ELECTROWEAK STRUCTURE OF THE NUCLEON

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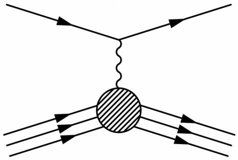
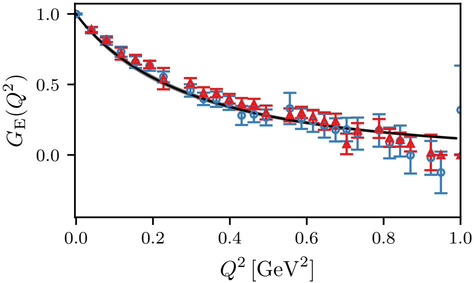
IFIC Scientific Session dedicated to L3: ”Flavour and Quark Matter”

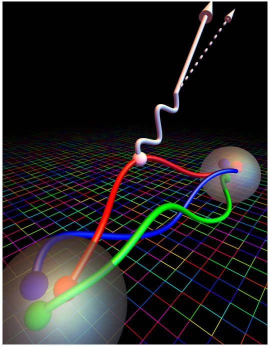
Electroweak structure of the nucleon

- Structure of protons and neutrons is encoded in form factors
  $F_{\mathrm{EM}} \to$ charge distribution
  - $V^\mu = \bar{q} Q \gamma^\mu q$,
    $\langle N | V^\mu(0) | N \rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] u$
    - $G_E = F_1 - \frac{Q^2}{4m_N^2} F_2, \quad G_M = F_1 + F_2$
    - slope=charge radius: $\langle r_E^2 \rangle = 6 \left. \frac{\mathrm{d}G_E}{\mathrm{d}q^2} \right|_0$
  - $A^\mu = \bar{q} \gamma^\mu \gamma_5 q$,
    $\langle N | A^\mu | N \rangle = \bar{u}' \left\{ \gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} G_P(q^2) \right\} \gamma_5 u$
  - $F_A \to$ spin distribution
    Weak interaction is $V - A$
    relevant for ν experiments





Electromagnetic form factor

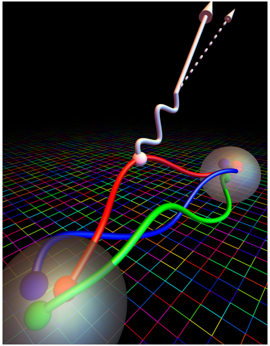


- Experimental determinations
  - $\langle r_E^2 \rangle$: proton radius puzzle [Pohl et al., Nature 466, 213 (2010)]
- Lattice QCD $\longrightarrow$ Alberto Ramos' talk, 10:25 tomorrow!
  - Nucleon f.f. is a benchmark for LQCD
  - Uncertainties reduced for unphysical large M_π
  - Technical difficulties $\rightarrow$ recent progress
  - Experimental and lattice q^2 parametrisation:

$$\left.\begin{array}{l} \text{- dipole ansatz} \\ \text{- z-expansion} \\ \text{- ...} \end{array}\right\} \Longrightarrow \text{different } \langle r_1^2 \rangle \text{, and } F_{\mathrm{EM}} \text{ in general}$$

- Theoretical input needed

Electromagnetic form factor



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    - dipole ansatz
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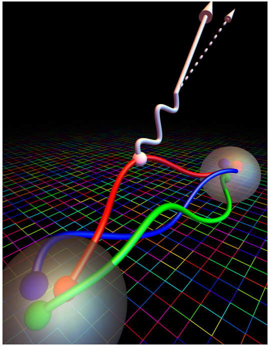
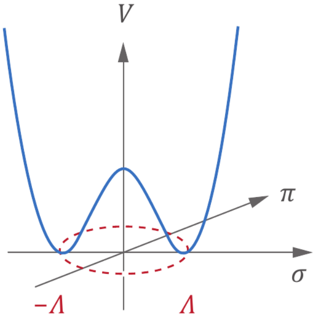
    - ...

    $\Longrightarrow$ different $\langle r_1^2 \rangle$, and F_{EM} in general
- Theoretical input needed
  - **Chiral Perturbation Theory** (χPT) $\rightarrow$ parametrise M_π and q^2 dep.
  - **Dispersion theory** $\rightarrow$ enlarge q^2 range
  - **Goal**: Disp+χPT $=$ good q^2 and M_π description

Electromagnetic form factor



- Lattice QCD parametrisation issue
- **Chiral Perturbation Theory** (χPT)
  - EFT for QCD at low energy
  - QCD based parametrisation of q^2 and M_π dependencies $\Longrightarrow$ extrapolate lattice results to the phys. point and extract $\langle r_i^2 \rangle$ and κ from the lattice simulations
  - Account for finite volume, lattice spacing and excited states
  - Determining χPT LECs from the lattice $\Longrightarrow$ predicting other observables



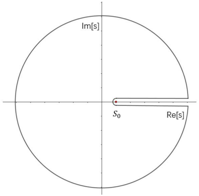
- QCD for light quarks ($m_q \to 0$):
  - $\mathscr{L}^0_{QCD} = i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R \ , \quad q^T = (u,d,s)$
  - $q \longrightarrow e^{i\varepsilon^a \frac{\lambda^a}{2}} q$ invariance for each chirality
- Spontaneous symmetry breaking < 1 GeV:
  - $\langle 0|\bar{q}q|0\rangle \neq 0$
  - $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \longrightarrow \mathrm{SU}(3)_V$
  - 8 Goldstones: $\pi^\pm$, π^0, η, $K^\pm$, K^0, $\overline{K^0}$
- Effective $\mathscr{L}$agrangian:
  - dofs: Goldstones
  - Symmetries: $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ (+Lorentz, C and P)
  - $\mathscr{L}_2 = \frac{F_0^2}{4}\langle D_\mu U \left(D^\mu U^\dagger\right)\rangle + \frac{F_0^2}{4}\langle \chi U^\dagger + U\chi^\dagger\rangle \ , \quad U = e^{\frac{i\phi^a\lambda^a}{F_0}}$
- We work with baryon relativistic ChPT

Dispersion theory

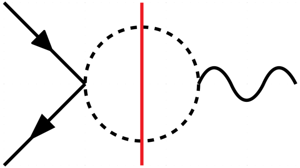
- Enlarge the q^2 range of χPT (ρ dynamics)

1. Analiticity $\Rightarrow$ Disp. rel. (Cauchy)

$$F(q^2) = \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{\mathrm{Im}\, F(s)}{s - q^2 - i\varepsilon}$$

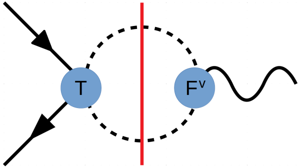


2. Unitarity$\Rightarrow$ $\mathrm{Im}\, F = \frac{1}{2} \sum_n T_{\gamma^* n} T^\dagger_{n \bar{N} N}$, $n = \pi^+ \pi^-, \ldots$
   - $\ell = 1$, $\pi\pi$ must be iso-vector (ρ channel)
   $\Rightarrow F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}$, $\quad F_i^{(v)} = F_i^p - F_i^n$



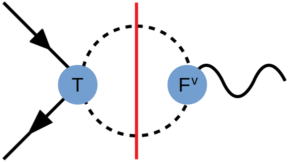
3. Using full $N\bar{N}\pi\pi$ and $\gamma^* \pi\pi$ vertices with M_π dep.

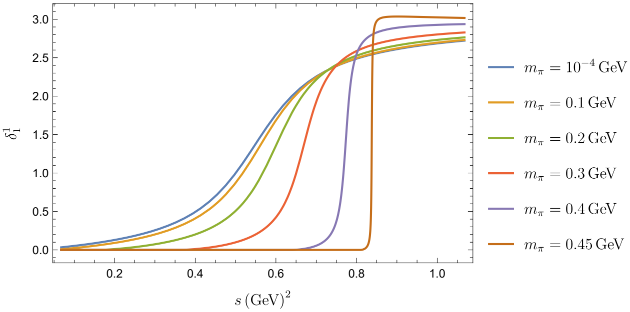
$$F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\, p_{\mathrm{cm}}^3\, F_\pi^{V*}}{s^{1/2}(s - q^2 - i\varepsilon)} ,$$



[Granados et al, EPJ A 53 (2017)]

- Our two vertices, T and F_π^V, include nonperturbatively the $\pi\pi$ scattering amplitude, t, thanks to IAM (resummation)
  - $\mathscr{A}(s) = \frac{8}{\pi}\sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell(s)$
  - $\ell = 1$, $t = \frac{\sqrt{s}}{2p_{\rm cm}} \sin\delta e^{i\delta}$
  - $\Omega(s) = \exp\left\{ s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s'-s-i\varepsilon)} \right\}$
  - We fit $t_{\rm IAM}$ to physical δ from [Garcia-Martin PRD 83(2011)]
  - We check that the M_π dependence is realistic
  - $\boxed{F_\pi^V(s) = [1+\alpha_V s]\,\Omega(s)}$





$\rho \nrightarrow \pi\pi$

Dispersion theory

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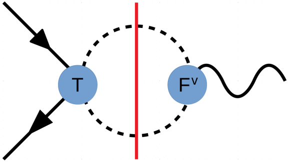

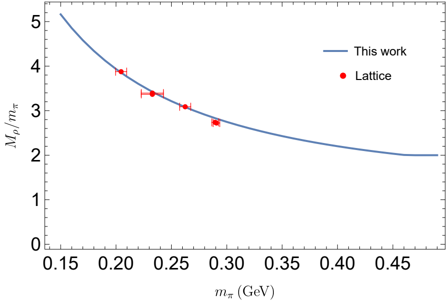

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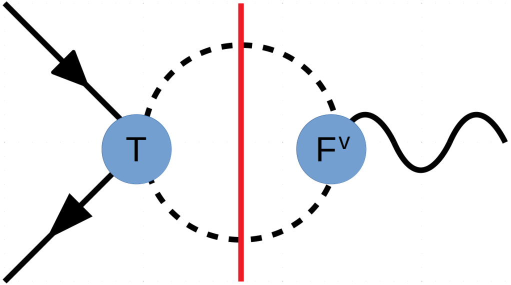
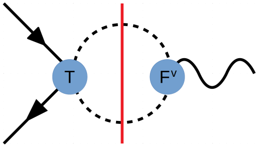
Dispersion theory

- Our two vertices, T and F_π^V
  - $\boxed{F_\pi^V(s) = [1+\alpha_V s]\,\Omega(s)}$
  - $\boxed{T_i(s) = K_i(s) + \Omega(s)\,P_i + I_i(s)}$,
    - $I_i(s) = \Omega(s)\,s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{K_i(s')\sin\delta(s')}{|\Omega(s')|\,(s'-s-i\varepsilon)\,s'}$
    - K and P from $N\bar{N} \to \pi\pi$ in χPT
      $\mathscr{M} = A\bar{v}u - \frac{1}{2}B\bar{v}\not{k}u \longleftrightarrow K, P$
  -

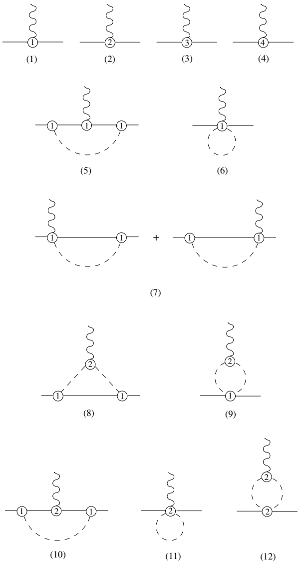
$$\Rightarrow \boxed{F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T\,p_{\mathrm{cm}}^3\,F_\pi^{V*}}{s^{1/2}(s-q^2-i\varepsilon)}},$$



- Disp+χPT$=F_{\rm EM}^{\rm disp}+$ diagrams w/o 2π cut from χPT
  - $F_{\rm EM}^{\rm disp}$



  - $+\chi$PT diagrams



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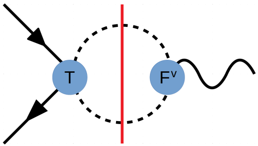
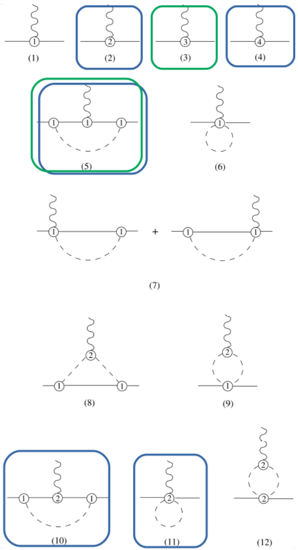


  - $+\chi$PT diagrams
    - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
    - green: $F_1 - F_1(0)$ (the charge is trivial)
    - blue: $F_2 \to$ we add the $\mathcal{O}(p^4)$ $\not{\Delta}$ terms



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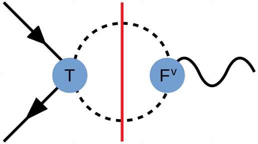

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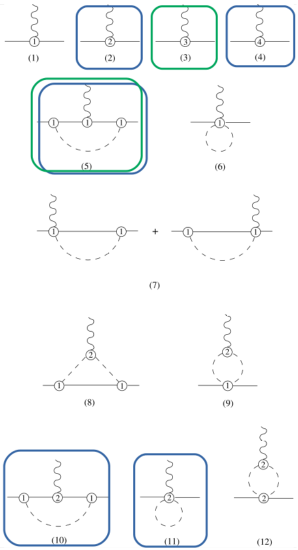
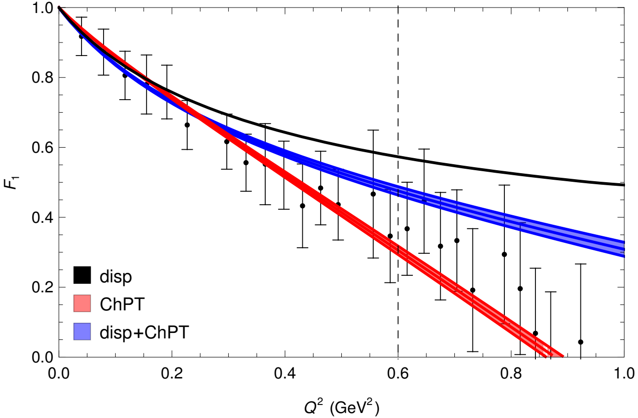
    - green: $F_1 - F_1(0)$ (the charge is trivial)
    - blue: $F_2 \rightarrow$ we add the $\mathcal{O}(p^4)$ $\not\Delta$ terms
  - Disp and χPT differ in the renormalization (UV)
    - At $\mathcal{O}(p^3)$ disp and χPT agree on the M_π nonanalyticities
    - Example: the dispersive contribution from $T^{\rm point} \sim \frac{1}{F^2}$ agrees with χPT $F_1^{\rm point} \sim F_1^{(9)} \sim q^2 \log M_\pi$
    - differences absorbed in LECs



- Dirac f.f., F_1

FA, An, Alvarez-Ruso & Leupold PRD 108 (2023)

  - $\langle N| V^\mu |N\rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2\right] u$
  - $F_1 = 1 + \frac{q^2}{6}\left[-12 d_6 + \langle r_1^{2(\log M_\pi)}\rangle \log M_\pi\right] + \mathcal{O}(p^4)$
  - Comparison with LQCD data [Djukanovic PRD 103(2021)] $\longleftarrow$ controlled FV and discret. effects
  - In the χPT and disp+χPT F_1, d_6 is fitted to LQCD
  - Disp $\longrightarrow$ q^2 curvature



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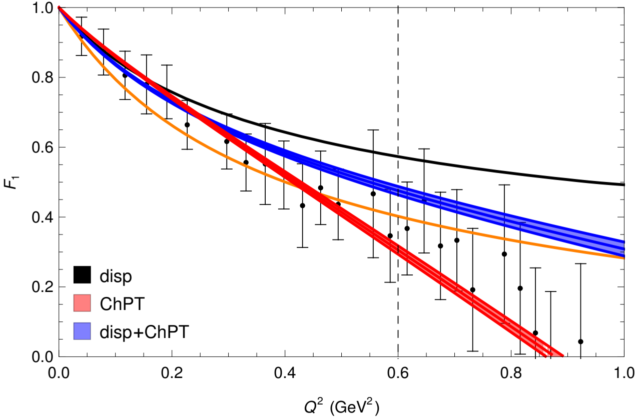
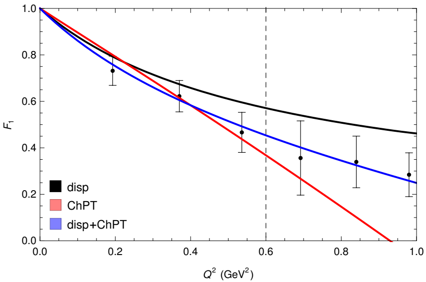
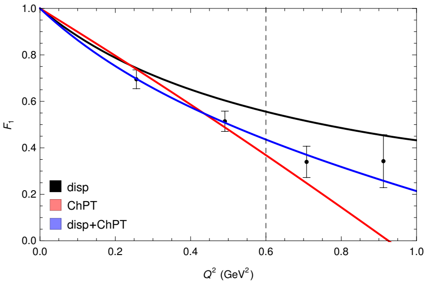




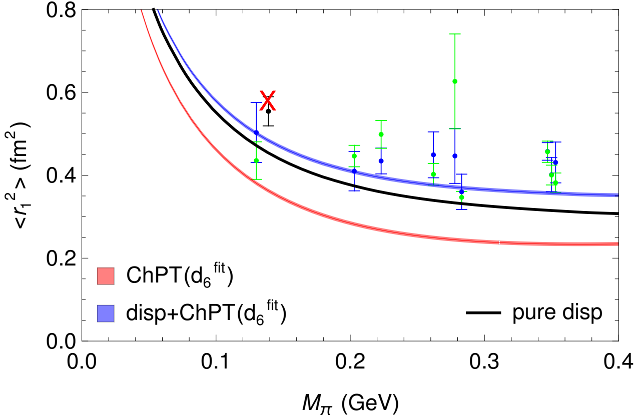
(h) H105 $M_\pi = 0.278$ GeV



(i) N302 $M_\pi = 0.353$ GeV

- **Disp+χPT describes well the M_π dep.**
  - good d_6 fit for $Q^2 < 0.6$ GeV^2 and $M_\pi \lesssim 350$ MeV
  - outperforms the pure dispersive and plain χPT descriptions

	Disp (prediction)	χPT	Disp+χPT
$d_6(\mu = m_\rho)$ (GeV^{-2})	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV^{-2})	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47-1) = 1.61$	$24.6/(47-1) = 0.53$
$\langle r_1^2 \rangle_{\mathrm{phys}}$ (fm^2)	0.4541	0.3626 ± 0.0047	**0.4838 ± 0.0047**



- $F_1^{(v)} = 1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathscr{O}(q^4)$, $\langle r_1^2\rangle^{\mathrm{PDG}} = 0.577\ \mathrm{fm}^2$
- Heavy baryon fit to LQCD from [Djukanovic PRD 103(2021)]: $\langle r_1^2\rangle^{\mathrm{HB}} = 0.554 \pm 0.035\ \mathrm{fm}^2$

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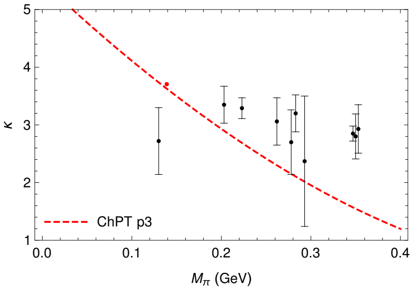
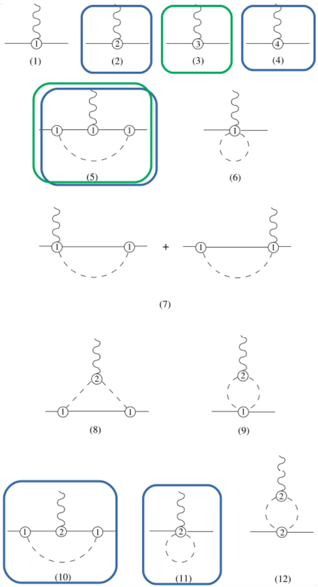
- Pauli f.f., F_2
  - $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2 \right] u$, $F_2 = \frac{F_M - F_E}{(1+Q^2/(4m^2))}$
  - $F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathscr{O}(q^4) \right]$
  - $\mathscr{O}(p^3)$ χPT is not enough (and Δ subtleties)

    $F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2} \right) M_\pi$

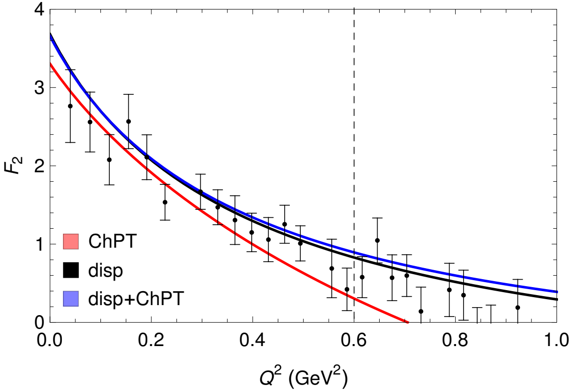


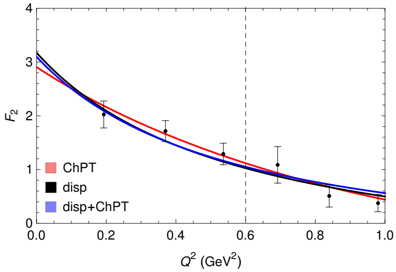
  - $\Longrightarrow$ include $\Delta\!\!\!/$ $\mathscr{O}(p^4)$
  - disp+χPT $\mathscr{O}(p^4)$: $F_2 = F_2^{\text{disp}} + F_2^{\text{tree}} + F_2^{\chi\text{PTloop}}$,

    $F_2^{\text{tree}} = c_6 - 16 e_{106} m_N M_\pi^2 + 2q^2 (d_6 + 2 e_{74} m_N)$

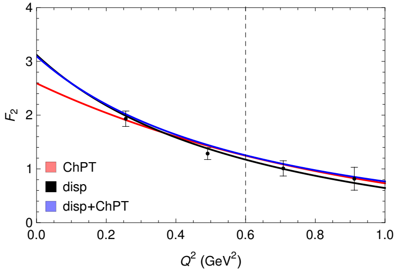


- F_2 **fit to LQCD**
  - In $F_2^{\rm disp}$ free c_6
  - In $F_2^{\chi\rm PT}$ and $F_2^{disp+\chi\rm PT}$, free c_6, e_{106}, e_{74}
  - χPT $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data





(a) H105 $M_\pi = 0.278$ GeV



(b) N302 $m_\pi = 0.353$ GeV

- F_2 **fit to LQCD**
  - In F_2^{disp} free c_6
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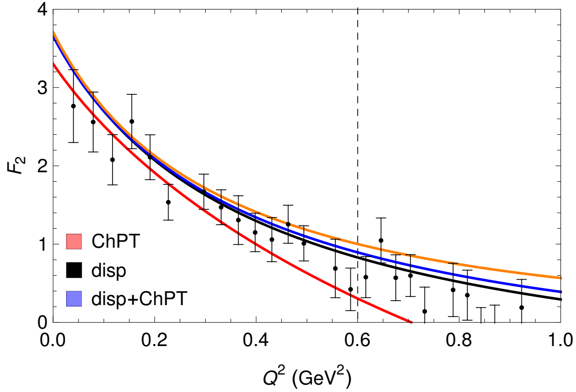

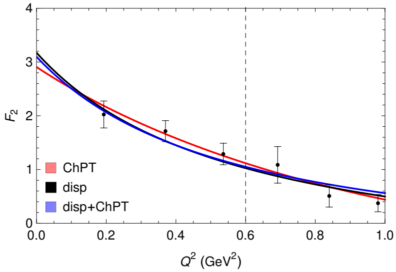

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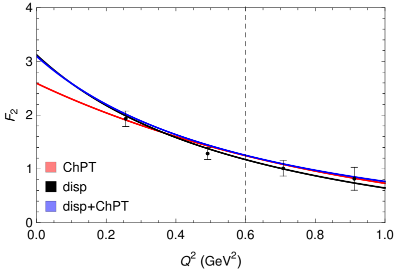
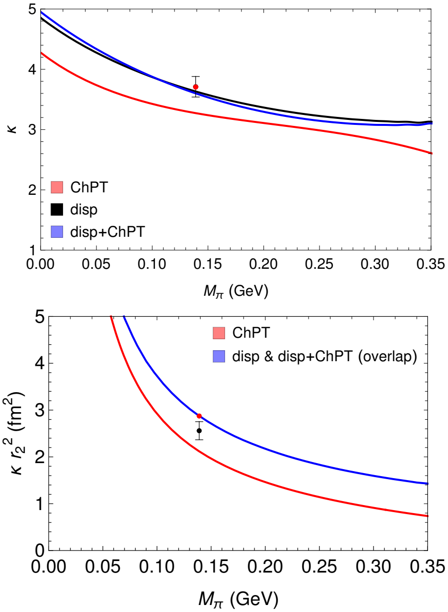


(b) N302 $m_\pi = 0.353$ GeV



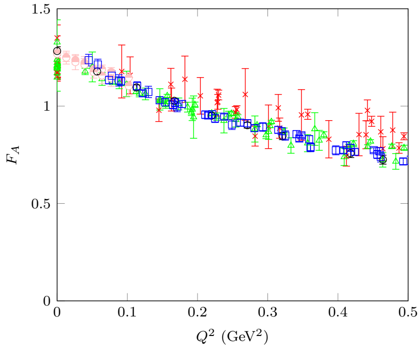
- $F_2^{(v)} = \kappa^{(v)}\left[1+\frac{1}{6}\langle r_2^2\rangle^{(v)}q^2+\mathcal{O}(q^4)\right]$
- . $\kappa_{\mathrm{PDG}} = 3.706,$
  $\kappa_{\mathrm{HB}} = 3.71 \pm 0.17,$
  $\langle r_2^2\rangle_{\mathrm{PDG}} = 0.7754\ \mathrm{fm}^2,$
  $\langle r_2^2\rangle_{\mathrm{HB}} = 0.690 \pm 0.042\ \mathrm{fm}^2.$

	Disp+c_6	χPT	**Disp+χPT**
χ^2/dof	$\frac{49.95}{47-2}=1.11$	$\frac{44.18}{47-4}=1.027$	$\frac{56.08}{47-4}=1.304$
χ_0^2/dof	1.09	1.027	1.283
κ_{phys}	3.64	3.42	**3.61**
$\langle r_2^2\rangle_{\mathrm{phys}}$ (fm^2)	0.673	0.619	**0.668**

- In general good $F_{1,2}$ description
- Our extraction from LQCD is in line with PDG
  - In this case HB yields also reasonable results

Axial form factor

- Analogously to the F_{EM} work:
  1. calculate F_A in ChPT
  2. analyse LQCD data



AXIAL FORM FACTOR

- $F_A \rightarrow$ spin distribution
  Weak interaction is $V - A$
  - $A^{i\mu} = \bar{q}\gamma^\mu\gamma_5\frac{\tau^i}{2}q$, $\langle N|A^\mu|N\rangle = \bar{u}'\left[\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N}G_P(q^2)\right]\gamma_5\frac{\tau^i}{2}u$
- **Nucleon axial isovector form factor**
  - $F_A(q^2) = g_A\left[1 + \frac{1}{6}\langle r_A^2\rangle q^2 + \mathcal{O}(q^4)\right]$
  - g_A and F_A dependence in q^2 are necessary in ν oscillations experiments
  - μ capture, β-decay
  - χPT calculation of F_A
    $\Longrightarrow$ extract $\langle r_A^2\rangle$ from lattice QCD without ad-hoc parametrization
  - $g_A(M_\pi)$: test of πN scatt. in χPT FA & Alvarez-Ruso PRD 105 (2021)
- **$\mathcal{O}(p^4)$ F_A in relativistic χPT**
  - $F_A = \mathring{g}_A + 4d_{16}M_\pi^2 + d_{22}q^2 + \mathrm{loops}(M_\pi, q^2)$

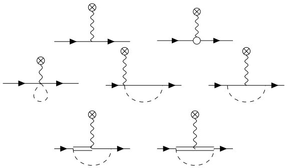


Figure: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

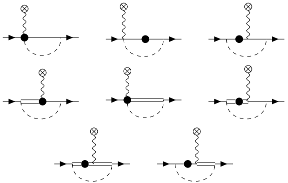
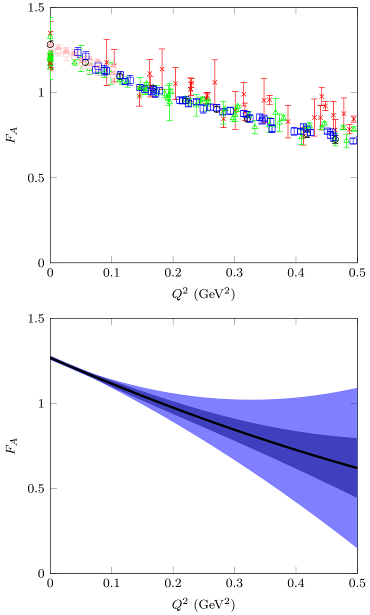


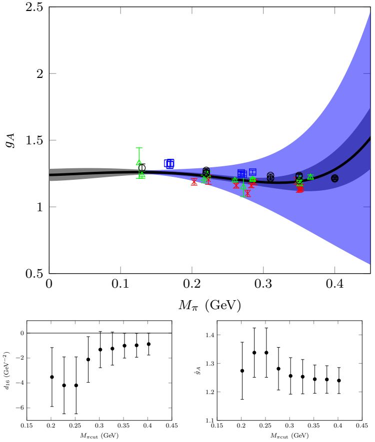
Figure: $\mathcal{O}(p^4)$

- F_A: **Meta-analysis of large set of recent LQCD results**
  - Many recent works ⇒ substantial improvements
  - RQCD[1] + PNDME[2] + ”Mainz”[3] + PACS[4] + ETMC[5]



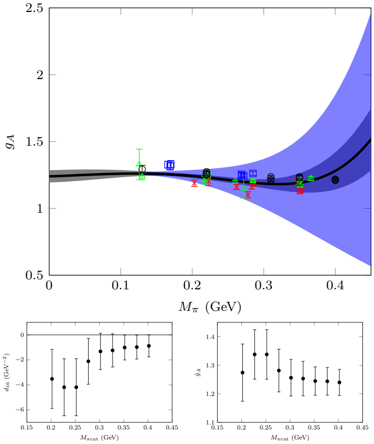
[1] Bali et al. JHEP 05 (2020)
[2] Park et al. 2103.05599
[3] Meyer et al. Modern Phys. A 34 (2019)
[4] Shintani et al. PRD 102 (2020)
[5] Alexandrou et al. PRD 103 (2021)

$F_A(q^2 = 0) = \boxed{g_A}$



- $g_A(M_\pi)$
  - we saw that $\Delta\!\!\!/$ LECs from πN elastic and inelastic scattering fail to describe its M_π dependence Alvarado & Alvarez-Ruso PRD 105 (2021)
- F_A fit procedure:
  - **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum EPJA 53 (2015)]
    $$\Delta g_{A\chi}^{(4)} = \max\left\{\left(\frac{M_\pi}{\Lambda}\right)^4 |\mathring{g}_A|, \left(\frac{M_\pi}{\Lambda}\right)^2 \left|g_A^{(3)}\right|, \frac{M_\pi}{\Lambda}\left|g_A^{(4)}\right|\right\}$$
  - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
  - LECs have naturalness priors
  - fit range: χ^2 plateau $\Longrightarrow M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $Q^2_{\mathrm{cut}} = 0.36$ GeV^2

$F_A(q^2 = 0) = \boxed{g_A}$



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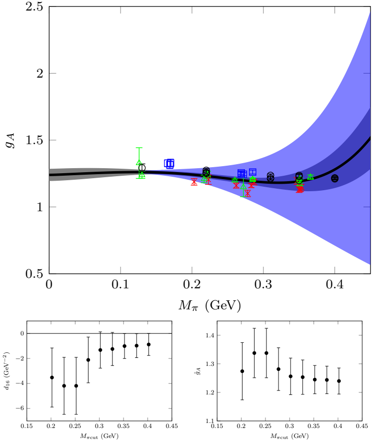
  - LECs have naturalness priors
  - fit range: χ^2 plateau $\Longrightarrow M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $Q^2_{\mathrm{cut}} = 0.36$ GeV^2
- Fit results: good description
  - accurate description at the physical point
  - Δ baryon is a necessary d.o.f.
  - $\mathcal{O}(p^5)$ still needed for full convergence

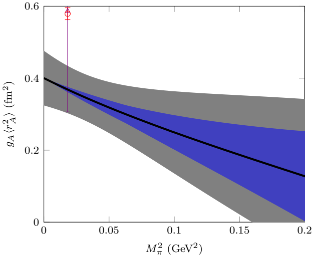
$F_A(q^2 = 0) = \boxed{g_A}$

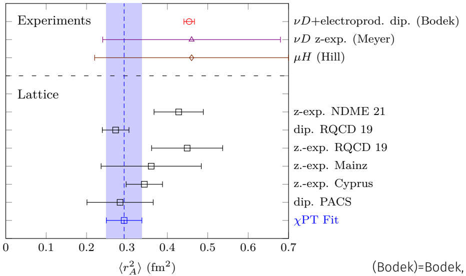


- **Axial charge results** from $F_A(q^2)$ fit
  - $g_A(M_{\pi\text{phys}}) = 1.273 \pm 0.014$ vs $g_A^{\text{exp}} = 1.2754(13)_{\text{exp}}(2)_{\text{RC}}$
    $\Rightarrow$ excellent agreement with exp.
    vs $g_A^{\text{FLAG}} = 1.246 \pm 0.028$
- $g_A(M_\pi) = \mathring{g}_A + 4d_{16}M_\pi^2 + \text{loop}(M_\pi)$
- $\boxed{d_{16} = -1.46 \pm 1.00\ \text{GeV}^{-2}}$

$\longrightarrow$ M_π dependence of long range nuclear forces
  - Can not be extracted from πN elastic scattering
  - In line with $d_{16} = -1.0 \pm 1.0\ \text{GeV}^{-2}$ from $\pi N \to \pi\pi N$
    [Siemens et al. PRC 96 (2017)] (EOMS corrected)

$F_A = g_A\left(1+\frac{1}{6}\boxed{\langle r_A^2\rangle}\,q^2\right)$ AXIAL RADIUS





(Bodek)=Bodek, Eur. Phys. J. C 53, 349 (2008)

(Meyer)=Meyer, PRD 93, 113015 (2016)

(Hill)=Hill, Rept. Prog. Phys. 81 (2018)

- **Our $\mathcal{O}(p^4)$ χPT extraction:**
  - M_π slope driven by loops with Δ
  - $d_{22} = 0.29 \pm 1.69$ GeV^{-2} (no assumptions on $\Delta\Delta\pi$ coupling enlarges error)
    - d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. Guerrero et al. PRD, 102 (2020)
  - $\boxed{\langle r_A^2\rangle(M_{\text{phys}}) = 0.293 \pm 0.044\ \text{fm}^2}$
  - Empirical determinations (model dependent) are in tension with ours and with most of LQCD extractions
  - Tipically the extracted $\langle r_A^2\rangle^{\text{phys}}$ value varies depending on the parametrisation
  - **Our QCD-based parametrisation leads to a value in line with most individual LQCD extractions**

Conclusions

- F_{EM}
  - **Dirac f.f., F_1**
    - The dispersive calculation supplemented with χPT contributions outperforms the pure dispersive and plain χPT descriptions
    - it fits well the LQCD F_1 at least for $Q^2 < 0.6$ GeV^2 and $M_\pi \lesssim 350$ MeV
    - $\langle r_1^2 \rangle_{\mathrm{phys}} = 0.4838 \pm 0.0047$ fm^2
    - value close to the LQCD HB [Djukanovic PRD 103(2021)] extraction and to the experimental one
  - **Pauli f.f., F_2**
    - Disp, $\mathcal{O}(p^4)$ χPT and disp+χPT describe the data well
    - $\kappa_{\mathrm{phys}} = 3.61$ and $\langle r_2^2 \rangle_{\mathrm{phys}} = 0.668$ fm^2
    - values in line with the HB and the experimental ones

- F_A
  - Succesful description of LQCD $F_A(q^2)$ using $\mathcal{O}(p^4)$ relativistic χPT
    - Fit describes data in $M_\pi^{\mathrm{cut}} \simeq 400$ MeV, $Q^2_{\mathrm{cut}} = 0.36$ GeV^2
  - There is tension between the experimental and lattice extraction of $\langle r_A^2 \rangle$
  - We extract $\langle r_A^2 \rangle^{\mathrm{phys}} = 0.291 \pm 0.052\ \mathrm{fm}^2$ without ad hoc parametrisations
- Useful LEC values extracted from both calculations

Backup

- EM f.f.
  - $V^\mu = \bar{q} Q \gamma^\mu q$,
    $\langle N(p')|\, V^\mu(0)\, |N(p)\rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2)\right] u$
  - $G_E^N = F_1^N - \frac{Q^2}{4m_N^2} F_2^N, \quad G_M^N = F_1^N + F_2^N$
  - $F_1 = 1 \times \left[1 + \frac{1}{6}\langle r_1^2\rangle q^2 + \mathcal{O}(q^4)\right]$
    $F_1(0) = F_E(0) = 1$ electric charge
  - $F_2 = \boldsymbol{\kappa} \left[1 + \frac{1}{6}\langle r_2^2\rangle q^2 + \mathcal{O}(q^4)\right]$
    $\boldsymbol{\kappa} = F_M(0) = \boldsymbol{\mu} - 1$ anom. magn. mom.
  - $\langle r_E^2\rangle = \langle r_1^2\rangle + \frac{3}{2m_N^2} \boldsymbol{\kappa}$ proton radius puzzle
  - $F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\boldsymbol{\sigma}^3}{2}$
  - $F_1^{(v)} = 1 \times \left[1 + \frac{1}{6}\langle r_1^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
    $F_1^{(v)}(0) = F_E^{(v)}(0) = 1$ electric charge
  - $F_2^{(v)} = \boldsymbol{\kappa}^{(v)} \left[1 + \frac{1}{6}\langle r_2^2\rangle^{(v)} q^2 + \mathcal{O}(q^4)\right]$
    $\boldsymbol{\kappa}^{(v)} = F_M^{(v)}(0) = \boldsymbol{\mu}^{(v)} - 1$ magnetic moment
  - $\langle r_E^2\rangle^{(v)} = \langle r_1^2\rangle^{(v)} + \frac{3}{2m_N^2} \boldsymbol{\kappa}^{(v)}$ proton radius puzzle

Backup

	Disp (prediction)	χPT	**Disp+χPT**
$d_6(\mu = m_\rho)$ (GeV^{-2})	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV^{-2})	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47-1) = 1.61$	$24.6/(47-1) = 0.53$
$\langle r_1^2 \rangle_{\mathrm{phys}}$ (fm^2)	0.4541	0.3626 ± 0.0047	**0.4838 ± 0.0047**

Extra

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV^{-2}) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\widetilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166

Extra 2

	$\mathcal{O}(p^3)$ $\not\Delta$	$\mathcal{O}(p^4)$ $\not\Delta$	$\mathcal{O}(p^3)$ Δ	$\mathcal{O}(p^4)$ Δ
$\mathring{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV^{-2}) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV^{-2}) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV^{-1})	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV^{-1})	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV^{-1})	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV^{-1})	-	3.31 ± 0.13	-	2.61 ± 0.10
a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV^{-2}) (free)	-	-	-	2.27 ± 2.28
$\widetilde{b}_4$ (GeV^{-2}) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm^{-2}) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm^{-2}) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm^{-1}) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm^{-2} GeV^{-2}) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm^{-2} GeV^{-2}) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm^{-1} GeV^{-2}) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\mathring{m}$ (GeV)	0.874	0.874	0.855	0.855
$\mathring{m}_\Delta$ (GeV)	-	-	1.166	1.166
χ_0^2/dof	$857.31/(127-9) = 7.27$		$533.87/(127-10) = 4.45$	$196.58/(127-13) = 1.724$