Meson and Glueball spectroscopy within the Graviton Soft Wall Model

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Outline

- Some words about Glueballs
- Holographic models generalities
- The Graviton Soft Wall Model
- QCD Phase Transition in holography
- Concluding Remarks

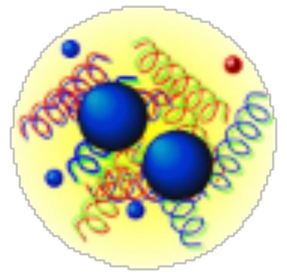
· Some words about Glueballs

From QCD:

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum \bar{\Psi}(i\gamma \cdot D - m)\Psi \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abs} A^b_\mu A^c_\nu$$

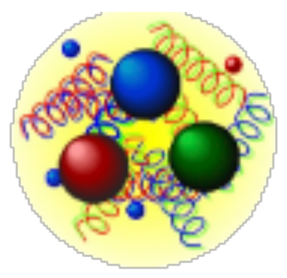
Mesons

$\mathbf{3} \otimes \bar{\mathbf{3}}$



Baryons

$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$



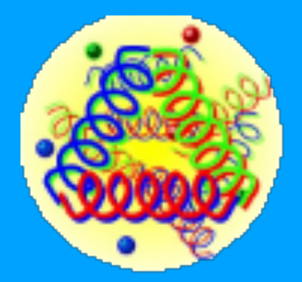
Exotic states

$\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{8}$ — HYBRIDS

$\mathbf{8} \otimes \mathbf{8}$

$\mathbf{8} \otimes \cdots\cdots \otimes \mathbf{8}$

} GLUEBALLS

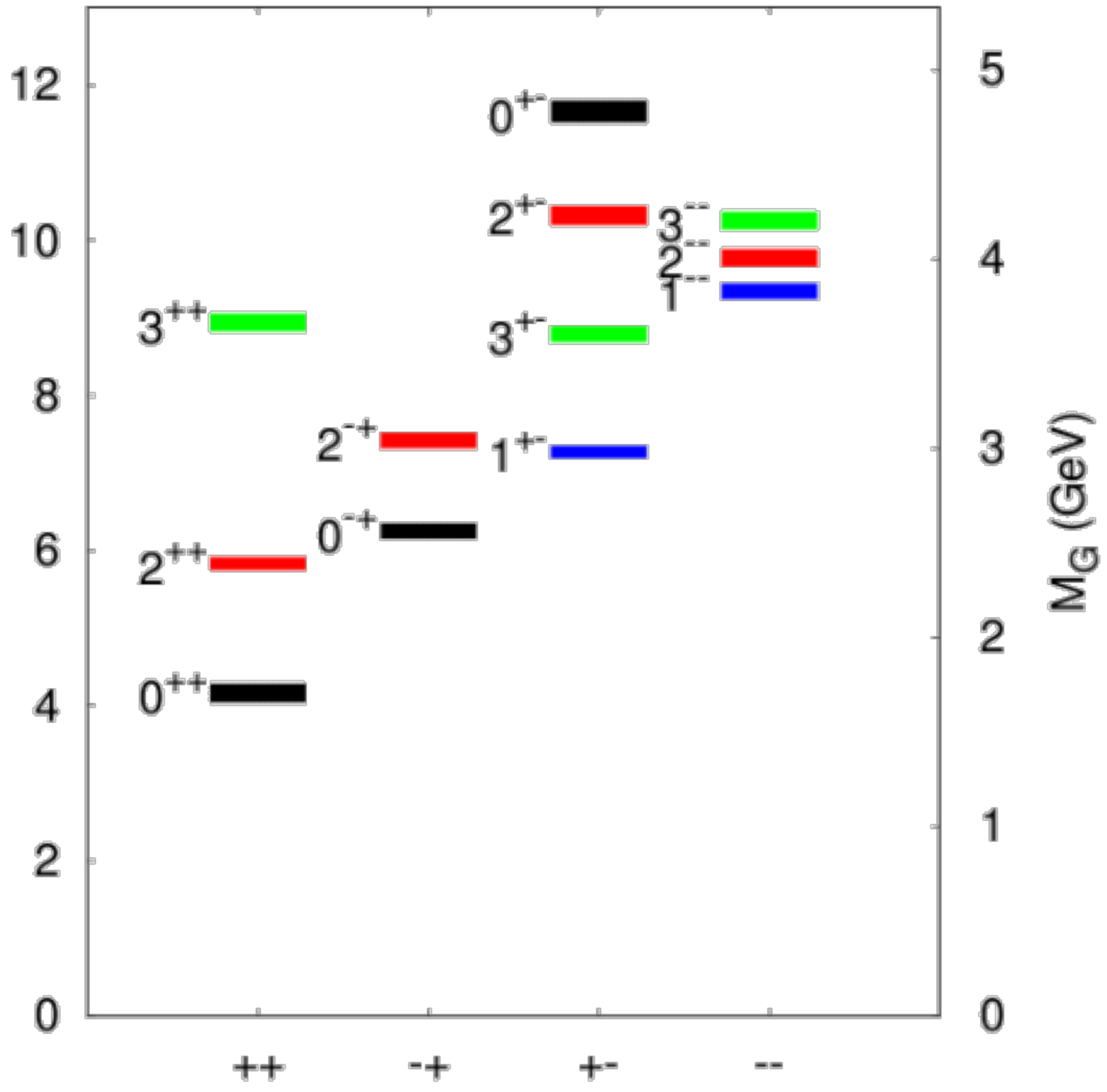


Glueball spectroscopy is a unique laboratory to test non perturbative QCD and CONFINEMENT

However :

- Lattice and Model calculations lead to masses and quantum numbers which are similar to those of mesons ➡ **MIXING**
- Model calculation of decays give varied scenarios.
- Lattice calculations of decays are difficult

Lattice Masses



MP: C.J. Morningstar et al, PRD 60, 034509 (1999)
YC: Y. Chen et al, PRD 73, 014516 (2006)

Different lattice collaborations predict different masses, in particular for the ground state:

LTW: B. Lucini et al, JHEP 06, 012 (2004)

A model dependent extraction from J/ψ decay:

$$M_0 \sim 1865 \pm 25^{+10}_{-30}\ \mathrm{MeV}$$

SDTK: E. Klempt et al PLB 816, 136227 (2021)

We will discuss here only scalar and tensor glueballs

J^{PC}	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277
SDTK	$1865 \pm 25^{+10}_{-30}$					

MP: C.J. Morningstar et al, PRD 60, 034509 (1999)
YC: Y. Chen et al, PRD 73, 014516 (2006)
LTW: B. Lucini et al, JHEP 06, 012 (2004)
SDTK: E. Klempt et al PLB 816, 136227 (2021)

Models do not seem to clarify the issue because they fit lattice data:
- the ball park 1500-1900 GeV for the lowest scalar glueball

J^{PC}	0^{++}
MP	1730 ± 94
YC	1719 ± 94
LTW	1475 ± 72
SDTK	$1865 \pm 25^{+10}_{-30}$

Meson	$f_0(1500)$	$f_0(1710)$
PDG	1504 ± 6	1723 ± 6

Are the PDG scalar mesons pure mesonic states, glueballs or mixed states?

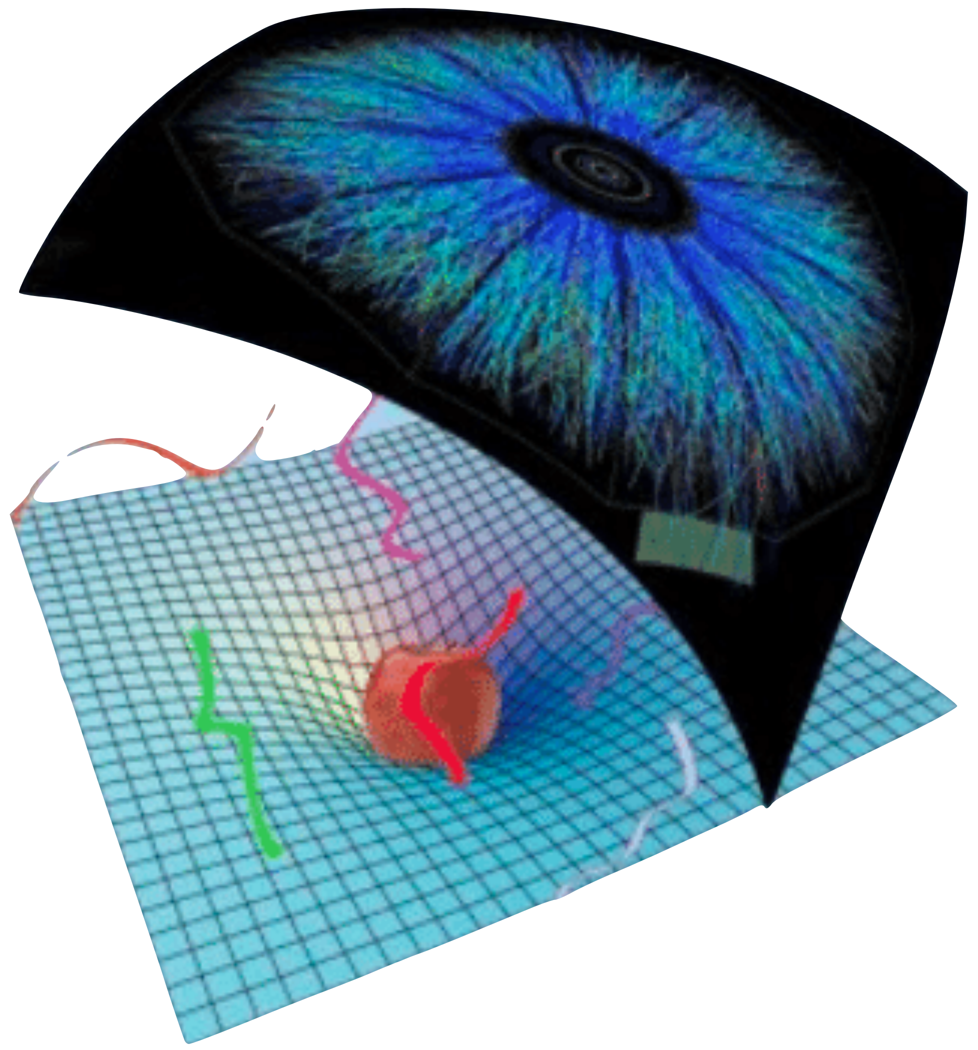
Comment : a fully agreed upon description of glueball decays is not yet in the market

· Holographic models generalities

Maldacena's conjecture:

N=4 SU(N) SYM ⟷ String theory on $AdS_5 \times S_5$

Observables of a QFT in the non-perturbative regime can be calculated exactly in a theory of anti-gravity!



QCD:

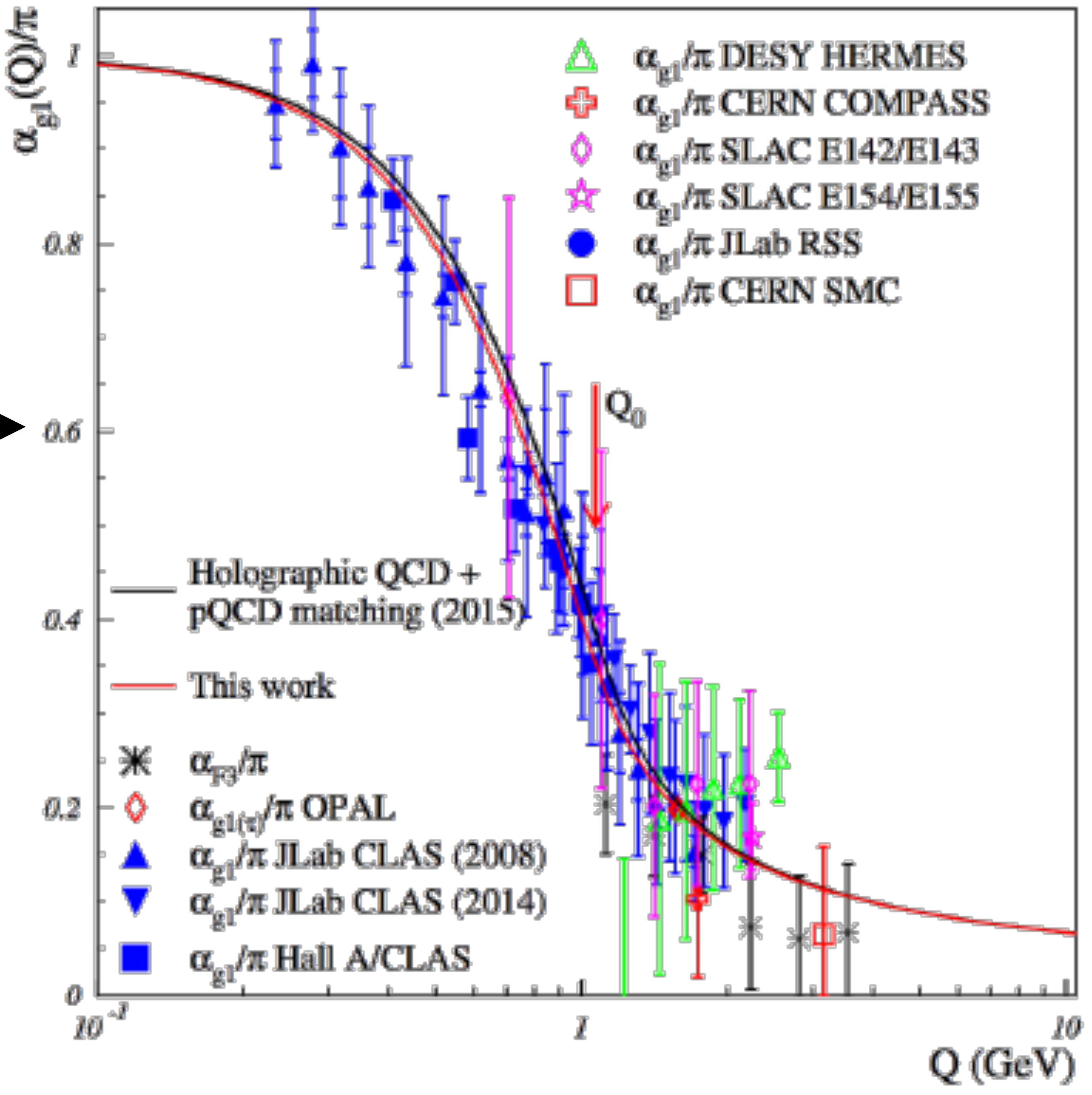
- No supersymmetry
- Conformal symmetry broken
- N is finite
- Confinement

One is able to construct a theory:

· with supersymmetry broken via compactification (Witten)
· approximate conformal symmetry: flattening of constants in the low energy regime and no quark masses

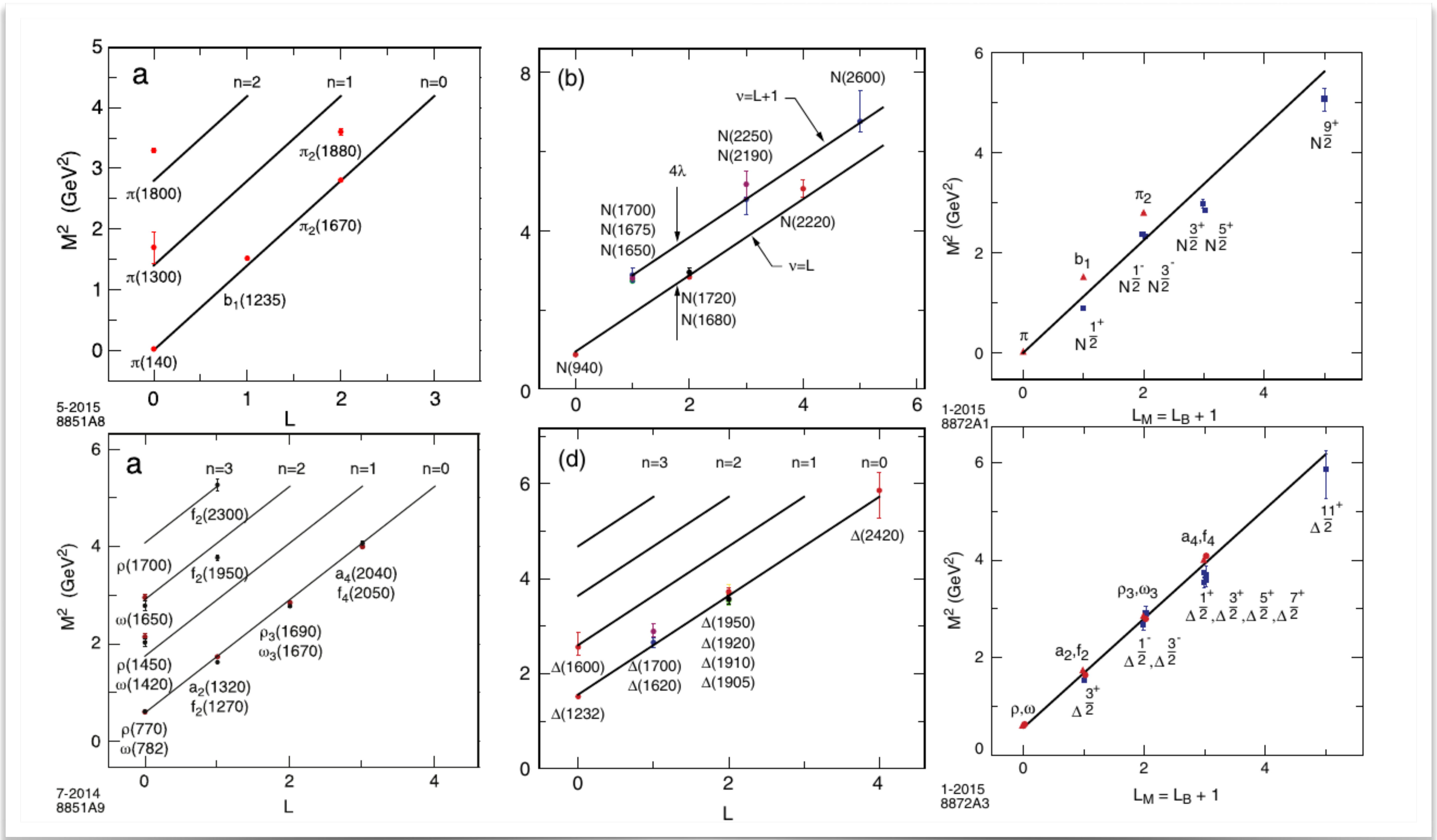
· approximate results in the large N limit ('t Hooft, Witten)

· modify the gravity theory to introduce confinement (bottom up approach) (Polchinsky)



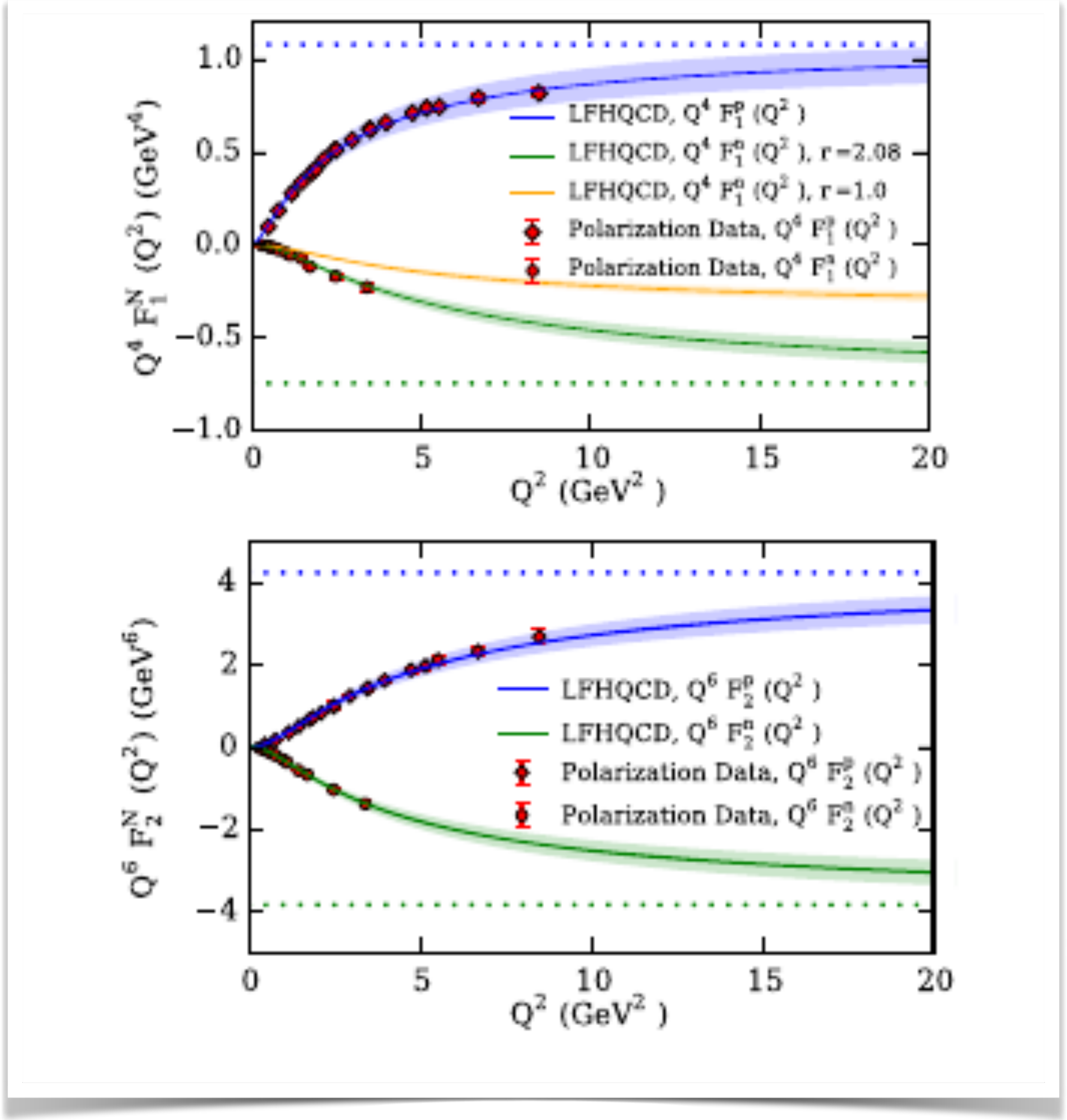
This theory is **NOT** QCD but is **almost** like QCD

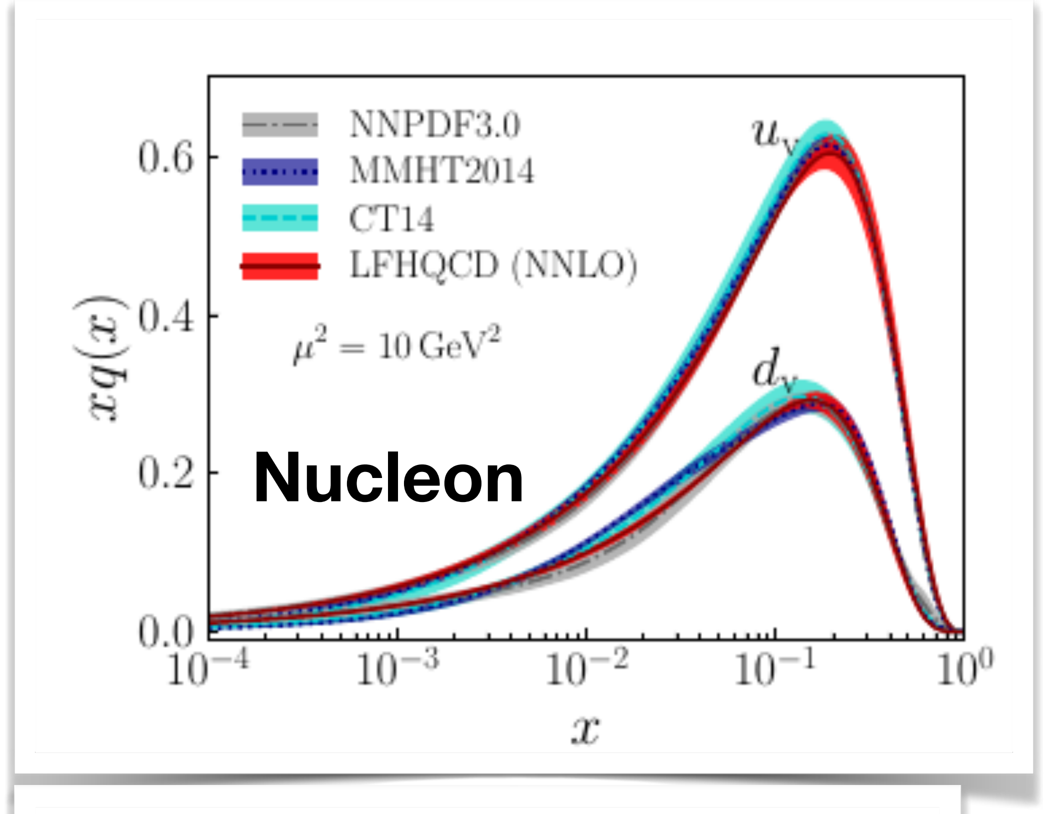
With these ingredients using the Maldacena conjecture one obtains qualitative agreement with conventional hadron spectroscopy and parton distributions

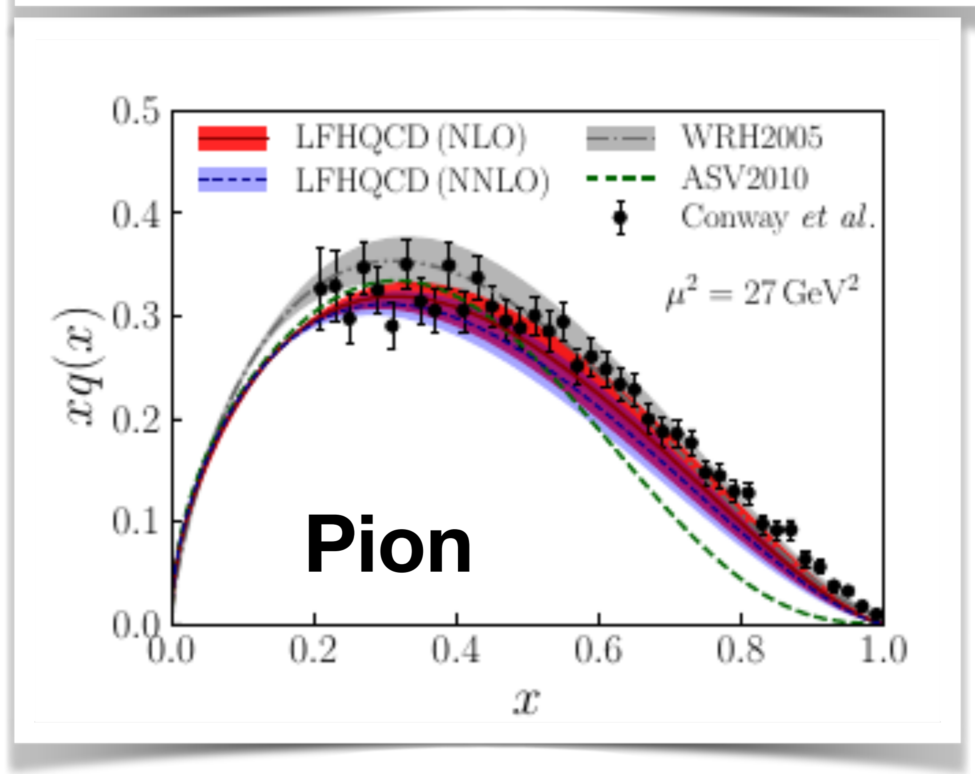


Hadron Spectrum

S.J. Brodsky et al, Phys. Rep. 584 (2015)
H.G. Dosh et al PRD 91, 045040 (2015), 085016 (2015)







Form Factors and Pdfs

S.J. Brodsky et al, Phys. Rep. 584 (2015)
H.G. Dosh et al PRD 91, 045040 (2015), 085016 (2015)

What about Glueballs?

· The Graviton Soft Wall Model

Brief Introduction to Soft Models

Karch et al, PRD 74, 015005 (2006)

The gravitational theory is based on Anti-De Sitter space in (4+1) dimensions:

$$g_{MN}dx^M dx^N = \frac{R^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$

The confinement can be implemented by adding a **Dilaton** in the action:

$$I = \int d^4x dz \sqrt{-g}\, e^{\varphi(z)} \mathscr{L} \qquad \varphi(z) = k^2 z^2$$

From the Euler-Lagrange equations for scalars, vectors… we get the mode functions and the spectrum

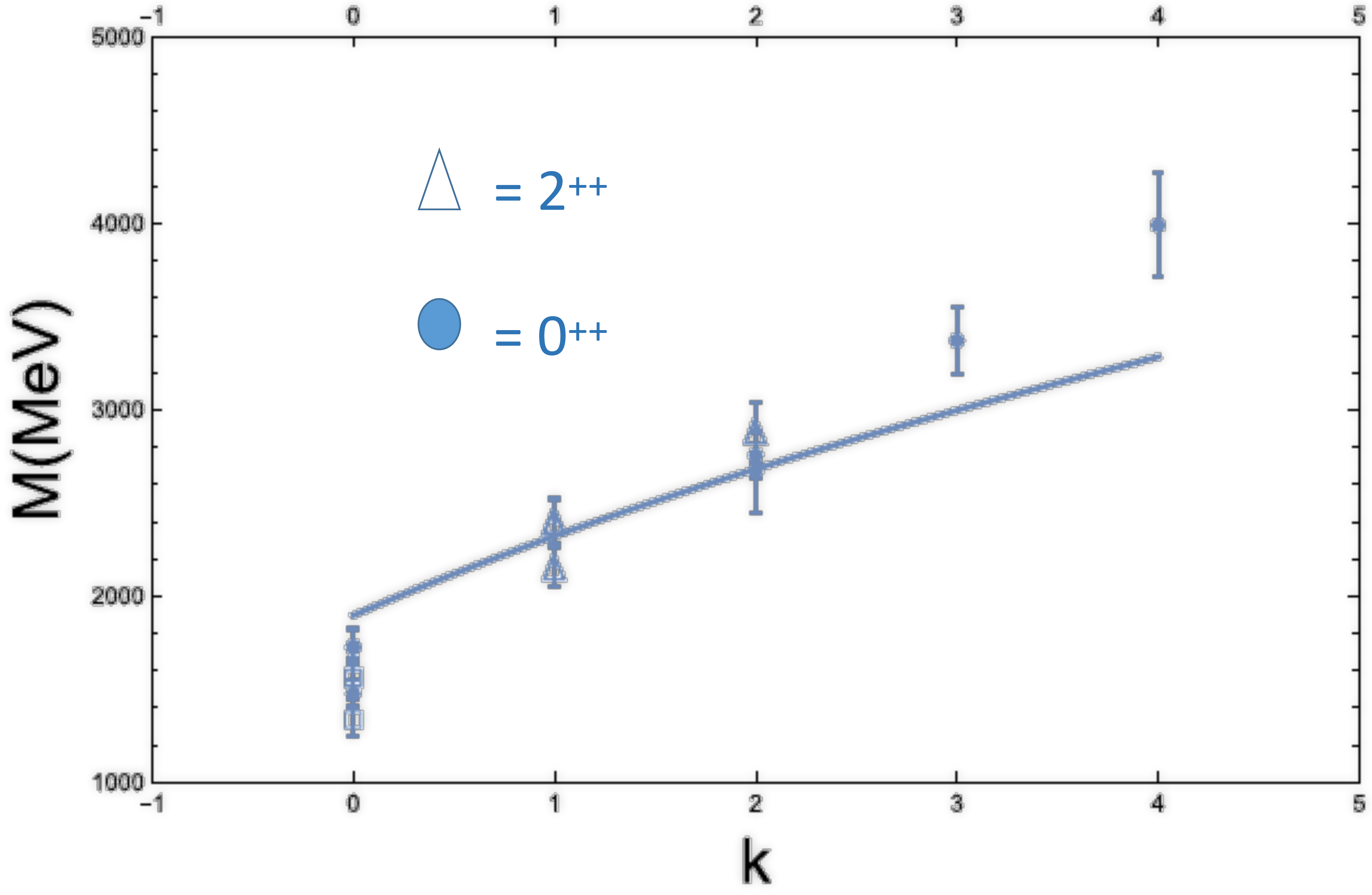
Successful in describing the Regge behavior of the spectrum: $M^2_{n,J} \sim n + j, \qquad j \geq 0$

Be more specific for glueballs

We have the field equations (for the the scalar glueball):

5° dimensional mass $\neq$ the physical one

$$I = \int d^5x \, \sqrt{g} \, e^{-\varphi(z)} \left[g^{MN} \partial_M \mathcal{G} \partial_N \mathcal{G} + M_5^2 \mathcal{G}^2 \right]$$



Eduardo Folco Capossoli et al, PLB 753 (2019) 419-423

$$M_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$$

where Δ is the conformal dimension of the fields,
p the character of the p-form describing the fields

4D	5D	p	Δ	$M_5^2 R^2$
$\overline{q}(x)q(x)$	$\psi(x,z)$	0	3	-3
$Tr(F(x)^2)$	G(x,z)	0	4	0
$\overline{q}\gamma^\mu q(x)$	$V^\mu(x,z)$	1	3	0
......	......	......	......	......

The model needs to be improved and a lot of people tried by modying the metric:

E. F. Capossoli et al, PLB 753, 419-423 (2006)
O. Andreev, PRD 100 (2019) 2, 026013
E. F. Capossoli et al, Chin. Phys. C 44 (2020) 6, 064194
W. de Paula et al, PRD 79, 075019 (2009)
S. Afonin et al, JPG, 49 (2022) 10, 105003

The Graviton Soft Wall Model

We introduced two modifications:

i) a new metric, but most important

ii) the graviton propagating in this modified space becomed the dual of the glueball,

M. Rinaldi and V. Vento EPJA 54 (2018) 151.

i) New metric:

$$\tilde{g}_{MN} dx^M dx^N = e^{-\alpha\varphi(z)} \frac{R^2}{z^2} \left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right) \qquad \varphi(z) = k^2 z^2$$

and we keep the dilaton

$$\tilde{\mathcal{I}} = \int d^5x \, \sqrt{-\tilde{g}} \; e^{-\beta\varphi(x)} \mathcal{L}$$

we fix β to have the same kinetic energy as all the SW models, thus keeping only one free parameter

ii) The glueballs arise from the components of the graviton field (h is the perturbation on the metric $\tilde{g}$ to produce the graviton)

$$-\frac{1}{2}\tilde{h}_{ab;c}^{;c} - \frac{1}{2}\tilde{h}_{c;ab}^{c} + \frac{1}{2}\tilde{h}_{ac;b}^{;c} + \frac{1}{2}\tilde{h}_{bc;a}^{;c} + 4\tilde{h}_{ab} = 0$$

Leading for the scalar glueball to the bound state equation

$$\frac{d^2\phi}{dz^2} + \left(\alpha^2 z - \frac{3}{z}\right)\frac{d\phi}{dz} + \left(\frac{8}{z^2} + 6\alpha^2 + M^2 + 4\alpha^2 z^2\right)\phi - \frac{8}{z^2}e^{-\alpha^2 z^2}\phi = 0.$$

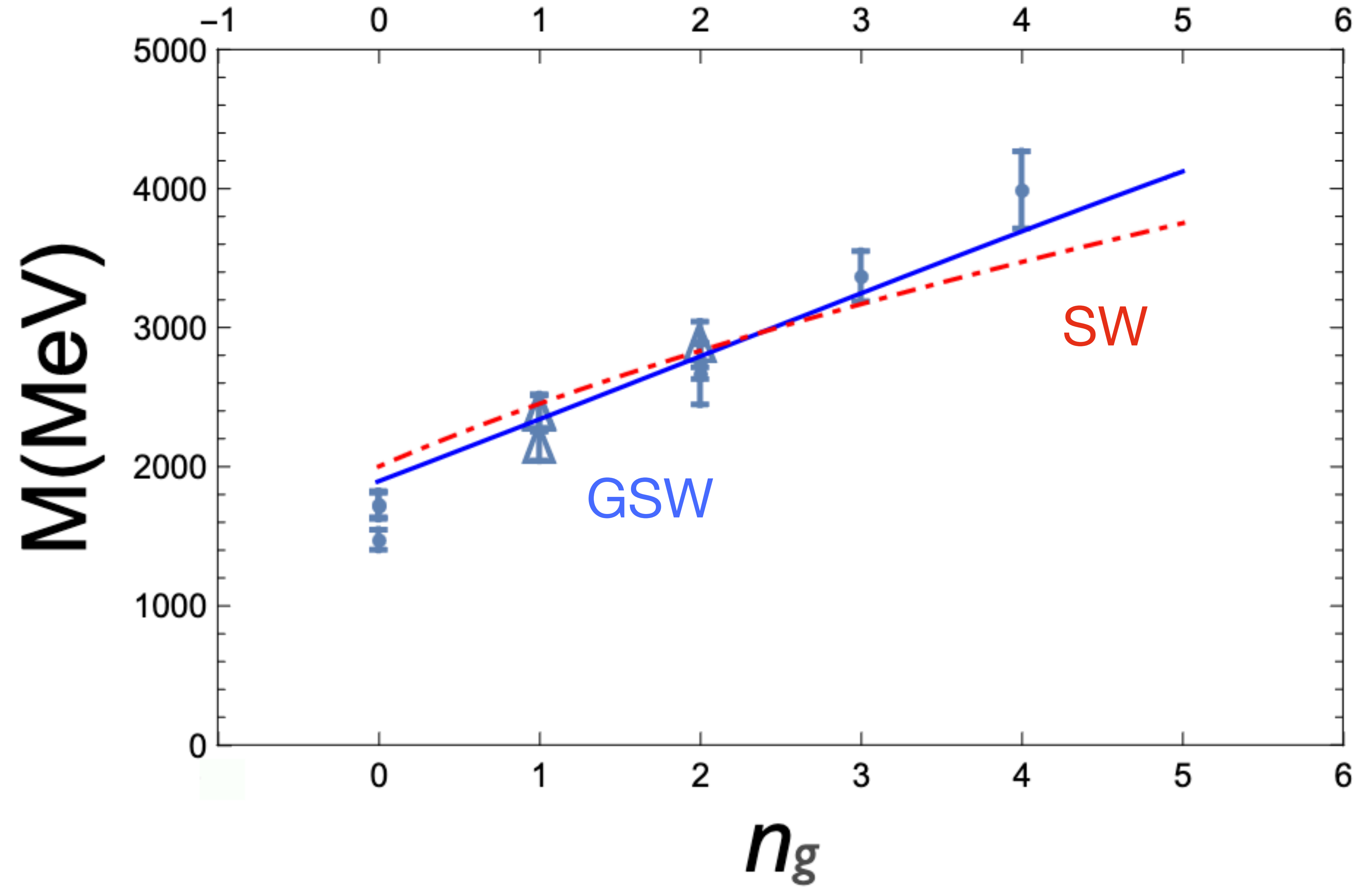
which can be reduced to a Schrödinger type equation

$$-\Psi''(z) + V_{GS}(z)\Psi(z) = M^2\Psi(z),$$

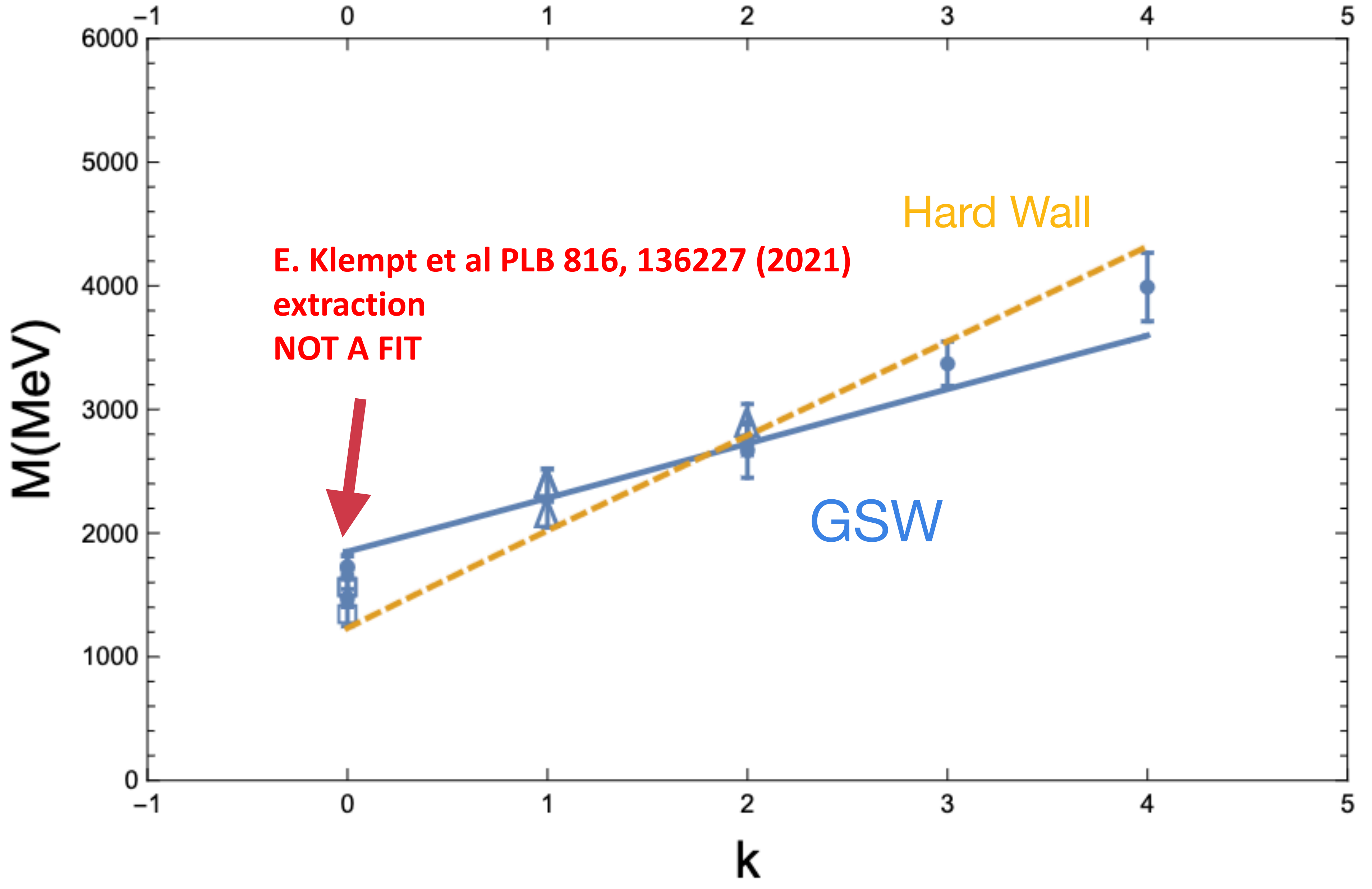
where

$$V_{GS}(z) = \frac{8e^{\alpha^2 z^2}}{z^2} - \frac{17}{4z^2} - 7\alpha^2 - \frac{15\alpha^4 z^2}{4}.$$

Here M is the mass of the mode, which appears after separating the trivial $(\overrightarrow{x}, t)$ part, and z is the fifth dimensional variable.



$\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$



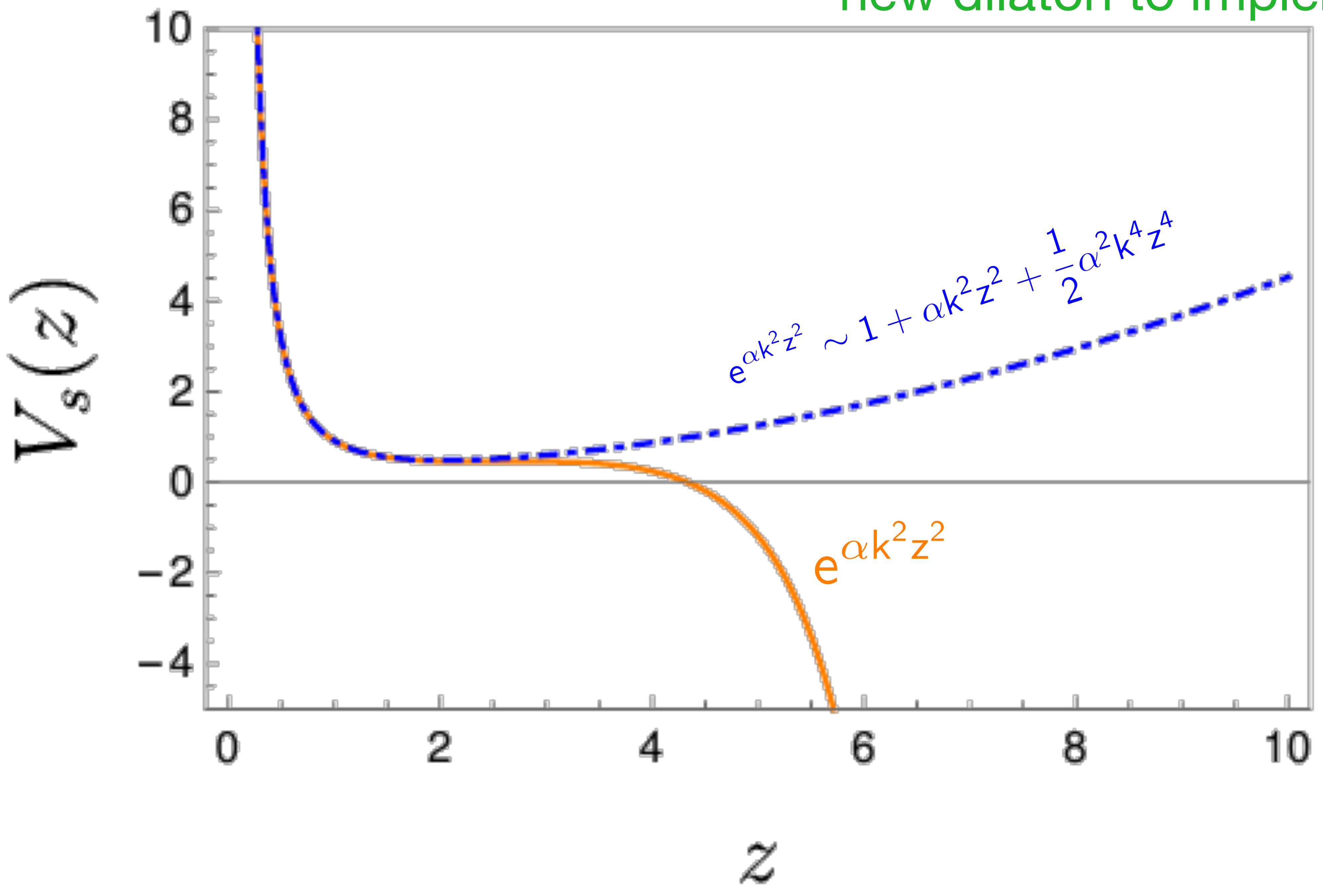
Once the glueballs are described by the metric we proceed to extend the model to the mesons with the metric already fixed.

M. Rinaldi and V. Vento JPG 47 (2020), 5, 055104
M. Rinaldi and V. Vento JPG 47 (2020), 12, 125003

The action for the scalar field becomes,

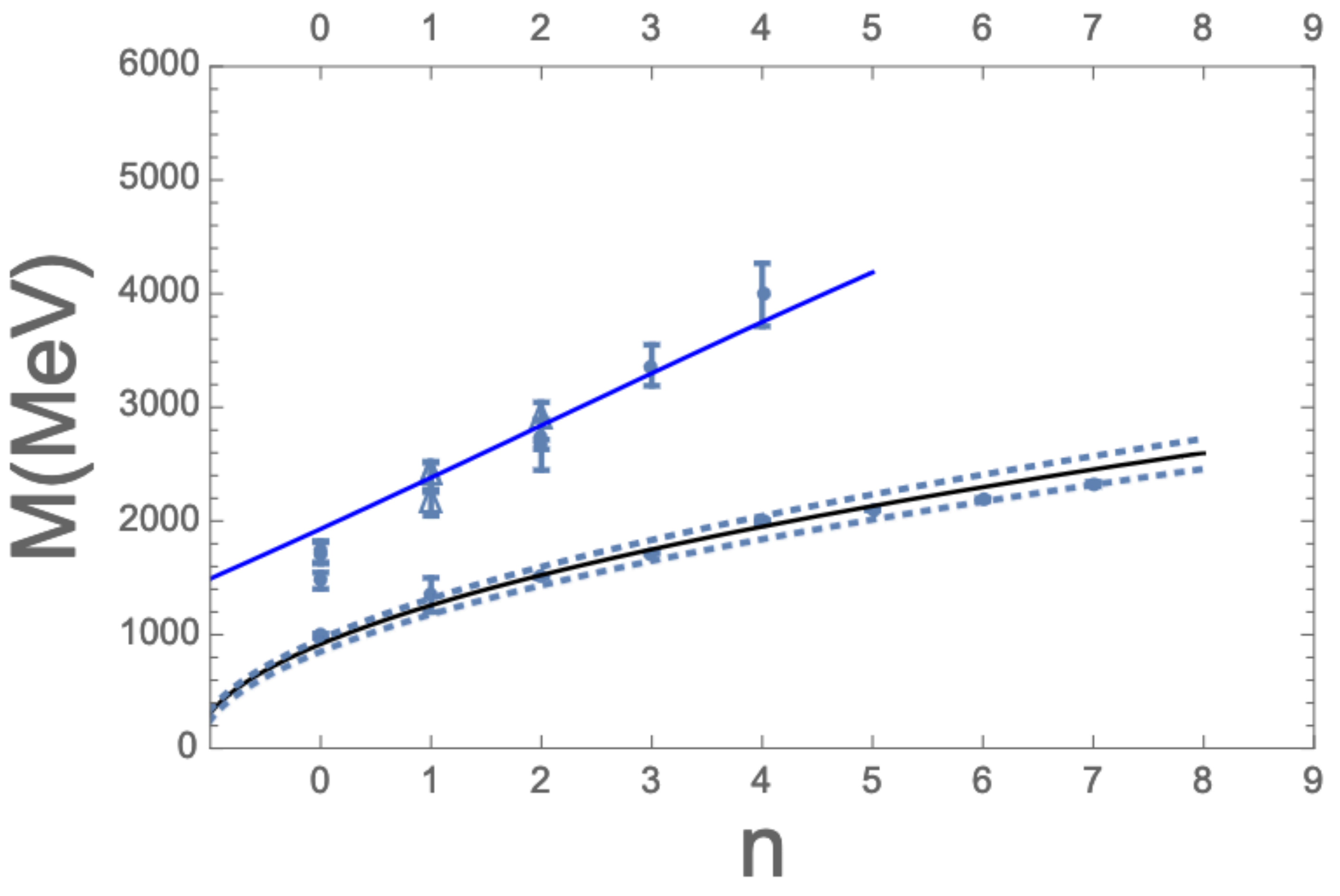
$$\tilde{I} = \int d^5x \sqrt{g}\, e^{-\varphi(z)-\varphi_n(z)} \left[g^{MN}\partial_M \mathcal{S} \partial_N \mathcal{S} + e^{\alpha\varphi(z)} M_5^2 \mathcal{S}^2 \right]$$

new dilaton to implement confinement   contribution from the new metric

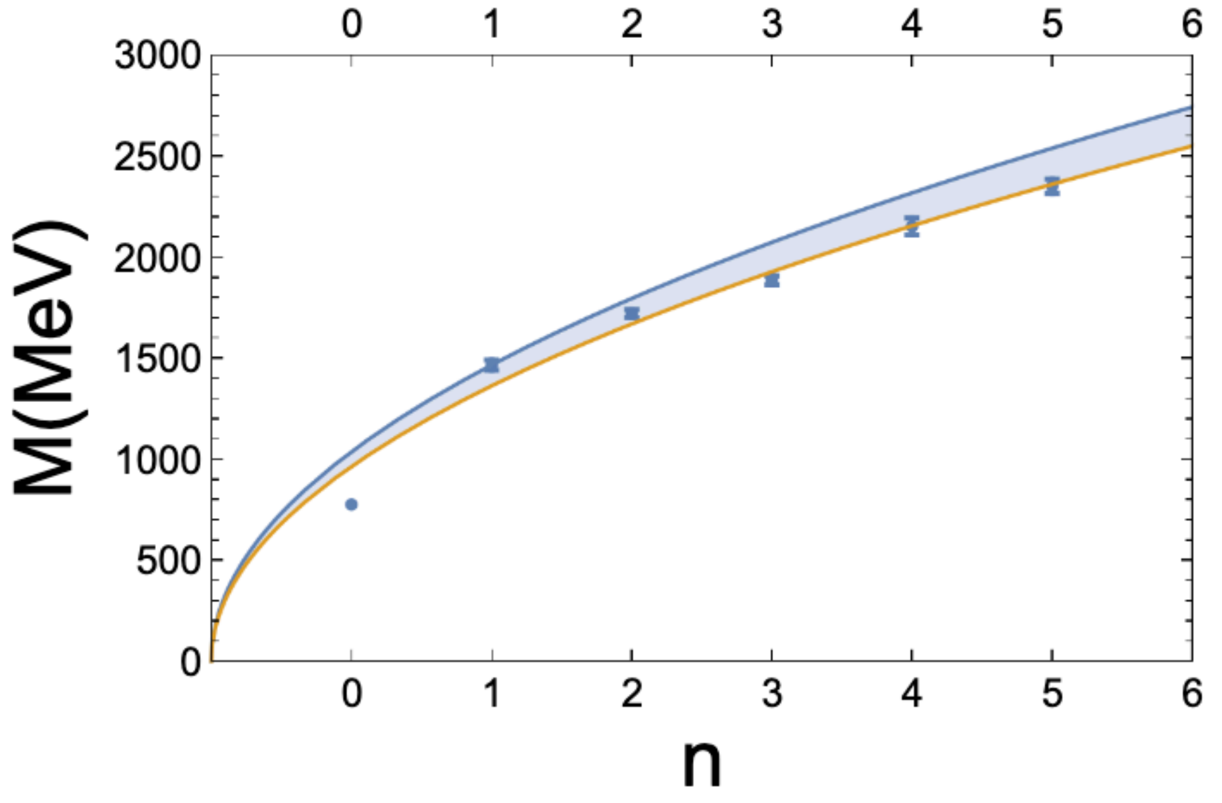


- The additional dilaton guarantees that the potential is binding
- No additional free parameters are introduced

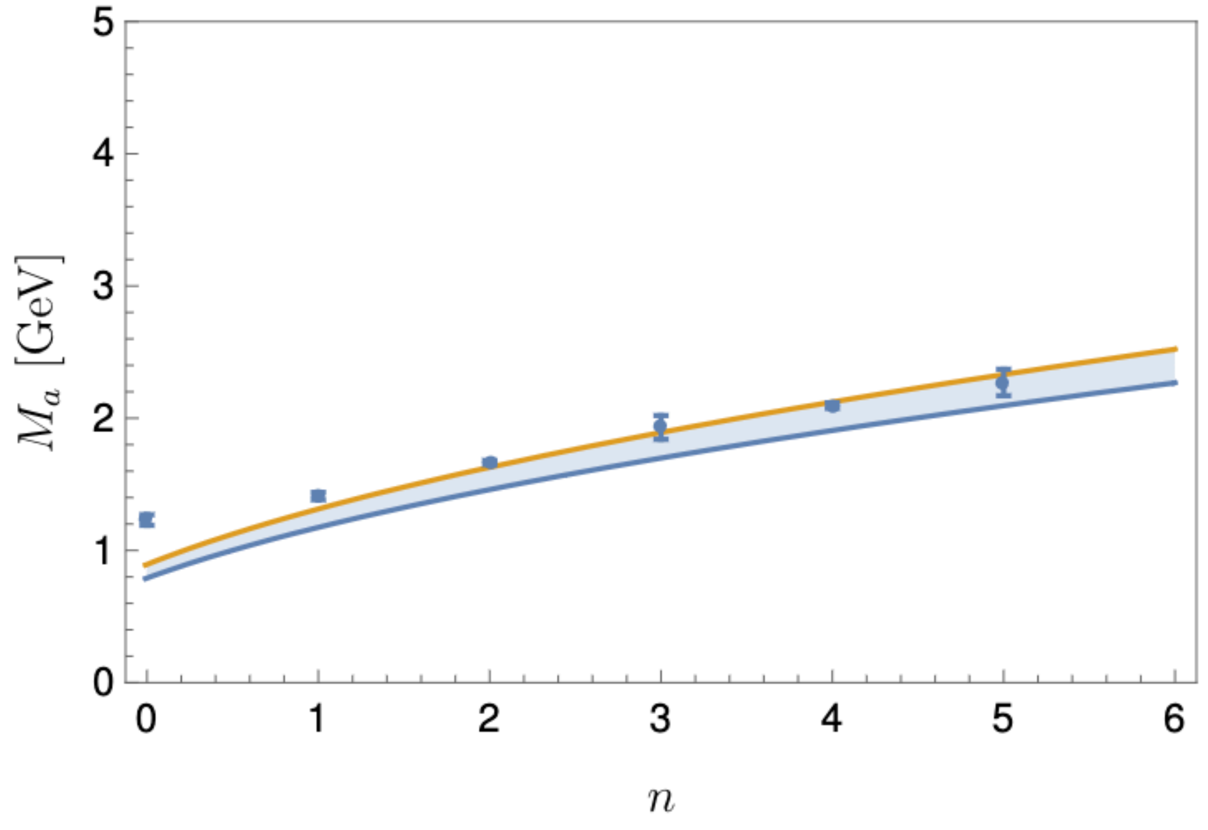
Excellent results for all low lying mesons: the two parameters fixed by the scalar glueball and the scalar mesons $\alpha k^2 = 0.37^2\ \mathrm{GeV}^2$ and $0.51 \leq \alpha \leq 0.59$ M.Rinaldi and V. Vento PRD 104 (2021) 3,034016



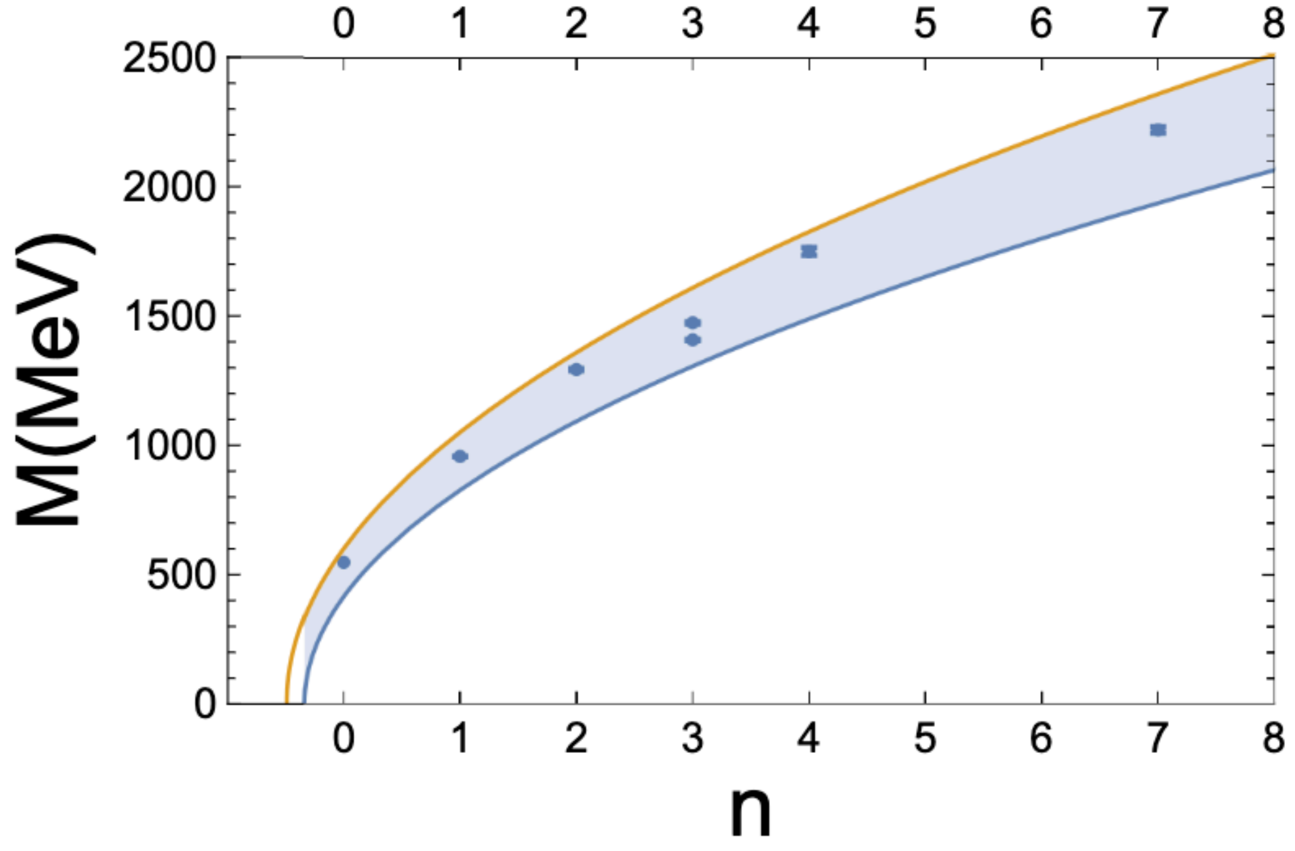
Scalar glueballs and f_0's (not $f_0(500)$)



ρ



a_1



η

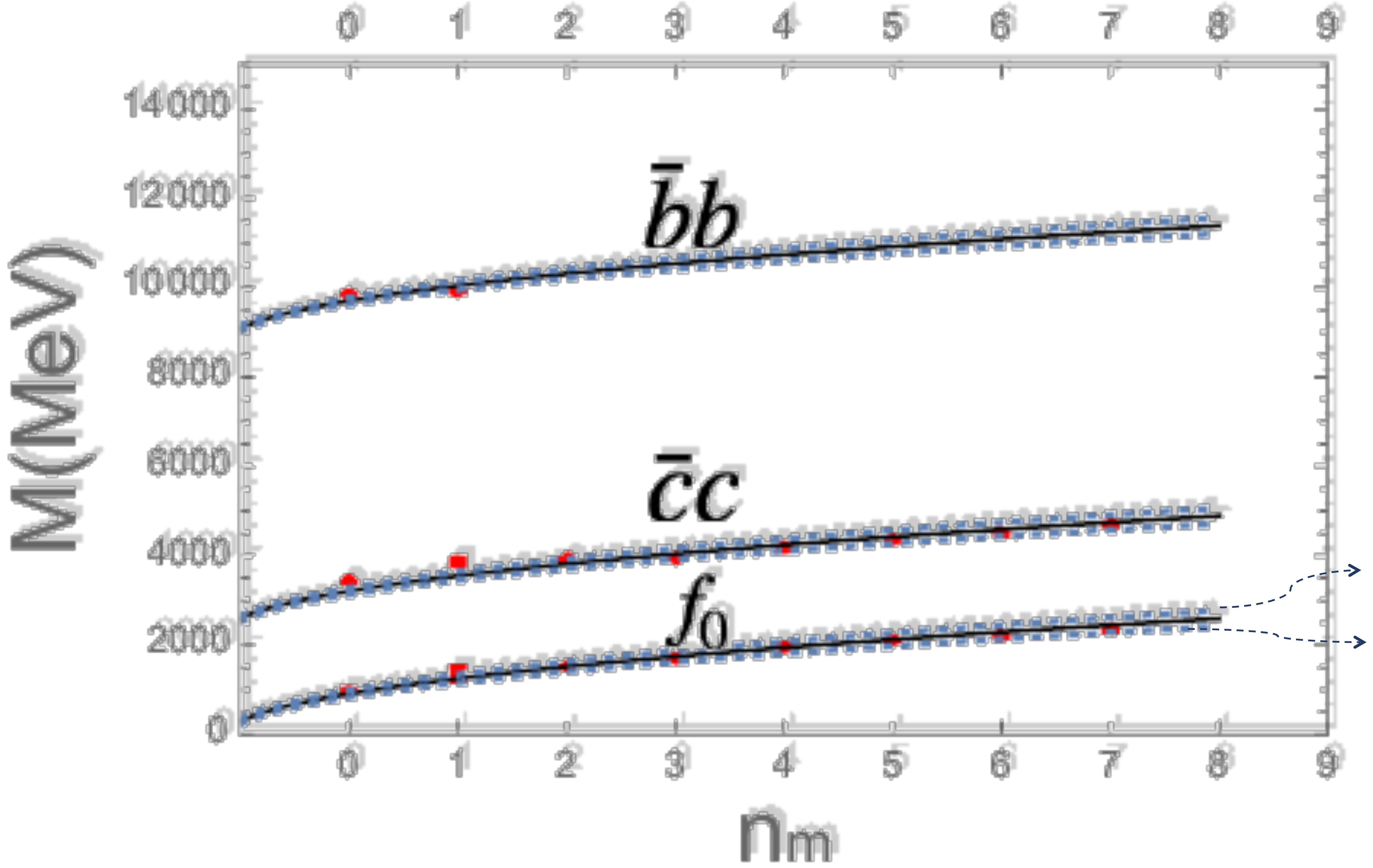
The pion requires a special dilaton (qualitative) or a more sophisitcated treatment introducing trasversal dynamics (quantitative) to incorporate spontaneous chiral symmetry breaking: M. Rinaldi, F. Ceccopieri and V. Vento EPJC 82 (2022) 7, 626.

Heavy Mesons :

$$M_{q\bar{q}} \sim M_{f_0} + C_{q\bar{q}}$$

$C_{b\bar{b}} \sim 2m_b$

$C_{c\bar{c}} \sim 2m_c$

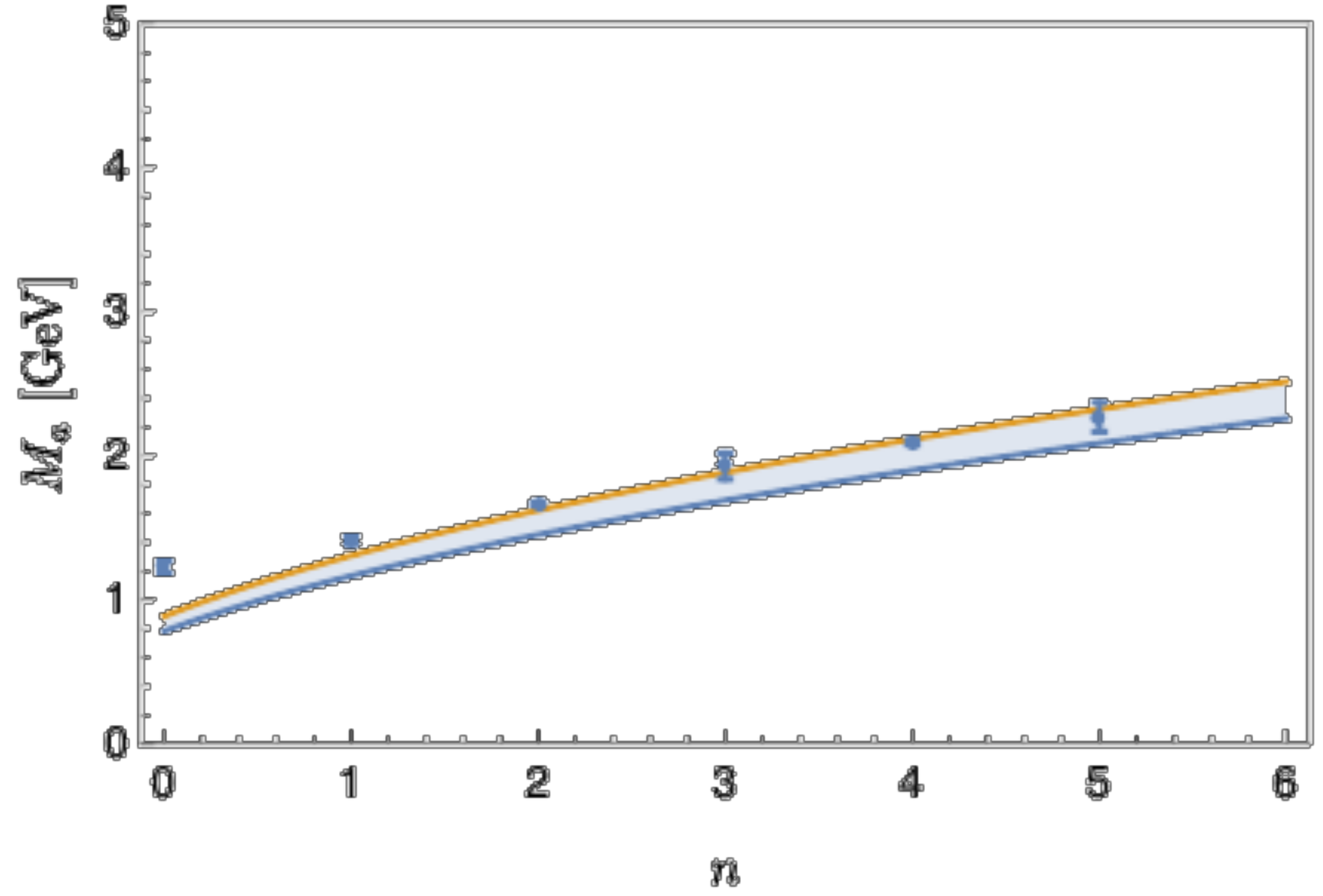


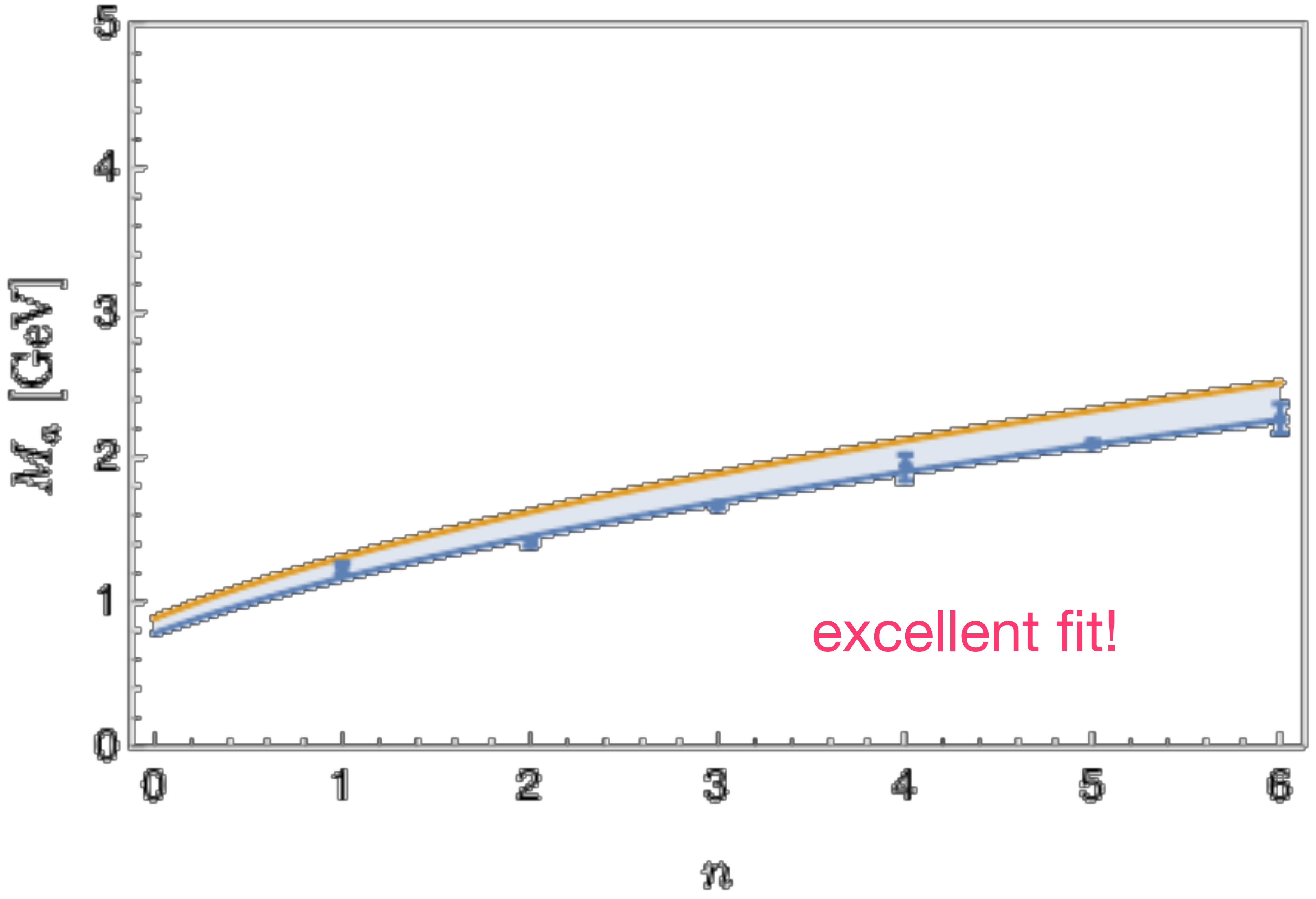
$\alpha \sim 0.51$

$\alpha \sim 0.59$

Possible Predictions

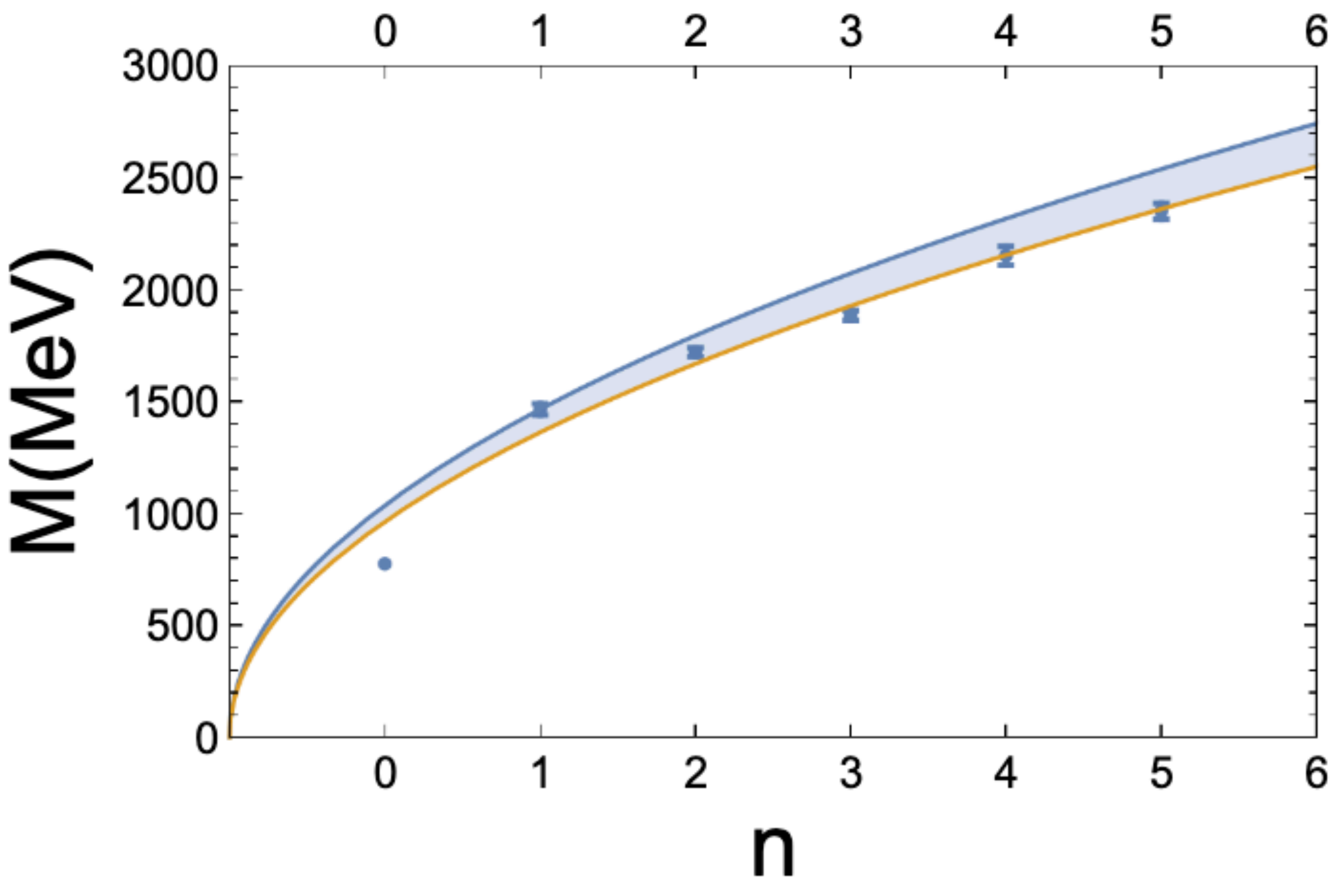
a_1

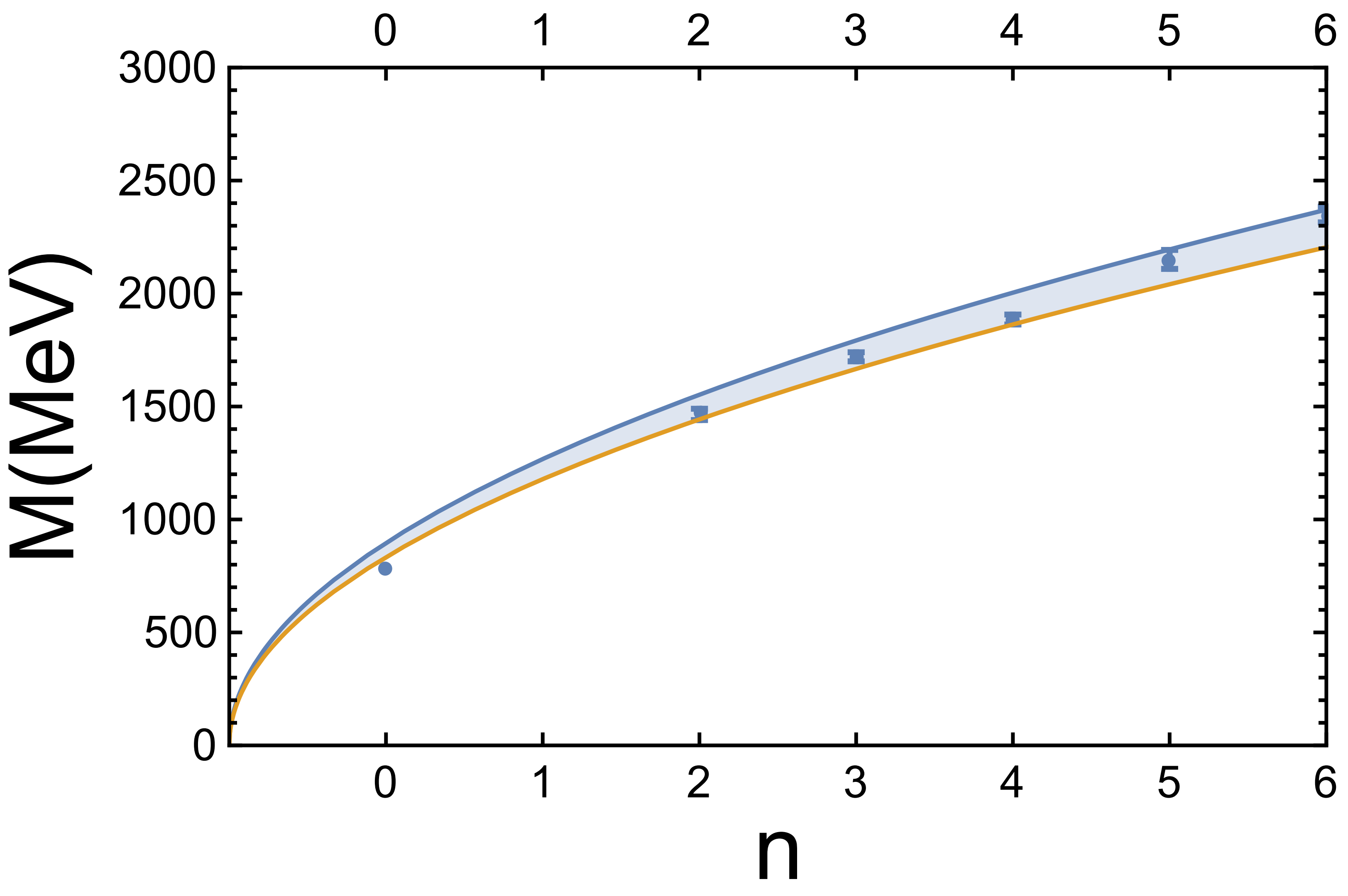




	$a_1(1260)$	$a_1(1420)$	$a_1(1640)$	$a_1(1930)$	$a_1(2095)$	$a_1(2270)$
PDG & Av	1230 ± 40	1411^{+15}_{-13}	1655 ± 16	1930^{+19}_{-70}	2096^{+17}_{-121}	2270^{+55}_{-40}
This work	833 ± 53	1235 ± 72	1535 ± 87	1785 ± 100	2005 ± 111	2202 ± 122

ρ

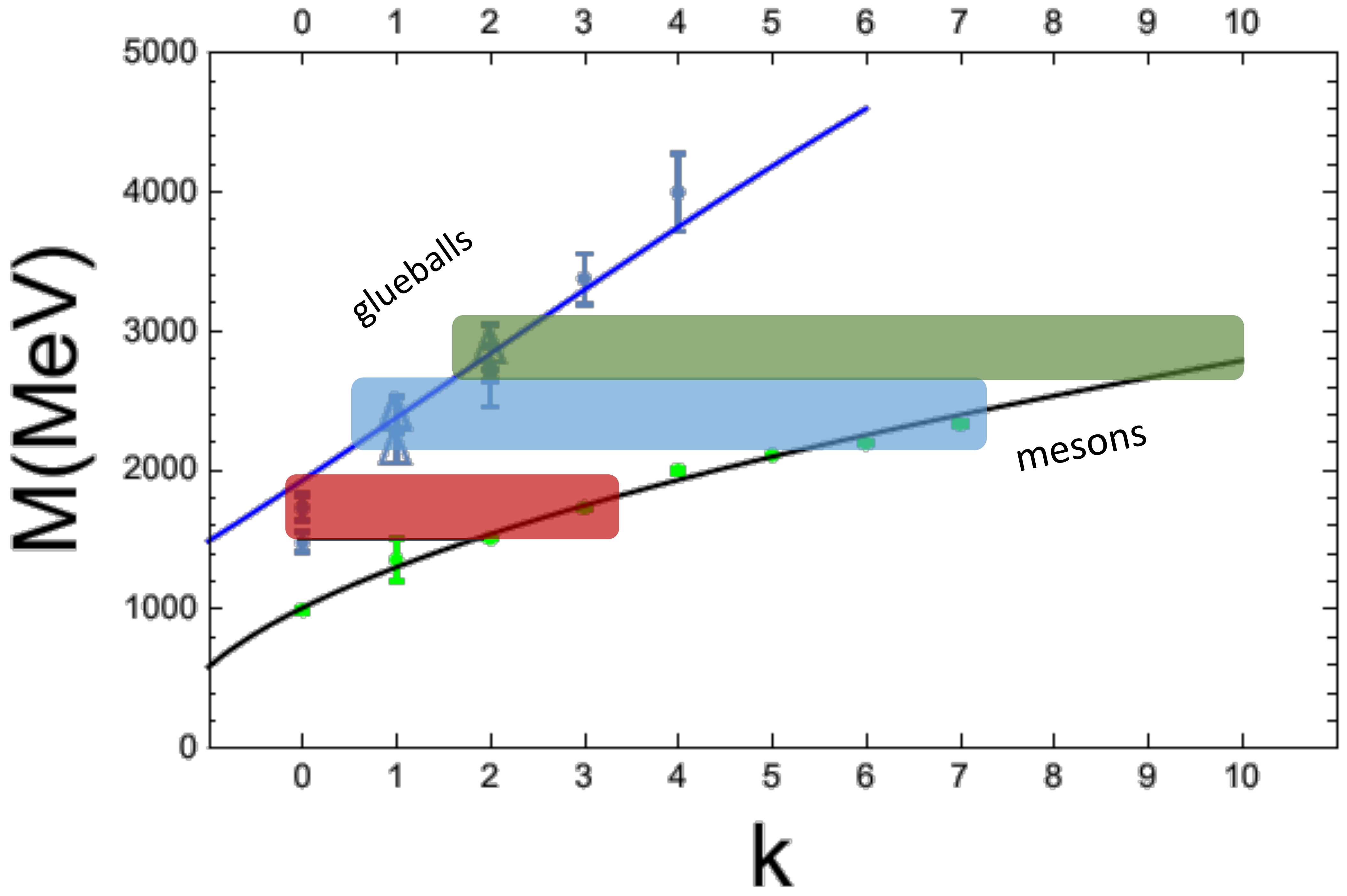


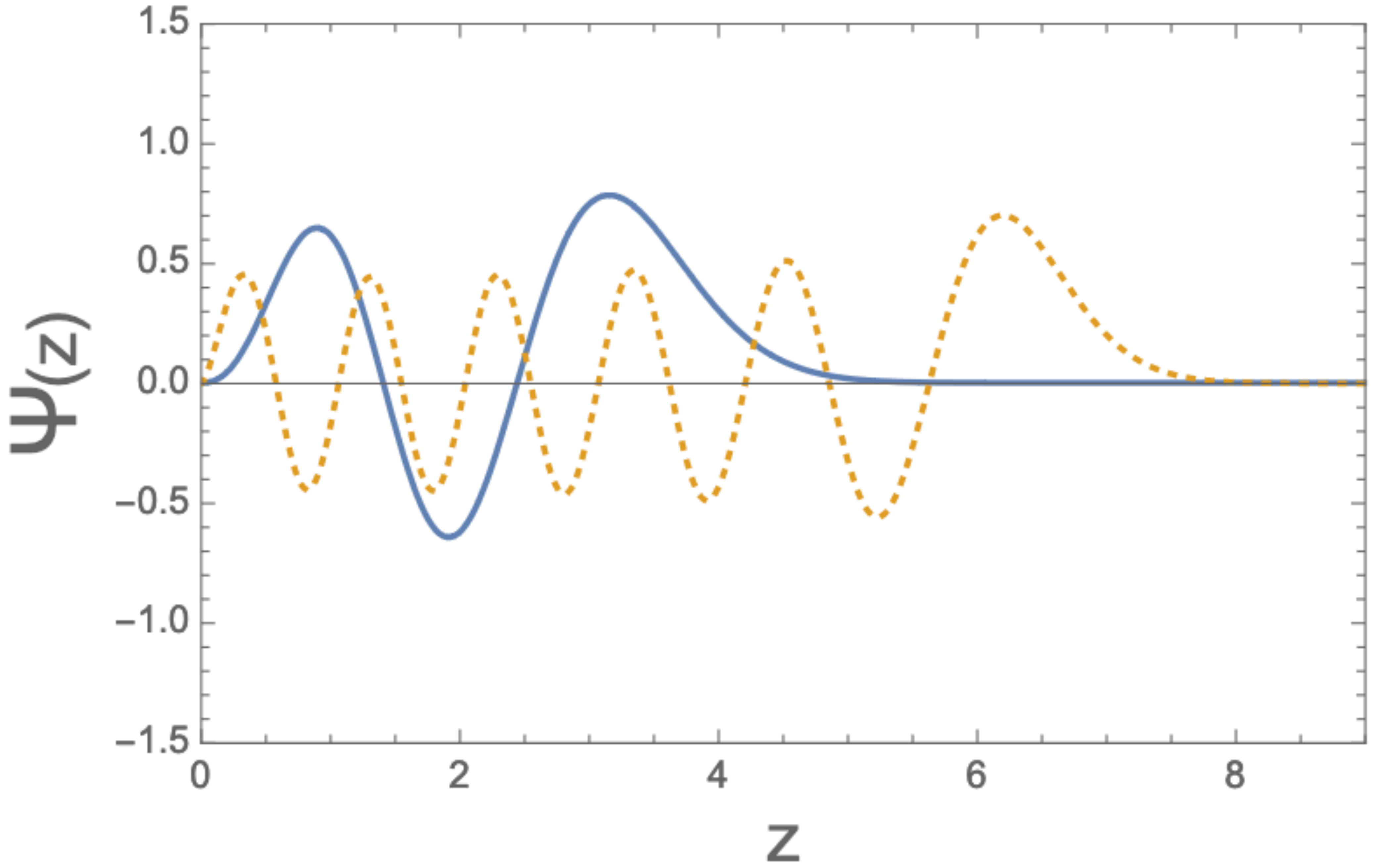


Meson-glueball mixing

We establish a relationship between the mode functions of AdS and a quantum mechanical lightcone formalism in such a way that **the mode functions define wave functions.** If we assume a non diagonal hamiltonian providing mixing, the mixing probability is described by t**he overlap of the wave functions**.

Let us look at the scalar sector. Mixing occurs between states of almost equal mass. We see that for very high masses the mode number of glueballs and mesons is very different and therefore the ovelap will be very small → tiny mixing.





Conclusion: A scenario for studying non perturbative QCD : high lying glueball and meson states will be almost pure states (masses > 2000 MeV).

Extensions

The calculation can now be extended to any hadronic system. We are interested in exotic objects: hybrids and multiquarks (work in progress): for hybrids leading N currents with the lowest conformal dimension

Mesonic quantum numbers

$$
\begin{array}{lll}
0^{-+} & S = \bar{\Psi}\gamma^i\lambda^a\Psi B_i^a & (\Delta = 5, \Delta_p = \ \ 0, p = 0, M_5^2R^2 = \ 5), \\
0^{++} & S' = \varepsilon_{ijk}\bar{\Psi}\sigma^{ij}\lambda^a\Psi B^{ka} & (\Delta = 5, \Delta_p = \ \ 0, p = 0, M_5^2R^2 = \ 5), \\
1^{--} & V_i = \bar{\Psi}\gamma_5\lambda^a\Psi B_i^a & (\Delta = 5, \Delta_p = -1, p = 1, M_5^2R^2 = \ 3), \\
1^{+-} & A_i' = \varepsilon_{ijk}\bar{\Psi}\gamma_5\gamma^j\lambda^a\Psi B^{ka} & (\Delta = 5, \Delta_p = -1, p = 1, M_5^2R^2 = \ 3), \\
1^{++} & A_i = \varepsilon_{ijk}\bar{\Psi}\gamma^j\lambda^a\Psi E^{ka} & (\Delta = 5, \Delta_p = \ \ 0, p = 1, M_5^2R^2 = \ 8).
\end{array}
$$

Non mesonic quantum numbers

$$
\begin{array}{lll}
0^{+-} & \Sigma = \bar{\Psi}\gamma_5\gamma^i\lambda^a\Psi B_i^a & (\Delta = 5, \Delta_p = \ -1, p = 0, M_5^2R^2 = \ 0), \\
0^{--} & \Sigma' = \bar{\Psi}\gamma_5\lambda^a\Psi E_i^a & (\Delta = 5, \Delta_p = \ -1, p = 0, M_5^2R^2 = \ 0), \\
1^{-+} & W_i = \varepsilon_{ijk}\bar{\Psi}\gamma^j\lambda^a\Psi B^{ka} & (\Delta = 5, \Delta_p = \ \ \ 0, p = 1, M_5^2R^2 = \ 8), \\
1^{-+} & W_i' = \bar{\Psi}\gamma_0\lambda^a\Psi E_i^a & (\Delta = 5, \Delta_p \ \ \ = 0, p = 1, M_5^2R^2 = \ 8).
\end{array}
$$

mesonic hybrids

	n=0	n=1	n=2	n=3
0^{-+}	2074	2536	2986	3429
0^{++}	2074	2536	2986	3429
1^{--}	1562	2045	2501	2943
1^{+-}	1562	2045	2501	2943
1^{++}	2149	2647	3125	3592

exotic hybrids

	n=0	n=1	n=2	n=3
0^{+-}	1411	1728	1995	2231
0^{--}	1411	1728	1995	2231
1^{-+}	2149	2647	3125	3592

Once we relate the Gravity Model to a Quantum Mechanical description any observable can be calculated: magnetic moments, form factors, pdfs,….

· QCD Phase Transition in holography

M. Rinaldi and V. Vento *Phys.Rev.D* 108 (2023) 11, 114020, *Eur.Phys.J.C* 82 (2022) 2, 140

One defines two phases (Hard Model), either with different métrics (C.P. Herzog) or with the same metric (us). We use two Black Hole (BH) metrics.

$$ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 - d\vec{x}^2 - f^{-1}(z)dz^2\right),$$

where $f(z) = 1 - z^4/z_h^4$. Note that z_h determines the Hawking's temperature of the black hole $T_h = 1/(\pi z_h)$.

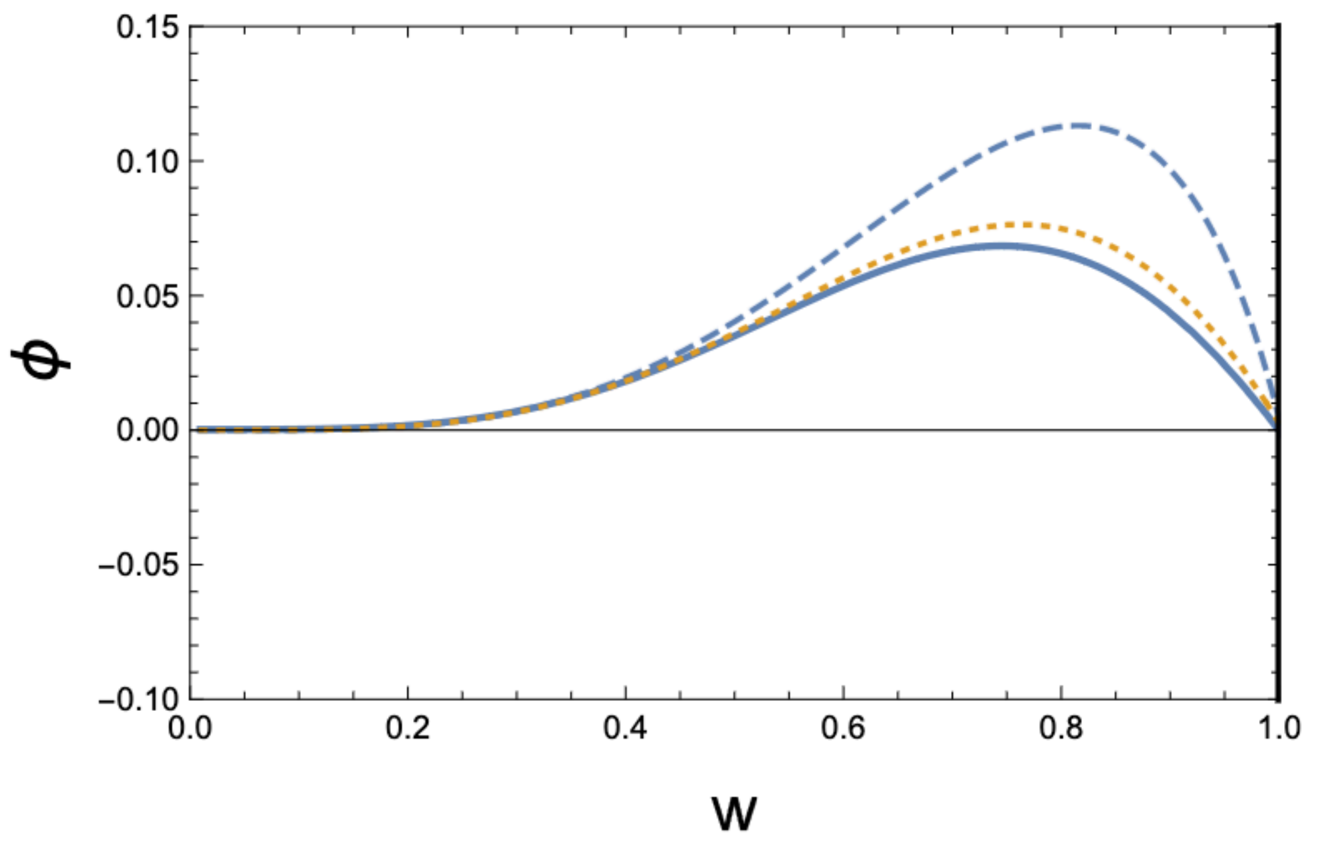
We define two sectors in z_h in relation with the confinement temperature z_0 :

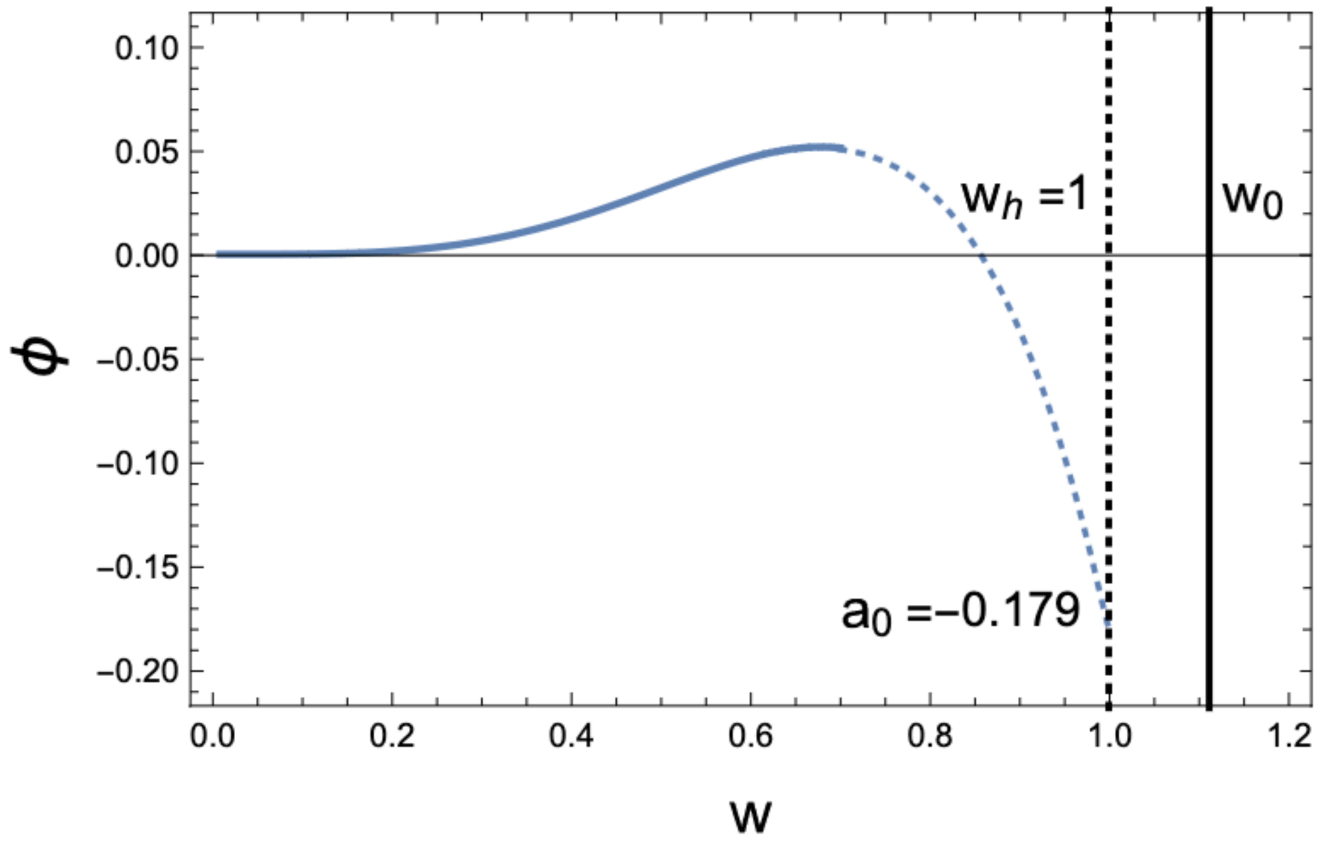
i) a low temperature sector where $z_h > z_0$ which we will characterize by Dirichlet or Neumann boundary conditions at z_0

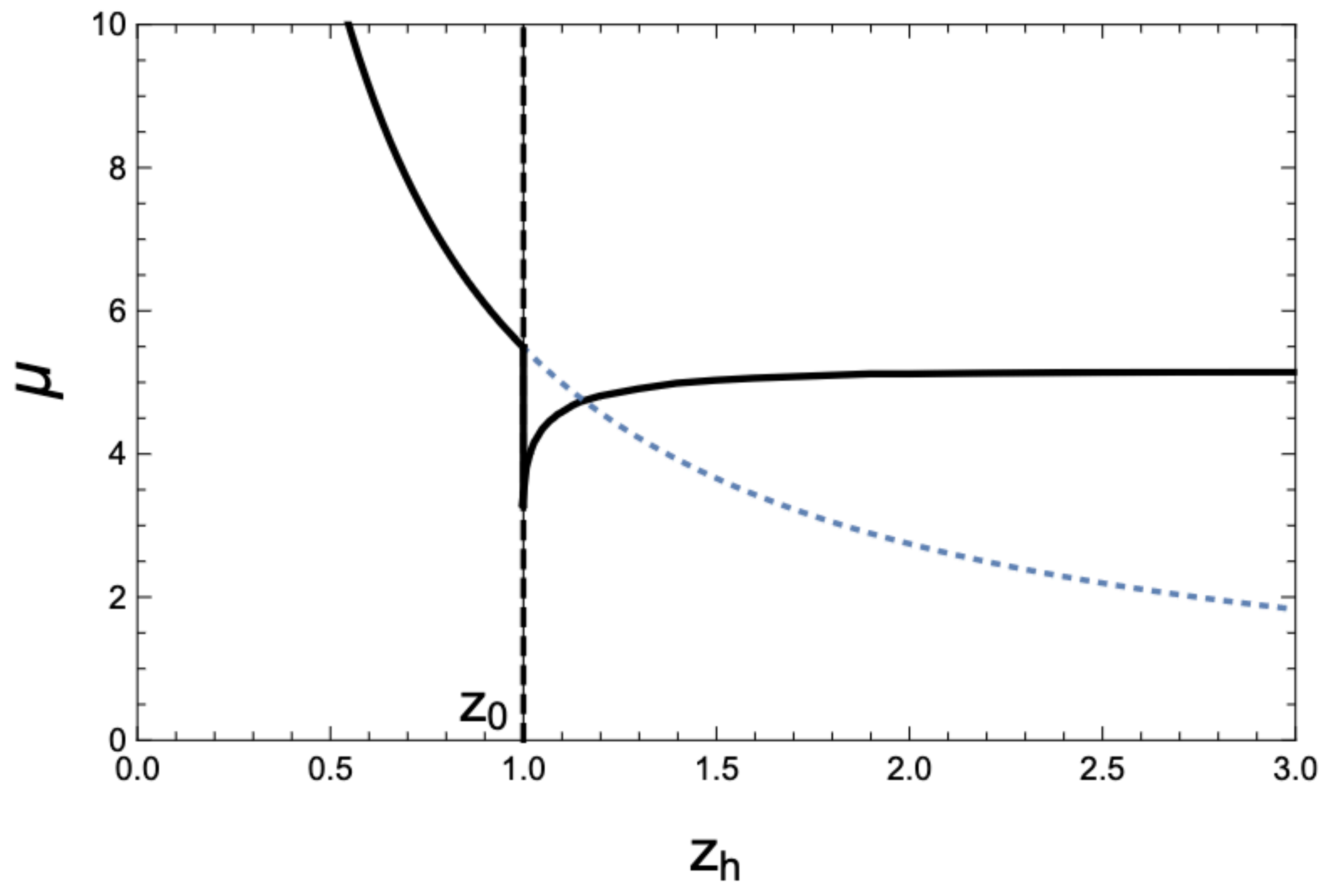
ii) a high temperature sector $z_h < z_0$ which satisfies the appropriate boundary conditions at the BH horizon z_h.

By calculating the free energy one finds that this description defines a first order phase transition at $T \sim 100$ MeV (too low)

We now study the behavior of the glueballs (gravitons) across the phase transition :

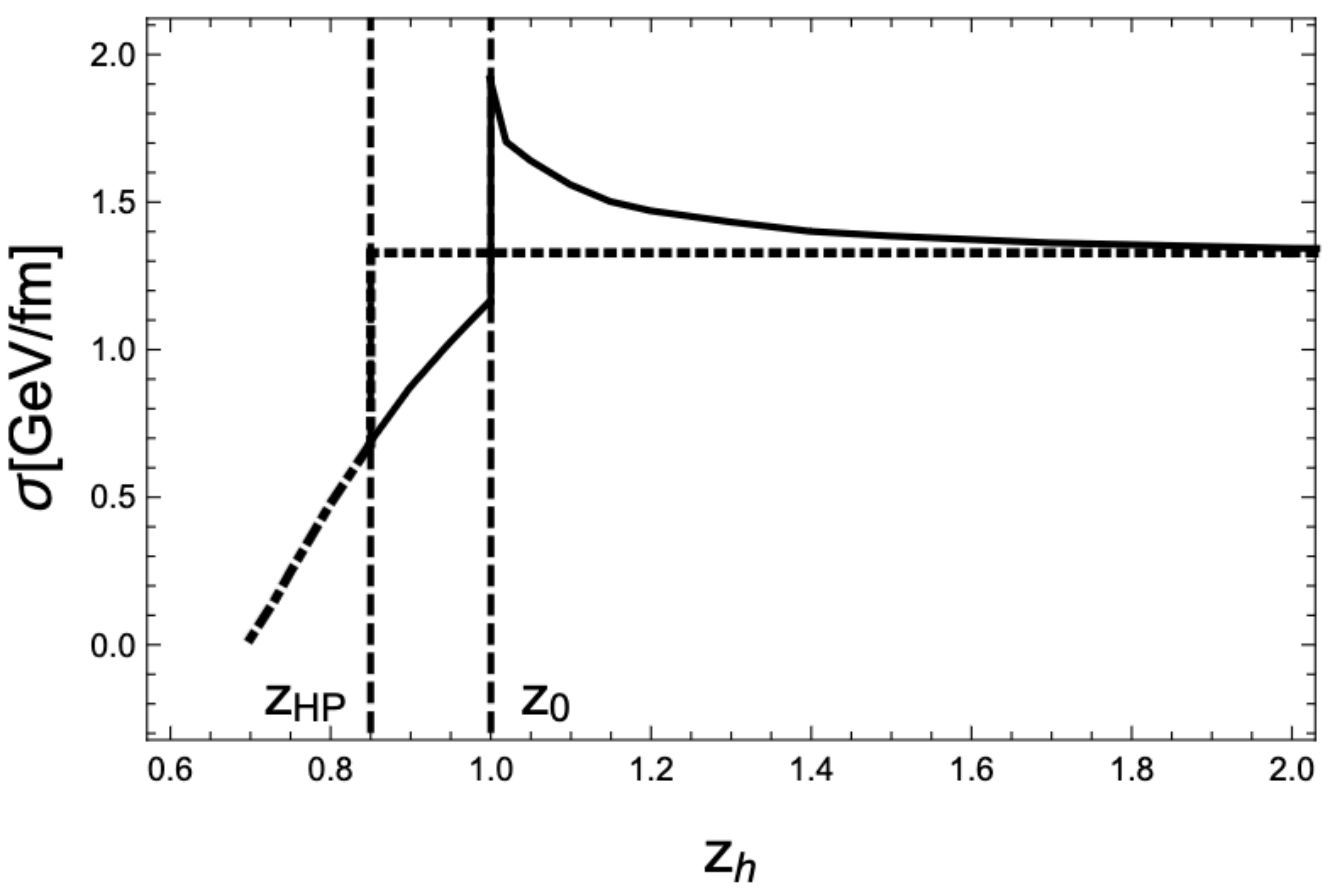






Mode Energy as a function of temperature

We associate the model to a Potential Model with deconfinement



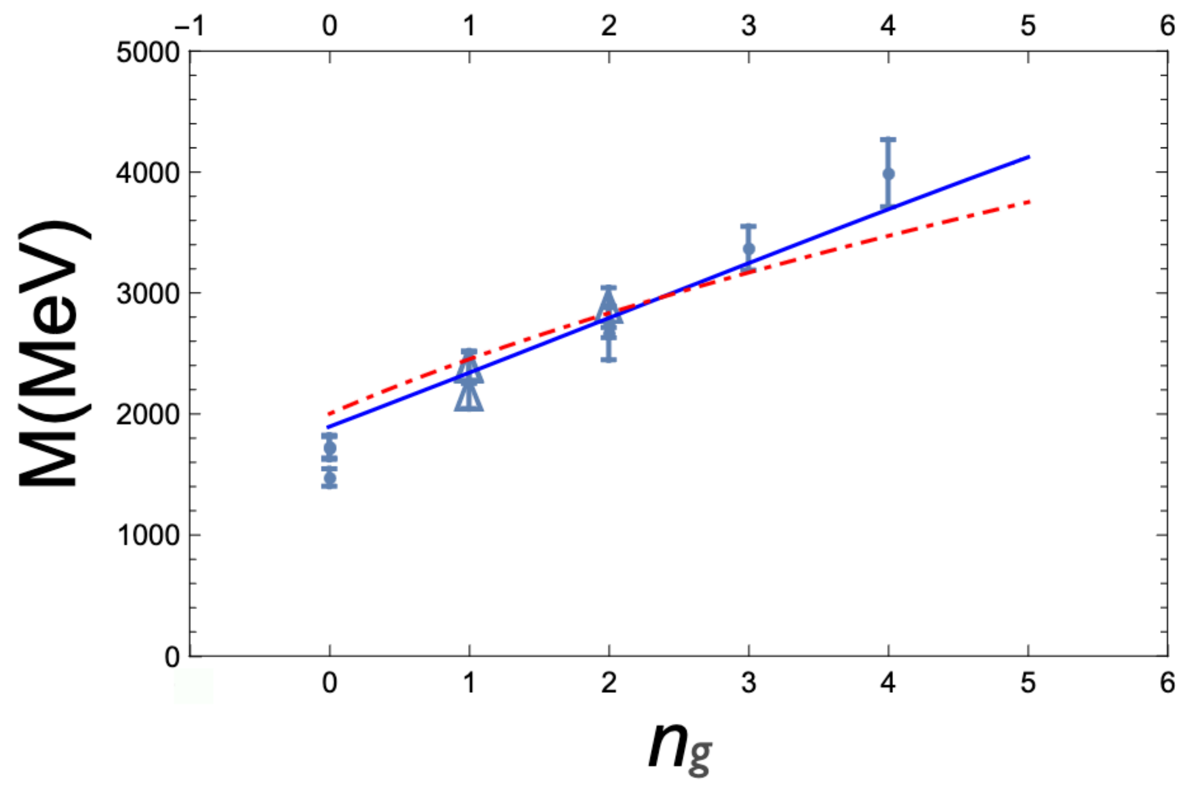
String tension as a function of temperature

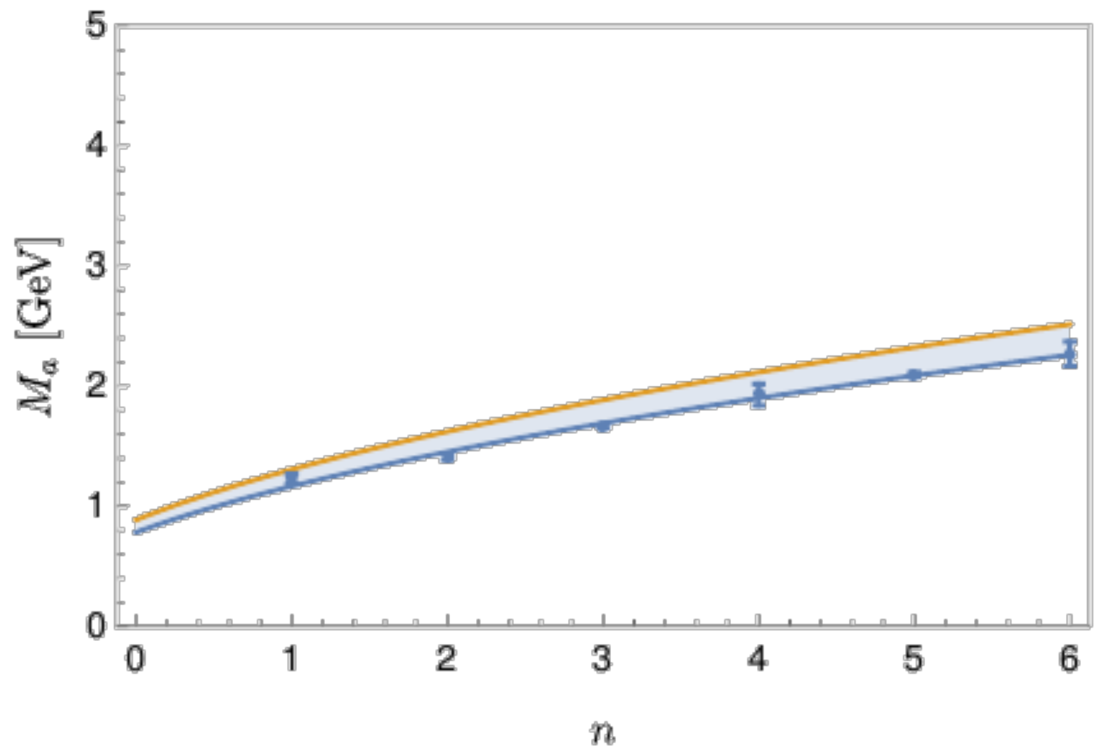
- Concluding Remarks

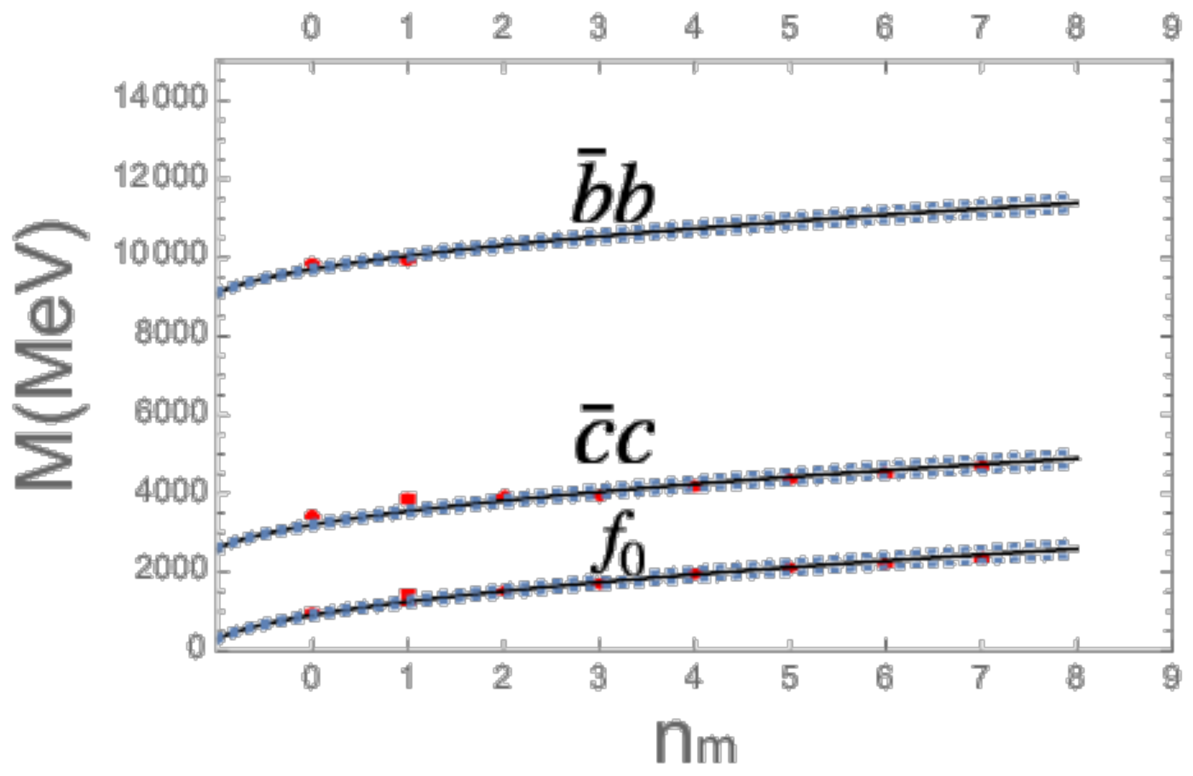
We have established a correspodence between a theory “analogous” to QCD and a fifth dimensional AdS gravity. The dynamics is dominated by the couplings at the four dimensional surface, the AdS metric and the confinement mechanism (dilaton).

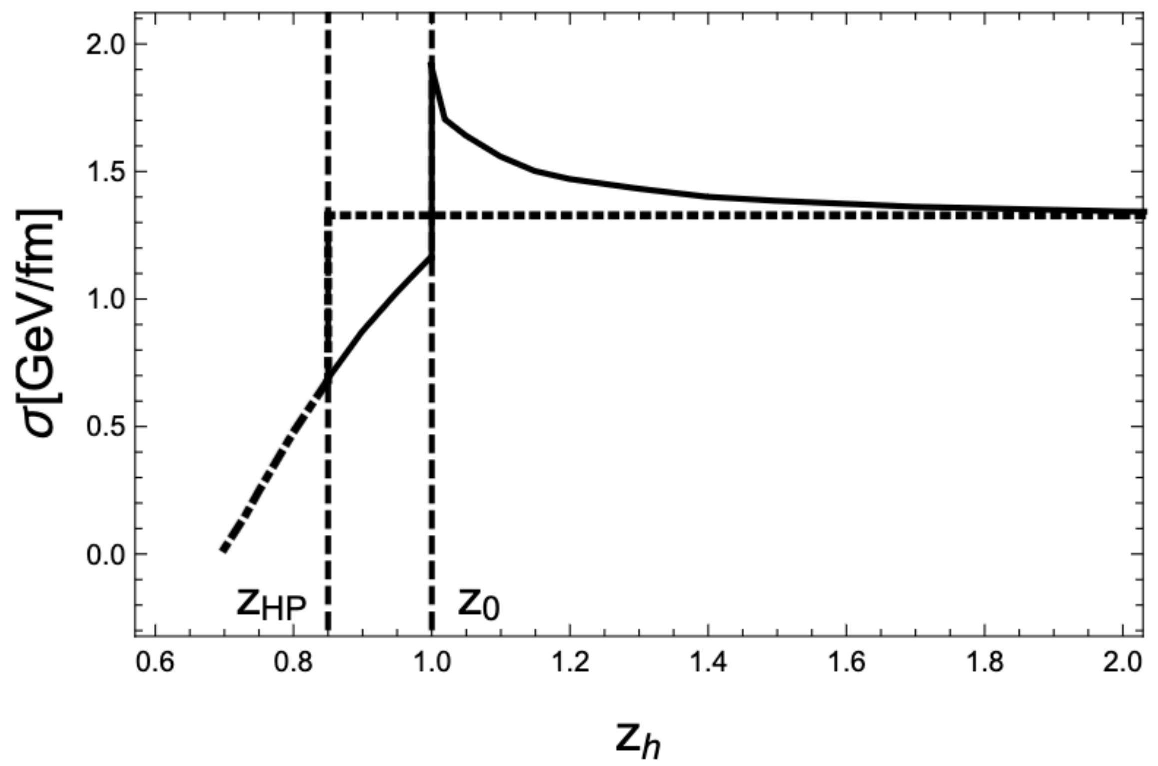
The results are excellent within the limitations associated to large N. They have though a strong predictive power in those sectors where confinement might play a major role: glueballs and hybrids.

In this correspondence the glueballs play a special role acting as gravitons of the theory. We should further investigate what this really implies in QCD.







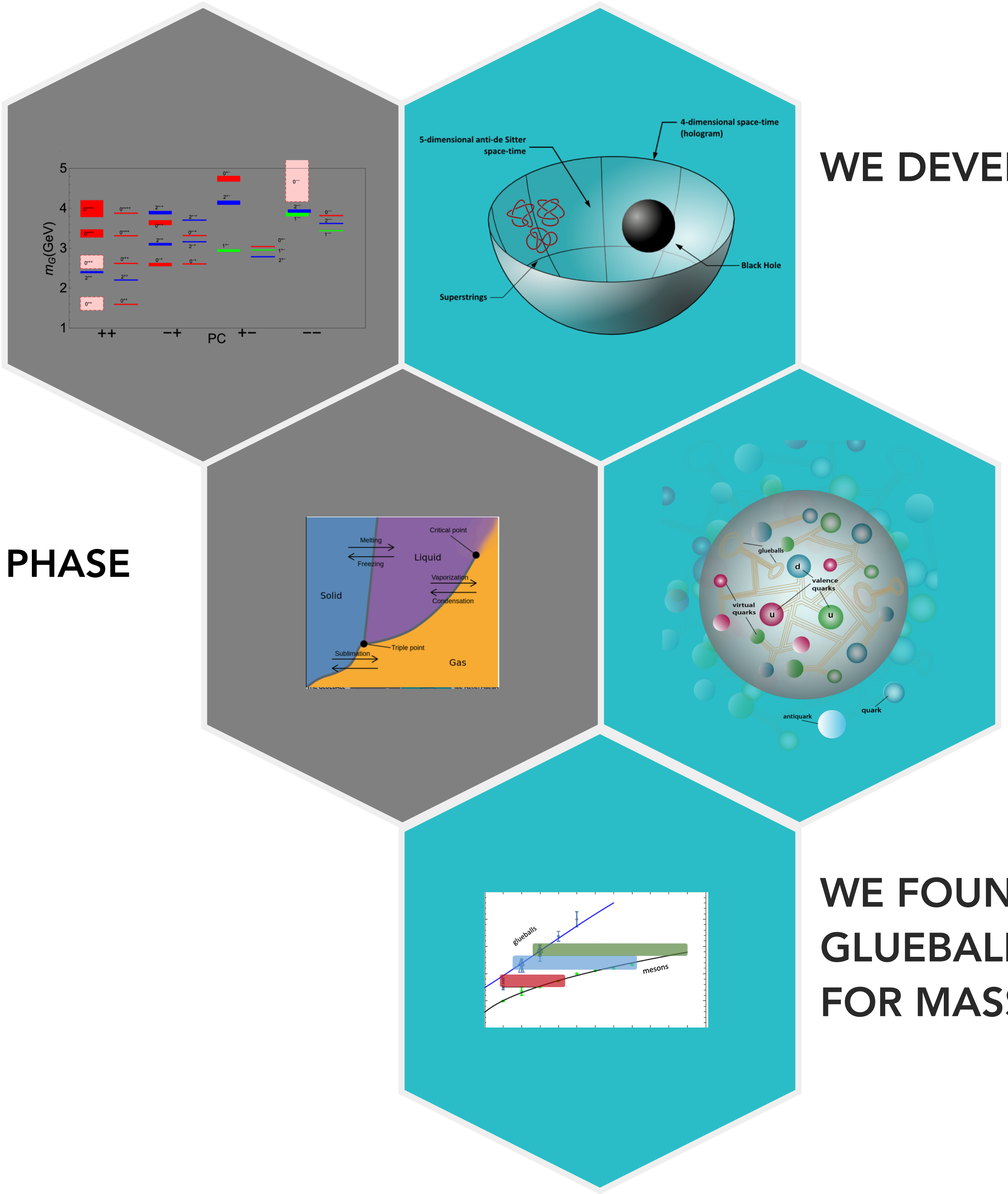


WE CONSIDER THE GLUEBALL & MESON SPECTRA

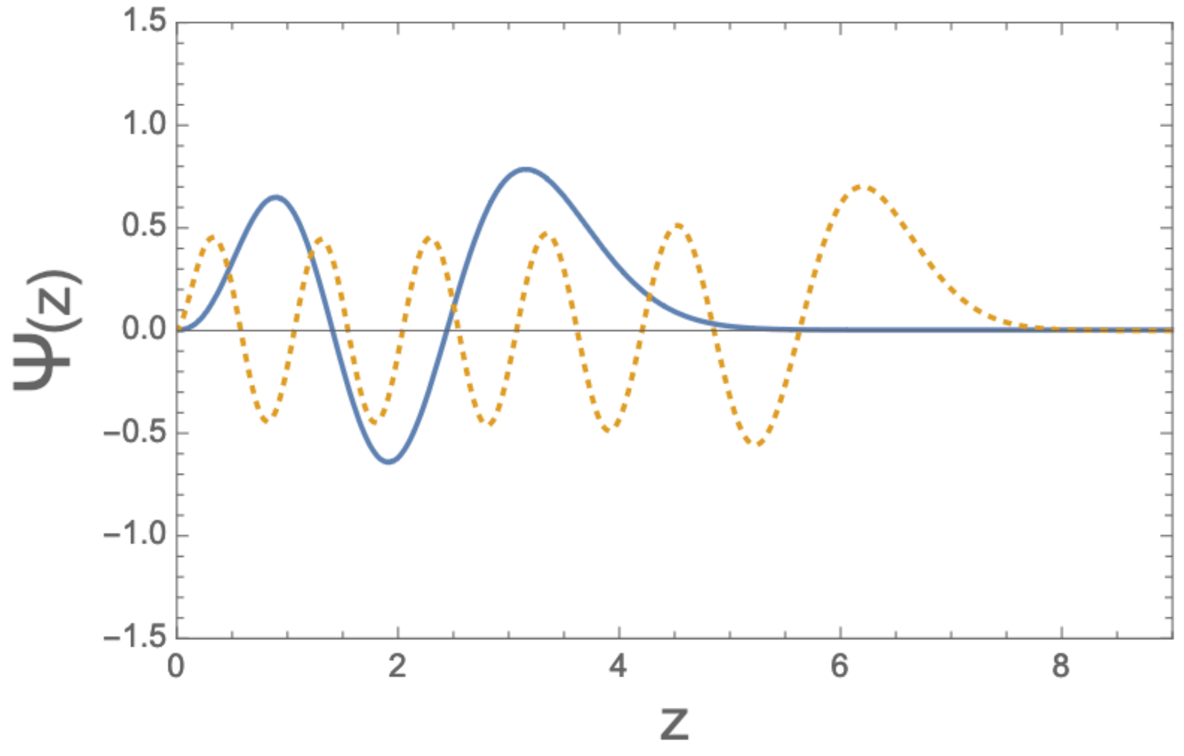
WE DEVELOPEPD THE GSW AdS/QCD MODEL

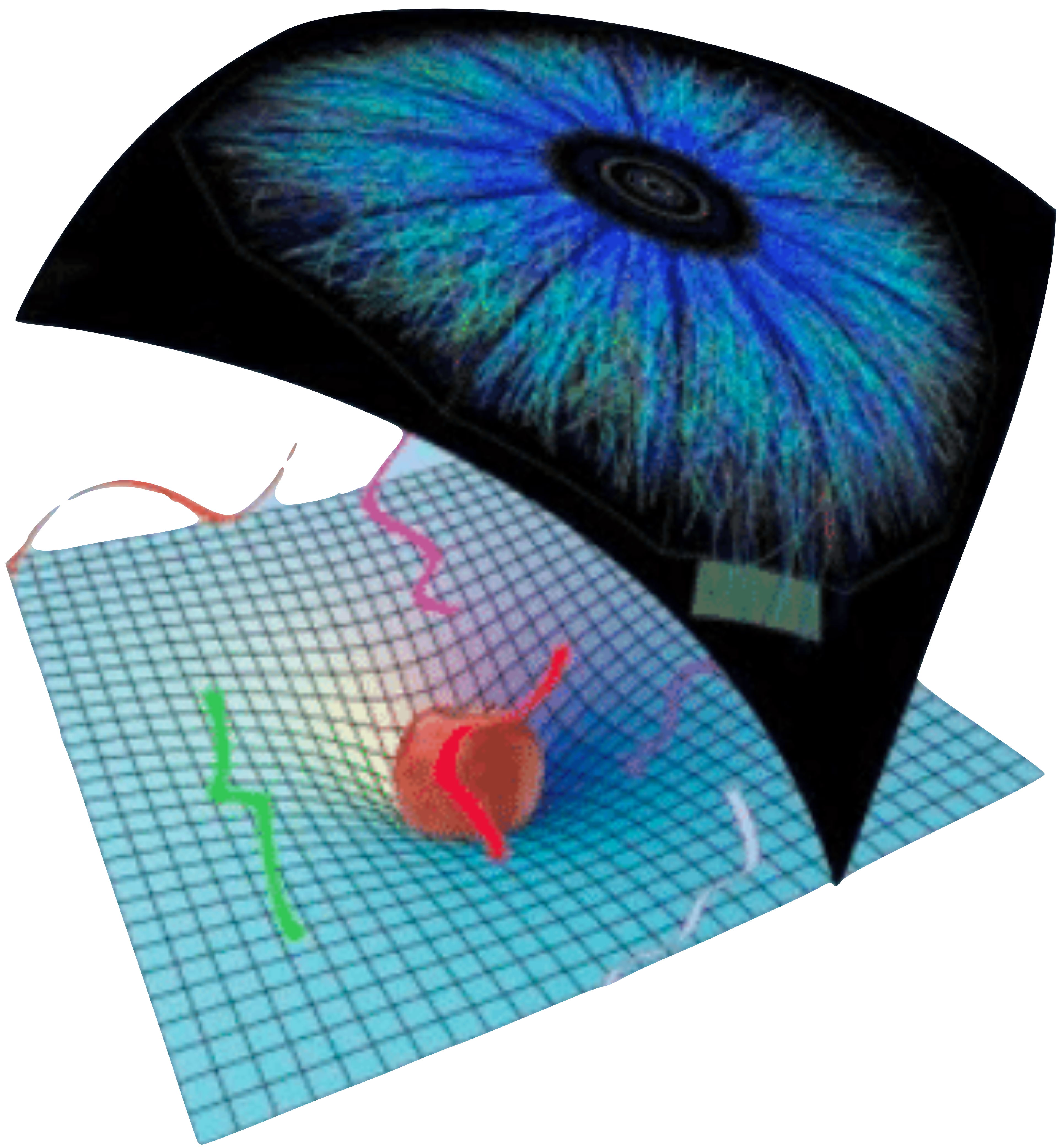
WE DESCRIBED A PHASE TRANSITION

WE DESCRIBED QUITE WELL GLUEBALL & MESON SPECTRA WITH 2 PARAMETERS



WE FOUND THAT PURE SCAI GLUEBALLS COULD BE FOUN FOR MASSES ABOVE 2 GeV





Thank you !