

ELECTROWEAK STRUCTURE OF THE NUCLEON

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ELECTROWEAK STRUCTURE OF THE NUCLEON

- Structure of protons and neutrons is encoded in form factors

F_{EM} → charge distribution

- $V^\mu = \bar{q} Q \gamma^\mu q$,

$$\langle N | V^\mu(0) | N \rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] u$$

- $G_E = F_1 - \frac{Q^2}{4m_N^2} F_2$, $G_M = F_1 + F_2$

- slope=charge radius: $\langle r_E^2 \rangle = 6 \left. \frac{dG_E}{dq^2} \right|_0$

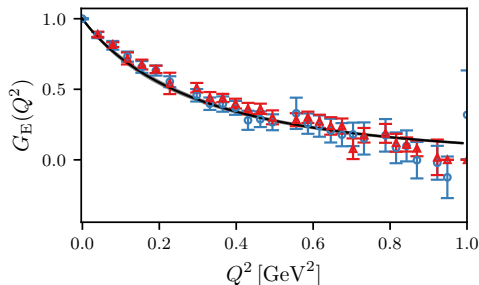
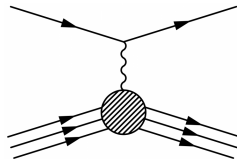
- $A^\mu = \bar{q} \gamma^\mu \gamma_5 q$,

$$\langle N | A^\mu | N \rangle = \bar{u}' \left\{ \gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} G_P(q^2) \right\} \gamma_5 u$$

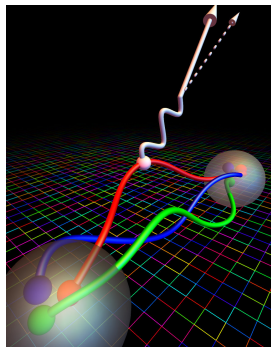
- F_A → spin distribution

Weak interaction is $V - A$

relevant for ν experiments

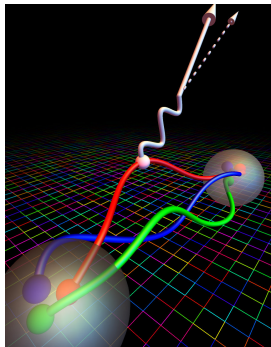


ELECTROMAGNETIC FORM FACTOR



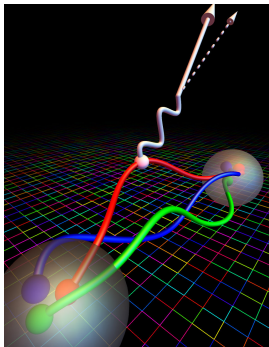
- Experimental determinations
 - $\langle r_E^2 \rangle$: proton radius puzzle [Pohl et al., Nature 466, 213 (2010)]
- Lattice QCD → Alberto Ramos' talk, 10:25 tomorrow!
 - Nucleon f.f. is a benchmark for LQCD
 - Uncertainties reduced for unphysical large M_π
 - Technical difficulties → recent progress
 - Experimental and lattice q^2 parametrisation:
 - dipole ansatz
 - z-expansion
 - ... } $\Rightarrow$ different $\langle r_1^2 \rangle$, and F_{EM} in general
- Theoretical input needed

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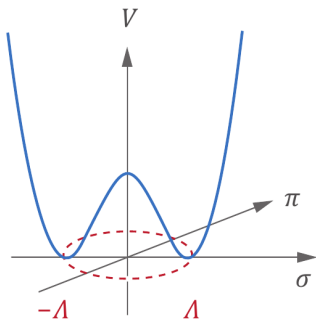
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 - ... $\left. \vphantom{\begin{matrix} - \\ - \\ - \end{matrix}} \right\} \Rightarrow \text{different } \langle r_1^2 \rangle, \text{ and } F_{EM} \text{ in general}$
- Theoretical input needed
 - Chiral Perturbation Theory (χ PT) → parametrise M_π and q^2 dep.
 - Dispersion theory → enlarge q^2 range
 - Goal: Disp+ χ PT = good q^2 and M_π description

ELECTROMAGNETIC FORM FACTOR



- Lattice QCD **parametrisation issue**
- **Chiral Perturbation Theory (χ PT)**
 - EFT for QCD at low energy
 - QCD based parametrisation of q^2 and M_π dependencies $\implies$ extrapolate lattice results to the phys. point and extract $\langle r_i^2 \rangle$ and κ from the lattice simulations
 - Account for finite volume, lattice spacing and excited states
 - Determining χ PT LECs from the lattice $\implies$ predicting other observables

CHIRAL PERTURBATION THEORY



- QCD for light quarks ($m_q \rightarrow 0$):
 - $\mathcal{L}_{QCD}^0 = i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R$, $q^T = (u, d, s)$
 - $q \rightarrow e^{i\epsilon^a \frac{\lambda^a}{2}} q$ invariance for each chirality
- Spontaneous symmetry breaking < 1 GeV:
 - $\langle 0 | \bar{q} q | 0 \rangle \neq 0$
 - $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$
 - 8 Goldstones: $\pi^\pm, \pi^0, \eta, K^\pm, K^0, \bar{K}^0$
- Effective Lagrangian:
 - dofs: Goldstones
 - Symmetries: $SU(3)_L \otimes SU(3)_R$ (+Lorentz, C and P)
 - $\mathcal{L}_2 = \frac{F_0^2}{4} \langle D_\mu U (D^\mu U^\dagger) \rangle + \frac{F_0^2}{4} \langle \chi U^\dagger + U \chi^\dagger \rangle$, $U = e^{\frac{i\phi^a \lambda^a}{F_0}}$
- We work with baryon relativistic ChPT

DISPERSION THEORY

- Enlarge the q^2 range of χ PT (ρ dynamics)

1. Analyticity $\Rightarrow$ Disp. rel. (Cauchy)

$$F(q^2) = \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{\text{Im} F(s)}{s - q^2 - i\epsilon}$$

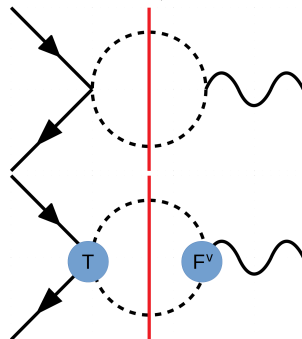
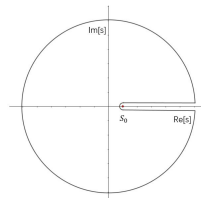
2. Unitarity $\Rightarrow \text{Im} F = \frac{1}{2} \sum_n T_{\gamma^* n} T_{n \bar{N} N}^\dagger$, $n = \pi^+ \pi^-, \dots$

- $\ell = 1$, $\pi\pi$ must be iso-vector (ρ channel)

$$\Rightarrow F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}, \quad F_i^{(v)} = F_i^p - F_i^n$$

3. Using full $N\bar{N}\pi\pi$ and $\gamma^*\pi\pi$ vertices with M_π dep.

$$F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T p_{\text{cm}}^3 F_\pi^{V*}}{s^{1/2}(s - q^2 - i\epsilon)},$$



DISPERSION THEORY

- Our two vertices, T and F_π^V , include nonperturbatively the $\pi\pi$ scattering amplitude, t , thanks to IAM (resummation)

- $\mathcal{A}(s) = \frac{8}{\pi} \sum_\ell (2\ell+1) P_\ell(\cos\theta) t_\ell(s)$

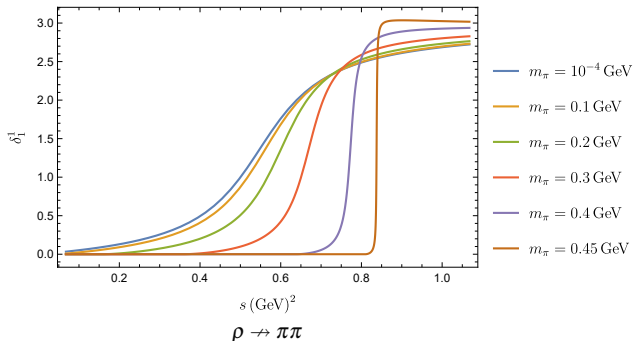
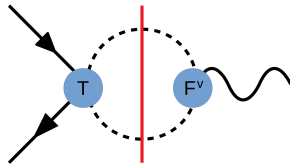
- $\ell=1$, $t = \frac{\sqrt{s}}{2p_{\text{cm}}} \sin\delta e^{i\delta}$

- $\Omega(s) = \exp \left\{ s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{\delta(s')}{s'(s'-s-i\epsilon)} \right\}$

- We fit t_{IAM} to physical δ from [Garcia-Martin PRD 83(2011)]

- We check that the M_π dependence is realistic

- $F_\pi^V(s) = [1 + \alpha_V s] \Omega(s)$



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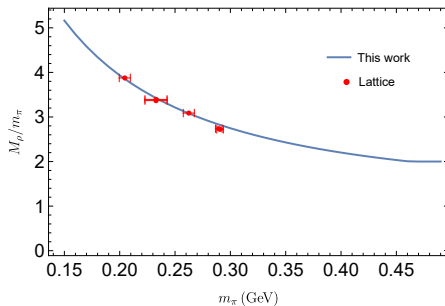
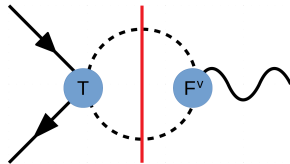
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$\rho \rightarrow \pi\pi$

DISPERSION THEORY

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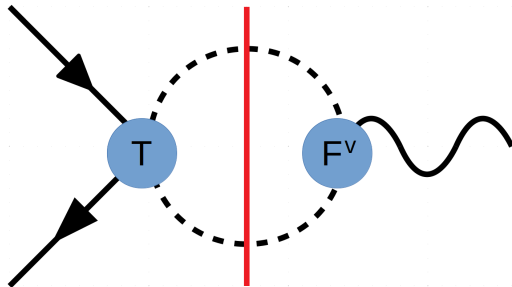
- $T_i(s) = K_i(s) + \Omega(s) P_i + I_i(s)$,

- $I_i(s) = \Omega(s) s \int_{4M_\pi^2}^{\infty} \frac{ds'}{\pi} \frac{K_i(s') \sin \delta(s')}{|\Omega(s')| (s' - s - i\varepsilon) s'}$

- K and P from $N\bar{N} \rightarrow \pi\pi$ in χ PT
 $\mathcal{M} = A\bar{v}u - \frac{1}{2}B\bar{v}\not{k}u \longleftrightarrow K, P$

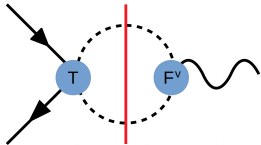
.

$$\Rightarrow F(q^2) = \frac{1}{12\pi} \int_{4M_\pi^2}^{\infty} \frac{ds}{\pi} \frac{T p_{\text{cm}}^3 F_\pi^{V*}}{s^{1/2}(s - q^2 - i\varepsilon)},$$

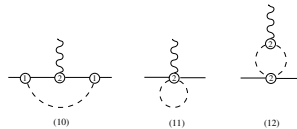
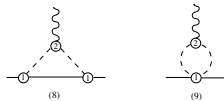
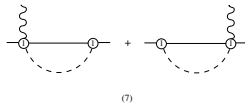
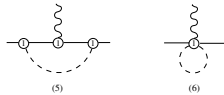
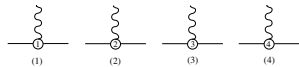


- **Disp+ χ PT** = $F_{\text{EM}}^{\text{disp}}$ + diagrams w/o 2π cut from χ PT

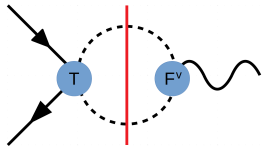
- $F_{\text{EM}}^{\text{disp}}$



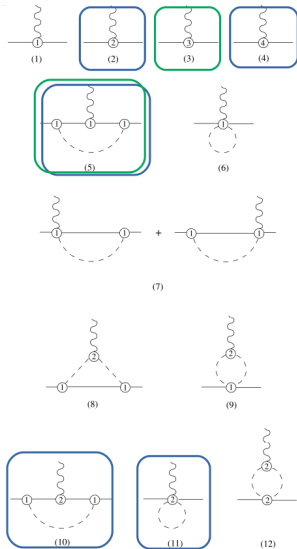
- + χ PT diagrams



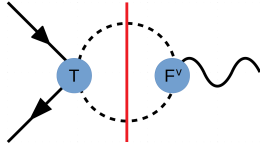
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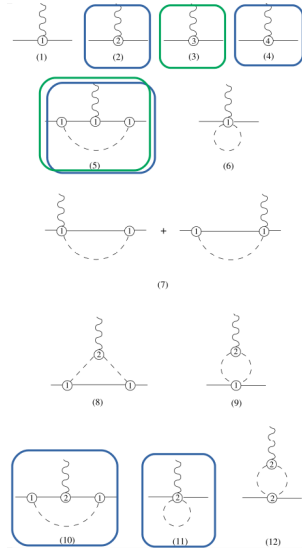
- + χ PT diagrams
 - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
 - green: $F_1 - F_1(0)$ (the charge is trivial)
 - blue: $F_2 \rightarrow$ we add the $\mathcal{O}(p^4)$ Δ terms



- $\text{Disp} + \chi\text{PT} = F_{\text{EM}}^{\text{disp}} + \text{diagrams w/o } 2\pi \text{ cut from } \chi\text{PT}$
- $F_{\text{EM}}^{\text{disp}}$



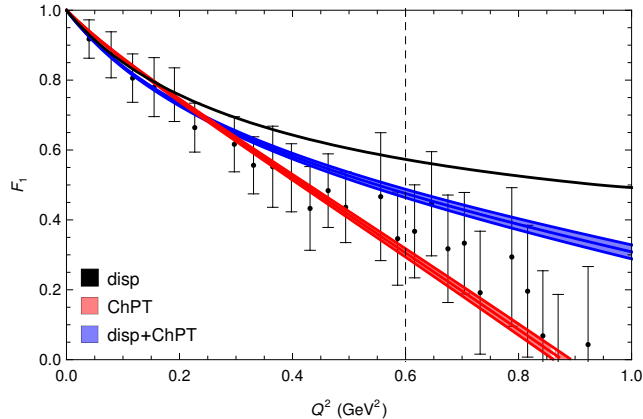
- $+\chi\text{PT}$ diagrams
 - Relativistic and with explicit $\Delta(1232)$ [Bauer et al., PRC 86 (2012)]
 - green: $F_1 - F_1(0)$ (the charge is trivial)
 - blue: $F_2 \rightarrow$ we add the $\mathcal{O}(p^4)$ Δ terms
- Disp and χPT differ in the renormalization (UV)
 - At $\mathcal{O}(p^3)$ disp and χPT agree on the M_π nonanalyticities
 - Example: the dispersive contribution from $T^{\text{point}} \sim \frac{1}{F^2}$ agrees with χPT
 $F_1^{\text{point}} \sim F_1^{(9)} \sim q^2 \log M_\pi$
 - differences absorbed in LECs



· Dirac f.f., F_1

[FA, An, Alvarez-Ruso & Leupold PRD 108 \(2023\)](#)

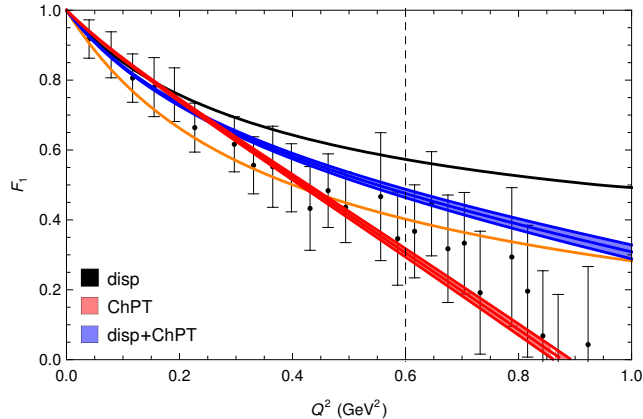
- $\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2 \right] u$
- $F_1 = 1 + \frac{q^2}{6} \left[-12d_6 + \langle r_1^{2(\log M_\pi)} \rangle \log M_\pi \right] + \mathcal{O}(p^4)$
- Comparison with LQCD data [Djukanovic PRD 103(2021)] ← controlled FV and discret. effects
- In the χ PT and disp+ χ PT F_1 , d_6 is fitted to LQCD
- Disp $\rightarrow q^2$ curvature

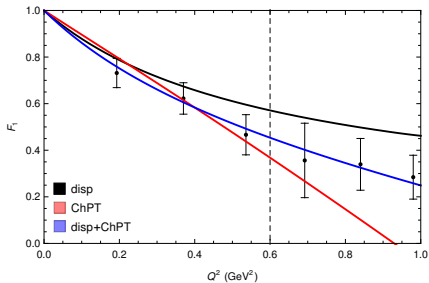


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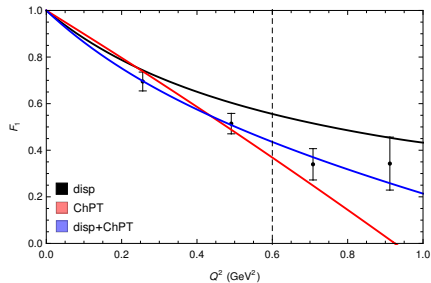
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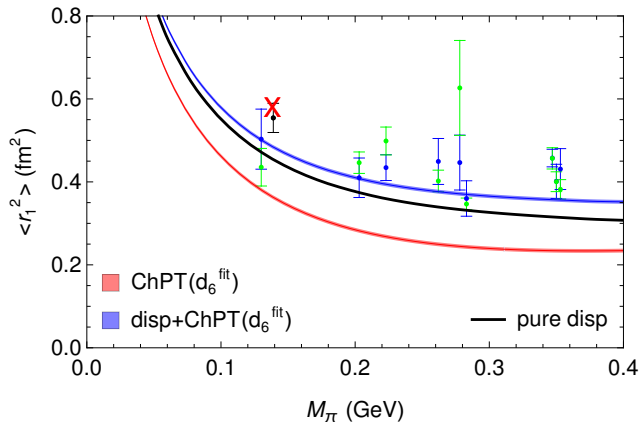
(h) H105 $M_\pi = 0.278$ GeV



(i) N302 $M_\pi = 0.353$ GeV

- Disp+ χ PT describes well the M_π dep.
- good d_6 fit for $Q^2 < 0.6$ GeV² and $M_\pi \lesssim 350$ MeV
- outperforms the pure dispersive and plain χ PT descriptions

	Disp (prediction)	χ PT	Disp+ χ PT
$d_6(\mu = m_\rho)$ (GeV ⁻²)	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV ⁻²)	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47 - 1) = 1.61$	$24.6/(47 - 1) = 0.53$
$\langle r_1^2 \rangle_{\text{phys}}$ (fm ²)	0.4541	0.3626 ± 0.0047	0.4838 ± 0.0047



- $F_1^{(\nu)} = 1 + \frac{1}{6} \langle r_1^2 \rangle^{(\nu)} q^2 + \mathcal{O}(q^4)$,
 $\langle r_1^2 \rangle^{\text{PDG}} = 0.577 \text{ fm}^2$
- Heavy baryon fit to LQCD from [Djukanovic PRD 103(2021)]:
 $\langle r_1^2 \rangle^{\text{HB}} = 0.554 \pm 0.035 \text{ fm}^2$

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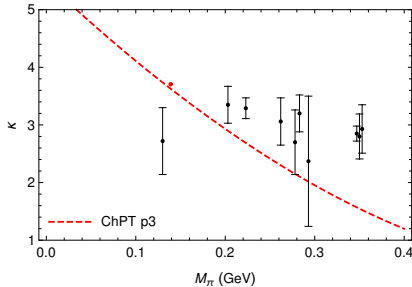
• Pauli f.f., F_2

$$\langle N | V^\mu | N \rangle = \bar{u}' \left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} F_2 \right] u, \quad F_2 = \frac{F_M - F_E}{(1 + Q^2/(4m^2))}$$

$$F_2^{(\nu)} = \kappa^{(\nu)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(\nu)} q^2 + \mathcal{O}(q^4) \right]$$

• $\mathcal{O}(p^3)$ χ PT is not enough (and Δ subtleties)

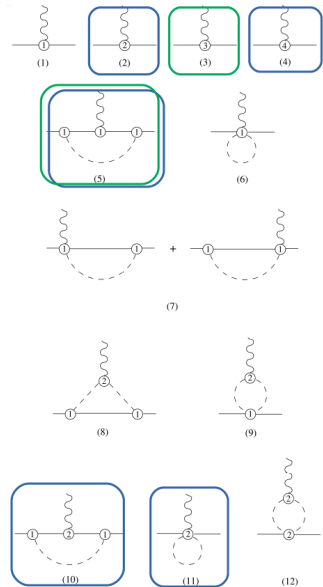
$$F_2(0)^{o(3)} = c_6 - \left(\frac{\pi m_N g_A^2}{4\pi^2 F^2} \right) M_\pi$$



• $\Rightarrow$ include $\Delta \mathcal{O}(p^4)$

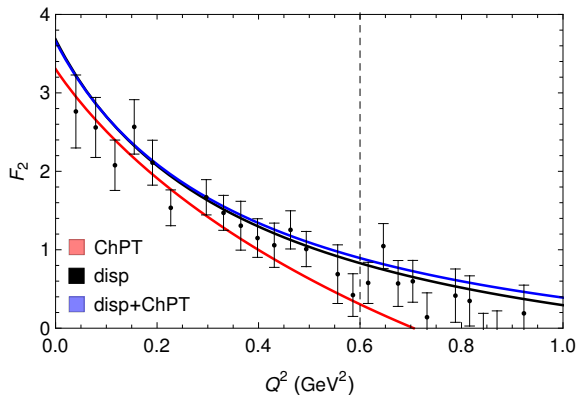
$$\text{disp} + \chi\text{PT } \mathcal{O}(p^4): F_2 = F_2^{\text{disp}} + F_2^{\text{tree}} + F_2^{\chi\text{PT loop}},$$

$$F_2^{\text{tree}} = c_6 - 16e_{106}m_N M_\pi^2 + 2q^2(d_6 + 2e_{74}m_N)$$

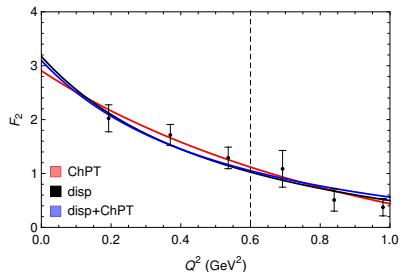


- F_2 fit to LQCD

- In F_2^{disp} free c_6
- In $F_2^{\chi^{\text{PT}}}$ and $F_2^{\text{disp}+\chi^{\text{PT}}}$, free c_6, e_{106}, e_{74}
- χ^{PT} $\mathcal{O}(p^4)$ and disp separately are good enough to describe the data



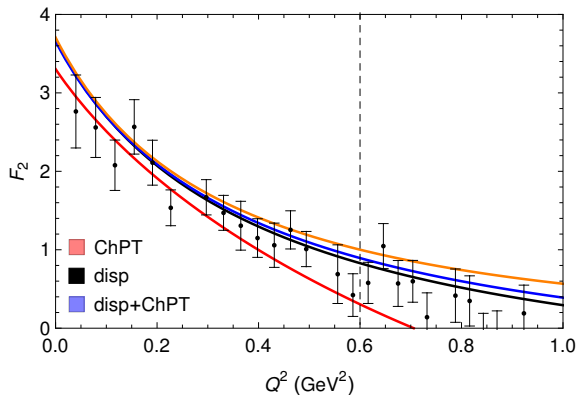
(a) H105 $M_\pi = 0.278$ GeV



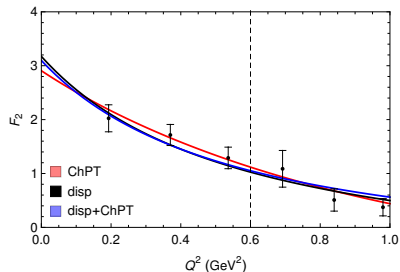
(b) N302 $m_\pi = 0.353$ GeV

- F_2 fit to LQCD

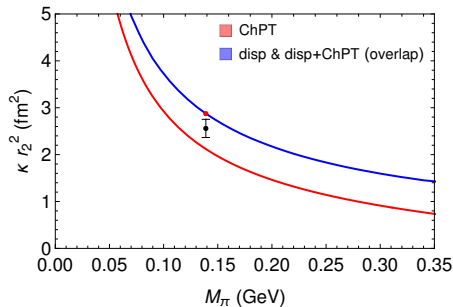
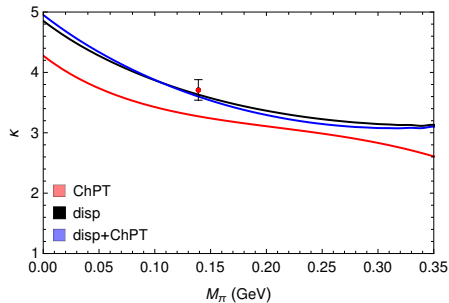
- In F_2^{disp} free c_6
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(a) H105 $M_\pi = 0.278$ GeV



(b) N302 $m_\pi = 0.353$ GeV



$$F_2^{(\nu)} = \kappa^{(\nu)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(\nu)} q^2 + \mathcal{O}(q^4) \right]$$

$$\kappa_{\text{PDG}} = 3.706,$$

$$\kappa_{\text{HB}} = 3.71 \pm 0.17,$$

$$\langle r_2^2 \rangle_{\text{PDG}} = 0.7754 \text{ fm}^2,$$

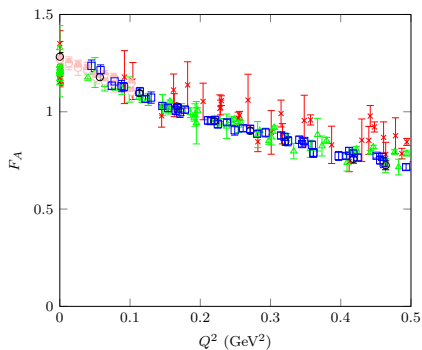
$$\langle r_2^2 \rangle_{\text{HB}} = 0.690 \pm 0.042 \text{ fm}^2.$$

	Disp+c ₆	χ^{PT}	Disp+ χ^{PT}
χ^2/dof	$\frac{49.95}{47-2} = 1.11$	$\frac{44.18}{47-4} = 1.027$	$\frac{56.08}{47-4} = 1.304$
χ_0^2/dof	1.09	1.027	1.283
κ_{phys}	3.64	3.42	3.61
$\langle r_2^2 \rangle_{\text{phys}} (\text{fm}^2)$	0.673	0.619	0.668

- In general good $F_{1,2}$ description
- Our extraction from LQCD is in line with PDG
 - In this case HB yields also reasonable results

AXIAL FORM FACTOR

- Analogously to the F_{EM} work:
 1. calculate F_A in ChPT
 2. analyse LQCD data



AXIAL FORM FACTOR

- $F_A \rightarrow$ spin distribution

Weak interaction is $V-A$

$$\cdot A^{i\mu} = \bar{q} \gamma^\mu \gamma_5 \frac{\tau^i}{2} q, \langle N | A^\mu | N \rangle = \bar{u}' \left[\gamma_\mu F_A(q^2) + \frac{q_\mu}{2m_N} G_P(q^2) \right] \gamma_5 \frac{\tau^i}{2} u$$

- Nucleon axial isovector form factor**

$$\cdot F_A(q^2) = g_A \left[1 + \frac{1}{6} \langle r_A^2 \rangle q^2 + \mathcal{O}(q^4) \right]$$

- g_A and F_A dependence in q^2 are necessary in ν oscillations experiments

- μ capture, β -decay

- χ PT calculation of F_A

$\Rightarrow$ extract $\langle r_A^2 \rangle$ from lattice QCD without ad-hoc parametrization

- $g_A(M_\pi)$: test of πN scatt. in χ PT [FA & Alvarez-Ruso PRD 105 \(2021\)](#)

- $\mathcal{O}(p^4) F_A$ in relativistic χ PT

$$\cdot F_A = \hat{g}_A + 4d_{16}M_\pi^2 + d_{22}q^2 + \text{loops}(M_\pi, q^2)$$

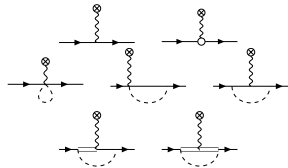


Figure: $\mathcal{O}(p)$ and $\mathcal{O}(p^3)$ (w. f. renormalisation not shown)

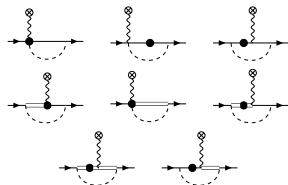
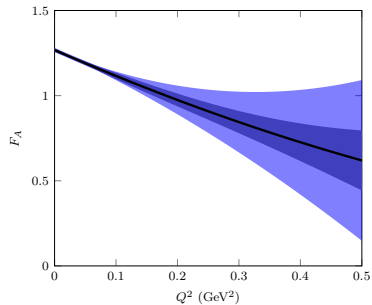
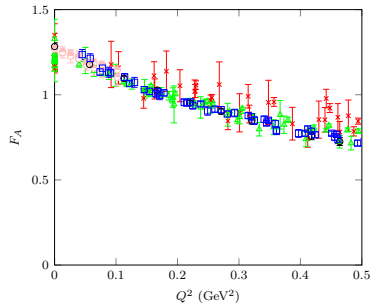


Figure: $\mathcal{O}(p^4)$

- F_A : Meta-analysis of large set of recent LQCD results

- Many recent works $\Rightarrow$ substantial improvements
- RQCD^[1] + PNDME^[2] + "Mainz"^[3] + PACS^[4] + ETMC^[5]



[1] [Bali et al. JHEP 05 \(2020\)](#)

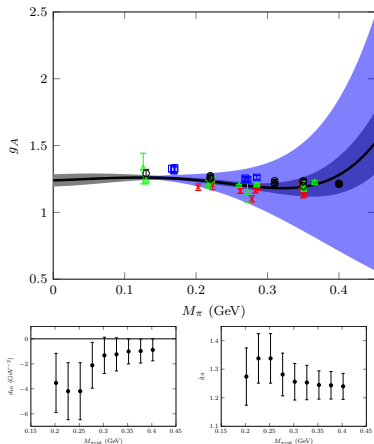
[2] [Park et al. 2103.05599](#)

[3] [Meyer et al. Modern Phys. A 34 \(2019\)](#)

[4] [Shintani et al. PRD 102 \(2020\)](#)

[5] [Alexandrou et al. PRD 103 \(2021\)](#)

$$F_A(q^2 = 0) = \boxed{g_A}$$



- $g_A(M_\pi)$

- we saw that Δ LECs from πN elastic and inelastic scattering fail to describe its M_π dependence [Alvarado & Alvarez-Ruso PRD 105 \(2021\)](#)

- F_A fit procedure:

- **Difference between orders $\simeq$ theoretical uncertainty** [Epelbaum EPJA 53 (2015)]

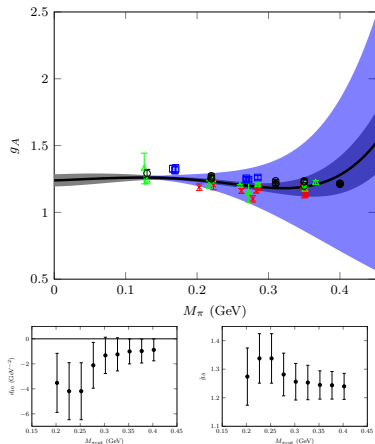
$$\Delta g_{A\chi}^{(4)} = \max \left\{ \left(\frac{M_\pi}{\Lambda} \right)^4 |\dot{g}_A|, \left(\frac{M_\pi}{\Lambda} \right)^2 |g_A^{(3)}|, \frac{M_\pi}{\Lambda} |g_A^{(4)}| \right\}$$

- $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2

- LECs have naturalness priors

- fit range: χ^2 plateau $\Rightarrow M_\pi^{\text{cut}} \simeq 400 \text{ MeV}$, $Q_{\text{cut}}^2 = 0.36 \text{ GeV}^2$

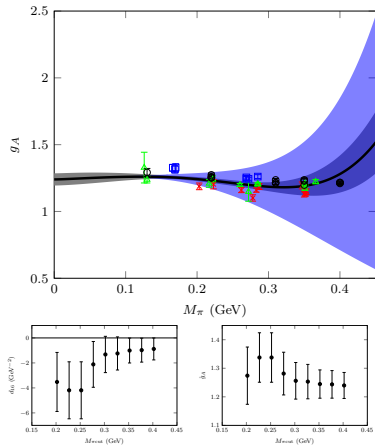
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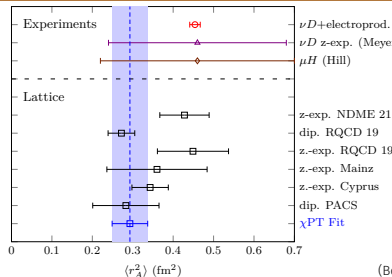
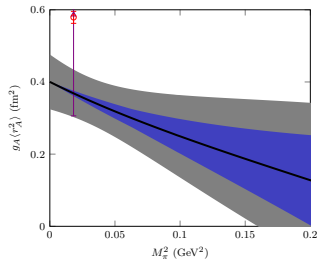
$$\Delta g_{A\chi}^{(4)} = \max \left\{ \left(\frac{M_\pi}{\Lambda} \right)^4 |\dot{g}_A|, \left(\frac{M_\pi}{\Lambda} \right)^2 |g_A^{(3)}|, \frac{M_\pi}{\Lambda} |g_A^{(4)}| \right\}$$
 - $\Delta F_{A\chi}$ is added to LQCD errors in the χ^2
 - LECs have naturalness priors
 - fit range: χ^2 plateau $\Rightarrow M_\pi^{\text{cut}} \simeq 400$ MeV, $Q_{\text{cut}}^2 = 0.36$ GeV²
- Fit results: good description
 - accurate description at the physical point
 - Δ baryon is a necessary d.o.f.
 - $\mathcal{O}(p^5)$ still needed for full convergence

$$F_A(q^2 = 0) = \boxed{g_A}$$



- Axial charge results from $F_A(q^2)$ fit
 - $g_A(M_{\pi\text{phys}}) = 1.273 \pm 0.014$ vs $g_A^{\text{exp}} = 1.2754(13)_{\text{exp}}(2)_{\text{RC}}$
 $\Rightarrow$ excellent agreement with exp.
 vs $g_A^{\text{FLAG}} = 1.246 \pm 0.028$
 - $g_A(M_\pi) = \hat{g}_A + 4d_{16}M_\pi^2 + \text{loop}(M_\pi)$
 - $\boxed{d_{16} = -1.46 \pm 1.00 \text{ GeV}^{-2}}$
- $\rightarrow M_\pi$ dependence of long range nuclear forces
 - Can not be extracted from πN elastic scattering
 - In line with $d_{16} = -1.0 \pm 1.0 \text{ GeV}^{-2}$ from $\pi N \rightarrow \pi\pi N$
 [Siemens et al. PRC 96 (2017)] (EOMS corrected)

$$F_A = g_A \left(1 + \frac{1}{6} \langle r_A^2 \rangle q^2 \right) \quad \text{AXIAL RADIUS}$$



(Bodek)=Bodek, Eur. Phys. J. C 53, 349 (2008)

(Meyer)=Meyer, PRD 93, 113015 (2016)

(Hill)=Hill, Rept. Prog. Phys. 81 (2018)

• Our $\mathcal{O}(p^4)$ χ PT extraction:

- M_π slope driven by loops with Δ
- $d_{22} = 0.29 \pm 1.69 \text{ GeV}^{-2}$ (no assumptions on $\Delta\Delta\pi$ coupling enlarges error)
 - d_{22} compatible with $\mathcal{O}(p^3)$ π electroprod. [Guerrero et al. PRD, 102 \(2020\)](#)

$$\langle r_A^2 \rangle(M_{\text{phys}}) = 0.293 \pm 0.044 \text{ fm}^2$$

- Empirical determinations (model dependent) are in **tension** with ours and with most of LQCD extractions
- Typically the extracted $\langle r_A^2 \rangle^{\text{phys}}$ value varies depending on the parametrisation
- Our QCD-based parametrisation leads to a value in line with most individual LQCD extractions

CONCLUSIONS

- F_{EM}
 - Dirac f.f., F_1
 - The dispersive calculation supplemented with χ PT contributions outperforms the pure dispersive and plain χ PT descriptions
 - it fits well the LQCD F_1 at least for $Q^2 < 0.6 \text{ GeV}^2$ and $M_\pi \lesssim 350 \text{ MeV}$
 - $\langle r_1^2 \rangle_{\text{phys}} = 0.4838 \pm 0.0047 \text{ fm}^2$
 - value close to the LQCD HB [Djukanovic PRD 103(2021)] extraction and to the experimental one
 - Pauli f.f., F_2
 - Disp, $\mathcal{O}(p^4)$ χ PT and disp+ χ PT describe the data well
 - $\kappa_{\text{phys}} = 3.61$ and $\langle r_2^2 \rangle_{\text{phys}} = 0.668 \text{ fm}^2$
 - values in line with the HB and the experimental ones
- F_A
 - Successful description of LQCD $F_A(q^2)$ using $\mathcal{O}(p^4)$ relativistic χ PT
 - Fit describes data in $M_\pi^{\text{cut}} \simeq 400 \text{ MeV}$, $Q_{\text{cut}}^2 = 0.36 \text{ GeV}^2$
 - There is tension between the experimental and lattice extraction of $\langle r_A^2 \rangle$
 - We extract $\langle r_A^2 \rangle^{\text{phys}} = 0.291 \pm 0.052 \text{ fm}^2$ without ad hoc parametrisations
 - Useful LEC values extracted from both calculations

· EM f.f.

$$\cdot V^\mu = \bar{q} Q \gamma^\mu q,$$

$$\langle N(p') | V^\mu(0) | N(p) \rangle = \bar{u}' \left[\gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m_N} F_2(q^2) \right] u$$

$$\cdot G_E^N = F_1^N - \frac{Q^2}{4m_N^2} F_2^N, \quad G_M^N = F_1^N + F_2^N$$

$$\cdot F_1 = 1 \times \left[1 + \frac{1}{6} \langle r_1^2 \rangle q^2 + \mathcal{O}(q^4) \right]$$

$$F_1(0) = F_E(0) = 1 \text{ electric charge}$$

$$\cdot F_2 = \kappa \left[1 + \frac{1}{6} \langle r_2^2 \rangle q^2 + \mathcal{O}(q^4) \right]$$

$$\kappa = F_M(0) = \mu - 1 \text{ anom. magn. mom.}$$

$$\cdot \langle r_E^2 \rangle = \langle r_1^2 \rangle + \frac{3}{2m_N^2} \kappa \text{ proton radius puzzle}$$

$$\cdot F_i = \frac{1}{2} F_i^{(s)} + F_i^{(v)} \frac{\sigma^3}{2}$$

$$\cdot F_1^{(v)} = 1 \times \left[1 + \frac{1}{6} \langle r_1^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$$

$$F_1^{(v)}(0) = F_E^{(v)}(0) = 1 \text{ electric charge}$$

$$\cdot F_2^{(v)} = \kappa^{(v)} \left[1 + \frac{1}{6} \langle r_2^2 \rangle^{(v)} q^2 + \mathcal{O}(q^4) \right]$$

$$\kappa^{(v)} = F_M^{(v)}(0) = \mu^{(v)} - 1 \text{ magnetic moment}$$

$$\cdot \langle r_E^2 \rangle^{(v)} = \langle r_1^2 \rangle^{(v)} + \frac{3}{2m_N^2} \kappa^{(v)} \text{ proton radius puzzle}$$

	Disp (prediction)	χ PT	Disp+ χ PT
$d_6(\mu = m_\rho)$ (GeV $^{-2}$)	-	0.074 ± 0.010	0.416 ± 0.010
$d_6(\mu = m_N)$ (GeV $^{-2}$)	-	-0.422 ± 0.010	0.155 ± 0.010
χ^2/dof	$108.9/47 = 2.32$	$73.9/(47 - 1) = 1.61$	$24.6/(47 - 1) = 0.53$
$\langle r_1^2 \rangle_{\text{phys}}$ (fm 2)	0.4541	0.3626 ± 0.0047	0.4838 ± 0.0047

	$\mathcal{O}(p^3) \Delta$	$\mathcal{O}(p^4) \Delta$	$\mathcal{O}(p^3) \Delta$	$\mathcal{O}(p^4) \Delta$
$\tilde{g}_A$ (free)	1.1782 ± 0.0073		1.2041 ± 0.0074	1.274 ± 0.041
d_{16} (GeV $^{-2}$) (free)	-1.021 ± 0.048		0.983 ± 0.062	-1.46 ± 1.00
d_{22} (GeV $^{-2}$) (free)	1.275 ± 0.086		3.77 ± 1.96	0.29 ± 1.69 (free g_1)
h_A	-	-	1.35	1.35
g_1 (free)	-	-	-0.69 ± 0.69	0.66 ± 0.56
c_1 (GeV $^{-1}$)	-	-0.89 ± 0.06	-	-1.15 ± 0.05
c_2 (GeV $^{-1}$)	-	3.38 ± 0.15	-	1.57 ± 0.10
c_3 (GeV $^{-1}$)	-	-4.59 ± 0.09	-	-2.54 ± 0.05
c_4 (GeV $^{-1}$)	-	3.31 ± 0.13	-	2.61 ± 0.10
d_1 (GeV $^{-1}$)	-	-	-	0.90
b_1 (GeV $^{-2}$) (free)	-	-	-	-0.27 ± 4.96
b_2 (GeV $^{-2}$) (free)	-	-	-	2.27 ± 2.28
$\tilde{b}_4$ (GeV $^{-2}$) (free)	-	-	-	-12.48 ± 1.28
x_1 (fm $^{-2}$) (free)	-8.4 ± 5.8	-	-5.6 ± 5.9	-0.25 ± 16.5 (consistent)
x_2 (fm $^{-2}$) (free)	-8.6 ± 2.6	-	-7.1 ± 2.6	-6.36 ± 4.20
x_3 (fm $^{-1}$) (free)	-0.25 ± 0.21	-	-0.08 ± 0.22	0.36 ± 0.47
y_1 (fm $^{-2}$ GeV $^{-2}$) (free)	-100 ± 40	-	-76 ± 44	-64 ± 121
y_2 (fm $^{-2}$ GeV $^{-2}$) (free)	-31 ± 21	-	-21 ± 22	-15 ± 46
y_3 (fm $^{-1}$ GeV $^{-2}$) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\tilde{m}$ (GeV)	0.874	0.874	0.855	0.855
$\tilde{m}_\Delta$ (GeV)	-	-	1.166	1.166

	$\mathcal{O}(p^3) \Delta$	$\mathcal{O}(p^4) \Delta$	$\mathcal{O}(p^3) \Delta$	$\mathcal{O}(p^4) \Delta$
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a_1 (GeV^{-1})	-	-	-	0.90
b_1 (GeV^{-2}) (free)	-	-	-	-0.27 ± 4.96
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y_3 ($\text{fm}^{-1} \text{GeV}^{-2}$) (free)	-0.63 ± 1.49	-	0.36 ± 1.63	2.54 ± 3.98
$\tilde{m}$ (GeV)	0.874	0.874	0.855	0.855
$\tilde{m}_\Delta$ (GeV)	-	-	1.166	1.166
χ^2_0/dof	$857.31/(127 - 9) = 7.27$		$533.87/(127 - 10) = 4.45$	$196.58/(127 - 13) = 1.724$