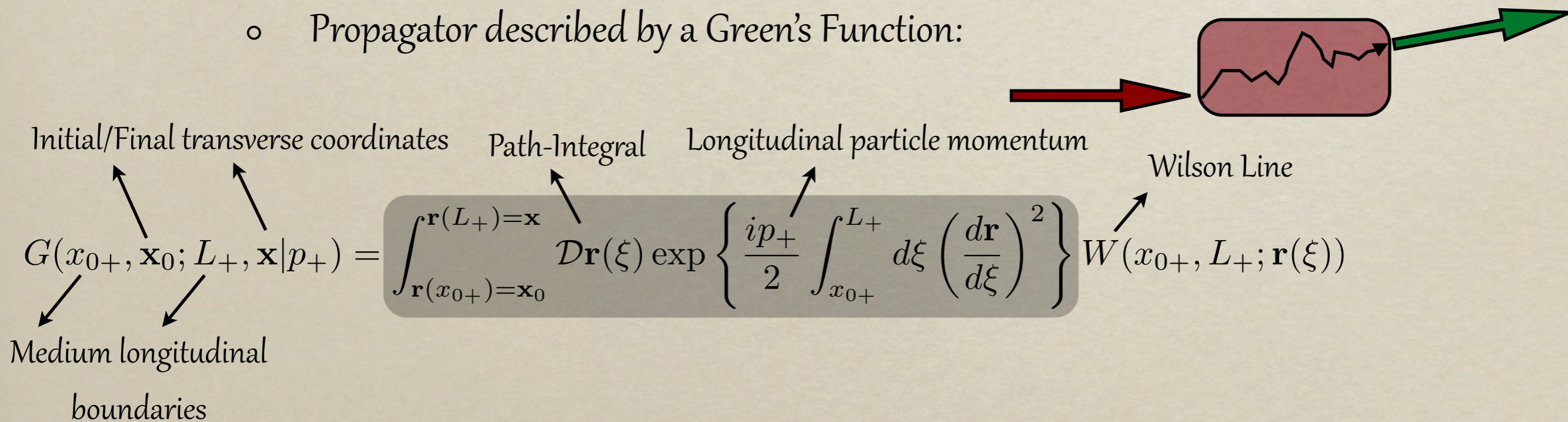
$$G(x_{0+}, \mathbf{x}_0; L_+, \mathbf{x} | p_+) = \int_{\mathbf{r}(x_{0+})=\mathbf{x}_0}^{\mathbf{r}(L_+)=\mathbf{x}} \mathcal{D}\mathbf{r}(\xi) \exp \left\{ \frac{ip_+}{2} \int_{x_{0+}}^{L_+} d\xi \left(\frac{d\mathbf{r}}{d\xi} \right)^2 \right\} W(x_{0+}, L_+; \mathbf{r}(\xi))$$

Initial/Final transverse coordinates Path-Integral Longitudinal particle momentum Wilson Line

Medium longitudinal boundaries

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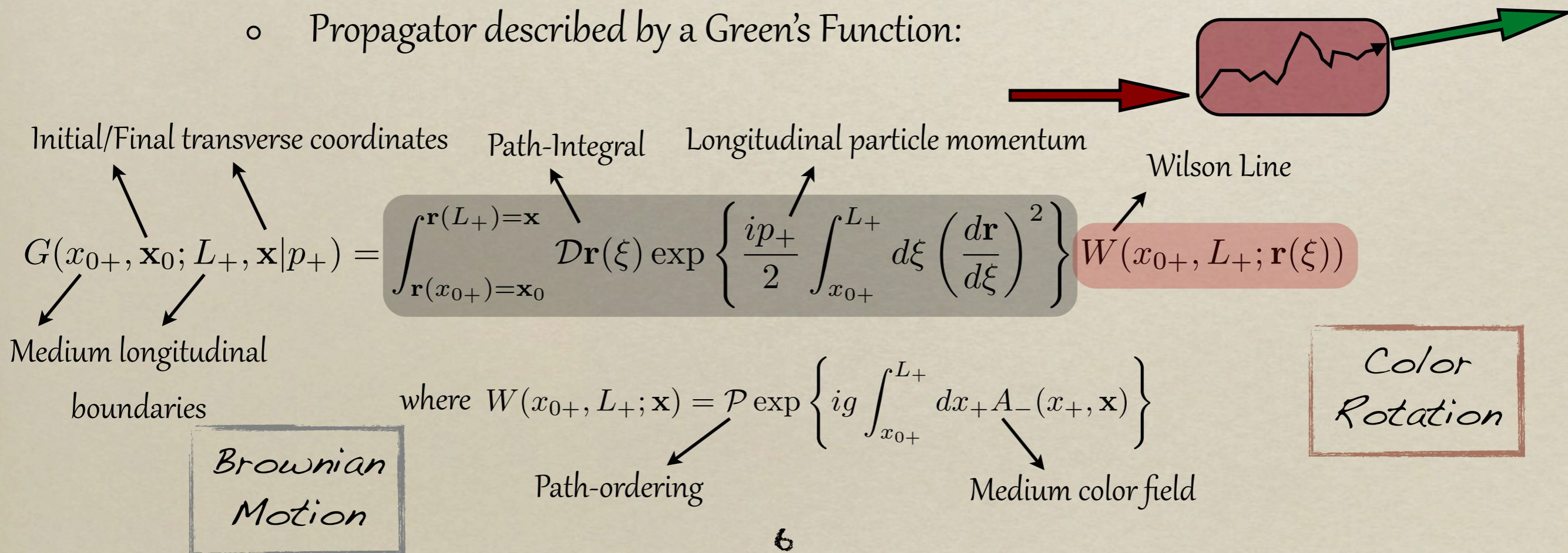


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Brownian Motion

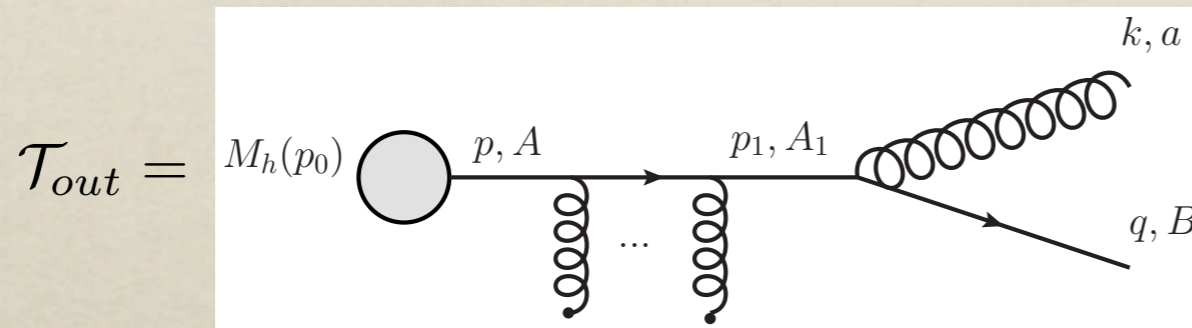
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Gluon emission spectrum

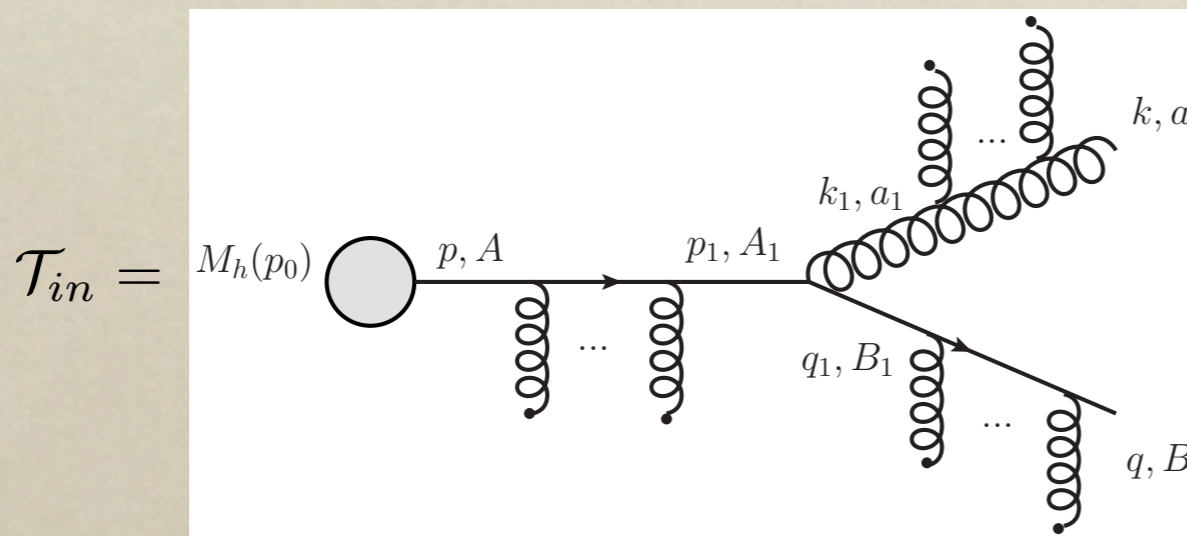
- Finite medium \Rightarrow 2 configurations



Kinematics:

$$k_+ = z p_+ \quad k_T = \mathbf{k}$$

$$q_+ = (1-z) p_+ \quad q_T = \mathbf{q}$$



$|\dots| \equiv$ Average over initial spin, color and gluon polarization

$\langle \dots \rangle \equiv$ Medium averages

- Emission cross section:

$$\frac{d^2\sigma}{d\Omega_k d\Omega_q} = \langle |\mathcal{T}_{tot}|^2 \rangle, \quad \text{where} \quad |\mathcal{T}_{tot}|^2 = |\mathcal{T}_{out}|^2 + |\mathcal{T}_{in}|^2 + 2\text{Re} \left\{ \mathcal{T}_{in} \mathcal{T}_{out}^\dagger \right\}$$

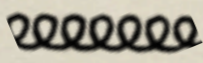
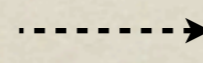
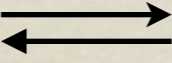
$$d\Omega_k = \frac{d\mathbf{k}}{(2\pi)^3} \frac{dk_+}{2k_+}$$

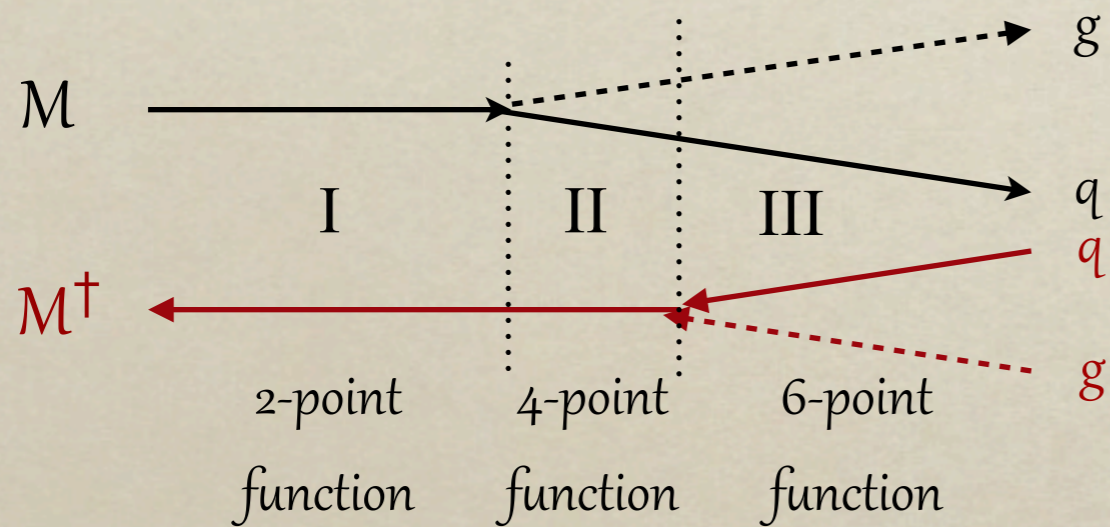
Medium averages

- High energy limit \Rightarrow Medium averages can be performed locally
 - \Rightarrow Decomposition into regions where the number of Wilson lines is fixed

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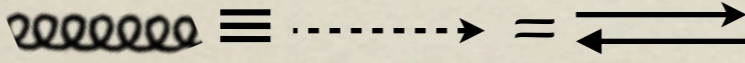
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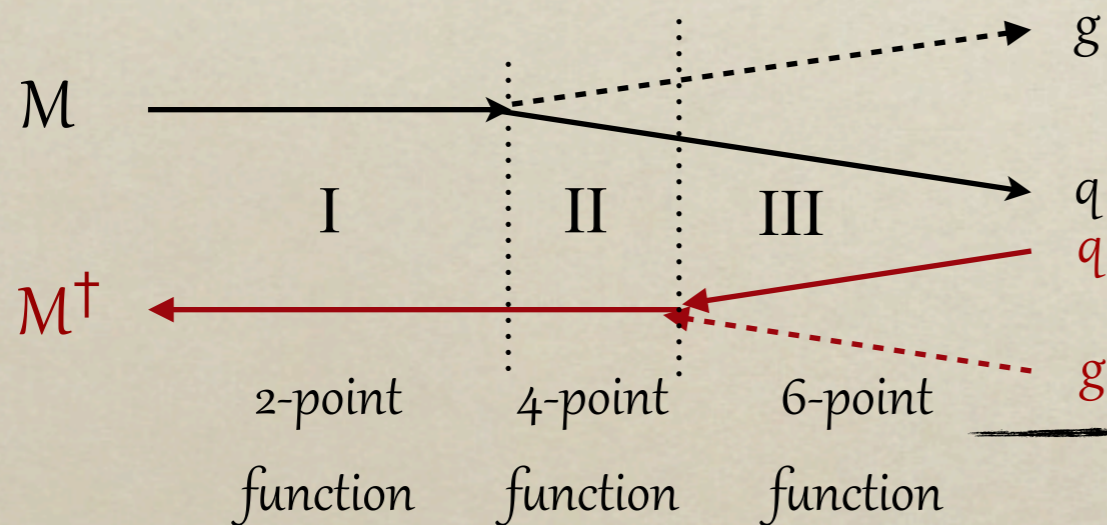
- Large N_c limit \Rightarrow  \equiv  \Rightarrow 



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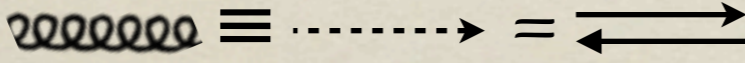
Correlators calculated by doing an infinitesimal expansion up to 2nd order of the fields:

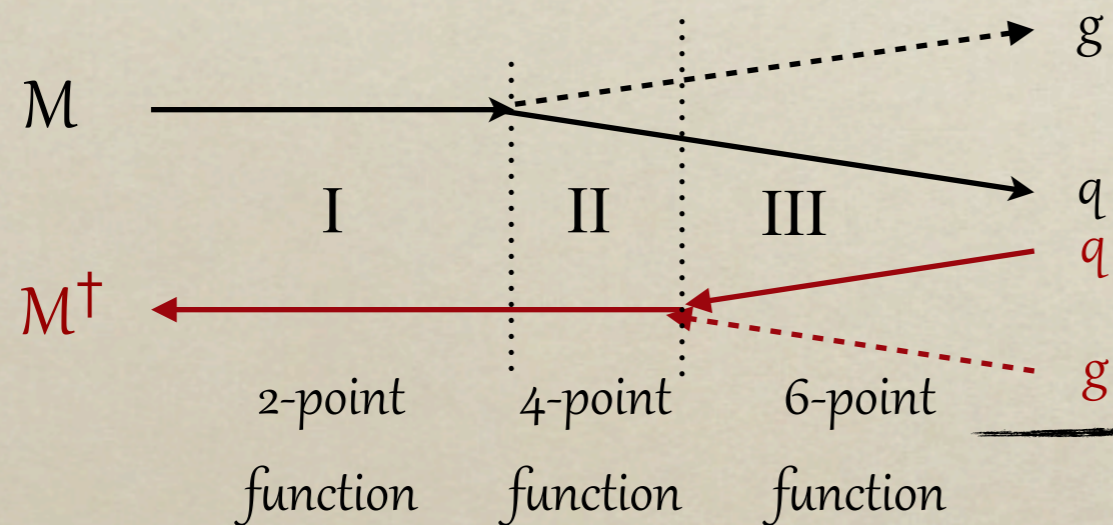
$$W_{ij}(L_+, x_+; \mathbf{x}) = V_{i\alpha}(L_+, \zeta; \mathbf{x}) \left[\delta_{\alpha j} \left(1 - \frac{C_F}{2} B(\zeta, \mathbf{0}) \right) + iT_{\alpha j}^a A_{\alpha j}^a(\zeta, \mathbf{x}) \right]$$

$$\text{with } \delta^{ab} B(x_+, \mathbf{x} - \mathbf{y}) = \langle A^a(x_+, \mathbf{x}) A^b(x_+, \mathbf{y}) \rangle$$

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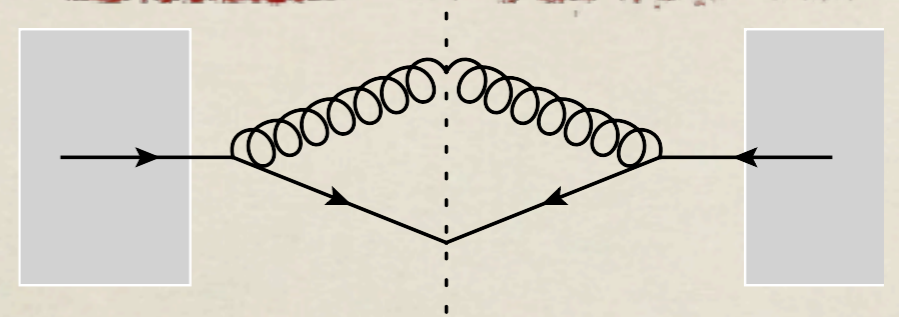
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Results

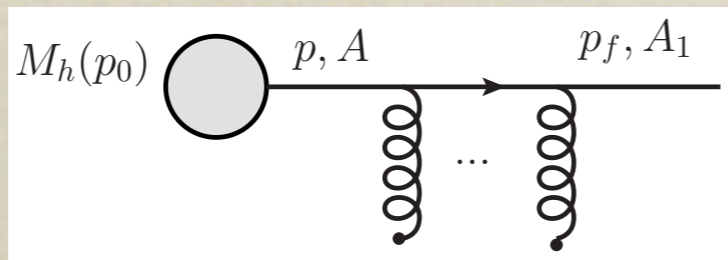
- Out-out term: $|\mathcal{T}_{out}|^2$

$$\frac{d^2 I_{out}}{d\Omega_q d\Omega_k} = \frac{1}{\sigma_{el}} \frac{d^2 \sigma_{out}}{d\Omega_q d\Omega_k} =$$

$$= g^2 (2\pi)^3 \frac{4z(1-z)p_{0+}}{((1-z)\mathbf{k} - z\mathbf{q})^2} P_{g \leftarrow q}(z) P(\mathbf{p}_0 \rightarrow \mathbf{k} + \mathbf{q}) \delta(k + q - p_0)_+$$



where $\sigma_{el} =$



Brownian motion of the initial quark

$$P(\mathbf{p}_0 \rightarrow \mathbf{p}_f) = \frac{1}{\pi(L_+ - x_{0+})\hat{q}_F} \exp \left\{ -\frac{(\mathbf{p}_f - \mathbf{p}_0)^2}{(L_+ - x_{0+})\hat{q}_F} \right\}$$

Transverse momentum broadening

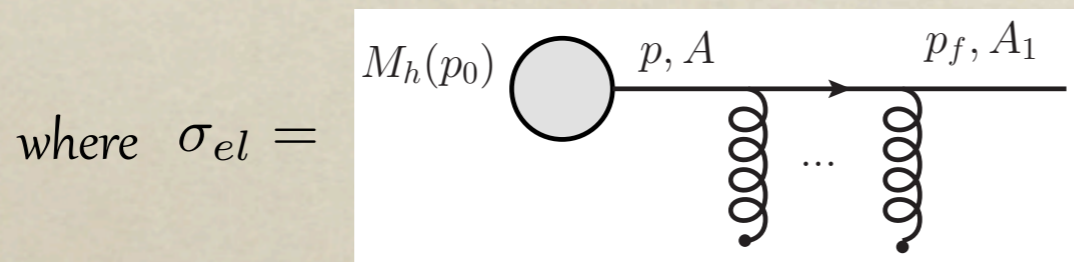
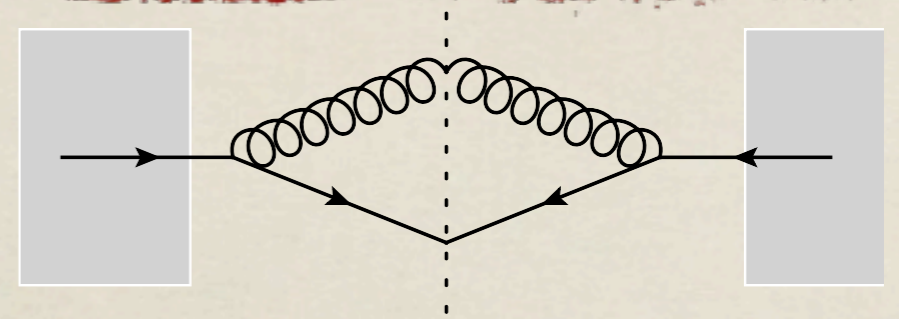
$$\text{Transport coefficient: } \hat{q} = \frac{\langle \mathbf{k}^2 \rangle}{\lambda}$$

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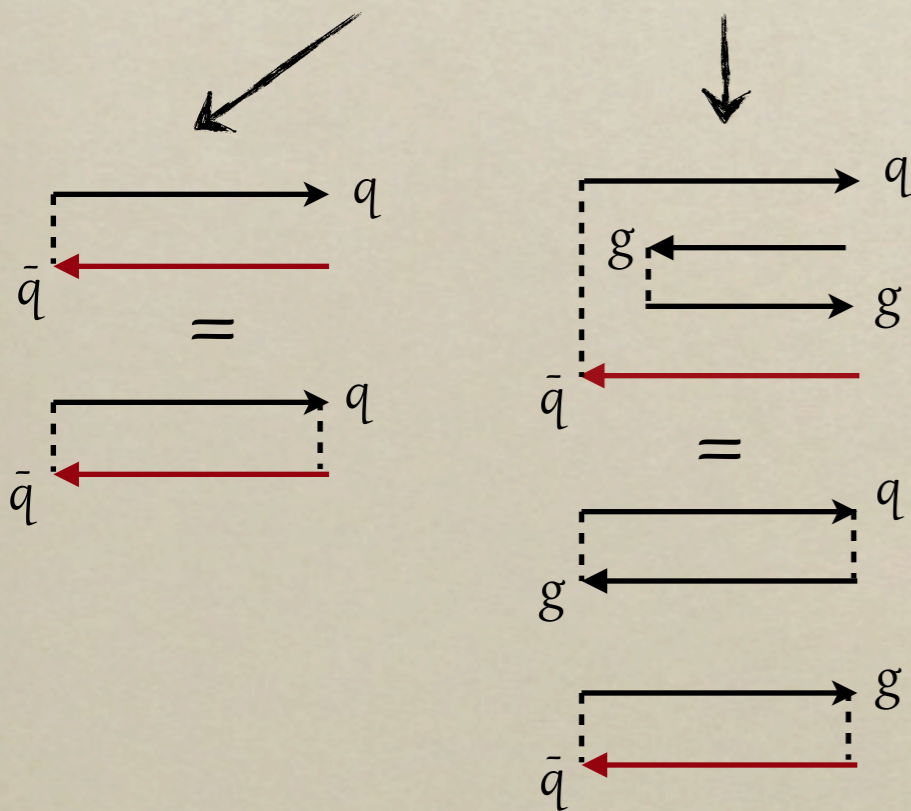
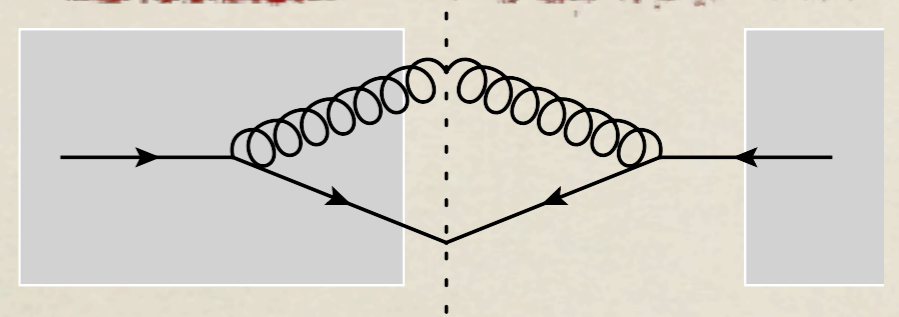
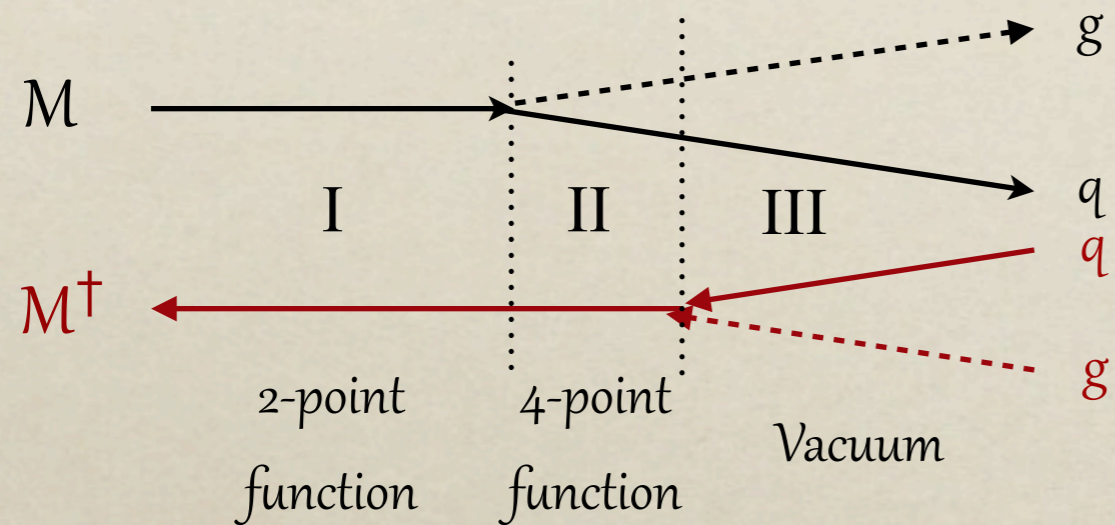
- Taking the limit $\hat{q}_F L_+ \rightarrow 0$ (no medium) and integrating over $d\Omega_q$:

$$\left. \frac{d^2 I_{out}}{dz d\mathbf{k}} \right|_{\hat{q}_F L_+ \rightarrow 0} = \frac{d^2 I_{vac}}{dz d\mathbf{k}} = \frac{\alpha_s}{2\pi^2} \frac{1}{\mathbf{k}^2} P_{g\leftarrow q}(z)$$

⇒ Complete vacuum radiation spectrum is recovered!

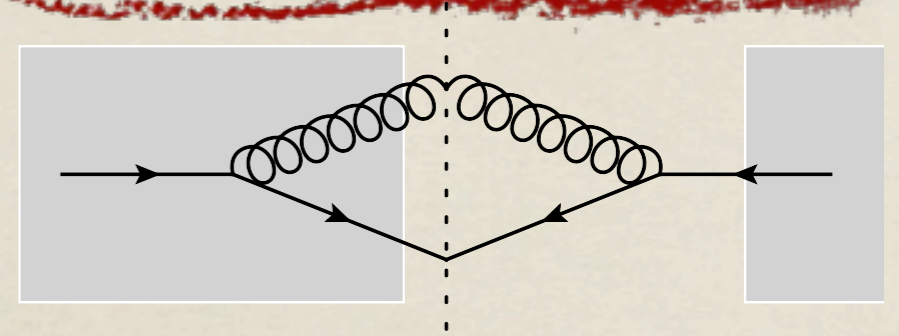
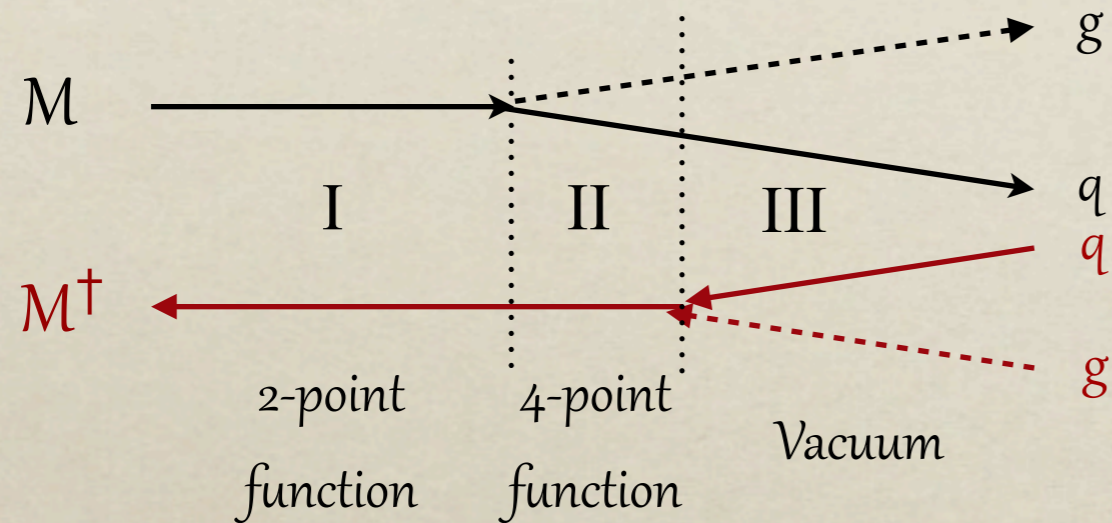
Results

- “Medium” term: in-out contribution

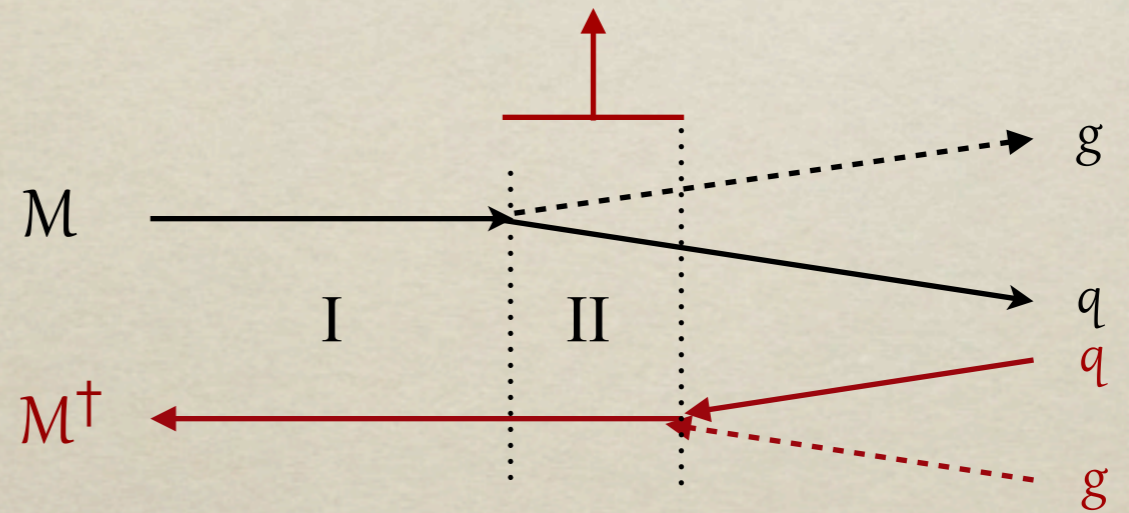
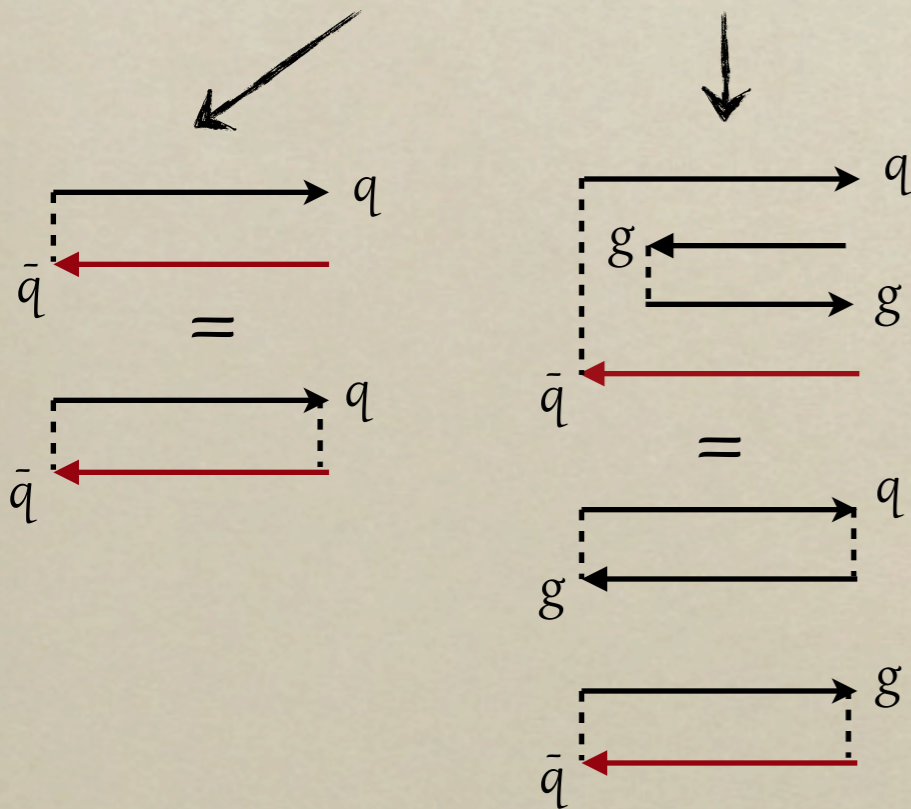


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Quark and gluon are correlated
(gluon formation time)

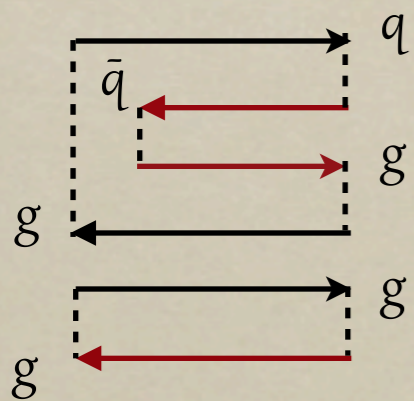
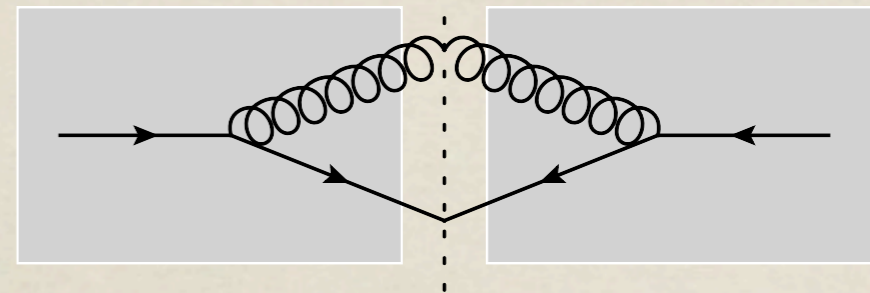
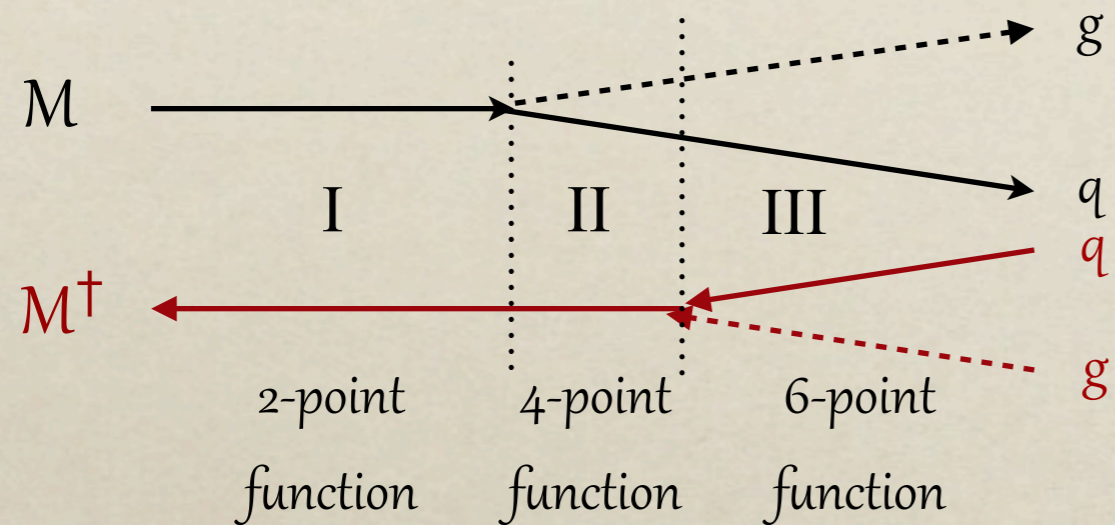


Random walk of initial quark

Vacuum propagation

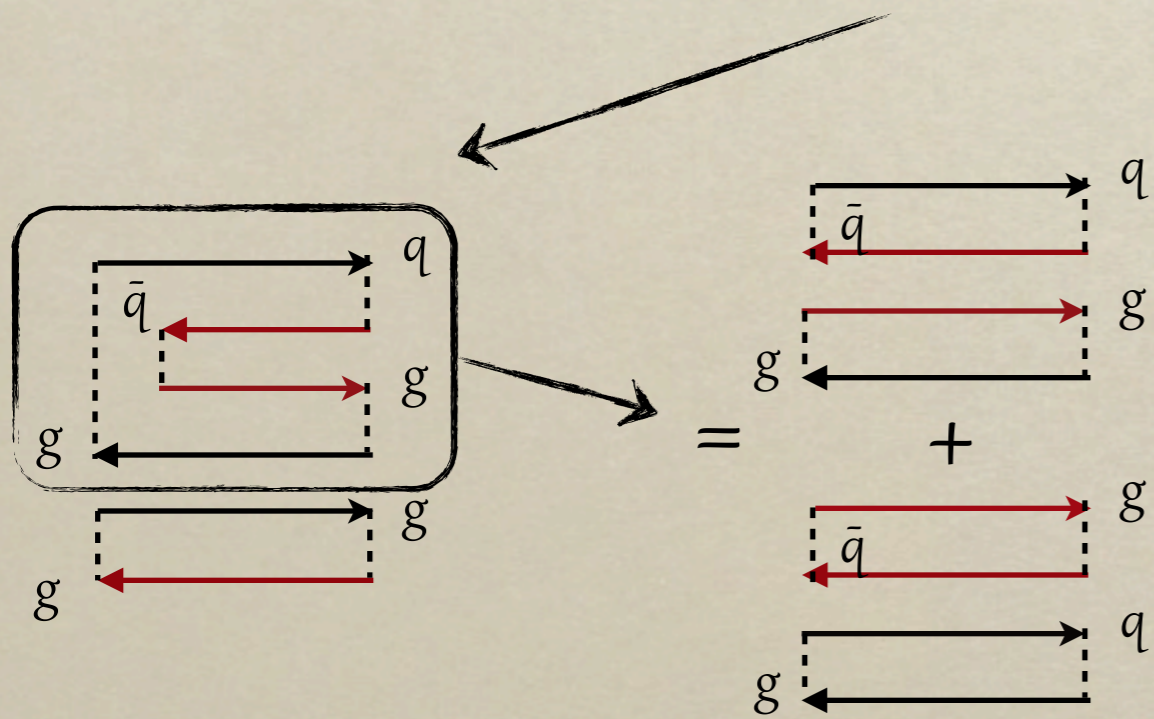
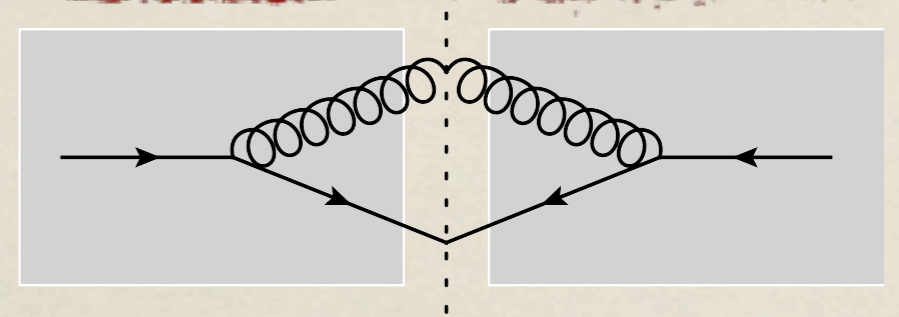
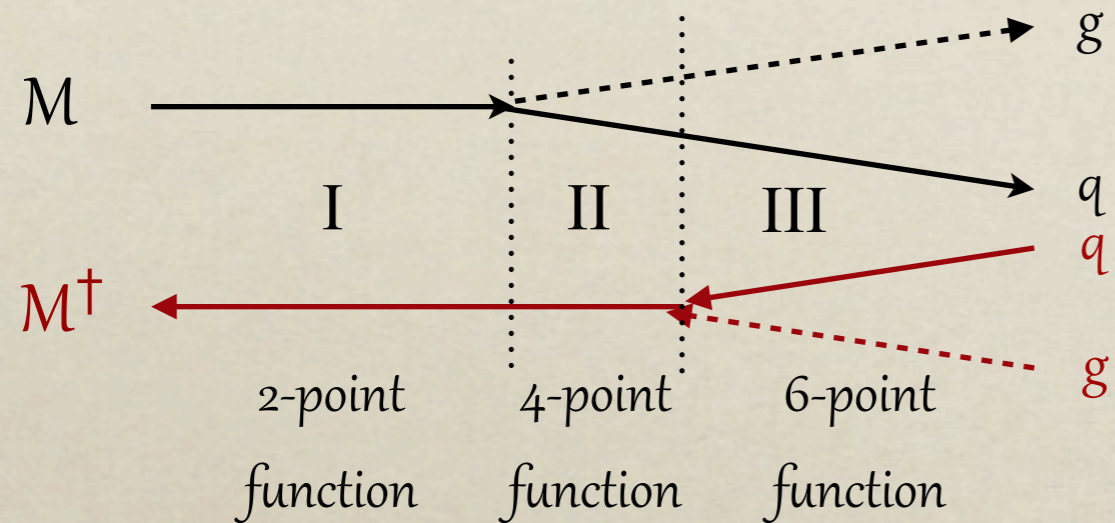
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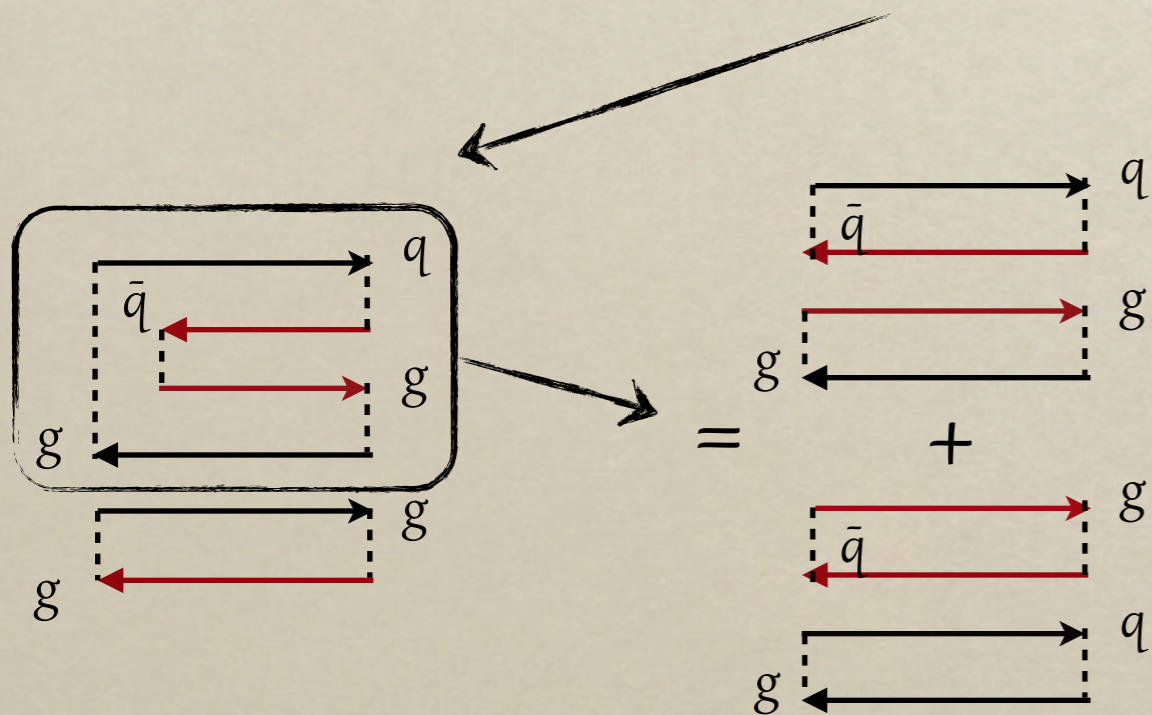
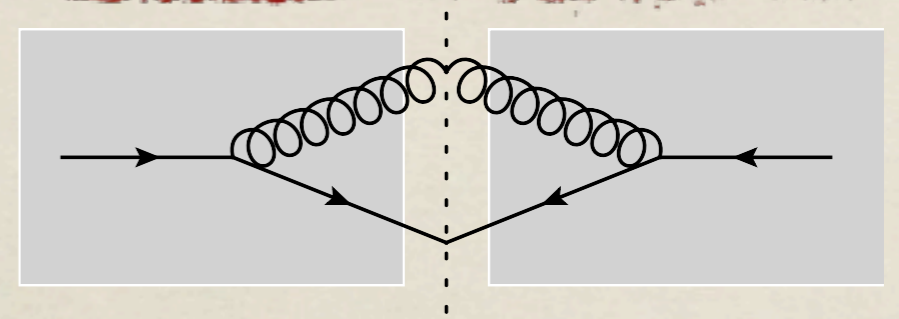
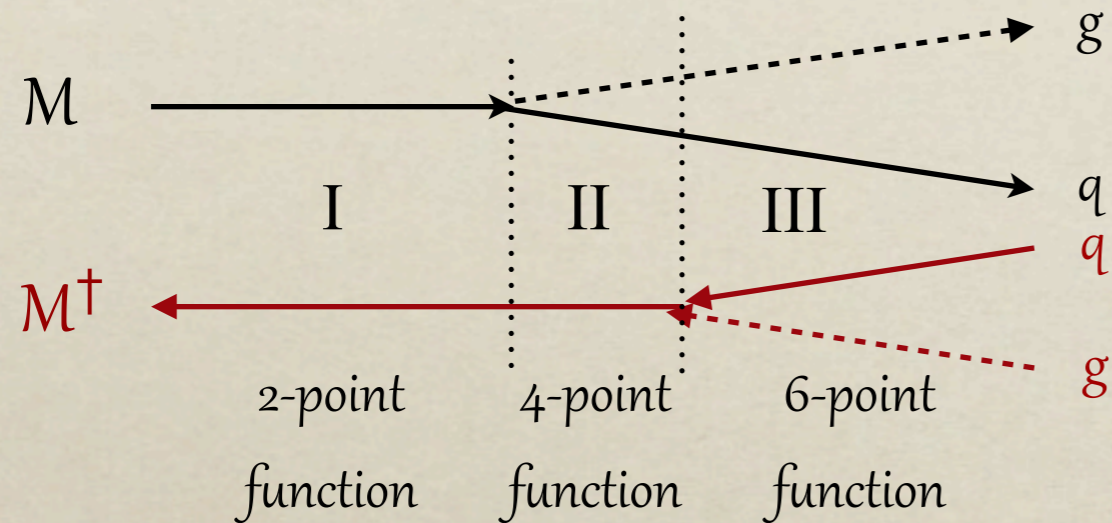
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o "Medium" term: in-in contribution



$$\langle \text{Tr} (W^\dagger(\mathbf{x}_g)W(\mathbf{x}_{\bar{g}})W^\dagger(\mathbf{x}_{\bar{q}})W(\mathbf{x}_q)) \rangle$$

$$= N \exp \left\{ -\frac{C_F}{4} \hat{q}(L_+ - x_{2+}) [(\mathbf{x}_{\bar{g}} - \mathbf{x}_g)^2 + (\mathbf{x}_q - \mathbf{x}_{\bar{q}})^2] \right\} \Delta_{med}$$

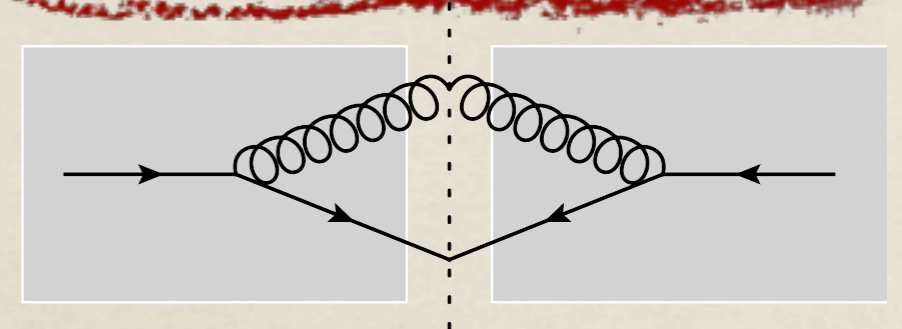
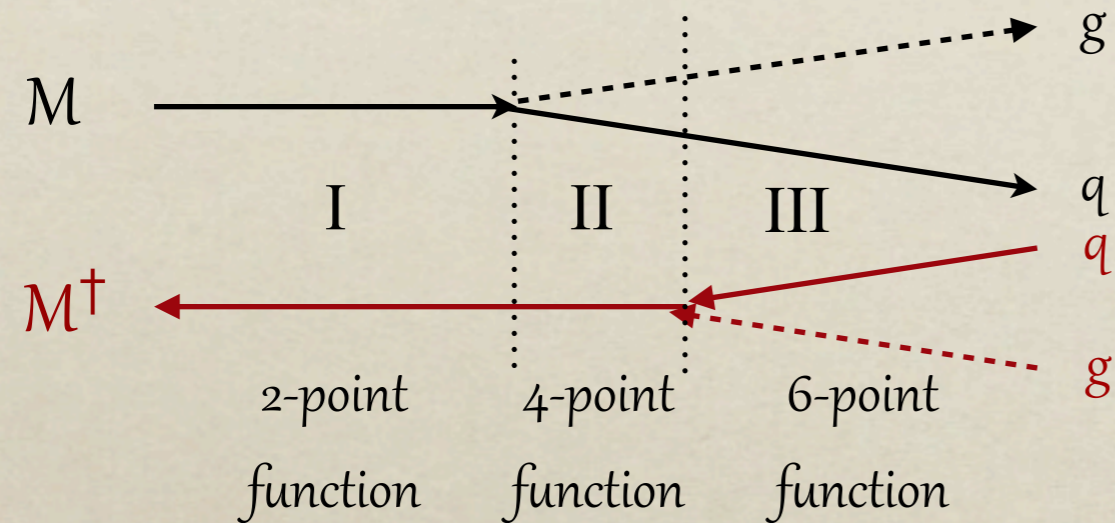
where

$$\Delta_{med} \equiv 1 - \exp \left\{ -\frac{C_F}{2} \hat{q}(L_+ - x_{2+})(\mathbf{x}_{\bar{q}} - \mathbf{x}_g) \cdot (\mathbf{x}_q - \mathbf{x}_{\bar{g}}) \right\}$$

Transport coefficient: $\hat{q} = \frac{\langle \mathbf{k}^2 \rangle}{\lambda}$

Results

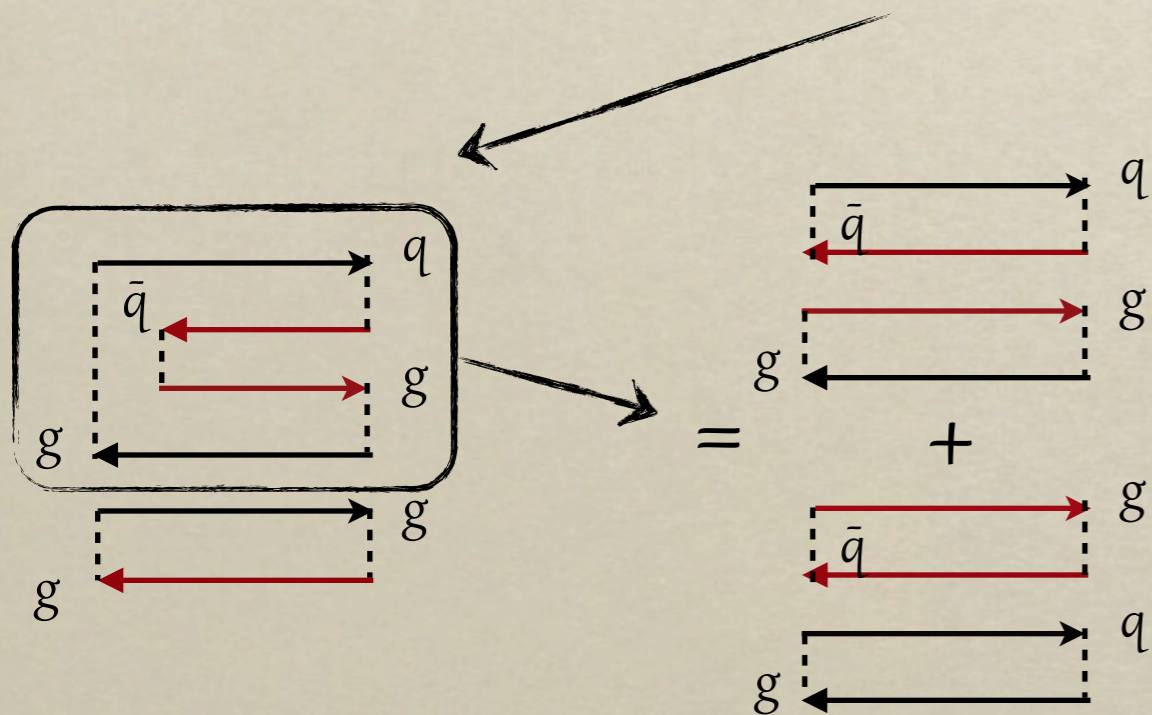
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Δ_{med} controls the decoherence of color sources:

$$\hat{q}_F L_+ \rightarrow \infty \Rightarrow \Delta_{med} \rightarrow 1$$

⇒ Quadrupole factorizes into two independent dipoles!



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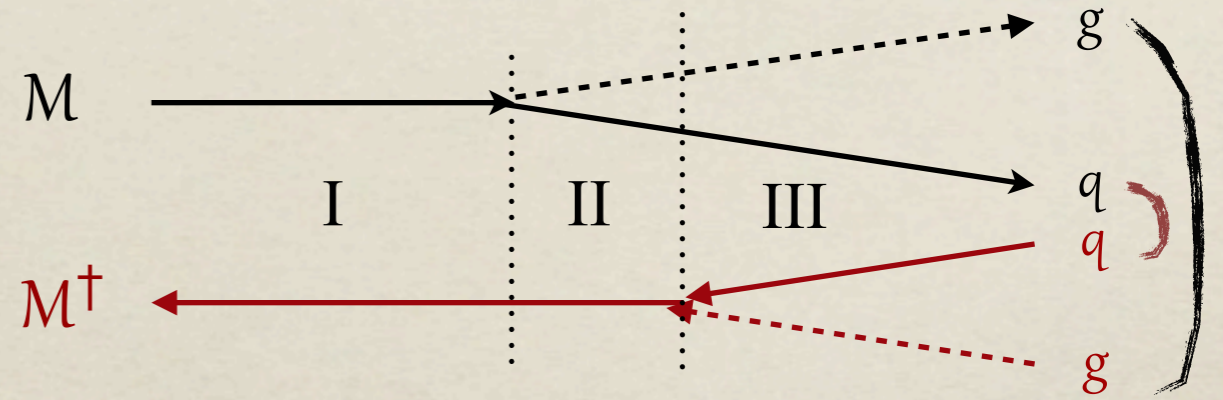
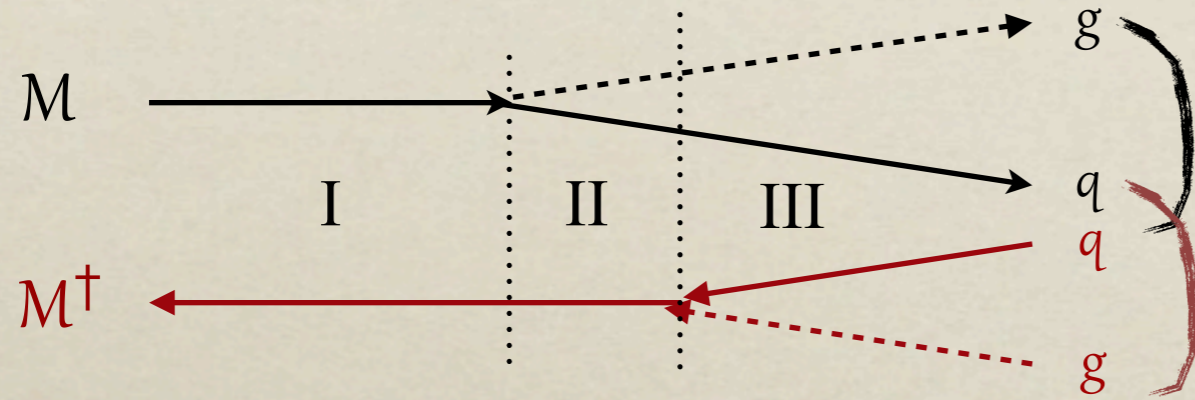
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⇒ Generalized Δ_{med} parameter to account for the transverse walk of both particles

Transport coefficient: $\hat{q} = \frac{\langle \mathbf{k}^2 \rangle}{\lambda}$

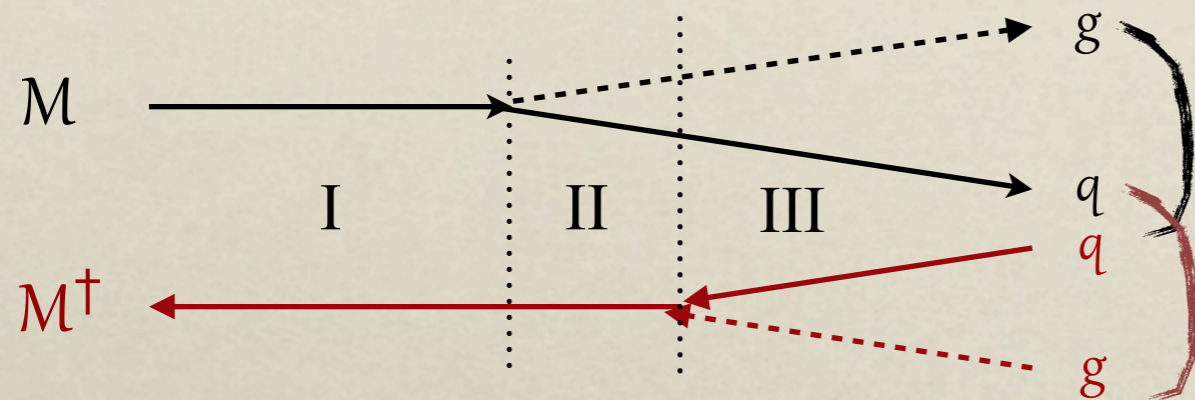
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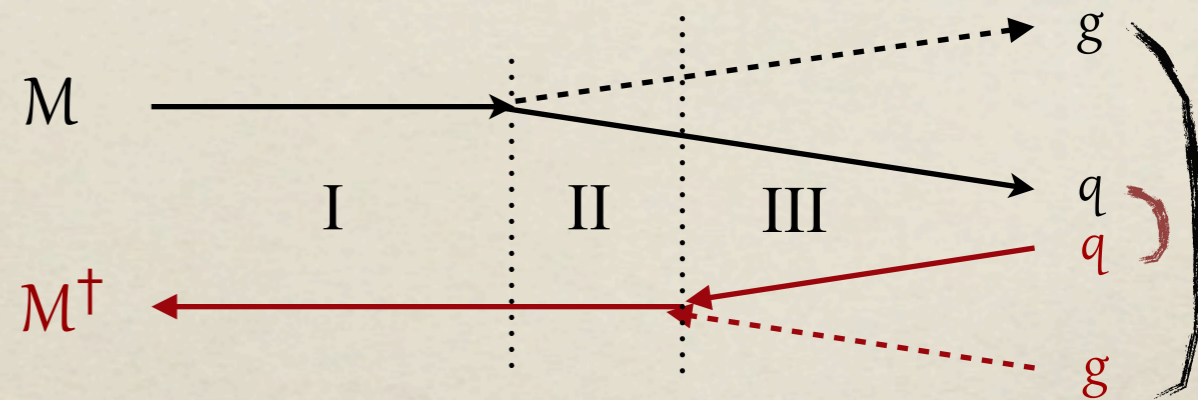


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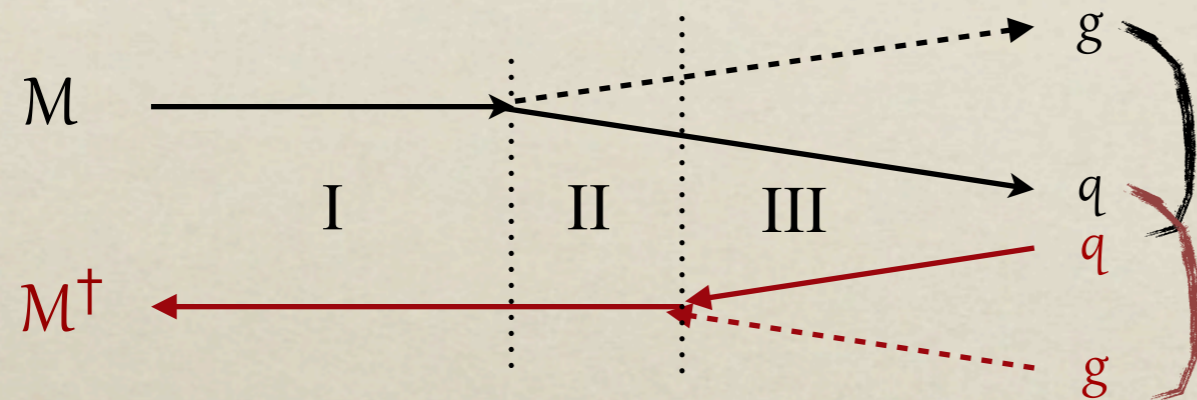
Random walk of initial quark



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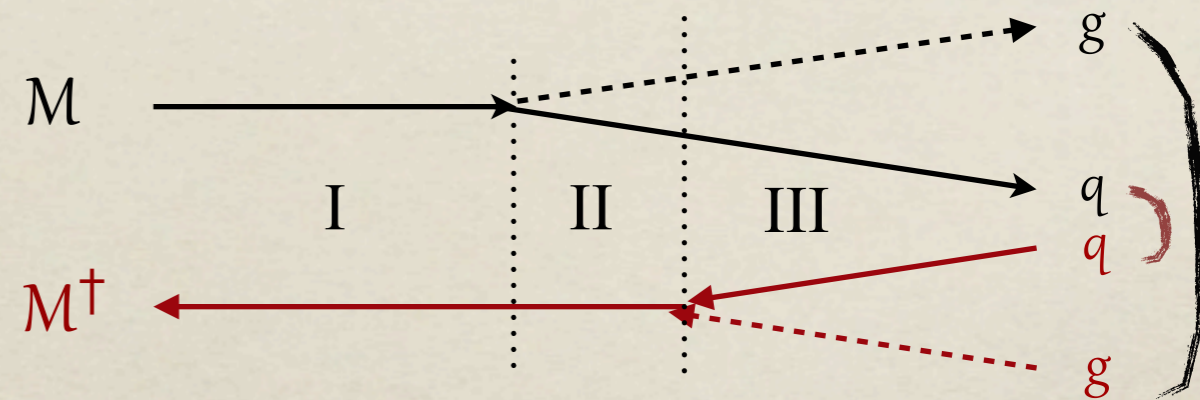
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Random walk of initial quark

Quark and gluon are correlated
(gluon formation time)

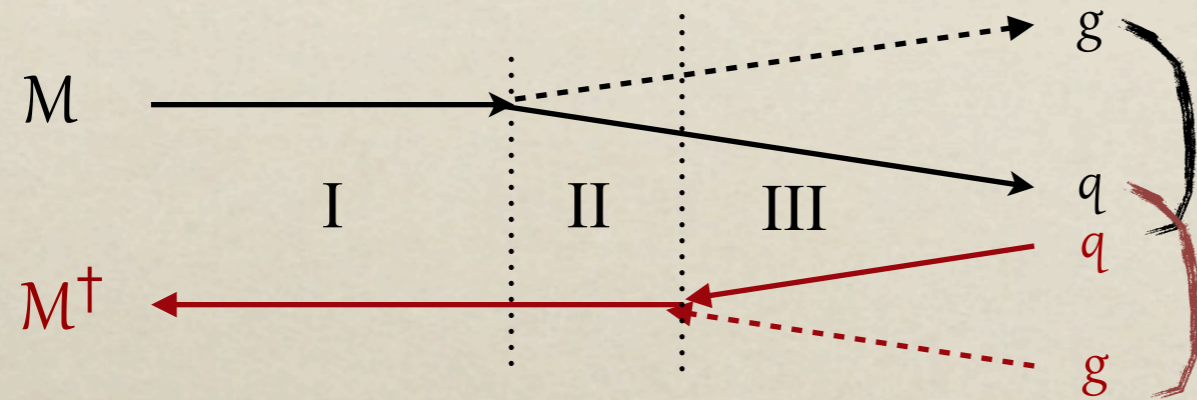


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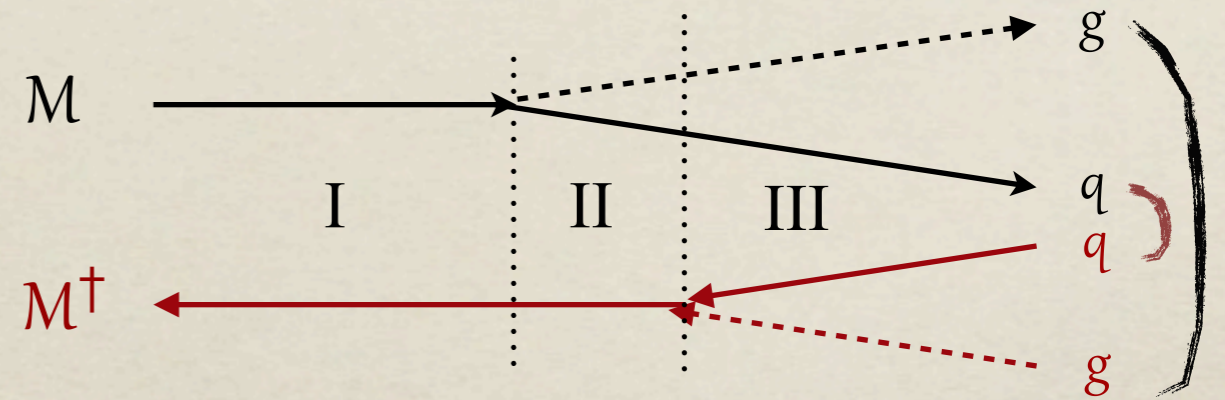
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Random walk of initial quark

Quark and gluon are correlated
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Quark and gluon act coherently
(coherent propagation controlled by dipole distance)



Random walk of initial quark

Quark and gluon are correlated
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Independent broadening of final partons

Conclusions

- Parton shower is modified in the presence of a medium:
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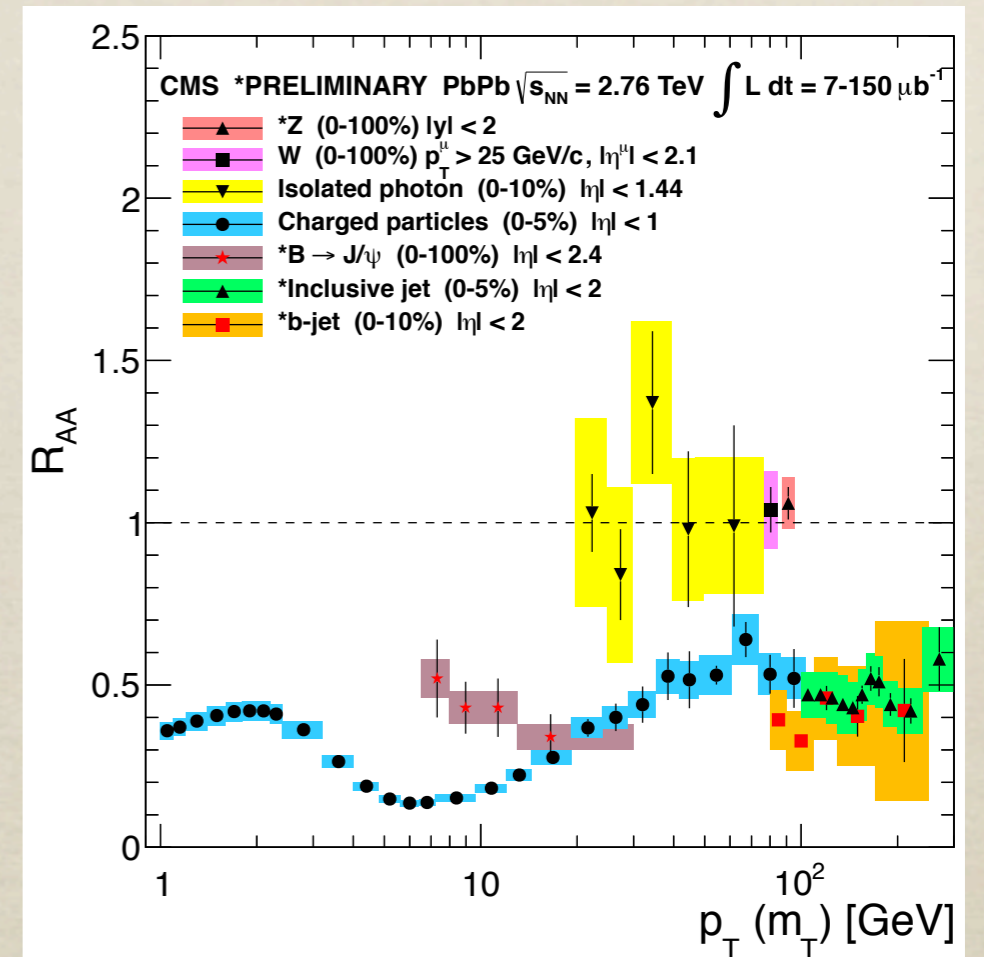
Backup Slides

Motivation

- Energy loss processes in PbPb collisions?
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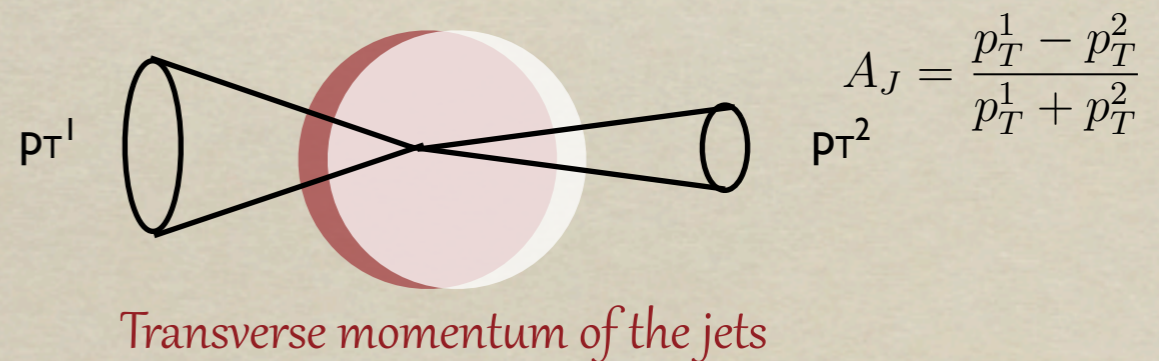
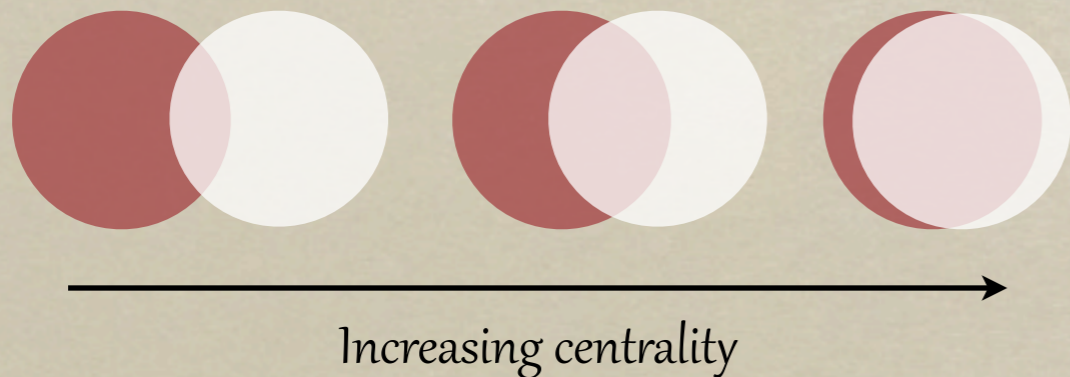
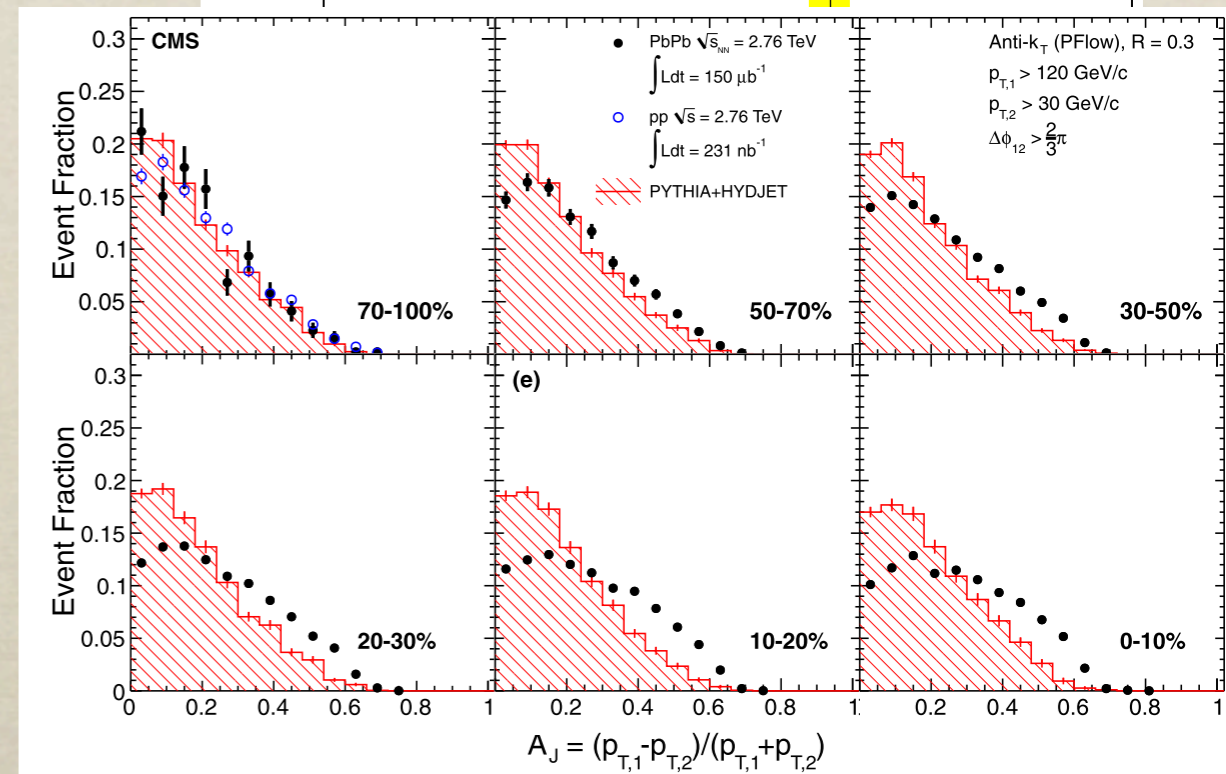
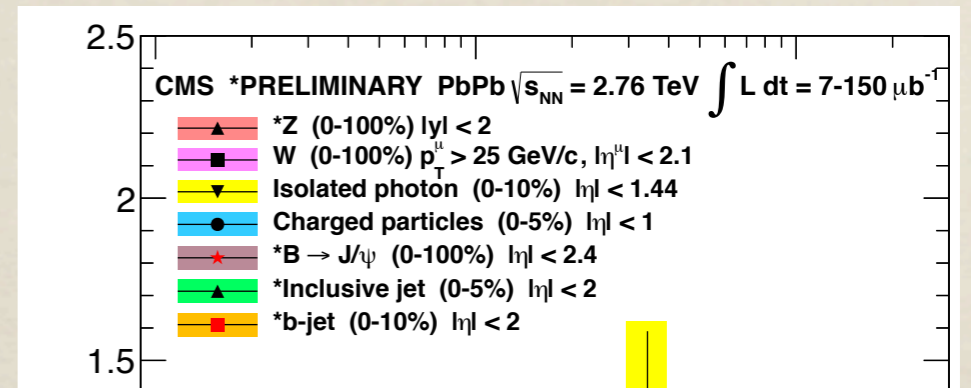


$$R_{AA}(p_T) = \frac{(1/N_{\text{evt}}^{AA}) d^2 N^{AA} / d\eta dp_T}{\langle N_{\text{coll}} \rangle (1/N_{\text{evt}}^{pp}) d^2 N^{pp} / d\eta dp_T}$$

Ratio of the particle yield in AA collisions over pp collisions

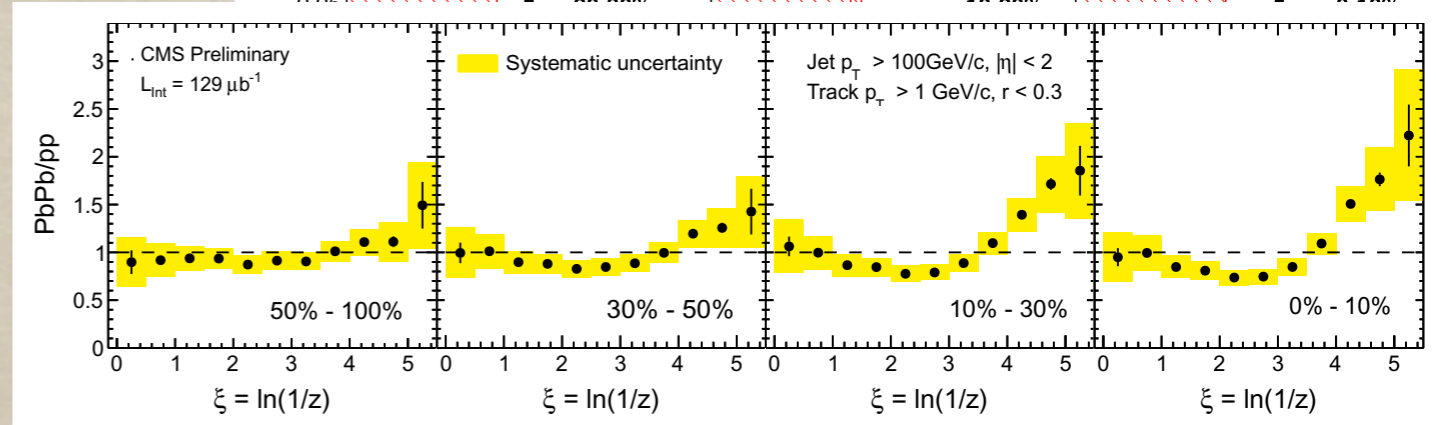
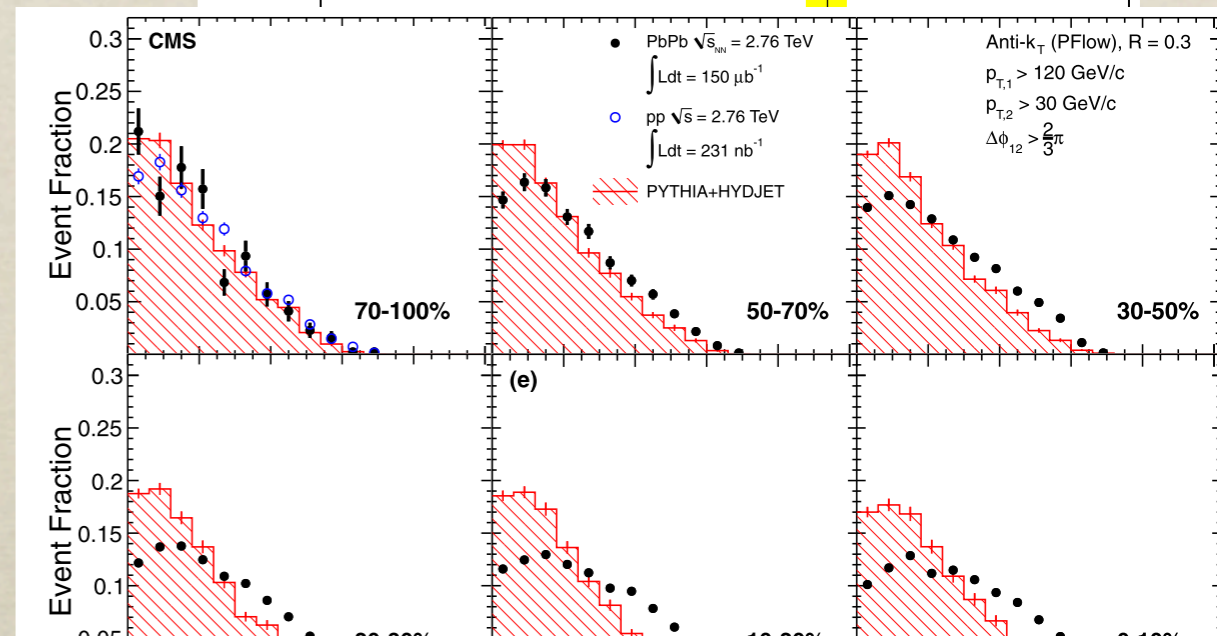
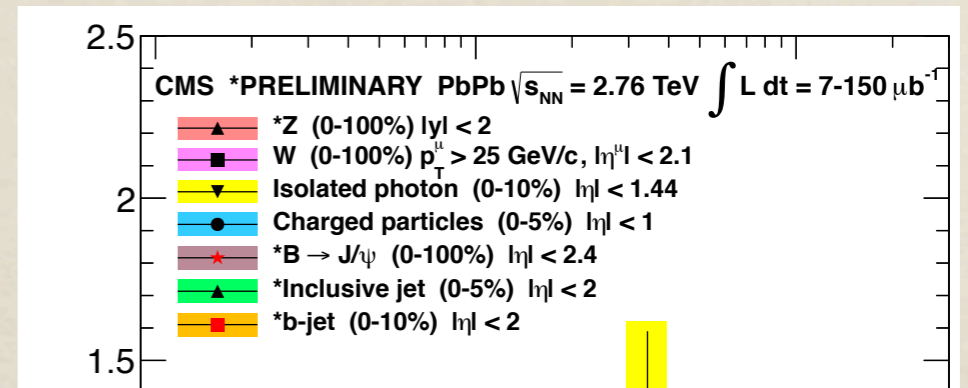
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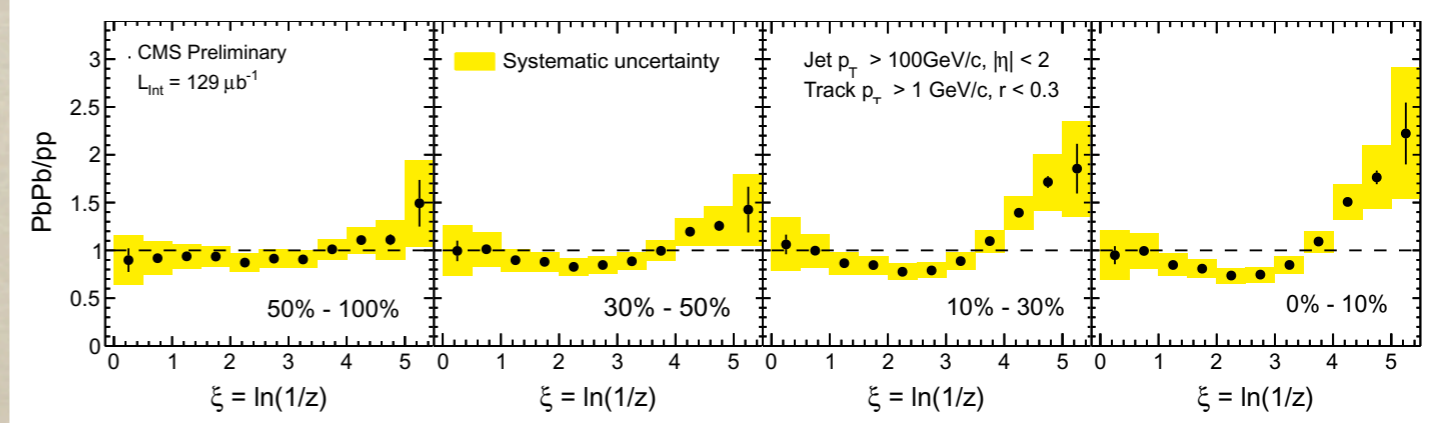
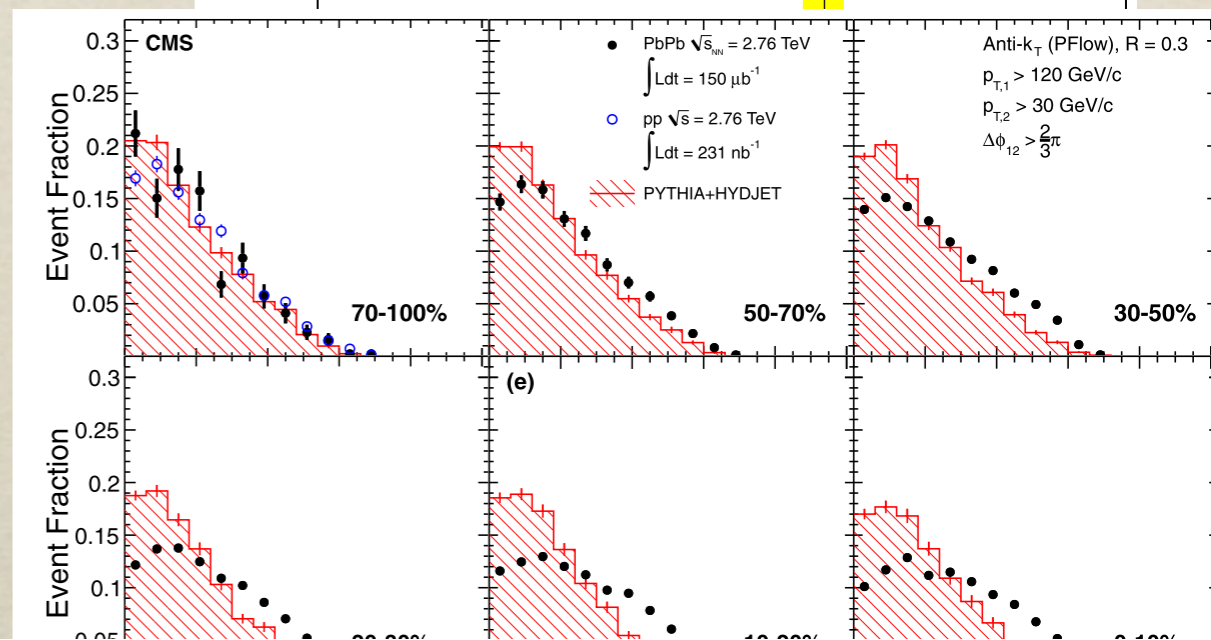
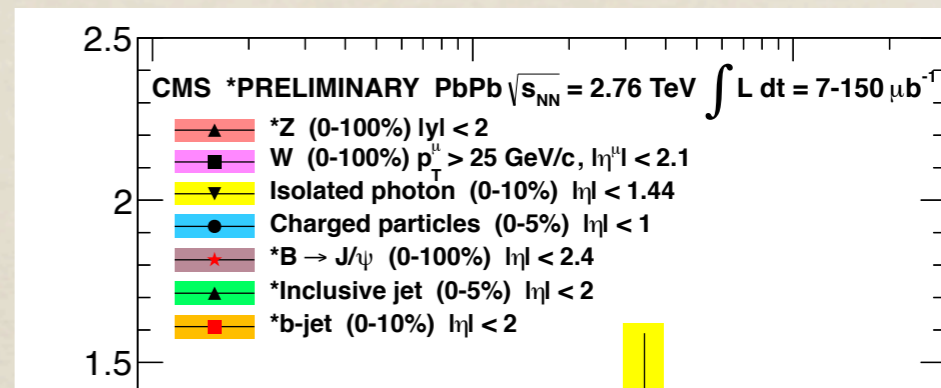
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⇒ Important to have a good description of energy loss mechanisms



Particularly relevant for Monte Carlo codes (usually employ phenomenological assumptions without a firm theoretical basis)

$$\xi = \ln\left(\frac{1}{z}\right), \quad z = \frac{p_{||}^{tr}}{p^{jet}}$$



Medium averages

- 2-point function:

- Result depends only on the transverse separation between the partons:

$$\frac{1}{N} \langle W(\mathbf{x}) W^\dagger(\mathbf{y}) \rangle = \exp \left\{ -\frac{C_F}{2} \int dx_+ \sigma(\mathbf{x} - \mathbf{y}) n(x_+) \right\}$$

$$n(x_+) \sigma(\mathbf{x}) \simeq \frac{1}{2} \hat{q} r^2 + \mathcal{O}(r^2 \ln r^2) \quad \Rightarrow \text{Valid for opaque media and small } r$$

$$\text{Transport coefficient: } \hat{q} = \frac{\langle \mathbf{k}^2 \rangle}{\lambda}$$

- 4-point function:

$$\begin{aligned} \langle [W(\mathbf{x}_1) W^\dagger(\mathbf{x}_2)]_{ij} [W(\mathbf{x}_2) W^\dagger(\mathbf{x}_3)]_{kl} \rangle &= \delta_{ij} \delta_{kl} \exp \left\{ \frac{1}{2N} v(\mathbf{x}_1 - \mathbf{x}_3) - \frac{N}{2} [v(\mathbf{x}_1 - \mathbf{x}_2) + v(\mathbf{x}_2 - \mathbf{x}_3)] \right\} \\ &+ \frac{1}{N} \delta_{il} \delta_{jk} \exp \{ -C_F v(\mathbf{x}_1 - \mathbf{x}_3) \} - \frac{1}{N} \delta_{il} \delta_{jk} \exp \left\{ \frac{1}{2N} v(\mathbf{x}_1 - \mathbf{x}_3) - \frac{N}{2} [v(\mathbf{x}_1 - \mathbf{x}_2) + v(\mathbf{x}_2 - \mathbf{x}_3)] \right\} \end{aligned}$$

\Rightarrow At large N_c , factorizes into a dipoles

- 6-point function:

$$\begin{aligned} &\langle \text{Tr} (W^\dagger(\mathbf{x}_1) W(\mathbf{x}_2)) \text{Tr} (W^\dagger(\mathbf{x}_2) W(\mathbf{x}_1) W^\dagger(\mathbf{x}_3) W(\mathbf{x}_4)) \rangle \\ &\underset{N \rightarrow \infty}{\simeq} \langle \text{Tr} (W^\dagger(\mathbf{x}_1) W(\mathbf{x}_2)) \rangle \langle \text{Tr} (W^\dagger(\mathbf{x}_2) W(\mathbf{x}_1) W^\dagger(\mathbf{x}_3) W(\mathbf{x}_4)) \rangle \end{aligned}$$

\Rightarrow At large N_c , factorization into a dipole + quadrupole