THE STRONG INTERACTION AND SYMMETRY

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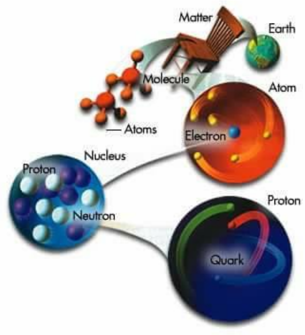
Students' seminar 2024

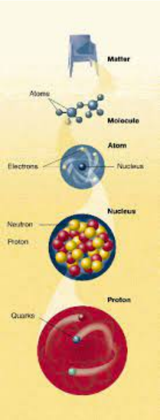
https://physicstravelguide.com

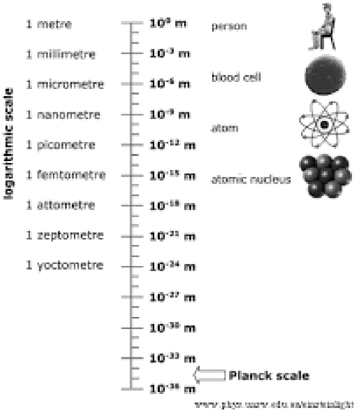
- Please interrupt me if there are questions
- What makes matter? What makes protons and neutrons exist?

- Outline

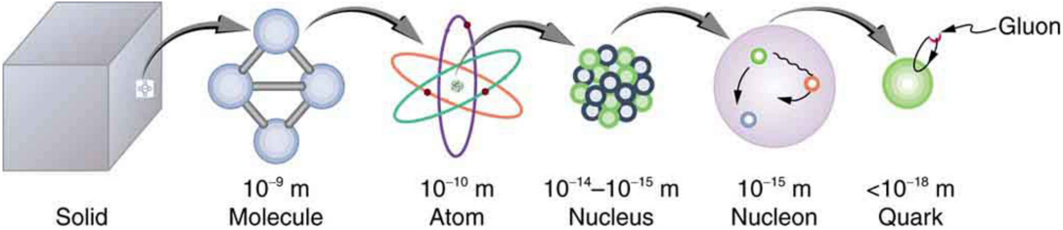
  1. Strong nuclear force and its symmetries
  2. Symmetry breaking
     - EFT: Chiral Perturbation Theory (ChPT)
  3. ChPT nowadays and similar EFTs even for new physics



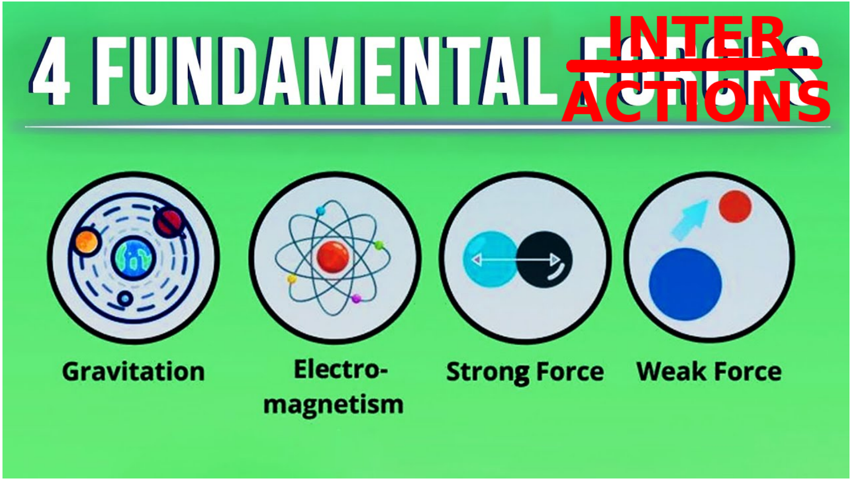




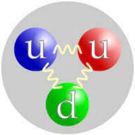
- What makes matter?
- chairs.

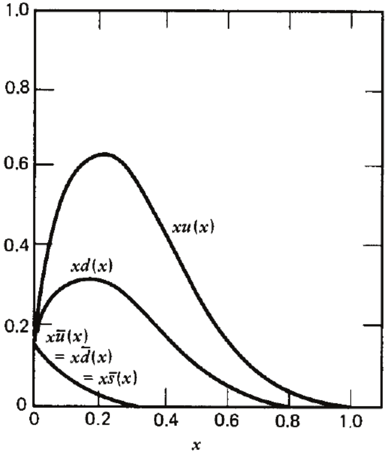


- 10^{-25} s, time for a quark traveling at c to cross the proton, relativistic
- Applications?
  - Fundamental knowledge is valuable per se
    + possible future applications:
    - Just studying the candle one would not invent the light bulb

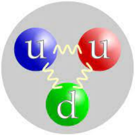


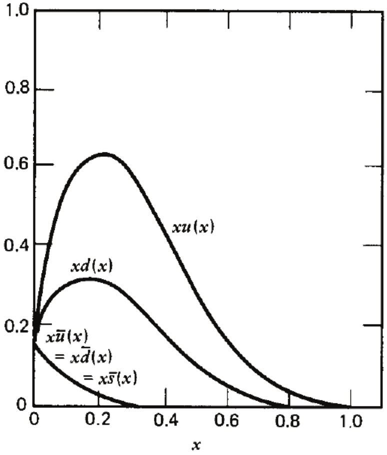
- weak: to have a neutrino-electron bound state, their masses should be comparable to the Weak boson ones (a possible weak bound state is WIMP-onium)





- Baryon: hadron made of three quarks, e.g. **proton**: $|\mathrm{u\ u\ d}\rangle$
  - But in which sense?





- Baryon: hadron made of three quarks, e.g. **proton**: $|u\ u\ d\rangle$
  - But in which sense?
  - $N_q = \int_0^1 dx\, q(x)$
  - $N_q \to \infty$, $N_{\bar{q}} \to \infty$, $N_q - N_{\bar{q}} = 3$
  - $|B\rangle = |qqq\rangle + |qqqq\bar{q}\rangle + |qqqqq\bar{q}\bar{q}\rangle + \ldots$
  - $|\text{proton}\rangle = |uud\rangle + \ldots$

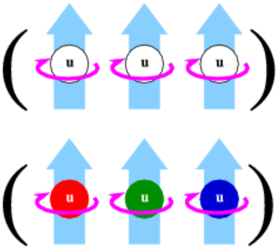
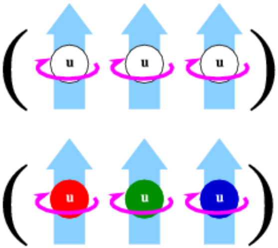


Figure: Δ^{++}

- Quantum mechanics
  - Δ^{++} baryon = $|q, q, q\rangle$
  - charge$(u) = 2/3$, charge$(d) = -1/3$
  - charge$= 2$, spin$= 3/2 \Rightarrow \Delta^{++}(a, b, c) = \left| \underbrace{u\uparrow}_{a}, \underbrace{u\uparrow}_{b}, \underbrace{u\uparrow}_{c} \right\rangle$
  - spin$= 3/2$ fermion $\Rightarrow$ why symmetric?



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  - $\mathrm{spin} = 3/2$ fermion $\Rightarrow$ why symmetric?
  - Color quantum number!
  - $\Delta^{++} = |u\uparrow, u\uparrow, u\uparrow\rangle \otimes \varepsilon_{ijk} q^i q^j q^k, \quad i,j,k = r - b$

    $= |u\uparrow r, u\uparrow g, u\uparrow b\rangle - |u\uparrow g, u\uparrow r, u\uparrow b\rangle + \ldots$
  - antisymmetric under $a \leftrightarrow b$:

    $\Delta^{++}(b,a,c) = |u\uparrow g, u\uparrow r, u\uparrow b\rangle - |u\uparrow r, u\uparrow g, u\uparrow b\rangle + \ldots$

    $= -\Delta^{++}(a,b,c)$

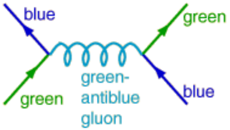
- - What makes the proton exist?

  - Quantum Chromodynamics (QCD)

  $\Rightarrow$ this is the Strong Nuclear Force

  binding $|uud\rangle$ to make the proton

  - other quark configurations lead to the rest of hadrons (neutron, pions etc)
  - Also binds nucleus
  - Force carriers: gluons
    - have color charge = Selfinteraction



Feynman diagram for an interaction between quarks generated by a gluon.

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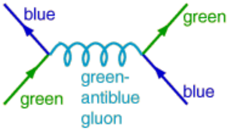
- Let's calculate the proton mass
  -



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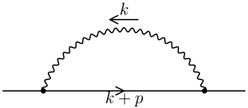
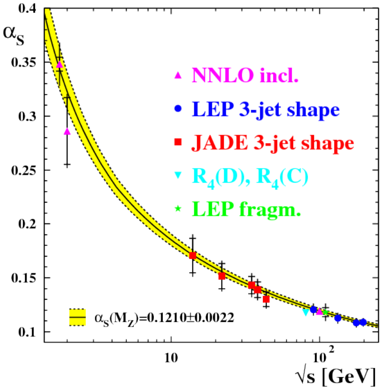
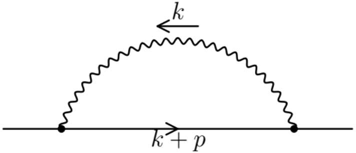


- Running of α_s
  - $\alpha_s > 1$ at low energy/long distance

    $\Rightarrow$ breaks perturbative calculation





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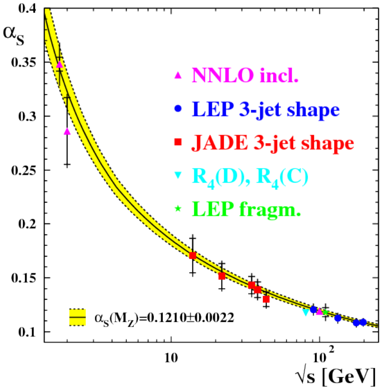

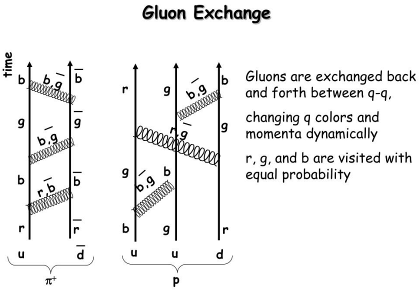

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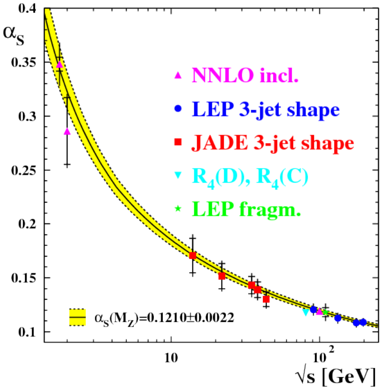
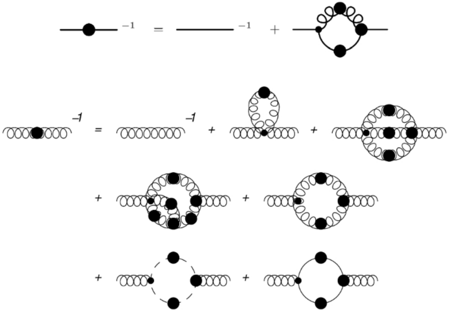
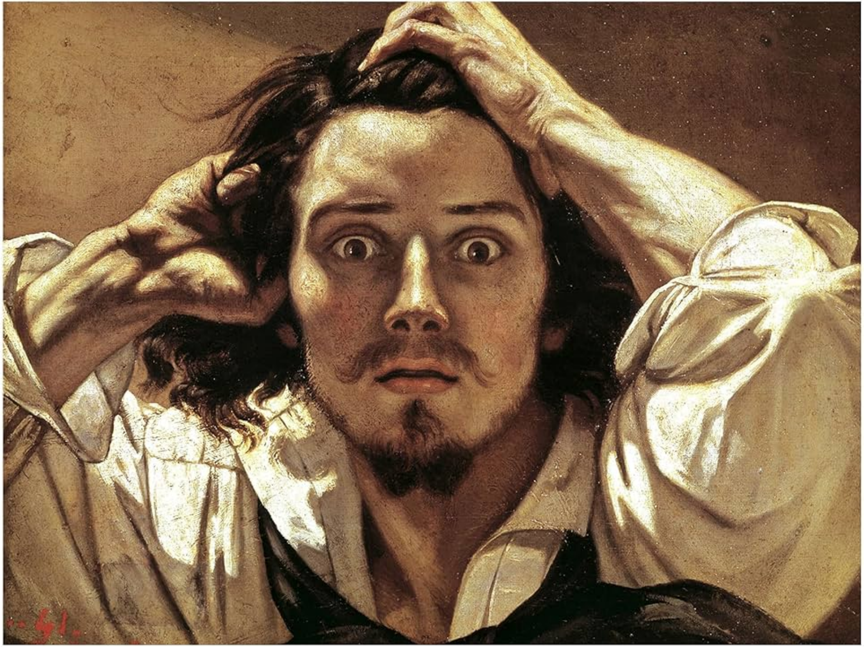


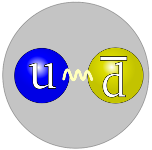


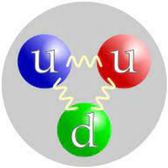


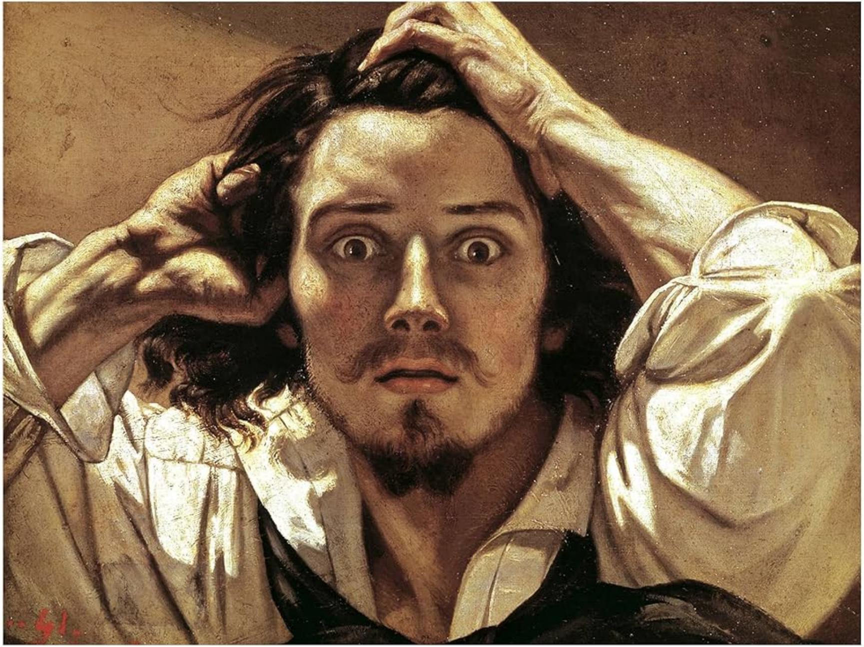
- Confinement
  1. $\alpha_s > 1$
  2. Gluon selfinteraction

  ! $\Rightarrow$ quarks confined in hadrons,

  like proton, pion etc (QCD bound states)
- calculation nightmare







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  1. $\alpha_S > 1$
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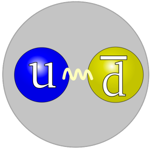
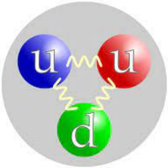
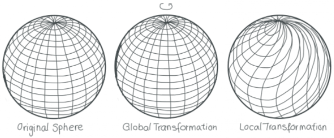
- Way out
  - Lattice simulations
  - Effective field theories: at low E, Chiral Perturbation Theory





- First: Symmetry is not just nice, QED and QCD examples
  - QED
    - $\mathscr{L} = \bar{\Psi}\left(\mathrm{i}\gamma^{\mu}\partial_{\mu} - \mathrm{m}\right)\Psi$, inv. under $\Psi \to \exp(\mathrm{i}\theta)\Psi$



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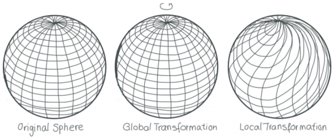
    - gauge it= make transform. depend on spacetime: $\Psi \to \exp(\mathrm{i}\theta(\mathrm{x}))\Psi$ (Omar style)
    - Tuning the Lagrangian to recover the invariance with extra field predicts the photon-electron interaction!
    - $\mathrm{D}_{\mu}\Psi = \left(\partial_{\mu} - \mathrm{ieA}_{\mu}\right)\Psi$, $\mathrm{A}_{\mu} \to \mathrm{A}_{\mu} + \partial\theta/\mathrm{e}$ (gauge transform), $\mathrm{D}_{\mu}\Psi \to \exp(\mathrm{i}\theta(\mathrm{x}))\mathrm{D}_{\mu}\Psi$
      $\mathscr{L} = \bar{\Psi}\left(\mathrm{i}\gamma^{\mu}\mathrm{D}_{\mu} - \mathrm{m}\right)\Psi + \mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu}/4$
      $\implies \mathscr{L}_{\mathrm{int}} = \mathrm{e}\bar{\psi}\gamma^{\mu}\psi\mathrm{A}_{\mu}$ """naturally"""



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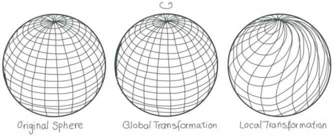
      $\mathscr{L} = \bar{\Psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\Psi + \mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu}/4$
      $\Longrightarrow \mathscr{L}_{\rm int} = e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ """naturally"""
  - QCD
    - $q \to \exp\left(-i\theta_a \frac{\lambda_a^c}{2}\right) q$ $SU(3)_c$, gauging it $\theta_a(x)$ (non Abelian):
    - $\mathscr{L} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m\right)q - \mathscr{G}_{a\mu\nu}\mathscr{G}^{a\mu\nu}/4$
      yields correct theory
      `https://physicstravelguide.com`



- Chiral Perturbation Theory (ChPT)
  - light quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$
  - $m_q \to 0$, OK because $m_q/m_p \ll 1 \Rightarrow \mathscr{L}^0_{\mathrm{QCD}} = i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R$

    $q_{L/R} = P_{L/R} q = \frac{1}{2}(1 \mp \gamma_5) q$

    (chirality $\sim$ spin orientation vs momentum)

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    $\{q_L, q_R\} \to \left\{\exp\left(i\varepsilon^i_L \frac{\tau^i}{2}\right) q_L, \exp\left(i\varepsilon^i_R \frac{\tau^i}{2}\right) q_R\right\}$, $\boldsymbol{\tau}^i$ Pauli matrices

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    - Symm. = hadrons of same mass (Ham. eigenstates)

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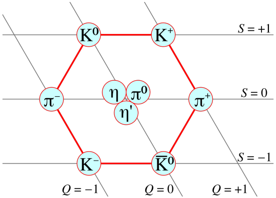
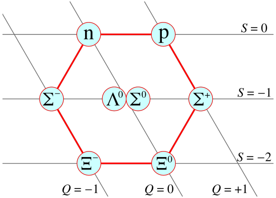
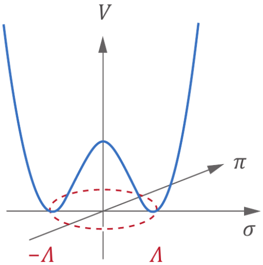
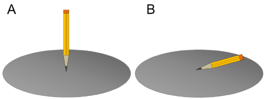


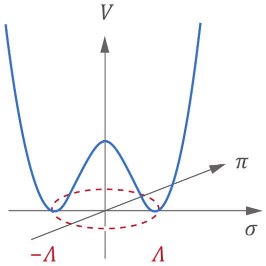
- Spontaneous Symmetry Breaking
  - $\bar{q}q$ hadrons: mesons (light), qqq hadrons: baryons (heavy)
  - However in the spectrum we only see one SU(2) (or SU(3)) symmetry
  - π^-, π^0, π^+ Isospin (u, d) SU(2) triplet ($J^P = 0^-$)
  - No other SU(2) mesons with opposite parity ($J^P = 0^+$)
  - The same happens with the rest of hadrons
  - $\Rightarrow$ Symmetry spontaneously broken (SSB)

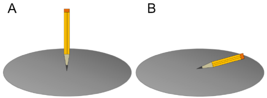






- Pions: linear $\boldsymbol{\sigma}$ model
  - $\boldsymbol{\pi}^i = i\bar{q}\boldsymbol{\tau}^i\gamma_5 q$, $\boldsymbol{\sigma} = \bar{q}q$ rotating q as before, $\boldsymbol{\Phi}^T = (\vec{\boldsymbol{\pi}}, \boldsymbol{\sigma})$ 4D space
  - $\mathscr{L}_{\boldsymbol{\sigma}} = \frac{1}{2}\partial_\mu \boldsymbol{\Phi}^T \partial^\mu \boldsymbol{\Phi} - \frac{\lambda}{4}\left(\boldsymbol{\Phi}^T\boldsymbol{\Phi} - v^2\right)^2$
  - inv. under rotations SO(4)
  - $\boldsymbol{\Phi}^T\boldsymbol{\Phi} = v^2$, chosen ground state, like Higgs (SSB→ quark mass)
    $\langle 0|\boldsymbol{\Phi}|0\rangle^T = (\vec{0}, v) = \boldsymbol{\Phi}_0^T$, $(\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - v)$





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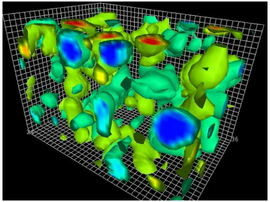
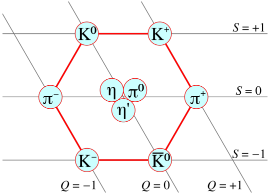
    $\langle 0|\boldsymbol{\Phi}|0\rangle^T = (\vec{0}, v) = \boldsymbol{\Phi}_0^T$, $(\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - v)$
  - $\mathscr{L}_\sigma = \frac{1}{2}\left(\partial_\mu\vec{\boldsymbol{\pi}}\partial^\mu\vec{\boldsymbol{\pi}} + \partial_\mu\hat{\boldsymbol{\sigma}}\partial^\mu\hat{\boldsymbol{\sigma}} - 2\lambda v^2\hat{\boldsymbol{\sigma}}^2\right) - \lambda v\hat{\boldsymbol{\sigma}}\left(\hat{\boldsymbol{\sigma}}^2 + \vec{\boldsymbol{\pi}}^2\right) - \frac{\lambda}{4}\left(\hat{\boldsymbol{\sigma}}^2 + \vec{\boldsymbol{\pi}}^2\right)^2$
  - SSB: SO(4)$\rightarrow$SO(3)$\Rightarrow$ $\vec{\boldsymbol{\pi}}$ Goldstones (massless)
  - $\sim \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \rightarrow \mathrm{SU}(2)_V$
    - $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R = \mathrm{SU}(2)_V \otimes \mathrm{SU}(2)_A$:

      $G \cdot q = \exp\left(i\varepsilon_V^i \frac{\tau^i}{2}\right)\exp\left(i\varepsilon_A^i \frac{\tau^i}{2}\gamma_5\right) q$,

      $V \Rightarrow L = R$





- Pions: true QCD
  - Dynamical Spontaneous Symmetry Breaking
    - QCD nontrivial crazy vacuum
    - Instead of $\langle 0|\,\mathbf{\Phi}\,|0\rangle^{\mathsf{T}} = (\vec{0}, \mathrm{v})$,
      nonzero v.e.v. of operator which is not invariant under G:
      $\langle 0|\,\bar{q}q\,|0\rangle^{\mathsf{T}} = \mathrm{v.e.v.} \neq 0$ (still invariant under $SU(2)_V$ ($L = R$)) (lattice check)
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    - QCD nontrivial crazy vacuum
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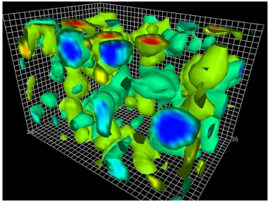

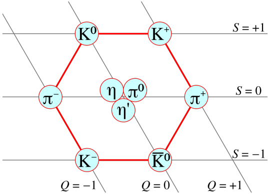

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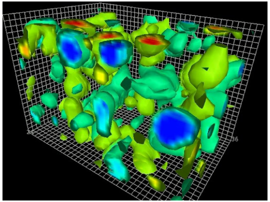
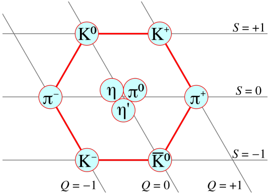
    - $\Rightarrow \vec{\pi}$ Goldstones ($\sim$ massless) right quantum numbers
    - $\Lambda_\chi \sim 1$ GeV hadronic scale: m_p, SSB, confinement, $4\pi F_0$, $m_\pi/\Lambda_\chi \sim 0.1 \ll 1$ OK!
    - With s quark: (u, d, s) $SU(3) \Rightarrow \pi^\pm$, π^0, η, $K^\pm$, K^0, $\bar{K}^0$ Goldstones!





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  - Effective $\mathscr{L}$agrangian:
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    - QCD Symmetries: $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ (+Lorentz, C and P)
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      ($\pi^i = i\bar{q}\tau^i\gamma_5 q$ and $q \to Gq$) $\Rightarrow U \to g_R U g_L^\dagger$ (nonlinear in π)
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  - In practice:
    - Pion decay $\pi \to \mu\nu$ (helicity and m_ℓ) gives F_0, then one has $4\pi, 6\pi$ etc
      $\langle 0|A_i^\mu(0)\,|\pi_j(p)\rangle = ip^\mu F_0 \delta_{ij}$
      $F_0 = 92$ MeV
    - Predicted $\pi\pi \to \pi\pi$ scatt. (and 6π etc)
      $\mathscr{A}(\pi^+\pi^0 \to \pi^+\pi^0) = t/F_0^2$, OK experimentally!
      $t = (p-p')^2$
    - Corrections can be implemented systematically

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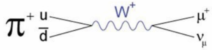

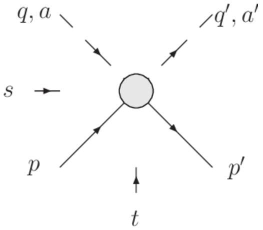

- Questions?

- Next: Explicit symmetry breaking

- FYI, I learnt this during the PhD

1. papers have mistakes/lies
2. Check out the fellowships for research stay and postdoc from Generalitat
3.

- Explicit symmetry breaking
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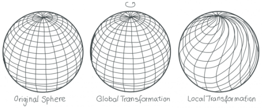

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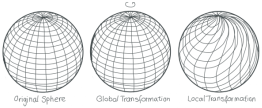

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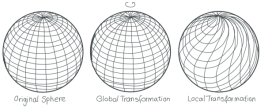

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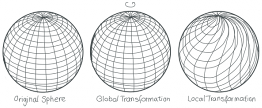

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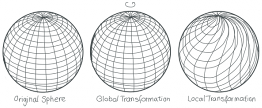

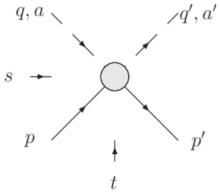

$\mathscr{A}(\pi^+\pi^0 \to \pi^+\pi^0) =$

$(t - m_\pi^2)/F_0^2$

Even better

experimentally!

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    - fake gauge symm. broken exactly as in QCD:
    - $s = \mathscr{M}$ (masses)
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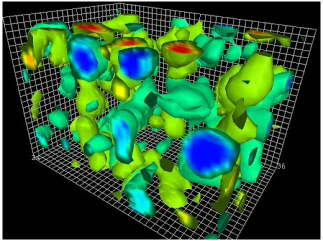
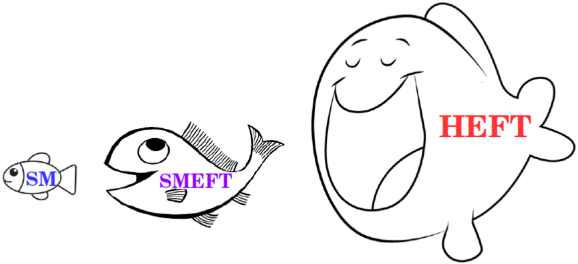
- Summary of Chiral $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ Symmetry on $q = \begin{pmatrix} u \\ d \end{pmatrix}$ (or SU(3) with s)

$\mathscr{L}_{\mathrm{QCD}}$ symm.	**Global**	**Local (gauge)**
Breaking	Spontaneous $\langle 0 \vert \bar{q}q \vert 0 \rangle \neq 0$	Explicit $s(x) \to \mathscr{M} = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$
Outcome	π Goldstones, $m_\pi \sim 0$ ChPT predictions	m_π, EM and other effects



- Chiral Perturbation Theory
  - Pion mass and quark masses
    - $F_0^2 m_\pi^2 = -(m_u + m_d)^{(\mu)} \langle 0 | (\bar{u}u + \bar{d}d) | 0 \rangle^{(\mu)}$ (ChPT GMOR)
    - $\underbrace{F_0^2}_{\text{obs.}} \underbrace{m_\pi^2}_{\text{obs.}} = - \underbrace{(m_u + m_d)^{(\mu)}}_{\to 0 \text{ ChPT } (\propto v_H, \text{ expl. SB})} \times \underbrace{\langle 0 | (\bar{u}u + \bar{d}d) | 0 \rangle^{(\mu)}}_{\text{SSB (QCD confinement)}}$
    - $\underbrace{F_0^2}_{\text{obs.}} \underbrace{m_\pi^2}_{\text{obs.}} = - \underbrace{(m_u + m_d)^{(\mu)}}_{\text{GUT?}(\mu)} \times \underbrace{\langle 0 | (\bar{u}u + \bar{d}d) | 0 \rangle^{(\mu)}}_{\text{lattice}(\mu) \text{ check}}$
  - Why so small? $m_\pi / m_p \sim 0.1$
    - QCD: bound state of massive quarks$\to$ too low mass to work in quark model
    - SSB (related to QCD confinement): Goldstone $m \to 0$
    - lattice gives hadron mass ratios
    - Still far from understanding fully

- ChPT still fit
  - More precision, other hadrons like proton-neutron, include resonances (unitarization), ChPT inspired theories etc
  - HEFT/EWET
    - Chiral theory for new physics



- EWSB
  - $\mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{\mathrm{Y}} \to \mathrm{U}(1)_{\mathrm{EM}}$
  - Higgs, EWSB Goldstones, EW gauge bosons
    - (gauge bosons eat the Goldstones)

- SM
  - $\Phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}$,
    $\phi^{+} = \phi_1 + i\phi_2$, $\phi^{0} = \phi_3 + i\phi_4$

- HEFT
  - $\Sigma = \frac{(v+h)}{\sqrt{2}} \exp\left\{ i\phi_i \tau^i/2 \right\}$, $i = 1-3$
    - Does not assume the Higgs to be inside an $\mathrm{SU}(2)_{\mathrm{L}}$ doublet
    - inputs it as a singlet and builds an EFT Lagrangian
      $\mathscr{L}_{\phi} = \frac{v^2}{4} \mathscr{F}_u(h)\, \mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) + \frac{1}{2}\left(\partial_\mu h\, \partial^\mu h\right) - V(h)$

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- Proton and pion exist because of strong nuclear force = Chromodynamics
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- Proton and pion exist because of strong nuclear force = Chromodynamics
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- Denounce Israel's genocide of the Palestinian people 
- Thank you!

Electroweak structure of the nucleon

- Structure of protons and neutrons is encoded in form factors
  $F_{\mathrm{EM}} \to$ charge distribution
  - $V^\mu = \bar{q} Q \gamma^\mu q$,
    $\langle N | V^\mu(0) | N \rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] u$
    - $G_E = F_1 - \frac{Q^2}{4m_N^2} F_2, \quad G_M = F_1 + F_2$
    - slope=charge radius: $\langle r_E^2 \rangle = 6 \left. \frac{dG_E}{dq^2} \right|_0$
  - $A^\mu = \bar{q} \gamma^\mu \gamma_5 q$,
    $\langle N | A^\mu | N \rangle = \bar{u}' \left\{ \gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} G_P(q^2) \right\} \gamma_5 u$
  - $F_A \to$ spin distribution
    Weak interaction is $V - A$
    relevant for ν experiments