Towards a global top-quark pole mass from tt+1@NLO

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IFIC

Top quark mass workshop
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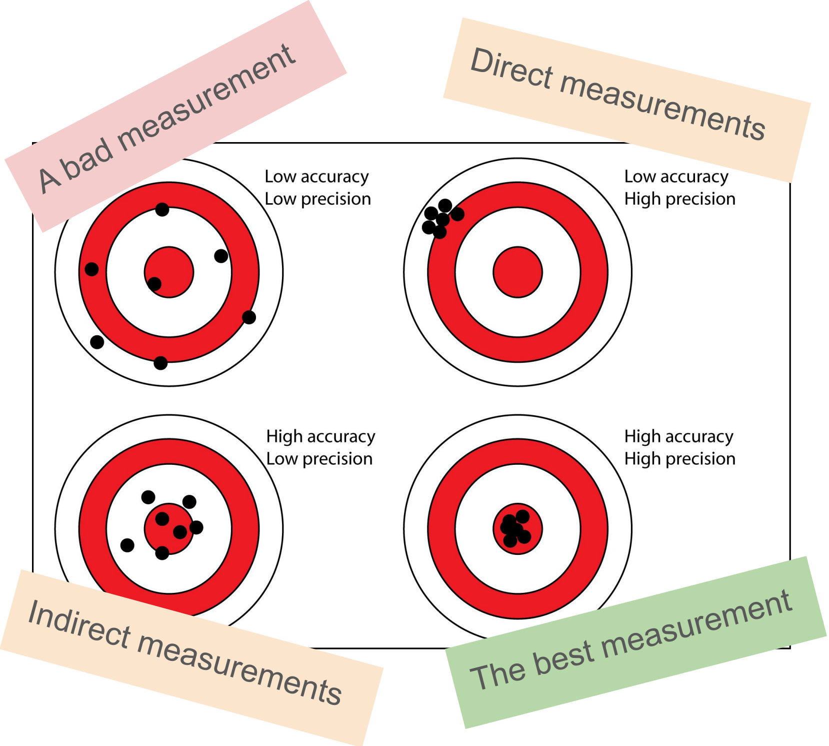
Introduction

We all want to measure top-quark mass as accurately and precisely as possible:

- direct measurement experimentally precise
- benefits from all advanced experimental techniques (NN, complicated evt sel defs,...)
- lack of accuracy in top-quark mass definition

- indirect measurements less precise
- can’t use advanced MC techniques to define xsecs for a fixed theory level
- top-quark mass definition accuracy is “well” defined



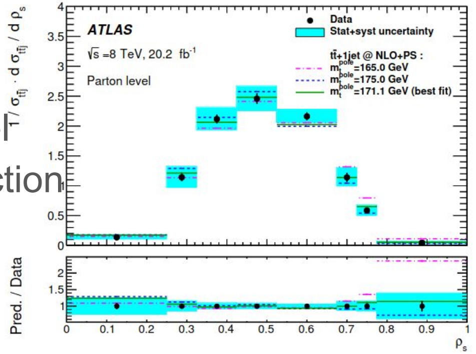
From the info of high accuracy measurements, get a higher precision mTop value

Towards the “best” top-quark mass measurement with tt+1j

Single observables providing a precise and accurate value for Mtop are obtained from tt+1j events [ATLAS & CMS] . Want to combine those (and the coming measurements expecially) in a common observable/fit.

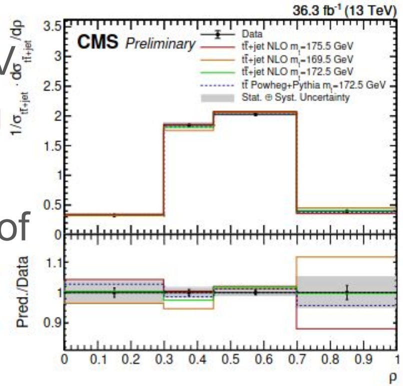
ATLAS 8TeV

- mTpole= 171.1+-1.2 GeV
- iterative bayesian unfolding
- semileptonic channel
- kinematic reconstruction of tt+1jet system
- pT_extrajet>50GeV



CMS 13TeV (data2015+2016)

- mTpole= 172.9 +- 1.3 GeV
- profile likelihood unfolding
- dileptonic channel
- NN-based reconstruction of tt+1jet system
- pT_extrajet>30GeV



Different approaches for a common/similar observable
Combining may reduce total uncertainty noticeably

Towards the “best” top-quark mass measurement with tt+1j

ATLAS 8TeV measurement

- experimental systematics evaluated on extracted top-quark mass value directly, by repeating unfolding&fitting on alternative ttbar MC samples
- systematics not fitted/constrained
- a format which allows combination “a la”-direct measurements (i.e. using mass values) but not ideal for differential cross-sections combinations.

CMS 13TeV measurement

- experimental systematics implemented as nuisance parameters (NP) in the unfolding likelihood
- NPs constrained/pulled in the fit
- diff xsec combination-ready format

Need to use the ATLAS 8TeV info (~5 years ago) with an alternative approach

Re-deriving the ATLAS 8TeV result - CovMat definitions

With some archeology, found old folder with 8TeV results.

The goal is to re-obtain the measured Mtop value from the ATLAS8TeV result, but implementing systematic uncertainties in the covariance matrix of the Mtop fit:

$$\chi^2 = [R^{\text{unfolded}}_{\text{syst}} - R^{\text{theo}}(m_t)][COV^{\text{stat}}]^{-1}[R^{\text{unfolded}}_{\text{syst}} - R^{\text{theo}}(m_t)]$$

$$COV^{\text{tot}} = COV^{\text{stat}} + COV^{\text{syst}}$$

It’s possible to derive covariance matrices from the alternative MC samples distributions as

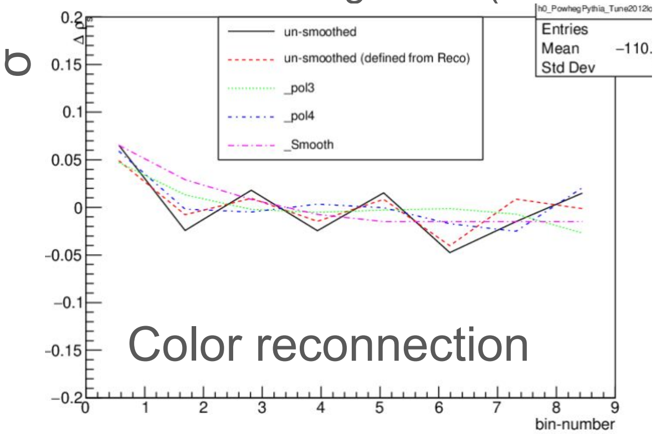
$$COV^{\text{syst}} = \sum_{\text{syst}} \sigma_i^{\text{syst}} \sigma_j^{\text{syst}}$$

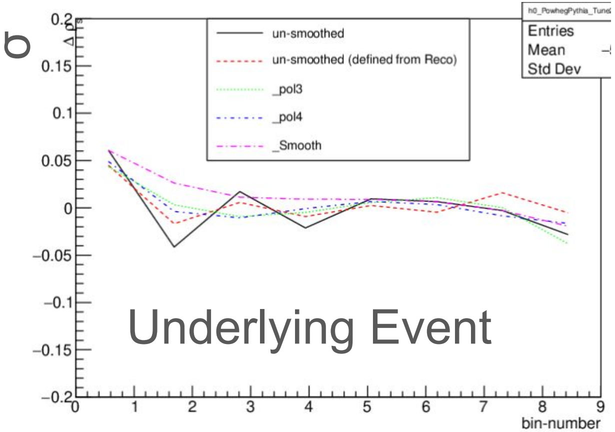
Assumption: systematics independent on each other and fully correlated across bins.

Can also test different assumptions too adding correlations “by hand” or use alternative statistical approaches

Re-deriving the ATLAS 8TeV result - Smoothing

At 8TeV, MC statistic was not great… (tested on 13TeV MC samples, too. There much better!)





Fitting one MCstat-limited distribution at the time, the obtained effect was like smoothing/interpolating across bins -> not a huge issue back then.

But now if $\Delta\sigma^{syst}_i$ get summed together into a single covariance matrix, fluctuations get summed too! -> need smoothing

Example:

- with smoothing : unfoldingPDFunc (100components) =100MeV
- without smoothing: unfolding PDFunc (100components) =800MeV

Re-deriving the ATLAS 8TeV result - fitted mass value

Published result (0.1 stat unc on systs from nominal)

Extracted value	171.1
Statistics	0.4
MC statistics	0.2
Shower and hadronisation	0.4
Colour reconnection	0.4
Underlying event	0.3
Signal Monte Carlo generator	0.2
Proton PDF	0.2
Initial- and final-state radiation	0.2
Jets	0.4
b-tagging efficiency and mistag	0.1
Lepton & MET	< 0.1
Total experimental systematic	0.9

Re-derived result

pol4-smooth σ_i^{syst}	172.13 ± 1.14
Uncertainty	$\Delta_m^{syst} = \sqrt{\lvert all^2 - allNOsyst^2 \rvert}$
jets	0.445
leptons	0.127
btag	0.115
ShowerHadr	0.099
Radiation	0.098
ColorRec	0.363
UnderEv	0.32
MatEl	0.2
PDF	0.243

Shifted central value (at the limits of uncs)

Only non-reproduced unc is PS and Rad

(PS because of smoothing, Rad not clear but close to right)

Re-deriving the ATLAS 8TeV result - shift in fitted mass value

screenshots from running code…

```
MASS BIN BY BIN!!
0   156.394 +- 3.02014
1   164.082 +- 4.62507
2   171.192 +- 3.35251
3   169.115 +- 5.78706
4   nan +- nan
5   167.79 +- 3.32384
6   172.259 +- 1.62035
7   172.571 +- 1.14616
```

COV^{stat}

0	0	0	0	0	0			
0	472.01	262.31	295.39	371.98	50.624	80.952	625.65	
0	262.31	379.2	187.99	231.83	60.531	71.486	476.48	
0	295.39	187.99	279.07	196.02	45.494	70.598	381.26	
0	371.98	231.83	196.02	383.37	6.957	62.871	474.52	
0	50.624	60.531	45.494	6.957	123.59	70.37	297.52	
0	80.952	71.486	70.598	62.871	-70.37	297.71	-1150.2	
0	625.65	476.48	381.26	474.52	297.52	-1150.2	20693	

x 170

x 70

shift due to different relative weights in fit for the high-sensivity bins

0	0	0	0	0	0	0	0
0	624.462	683.027	716.644	1091.57	299.413	325.38	1639.59
0	683.027	1119.38	1064.95	1583.99	445.378	472.292	2343.63
0	716.644	1064.95	1305.42	1813.42	507.504	528.45	2661.18
0	1091.57	1583.99	1813.42	3038.01	785.065	829.861	4164.04
0	299.413	445.378	507.504	785.065	505.49	163.676	1422.51
0	325.38	472.292	528.45	829.861	163.676	1300.91	769.186
0	1639.59	2343.63	2661.18	4164.04	1422.51	-769.186	59632.2

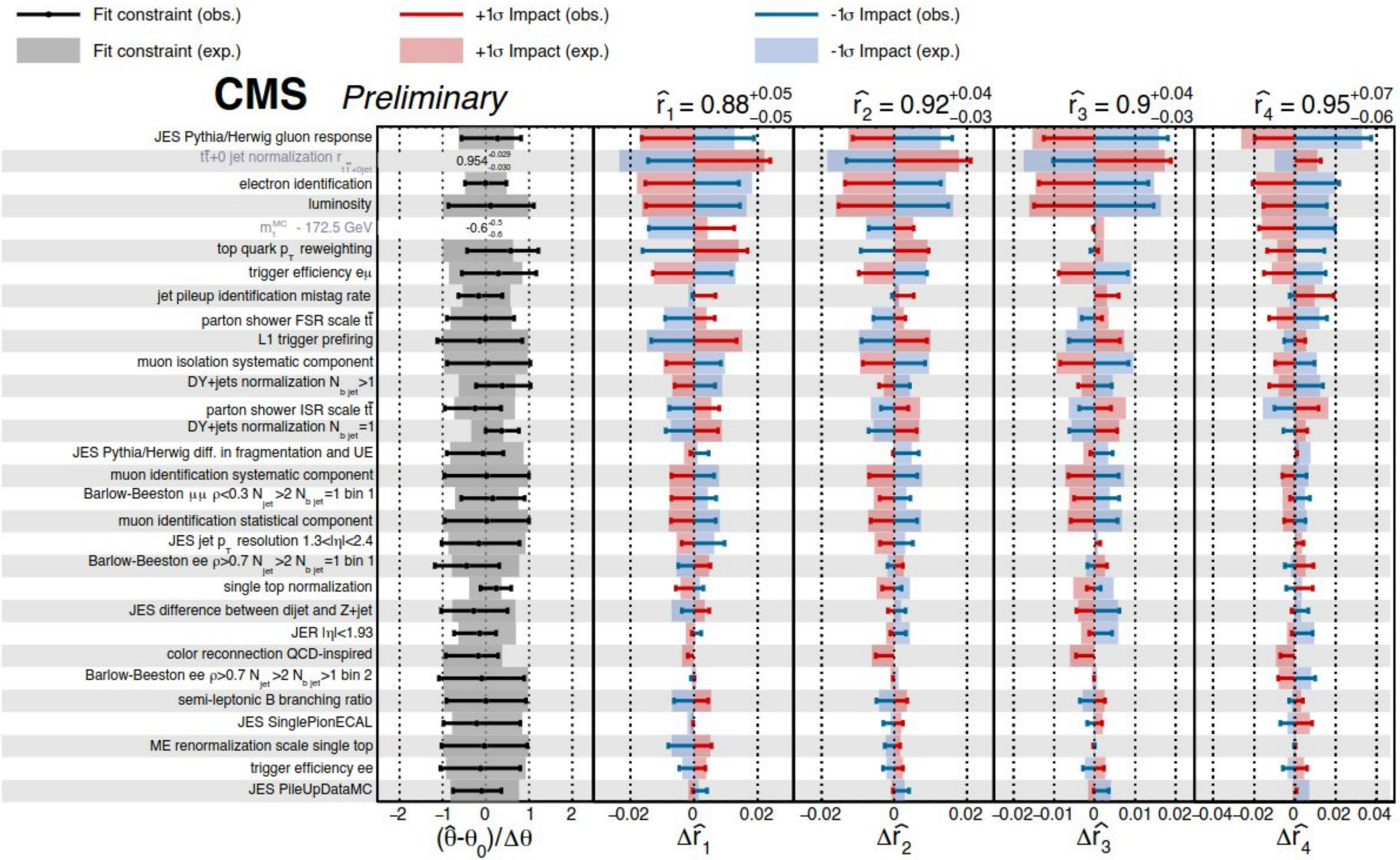
x 120

x 40

$COV^{\text{stat}} + COV^{\text{syst}}$:

Is this there also for CMS13TeV?

CMS 13TeV



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CMS Preliminary
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$(\hat{\theta}-\theta_0)/\Delta\theta$
$\Delta\hat{r}_1$
$\Delta\hat{r}_2$
$\Delta\hat{r}_3$
$\Delta\hat{r}_4$

Reproducing CMS13TeV result with ATLAS fitting code

CMS provided syst and full covariance matrices publicly on HepData. Can try reproduce the result

In my fits using CT18 pdf, μ_0=Et/2:

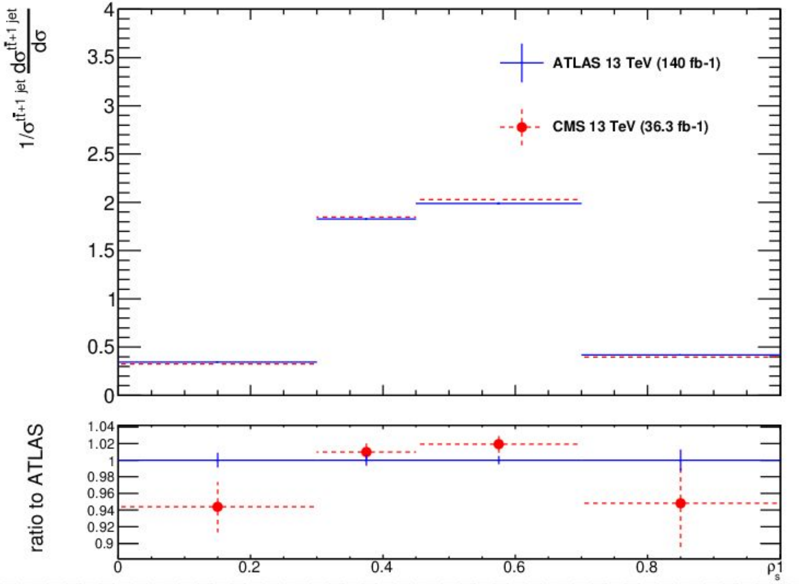
- no PDF NPs or extrapolated uncs in cov matrix, but they should have a small effect

my stat-only fit of ttj@NLO to CMS data :
169.53 +- 0.41 GeV

my syst+stat fit of ttj@NLO to CMS data:
171.621 +- 1.29 GeV

public CMS value (CT18, μ_0=Ht/2 I think):

172.16 ± 1.35(fit+PDF+extr) $\pm^{0.50}_{0.40}$ (scale)



one check of compatibility among ATLAS and CMS 13 TeV
(ATLAS13TeV mass value still blinded)

Tools for combination

Available on the market:

- BLUE
  - faster/lessCPU
  - used for the recent ATLAS+CMS direct top mass combination (ref)
- Convino
  - more functionalities
  - slower,but still ok
  - decided to pick this!

from a 2016 talk

	BLUE	BLUE tool	Convino
Absolute uncertainties	X	X	X
Relative uncertainties		*	X
Log-normal priors			X
Can combine 'sim. fit measurements'			X
Access to pulls of all estimates	X	X	X
Access to pulls of all uncertainties			X
Automated correlation scans		X	X
Creates figures for scans		X	X
Creates LaTeX tables		X	#
CPU time (for about 200 parameters)	<<10 min	<10 min✣	~10 min
Statistical bias	Neyman	Neyman	Pearson Neyman❖

Want to use full information on correlations, typically not/hardly available outside exps.
Will combine cross-sections first, then extract top mass

Statistical models

Convino

$$\chi^2 = \vec{\Delta}(\vec{x},\vec{\lambda})^T M^{-1}(\vec{x})\ \vec{\Delta}(\vec{x},\vec{\lambda}) + \vec{P}(\vec{\lambda})^T C^{-1} \vec{P}(\vec{\lambda}) + \sum_{i=1}^{N_{sys}} \omega_i^{-2}\lambda_i^2$$

correlations among measured bins (from unfolding for example)

correlations among systematics

constrains on systematics

close to was suggested in https://www.slac.stanford.edu/econf/C030908/papers/MOET002.pdf

Stat+syst covariance sum

$$\chi^2 = [R^{\text{unfolded}}_{\text{syst}} - R^{\text{theo}}(m_t)] \left[COV^{\text{stat}} + COV^{\text{syst}} \right]^{-1} [R^{\text{unfolded}}_{\text{syst}} - R^{\text{theo}}(m_t)]$$

have assumptions in the building of cov matrix

Differences in the approximations used in the statistical approach can take to slightly different results -> can this have an impact on our final mTop value?

Conclusions

Single tt+1jet Mtop measurements reaching their limit

Further improvements can come from combining the various measurements

Need to re-interpret old measurements with a “fresh look”

- reproduced ATLAS 8 TeV result
    - features of the re-produced result also spot in re-deriving CMS13TeV result

Adopting different statistical approaches can bring to slightly different answers

- profile likelihood: the ensemble of values of parameters which best describe data
- single fits: which mass would I get if I have a different (pseudo)data?

what if this become of the size of theo/exp uncertainties?

Back-up

Smooth in syst cov mat and re-obtain ATLAS8TeV result

No-smooth $\sigma_i^{\rm syst}$	172.55 ± 1.31
Uncertainty	$\Delta_m^{\rm syst} = \sqrt{\lvert all^2 - allNOsyst^2 \rvert}$
jets	0.351
leptons	0.256
btag	0.233
ShowerHadr	0.214
Radiation	0.208
ColorRec	0.355
UnderEv	0.467
MatEl	0.384
PDF	0.762

ROOT-smooth $\sigma_i^{\rm syst}$	172.32 ± 0.94
Uncertainty	$\Delta_m^{\rm syst} = \sqrt{\lvert all^2 - allNOsyst^2 \rvert}$
jets	0.3
leptons	0.166
btag	0.234
ShowerHadr	0.13
Radiation	0.126
ColorRec	0.083
UnderEv	0.055
MatEl	0.186
PDF	0.063

pol4-smooth $\sigma_i^{\rm syst}$	172.13 ± 1.14
Uncertainty	$\Delta_m^{\rm syst} = \sqrt{\lvert all^2 - allNOsyst^2 \rvert}$
jets	0.445
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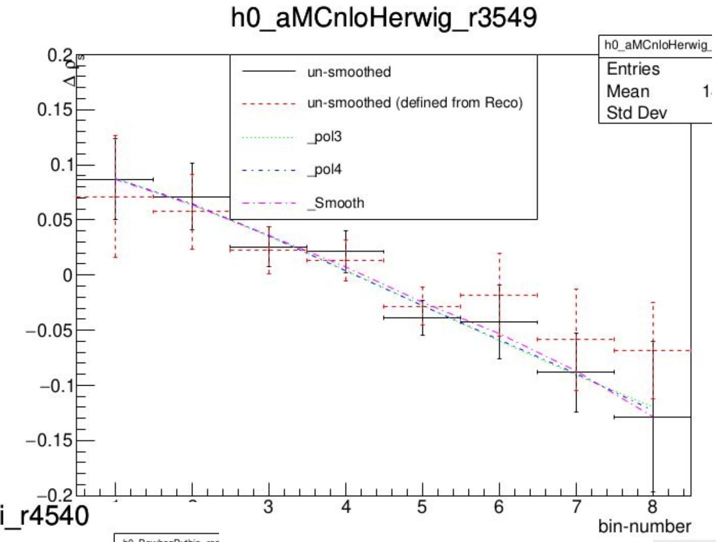
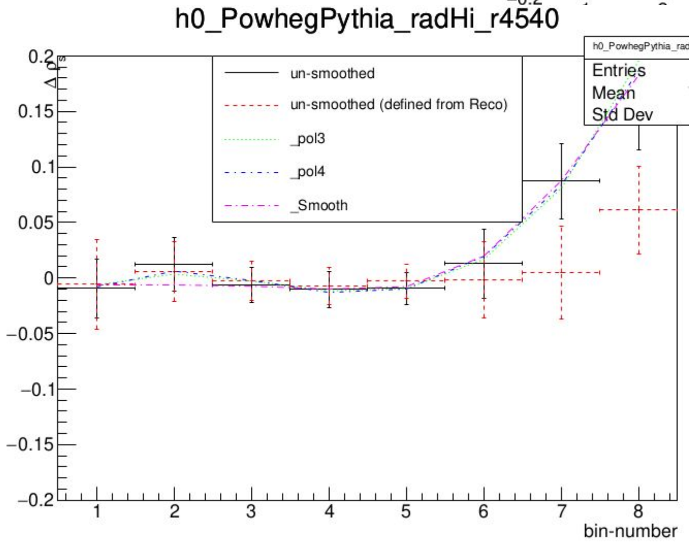


h0_aMCnloHerwig_r3549
h0_aMCnloHerwig_
Entries
Mean
Std Dev
un-smoothed
un-smoothed (defined from Reco)
_pol3
_pol4
_Smooth
bin-number



h0_PowhegPythia_radHi_r4540
Entries
Mean
Std Dev
un-smoothed
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_pol3
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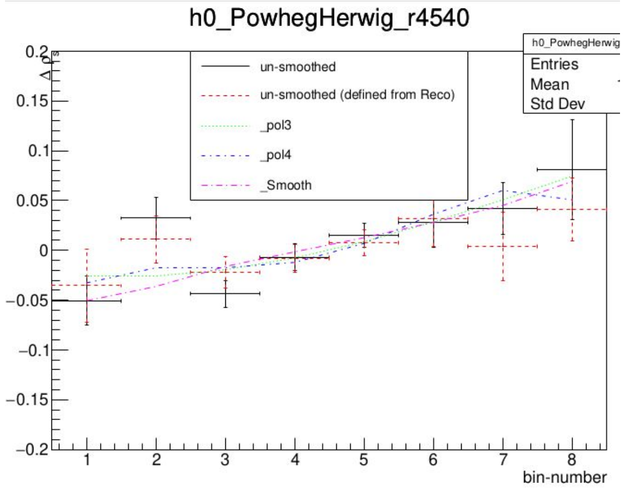
bin-number



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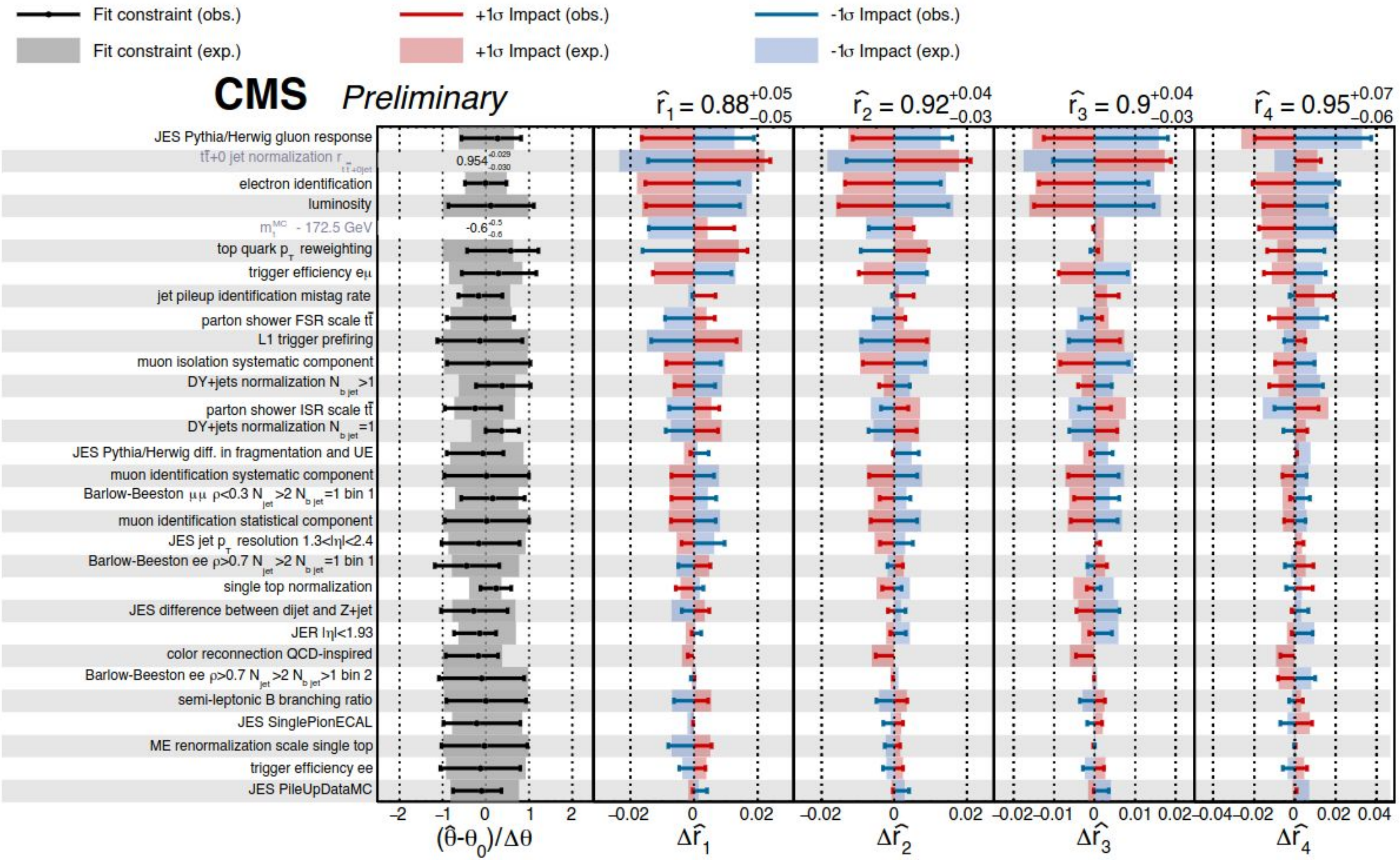
CMS 13TeV



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$\Delta\hat{r}_3$
$\Delta\hat{r}_4$

In a publication (NIM A) the following measurement for two data points x_1, x_2 and a common normalization factor α with uncertainty ϵ is given:

$$x_1 = 8.0 \pm 2\% \qquad x_2 = 8.5 \pm 2\% \qquad \alpha = 1 \pm \epsilon \qquad \text{with} \quad \epsilon = 0.1$$

"Assuming that the two measurements refer to the same physical quantity, the best estimate of its true value can be obtained by fitting the points to a constant" (from the publication).

A simple straightforward average would be $x_{\text{ave}} = (x_1 + x_2)/2 = 8.25$, but …

Publication: average x_{ave} by "χ^2-function minimization", the covariance matrix $\boldsymbol{V}$ is defined to include the normalization uncertainty:

Simon David Badger has started screen sharing

$$\chi^2 = \boldsymbol{\Delta}^{\mathrm{T}} \boldsymbol{V}^{-1} \boldsymbol{\Delta} = \text{minimum} \qquad \text{with} \quad \boldsymbol{V} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} + \epsilon^2 \cdot \begin{pmatrix} x_1^2 & x_1 x_2 \\ x_1 x_2 & x_2^2 \end{pmatrix}$$

($\boldsymbol{\Delta}$ is "the vector of the differences" between x_i and average x_{ave}).

Resulting average is $x_{\text{ave}} = 7.87 \pm 0.81$, outside (!) the range of the two input values

…apparently wrong ⇝ large bias with constructed non-diagonal covariance matrix. ⇒**more**

Note: weights $w_1 = +1.25$ and $w_2 = -0.25$ because $\sigma_1 < \sigma_2$;

link

link2

With two constraints the average is forced to agree with the two measurements, multiplied by the normalization factor (NP in the fit) α:

variable	measured		fit result		pull
x_1	8.0	±2%	8.235	±0.116	2.14
x_2	8.5	±2%	8.235	±0.116	−2.14
α	1	±10%	1.000	±0.100	−2.14
x_{ave}			8.235	±0.832	

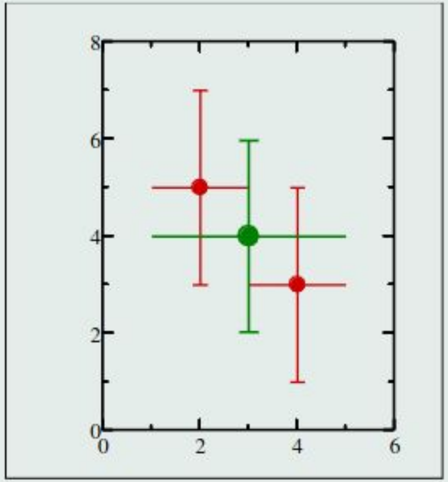
The wonderful world of correlations

Average of two *correlated* numbers d_1 and d_2 (assuming $\sigma_1 = \sigma_2$) with positive/negative correlation:

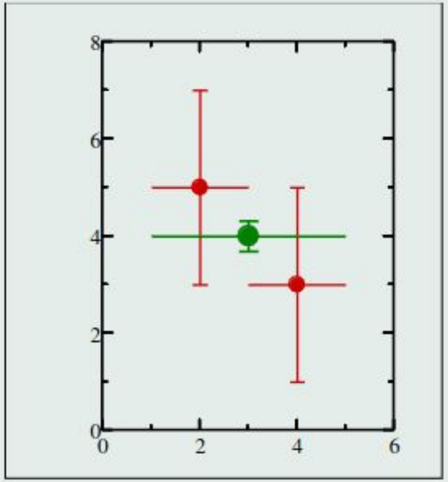
$$\text{average} \quad \overline{\boldsymbol{d}} = \tfrac{1}{2}\,(d_1 + d_2)$$
$$\boldsymbol{V}_{\overline{d}} = \tfrac{1}{2}\,(1 + \rho_{12})\,\sigma^2$$

$$\boldsymbol{V} = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

$\rho_{12} = +0.95$



$\rho_{12} = -0.95$



Averaged value has almost the same error as each single data value (0.987σ).

Averaged value has much smaller error than each single data value (0.158σ).

$$\chi^2 = \boldsymbol{\Delta}^{\mathrm{T}} \boldsymbol{V}^{-1} \boldsymbol{\Delta} = \text{minimum} \qquad \text{with} \quad \boldsymbol{V} = \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix} + \epsilon^2 \cdot \begin{pmatrix} x_1^2 & x_1 x_2 \\ x_1 x_2 & x_2^2 \end{pmatrix}$$

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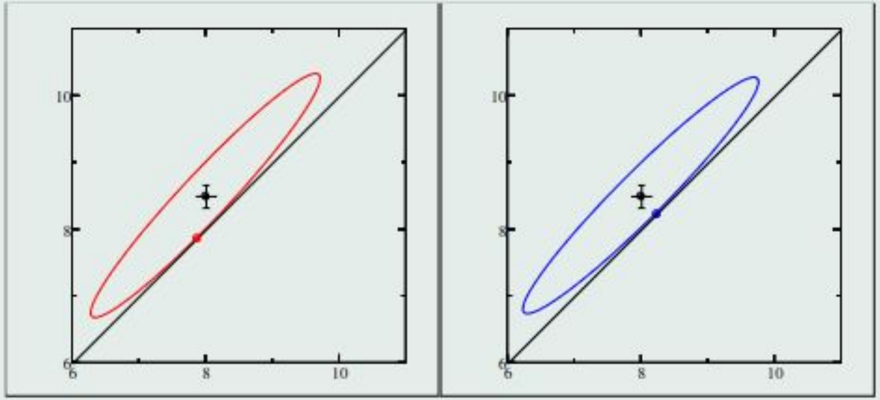

Axis of covariance ellipse is not tilted for $\sigma_1 = \sigma_2$ (right).

biases in chi2 minimum with inputs with different values/uncertainties

Uncertainty table for ATLAS 8TeV

Mass scheme	m_t^{pole} [GeV]	$m_t(m_t)$ [GeV]
Value	**171.1**	**162.9**
Statistical uncertainty	**0.4**	**0.5**
Simulation uncertainties		
Shower and hadronisation	0.4	0.3
Colour reconnection	0.4	0.4
Underlying event	0.3	0.2
Signal Monte Carlo generator	0.2	0.2
Proton PDF	0.2	0.2
Initial- and final-state radiation	0.2	0.2
Monte Carlo statistics	0.2	0.2
Background	<0.1	<0.1
Detector response uncertainties		
Jet energy scale (including b-jets)	0.4	0.4
Jet energy resolution	0.2	0.2
Missing transverse momentum	0.1	0.1
b-tagging efficiency and mistag	0.1	0.1
Jet reconstruction efficiency	<0.1	<0.1
Lepton	<0.1	<0.1
Method uncertainties		
Unfolding modelling	0.2	0.2
Fit parameterisation	0.2	0.2
Total experimental systematic	**0.9**	**1.0**
Scale variations	$(+0.6, -0.2)$	$(+2.1, -1.2)$
Theory PDF$\oplus\alpha_s$	0.2	0.4
Total theory uncertainty	$(\mathbf{+0.7}, -0.3)$	$(\mathbf{+2.1}, -1.2)$
Total uncertainty	$(\mathbf{+1.2}, -1.1)$	$(\mathbf{+2.3}, -1.6)$

Breakdown of modelling unc at particle level - ATLAS 8TeV