A 2-LOOP CALCULATION OF $H \to \gamma\gamma$ IN 4 DIMENSIONS: THE FDR APPROACH

- AN EASIER WAY TO NNLO CALCULATIONS -

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FDR

- AN EASIER WAY TO NNLO CALCULATIONS -

INTRODUCTION

- PRECISION PHYSICS as a powerful probe to NEW PHYSICS

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- multi-loop results essential for the studies of the HIGGS PROPERTIES

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ATTACK THE NNLO PROBLEM

BY **ABANDONING DR**
FOR A **FULLY 4-DIM. APPROACH**

D R

gauge invariant

one regulator
for UV and IR/CL
divergences

FDR

- finite
- four dimensional
- one regulator for UV and IR/CL divergences
- gauge invariant

FDR

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REINTERPRETATION OF THE LOOP INTEGRAL:

- UV infinities dropped at the integrand level
- IR/CL infinities cured by off-shell external momenta

FINITE BEFORE INTEGRATING!

FDR

finite

four dimensional

one regulator for UV and IR/CL divergences

gauge invariant

REINTERPRETATION OF THE LOOP INTEGRAL:

- UV infinities dropped at the integrand level
- IR/CL infinities cured by off-shell external momenta

- NNLO required, but a general solution is lacking
- great potential of **FDR**

THE METHOD

notation
$D = q^2 - m^2$

partial fraction identity $\frac{1}{D} = \frac{1}{q^2}\left(1 + \frac{m^2}{D}\right)$

$$\frac{1}{D^2}$$

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IR divergent!

notation	add a small mass μ	partial fraction identity
$D = q^2 - m^2$	$q^2 \to \bar{q}^2 = q^2 - \mu^2$ $D \to \bar{D} = D - \mu^2$	$\frac{1}{\bar{D}} = \frac{1}{\bar{q}^2}\left(1 + \frac{m^2}{\bar{D}}\right)$

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IR safe!

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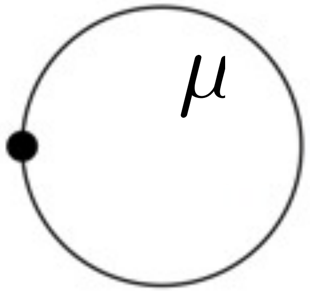

VACUUM BUBBLE
- divergent in UV
- regular in IR
- universal

FINITE PART
- convergent in 4D
- contains all kinematics

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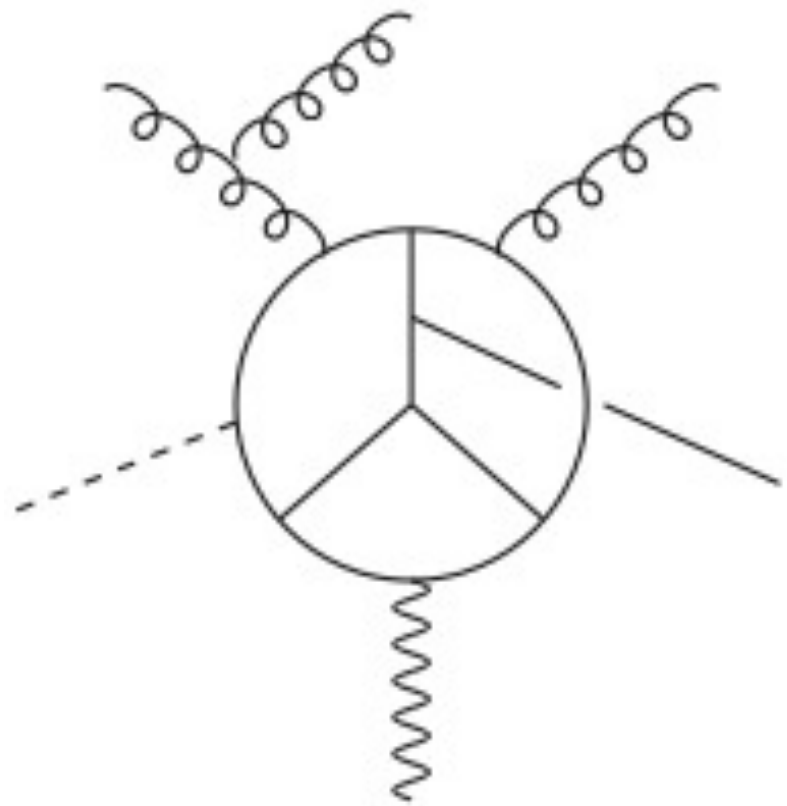
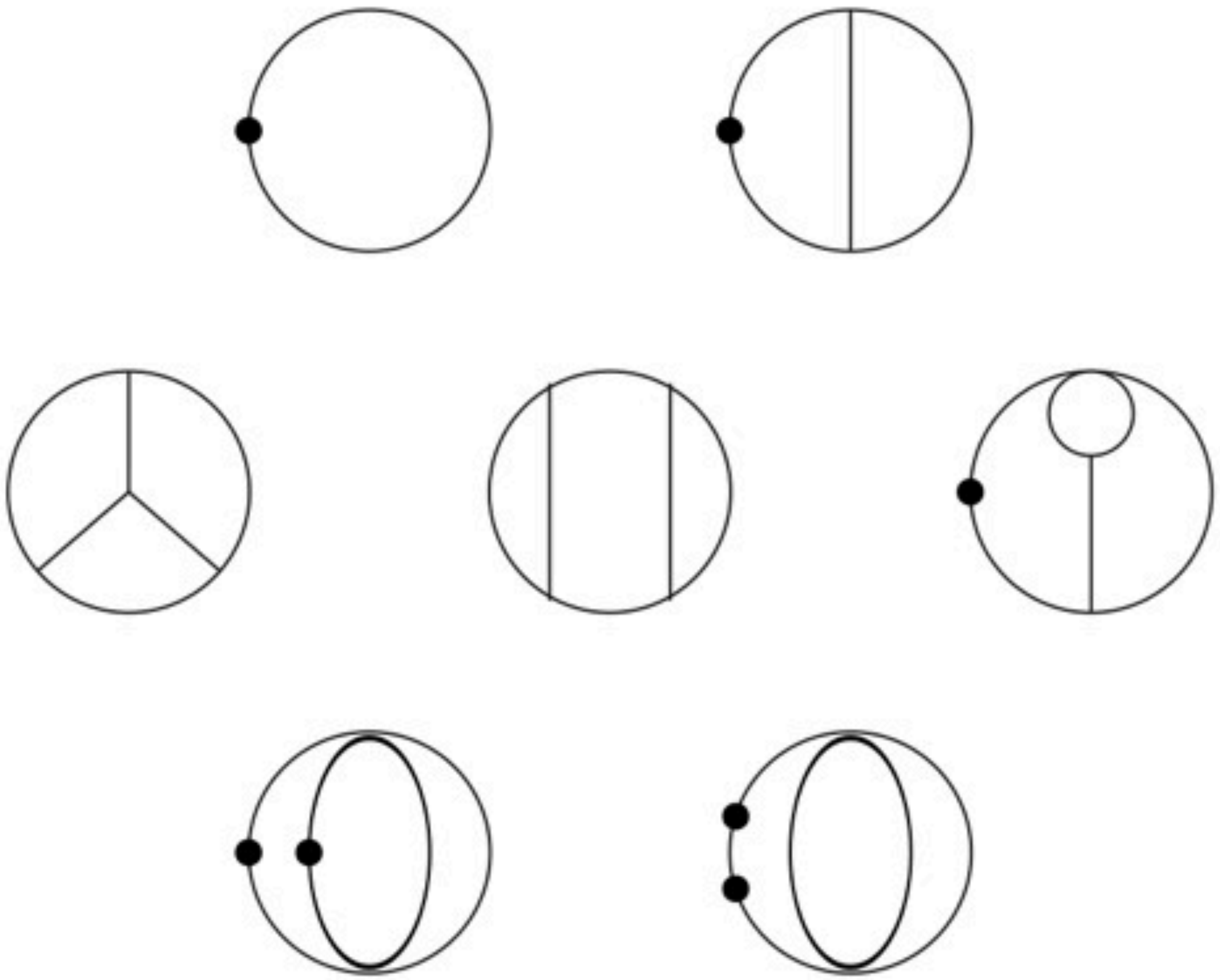
FDR INTEGRAL

FINITE PART

- convergent in 4D
- contains all kinematics

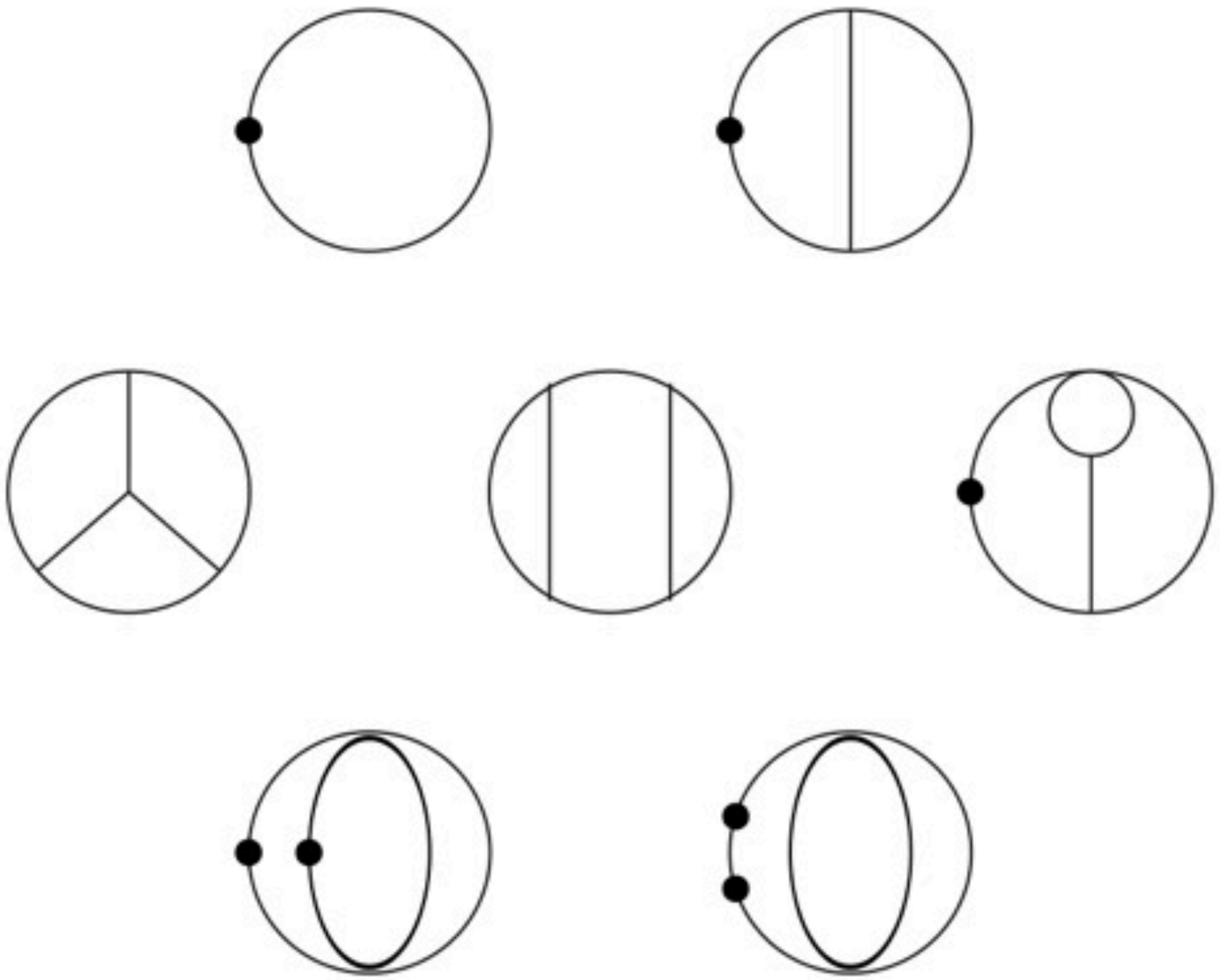


in UV limit all physical scales are irrelevant with respect to loop momenta: the diagram behaves as a vacuum bubble!
(~ Gell-Mall-Low formula)



Four Dimensional Renormalization

Order by order redefinition of the vacuum to absorb infinities



Four
Dimensional
Renormalization

Order by order redefinition of the vacuum to absorb infinities

FINITE RENORMALIZATION: $\mathcal{L}(p_1, \ldots, p_n) \leftrightarrow \left(O_1^{\text{exp}}, \ldots, O_n^{\text{exp}}\right)$

FDR

$$\int \prod [\mathrm{d}^4 q_i] J(\{q_i\}) = \lim_{\mu \to \infty} \int \prod \mathrm{d}^4 q_i \; J_F(\{q_i\}; \mu)$$

4D

FINITE

FDR

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4D

FINITE

only logarithmic dependence on μ:
$\mu = \mu_R$ as renormalization scale

FDR

$$\int \prod [\mathrm{d}^4 q_i] J(\{q_i\}) = \lim_{\mu\to\infty} \int \prod \mathrm{d}^4 q_i \; J_F(\{q_i\};\mu)$$

JUST AN INTEGRAL!

- linear
- shift-invariant
- gauge-invariant

only logarithmic dependence on μ: μ=μR as renormalization scale

4D

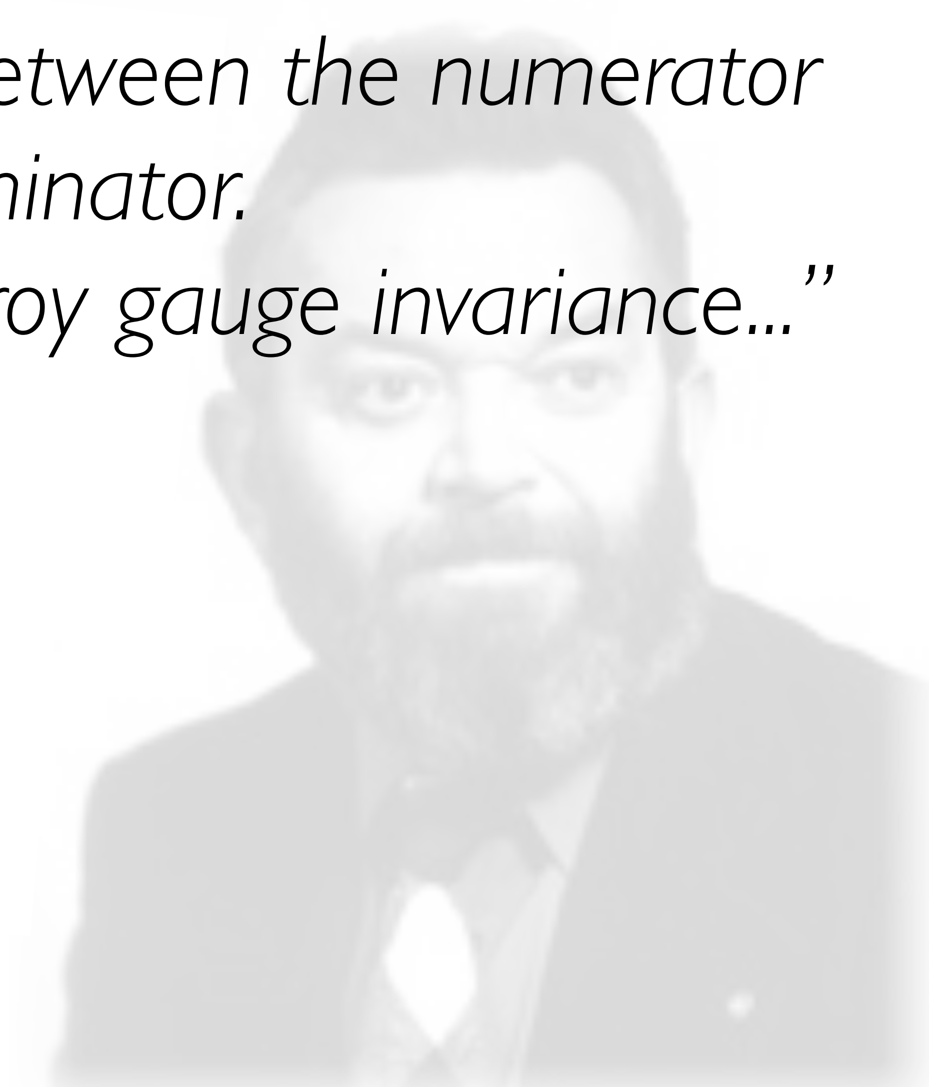
FINITE

GAUGE INVARIANCE

RESPECTED BY CONSTRUCTION

"Gauge Invariance implies a tight interplay between the numerator of an integrand and its denominator.
Changing either of the two will generally destroy gauge invariance..."

- M.Veltman (1974) -



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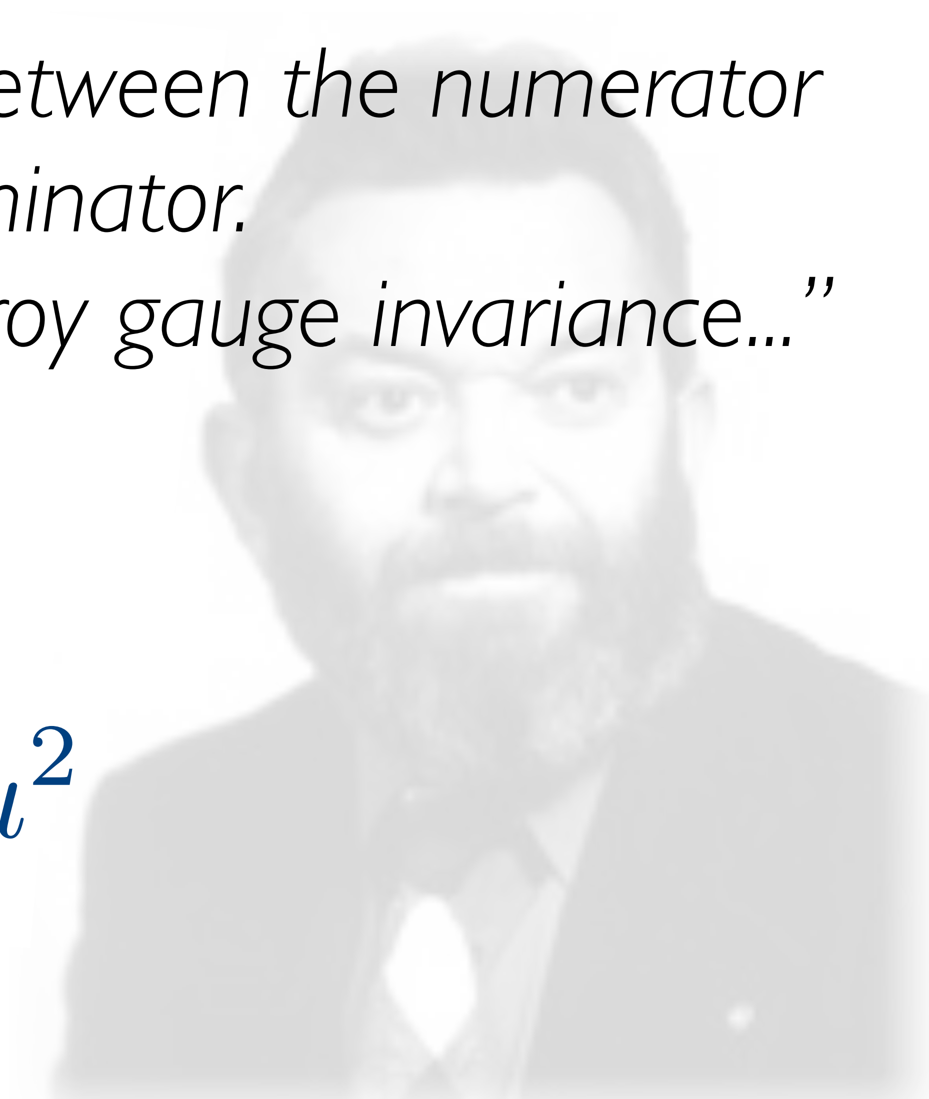

$$q^2 \to \bar{q}^2 = q^2 - \mu^2$$

GAUGE INVARIANCE

RESPECTED BY CONSTRUCTION

- propagators $$\frac{1}{\bar{q}^2 - m^2} = \frac{1}{q^2 - m^2 - \mu^2}$$
- q^2 coming from Feynman rules $$\bar{q}^2 = q^2 - \mu^2$$

} simplifications between numerator and denominator

- tensorial reduction $$q^\mu q^\nu g_{\mu\nu} \to \bar{q}^2 + \mu^2$$

} finite contributions

$$\frac{1}{\bar{q}^2 - m^2} = \frac{1}{q^2 - m^2 - \mu^2}$$

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$$\frac{1}{\bar{q}^2 - m^2} = \frac{1}{q^2 - m^2 + i\varepsilon}$$

DR vs FDR

- equivalent at **one loop**: $\log \mu_R^2 + \frac{2}{\epsilon} + UC \quad \Leftrightarrow \quad \log \mu^2$

  (diagram by diagram)

DR vs FDR

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RG eqs. unchanged in **FDR**

DR vs FDR

- equivalent at **one loop**: $\log \mu_R^2 + \frac{2}{\epsilon} + UC \quad \Leftrightarrow \quad \log \mu^2$

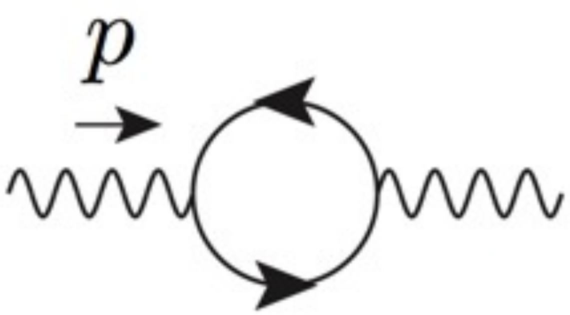
  (diagram by diagram)

- completely different at **two loops**: **FDR** is simpler .

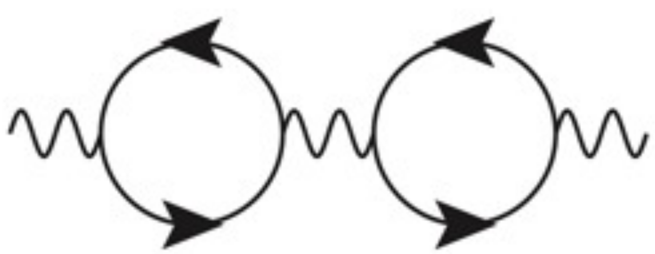
  $\left\{ \frac{1}{\epsilon^2}, \frac{1}{\epsilon}, \log \mu_R^2, \log^2 \mu_R^2, \frac{1}{\epsilon} \log \mu_R^2 \right\} \qquad \log \mu^2, \; \log^2 \mu^2$

- **FDR** integral = integral in 4 dim. of the finite part only
- gauge-invariance naturally respected
- infinities absorbed in the vacuum (not in the Lagrangian)
- only finite renormalization required
- at one loop: **FDR** equivalent to DR diagram by diagram

TWO LOOP

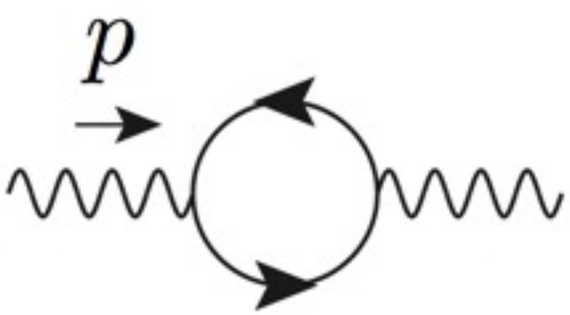


$$\propto \Pi(p^2)$$

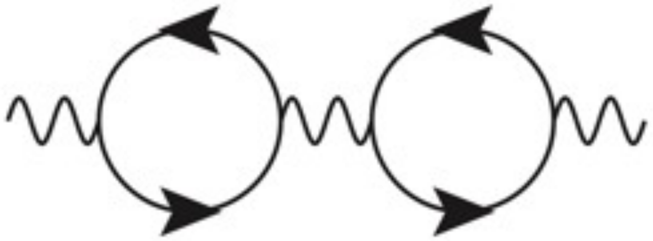


$$\propto \left[\Pi(p^2)\right]^2$$

DR



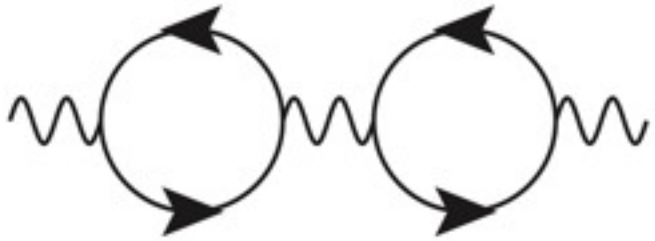
$$\propto \Pi(p^2) = \frac{\Pi_{-1}}{\epsilon} + \Pi_0 + \epsilon\, \Pi_1$$



$$\propto \left[\Pi(p^2)\right]^2 = 2\Pi_{-1}\Pi_1 + \Pi_0^2 + \frac{2\Pi_{-1}\Pi_0}{\epsilon} + \frac{\Pi_{-1}^2}{\epsilon^2}$$

DR

Can we simply remove the UV poles ?



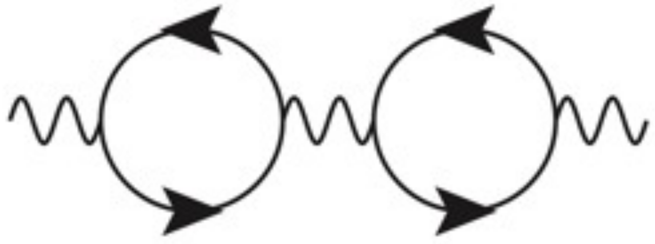
?

$$2\Pi_{-1}\Pi_1 + \Pi_0^2$$

$$+\frac{2\Pi_{-1}\Pi_0}{\epsilon} + \frac{\Pi_{-1}^2}{\epsilon^2}$$

DR

Can we simply remove the UV poles ? No.



$$\neq \quad 2\Pi_{-1}\Pi_1 + \Pi_0^2$$

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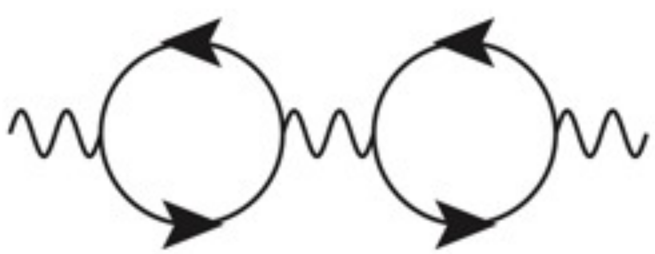
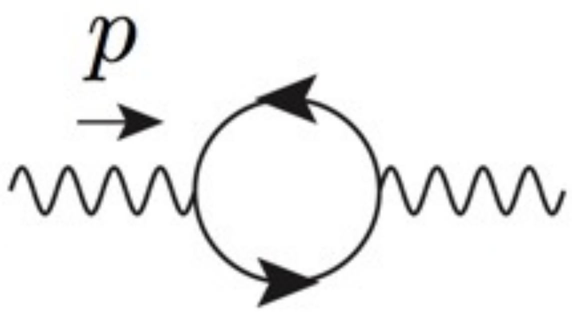


$$\neq \quad 2\Pi_{-1}\Pi_1 + \Pi_0^2$$

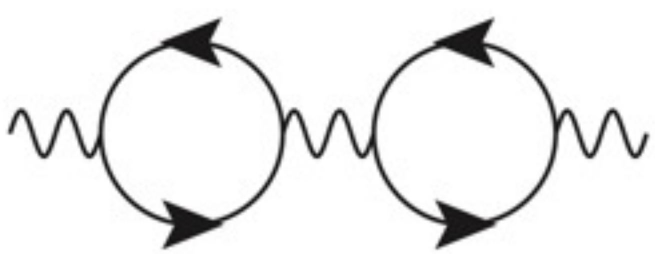
$$2\Pi_{-1}\Pi_1 \to \frac{1}{\epsilon} \times \mathcal{O}(\epsilon)$$

DR



$+$

$= \Pi_0$



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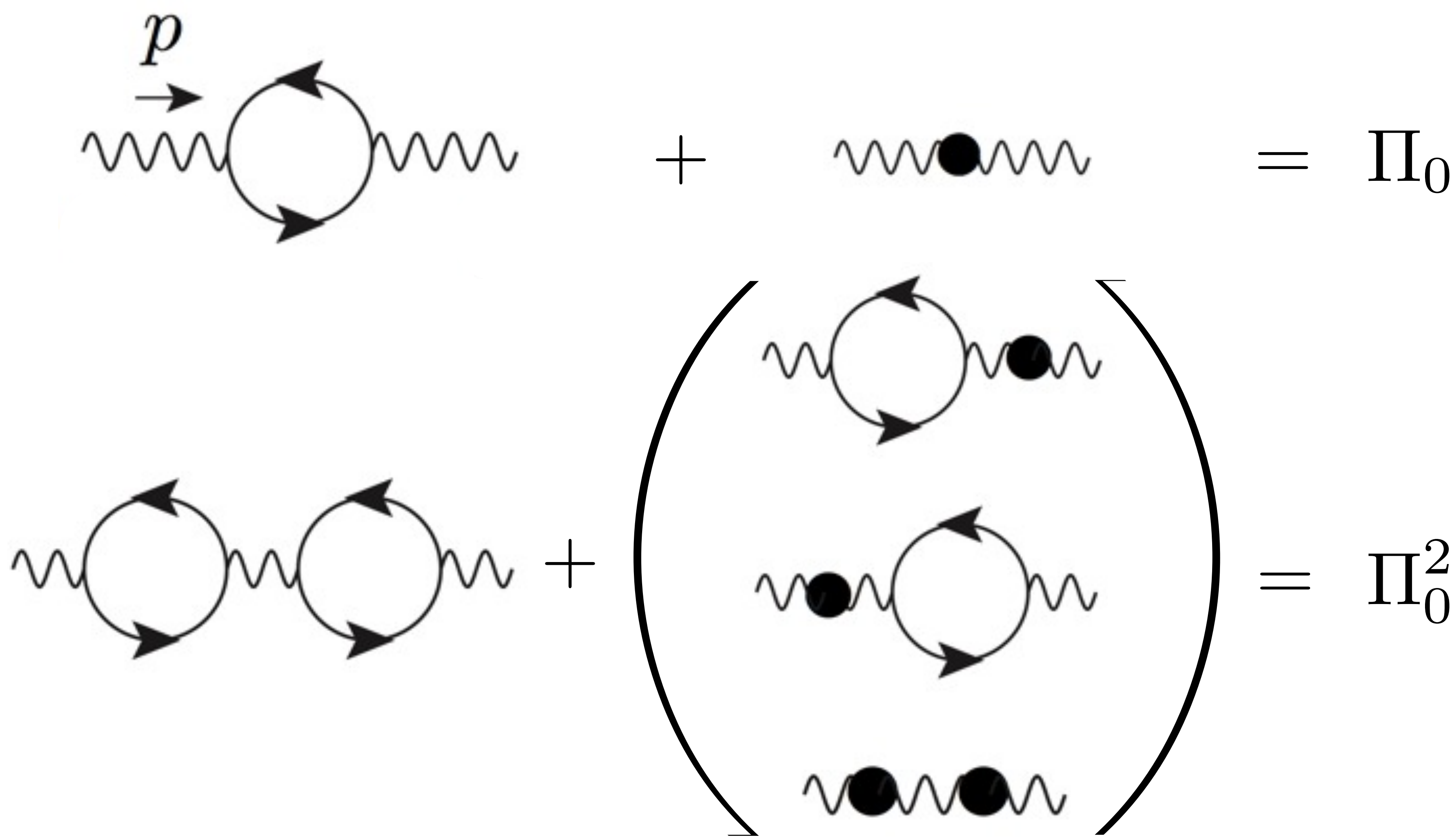
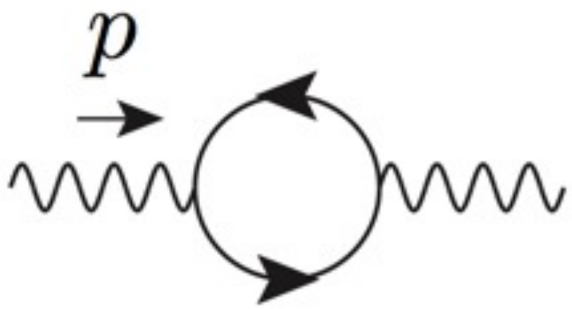


FDR



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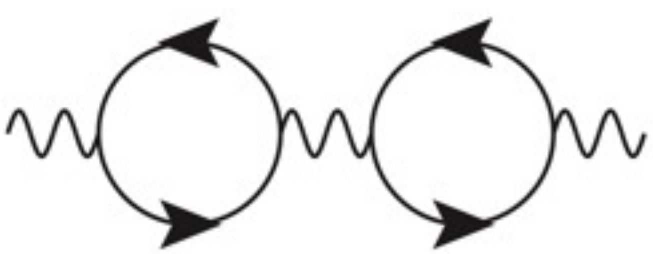
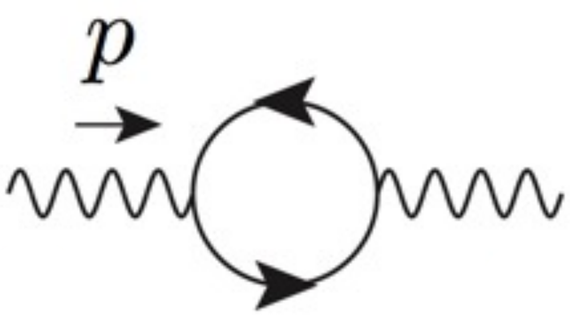


$$= \Pi_0^2$$

FDR



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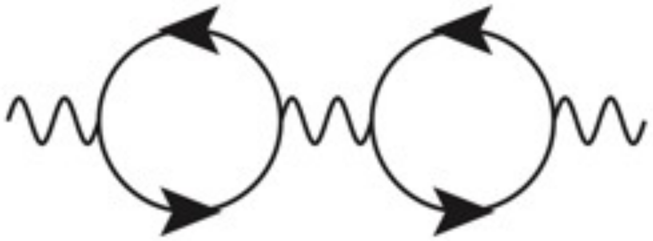

$= \Pi_0^2$

NO ORDER-BY-ORDER RENORMALIZATION NEEDED IN FDR !

FDR

factorizable *n*-loop integrals

$$\int [\mathrm{d}^4 q] \, \frac{1}{\overline{D}^{\alpha}} = A$$

$$\int [\mathrm{d}^4 q] \, \frac{1}{\overline{D}^{\beta}} = B$$

$$\int [\mathrm{d}^4 q_1][\mathrm{d}^4 q_2] \, \frac{1}{\overline{D}_1^{\alpha} \overline{D}_2^{\beta}} = AB$$

DR

factorizable *n*-loop integrals

$$\int d^n q \, \frac{1}{\overline{D}^{\alpha}} = \frac{A_{-1}}{\epsilon} + A_0 + \epsilon A_1$$

$$\int d^n q \, \frac{1}{\overline{D}^{\beta}} = \frac{B_{-1}}{\epsilon} + B_0 + \epsilon B_1$$

$$\frac{1}{\epsilon} \times \mathcal{O}(\epsilon)$$

FDR

factorizable *n*-loop integrals

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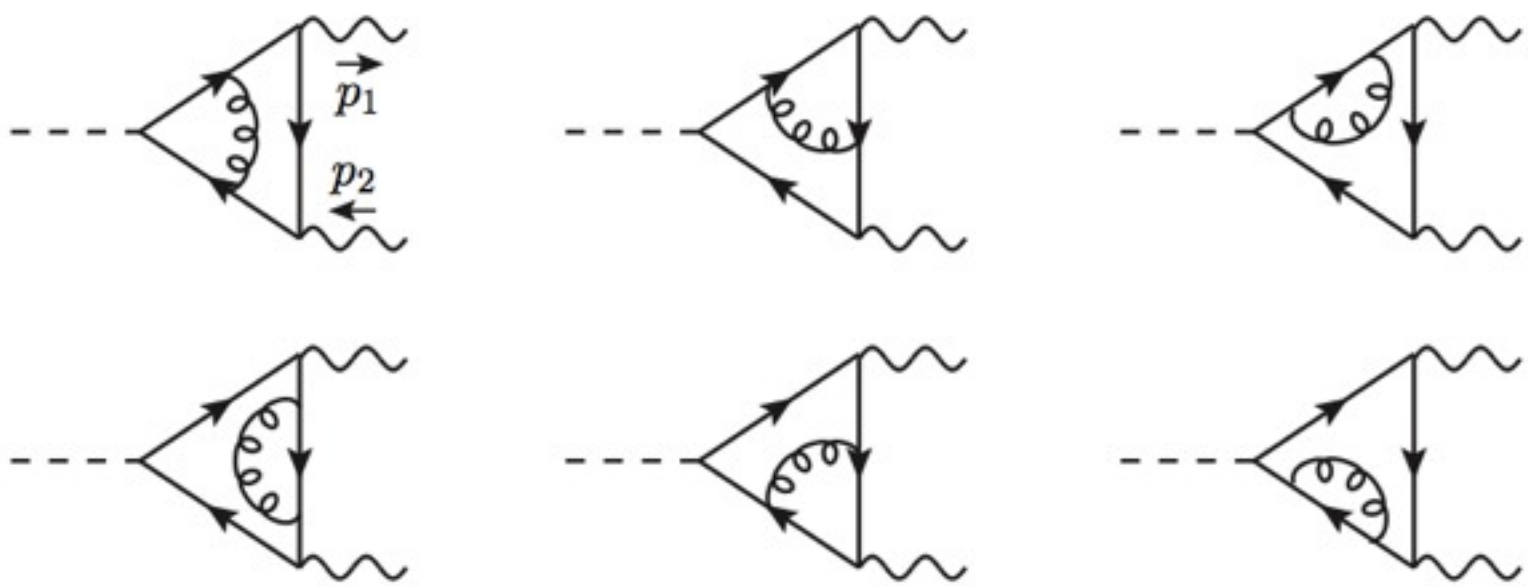
(n-1)-loop results directly re-usable in n-loop factorizable integrals

- at $\geq$2 loop, no direct correspondence between **FDR** and DR
- **FDR** is better:
  - no order by order renormalization
  - lower order results directly re-usable

$H \to \gamma\gamma$

FIRST 2-LOOP CALCULATION IN FDR

Amplitude for $H \to \gamma\gamma$ in the limit $m_{\rm top} \to \infty$



$$\mathcal{M}^{\mu\nu} = \mathcal{M}^{\mu\nu}_{\rm Born}\left(1 - \frac{\alpha_S}{\pi}\right)$$

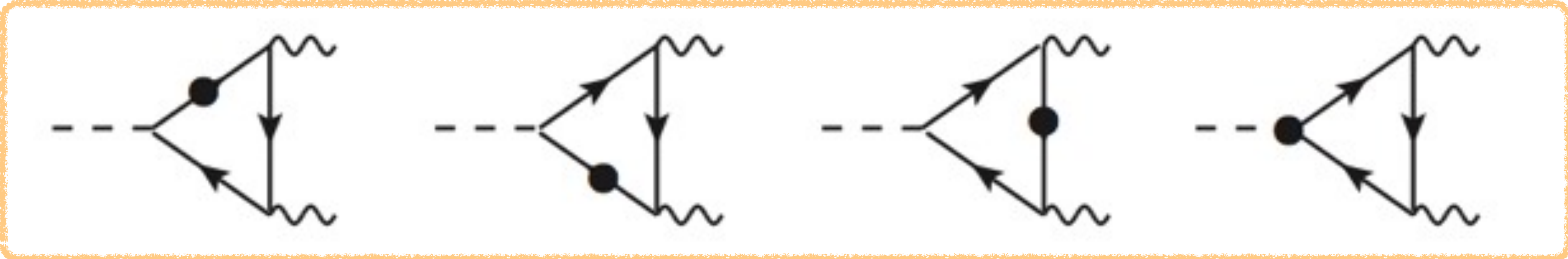
$$\mathcal{M}^{\mu\nu}_{\rm Born} = \frac{\alpha}{\pi}\frac{i c_N}{3v}T^{\mu\nu}$$

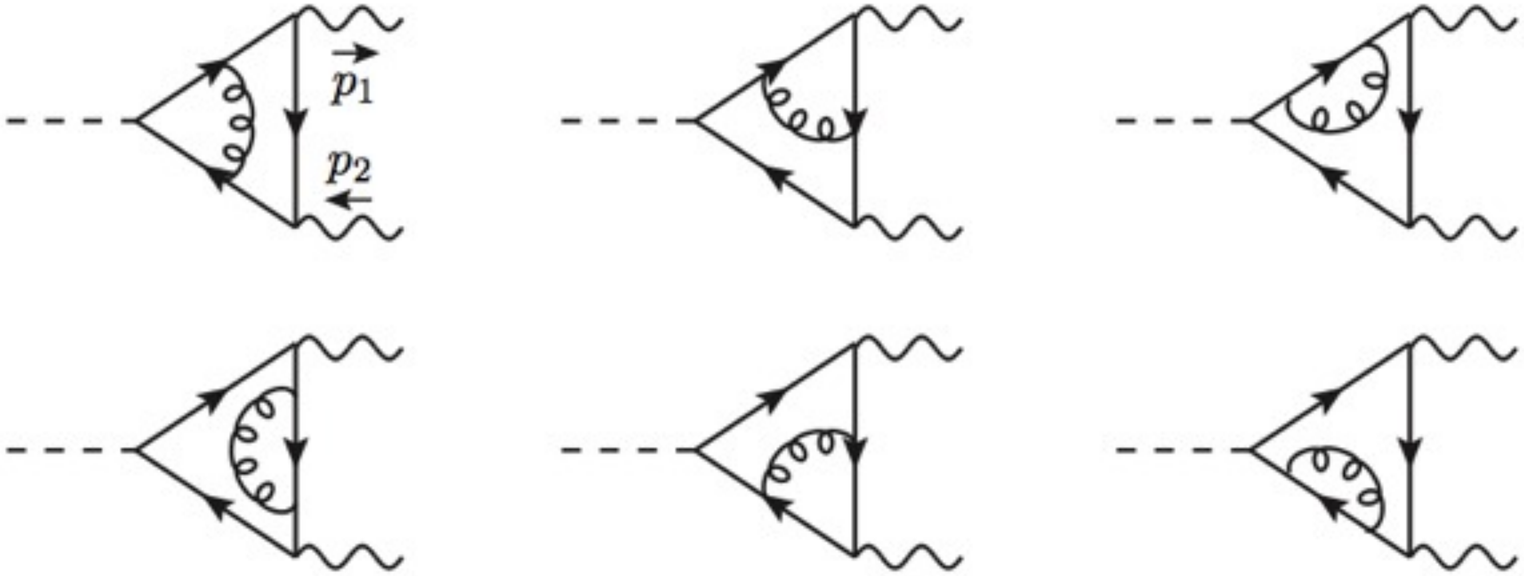
$$T^{\mu\nu} = p_2^{\mu} p_1^{\nu} + \frac{s}{2} g^{\mu\nu}$$

COUNTER-TERMS

DR

top self energy





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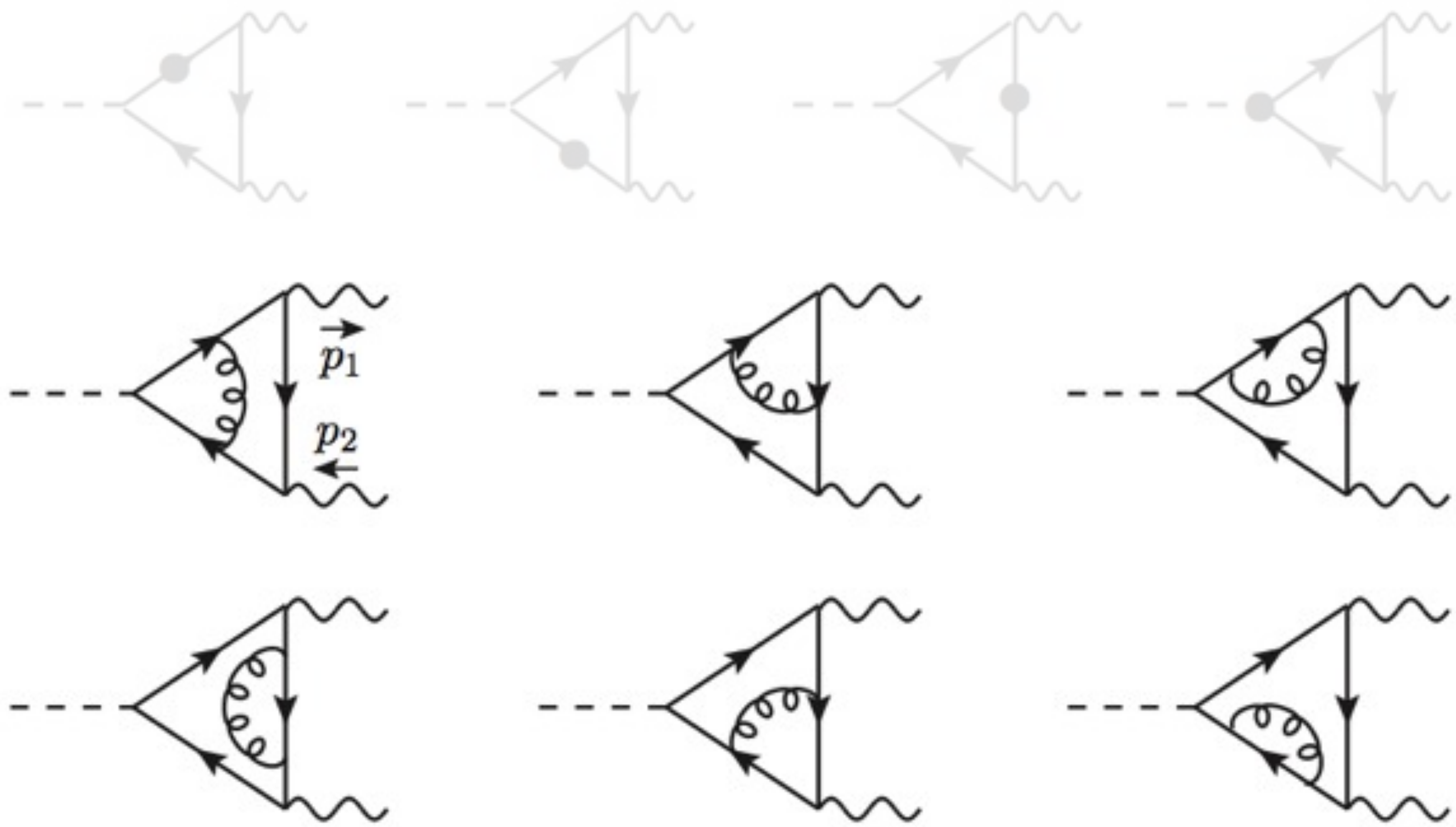

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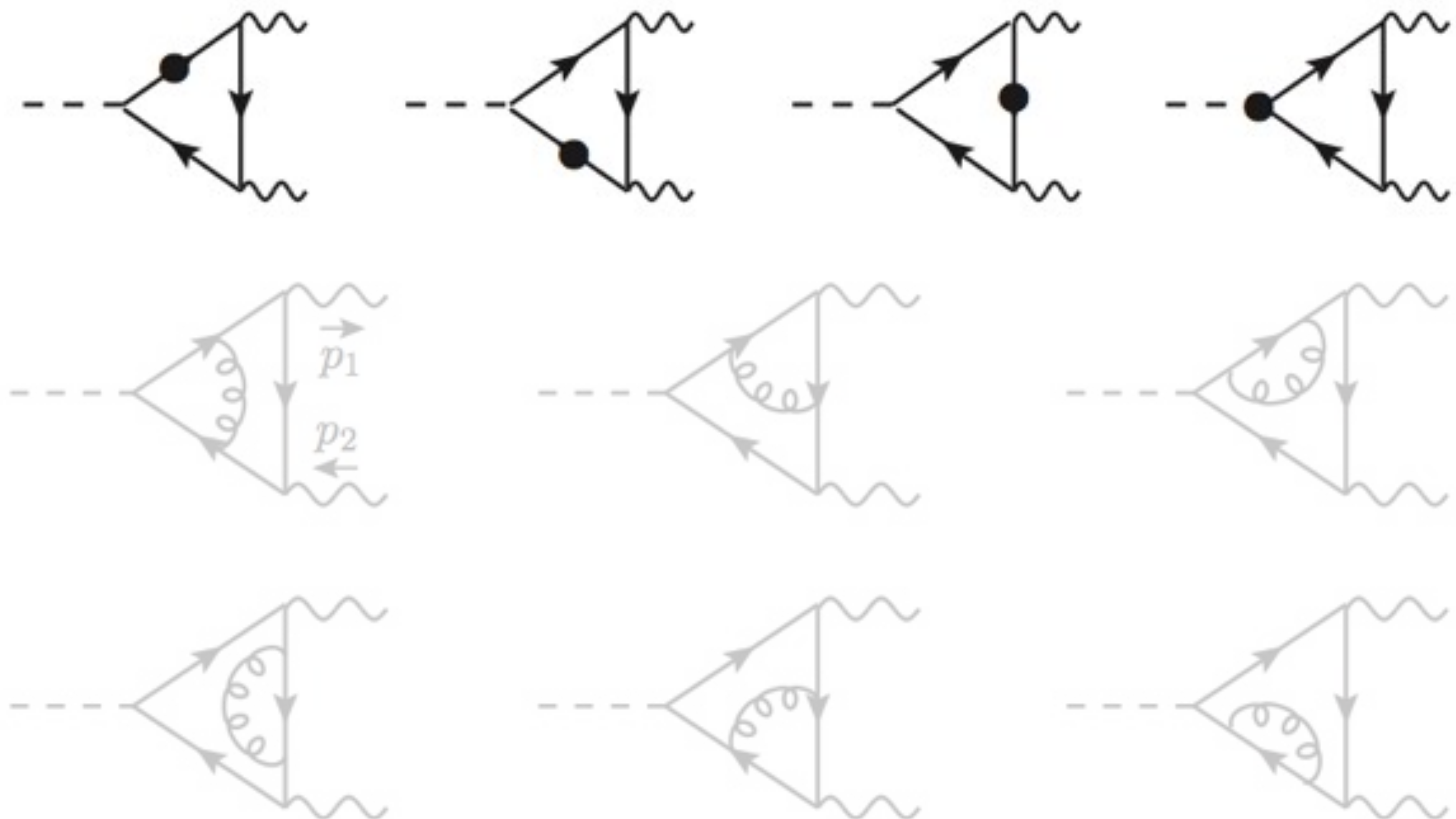

$$\equiv 0$$

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FDR

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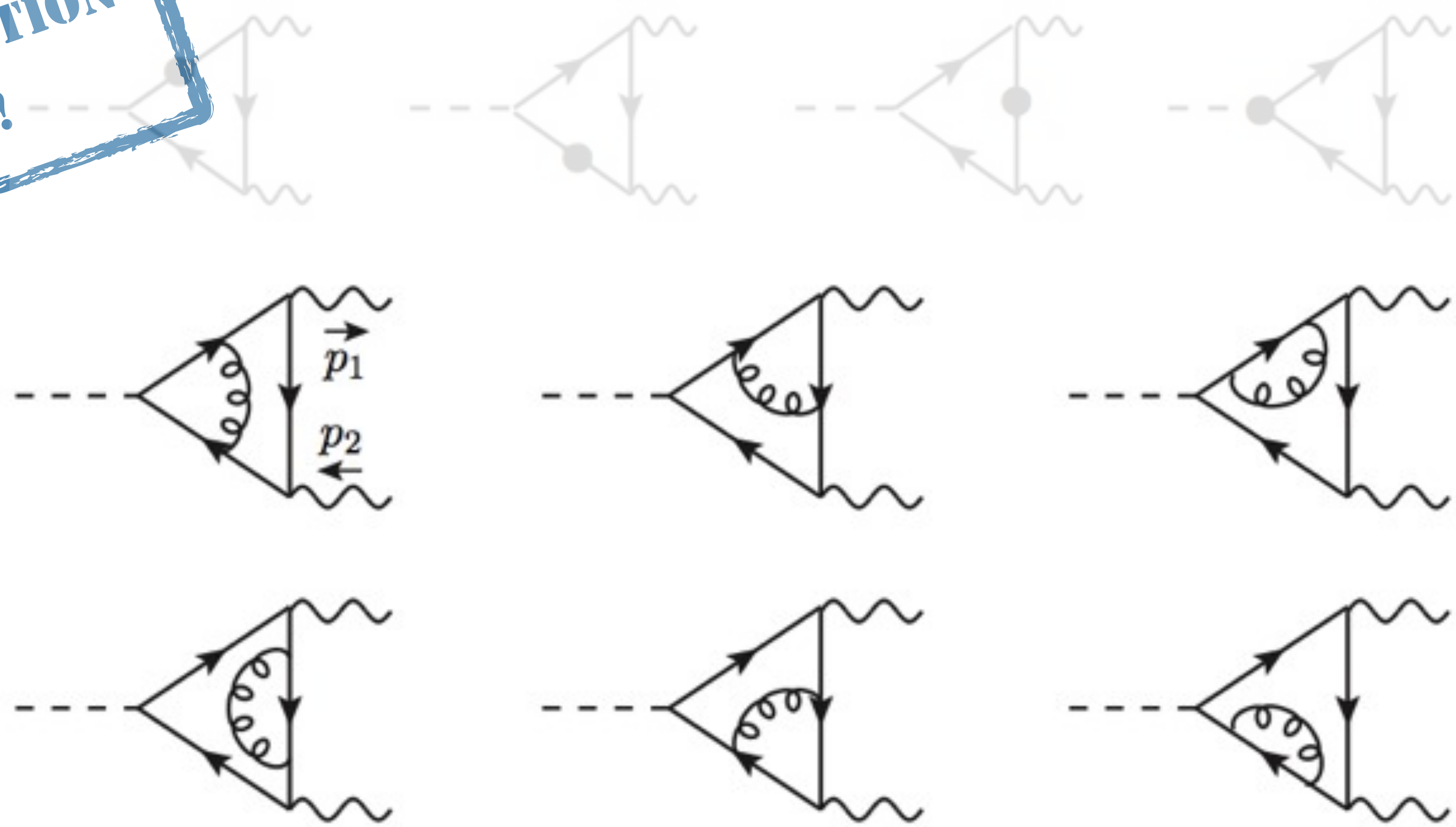

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CONCLUSIONS

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- AN EASIER WAY TO NNLO CALCULATIONS -

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FDR

- finite
- four dimensional
- one regulator for UV and IR/CL divergences
- gauge invariant

NNLO
FDR

finite

↓

only a **finite**
“renormalization”
is required

four dimensional

↓

lower order results
directly reusable in
higher loop calculations

one regulator for UV and IR/CL divergences

gauge invariant

↓

no order by order renormalization needed

NNLO
FDR

finite

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four dimensional

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NEXT!

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no order by order renormalization needed

NUMERICAL TECHNIQUES + NNLO FDR

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↓

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NUMERICAL TECHNIQUES + NNLO FDR ⇨ SIMPLIFICATION AND AUTOMATION

THANKS

Read more about **FDR**

JHEP 11 (2012) 151 arXiv:1208.5457	"A Four Dimensional approach to Quantum Field Theories" - Roberto Pittau -
JHEP 4 (2013) arXiv:1302.5668	"Gauge Invariance at work in FDR: H$\rightarrow\gamma\gamma$" - Alice M. Donati, Roberto Pittau -
arXiv:1305.0419	"On the predictivity of non-renormalizable Quantum Field Theories" - Roberto Pittau -
arXiv:1307.0705	"QCD corrections to H$\rightarrow$gg" - Roberto Pittau -
arXiv:1311.3551	**"FDR, an easier way to NNLO calculations: a two loop case study" - Alice M. Donati, Roberto Pittau -**

μ and the renormalization scale

notation $D = q^2 - m^2$

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$$\int [\mathrm{d}^4 q] \frac{1}{\bar{D}^2} = \lim_{\mu \to 0} \int \mathrm{d}^4 q \left(\frac{2m^2}{\bar{q}^4 \bar{D}} + \frac{m^4}{\bar{q}^4 \bar{D}^2} \right) \Bigg|_{\mu = \mu_R}$$

$$= -i\pi^2 \log \frac{m^2}{\mu_R^2}$$

if the infinity is logarithmic,
the limit cannot be taken:
a μ-dependence is left

but it can be interpreted as
the renormalization scale

INDEPENDENT OF THE UV REGULATOR

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FDR INTEGRAL — INTEGRAL IN CUT-OFF SCHEME — VACUUM

$$\int [\mathrm{d}^4 q] \frac{1}{\bar{D}^2} = \int_\Lambda \mathrm{d}^4 q \, \frac{1}{\bar{D}^2} - \lim_{\mu \to 0} \int_\Lambda \mathrm{d}^4 q \left[\frac{1}{\bar{q}^4} \right]$$

physical!

$$2 i \pi^2 \left(\int_0^{\mu_R} \mathrm{d}q + \int_{\mu_R}^{\Lambda} \mathrm{d}q \right) \frac{q^3}{(q^2 + \mu^2)^2}$$

UV part

$$= -i\pi^2 \log \frac{m^2}{\mu^2}$$

$$= -i\pi^2 \left(1 + \log \frac{\mu^2}{\mu_R^2} \right)$$

Hierarchy problem ?

the natural scale dependence is only logarithmic

$$\log \frac{M_H}{GeV} \sim 5$$

$$\log \frac{\Lambda_{\text{Planck}}}{GeV} \sim 44$$

IR and CL divergences in massless theories

VIRTUAL

naturally regulated by μ

vanishing scaleless integrals

$$B_0(p^2,0,0) \underset{p^2 \to 0}{\to} \begin{cases} 0 & \text{in FDR} \\ \frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}} & \text{in DR} \end{cases}$$

REAL

would-be-massless external momenta are given a mass μ when integrating in the PS

(optical theorem)

WHAT GUARANTEES GAUGE INVARIANCE?

solitary μ : a μ^2 must be treated as a $\bar{q}^2$ within an integral

$$\int [\mathrm{d}^4 q] \frac{\mu^2}{\bar{D}^2} = i\pi^2 m^2$$

⇒ correct constant part (rational term)

FDR

$$\int [\mathrm{d}^2 q] \frac{\mu^2}{\bar{D}^2} \to \text{constant}$$

⟷

DR

$$\frac{1}{\epsilon} \times O(\epsilon) \to \text{constant}$$

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where can we meet a **solitary μ** ?

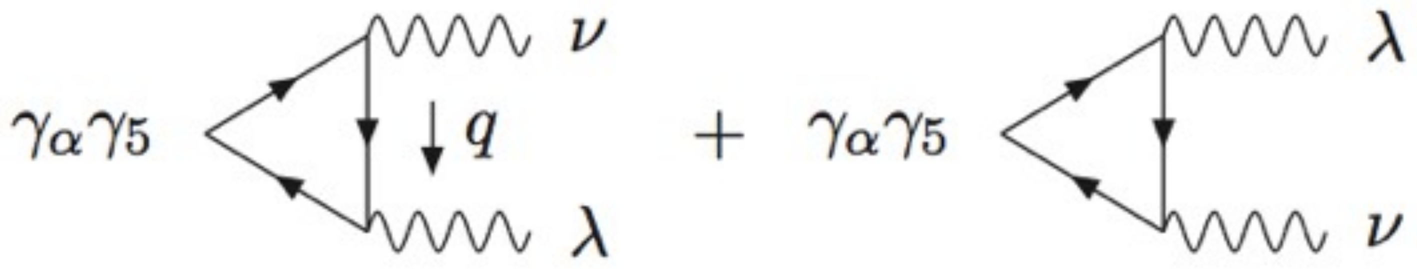
during the TENSORIAL REDUCTION.

$$\frac{q^\mu q^\nu}{\bar{D}^2} = \frac{1}{4} g^{\mu\nu} \frac{q^2}{\bar{D}^2} = \frac{1}{4} g^{\mu\nu} \left(\frac{\bar{q}^2}{\bar{D}^2} + \frac{\mu^2}{\bar{D}^2} \right)$$

tensorial reduction (Passarino-Veltman, OPP, ...) works fine in **FDR** what about **the chiral matrix γ_5** ?

- all γ_5's should be anticommutated at the beginning of the string first
- all γ_5's should be anticommutated next to the non-conserved current, in closed loops

⇒ ABJ anomaly reproduced in **FDR** (JHEP1211)



an example at 2-loop

$$\int [d^4q_1][d^4q_2]\,\frac{1}{\bar{D}_1^2\bar{D}_2\bar{q}_{12}^2} = \pi^4\left\{f - \mathrm{Li}_2\left(1-\frac{m_2^2}{m_1^2}\right) - \frac{1}{2}\ln^2\frac{\mu^2}{m_1^2} - \ln\frac{\mu^2}{m_1^2}\right\}$$

in DR

$$\int d^nq_1 d^nq_2\,\frac{1}{\bar{D}_1^2\bar{D}_2\bar{q}_{12}^2} = \pi^4\left\{-\mathrm{Li}_2\left(1-\frac{m_2^2}{m_1^2}\right) - \ln^2\frac{\mu^2}{m_1^2} - \ln\frac{\mu^2}{m_1^2} + \mathrm{const.}\right\}$$