# Towards NNLO QCD predictions for pp→tt+j

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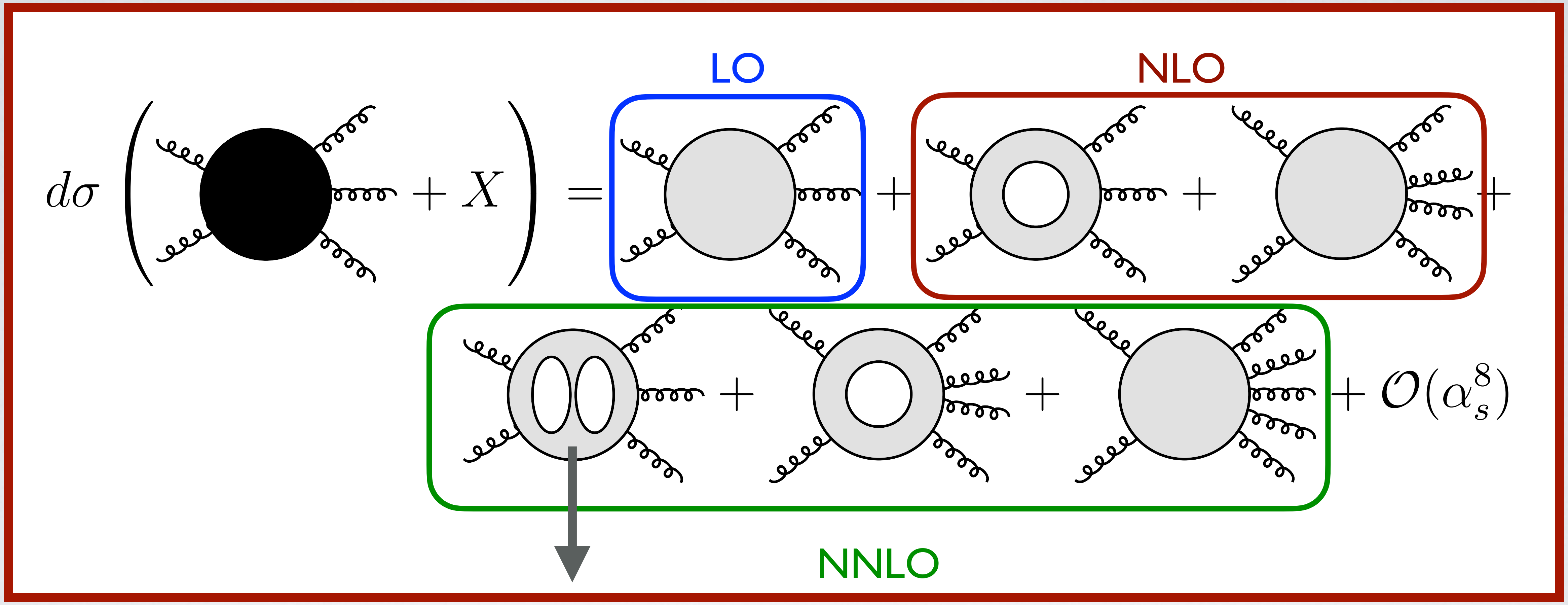
based on work with:

Becchetti, Brancaccio, Hartanto, Giraudo, Zoia

Workshop on top quark mass measurements

21st May 2024

FARE
RICERCA IN ITALIA
FRAMEWORK PER L'ATTRAZIONE E IL RAFFORZAMENTO DELLE ECCELLENZE PER LA RICERCA IN ITALIA




**missing**

Theory predictions for tt+(massless) signals (i.e. tt+j / tt+γ) present a **serious** challenge

⇒ we need to develop new calculational techniques

[parallel on-going theory efforts for ttH, ttW, ttZ]

# Short version

# When will it be available?

1. numerical evaluation of the amplitude at benchmark points - **soon**
2. integration over full phasespace and combination with RV and VV - **not so soon**

The simulations will be very expensive - we should make sure to provide precisely the right things needed for the analyses

The relative size of the double virtual may not be that large (c.f. other 2→3 processes) - leading colour likely good enough.

There has been **significant progress** for **2→3 scattering problems** in recent years

Agarwal, Buccioni, Devoto, Gambuti, von Manteuffel, Tancredi, Abreu, De Laurentis, Dormans, Febres Cordero, Ita, Kraus, Klinkert, Page, Pascual, Ruf, Sotnikov, SB, Brønnum-Hansen, Chicherin, Gehrmann, Hartanto, Henn, Krys, Marcoli, Moodie, Peraro, Zoia,…

**recent results**

3j full colour 2L

γ+2j full colour NNLO

5pt one-mass master integrals (V+2j, H+2j,…)

processes with **massless internal** particles no longer present the same challenge they did ~ 5 years ago

**This talk**: before we can tackle the summit we need to make sure we have the right equipment

- NEW! master integrals for ttj in the large $N_c$ limit

# top quark helicity amplitudes

colour decomposition

$$\mathcal{A} = \sum_k C_k A_k$$

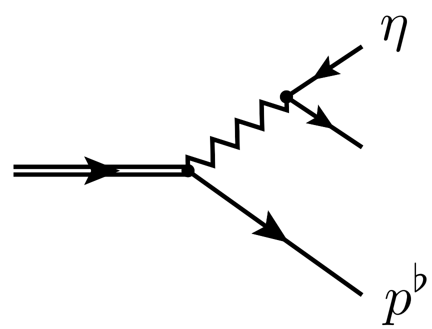
consider **leading colour** only for now

$$A_k = \sum_l c_{kl} I_l$$

rational coefficients

basis of linearly independent basis integrals closed under IBP identities

helicity amplitudes encode spin correlations in the narrow width approximation




$$\overline{u_\pm}(p, m; n) = \frac{\langle \eta \mp | (\not{p} + m)}{\langle \eta \mp | p^\flat \pm \rangle}$$

e.g. Melnikov, Schulze '09

corrections at **one-loop** in QCD now standard in MC event generators
[Integrals: QCDloop, OneLoop… Amplitudes: OpenLoops, aMC@NLO, Recola,…]

# canonical form differential equations (DEQs)

[Henn (2013)]

$$\partial_x M_i = \epsilon A_{ij}(x) M_j$$

$d = 4 - 2\epsilon$

$M_i$ integral basis usually called 'master integrals' (MIs)

$A_{ij}$ matrix depends on kinematic invariants

if $dA = \sum_i d\log(W_i)$ it is (relatively) easy to define a special function basis from iterated integrals

e.g. 1M pentagon functions: Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia [2306.15431]

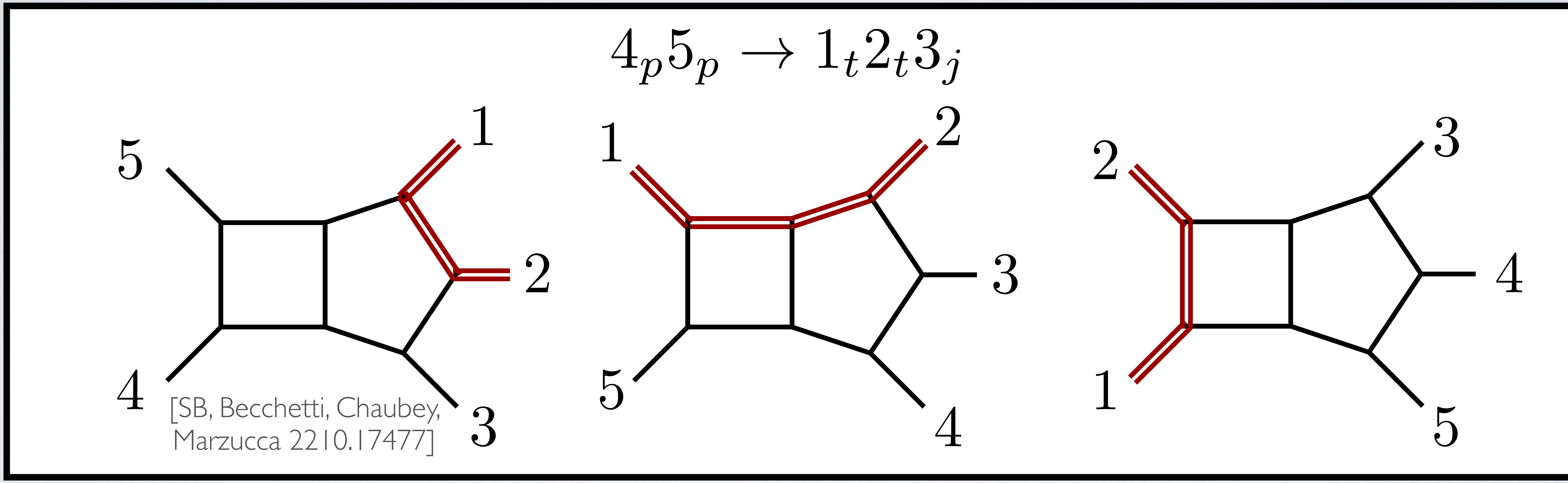
$W_i$ alphabet

automated approaches to find canonical bases (Fuchsia, epsilon, initial, dlog…) not sufficient to handle complicated kinematics of ttj

Analytic forms for DEQ matrices, A, have been obtained efficiently using the latest techniques for **Finite Field** (FINITEFLOW [Peraro '19]) solutions of **optimized integration by parts identities** (NEATIBP [Wu et al. '23])

# integral basis for leading colour ttj

c




| 88 MIs | 121 MIs | 109 MIs |
|---|---|---|
| dlog ✓ | dlog ✗ | dlog ✓ |
| 71 letters | | 79 letters |

high precision boundary values with Auxiliary Mass Flow (AMFLOW Liu '22)

# integral basis for leading colour ttj

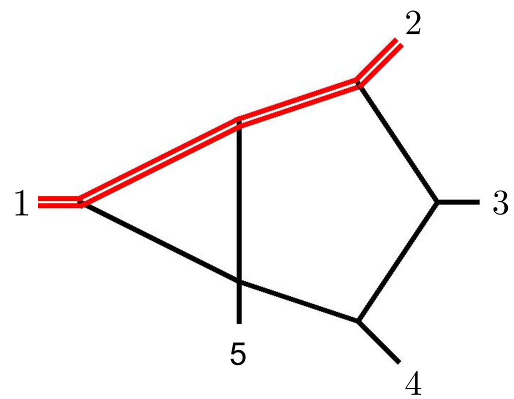
[SB, Bechetti, Giraudo, Zoia 2404.12325]

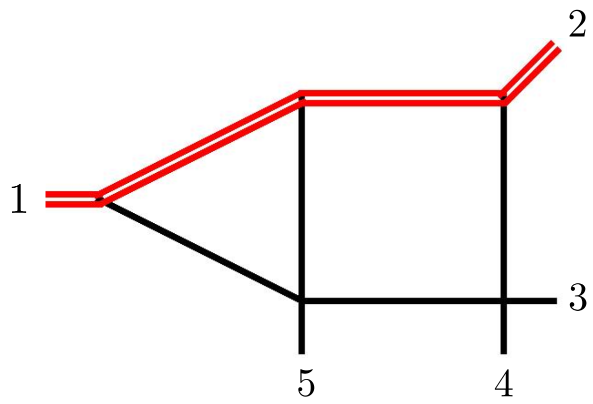
dlog candidates for most integrals follow patterns seen in previous examples:

- local and extra-dim. numerator insertions
- square roots (e.g. Gram determinants)
  8 square roots in total
- algebraic letters of the form

$$\frac{a+\sqrt{b}}{a-\sqrt{b}}$$

**problem sectors**



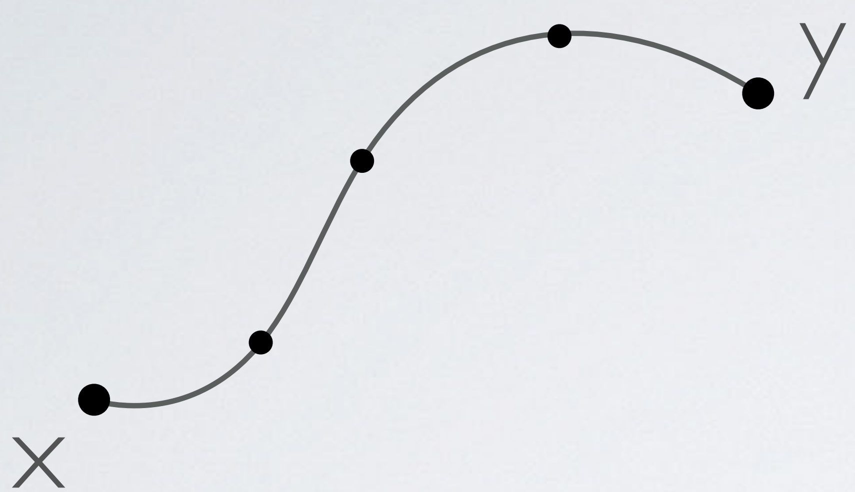




nested square root required to rotate into **ε** factorised form

Picard-Fuchs analysis confirms it to contain an elliptic curve

technology for analytic representation of elliptic integrals not yet ready for example of this complexity: resort to numerical evaluation of the DEQs

# solving differential equations numerically




transport point x to point y using
**generalised series expansion**
of the differential equations

[Moriello 1907.13234]
[DiffExp Hidding 2006.05510]

B topology basis chosen such that $k_{max}=2$

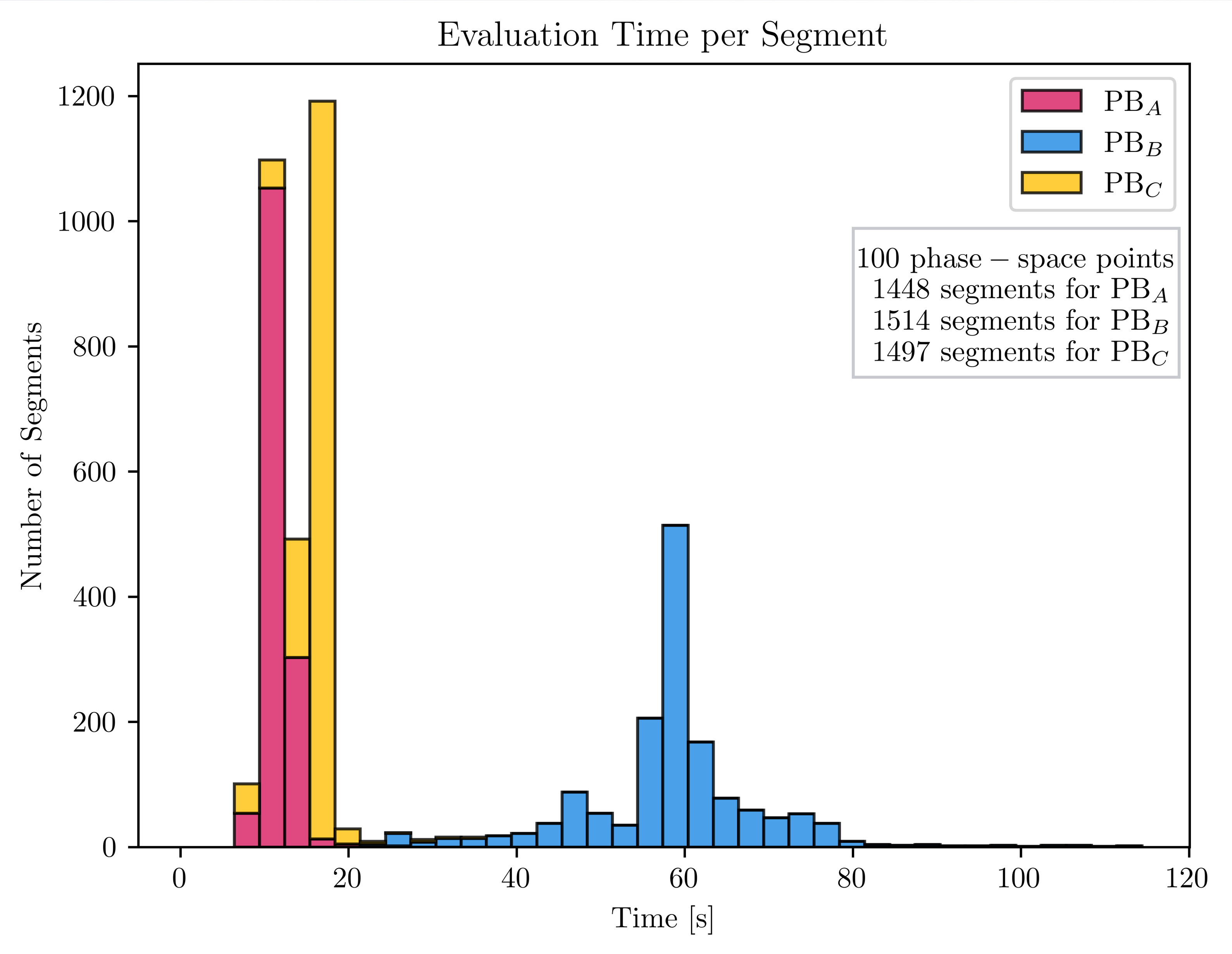
$$\mathrm{d}A^{(B)}(\vec{x},\varepsilon) = \sum_{k=0}^{k_{\max}} \sum_i \varepsilon^k \, \omega_i(\vec{x}) \, c_{k,i}^{(B)}$$

ω are linearly independent one-forms (135 of which 72 are dlog)

compact analytic representation (for all topologies)

Efficient phase-space integration possible if the number of segments between points is minimized
(e.g. [Becchetti et al. 2010.09451])

# solving differential equations numerically




simplest DiffExp setup

target accuracy $10^{-16}$

transport all points
from same boundary

# outlook: route to the two-loop amplitudes

[work in progress: SB, Becchetti, Brancaccio, Hartanto, Zoia]

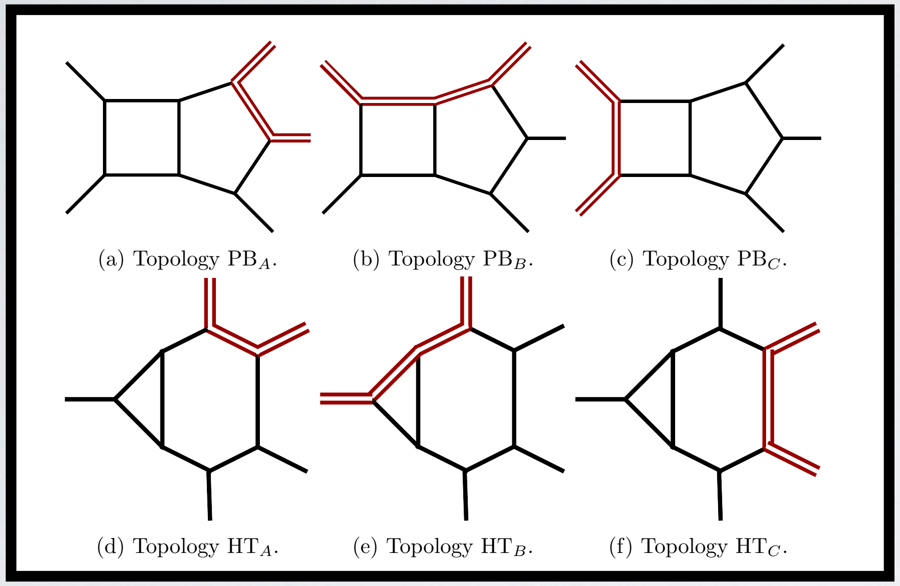
**Finite field compatible computations** for helicity amplitude with spin projection basis (ground work layed out using ttj @ 1L to O($\epsilon^2$) [SB et al. 2201.12188])

$$A_k = \sum_l c_{kl} I_l$$

have we reached the limit of analytic representations?

- setup FiniteFlow graph using rational parametrisation of kinematics (e.g. momentum twistors)
- coefficients evaluated modulo large prime number - multiple evaluations + chinese remainder theorem for rational (number OR function) reconstruction
- analytic reconstruction expensive - numerical evaluation for physical space requires complex input parameters. Proceed through rational reconstruction mod primes for which $x^2+1=0$ factorizes (2x cost of real reconstruction)
- unknown aspect: how many prime numbers are needed

# outlook: route to the two-loop amplitudes




IBP reduction (optimized with NeatIBP) via finite fields (FiniteFlow) over a **rationalised phase-space**

**checked**: all other planar diagram topologies can be mapped to the pentagon-boxes

how many points do we need to cover the full phase-space?

Numerical solution of DEQs: **need better optimized** strategy for DiffExp method

We have mapped out a route to the top…
hopefully we'll make it