

SCHWINGER MECHANISM IN QCD

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J.P., CHIN. PHYS. C 46 (2022) 11, 112001 [HEP-PH 2207.04977]

M.N.FERREIRA AND J.P., PARTICLES 6, NO.1, 312-363 (2023) [HEP-PH 2301.02314]

L3: FLAVOUR AND QUARK MATTER

JANUARY 8-9, 2024

DYNAMICAL GENERATION OF A GLUON MASS

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \underbrace{\frac{1}{2\xi}(\partial^\mu A_\mu^a)^2}_{\text{GAUGE-FIXING}} - \underbrace{\bar{c}^a \partial^\mu D_\mu^{ab} c^b}_{\text{GHOSTS}} + \mathcal{L}_{\text{quarks}}$$

ANTISYMMETRIC FIELD TENSOR $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$

COVARIANT DERIVATIVE IN THE ADJOINT $D_\mu^{ab} = \partial_\mu \delta^{ac} + g f^{amb} A_\mu^m$

GAUGE SYMMETRY FIRST, AND, AFTER THE GAUGE-FIXING, BRST SYMMERTY
PROHIBIT A MASS TERM $m^2 A_\mu^a A^{a\mu}$ FOR THE GLUON

PROPERLY REGULARIZED PERTURBATION THEORY
CANNOT GENERATE A GLUON MASS AT ANY FINITE ORDER

$$\int \frac{d^d k}{k^2} = 0$$

HOWEVER: GLUON SELF-INTERACTIONS GENERATE A NON-PERTURBATIVE MASS

THE FIRST PLACE WHERE THE EMERGENCE OF A MASS IS FELT IS THE **GLUON PROPAGATOR**

ACCORDING TO CORNWALL: GLUON PROPAGATOR **FINITE AT THE ORIGIN**

NON-PERTURBATIVE METHODS (E.G. SCHWINGER-DYSON EQUATIONS) ARE REQUIRED

HISTORICALLY DURING A **QUARTER OF A CENTURY** MANY DIFFERENT STUDIES PREDICTED
A VARIETY OF DISTINCT BEHAVIORS FOR THE GLUON PROPAGATOR
(MANDELSTAM, STINGL, KUGO-OJIMA, GRIBOV-ZWANZIGER, GHOST DOMINANCE ...)

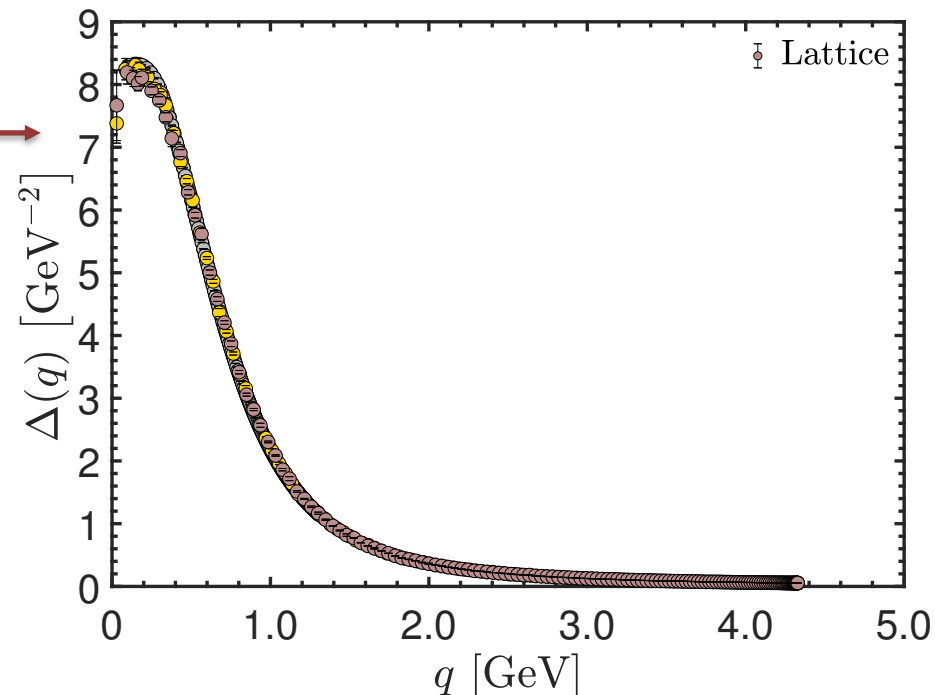
TURNING POINT: **LARGE-VOLUME LATTICE SIMULATIONS (CIRCA 2007)**

I. BOGOLUBSKY ET AL, PROC. SCI., LATTICE2007 (2007) 290

$$\Delta(0) = \frac{1}{m^2} \longrightarrow$$

THE SATURATION OF THE GLUON PROPAGATOR
IN THE DEEP INFRARED IS THE UNEQUIVOCAL SIGNAL
OF GLUON MASS GENERATION

MUST EXPLAIN THIS FROM WITHIN QCD,
WITHOUT MODIFYING THE LAGRANGIAN !



SCHWINGER MECHANISM

GAUGE BOSON
PROPAGATOR

$$\Delta_{\mu\nu}(q) = -i\Delta(q)P_{\mu\nu}(q), \quad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

TREE LEVEL :

$$\Delta^{-1}(q^2) = q^2 + m^2 = q^2 \left[1 + \frac{m^2}{q^2} \right]$$



ACTS AS A POLE

IN GENERAL :

$$\Delta^{-1}(q^2) = q^2 + m^2 + \Pi(q^2)$$



VACUUM POLARIZATION

SCHWINGER-DYSON EQUATION FOR GAUGE BOSON PROPAGATOR

$$\left(\text{wavy line} \text{---} \text{blue circle} \text{---} \text{wavy line} \right)^{-1} = \left(\text{wavy line} \right)^{-1} + \text{wavy line} \text{---} \left(\text{fermion loop with blob} \right) \text{---} \text{wavy line}$$

$\Pi(q^2)$

WHEN THE SYMMETRY PROHIBITS A TREE-LEVEL MASS: $\longrightarrow m^2 = 0$

$$\Delta^{-1}(q^2) = q^2[1 + \Pi(q^2)]$$

IF, FOR SOME REASON $\lim_{q^2 \rightarrow 0} \Pi(q^2) = \frac{c}{q^2}$, $c > 0$

$\longrightarrow \Delta^{-1}(0) = c > 0$

GAUGE INVARIANCE AND MASS

A GAUGE BOSON MAY ACQUIRE A MASS, EVEN IF THE GAUGE SYMMETRY FORBIDS A MASS TERM AT THE LEVEL OF THE FUNDAMENTAL LAGRANGIAN, PROVIDED THAT ITS VACUUM POLARIZATION FUNCTION DEVELOPS A POLE AT ZERO MOMENTUM TRANSFER

J. S. Schwinger, Phys. Rev.125, 397 (1962); Phys.Rev.128, 2425 (1962)



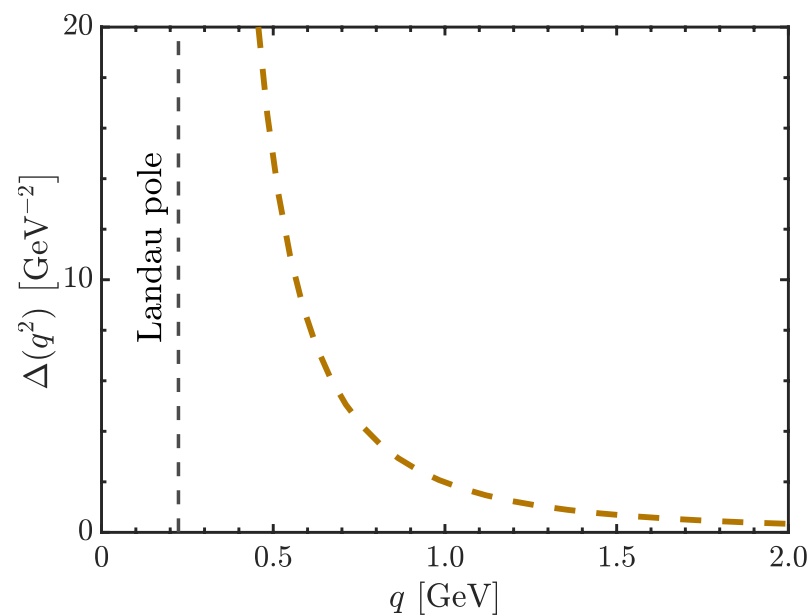
SYSTEM OF SCHWINGER-DYSON EQUATIONS (SDEs)

$$\left(\text{wavy line with circle} \right)^{-1} = \left(\text{wavy line} \right)^{-1} + \text{wavy line } q \text{ with loop and red dot} \text{ wavy line } q + \dots$$

$$\underbrace{\text{wavy line } q \text{ with red dot and wavy lines } r, p}_{\Pi_{\alpha\mu\nu}(q, r, p)} = \text{wavy line } q \text{ with wavy lines } r, p + \text{wavy line } q \text{ with loop and red dots } r, k, \nu, n + \dots$$

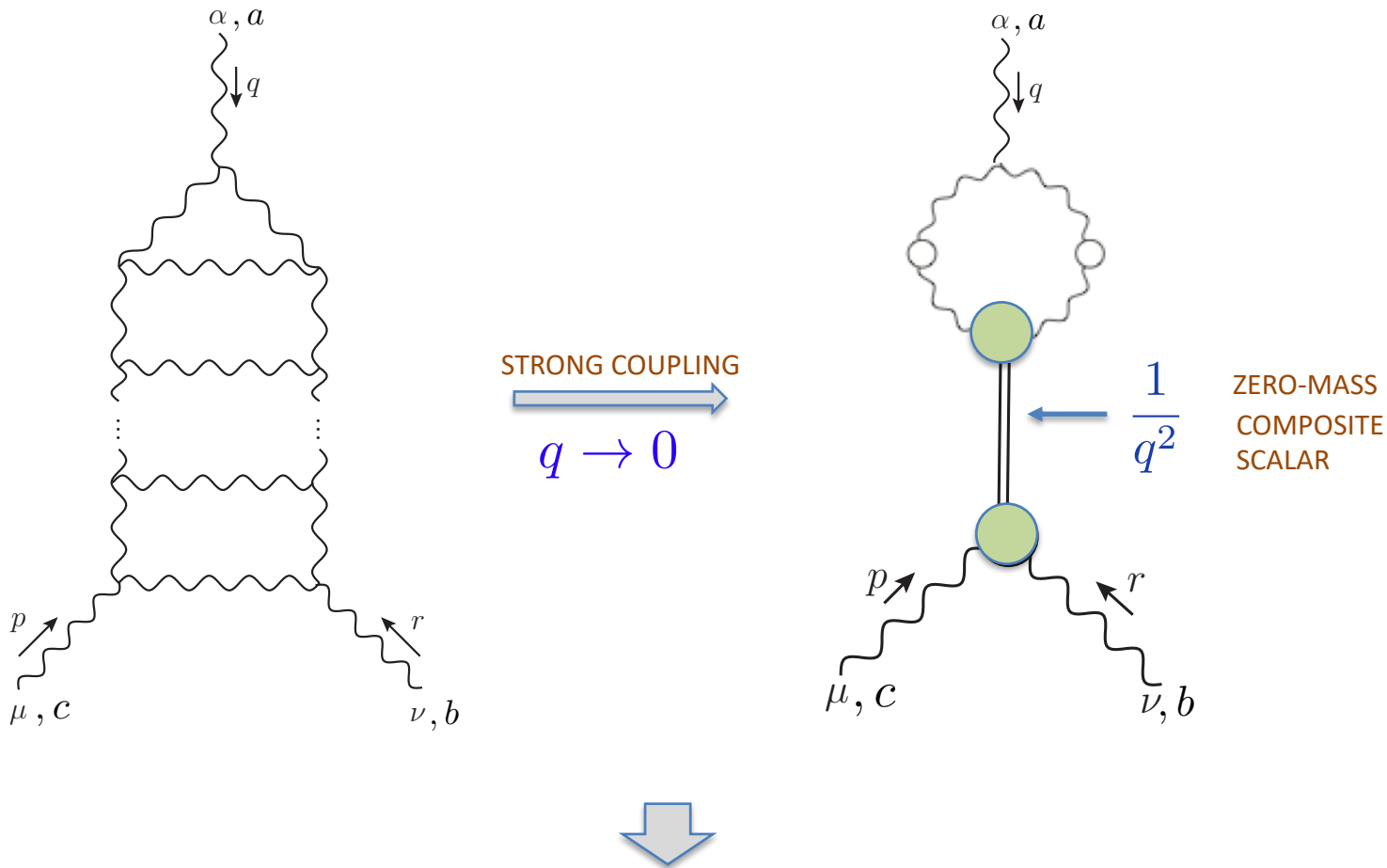
$\Pi_{\alpha\mu\nu}(q, r, p)$

IN THE ABSENCE OF A MASS, THE SYSTEM
HAS NO INFRARED STABLE SOLUTIONS



BUT THE THEORY FINDS THE WAY TO **STABILIZE** ITSELF :

DYNAMICAL FORMATION OF **COLOR-CARRYING** MASSLESS SCALAR EXCITATIONS



THE MASSLESS POLE REQUIRED FOR TRIGGERING THE **SCHWINGER MECHANISM**
IN QCD IS PRODUCED FROM THE **INTERACTIONS**

GENERAL STRUCTURE OF THE THREE-GLUON VERTEX

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q, r, p) = \underbrace{\Gamma_{\alpha\mu\nu}(q, r, p)}_{\text{POLE-FREE}} + \underbrace{V_{\alpha\mu\nu}(q, r, p)}_{\text{POLE}}$$

REMEMBER

$$V_{\alpha\mu\nu}(q, r, p) = \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q, r, p)$$

BOSE SYMMETRY IMPOSES

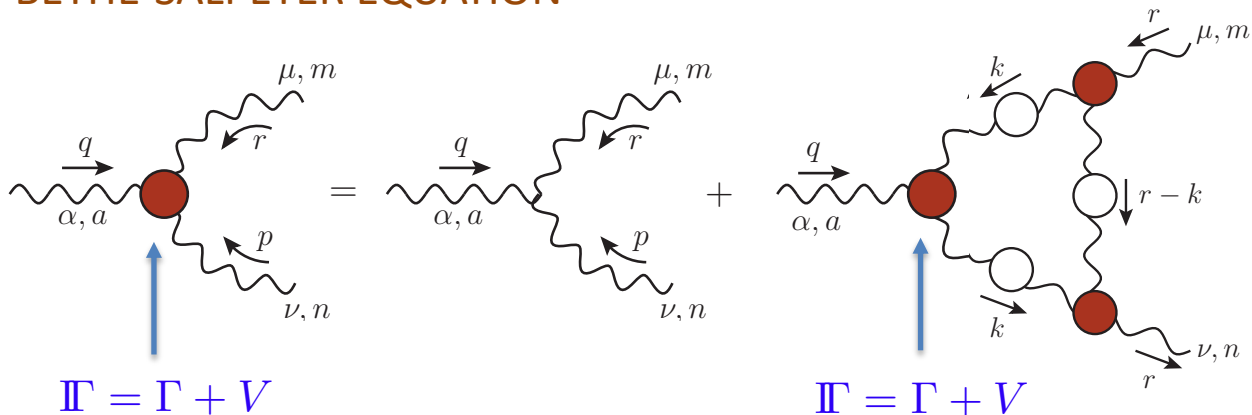
$$C_1(q, r, p) = -C_1(q, p, r) \quad \Longrightarrow \quad C_1(0, r, -r) = 0$$

$$\Longrightarrow \quad \lim_{q \rightarrow 0} C_1(q, r, p) = 2(q \cdot r) \underbrace{\left[\frac{\partial C_1(q, r, p)}{\partial p^2} \right]_{q=0}}_{\mathbb{C}(r^2)} + \mathcal{O}(q^2)$$

RESIDUE FUNCTION

BETHE-SALPETER EQUATION

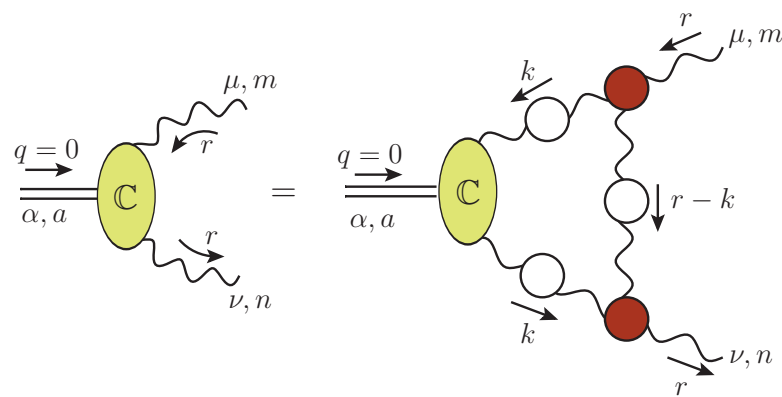
START WITH VERTEX SDE:



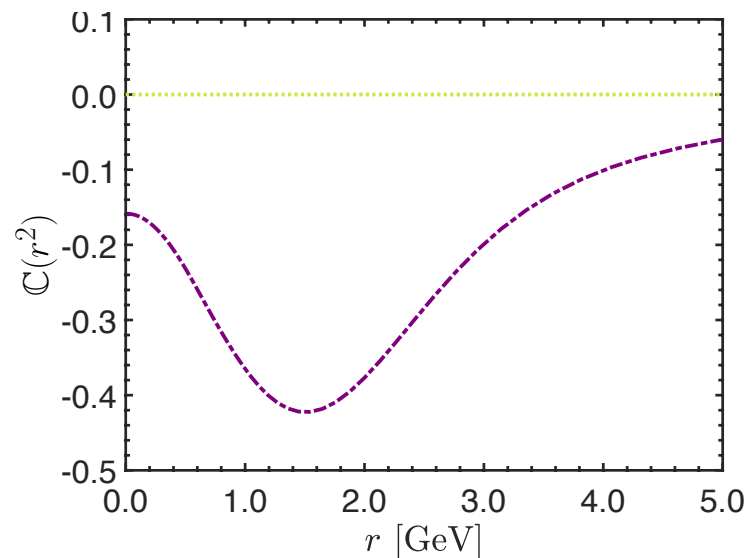
SUBSTITUTE:

TAKE THE LIMIT $q \rightarrow 0$ AND USE $\lim_{q \rightarrow 0} V(q, r, p) = 2(q \cdot r)\mathbb{C}(r^2)$

BSE



NON-TRIVIAL
SOLUTION



$\mathbb{C}(r^2)$ ACTS AS BETHE-SALPETER AMPLITUDE

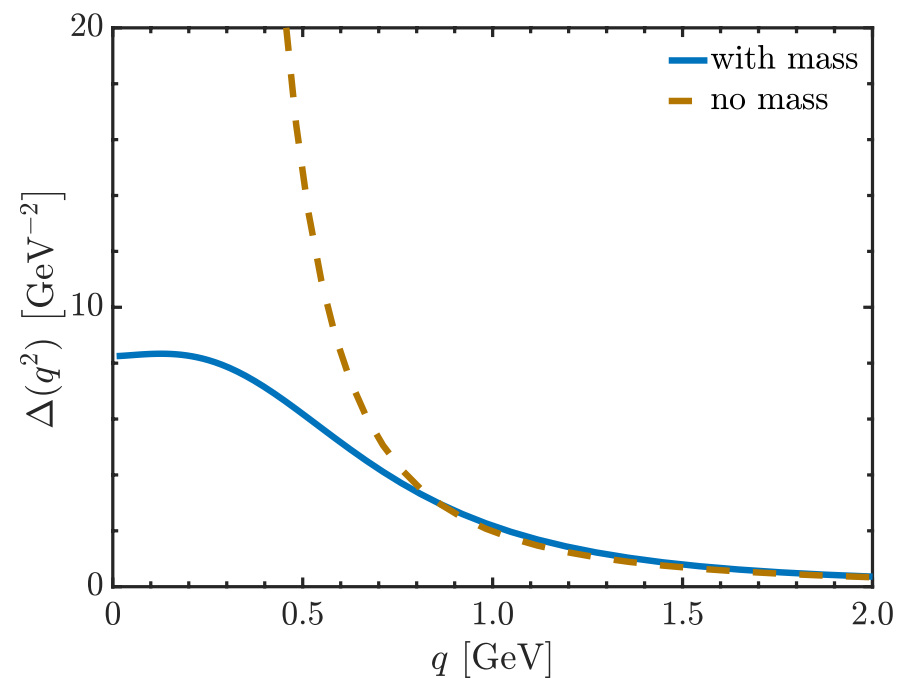
EMERGENCE OF A GLUON MASS

$$\Delta^{-1}(0) = \lim_{q \rightarrow 0} \text{diagram} = \lim_{q \rightarrow 0} \text{diagram} = \lim_{q \rightarrow 0} \text{diagram}$$

The first diagram shows a wavy line with momentum q entering a loop of wavy lines, which then exits with momentum q . A red dot is placed on the right wavy line. The second diagram is identical but with a green dot on the right wavy line. The third diagram shows two such loops connected by a double line with momentum $1/q^2$ between two green dots.

$$= -\frac{3C_A\alpha_s}{8\pi} \int_0^\infty dy y^2 \Delta^2(y) \mathbb{C}(y)$$

$$= m^2$$



THE DECISIVE FEATURE: WARD IDENTITY DISPLACEMENT

A.C.AGUILAR, D. BINOSI, C.T. FIGUEIREDO, AND JP, PHYS.REV.D 94 (2016) 045002

THE LONGITUDINAL NATURE OF THE POLES MAKES THEIR DIRECT DETECTION IMPOSSIBLE

$$V_{\alpha\mu\nu}^{abc}(q, r, p) = f^{abc} \frac{q_\alpha}{q^2} \left[g_{\mu\nu} C_1(q, r, p) + \cdots \right]$$

PHYSICAL QUANTITIES AND (LANDAU-GAUGE) LATTICE OBSERVABLES ARE ALWAYS FINITE:
THE SCHWINGER POLES DO NOT INTRODUCE ANY DIVERGENCES

HOWEVER

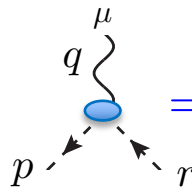
THERE IS A CHARACTERISTIC FEATURE, UNIQUE TO THIS MECHANISM,
WHICH ALLOWS THE UNEQUIVOCAL CONFIRMATION OF THIS SCENARIO

THE WARD IDENTITIES UNDERGO A FINITE DISPLACEMENT WITH RESPECT
TO THE RESULT FOUND IN THE ABSENCE OF THE SCHWINGER MECHANISM

QUITE REMARKABLY, THE QUANTITY THAT DETERMINES THIS EFFECT IS NONE OTHER THAN

$$\mathbb{C}(r^2)$$

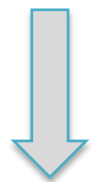
SCHWINGER MECHANISM OFF



$$= \Gamma_\mu(q, r, p)$$

TAKAHASHI IDENTITY

$$q^\mu \Gamma_\mu(q, r, p) = D^{-1}(p^2) - D^{-1}(r^2)$$

$\begin{matrix} q \rightarrow 0 \\ p \rightarrow -r \end{matrix}$

TAYLOR EXPANSION

WARD IDENTITY

$$\Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

TENSORIAL DECOMPOSITION

$$\Gamma_\mu(0, r, -r) = L_{sg}^*(r^2) r_\mu$$



$$L_{sg}^*(r^2) = 2 \frac{\partial D^{-1}(r^2)}{\partial r^2}$$

SCHWINGER MECHANISM ON

$$\mathbb{\Gamma}_\mu(q, r, p) = \underbrace{\Gamma_\mu(q, r, p)}_{\text{POLE-FREE}} + \frac{q_\mu}{q^2} C(q, r, p)$$

THE TAKAHASHI IDENTITY DOES NOT CHANGE

$$\begin{aligned} q^\mu \mathbb{\Gamma}_\mu(q, r, p) &= q^\mu \Gamma_\mu(q, r, p) + C(q, r, p) \\ &= D^{-1}(p^2) - D^{-1}(r^2) \end{aligned}$$

$q \rightarrow 0$

TAYLOR EXPANSION

WARD IDENTITY

$$\Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q, r, p)}{\partial p^2} \right]_{q=0}}_{\mathbb{C}(r^2)}$$

TENSORIAL DECOMPOSITION

$$\Gamma_\mu(0, r, -r) = L_{sg}(r^2) r_\mu$$



$$L_{sg}(r^2) = L_{sg}^*(r^2) - \underbrace{2 \mathbb{C}(r^2)}_{\text{DISPLACEMENT}}$$

DISPLACEMENT FROM LATTICE QCD : APPLY THIS IDEA ON THE THREE-GLUON VERTEX

A.C.AGUILAR, F. DE SOTO, M.N.FERREIRA, J.P., F. PINTO-GOMEZ, C.D. ROBERTS, J. RODRIGUEZ-QUINTERO, PHYS.LETT. B 841 (2023) 137906

SLAVNOV-TAYLOR IDENTITY:

$$q^\alpha \Gamma_{\alpha\mu\nu}(q, r, p) + C_1(q, r, p) g_{\mu\nu} = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

TAYLOR EXPANSION



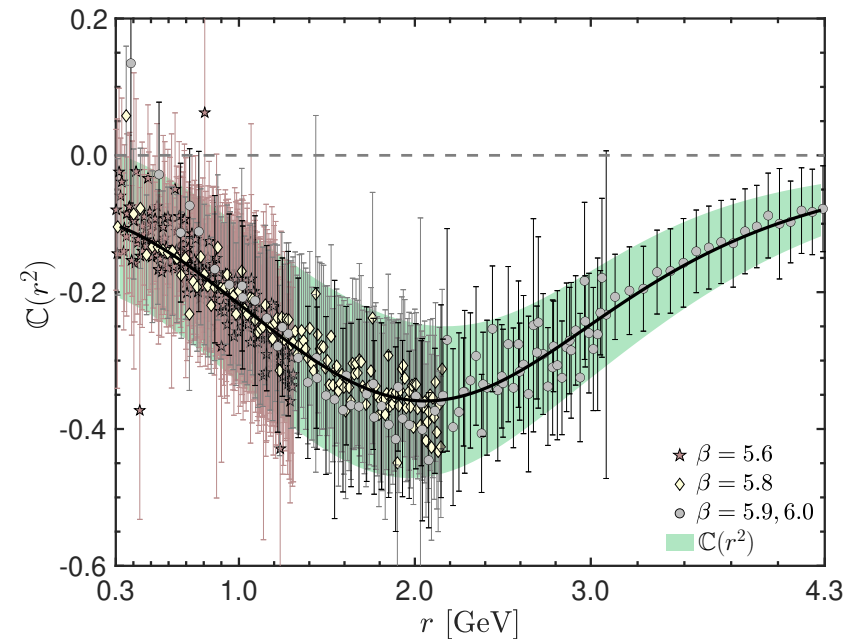
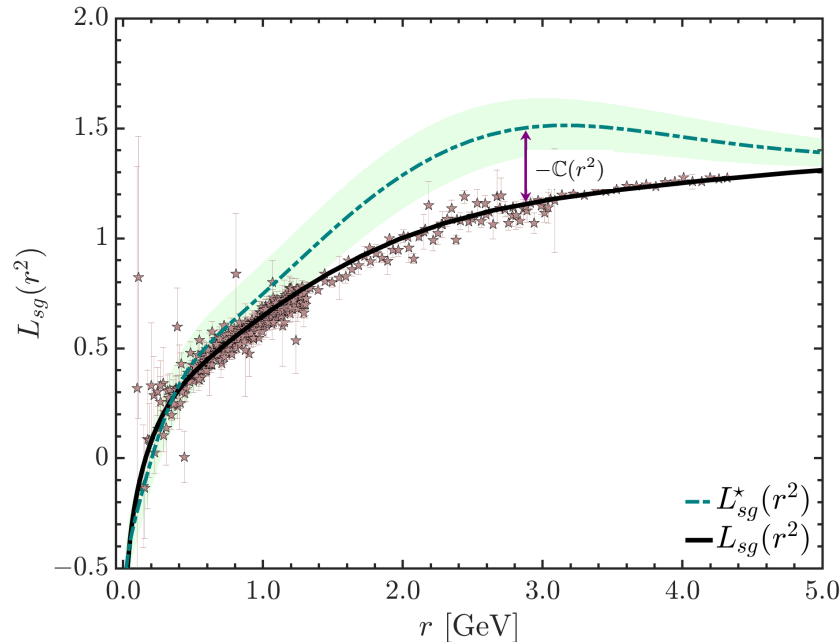
WARD IDENTITY

$$\underbrace{L_{sg}(r^2)}_{\text{LATTICE QCD}} = \underbrace{L_{sg}^*(r^2)}_{\text{COMPUTE WITH INPUTS FROM LATTICE QCD}} + \underbrace{\mathbb{C}(r^2)}_{\text{DISPLACEMENT FUNCTION}}$$

LATTICE QCD

COMPUTE WITH INPUTS
FROM LATTICE QCD

DISPLACEMENT
FUNCTION



35.7 σ AWAY FROM THE NULL HYPOTHESIS!

CONCLUSIONS

GLUON SELF-INTERACTIONS GENERATE A DYNAMICAL MASS SCALE IN THE GAUGE SECTOR OF QCD

DYNAMICS AND SYMMETRY ARE TIGHTLY INTERTWINED

FULL CONFIRMATION OF THE ACTION OF THE SCHWINGER MECHANISM FROM LATTICE QCD

TRULY MASS FROM NOTHING

