Flavour, Higgs, $g-2$

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IFIC Scientific Day, L3:
flavour and quark matter
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Outline

- Part I – $(g-2)_{e,\mu}$
  1. Motivation, 2HDMs I-gℓFC and II-gℓFC
  2. New contributions and solution regions
  3. Constraints and results
- Part II – Spontaneous CP violation and flavour conservation
  1. Motivation and the "CP conserving argument"
  2. Counterexample and general aspects
  3. Dirac neutrino example, μ–τ symmetric PMNS

Based on work done in collaboration with:
Part I: F.J. Botella, F. Cornet-Gómez & C. Miró

☞ arXiv:2302.05471, JPhysG51 (2024)
☞ arXiv:2205.01115, EPJC82 (2022)
☞ arXiv:2006.01934, PRD102 (2020)

Part II: J.A. Alves, F.J. Botella & C. Miró

☞ arXiv:2306.14952, EPJC83 (2023)

I Motivation: $(g-2)_{e,\mu}$

"Anomalies" in the anomalous magnetic moments of μ and e

$$\delta a_\mu^{\text{Exp}} \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = +(2.5 \pm 0.6) \times 10^{-9}$$

Muon $g-2$ collaboration, *Phys. Rev. Lett. 126 (2021) 14*

$$\delta a_e^{\text{Exp,Cs}} \equiv a_e^{\text{Exp,Cs}} - a_e^{\text{SM}} = -(8.7 \pm 3.6) \times 10^{-13}$$

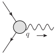
Parker et al. *Science (2018) 360:191*, α from ^{133}Cs recoil

Also

$$\delta a_e^{\text{Exp,Rb}} \equiv a_e^{\text{Exp,Rb}} - a_e^{\text{SM}} = +(4.8 \pm 3.0) \times 10^{-13}$$

Morel et al. *Nature (2020) 588:61*, α from ^{87}Rb recoil

N.B. $a_\ell = (g_\ell - 2)/2$

Vertex $\bar{\ell}\ell A^\mu$  $\gamma^\mu \to \Gamma^\mu = \gamma^\mu F_1(q^2) + i\dfrac{\sigma^{\mu\nu}q_\nu}{2m_\ell}F_2(q^2),\ a_\ell = F_2(0)$

I Motivation: $(g-2)_{e,\mu}$

$$\frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}} \simeq -\left(\frac{m_e}{m_\mu}\right)^{1.494}$$

- $\delta a_\mu^{\text{Exp}}$ and $\delta a_e^{\text{Exp,Cs}}$ have *opposite* signs!
- Not only the sign,
  - if NP model gives $\delta a_\ell \propto m_\ell$

$$\frac{\delta a_e}{\delta a_\mu} \sim \frac{m_e}{m_\mu} = \left(\frac{m_e}{m_\mu}\right)^{-0.494} \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}} \simeq -13.9\, \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}}$$

  - if NP model gives $\delta a_\ell \propto m_\ell^2$

$$\frac{\delta a_e}{\delta a_\mu} \sim \left(\frac{m_e}{m_\mu}\right)^2 = \left(\frac{m_e}{m_\mu}\right)^{0.506} \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}} \simeq -\frac{1}{14.8}\, \frac{\delta a_e^{\text{Exp,Cs}}}{\delta a_\mu^{\text{Exp}}}$$

✎ Serious obstacle for many New Physics solutions

I Motivation: $(g-2)_{e,\mu}$

- If the origin of both anomalies is beyond SM, some sort of *effective decoupling* between e and μ should be in place
- 2 Higgs Doublets Models (2HDMs) incorporate *new flavour structures* that can implement that property but
  - not in symmetry-shaped 2HDMs of types I, II, X, Y
    (new couplings proportional to masses)
  - not in "aligned 2HDMs" (proportionality to masses again)
  - maybe in general flavour conserving 2HDMs (gFC-2HDMs)! ✍

2HDMs

- In 2HDMs the Yukawa sector is

$$\mathscr{L}_{\rm Y} = -\overline{Q}_L^0 \left(\Phi_1 Y_{d1} + \Phi_2 Y_{d2}\right) d_R^0 - \overline{Q}_L^0 \left(\tilde{\Phi}_1 Y_{u1} + \tilde{\Phi}_2 Y_{u2}\right) u_R^0 - \overline{L}_L^0 \left(\Phi_1 Y_{\ell 1} + \Phi_2 Y_{\ell 2}\right) \ell_R^0 + \text{H.c.}$$

N.B. $\tilde{\Phi}_j = i\sigma_2 \Phi_j^*$, neutrinos are massless

- Going to the Higgs and fermion mass bases

$$\mathscr{L}_{\rm Y} = -\frac{\sqrt{2}}{v}\bar{Q}_L \left(H_1 \mathrm{M}_d + H_2 \mathrm{N}_d\right) d_R - \frac{\sqrt{2}}{v}\bar{Q}_L \left(\tilde{H}_1 \mathrm{M}_u + \tilde{H}_2 \mathrm{N}_u\right) u_R - \frac{\sqrt{2}}{v}\bar{L}_L \left(H_1 \mathrm{M}_\ell + H_2 \mathrm{N}_\ell\right) \ell_R + \text{H.c.}$$

where

- M_f are the diagonal fermion mass matrices
- N_f are the new flavour structures

(the ones that may explain the anomalies!)

Higgs basis

- Expansion around vacuum appropriate for electroweak symmetry breaking

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{v_j + \rho_j + i\eta_j}{\sqrt{2}} \end{pmatrix}, \quad \langle \Phi_j \rangle = \frac{e^{i\theta_j} v_j}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Higgs basis, $c_\beta \equiv \cos\beta = \frac{v_1}{v}$, $s_\beta \equiv \sin\beta = \frac{v_2}{v}$, $t_\beta \equiv \tan\beta$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix}, \quad \text{with} \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \; \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

$$\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad v^2 = v_1^2 + v_2^2 = \frac{1}{\sqrt{2} G_F}$$

I 2HDMs

- Higgs basis

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+H^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \mathrm{H}^+ \\ \frac{R^0+iI^0}{\sqrt{2}} \end{pmatrix}$$

  - would-be Goldstone bosons G^0, $G^\pm$
  - physical charged scalar $H^\pm$
  - neutral scalars $\{H^0, R^0, I^0\}$, not the mass eigenstates

The I-gℓFC and II-gℓFC models

Finally

- Model I-gℓFC is defined by

$$\mathrm{N}_u = t_\beta^{-1}\mathrm{M}_u, \quad \mathrm{N}_d = t_\beta^{-1}\mathrm{M}_d, \quad \mathrm{N}_\ell = \mathrm{diag}(n_e, n_\mu, n_\tau)$$

  The couplings $\mathrm{N}_u, \mathrm{N}_d$ are the same as in 2HDMs of types I or X

- Model II-gℓFC is defined by

$$\mathrm{N}_u = t_\beta^{-1}\mathrm{M}_u, \quad \mathrm{N}_d = -t_\beta\mathrm{M}_d, \quad \mathrm{N}_\ell = \mathrm{diag}(n_e, n_\mu, n_\tau)$$

  The couplings $\mathrm{N}_u, \mathrm{N}_d$ are the same as in 2HDMs of types II or Y

- N_ℓ is diagonal, arbitrary and one loop RGE stable
- Effective decoupling among new e and μ couplings required for the $g-2$ anomalies $\leftrightarrow$ independence of n_e and n_μ

Completing the scalar sector

- since the quark sector is a type I or type II 2HDM, adopt a $\mathbb{Z}_2$ symmetric scalar potential

$$\mathcal{V}(\Phi_1,\Phi_2) = \mu_{11}^2\Phi_1^\dagger\Phi_1 + \mu_{22}^2\Phi_2^\dagger\Phi_2 + \left(\mu_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c.}\right) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2$$
$$+\, 2\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + 2\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left(\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{H.c.}\right)$$

$\mu_{12}^2 \neq 0 \Rightarrow$ softly broken $\mathbb{Z}_2$ symmetry

- Mass matrix of the neutral scalars $\mathcal{M}_0^2$, diagonalised by a 3×3 real orthogonal matrix $\mathcal{R}$

$$\mathcal{R}^T\,\mathcal{M}_0^2\,\mathcal{R} = \text{diag}(m_{\text{h}}^2, m_{\text{H}}^2, m_{\text{A}}^2)\,,\quad \mathcal{R}^{-1} = \mathcal{R}^T$$

- Physical neutral scalars $\{\text{h},\text{H},\text{A}\}$:

$$\begin{pmatrix}\text{h}\\ \text{H}\\ \text{A}\end{pmatrix} = \mathcal{R}^T\begin{pmatrix}H^0\\ R^0\\ I^0\end{pmatrix}$$

The I-gℓFC and II-gℓFC models

- Further simplifications
  1. the new Yukawa couplings are real, $\mathrm{Im}\,(n_\ell) = 0$
  2. there is no CP violation in the scalar sector,

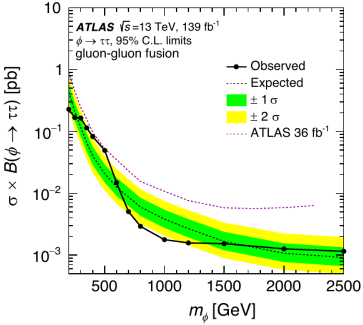
$$\mathcal{R} = \begin{pmatrix} s_{\alpha\beta} & -c_{\alpha\beta} & 0 \\ c_{\alpha\beta} & s_{\alpha\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \begin{cases} s_{\alpha\beta} \equiv \sin(\alpha-\beta) \\ c_{\alpha\beta} \equiv \cos(\alpha-\beta) \end{cases}$$

$\alpha - \frac{\pi}{2}$: mixing angle in $\{\rho_j, \eta_j\} \to \{G^0, \mathrm{h}, \mathrm{H}, \mathrm{A}\}$

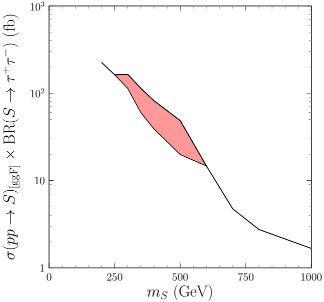
  - h, H are *scalars* and A *pseudoscalar*
  - h is assumed to be the 125 GeV SM-like Higgs

I Motivation: the τ hint

- We have 3 new couplings n_e, n_μ, n_τ,
  and 2 "New Physics observations", δa_e, δa_μ
- …is there some other "New Physics hint" for the τ? YES

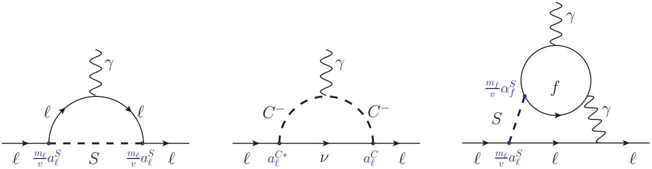


$\rightarrow$

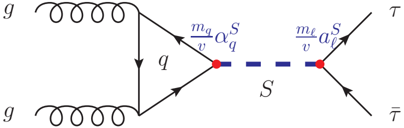


ATLAS collaboration, *Phys. Rev. Lett. 125 (2020) 051801*, 2002.12223

- New contributions to δa_ℓ at one loop and at two loops (Barr-Zee)



- $[pp]_{\mathrm{ggF}} \to S \to \tau\bar{\tau}$



The new contributions to δa_ℓ

Two types of solutions

- ✎ "Solution [A]/heavy": scalars with masses in the 1–2 TeV range, $t_\beta \sim 1$, and $(g-2)_{e,\mu}$ anomalies produced by two loop Barr-Zee contributions.
  $\mathrm{Re}\,(n_e)$ in the few GeV range, $\mathrm{Re}\,(n_\mu) \sim -15\mathrm{Re}\,(n_e)$
  Solution a priori present in *both* I-gℓFC and II-gℓFC
- ✎ "Solution [B]/light": $t_\beta \gg 1$, lighter H, $m_{\mathrm{H}} \in [200; 400]$ GeV, and a heavier A. δa_e is obtained with two loop contributions while δa_μ is one loop controlled. Contrary to solution [A], there is *no* linear relation among $\mathrm{Re}\,(n_\mu)$ and $\mathrm{Re}\,(n_e)$, and in fact both signs of $\mathrm{Re}\,(n_\mu)$ can work.
- \+ "Intermediate regions"
- ? Impact of the $[pp]_{\mathrm{ggF}} \to S \to \tau\bar{\tau}$ excess

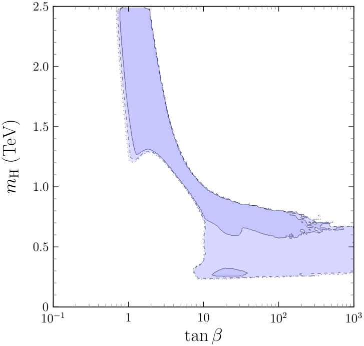
N.B. Solution [B] available in the I-gℓFC model, but not in II-gℓFC.

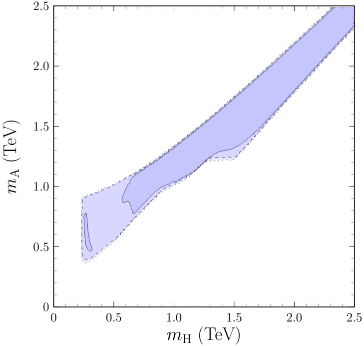
I Constraints for full numerical analysis

Shopping list

- $\delta a_\ell^{\mathrm{Exp}}$ constraints
- $[pp]_{\mathrm{ggF}} \to S \to \tau\bar{\tau}$ excess
- Scalar sector
- Fermion sector
- Higgs signal strengths
- $\mathrm{H}^\pm$ mediated contributions
  - Lepton flavour universality
  - $b \to s\gamma$, B_q^0–$\bar{B}_q^0$ mixing
- $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ at LEP
- LHC searches
  - searches of dilepton resonances: $\sigma(pp \to S)_{[\mathrm{ggF}]} \times \mathrm{Br}\left(S \to \ell^+\ell^-\right)$, $S = \mathrm{H}, \mathrm{A}$ and $\ell = \mu, \tau$
  - searches of charged scalars: $\sigma(pp \to \mathrm{H}^\pm tb) \times \mathrm{Br}\left(\mathrm{H}^\pm \to f\right)$, $f = \tau\nu, tb$
- CDF W mass shift (optional)

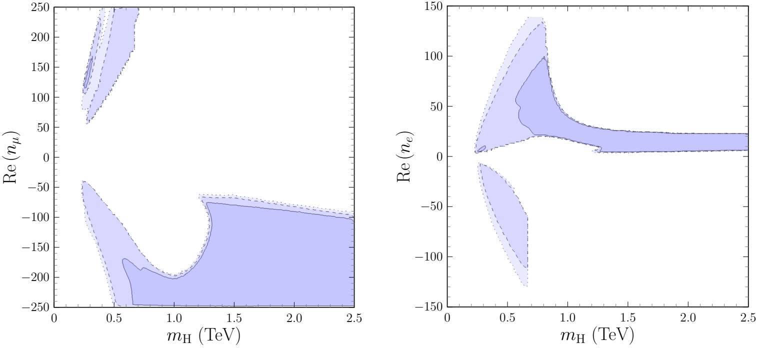
Results, model I-gℓFC – without $\tau\bar{\tau}$ excess





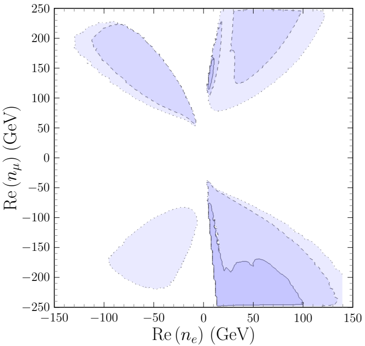
Scalar masses & t_β

I Results, model I-gℓFC – without $\tau\bar{\tau}$ excess



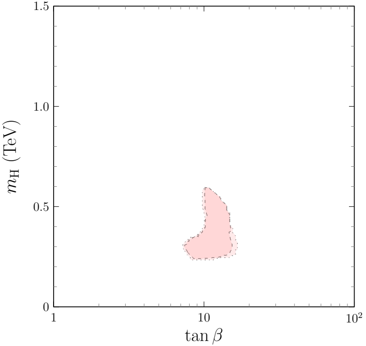
$n_{e,\mu}$ (GeV) & m_H

Results, model I-gℓFC – without $\tau\bar{\tau}$ excess



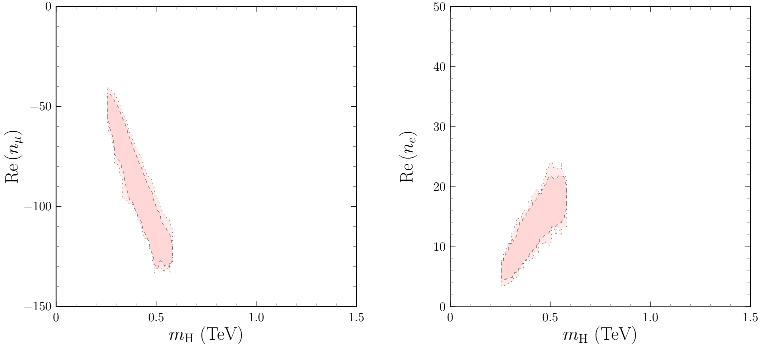
n_ℓ couplings

Results, model I-gℓFC – with $\tau\bar{\tau}$ excess



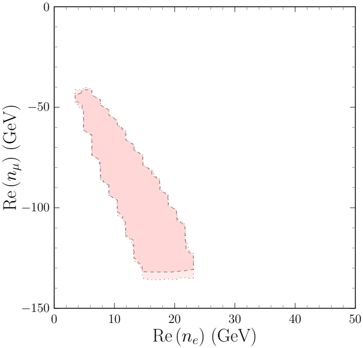
Scalar masses & t_β

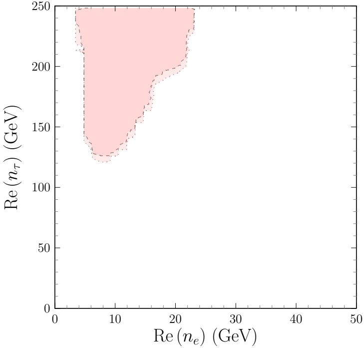
Results, model I-gℓFC – with $\tau\bar{\tau}$ excess



$n_{e,\mu}$ (GeV) & m_{H}

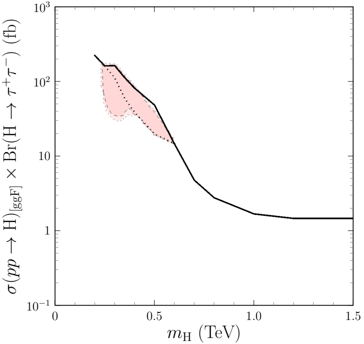
Results, model I-gℓFC – with $\tau\bar{\tau}$ excess





n_ℓ couplings

Results, model I-gℓFC – with $\tau\bar{\tau}$ excess



$\tau\bar{\tau}$ excesses

I Conclusions – Part I

- General 2HDMs without SFCNC in the lepton sector are a robust framework (stable under RGE)
- Lepton flavour universality is broken beyond $\propto$ mass
- Two models, I-gℓFC & II-gℓFC, to address the δa_ℓ anomalies
- Quark & scalar sector as type I, II 2HDMs, with softly broken $\mathbb{Z}_2$
- Solutions in agreement with constraints without $\tau\bar{\tau}$ excess
  1. "Heavy", present in both models
     - new scalars have masses in the 1–2.5 TeV range,
     - $v_1 \sim v_2$,
     - both δa_ℓ from two loop Barr-Zee contributions
  2. "Light", present in I-gℓFC, not in II-gℓFC
     - new scalars have masses below 1 TeV,
     - $v_1 \ll v_2$,
     - δa_e from two loop Barr-Zee contributions, δa_μ from one loop
  3. "Intermediate" regions

I Conclusions – Part I

- Including $[pp]_{\mathrm{ggF}} \to S \to \tau\bar{\tau}$ excess
  - obtained with H, or A, or both (two $\neq$ excesses within the region)
  - selects the "Intermediate"-"Light" region
  - only model I-gℓFC can do the work
- Involved numerical analysis (subtleties & unexpected regions)
  - Gluon-gluon fusion production of scalar vs pseudoscalar
  - Role of A $\to$ HZ
  - τ loop in Barr-Zee contributions
- Extra balls
  - Role of different $\delta a_e^{\mathrm{Exp}}$ values
  - CDF M_W through ΔS and ΔT
  - $(g-2)_\tau$
  - Example points

II Motivation

- In the context of 2HDM, a model with
  - CP invariant lagrangian, in particular real Yukawa coupling matrices,
  - spontaneous CP violation producing sourcing all CP violation, hfill including a realistic CP violating CKM matrix,
  - controlled SFCNC, if absent, CKM is not CP violating

MN, F.J. Botella & G.C. Branco

☞ `arXiv:1808.00493`, EPJC79 (2019)

The "CP conserving mixing" argument

Flavour conservation means that the matrices M_q^0 *and* N_q^0 , $q = u, d$, *are simultaneously bidiagonalized. This is equivalent to* $Y_1^{(\mathrm{q})}$ *and* $Y_2^{(\mathrm{q})}$ *being bidiagonalized simulaneously.* $Y_1^{(\mathrm{q})}$ *and* $Y_2^{(\mathrm{q})}$ *are real, and thus the bidiagonalization is achieved with real orthogonal matrices,* $O_{q_L}^T Y_j^{(\mathrm{q})} O_{q_R} = \mathrm{diag}(y_{j1}^{(\mathrm{q})}, y_{j2}^{(\mathrm{q})}, y_{j3}^{(\mathrm{q})})$, $y_{jk}^{(\mathrm{q})} \in \mathbb{R}$, *implying that* $M_q = O_{q_L}^T M_q^0\, O_{q_R}$ *and* $N_q = O_{q_L}^T N_q^0\, O_{q_R}$ *are diagonal. Then, the CKM matrix is* $V = R_U\, O_{u_L}^T O_{d_L}\, R_D$ *with* R_U, R_D *diagonal rephasing matrices, which can be absorbed in a redefinition of the fields: the CKM matrix is thus essentially real, not CP violating.*

G.C. Branco ☞ PRL44 (1980)

II Motivation

The "CP conserving mixing" argument is convincing **but** it has a loophole: even if $Y_1^{(\mathrm{q})}$ and $Y_2^{(\mathrm{q})}$ are real,

- they can have complex eigenvalues and in that case they are not necessarily bidiagonalised simultaneously with real orthogonal matrices

Counterexample

G. Ecker, W. Grimus & H. Neufeld, ☞ PLB194 (1987)

Complex conjugate eigenvalues and mixing moduli relations

M. Gronau, A. Kfir, G. Ecker, W. Grimus & H. Neufeld,

☞ PRD37 (1988)

II Counterexample

- Model with 2HDM and 4 generations "out of the blue"
  G. Ecker, W. Grimus & H. Neufeld, ☞ PLB194 (1987)
  Yukawa matrices

$$Y_j^{(\mathrm{d})} = \mathrm{diag}(y_{j1}^{(\mathrm{d})}, y_{j2}^{(\mathrm{d})}, y_{j3}^{(\mathrm{d})}, y_{j4}^{(\mathrm{d})}), \quad Y_j^{(\mathrm{u})} = O^T \begin{pmatrix} y_{j1}^{(\mathrm{u})} & 0 & 0 & 0 \\ 0 & y_{j2}^{(\mathrm{u})} & 0 & 0 \\ 0 & 0 & a_j & b_j \\ 0 & 0 & -b_j & a_j \end{pmatrix}$$

with real $Y_j^{(\mathrm{d})}$, $Y_j^{(\mathrm{u})}$ (O orthogonal)

- Crucial ingredient: the blocks

$$B_j = \begin{pmatrix} a_j & b_j \\ -b_j & a_j \end{pmatrix}$$

(N.B. $b_1/a_1 \neq b_2/a_2$, otherwise degenerate mass eigenstates)

Special blocks

- they obey (no sum over j)

$$B_j B_j^T = B_j^T B_j = (a_j^2 + b_j^2)\mathbf{1}_2$$

- B_j has two *complex conjugate* eigenvalues $a_j \pm i b_j$ while $B_j B_j^T$ have two *degenerate* eigenvalues $a_j^2 + b_j^2$
- The simultaneous real orthogonal bidiagonalization of both $Y_1^{(\mathrm{u})}$ and $Y_2^{(\mathrm{u})}$ *fails*
- However

$$U^\dagger B_j U = \begin{pmatrix} a_j + i b_j & 0 \\ 0 & a_j - i b_j \end{pmatrix}, \quad \text{with} \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

II Counterexample

- $Y_1^{(\mathrm{u})}$ and $Y_2^{(\mathrm{u})}$ are simultaneously diagonalized *unitarily*

$$U_{u_L}^\dagger Y_j^{(\mathrm{u})} U_{u_R} = \mathrm{diag}(y_{1j}^{(\mathrm{u})}, y_{2j}^{(\mathrm{u})}, a_j + ib_j, a_j - ib_j)$$

$$U_{u_L} = O^T\, U_{34}\,,\ U_{u_R} = U_{34}, \quad U_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix}$$

- The resulting CKM matrix, up to rephasings, is

$$V = U_{34}^\dagger\, O$$

- It follows that $V_{3j} = V_{4j}^* = (O_{3j} - iO_{4j})/\sqrt{2}$, i.e. the rephasing invariant relation

$$|V_{3j}| = |V_{4j}|, \quad j = 1, 2, 3, 4$$

II Counterexample

No real orthogonal simulaneously bidiagonalisation

- Notice that

$$B_j = \begin{pmatrix} a_j & b_j \\ -b_j & a_j \end{pmatrix} = \lambda_j \begin{pmatrix} \cos\theta_j & \sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{pmatrix}$$

with $\lambda_j = \sqrt{a_j^2 + b_j^2}$, $\frac{a_j}{\lambda_j} = \cos\theta_j$, $\frac{b_j}{\lambda_j} = \sin\theta_j$

B_1 and B_2 not proportional $\Leftrightarrow \theta_2 \neq \theta_1\ [\pi]$

- $SO(2,\mathbb{R})$

$$O_2(\theta) \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$[O_2(\theta)]^{-1} = [O_2(\theta)]^T = O_2(-\theta), \quad O_2(\theta_a)O_2(\theta_b) = O_2(\theta_a + \theta_b)$$

- Orthogonal bidiagonalisation

$$O_2(-\theta_L)B_jB_j^TO_2(\theta_L) = O_2(-\theta_R)B_jB_j^TO_2(\theta_R) = \lambda_j^2\mathbf{1}_2$$

it looks like we have full freedom to choose θ_L and θ_R

II Counterexample

No real orthogonal simulaneously bidiagonalisation

- It looks like we have full freedom to choose θ_L and θ_R ... but

$$O_2(-\theta_L)\, B_1\, O_2(\theta_R) = \lambda_1\, O_2(-\theta_L + \theta_1 + \theta_R)$$

$$O_2(-\theta_L)\, B_2\, O_2(\theta_R) = \lambda_2\, O_2(-\theta_L + \theta_2 + \theta_R)$$

If B_1 bidiagonalised, $-\theta_L + \theta_1 + \theta_R = 0[\pi]$,
but then $-\theta_L + \theta_2 + \theta_R \neq 0[\pi]$ i.e. B_2 not bidiagonalised

II General analysis

In

$$U^\dagger \begin{pmatrix} \cos\theta_j & \sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{pmatrix} U = \begin{pmatrix} e^{i\theta_j} & 0 \\ 0 & e^{-i\theta_j} \end{pmatrix}, \quad \text{with} \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

important points:

- eigenvectors $\vec{v}_+ = \frac{1}{\sqrt{2}} \left(\begin{smallmatrix} 1 \\ i \end{smallmatrix}\right)$, $\vec{v}_- = (\vec{v}_+)^*$, corresponding to eigenvalues $e^{\pm i\theta}$, are orthonormal

  $\Rightarrow$ unitary diagonalisation exists
- eigenvectors "independent" of the eigenvalues

  (unique diagonalisation for all $SO(2,\mathbb{R})$ matrices)

With three generations in mind, notice that $O(3,\mathbb{R})$ matrices

- have two complex conjugate eigenvalues $e^{\pm i\alpha}$ and one real eigenvalue ± 1
- orthonormal eigenvectors ($\Rightarrow$ unitary diagonalisation exists), two are complex conjugate of each other
- the eigenvectors do not depend on the eigenvalues
  $\Rightarrow$ different $O(3,\mathbb{R})$ matrices can be diagonalised simultaneously

II Model with Dirac neutrinos

- Yukawa lagrangian

$$-\mathscr{L}_{Y\ell} = \overline{L}_L^0 \left(Y_1^{(\nu)} \tilde{\Phi}_1 + Y_2^{(\nu)} \tilde{\Phi}_2 \right) \nu_R^0 + \overline{L}_L^0 \left(Y_1^{(\ell)} \Phi_1 + Y_2^{(\ell)} \Phi_2 \right) \ell_R^0 + \text{H.c.}$$

with

$$Y_j^{(\nu)} = \text{diag}(y_{j1}^{(\nu)}, y_{j2}^{(\nu)}, y_{j3}^{(\nu)})$$

$$Y_j^{(\ell)} = O^T \lambda_j \begin{pmatrix} y_{j1}^{(\ell)}/\lambda_j & 0 & 0 \\ 0 & \cos\varphi_j & \sin\varphi_j \\ 0 & -\sin\varphi_j & \cos\varphi_j \end{pmatrix}$$

$$\text{M}_\nu^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}} \left(c_\beta Y_1^{(\nu)} + s_\beta e^{-i\theta} Y_1^{(\nu)} \right), \quad \text{M}_\ell^0 = \frac{v e^{i\theta_1}}{\sqrt{2}} \left(c_\beta Y_1^{(\ell)} + s_\beta e^{i\theta} Y_2^{(\ell)} \right)$$

$$\text{N}_\nu^0 = \frac{v e^{-i\theta_1}}{\sqrt{2}} \left(-s_\beta Y_1^{(\nu)} + c_\beta e^{-i\theta} Y_2^{(\nu)} \right), \quad \text{N}_\ell^0 = \frac{v e^{i\theta_1}}{\sqrt{2}} \left(-s_\beta Y_1^{(\ell)} + c_\beta e^{i\theta} Y_2^{(\ell)} \right)$$

II Model with Dirac neutrinos

- Simultaneous diagonalisation of $\{Y_1^{(\ell)}, Y_2^{(\ell)}\} \Leftrightarrow \{\mathrm{M}_\ell^0, \mathrm{N}_\ell^0\}$

$$U_{\ell_L}^\dagger\, \mathrm{M}_\ell^0\, U_{\ell_R} = \mathrm{M}_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$U_{\ell_L} = O^T\, U_{23} \qquad U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix}$$

$$U_{\ell_R} = U_{23} R_{\ell_R}$$

(N.B. rephasings R_{ℓ_R})

- M_ν^0 and N_ν^0 diagonal
- PMNS matrix (up to rephasings)

$$U = U_{23}^\dagger O$$

and then

$$U_{2j} = U_{3j}^* = \frac{1}{\sqrt{2}}(O_{2j} - iO_{3j}) \;\Rightarrow\; |U_{2j}| = |U_{3j}|,\; j = 1, 2, 3$$

II Model with Dirac neutrinos

- With
$$|U_{2j}| = |U_{3j}|,\ j = 1, 2, 3$$
the PMNS matrix has "$\mu - \tau$ symmetry"
(in standard PDG parameterisation $\theta_{23} = \pi/4$, $\delta = \pm\pi/2$)
- Freedom left in O_{1j} sufficient to have a realistic PMNS

II Model with Dirac neutrinos

Final comments

- CP violation in PMNS is in the end independent of the vacuum SCPV phase (!)
- This choice (generations 2 and 3 of charged leptons) is the only viable implementation of the idea, no other rows or columns could work
- Diagonalisation of the neutrino mass matrix has had almost no role ($\Rightarrow$ easy to do Majorana neutrinos with type I seesaw)

This is the basic picture, for further details, including phenomenological aspects, RGE, type I seesaw Majorana neutrinos: J.A. Alves, F.J. Botella, C. Miró & MN

☞ `arXiv:2306.14952`, EPJC83 (2023)

Conclusions – Part II

- In 2HDMs (or nHDM) one can reconcile flavour conservation and a spontaneous origin of all CP violation (including fermion mixing)
- However: OK for leptons but not OK for quarks
- Essentially only one implementation is phenomenologically viable (simple illustration with Dirac neutrinos, straightforward extension to type I seesaw Majorana neutrinos)
- This implementation implies a PMNS with μ–τ symmetry

Thank you!

Backup

- 2HDMs
- δa_ℓ in detail
- Constraints
- Scalar vs pseudoscalar gluon-gluon fusion
- M_W from CDF
- δa_τ
- Example points
- Different $\delta a_e^{\mathrm{Exp}}$
- Yukawa couplings
- $O(3,\mathbb{R})$ matrices
- μ–τ symmetry with $O(3,\mathbb{R})$ matrices

2HDMs

$$\mathscr{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\bar{Q}_L\left(H_1\mathrm{M}_d + H_2\mathrm{N}_d\right)d_R - \frac{\sqrt{2}}{v}\bar{Q}_L\left(\tilde{H}_1\mathrm{M}_u + \tilde{H}_2\mathrm{N}_u\right)u_R - \frac{\sqrt{2}}{v}\bar{L}_L\left(H_1\mathrm{M}_\ell + H_2\mathrm{N}_\ell\right)\ell_R + \mathrm{H.c.}$$

- Natural Flavour Conservation:
  only one Yukawa matrix $\neq 0$ in each sector
  $\mathbb{Z}_2$ symmetry, types I, II, X, Y, with $\mathrm{N}_f = \pm t_\beta^{\mp 1}\mathrm{M}_f$

  Glashow & Weinberg, *PRD15 (1977)*

- "Aligned" 2HDM: $\mathrm{N}_f = \zeta_f \mathrm{M}_f$

  Pich & Tuzón, *PRD80 (2009)*

  RGE: unstable quark sector, stable lepton sector

  Botella, Branco, Coutinho, Rebelo & Silva-Marcos, *EPJC75 (2015)*

- General flavour conserving: diagonal N_f

  Peñuelas & Pich, *JHEP 12 (2017)*

  RGE: unstable quark sector, stable lepton sector

  Botella, Cornet-Gómez & N *PRD98 (2018)*

The new contributions to δa_ℓ

- Full prediction

$$a_\ell^{\text{Th}} = a_\ell^{\text{SM}} + \delta a_\ell$$

a_ℓ^{SM}: SM contribution; δa_ℓ: corrections due to the model

- To solve the discrepancies, the aim is

$$\delta a_e \simeq \delta a_e^{\text{Exp,Cs}}, \qquad \delta a_\mu \simeq \delta a_\mu^{\text{Exp}}$$

within models I-gℓFC and II-gℓFC

- Introduce Δ_ℓ

$$\delta a_\ell = K_\ell \, \Delta_\ell, \qquad K_\ell = \frac{1}{8\pi^2}\left(\frac{m_\ell}{v}\right)^2 = \frac{1}{8\pi^2}\left(\frac{g m_\ell}{2M_W}\right)^2$$

K_ℓ are typical factors arising in one loop contributions

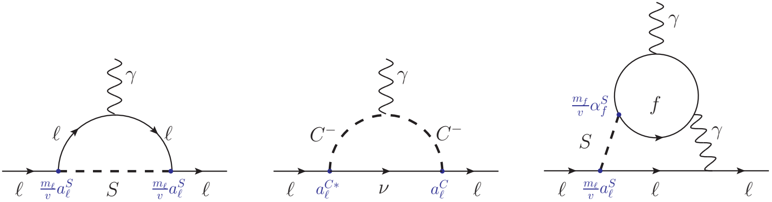
$$K_e \simeq 5.5 \times 10^{-14}, \qquad K_\mu \simeq 2.3 \times 10^{-9}$$

The new contributions to δa_ℓ

- With these values, $K_e \simeq 5.5 \times 10^{-14}$, $K_\mu \simeq 2.3 \times 10^{-9}$, we need

$$\Delta_e \simeq -16, \qquad \Delta_\mu \simeq 1$$

- Contributions at one loop and at two loops (Barr-Zee type) can be relevant



The new contributions to δa_ℓ

- To gain some insight consider the leading terms in $(m_\ell/m_\mathrm{S})^2$ of the one loop contributions in the alignment limit $s_{\alpha\beta} \to 1$

$$\Delta_\ell^{(1)} \simeq [\mathrm{Re}\,(n_\ell)]^2 \left(\frac{I_{\ell\mathrm{H}}}{m_\mathrm{H}^2} - \frac{I_{\ell\mathrm{A}} - 2/3}{m_\mathrm{A}^2} - \frac{1}{6 {m_{\mathrm{H}^\pm}}^2} \right)$$

where

$$I_{\ell\mathrm{S}} = -\frac{7}{6} - 2\ln\left(\frac{m_\ell}{m_\mathrm{S}}\right)$$

[N.B. Same in both models I-gℓFC & II-gℓFC]

- We do not consider light scalars/pseudoscalars,
  we assume $m_\mathrm{h} < m_\mathrm{H}, m_\mathrm{A}$
- Typical values of the loop function for $m_\mathrm{S} \in [0.2;\ 2.0]$ TeV

$$I_{e\mathrm{S}} \in [24.6; 29.2]\,, \qquad I_{\mu\mathrm{S}} \in [13.9; 18.5]$$

The new contributions to δa_ℓ

$$\Delta_\ell^{(1)} \simeq [\mathrm{Re}\,(n_\ell)]^2 \left(\frac{I_{\ell\mathrm{H}}}{m_\mathrm{H}^2} - \frac{I_{\ell\mathrm{A}} - 2/3}{m_\mathrm{A}^2} - \frac{1}{6 m_{\mathrm{H}^\pm}{}^2}\right)$$

$$I_{e\mathrm{S}} \in [24.6; 29.2]\,, \qquad I_{\mu\mathrm{S}} \in [13.9; 18.5]$$

- Dominant contributions from H and A (log enhanced), $\Delta_e^{(1)} \simeq -16$ can only come from A:

$$\Delta_e^{(1)} \simeq -[\mathrm{Re}\,(n_e)]^2 I_{e\mathrm{A}}/m_\mathrm{A}^2 \text{ requires } [\mathrm{Re}\,(n_e)]^2 \sim m_\mathrm{A}^2$$

  $\Rightarrow$ violate perturbativity requirements for Yukawa couplings or contraints from resonant dilepton searches

- ✎ We *do not* expect an explanation of $\delta a_e^{\mathrm{Exp,Cs}}$ in terms of one loop contributions

The new contributions to δa_ℓ

$$\Delta_\ell^{(1)} \simeq \mathrm{Re}\,(n_\ell)^2 \left(\frac{I_{\ell \mathrm{H}}}{m_{\mathrm{H}}^2} - \frac{I_{\ell \mathrm{A}} - 2/3}{m_{\mathrm{A}}^2} - \frac{1}{6 m_{\mathrm{H}^\pm}{}^2}\right)$$

$$I_{e\mathrm{S}} \in [24.6; 29.2]\,, \qquad I_{\mu \mathrm{S}} \in [13.9; 18.5]$$

- Dominant contributions from H and A (log enhanced), $\Delta_\mu^{(1)} \simeq 1$ can only come from H:

$$\Delta_\mu^{(1)} \simeq [\mathrm{Re}\,(n_\mu)]^2 I_{\mu\mathrm{H}}/m_{\mathrm{H}}^2 \text{ requires } [\mathrm{Re}\,(n_\mu)]^2 \sim [m_{\mathrm{H}}/4]^2$$

  $\Rightarrow$ a not too heavy H (reasonably perturbative n_μ)
  $\Rightarrow$ $m_{\mathrm{A}} > m_{\mathrm{H}}$ in order to avoid cancellations

✎ An explanation of $\delta a_\mu^{\mathrm{Exp}}$ in terms of one loop contributions *might be possible*

The new contributions to δa_ℓ

- Dominant two loop contributions: Barr-Zee diagrams
- In the same approximation (leading m_ℓ/m_{S} terms, $s_{\alpha\beta}\to 1$)

$$\Delta_\ell^{(2)} = -\left(\frac{2\alpha}{\pi}\right)\left(\frac{\mathrm{Re}\,(n_\ell)}{m_\ell}\right) F$$

F depends on

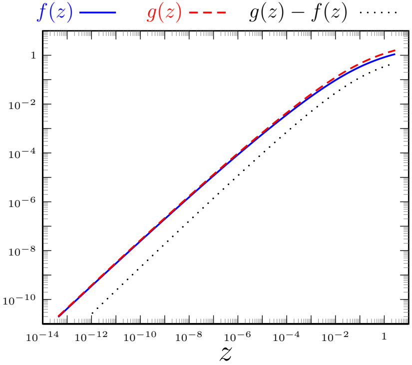
- masses of the fermions in the closed loop,
- couplings of those fermions to H and A,
- m_{H} and m_{A}

$$F_{\mathrm{I}} = \frac{\cot\beta}{3}\left[4(f_{t\mathrm{H}}+g_{t\mathrm{A}})+(f_{b\mathrm{H}}-g_{b\mathrm{A}})\right]+\frac{\mathrm{Re}\,(n_\tau)}{m_\tau}(f_{\tau\mathrm{H}}-g_{\tau\mathrm{A}})\,,$$

$$F_{\mathrm{II}} = \frac{\cot\beta}{3}\left[4(f_{t\mathrm{H}}+g_{t\mathrm{A}})-\tan^2\beta(f_{b\mathrm{H}}-g_{b\mathrm{A}})\right]+\frac{\mathrm{Re}\,(n_\tau)}{m_\tau}(f_{\tau\mathrm{H}}-g_{\tau\mathrm{A}})$$

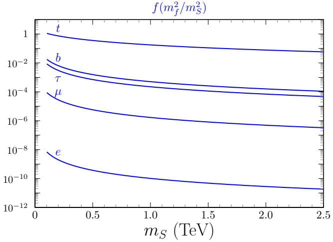
The new contributions to δa_ℓ

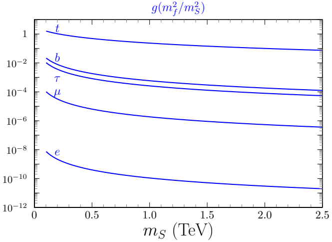
$$f_{\mathrm{fS}} \equiv f\left(\frac{m_{\mathrm{f}}^2}{m_{\mathrm{S}}^2}\right)\,, \quad g_{\mathrm{fS}} \equiv g\left(\frac{m_{\mathrm{f}}^2}{m_{\mathrm{S}}^2}\right)$$



The new contributions to δa_ℓ

$$f_{\mathrm{fS}} \equiv f\left(\frac{m_{\mathrm{f}}^2}{m_{\mathrm{S}}^2}\right)\,, \quad g_{\mathrm{fS}} \equiv g\left(\frac{m_{\mathrm{f}}^2}{m_{\mathrm{S}}^2}\right)$$





The new contributions to δa_ℓ

- Relevant aspects
  - $f(z) \simeq g(z)$ in the range of interest
  - the largest values correspond to the heavier fermion
  - the values of f and g for the top quark contributions vary between 0.1 and 1
- Considering the dominant top quark terms, for $t_\beta \simeq 1$ and $m_{\mathrm{H}} \simeq m_{\mathrm{A}}$, one can realize that for $m_{\mathrm{H}} \sim 1-2$ TeV,
  $\delta a_e^{\mathrm{Exp,Cs}}$ can be explained with $\mathrm{Re}\,(n_e) \sim 3-7$ GeV
- To obtain $\delta a_\mu^{\mathrm{Exp}}$ from the same type of contribution

$$\mathrm{Re}\,(n_\mu) = \frac{\delta a_\mu}{\delta a_e}\frac{m_e}{m_\mu}\,\mathrm{Re}\,(n_e) \simeq -15\mathrm{Re}\,(n_e)$$

  Different signs of δa_e and δa_μ $\rightarrow$ freedom to have opposite $\mathrm{Re}\,(n_e)$ and $\mathrm{Re}\,(n_\mu)$

  Same assumptions $t_\beta \sim 1$, $m_{\mathrm{A}} \sim m_{\mathrm{H}} \sim 1-2$ TeV $\rightarrow \mathrm{Re}\,(n_\mu) \in -[45;105]$ GeV

  Argument applies to both models I-gℓFC and II-gℓFC

The new contributions to δa_ℓ

Beyond $t_\beta \sim 1$

- $t_\beta \ll 1$ excluded in 2HDMs of types I and II by flavour constraints $\Rightarrow$ excluded in I-gℓFC and II-gℓFC as well
- What about $t_\beta \gg 1$ and δa_ℓ?
- The factor F

$$\Delta_\ell^{(2)} = -\left(\frac{2\alpha}{\pi}\right)\left(\frac{\mathrm{Re}\,(n_\ell)}{m_\ell}\right) F$$

is quite model dependent

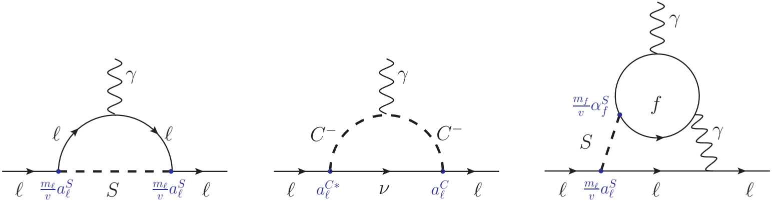
- We consider for reference $t_\beta \sim 1$ and $m_{\mathrm{A}} \sim m_{\mathrm{H}} \sim 1-2$ TeV, which can reproduce the anomalies, and analyse how to maintain that prediction if, for definiteness, $t_\beta \mapsto t_\beta = 50$

The new contributions to δa_ℓ

$$F_{\mathrm{I}} = \frac{\cot\beta}{3}\left[4(f_{t\mathrm{H}} + g_{t\mathrm{A}}) + (f_{b\mathrm{H}} - g_{b\mathrm{A}})\right] + \frac{\mathrm{Re}\,(n_\tau)}{m_\tau}(f_{\tau\mathrm{H}} - g_{\tau\mathrm{A}})$$

- In I-gℓFC, the $\cot\beta$ suppression can be compensated with *smaller* m_{H}, m_{A} and *larger* $\mathrm{Re}\,(n_e)$: e.g. $m_{\mathrm{A}} \sim m_{\mathrm{H}} \sim 200$ GeV gives a factor of 10 with respect to $m_{\mathrm{A}} \sim m_{\mathrm{H}} \sim 1-2$ TeV, $\mathrm{Re}\,(n_e) \mapsto 5\mathrm{Re}\,(n_e)$ required to fully compensate the factor of 50
- That is, $\delta a_e^{\mathrm{Exp,Cs}}$ can be reproduced by the two loop contributions in the $t_\beta \gg 1$ regime with light H, A and $\mathrm{Re}\,(n_e) \sim 15-35$ GeV
- What about δa_μ?
  $\mathrm{Re}\,(n_\mu) \mapsto 5\mathrm{Re}\,(n_\mu)$ gives $\mathrm{Re}\,(n_\mu) \in -[225; 505]$ GeV,
  in conflict with perturbativity requirements!
  Fortunately, for light m_{H}, e.g. $m_{\mathrm{H}} \in [200; 400]$ GeV and $|\mathrm{Re}\,(n_\mu)| \sim m_{\mathrm{H}}/4 \in [50; 100]$ GeV, the one loop contributions
  can reproduce $\delta a_\mu^{\mathrm{Exp}}$!

Loop contributions to δa_ℓ



One loop contributions to δa_ℓ

Yukawa interactions of the form

$$\mathscr{L}_{S\ell\ell} = -\frac{m_\ell}{v} S\bar{\ell}(a_\ell^S + i b_\ell^S \gamma_5)\ell$$

give one loop contributions

$$\Delta a_\ell^{(1)} = \frac{1}{8\pi^2}\frac{m_\ell^2}{v^2}\sum_S \left\{ [a_\ell^S]^2 \left(2 I_2(x_{\ell S}) - I_3(x_{\ell S})\right) - [b_\ell^S]^2 I_3(x_{\ell S}) \right\},$$

with $x_{\ell S} \equiv m_\ell^2/m_S^2$ and

$$I_2(x) = 1 + \frac{1-2x}{2x\sqrt{1-4x}} \ln\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right) + \frac{1}{2x}\ln x$$

$$I_3(x) = \frac{1}{2} + \frac{1}{x} + \frac{1-3x}{2x^2\sqrt{1-4x}} \ln\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right) + \frac{1-x}{2x^2}\ln x$$

One loop contributions to δa_ℓ

For $x \ll 1$,

$$I_2(x) \simeq x\left(-\frac{3}{2} - \ln x\right) + x^2\left(-\frac{16}{3} - 4\ln x\right) + \mathcal{O}(x^3)$$
$$I_3(x) \simeq x\left(-\frac{11}{6} - \ln x\right) + x^2\left(-\frac{89}{12} - 5\ln x\right) + \mathcal{O}(x^3)$$

For $m_\ell \ll m_S$,

$$\Delta a_\ell^{(1)} = \frac{1}{8\pi^2}\frac{m_\ell^2}{m_S^2}\frac{m_\ell^2}{v^2}\left\{-[a_\ell^S]^2\left(\frac{7}{6} + \ln\left(\frac{m_\ell^2}{m_S^2}\right)\right) + [b_\ell^S]^2\left(\frac{11}{6} + \ln\left(\frac{m_\ell^2}{m_S^2}\right)\right)\right\}$$

One loop contributions to δa_ℓ

Yukawa interactions of the form

$$\mathscr{L}_{C\ell\nu} = -C^- \bar{\ell}(a_\ell^C + i b_\ell^C \gamma_5)\nu - C^+ \bar{\nu}(a_\ell^{C*} + i b_\ell^{C*}\gamma_5)\ell$$

give one loop contributions

$$\Delta a_\ell^{(1)} = -\frac{1}{8\pi^2}\sum_C \left\{|a_\ell^C|^2 + |b_\ell^C|^2\right\} H(x_{\ell C})$$

where $x_{\ell C} = m_\ell^2/m_{C^\pm}^2$, and

$$H(x) = -\frac{1}{2} + \frac{1}{x} + \frac{1-x}{x^2}\ln(1-x)\,, \quad H(x) \simeq \frac{x}{6} + \frac{x^2}{12} + \mathcal{O}(x^3) \text{ for } x \ll 1$$

Two loop contributions to δa_ℓ

For quarks

$$\mathscr{L}_{S\bar{f}f} = -\frac{m_f}{v} S \bar{f}(\alpha_f^S + i\beta_f^S \gamma_5) f$$

The two loop Barr-Zee contributions to the anomalous magnetic moment of lepton ℓ

$$\Delta a_\ell^{(2)} = -\frac{\alpha^2}{4\pi^2 s_W^2} \frac{m_\ell^2}{M_W^2} \sum_f \sum_S N_c^f Q_f^2 \left\{ a_\ell^S \alpha_f^S f(z_{fS}) - b_\ell^S \beta_f^S g(z_{fS}) \right\}$$

f: fermions in the closed fermion loop

(N_c^f colour, Q_f electric charge, $z_{fS} = m_f^2/m_S^2$)

S: neutral scalar connecting the closed fermion loop with the external lepton line

Two loop contributions to δa_ℓ

Loop functions

$$f(z) = \frac{z}{2}\int_0^1 dx\, \frac{1-2x(1-x)}{x(1-x)-z}\, \ln\left(\frac{x(1-x)}{z}\right)$$
$$g(z) = \frac{z}{2}\int_0^1 dx\, \frac{1}{x(1-x)-z}\, \ln\left(\frac{x(1-x)}{z}\right)$$

Constraints: δa_ℓ

- The anomalies

$$\delta a_e^{\text{Exp,Cs}} = -(8.7 \pm 3.6) \times 10^{-13}\,, \quad \delta a_\mu^{\text{Exp}} = (2.5 \pm 0.6) \times 10^{-9}\,.$$

- The "natural" δa_ℓ constraint

$$\chi_0^2(\delta a_e, \delta a_\mu) = \left(\frac{\delta a_e - c_e}{\sigma_e}\right)^2 + \left(\frac{\delta a_\mu - c_\mu}{\sigma_\mu}\right)^2\,,$$

with $\delta a_\ell^{\text{Exp}} = c_\ell \pm \sigma_\ell$

- We impose a stronger requirement

$$\chi^2(\delta a_e, \delta a_\mu) = 16\chi_0^2(\delta a_e, \delta a_\mu)$$

that is $\sigma_\ell \mapsto \sigma_\ell/4$

Constraints

- Scalar sector
  - potential bounded from below
  - perturbativity and perturbative unitarity of $2 \to 2$ high energy scattering
  - electroweak precision (oblique parameters S, T)

☞ one can play with M_W and the CDF value!

- Fermion sector: perturbative Yukawa couplings

$$|n_\ell| \leq n_0$$

  with two different choices $n_0 = 100$ GeV or $n_0 = 250$ GeV
- Higgs signal strengths:
  - production × decay signal strengths of the usual channels
  - large lepton couplings: also include $\mathrm{h} \to \mu^+\mu^-, e^+e^-$ information

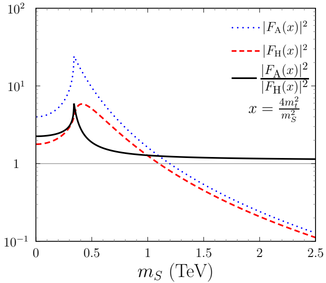
$\Rightarrow$ Higgs alignment

Constraints

- $\mathrm{H}^{\pm}$ mediated contributions
  - Lepton flavour universality
    - purely leptonic decays $\ell_j \to \ell_k \nu\bar{\nu}$
    - decays with light pseudoscalar mesons $K, \pi \to e\nu, \mu\nu$ and $\tau \to K\nu, \pi\nu$
  - $b \to s\gamma$, B_q^0–$\bar{B}_q^0$ mixing
- $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ at LEP

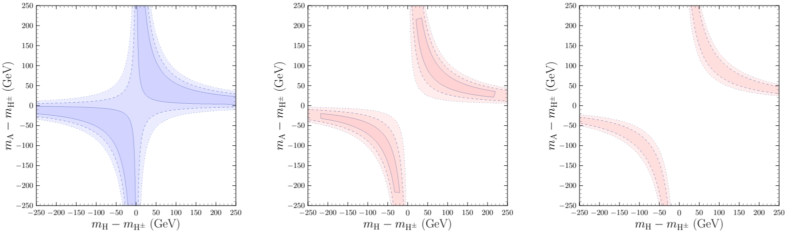
  (cross sections up to $\sqrt{s} = 208$ GeV)
- LHC searches
  - searches of dilepton resonance: $\sigma(pp \to S)_{[\mathrm{ggF}]} \times \mathrm{Br}\left(S \to \ell^+\ell^-\right)$, $S = \mathrm{H}, \mathrm{A}$ and $\ell = \mu, \tau$
  - searches of charged scalars: $\sigma(pp \to \mathrm{H}^{\pm} tb) \times \mathrm{Br}\left(\mathrm{H}^{\pm} \to f\right)$, $f = \tau\nu, tb$

Gluon-gluon-scalar vs gluon-gluon-pseudoscalar



$F_{\rm H}(x)$ from $\mathrm{S}\bar{t}t$, $F_{\rm A}(x)$ from $\mathrm{S}\bar{t}\gamma_5 t$

Oblique parameters and M_W from CDF



$m_A - m_{H^\pm}$ vs $m_H - m_{H^\pm}$ for $m_{H^\pm} = 1$ TeV

$\Delta S = 0.00 \pm 0.07$
$\Delta T = 0.05 \pm 0.06$
$\rho = 0.92$

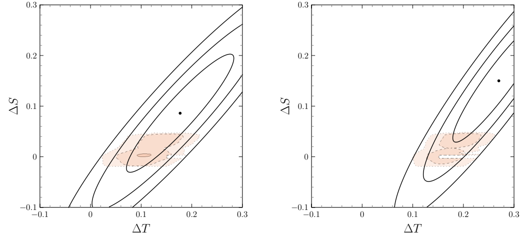
$\Delta S = 0.086 \pm 0.077$
$\Delta T = 0.177 \pm 0.070$
$\rho = 0.89$
"Conservative"
2204.04204

$\Delta S = 0.15 \pm 0.08$
$\Delta T = 0.27 \pm 0.06$
$\rho = 0.93$
2204.03796

Oblique parameters and M_W from CDF

Bottom line: since $m_{\mathrm{H}^\pm}$ is rather irrelevant for δa_ℓ,

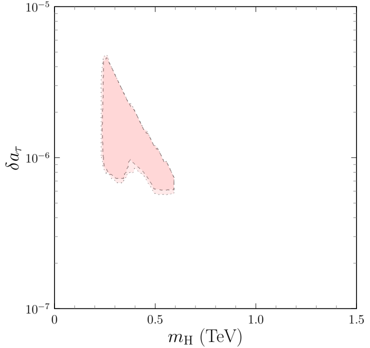
a simple shift in $m_{\mathrm{H}^\pm}$ may work



"Conservative"

ΔT vs ΔS

δa_τ



δa_τ & m_{H}, n_τ

Example points

Pt	m_{H}	m_{A}	$m_{\mathrm{H}^\pm}$	t_β	$\mathrm{Re}\left(\mu_{12}^2\right)$	$\mathrm{Re}\left(n_e\right)$	$\mathrm{Re}\left(n_\mu\right)$	$\mathrm{Re}\left(n_\tau\right)$
1	362	775	778	10.7	-1.19×10^4	8.16	-72.8	222.5
2	277	494	502	10.8	-6.61×10^3	9.12	-58.0	191.2
3	278	539	554	7.93	-8.90×10^5	6.71	-53.1	189.7

Parameters in GeV ($\mathrm{Re}\left(\mu_{12}^2\right)$ in GeV^2)

		1 loop			2 loop					
Pt	$\delta a_e^{(*)}$	H	A	$\mathrm{H}^\pm$	$t\mathrm{H}$	$t\mathrm{A}$	$\tau\mathrm{H}$	$\tau\mathrm{A}$	$\mu\mathrm{H}$	$\mu\mathrm{A}$
1	-7.24	0	0	0	0.307	0.199	0.779	-0.259	-0.036	0.010
2	-9.84	0	0	0	0.310	0.258	0.845	-0.388	-0.038	0.015
3	-8.87	0	0	0	0.347	0.264	0.683	-0.271	-0.028	0.010

Relative contributions to δa_e. $^{(*)}$: $10^{13}\delta a_e$

Example points

		1 loop			2 loop					
Pt	$\delta a_\mu^{(*)}$	H	A	$\mathrm{H}^\pm$	tH	tA	τH	τA	μH	μA
1	2.42	0.588	-0.136	-0.003	0.169	0.110	0.429	-0.143	-0.020	0.006
2	2.29	0.650	-0.218	-0.005	0.175	0.146	0.477	-0.219	-0.021	0.009
3	2.35	0.531	-0.147	-0.003	0.214	0.163	0.422	-0.168	-0.017	0.006

Relative contributions to δa_μ. $^{(*)}$: $10^9 \delta a_\mu$

Pt	$S \to$	$e\bar{e}$	$\mu\bar{\mu}$	$\tau\bar{\tau}$	$t\bar{t}$	HZ
1	H	1.2×10^{-3}	0.096	0.902	7×10^{-4}	–
	A	0.0004	0.028	0.265	0.004	0.703
2	H	0.002	0.084	0.914	–	–
	A	0.001	0.049	0.528	0.008	0.415
3	H	0.001	0.073	0.926	–	–
	A	0.0005	0.033	0.415	0.012	0.540

Decay branching ratios of H and A

Example points

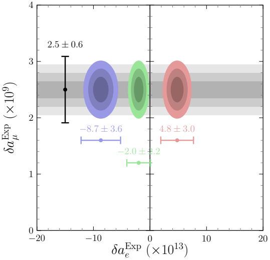
		$\sigma(pp \to S)_{[\mathrm{ggF}]} \times \mathrm{Br}\,(S \to \tau^+\tau^-)$ (fb)			
Point		Value	Expected bound	Observed bound	Excess
1	H	83.6	54.9	106.0	✓
	A	1.50	3.2	3.2	–
2	H	84.0	136.0	164.0	–
	A	27.3	20.8	50.4	✓
3	H	160.1	135.4	164.0	✓
	A	27.6	17.6	31.1	✓

Different δa_e^{Exp} assumptions

Analyses with different “measurements”

$$\delta a_e^{\text{Exp,Cs}} = -(8.7 \pm 3.6) \times 10^{-13},$$
$$\delta a_e^{\text{Exp,Rb}} = (4.8 \pm 3.0) \times 10^{-13}$$
$$\delta a_e^{\text{Exp,Avg}} = -(2.0 \pm 2.2) \times 10^{-13},$$
$$\delta a_e^{\text{Exp,Bound}} = 20 \times 10^{-13}$$

N.B. Last case: $|\delta a_e| < \delta a_e^{\text{Exp,Bound}}$
In all cases $n_0 = 250$ GeV



Different $\delta a_e^{\mathrm{Exp}}$ assumptions

Simple analysis

- For a point in parameter space explaining $\delta a_e^{\mathrm{Exp,Cs}}$,
  1. consider $\mathrm{Re}\,(n_e) \mapsto \mathrm{Re}\,(n_e)\,\frac{\delta a_e^{\mathrm{Exp}}}{\delta a_e^{\mathrm{Exp,Cs}}}$, which gives $\delta a_e \simeq \delta a_e^{\mathrm{Exp}}$
  2. analyse if this new value conflicts with other observables sensitive to $\mathrm{Re}\,(n_e)$: muon decay and pseudoscalar mesons decays
- Answer
  - Short version: $\delta a_e^{\mathrm{Exp,Cs}}$ is “worst case”
    because of absolute value and sign
  - Long version in the next slides

✎ all cases can be reproduced at least with the regions arising from the previous $\mathrm{Re}\,(n_e)$ transformation

Different δa_e^{Exp} assumptions

- For muon decay

$$\left|\frac{\text{Re}\,(n_e)\,\text{Re}\,(n_\mu)}{m_{\text{H}^\pm}^2}\right| < 0.035$$

$\Rightarrow$ $|\text{Re}\,(n_e)|$ for $\delta a_e^{\text{Exp,Cs}}$ is "worse" than other cases

- For pseudoscalar meson decays, consider

$$R_{\mu e}^P = \frac{\Gamma(P^+ \to \mu^+\nu)}{\Gamma(P^+ \to \mu^+\nu)_{\text{SM}}}\frac{\Gamma(P^+ \to e^+\nu)_{\text{SM}}}{\Gamma(P^+ \to e^+\nu)}$$

Experimental values:

$$R_{\mu e}^\pi = 1 + (4.1 \pm 3.3) \times 10^{-3}\,, \qquad R_{\mu e}^K = 1 - (4.8 \pm 4.7) \times 10^{-3}$$

Model prediction:

$$R_{\mu e}^P = \frac{|1-\Delta_\mu^P|^2}{|1-\Delta_e^P|^2}\,, \qquad |1-\Delta_\ell^P|^2 = \left|1 - \frac{M_P^2}{t_\beta m_{\text{H}^\pm}^2}\frac{\text{Re}\,(n_\ell)}{m_\ell}\right|^2$$

Different $\delta a_e^{\mathrm{Exp}}$ assumptions

Experimental values:

$$R_{\mu e}^{\pi} = 1 + (4.1 \pm 3.3) \times 10^{-3}\,, \qquad R_{\mu e}^{K} = 1 - (4.8 \pm 4.7) \times 10^{-3}$$

Model prediction:

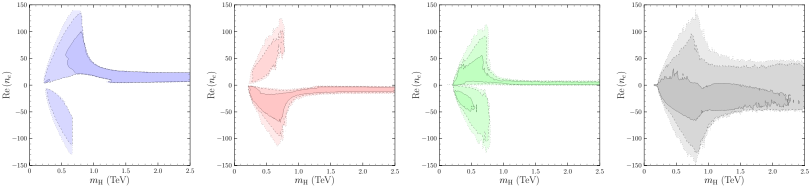
$$R_{\mu e}^{P} = \frac{|1-\Delta_\mu^P|^2}{|1-\Delta_e^P|^2}\,, \qquad |1-\Delta_\ell^P|^2 = \left|1 - \frac{M_P^2}{t_\beta m_{\mathrm{H}^\pm}^2}\frac{\mathrm{Re}\,(n_\ell)}{m_\ell}\right|^2$$

- For $\Delta_\ell^P \ll 1$,

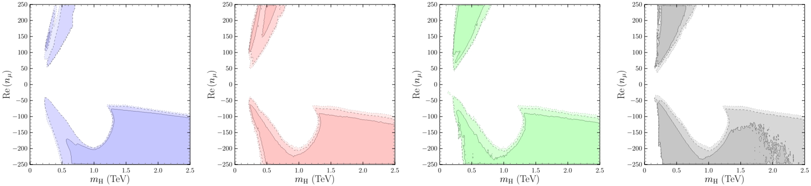
$$R_{\mu e}^{P} \simeq 1 + 2\frac{M_P^2}{t_\beta m_{\mathrm{H}^\pm}^2}\left(\frac{\mathrm{Re}\,(n_e)}{m_e} - \frac{\mathrm{Re}\,(n_\mu)}{m_\mu}\right)$$

- Concentrate on $R_{\mu e}^{K}$, neglect the $\mathrm{Re}\,(n_\mu)$ contribution:
  $\mathrm{Re}\,(n_e) > 0$ for $\delta a_e^{\mathrm{Exp,Cs}}$ is “worse” than other cases

Different $\delta a_e^{\mathrm{Exp}}$ assumptions

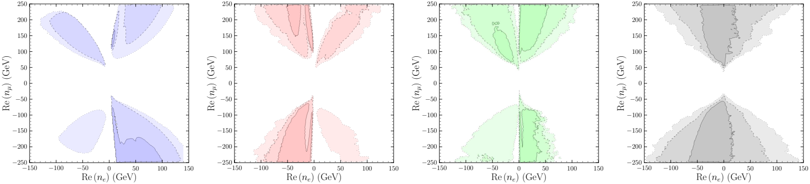


$\mathrm{Re}\,(n_e)$ vs m_{H}



$\mathrm{Re}\,(n_\mu)$ vs m_{H}

Different $\delta a_e^{\mathrm{Exp}}$ assumptions



$\mathrm{Re}\,(n_\mu)$ vs $\mathrm{Re}\,(n_e)$

Yukawa couplings

Neutral scalars

$$\mathscr{L}_{\mathrm{S}\bar{f}f} = -\frac{\mathrm{S}}{v}\,\bar{f}\,\left[\mathcal{R}_{1s}\mathrm{M}_f + \mathcal{R}_{2s}\frac{\mathrm{N}_f + \mathrm{N}_f^\dagger}{2} + i\epsilon_{(f)}\mathcal{R}_{3s}\frac{\mathrm{N}_f - \mathrm{N}_f^\dagger}{2}\right] f$$
$$- \frac{\mathrm{S}}{v}\,\bar{f}\,\gamma_5\,\left(\mathcal{R}_{2s}\frac{\mathrm{N}_f - \mathrm{N}_f^\dagger}{2} + i\epsilon_{(f)}\mathcal{R}_{3s}\frac{\mathrm{N}_f + \mathrm{N}_f^\dagger}{2}\right) f$$

where $s = 1, 2, 3$ in correspondence with $\mathrm{S} = \mathrm{h}, \mathrm{H}, \mathrm{A}$; $f = u, d, \ell$; in terms proportional to $\mathcal{R}_{3s}$, $\epsilon_{(d)} = \epsilon_{(\ell)} = -\epsilon_{(u)} = 1$

Yukawa couplings

Charged scalars

$$\mathscr{L}_{\mathrm{H}^\pm ud} = \frac{\mathrm{H}^-}{\sqrt{2}v}\bar{d}\left[V^\dagger \mathrm{N}_u - \mathrm{N}_d^\dagger V^\dagger + \gamma_5\left(V^\dagger \mathrm{N}_u + \mathrm{N}_d^\dagger V^\dagger\right)\right] u$$
$$+ \frac{\mathrm{H}^+}{\sqrt{2}v}\bar{u}\left[\mathrm{N}_u^\dagger V - V\mathrm{N}_d + \gamma_5\left(\mathrm{N}_u^\dagger V + V\mathrm{N}_d\right)\right] d$$

and

$$\mathscr{L}_{\mathrm{H}^\pm \ell\nu} = -\frac{\sqrt{2}}{v}\mathrm{H}^+\,\bar{\nu}_L\,U^\dagger \mathrm{N}_\ell\,\ell_R - \frac{\sqrt{2}}{v}\mathrm{H}^-\,\bar{\ell}_R\,\mathrm{N}_\ell^\dagger U\,\nu_L$$

V and U are, respectively, the CKM and PMNS mixing matrices (massless neutrinos assumed, one can set $U \to \mathbf{1}$)

Couplings in the I-gℓFC and II-gℓFC models

$$\mathscr{L}_N = -\sum_{S=\mathrm{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^{3} \frac{m_{f_j}}{v}\, S\, \bar{f}_j (a^S_{f_j} + i b^S_{f_j}\gamma_5) f_j$$

Quark couplings

		a_u^S	b_u^S	a_d^S	b_d^S
I-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0
	A	0	$-t_\beta^{-1}$	0	$+t_\beta^{-1}$
II-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta} t_\beta^{-1}$	0	$s_{\alpha\beta} - c_{\alpha\beta} t_\beta$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta} t_\beta^{-1}$	0	$-c_{\alpha\beta} - s_{\alpha\beta} t_\beta$	0
	A	0	$-t_\beta^{-1}$	0	$-t_\beta$

Couplings in the I-gℓFC and II-gℓFC models

$$\mathscr{L}_N = -\sum_{S=\mathrm{h,H,A}} \sum_{f=u,d,\ell} \sum_{j=1}^{3} \frac{m_{f_j}}{v}\, S\, \bar{f}_j (a^S_{f_j} + i b^S_{f_j}\gamma_5) f_j$$

Lepton couplings

		a^S_ℓ	b^S_ℓ
I-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	A	0	$\frac{\mathrm{Re}(n_\ell)}{m_\ell}$
II-gℓFC	h	$s_{\alpha\beta} + c_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	H	$-c_{\alpha\beta} + s_{\alpha\beta}\frac{\mathrm{Re}(n_\ell)}{m_\ell}$	0
	A	0	$\frac{\mathrm{Re}(n_\ell)}{m_\ell}$

Couplings in the I-gℓFC and II-gℓFC models back

Yukawa couplings of the charged scalar

$$\mathscr{L}_{Ch} = -\frac{1}{\sqrt{2}v} \sum_{f=q,\ell} \sum_{j,k=1}^{3} \Big\{ \mathrm{H}^- \bar{f}_{-\frac{1}{2},j} (\alpha^f_{jk} + i\beta^f_{jk}\gamma_5) f_{\frac{1}{2},k} + \mathrm{H}^+ \bar{f}_{\frac{1}{2},k} (\alpha^{f*}_{jk} + i\beta^{f*}_{jk}\gamma_5) f_{-\frac{1}{2},j} \Big\}$$

with $q_{+\frac{1}{2},j} = u_j$; $q_{-\frac{1}{2},j} = d_j$; $\ell_{+\frac{1}{2},j} = \nu_j$; $\ell_{-\frac{1}{2},j} = \ell_j$

	α^q_{ij}	β^q_{ij}
I-gℓFC	$V^*_{ji}\, t_\beta^{-1}(m_{u_j} - m_{d_i})$	$V^*_{ji}\, t_\beta^{-1}(m_{u_j} + m_{d_i})$
II-gℓFC	$V^*_{ji}\,(t_\beta^{-1} m_{u_j} + t_\beta m_{d_i})$	$V^*_{ji}\,(t_\beta^{-1} m_{u_j} - t_\beta m_{d_i})$

	α^ℓ_{ij}	β^ℓ_{ij}
I-gℓFC	$-\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$	$\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$
II-gℓFC	$-\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$	$\mathrm{Re}\,(n_{\ell_i})\,\delta_{ij}$

$O(3,\mathbb{R})$ matrices

- $O \in SO(3,\mathbb{R})$ are of the form $O = \exp(\alpha A)$ with real $\alpha \in [0; 2\pi[$ and A a normalized antisymmetric matrix

$$A = \begin{pmatrix} 0 & \hat{n}_3 & -\hat{n}_2 \\ -\hat{n}_3 & 0 & \hat{n}_1 \\ \hat{n}_2 & -\hat{n}_1 & 0 \end{pmatrix}, \quad \hat{n}_j \in \mathbb{R}, \quad \hat{n}_1^2 + \hat{n}_2^2 + \hat{n}_3^2 = 1$$

- A has eigenvalues $\lambda = 0, \pm i$; the normalized eigenvectors are

$$\lambda = 0,\ \vec{v}_0^T = (\hat{n}_1, \hat{n}_2, \hat{n}_3) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$

$$\lambda = i,\ \vec{v}_+^T = \frac{1}{\sqrt{2}}(-\cos\theta\cos\varphi + i\sin\varphi, -\cos\theta\sin\varphi - i\cos\varphi, \sin\theta),$$

$$\lambda = -i,\ \vec{v}_- = (\vec{v}_+)^*,$$

$O(3, \mathbb{R})$ matrices

- Diagonalisation

$$U^\dagger \, A \, U = \mathrm{diag}(0, i, -i), \qquad U = \begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \vec{v}_0 & \vec{v}_+ & \vec{v}_- \\ \downarrow & \downarrow & \downarrow \end{pmatrix}$$

- $O = \exp(\alpha A)$ has eigenvalues $\{1, e^{i\alpha}, e^{-i\alpha}\}$ and the same eigenvectors

$$U^\dagger \, O \, U = \mathrm{diag}(1, e^{i\alpha}, e^{-i\alpha})$$

- Geometrically O represents a rotation in $\mathbb{R}^3$ of angle α around the axis $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$
- The eigenvectors of O do not depend on $\alpha \Rightarrow O_1 = \exp(\alpha_1 A)$ and $O_2 = \exp(\alpha_2 A)$ with $\alpha_1 \neq \alpha_2$ can be unitarily diagonalised simultaneously.

μ–τ symmetry with $O(3,\mathbb{R})$ matrices

PMNS matrix U = unitary$^\dagger\times$orthogonal with $\vec{r}_k$ real

$$\begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \vec{r}_0 & \vec{v}_+ & (\vec{v}_+)^* \\ \downarrow & \downarrow & \downarrow \end{pmatrix}^\dagger \begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \vec{r}_1 & \vec{r}_2 & \vec{r}_3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} \vec{r}_0\cdot\vec{r}_1 & \vec{r}_0\cdot\vec{r}_2 & \vec{r}_0\cdot\vec{r}_3 \\ (\vec{v}_+)^*\cdot\vec{r}_1 & (\vec{v}_+)^*\cdot\vec{r}_2 & (\vec{v}_+)^*\cdot\vec{r}_3 \\ \vec{v}_+\cdot\vec{r}_1 & \vec{v}_+\cdot\vec{r}_2 & \vec{v}_+\cdot\vec{r}_3 \end{pmatrix}$$

and $U_{2j} = (U_{3j})^* = (\vec{v}_+)^*\cdot\vec{r}_j$, $j=1,2,3$, that is, again, μ–τ symmetry:

$$|U_{2j}| = |U_{3j}|,\ \ j=1,2,3$$