# Computers, hadrons and the QCD coupling

(LatticeQCD activities at IFIC)

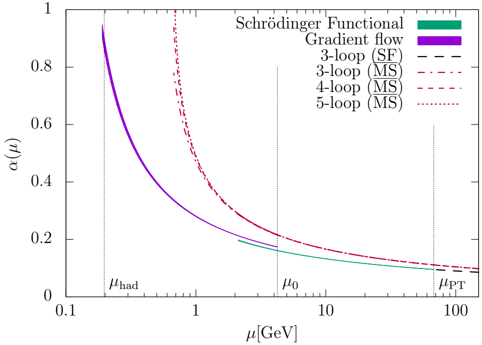
Alberto Ramos <alberto.ramos@ific.uv.es> IFIC (CSIC/UV)

# QCD: THE THEORY OF STRONG INTERACTIONS

**Very simple theory**

$$S = \int \mathrm{d}x^4 \left\{ \frac{1}{4g^2} F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i(\gamma_\mu D_\mu + m_i)\psi_i \right\}$$

- Only $N_f + 1$ free parameters: $g^2, m_i$
- Incredibly rich phenomena: full hadron spectrum




**Difficult theory: "free" at high eneries, strongly coupled at low energies**

- At high energies $g^2(\mu) \to 0$: perturbation theory is reliable
- At low energies $\alpha(\mu) \equiv g^2(\mu)/(12\pi) \approx 1$: perturbation theory breaks down
- We need "non-perturbative framework" to describe low energy physics

# Computing path integrals: Lattice field theory

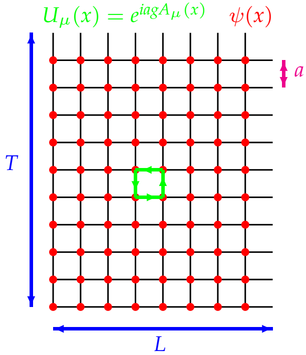
**Non Perturbative definition of QFT**

- Discretize space-time in an hyper-cubic lattice (spacing $a$)
- Path integral $\longrightarrow$ multiple integral (one variable for each field at each point)
- Compute the integral numerically $\to$ Monte Carlo sampling.

$$\langle O\rangle = \frac{1}{N_{\rm conf}}\sum_{i=1}^{N_{\rm conf}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\rm conf}})$$

Observable computed averaging over samples

- This works both in the perturbative and non-perturbative regimes!




$$S_G[U] = \frac{\beta}{6}\sum_{p\in\text{Plaquettes}} Tr(1 - U_p - U_p^+) \xrightarrow[a\to 0]{} -\frac{1}{2}\int d^4x\, \text{Tr}(F_{\mu\nu}F_{\mu\nu})$$

# Lattice QCD Timeline

| | |
|---|---|
| 1974 | First formulation of a non-Abelian gauge theory on a space-time lattice **[Wilson. Phys.Rev D10 (1974)]**. |
| 1980 | First lattice simulation: Pure $SU(2)$ gauge theory in a lattice up to $10^4$. **[Creutz. Phys.Rev D21 (1980)]**. |
| 1985 | Firsts unquenched simulations: $2 \times 4^3$ to $4 \times 8^3$ lattices. **[Duane, Kogut. Phys.Rev.Lett. 55 (1985)]**. |
| '90s | Quenched lattice QCD reign. Formally large $N_c$ limit of QCD. Error $\sim 1/N_c \approx 30\% \rightarrow$ Uncontrolled systematics. |
| '00s | Unquenched simulation at "large" volumes comes up. Many $N_\mathrm{f} = 2$ simulations. Large quark masses ($M_\pi \approx 600$ MeV) $\rightarrow$ uncontrolled chiral extrapolation |
| '10s | Reaching the physical point. |
| Present | Precision lattice QCD era: Large volumes, physical quark masses, QED (partially!), etc... |

# Reaching the physical point: The fall of the Berlin wall

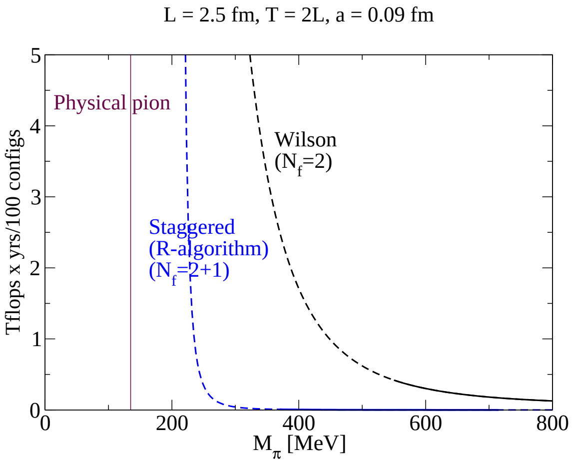



Figure: 2001 Berlin conference

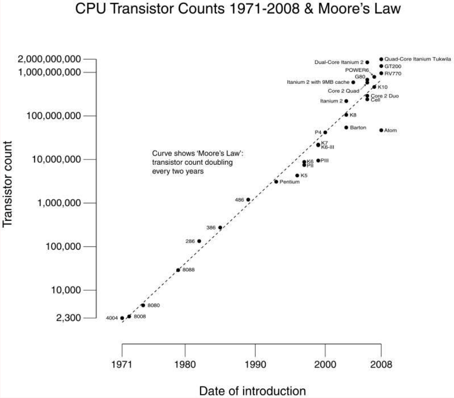
# Reaching the physical point: The fall of the Berlin wall

## Algorithmic improvements

- Schwartz-preconditioned Hybrid MC **[Lüscher '03-'04; PACS-CS '08]**
- Multiple time scale integration **[Hasenbusch '01;BMW '07]**
- Mass preconditioning **[Urbach '06; BMW '07]**
- Deflation acceleration **[Lüscher '07]**
- Better understanding of the simulations $\Rightarrow$ more intelligent choice of simulation parameters.

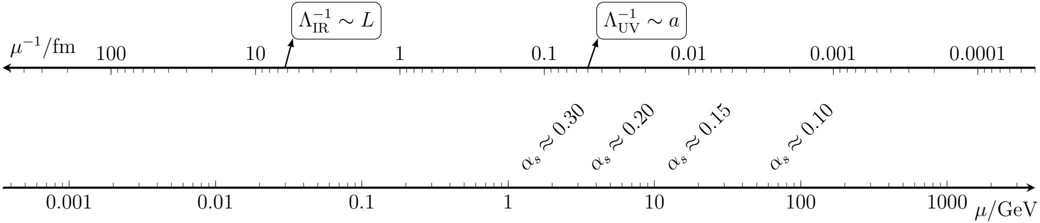
Figure: LUMI Supercomputer (EuroHPC/Finland)


## Computing power




Moore's law

# Systematic in Lattice QCD




**Lattice QCD simulations are performed**

- At non-zero lattice spacing $\Longrightarrow$ Continuum extrapolation
- In finite volume $\Longrightarrow$ Infinite volume extrapolation
- At non-physical values of quark masses $\Longrightarrow$ Chiral dependence

**Lattice QCD needs to understand**

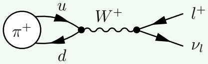
- Some observables require renormalization $\Longrightarrow$ NPT/PT
- Heavy quarks (i.e. is $am_h \ll 1$??) $\Longleftarrow$ EFT
- Multi-hadron states(?), QED, ...

# Lattice QCD state of the art

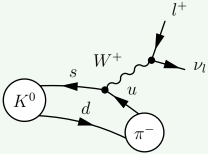
**Not all results are on an equal footing!**

- Key input for flavor physics. FLAG is your friend! [http://flag.unibe.ch/2021/]

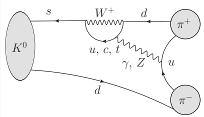
- "Very Easy": meson decay constants




- "Easy": semileptonic decays




- "Very hard": $K \to \pi\pi$ and $\epsilon'/\epsilon$




- Still need to understand how to many things!: QED corrections to hadronic processes!
- Enormous progress in determination of PDF's: Soon useful information for some kinematics!
- Large activities understanding multi-particle systems and resonances

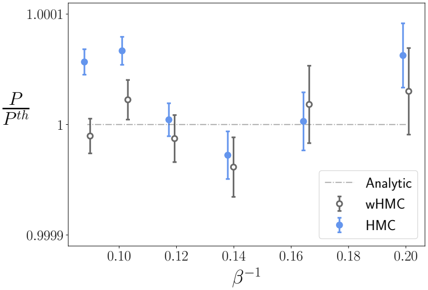
# Advances in lattice QCD

- **New ideas** (based on QFT)
- **New computational strategies**
- **Phenomenological relevant results** (largish collaborations)

# Topology freezing

David Albandea [Eur.Phys.J.C 81 (2021) 10, 873],

- Preferred algorithm in Lattice QCD (HMC) is a continuous transformation of the (lattice) fields
- As one approaches the continuum, Topology is difficult to change
- Might compromise correctness of the algorithm



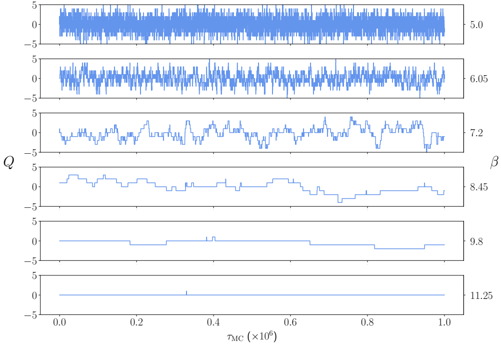




Figure: Topology freezing in $U(1)$ gauge theories

# Dependence of observables with action parameters

Guilherme Telo [2307.15406]

- Perturbation theory on top of non-perturbative bweckgrounds
  - QCD+QED
  - $\theta$ vacuum dependence on observables
  - Application to the quantum rotor [D. Albandea, G. Telo]
- Potential for large gains in statistical precision

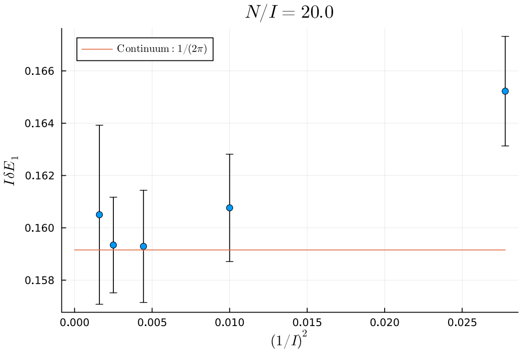



Figure: Dependence of the energy levels in the Quamtum rotor on $\theta$

# Understanding Multi-particle states

Jorge Baeza [JHEP 06 (2022) 049]

## $\pi\pi$ Scattering at large $N_c$

- Phase shift using Lüscher formalism
- shed some light to $\Delta I = 1/2$ rule
- Anticorrelation of the leading $O(1/N_c)$ and $O(N_f/N_c^2))$ corrections in $A_0, A_2$

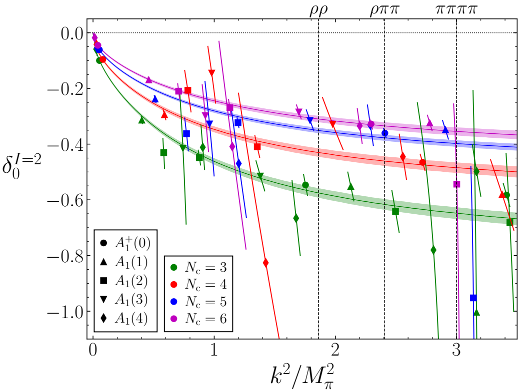



Figure: Phase shift for isospin=2

# The determination of $\alpha_s$

## Lattice determinations

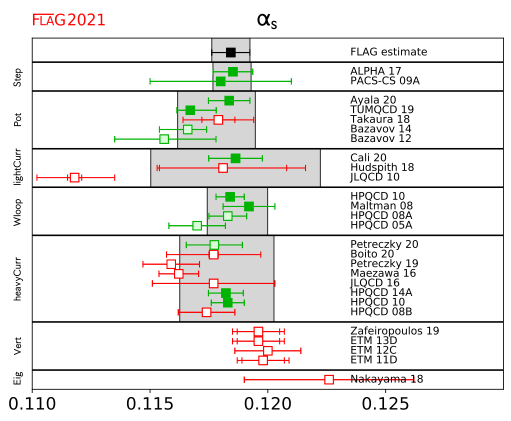



Figure: Source: FLAG

## Pheno determinations

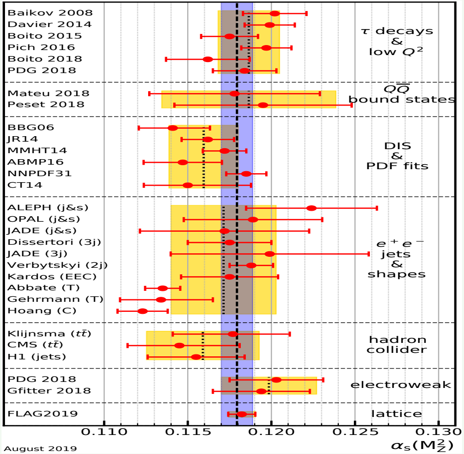



Figure: Source: PDG

# Why is so difficult? Systematic uncertainties related with PT

$$O(Q) \overset{Q\to\infty}{\sim} \alpha_s(Q) + \sum_{n=2} c_n \alpha_s^n(Q) + \mathcal{O}(\alpha_s^{N+1}(Q)) + \mathcal{O}\left(\frac{\Lambda^p}{Q^p}\right) + \dots$$

**Non-perturbative corrections**

- Difficult to compute (NP physics is difficult!)
- **Better use smaller $\alpha \implies$ (larger $Q$)**

**Perturbative corrections**

- Difficult to estimate (i.e. scale variation might fail)
- Main source of uncertainty in most lattice QCD extractions of $\alpha_s$
- **Better use smaller $\alpha \implies$ (exponentially larger $Q$)**

# The problem: $\alpha_s$ extractions are a multi-scale problem

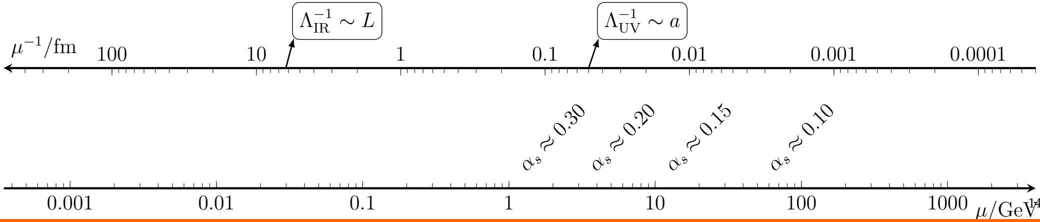
$$O(Q) \overset{Q\to\infty}{\sim} \alpha_s(Q) + \sum_{n=2} c_n \alpha_s^n(Q) + \mathcal{O}(\alpha_s^{N+1}(Q)) + \mathcal{O}\left(\frac{\Lambda^p}{Q^p}\right) + \dots$$

**Why not just use larger $Q$?**

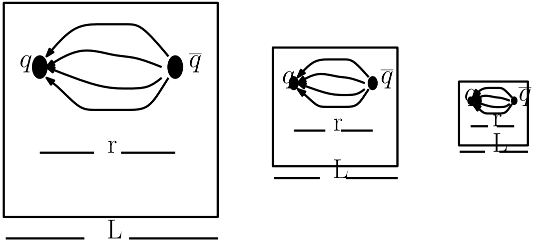
Experimentalist: At large $Q$ the effect you are trying to measure is "weak" $\Longrightarrow$ Larger uncertainties

Latticero: In all simulations $a^{-1} \gg Q \gg L^{-1}$. You need $m_\pi L \approx 4$, so with current computers ($L/a \approx 128$) we have $Q \ll 4$ GeV. In fact:

- Computer cost $\propto (L/a)^7$
- Non-perturbative uncertainties $\propto (a/L)^p$
- Perturbative uncertainties $\propto 1/\log(L/a)$



# The soution: Finite size scaling [Lüscher, Weisz, Wolff '91]



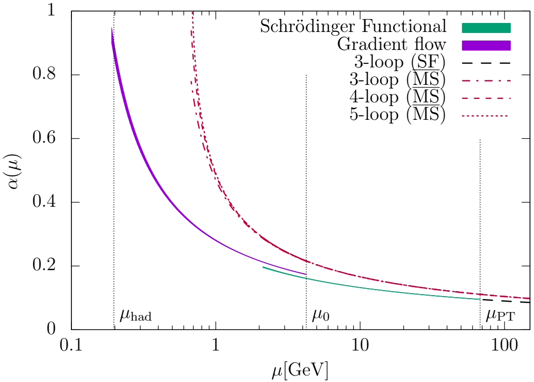

**Finite volume renormalization schemes: fix $QL$ = constant**

- Coupling $\alpha(Q)$ depends on no other scale but $L$ (Notation: $\alpha(L), \alpha(1/L)$).
- Small $L \Longrightarrow$ small $\alpha(L)$
- $a \ll 1/Q$ easily achieved: $L/a \sim 10 - 40$
- Step scaling function: How much changes the coupling when we change the renormalization scale:

$$\sigma(u) = g^2(Q/2)\Big|_{g^2(Q)=u}$$

achieved by simple changing $L/a \to 2L/a$!

# Results for $\alpha_s(M_Z)$ [ALPHA '17. Phys.Rev.Lett (2017) 119. [arXiv:1706.03821]]




- Non-perturbative running from 200 MeV to 140 GeV
- Very precise result

$$\alpha_s(M_Z) = 0.11852(84)\ [0.7\%]\,.$$

**Still dominated by statistics!**

- Many technical improvements:
  - Gradient flow coupling [P. Fritzsch, AR. '13]
  - Symanzik analysis of cutoff effects [AR, S. Sint '16]
  - …

**Many simulations of the femto-universe**

QCD in a small universe to determine the running coupling!

# 3M: universe with 3 heavy quarks $M \gg \Lambda$

**Alice uses fundamental theory**

$$S_{\rm fund}[A_\mu,\psi,\bar\psi] = \int \mathrm{d}^4x \left\{ -\frac{1}{2g^2}\mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu}) + \sum_{i=1}^{3}\bar\psi_i(\gamma_\mu D_\mu + M)\psi_i \right\}$$

**Bob uses effective theory**

$$S_{\rm eff}[A_\mu] = -\frac{1}{2g_{\rm eff}^2}\int \mathrm{d}^4x\,\{\mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu})\} + \cancel{\frac{1}{M^2}\sum_k \omega_k \int \mathrm{d}^4x\,\mathcal{L}_k^{(6)}} + \ldots$$

**Decoupling**

- Dimensionless "low energy quantities" $\sqrt{t_0}/r_0, w_0/\sqrt{8t_0}, r_0/w_0, \ldots$ from effective theory

$$\frac{\mu_1^{\rm fund}(M)}{\mu_2^{\rm fund}(M)} = \frac{\mu_1^{\rm eff}}{\mu_2^{\rm eff}} + \mathcal{O}\left(\frac{\mu^2}{M^2}\right)$$

**Strong coupling from pure gauge!**

$$\Lambda^{(3)} = \lim_{M\to\infty} \mu_{\rm dec}(M) \times \frac{\Lambda^{(0)}}{\mu'_{\rm dec}} \times \frac{1}{P(\Lambda/M)}$$

We need

- Running in pure gauge: $\Lambda^{(0)}/\mu'_{\rm dec}$
- A scale in a world with degenerate massive quarks: $\mu_{\rm dec}(M)$ in fm/MeV.

# $\alpha_s$ from decoupling [ALPHA: Phys.Lett.B 807 (2020) 135571, Eur.Phys.J.C 82 (2022) 12, 1092]

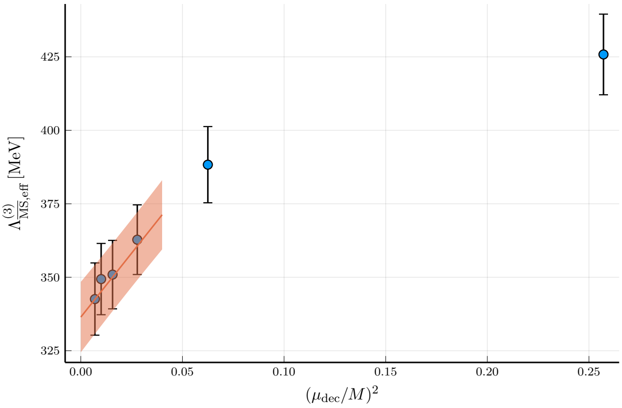
- Use NP coupling as matching condition ($\{N_f = 3, M\} \leftrightarrow \{N_f = 0\}$)

$$\bar{g}^2(\mu_{\text{dec}}(M))\Big|_{N_f=3,M,T=2L} = \bar{g}^2(\mu_{\text{dec}})\Big|_{N_f=0,T=2L}\,.$$

- Simulate quark masses $M \approx 2 - 10$ GeV
- Precise result

$$\alpha_s(m_Z) = 0.11823(84) \quad [0.7\%]\,.$$

- Dominated by **statistics**
- Can (will!) be improved with new pure gauge computations



# Conclusions

- Lattice QCD activities at IFIC
  - Development of new methods (toy models, small groups)
    - Fighting topology freezing
    - Using Machine Learning approaches
    - Computation of electromagnetic corrections
- Results in QCD of phenomenological relevance
  - Determination of $\alpha_s$ (currently dominate world average)
  - Applications in multi-scale problems:
    - Determination of quark masses
    - Heavy quarks
- Well connected internationally
  - ALPHA collaboration: Madrid, DESY, Wuppertal,...
  - Seattle, Edinburgh, ...

**Challenge**

Keep up with the high level brought by the PhD students!

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