Exotic spectroscopy and femtoscopy

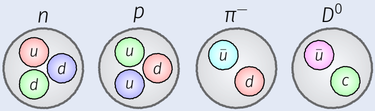
Miguel Albaladejo (IFIC-CSIC)

IFIC Scientific Day
L3: Flavour and quark matter
Valencia, Jan. 8-9, 2024

Outline

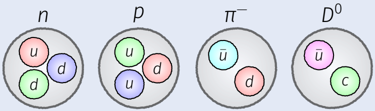
Outline

1. Introduction: spectroscopy, exotics, QCD and EFTs
2. Open charm: spectroscopy and femtoscopy
3. Double charm: spectroscopy and femtoscopy
4. Conclusions



- Conventional hadrons:
  - Mesons: $q\bar{q}'$: $\pi^+ = u\bar{d}$, $D^0 = c\bar{u}$, ...
  - Baryons: $q_1q_2q_3$: $p = uud$, $n = udd$, ...

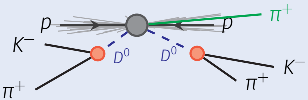
- Constituent quark models have succesfully described most of (but not all!) the hadrons discovered so far.
- Only possibilities? No, the only requirement is to be color singlets. There can be tetraquarks ($q_1q_2q_3q_4$), pentaquarks ($\bar{q}_1q_2q_3q_4$), hybrids ($\bar{q}_1q_2g$), glueballs (gg), hadronic molecules (MM', MB, BB'),...

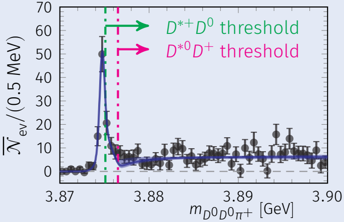


- Conventional hadrons:
  - Mesons: $q\bar{q}'$: $\pi^+ = u\bar{d}$, $D^0 = c\bar{u}$, ...
  - Baryons: $q_1q_2q_3$: $p = uud$, $n = udd$, ...

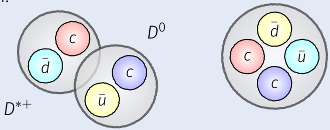
- Constituent quark models have succesfully described most of (but not all!) the hadrons discovered so far.
- Only possibilities? No, the only requirement is to be color singlets. There can be tetraquarks ($q_1q_2q_3q_4$), pentaquarks ($\bar{q}_1q_2q_3q_4$), hybrids ($\bar{q}_1q_2g$), glueballs (gg), hadronic molecules (MM', MB, BB'),...

[Nature Phys., **18**, 751('22); Nature Com., **13**, 3351('22)]





- In 2021, the LHCb collaboration discovers T_{cc}^+, with quark content $cc\bar{u}\bar{d}$, and very close to $D^{*+}D^0$ threshold
- Even if something is explicitly exotic, you still have to understand how are quarks distributed inside the hadron.



- [MA, PL,**B829**,137052('21); MA, Nieves, EPJ,**C82**,724('22)]
- Our analysis favours the molecular picture.
- Also, relevant to understand QCD confinement, and color combinations.

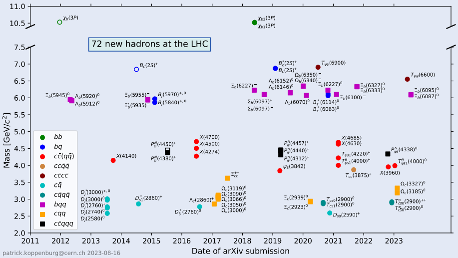
Spectroscopy: precision and exotics

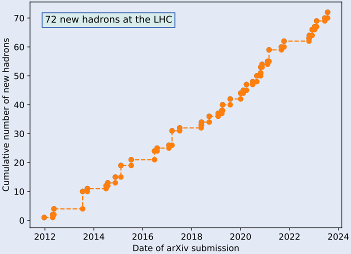
- The case of $T_{cc}^{+}(3875)$ is only a recent example...
- 2nd November Revolution started 2003 with the discovery of:
  - $X(3872)$ (hidden charm)
    Belle, Phys. Rev. Lett., **91**, 262001 (2003)
  - $D_{s0}^{*}(2317)^{+}$ (charm and strange)
    BaBar, Phys. Rev. Lett., **90**, 242001 (2003)

Observation of a narrow charmonium-like state in exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ decays #1
Belle Collaboration • S.K. Choi (Gyeongsang Natl. U.) et al. (Sep, 2003)
Published in: *Phys.Rev.Lett.* 91 (2003) 262001 • e-Print: hep-ex/0309032 [hep-ex]
pdf links DOI cite claim reference search 2,415 citations

Observation of a narrow meson decaying to $D_s^{+}\pi^{0}$ at a mass of 2.32-GeV/c^2 #2
BaBar Collaboration • B. Aubert (Annecy, LAPP) et al. (Apr, 2003)
Published in: *Phys.Rev.Lett.* 90 (2003) 242001 • e-Print: hep-ex/0304021 [hep-ex]
pdf links DOI cite claim reference search 990 citations

- We have entered the era of precision and exotic spectroscopy
- The LHC (mainly LHCb) is largely contributing on the experimental side, but also many other experiments: BES, Belle, GlueX...





- Theoreticians have a lot of work to do...

- QCD lagrangian built in terms of (degrees of freedom) quarks and gluons: $\mathcal{L}[q, \bar{q}, g]$
- Quarks are confined into colorless hadrons, which are ultimately the objects detected in experiments.
- EFTs: build QFT with hadrons (π, K, ..., D, B, ...) as degrees of freedom

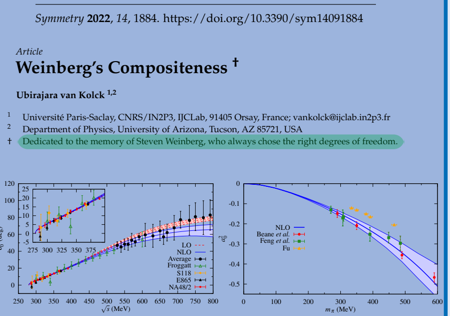
Example 1: Chiral Perturbation Theory

- Light quarks are... light: $m_u \simeq m_d \simeq m_s \simeq 0$
- 8 Goldstone bosons SU(3): $\phi_i \to \pi, K, \eta$
- [Weinberg, PRL,17,616('66); Phys. A96,327 ('79)]]
  [Gasser, Leutwyler, AP,158,142('84); NP,B250,465('85)]
- LO lagrangians:

$$\mathcal{L}_2 = \frac{f^2}{4}\left\langle D_\mu U^\dagger D^\mu U \right\rangle + \frac{f^2}{4}\left\langle \chi^\dagger U + \chi U^\dagger \right\rangle$$

$$U[\phi] = \exp\left(i \sum_{i=1}^{8} \phi_i \lambda_i\right)$$

$$\chi = 2B_0 \,\mathrm{diag}\left(m_u, m_d, m_s\right) \simeq \mathrm{diag}\left(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2\right)$$


Symmetry **2022**, *14*, 1884. https://doi.org/10.3390/sym14091884

Article

Weinberg's Compositeness †

Ubirajara van Kolck [1,2]

[1] Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France; vankolck@ijclab.in2p3.fr
[2] Department of Physics, University of Arizona, Tucson, AZ 85721, USA
† Dedicated to the memory of Steven Weinberg, who always chose the right degrees of freedom.

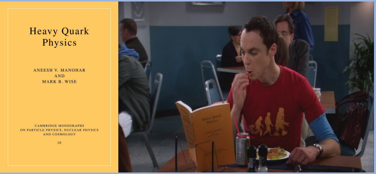
Effective Field Theories

- QCD lagrangian built in terms of (degrees of freedom) quarks and gluons: $\mathcal{L}[q,\bar{q},g]$
- Quarks are confined into colorless hadrons, which are ultimately the objects detected in experiments.
- EFTs: build QFT with hadrons (π, K, ..., D, B, ...) as degrees of freedom

Example 2: HQSS + SU(3) flavour

- Heavy quarks are... heavy: $m_c, m_b \to \infty$
- $H_a^{(Q)} = \frac{1+\not{v}}{2}\left(P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right)$
- HQSS and $SU(3)$ light flavour symmetry: $H_a^{(Q)} \sim (Q\bar{u}, Q\bar{d}, Q\bar{s}) \sim \left(D^{(*)0}, D^{(*)+}, D_s^{(*)+}\right)$

  [Grinstein *et al.*, NP,B380('92); Alfiky *et al.*, PL,B640('06), ...]



$H\bar{H} \to H\bar{H}$ LO lagrangian:

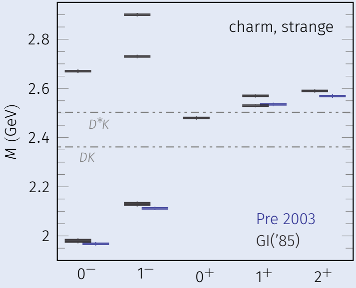
$$\begin{aligned}\mathcal{L}_{4H} &= \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\right]\left(F_A\,\delta_a^b\delta_c^d + F_A^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)\\ &+ \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\gamma_5\right]\left(F_B\,\delta_a^b\delta_c^d + F_B^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right),\end{aligned}$$

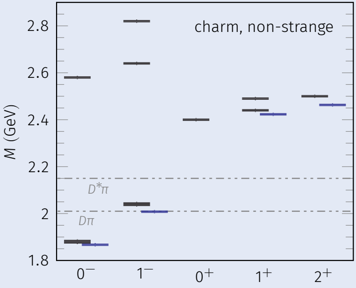
Outline

Quark model in the open-charm sector

- Quark model $c\bar{n}$ is still our baseline:

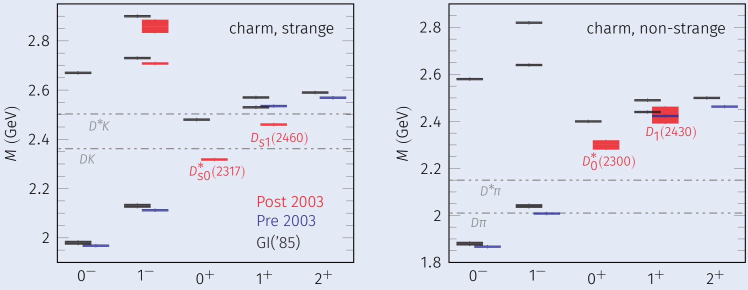
  «In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework.» [Godfrey, Isgur, PR,D32,189('85)]





- Quark model $c\bar{n}$ is still our baseline:

  «In this paper we present the results of a study of light and heavy mesons in soft QCD. We have found that all mesons–from the pion to the upsilon–can be described in a unified framework.» [Godfrey, Isgur, PR,D32,189('85)]



- The discovery of $D_{s0}^*(2317)$ in 2003 (and $D_{s1}(2460)$ later on) is "equivalent" to the discovery of $X(3872)$ in charmonium-like system.

[BABAR, PRL,90,242001('03)]
[CLEO, PR,D68,032002('03)]

Theoretical interpretations

$c\bar{q}$ states

Colangelo, De Fazio, Phys. Lett. B **570**, 180 (2003)
Dai *et al.* Phys. Rev. D **68**, 114011 (2003)
Narison, Phys. Lett. B **605**, 319 (2005)
Bardeen *et al.*, Phys. Rev. D **68**, 054024 (2003)
Lee *et al.*, Eur. Phys. J. C **49**, 737 (2007)
Wang, Wan, Phys. Rev. D **73**, 094020 (2006)

Pure tetraquarks

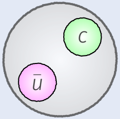
Cheng, Hou, Phys. Lett. B **566**, 193 (2003)
Terasaki, Phys. Rev. D **68**, 011501 (2003)
Chen, Li, Phys. Rev. Lett. **93**, 232001 (2004)
Maiani *et al,*, Phys. Rev. D **71**, 014028 (2005)
Bracco *et al.*, Phys. Lett. B **624**, 217 (2005)
Wang, Wan, Nucl. Phys. A **778**, 22 (2006)

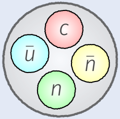
$c\bar{q}+$ tetraquarks or meson–meson

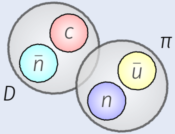
Browder *et al.*, Phys. Lett. B **578**, 365 (2004)
van Beveren, Rupp, Phys. Rev. Lett. **91**, 012003 (2003)

Heavy-light meson–meson molecules

Barnes *et al.*, Phys. Rev. D **68**, 054006 (2003)
Szczepaniak, Phys. Lett. B **567**, 23 (2003)
Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)
Hofmann, Lutz, Nucl. Phys. A **733**, 142 (2004)
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)
Gamermann *et al.*, Phys. Rev. D **76**, 074016 (2007)
Faessler *et al.*, Phys. Rev. D **76**, 014005 (2007)
Flynn, Nieves, Phys. Rev. D **75**, 074024 (2007)







Meanwhile, in the lattice...

- Masses larger than the physical ones if using $c\bar{s}$ interpolators only.

  Bali, Phys. Rev., **D68**, 071501 (2003)
  UKQCD Collab., Phys. Lett., **B569**, 41 (2003)

- Masses consistent with $D_0^*(2300)$ and $D_{s0}^*(2317)$ obtained when "meson-meson" interpolators are employed.

  Mohler, Prelovsek, Woloshyn, Phys. Rev.,**D87**, 034501 (2013)
  Mohler *et al.*, Phys. Rev. Lett., **111**, 222001 (2013)

- Close to the physical point: RQCD Collab., Phys. Rev., **D96**, 074501 (2017)

- More complete studies from the HadSpec collaboration:
  - $D\pi$, $D\eta$ and $D_s\bar{K}$ coupled-channel scattering. A bound state with large coupling to $D\pi$ is identified with $D_0^*(2300)$.

    HadSpec Collab., JHEP **1610**, 011 (2016)
  - $D_{s0}^*(2317)$: A bound state is found in the DK channel, with:
    - $\Delta E = 25(3)$ MeV ($m_\pi = 391$ MeV)
    - $\Delta E = 57(3)$ MeV ($m_\pi = 239$ MeV)
    - Compare with experimental, $\Delta E \simeq 45$ MeV (the dependence on m_π does not need to be monotonic!

[HadSpec Collab., JHEP **02 (2021)** 100; JHEP **07 (2021)** 123]

- $D^*_{s0}(2317)$ $(S,I)=(1,0)$ $M_{D^*_{s0}(2317)} = 2317.8 \pm 0.5$ MeV (PDG)
- $D^*_0(2300)$ $(S,I)=(0,1/2)$ Not so well stablished:

	Collab.	M (MeV)	$\Gamma/2$ (MeV)	Ref.
Neutral	Belle	2308 ± 36	138 ± 33	Phys. Rev.,**D69**, 112002 (2004)
Neutral	BaBar	2297 ± 22	137 ± 25	Phys. Rev.,**D79**, 112004 (2009)
Neutral	FOCUS [?]	2407 ± 41	120 ± 40	Phys. Lett. **B586**, 11 (2004)
Charged	LHCb	2360 ± 34	128 ± 29	Phys. Rev., **D92**, 032012 (2015) ($B^0 \to \bar{D}^0 K^+ \pi^-$)
Charged	LHCb	2349 ± 7	109 ± 9	Phys. Rev., **D92**, 012012 (2015) ($B^0 \to \bar{D}^0 \pi^+ \pi^-$)
Charged	LHCb	2354 ± 13	128 ± 13	Phys. Rev., **D92**, 012012 (2015) ($B^0 \to \bar{D}^0 \pi^+ \pi^-$)
Charged	FOCUS [?]	2403 ± 38	142 ± 21	Phys. Lett. **B586**, 11 (2004)
	PDG	2343 ± 10	115 ± 8	Neutral and charged

Three puzzles

1. Mass problem: Why are $D^*_{s0}(2317)$ and $D_{s1}(2460)$ masses much lower than the CQM expectations?
2. Splittings: Why $M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)} \simeq M_{D^*} - M_D$ (within a few MeV)?
3. Hierarchy: Why $M_{D^*_0(2300)} > M_{D^*_{s0}(2317)}$, *i.e.*, why $c\bar{u}$, $c\bar{d}$ heavier than $c\bar{s}$?

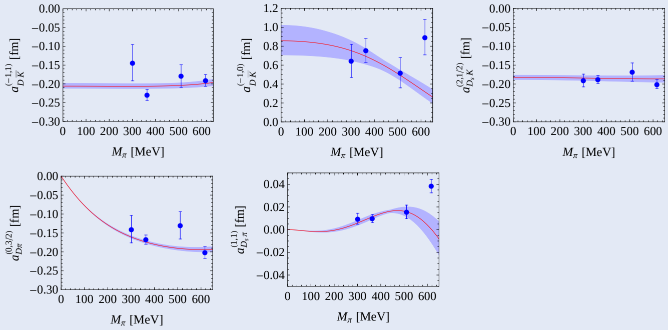
$D\pi$, $D\eta$, $D_s\overline{K}$ scattering amplitudes

- Coupled channel T-matrix: $D\pi$, $D\eta$, $D_s\overline{K}$ scattering $[J^P = 0^+, (S, I) = (0, \frac{1}{2})]$.
- Unitarity: $T^{-1}(s) = V^{-1}(s) - \mathcal{G}(s)$
- Chiral symmetry used to compute the $\mathcal{O}(p^2)$ potential:

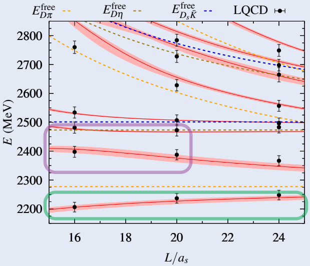
$$f^2 V_{ij}(s,t,u) = C^{ij}_{\rm LO} \frac{s-u}{4} + \sum_{a=0}^{5} h_a C^{ij}_a(s,t,u)$$

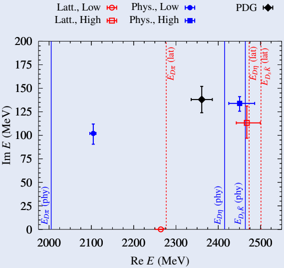
Guo *et al.*, Phys. Lett. B **666**, 251 (2008)
Liu *et al.*, Phys. Rev. D **87**, 014508 (2013)

- h_a (LECs), subtraction constants: free parameters previously fixed.
- Fitted to reproduce scattering lengths obtained in a LQCD simulation:



MA, P. Fernández-Soler, F.-K. Guo, J. Nieves, Phys. Lett. B **767**, 465 (2017)





- $E_n(L)$ are provided for $D\pi$, $D\eta$, $D_s\bar{K}$ in a recent LQCD simulation. [G. Moir *et al.*, JHEP 1610, 011 (2016)]
- Red Bands: Our amplitude in a finite volume. [MA *et al.*, Phys. Lett. B **767**, 465 (2017)]
- No fit is performed (LECs previously determined)
- Level below threshold, associated with a bound state.
- Second level has large shifts w. r. t. thresholds, non-interacting energy levels.
- For lattice masses, we find a bound state and a resonance.
- For physical masses, both evolve into resonances.

	M (MeV)	$\Gamma/2$ (MeV)
Low pole	2105^{+6}_{-8}	102^{+10}_{-12}
High pole	2451^{+36}_{-26}	134^{+7}_{-8}

- We also study DK, $D_s\eta$, $(S,I)=(1,0)$, $D^*_{s0}(2317)$ bound state: $M = 2315^{+18}_{-28}$ MeV.

The $D_0^*(2300)$ structure actually consists of two different states (with complicated interferences with the thresolds)

Previously reported in:

Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)
Guo *et al.*, Eur. Phys. J. A **40**, 171 (2009)

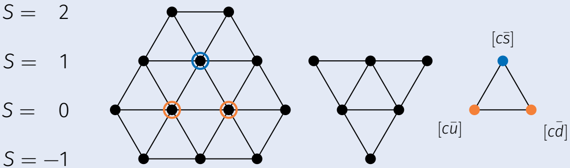
$SU(3)$ light–flavor limit

MA, P. Fernández-Soler, F.-K. Guo, J. Nieves, Phys. Lett. B **767**, 465 (2017)

- $SU(3)$ flavor limit: $m_i \to m = 0.49$ GeV, $M_i \to M = 1.95$ GeV.
- Irrep decomposition: $\bar{3} \otimes 8 = \overline{15} \oplus 6 \oplus \bar{3}$. T and V can be diagonalized:

$$V_d(s) = D^\dagger V(s) D = \text{diag}\left(V_{\overline{15}}(s)\,, V_6(s)\,, V_{\bar{3}}(s)\right) = A(s)\,\text{diag}\,(1, -1, -3)\,,$$

- $\overline{15}$ is repulsive. 6 and $\bar{3}$ are attractive. "Curiously", $\bar{3}$ admits a $c\bar{q}$ interpretation.



State	Channels	(S, I)	$\overline{15}$	6	$\bar{3}$
D_0^*	$D\pi$, $D\eta$, $D_s\bar{K}$	$\left(0, \frac{1}{2}\right)$	✓	✓	✓
$D_{s0}^*(2317)$	DK, $D_s\eta$	$(1, 0)$	✓	✗	✓

In the $SU(3)$ limit:

- low D_0^* and $D_{s0}^*(2317)$ connect with a bound state in $\bar{3}$
- high D_0^* connects with a virtual state in 6
- See also [Gregory *et al.*, 2106.15391]

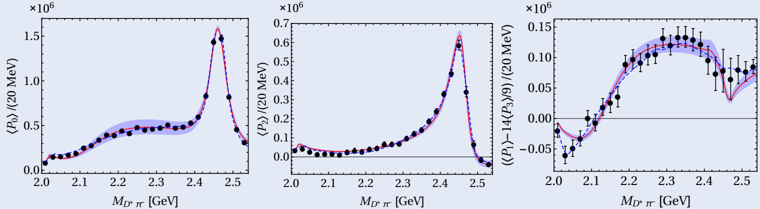
- A recent LQCD calculation by the HadSpec Collaboration finds a similar picture.

[HadSpec Collab., JHEP 02 (2021) 100; JHEP 07 (2021) 123]

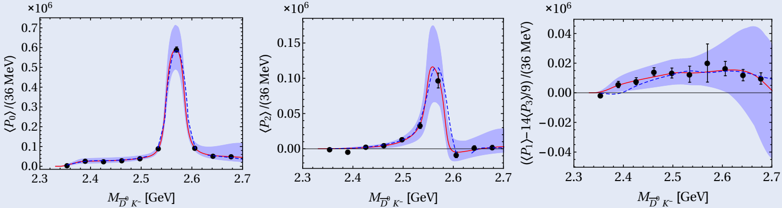
$D\pi$, $D\eta$, $D_s\overline{K}$ and $D^*(2300)$: Comparison with LHCb data

M.-L. Du, MA, P. Fernández-Soler, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. Nieves, D.-L. Yao, PR,D98,094018('18)

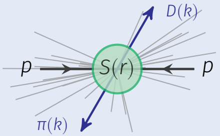
- $B^- \to D^+\pi^-\pi^-$ [LHCb Collab., PR,D94,072001('16)]



- $B_s^0 \to \bar{D}^0 K^- \pi^+$ [LHCb Collab., PR,D90,072003('14)]



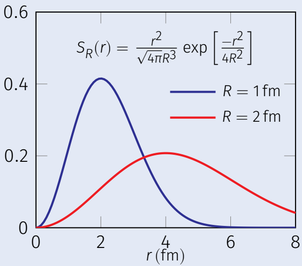
- Rapid movement in $\langle P_{13}\rangle$ [no $D_2(2460)$] at 2.4-2.5 GeV [$D\eta$ and $D_s\bar{K}$].
- Recall: these are the amplitudes with two states in the $D_0^*(2300)$ region, and no fit of the T-matrix parameters is done (production parameters are fitted).



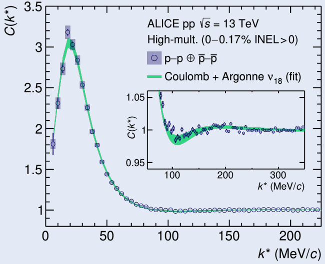
$$C_{\rm exp}(k) = \xi(k)\,\frac{N_{\rm same}(k)}{N_{\rm mixed}(k)}$$

$$C_{\rm th}(k) = \sum_{\ell=0}(2\ell+1)\int dr\, S_R(r)\,|\psi_\ell(r,k)|^2$$

$$\psi_\ell^{[\rm asy,nr]}(k,r) = j_\ell(kr) + f_\ell(k)\,\frac{e^{i(kr-\pi\ell/2)}}{r}$$



[ALICE PL,B805,135419('20)]



- Known the source $S(r)$, explore interactions (encoded in the wave function)
- A new method to explore hadron interactions
- Lot of attraction. In HADRON 2023:
  - M. Janik [Mon. 11:00]
  - V. Mantovani [Mon. 14:30]
  - D. Mihaylov [Mon. 17:40]
  - M. Albaladejo [Tue. 15:10]
  - L. Graczykowski [Wed. 14:00]
  - W. Rzesa [Wed. 14:24]
  - L. Serksnyte [Wed. 15:12]
  - E. Oset [Thu. 14:30]
  - M. Lesch [Thu. 15:12]
  - R. Lea [Thu. 15:42]

[Fabbietti, Mantovani, Vázquez-Doce, ARNPS,71,377('21)]

Check those talks for more references!

Open charm femtoscopy

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)

Channels (S-wave only):

- $\boldsymbol{D_0^*}$ ($S=0$, $I=1/2$): $D\pi$, $D_s\overline{K}$, $D\eta$.
- $\boldsymbol{D_{s0}^*}$ ($S=1$, $I=0,1$): $D_s^+\pi^0$, D^0K^+, D^+K^0, $D_s^+\eta$.

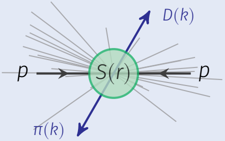
$$C_i(s) = 1 + \int_0^\infty dr\, S_R(r) \left[\sum_j \left| \psi_i^j(s,r) \right|^2 - j_0(p_i r)^2 \right]$$

$$\psi_i^j(s,r) = j_0(p_i r)\delta_{ij} + \tilde{G}_j(s,r) T_{ji}(s)$$

$$T^{-1}(s) = V^{-1}(s) - G(s) \quad \text{(previous slides)}$$

$$\tilde{G}_j(s,r) = \frac{1}{\pi} \int_{s_{\text{th}}}^\infty ds' \frac{p_j(s')}{8\pi\sqrt{s'}} \frac{j_0(p_j(s')r)}{s - s' + i\epsilon} \theta(\Lambda - p_i(s'))$$

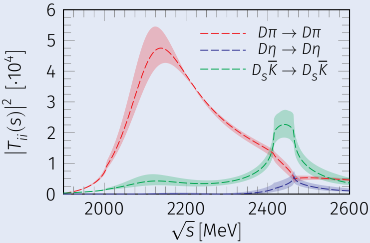
- $C(k)$ in terms of on-shell T-matrix:
  - Feijoo, Vidaña, MA, Nieves, Oset, PLB,846,138201('23)
- Coupled channels based on:
  - Lednicky, Lyuboshitz($\times 2$), PAN,61,2950('98)
  - Haidenbauer, NPA,981,1('19)
- Cut-off Λ:
  - Only used in $\tilde{G}(s,r)$, not in $G(s)$
  - Very soft effect, because $\tilde{G}$ would be convergent (extra $1/p$ from j_0)
  - Take $\Lambda \in [0.6, 0.9]$ GeV

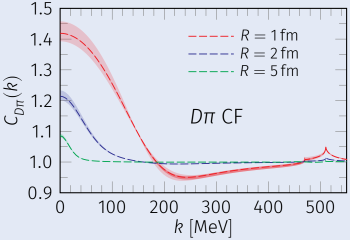


Goal of our paper [MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)]

To use our previously fixed T-matrices in the open-charm sector to predict correlation functions to be measured

- Lower pole, peak at 2135 MeV would correspond to $k_{D\pi} = 215$ MeV. However, we find a minimum at $C_{D\pi}(k)$.



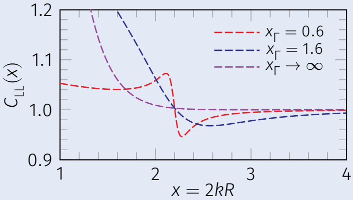


- Take the LL approximation to $C(k)$, and $f(k)$ in terms of $\delta(k)$:

$$C_{LL}(k) = 1 + \frac{2\sin^2\delta(k)}{x^2}\left(e^{-x^2} + \frac{2xF_1(x)}{\sqrt{\pi}}\cot\delta(k)\right)$$

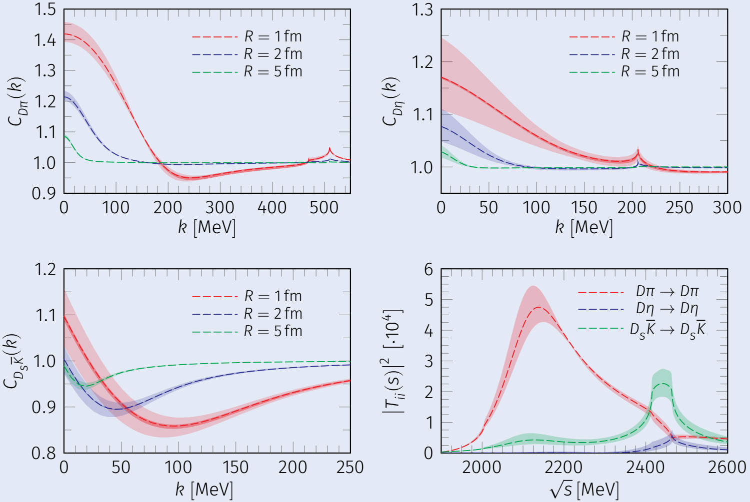
  - For a simple BW: $C_{LL}(k_R) = 1$, $\quad C'_{LL}(k_R) < 0$

Conclusion: the minimum at $k_{D\pi} = 215$ MeV is a clear signature of the lowest pole.



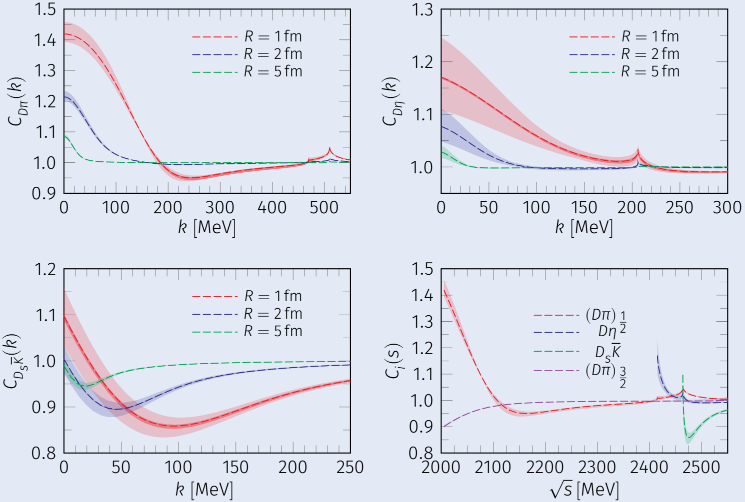
Open charm femtoscopy: results $S = 0$, $I = 1/2$ [$D\pi$, $D\eta$, $D_s\bar{K}$]

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)



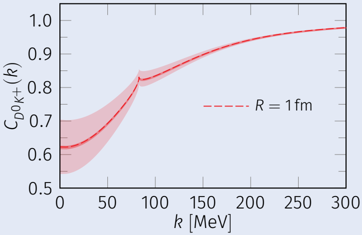
- Two different minima at $\sqrt{s} \simeq 2135$ MeV ($D\pi$ CF) and 2475 MeV ($D_s\bar{K}$ CF), produced by the two different D_0^* states, can be observed
- Their observation would constitute a strong additional support of the two-state pattern.

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)



- Two different minima at $\sqrt{s} \simeq 2135$ MeV ($D\pi$ CF) and 2475 MeV ($D_s\bar{K}$ CF), produced by the two different D_0^* states, can be observed
- Their observation would constitute a strong additional support of the two-state pattern.

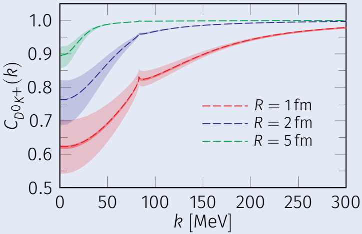
- $D_{s0}^*(2317)$ is a (mostly) DK $I=0$ bound state, with $B=45\,\text{MeV}$, $p_B=190i\,\text{MeV}$
- Why use physical channels?
  - $E^{\text{th}}_{D^+K^0} - E^{\text{th}}_{D^0K^+} \simeq 9\,\text{MeV}$ $[k_{D^0K^+} \simeq 83\,\text{MeV}]$
  - Also, $C_{D^0K^+} = C_{D^+K^0} = \dfrac{C_0^{DK} + C_1^{DK}}{2}$
- Clear depletion at threshold, related to the presence of $D_{s0}^*(2317)$
- Similar trends in recent works, only small differences in the values:
  - [Liu, Lu, Geng, PR,**D107**,074019('23)]
  - [Ikeno, Toledo, Oset, 2305.16431]



Open charm femtoscopy: results $S=1$, $I=0,1$ $[D_s^+\pi^0, D^0K^+, D^+K^0, D_s^+\eta]$

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)

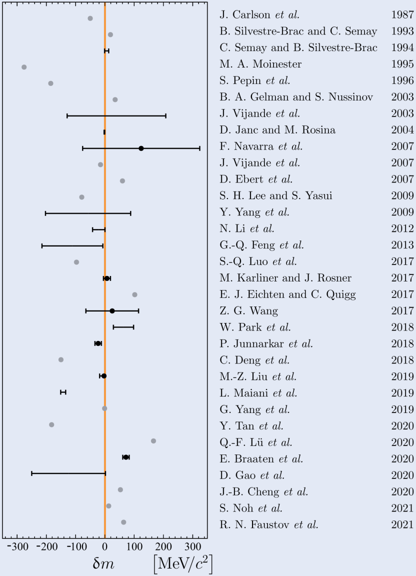
- $D_{s0}^*(2317)$ is a (mostly) DK $I=0$ bound state, with $B=45\,\text{MeV}$, $p_B=190i\,\text{MeV}$
- Why use physical channels?
  - $E^{th}_{D^+K^0} - E^{th}_{D^0K^+} \simeq 9\,\text{MeV}$ $[k_{D^0K^+} \simeq 83\,\text{MeV}]$
  - Also, $C_{D^0K^+} = C_{D^+K^0} = \dfrac{C_0^{DK} + C_1^{DK}}{2}$
- Clear depletion at threshold, related to the presence of $D_{s0}^*(2317)$
- Similar trends in recent works, only small differences in the values:
  - [Liu, Lu, Geng, PR,**D107**,074019('23)]
  - [Ikeno, Toledo, Oset, 2305.16431]



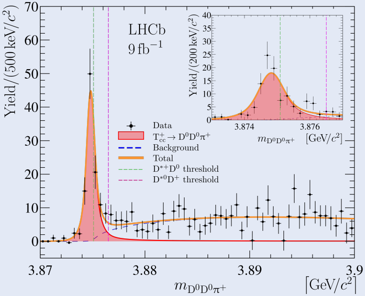
Outline

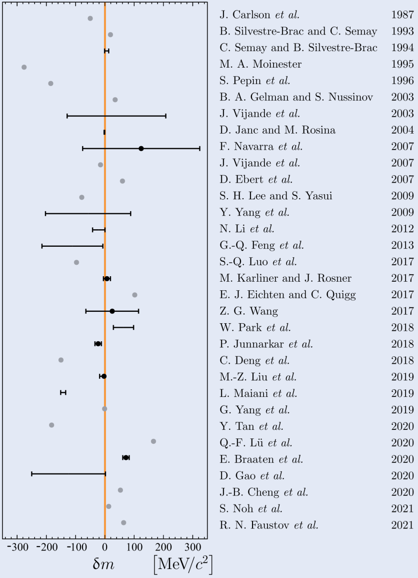
$T_{cc}(3875)^+$ and previous predictions

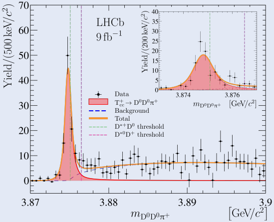
- $T_{cc}(3875)^+$ is a tetraquark with constituent $cc\bar{u}\bar{d}$
- Models give broad range of predictions.
- Not observed until now (only Ξ_{cc}^{++} [LHCb])
  [PRL,119,112001('17)]
- LQCD: not conclusive in the charm sector; more agreement in the bottom sector.
  [Leskovec *et al.*,PR,D100,014503('19)]
  [Bicudo *et al.*,PR,D103,114506('21)]



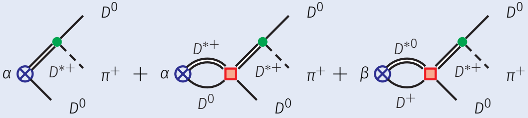
- $T_{cc}(3875)^+$ is a tetraquark with constituent $cc\bar{u}\bar{d}$
- Models give broad range of predictions.
- Not observed until now (only Ξ_{cc}^{++} [LHCb])
  [PRL,119,112001('17)]
- LQCD: not conclusive in the charm sector; more agreement in the bottom sector.
  [Leskovec *et al.*,PR,D100,014503('19)]
  [Bicudo *et al.*,PR,D103,114506('21)]
- Then comes LHCb... [2109.01038;2109.01056]







- LHCb spectrum is essentially T_{cc}^{+} signal and a D^*D phase space background
- Reasonable to assume that all $DD\pi$ events are produced through D^*D
- Small range ($\sim 30\,\text{MeV}$) $DD\pi$ invariant mass: assume D^*D in S-wave



$$\mathcal{N}_{\rm ev}(Q^2) = \mathcal{N}_0 \left(\frac{Q^2_{\rm th}}{Q^2}\right)^{\frac{3}{2}} \int_{s_{\rm th}}^{s_{\rm max}(Q^2)} ds \int_{t_-(s,Q^2)}^{t_+(s,Q^2)} dt \sum_\lambda \left|\mathcal{M}_\lambda(Q^2,s,t,u)\right|^2 ,$$

$$\mathcal{M}_\lambda(Q^2,s,t,u) = g_{D^*D\pi}\, p_\pi^\nu \epsilon_S^\mu(\lambda) \left[\frac{K_t(Q^2)}{t-m^2_{D^*_{(t)}}}\left(-g_{\mu\nu}+\frac{k_\mu^{(t)}k_\nu^{(t)}}{t}\right) + \frac{K_u(Q^2)}{u-m^2_{D^*_{(u)}}}\left(-g_{\mu\nu}+\frac{k_\mu^{(u)}k_\nu^{(u)}}{u}\right)\right] .$$

$$K_t(Q^2) = \alpha\left(1+G_1(Q^2)T_{11}(Q^2)\right) C_{D^{*+}\to D^0\pi^+} + \beta G_2(Q^2)T_{12}(Q^2)\, C_{D^{*+}\to D^0\pi^+} .$$

- Coupled T-matrix for the $D^{*+}D^0$, $D^{*0}D^+$ channels:

$$T^{-1}(E) = V^{-1}(E) - \mathcal{G}(E) \ ,$$

- $I_z = 0$: the isospin decomposition reads:

$$\left|D^{*+}D^0\right\rangle = -\frac{1}{\sqrt{2}}\left(\left|D^*D, I=1\right\rangle + \left|D^*D, I=0\right\rangle\right) \ ,$$
$$\left|D^{*0}D^+\right\rangle = -\frac{1}{\sqrt{2}}\left(\left|D^*D, I=1\right\rangle - \left|D^*D, I=0\right\rangle\right) \ ,$$

$V(E)$: interaction kernels written in terms of $C_{I=0,1}$ (constants):

$$V(E) = \frac{1}{2}\begin{pmatrix} C_0 + C_1 & C_1 - C_0 \\ C_1 - C_0 & C_0 + C_1 \end{pmatrix}$$

$\mathcal{G}(E)$: loop functions of the $D^{*+}D^0$, $D^{*0}D^+$ channels:

$$G_i(E) = \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{e^{-\frac{2\vec{k}^2}{\Lambda^2}}}{E - E^i_{\text{th}} - \frac{\vec{k}^2}{2\mu_i}}$$

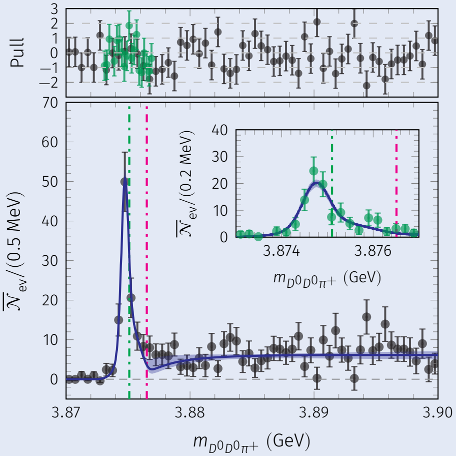
- Width of the D^*: the loop functions are analytically continued to complex values of the D^* mass, $m_{D^*} \to m_{D^*} - i\Gamma_{D^*}/2$.
- Two values for the cutoff, $\Lambda = 0.5$ GeV and $\Lambda = 1.0$ GeV.
- The V-matrix elements depend now on the cutoff, $C_I(\Lambda)$.

Results: Fit

MA, Phys. Lett., B829, 137052 (2021)

- Exp. resolution taken from LHCb ($\delta \simeq 400\,\text{keV}$):

$$\overline{\mathcal{N}}_{\text{ev}}(E) = \int dE'\, R_{\text{LHCb}}\,(E, E')\ \mathcal{N}_{\text{ev}}(E')$$



Parameter	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$C_0(\Lambda)$ [fm^2]	$-0.7008(22)$	$-1.5417(121)$
$C_1(\Lambda)$ [fm^2]	$-0.440(79)$	$-0.71(27)$
β/α	0.228(108)	0.093(79)
χ^2/dof	0.95	0.92

- Good agreement ($\chi^2/\text{dof} = \{0.92, 0.95\}$)
- Check: pull of the data seems randomly distributed.
- Statistical uncertainties obtained by MC bootstrap of the data

- Bound state pole in T-matrix, $\det(\mathbb{1} - VG) = 0$:

$$T_{ij}(E) = \frac{\tilde{g}_i \tilde{g}_j}{E^2 - \left(M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2\right)^2} + \cdots$$

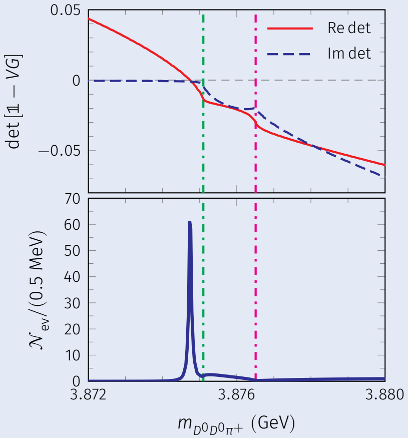
- Width: $m_{D^*} - i\,\Gamma_{D^*}/2 \Rightarrow M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2$
- Pole position (wrt $D^{*+}D^0$ threshold):

Λ (GeV)	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
1.0	−357(29)	77(1)
0.5	−356(29)	78(1)

- Good agreement with LHCb determination:

	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
[2109.01038]	−273(61)	410(165)
[2109.01056]	−360(40)	48(2)

- Our width is somewhat larger than the ~ 50 keV obtained by LHCb and [Feijoo *et al.*, 2108.02730], [Ling *et al.*, 2108.00947].
- [Du *et al.*, 2110.13765]: $\Gamma_{T_{cc}^+}$ depending on the model used.



- Results similar to [LHCb, 2109.0156] (top) and [Feijoo *et al.*, 2108.02730; Du *et al.*, 2110.13765] (bottom).

Molecular state?

MA, Phys. Lett., B829, 137052 (2021)

- Weinberg compositeness [Weinberg, PR,137,B672('65)]: $P = 1 - Z \simeq \frac{\mu^2 g^2}{2\pi\gamma_B} = -g^2 G'(E_B)$
- We get $P_{D^{*+}D^0} = 0.78(5)(2)$, $P_{D^{*0}D^+} = 0.22(5)(2) \to P_{I=0} = 1$ purely molecular state (model built-in!)
  - Single channel & isospin limit:

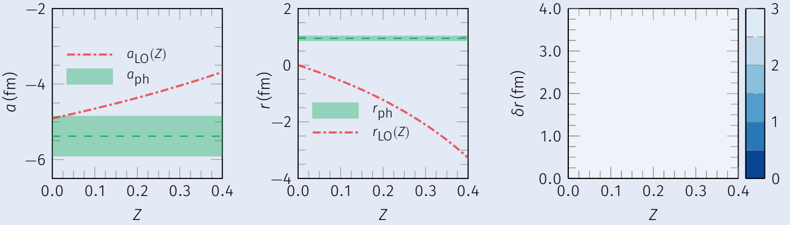
Λ (GeV)	0.5	1.0
E_B (keV)	833(67)	856(53)
$a_{I=0}$ (fm)	$-5.57(25)$	$-5.18(16)$
$r_{I=0}$ (fm)	0.63	1.26

  - Relation to ERE parameters a, r [Weinberg,PR,137,B672('65)]

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} + \cdots ,$$

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \cdots .$$

- Average values: $a_{\rm ph} = -5.38(30)$ fm, $r_{\rm ph} = 0.95(32)$ fm, $\gamma_{B_{\rm ph}} = 40.4(1.7)$ MeV.



The values obtained clearly support a molecular picture for T_{cc}^+

Molecular state?

MA, Phys. Lett., B829, 137052 (2021)

- Weinberg compositeness [Weinberg, PR,137,B672('65)]: $P = 1 - Z \simeq \frac{\mu^2 g^2}{2\pi\gamma_B} = -g^2 G'(E_B)$
- We get $P_{D^{*+}D^0} = 0.78(5)(2)$, $P_{D^{*0}D^+} = 0.22(5)(2) \to P_{I=0} = 1$ purely molecular state (model built-in!)
  - Single channel & isospin limit:

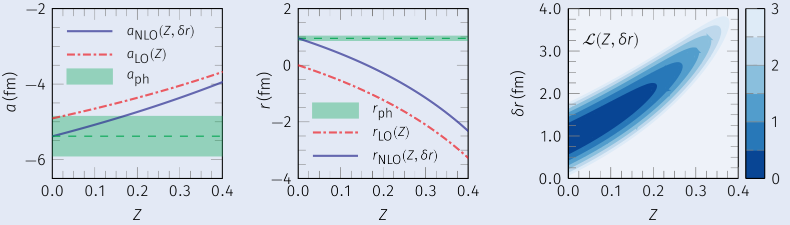
Λ (GeV)	0.5	1.0
E_B (keV)	833(67)	856(53)
$a_{I=0}$ (fm)	−5.57(25)	−5.18(16)
$r_{I=0}$ (fm)	0.63	1.26

  - Relation to ERE parameters a, r [Weinberg,PR,137,B672('65)] + [MA, J. Nieves, EPJ,C82,8('22)]

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} - 2\delta r\left(\frac{1-Z}{2-Z}\right)^2 + \cdots \,,$$

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \delta r + \cdots \,.$$

- Average values: $a_{\rm ph} = -5.38(30)$ fm, $r_{\rm ph} = 0.95(32)$ fm, $\gamma_{B_{\rm ph}} = 40.4(1.7)$ MeV. Minimum at $\delta r \simeq r_{\rm ph} \simeq 1$ fm



The values obtained clearly support a molecular picture for T_{cc}^+

HQSS partner

MA, Phys. Lett., B829, 137052 (2021)

- Heavy-Quark Spin Symmetry (HQSS) predicts that heavy-meson interactions are independent of the heavy-quark spin in the limit $m_Q \to \infty$.
- Relation between $D^*D^* \to D^*D^*$ and $D^*D \to D^*D$ amplitudes.
- The interaction kernels of the $I(J^P)$ D^*D^* systems are related to those of the D^*D ones as:

$$\left\langle D^*D^*,\, 0(1^+) \left|\hat{V}\right| D^*D^*,\, 0(1^+)\right\rangle = \left\langle D^*D,\, 0(1^+) \left|\hat{V}\right| D^*D,\, 0(1^+)\right\rangle = V_0\ ,$$
$$\left\langle D^*D^*,\, 1(2^+) \left|\hat{V}\right| D^*D^*,\, 1(2^+)\right\rangle = \left\langle D^*D,\, 1(1^+) \left|\hat{V}\right| D^*D,\, 1(1^+)\right\rangle = V_1\ .$$

- We predict the existence of T_{cc}^{*+}, a D^*D^* molecular state, HQSS partner of T_{cc}^+, with a binding energy (wrt the different D^*D^* thresholds) of 1.1–1.5 MeV.

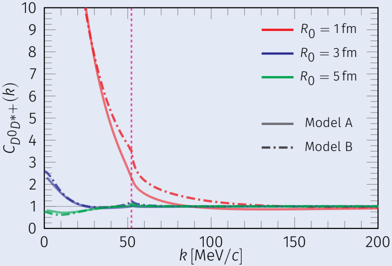
	$\delta M_{T_{cc}^*}$ (keV)			
	Isoscalar solution		Isovector solution	
	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$D^{*+}D^{*+}$			−1580(71)	−1156(79)
$D^{*+}D^{*0}$	−1561(71)	−1148(79)	−1561(71)	−1148(79)
$D^{*0}D^{*0}$			−1543(71)	−1140(79)

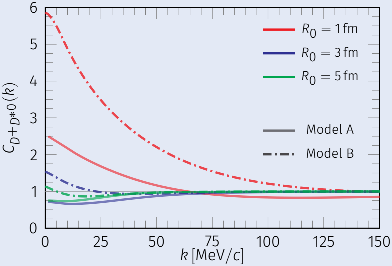
- Similar predictions are obtained in a later work [Dai *et al.*, PR,D105,016029('22)]
- Previous works predicting D^*D^* states: [Molina *et al.*, PR,D82,014010('10); Liu *et al.*, PR,D99,094018('19)].

Femtoscopic correlation functions for $T_{cc}(3875)^+$

Feijoo, Vidaña, MA, Nieves, Oset, PLB,846,138201('23)

- $D^0 D^{*+}$ channel: $C(k)$ positive and very large ($\simeq 30$) for $R = 1\,\text{fm}$ at the threshold.
- $D^+ D^{*0}$ channel: Also positive and larger than 1 (but substantially smaller than for $D^0 D^{*+}$)
- These predictions could be compared with future measurements by ALICE





Outline

Conclusions

- Constituent quark models are able to reproduce many of the discovered hadrons, but there's room for many more species of them.
- We have entered into the exotic and precision spectroscopy era.
- Femtoscopy can give new experimental insights into these challenges.
- Effective Field Theories are an ideal tool to study reactions and spectroscopy
  - Their combination with LQCD simulations can be very powerful
- For single charm studies, the combination of ChPT, $SU(3)$ and HQSS produces an economic description of scattering amplitudes.
  - Two states are predicted instead of a single $D_0^*(2300)$
  - This picture is compatible with different LQCD simulations and experimental (LHCb) data
  - We have also predicted CFs that can be measured in femtoscopy studies at ALICE
- We are having a first glimpse into the double charm sector.
  - The study of the spectra points to a molecular nature of the $T_{cc}(3875)^+$
  - A HQSS partner [$T_{cc}^{*+}(4016)$] is predicted. Its finding would constitute a strong support of this picture.
  - Femtoscopic correlation functions are also predicted for $T_{cc}(3875)^+$.

Thank you!

(Some) attempts to explain $D_{s0}^*(2317)$ as a $c\bar{s}$ state

[Ortega *et al.*, PR,D94,074037('16) (and references therein)]

- Problem: original Quark Model prediction mass is $\sim$ 150 MeV above experimental one.
- 1-loop correction to OGE potential ($\mathcal{O}(\alpha_s^2)$) reduces the mass to 2383 MeV, much closer to the experimental one.

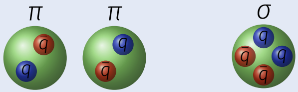
[Lakhina, Swanson, PL,B650,159('07)]

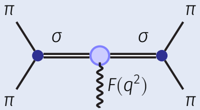
- ${}^3P_0$ mechanism to couple $c\bar{s}$ states to DK meson-pairs, $P_{DK} \sim 30\%$.
- Much better situation, but:
  - Still above DK threshold
  - This mechanism only affects the 0^+ sector, still problems with 1^+
  - Coupling to DK is included, but no DK "dynamics"

- Need of more collaboration between (and simultaneous use of!) different "subcommunities": LQCD, molecular/tetraquarks/QM models...
- Spectroscopy, mixing:
  Specific example of $D_{s0}^*(2317)$, take for granted the presence of a CQM $c\bar{s}$ state. Theoretical possibilities:
  - Genuine $c\bar{s}$, (very) renormalized by DK threshold. Or renormalized by DK interactions themselves?
  - Or, there is a $S=1$, $I=0$ state coming from DK interactions in addition to the $c\bar{s}$ state. If so, where are those two poles? Which is which?

[Cincioglu *et al.*, EPJ,C76,576('16); MA *et al.*, EPJ,C78,722('19)]

- Nature/size:
  - Can we address the question of $4q$, $q\bar{q}$, molecule based on the size of the object?





  - For $\pi\pi$ scattering, σ meson: MA, Oller, PR,D86,034003('12)
    - $\sqrt{\langle r^2\rangle_\sigma^S} \simeq 0.44$ fm vs $\sqrt{\langle r^2\rangle_\pi^S} \simeq 0.81$ fm
  - Perhaps only theoretical? Future lattice QCD calculations?

Briceño *et al.*, PR,D103,114512('21) [and refs. therein]

Connecting $SU(3)$ and physical limits Riemann sheets

Riemann sheets:

$$\mathcal{G}_{ii}(s) \to \mathcal{G}_{ii}(s) + i\,\frac{p_i(s)}{4\pi\sqrt{s}}\,\xi_i$$

$SU(3)$ limit:

$$m_i = m_i^{\rm phy} + x(m - m_i^{\rm phy})\,, \qquad (m = 0.49~\text{GeV})\,,$$
$$M_i = M_i^{\rm phy} + x(M - M_i^{\rm phy})\,, \qquad (M = 1.95~\text{GeV})\,.$$

- Physical case ($x = 0$): RS specified by $(\xi_1\xi_2\xi_3)$, $\xi_i = 0$ or 1.
- $SU(3)$ symmetric case ($x = 1$): all channels have the same threshold, so there are only two RS (000) and (111).
- To connect the lower pole with the T_6 virtual state,

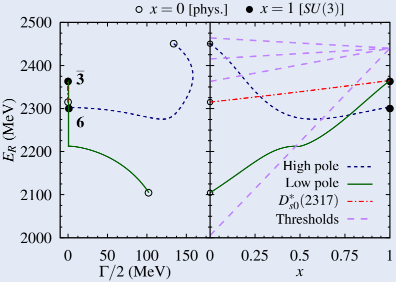
$$\xi_3 = x \qquad (1,1,0) \to (1,1,x)$$

- To connect the lower pole with the $T_{\bar{3}}$ bound state,

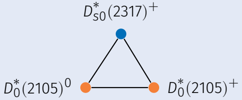
$$\xi_1 = 1 - x \qquad (1,0,0) \to (1-x,0,0)$$

Connecting physical ($x = 0$) and flavor $SU(3)$ ($x = 1$) limits:

$$m_i = m_i^{\text{phy}} + x(m - m_i^{\text{phy}}) \,, \quad (m = 0.49 \text{ GeV}) \,,$$
$$M_i = M_i^{\text{phy}} + x(M - M_i^{\text{phy}}) \,, \quad (M = 1.95 \text{ GeV}) \,.$$



- The high D_0^* connects with a 6 virtual state (unph. RS, below threshold).
- The low D_0^* connects with a $\overline{3}$ bound state (ph. RS, below threshold).
- The $D_{s0}^*(2317)$ also connects with the $\overline{3}$ bound state.



- The low D_0^* and the $D_{s0}^*(2317)$ are $SU(3)$ flavor partners.
- This solves the “puzzle” of $D_{s0}^*(2317)$ being lighter than $D_0^*(2300)$: it is not, the lower D_0^* pole ($M = 2105$ MeV) is lighter.

Form factors in semileptonic $D \to \pi \bar{\ell} \nu_\ell$

D.-L. Yao, P. Fernández-Soler, MA, F.-K. Guo, J. Nieves, Eur. Phys. J. C **78**, 310 (2018)

- General definitions:

$$\frac{d\Gamma(D \to \pi \bar{\ell} \nu_\ell)}{dq^2} = \frac{G_F^2}{24\pi^3} |\vec{p}_\pi|^3 |V_{cd}|^2 |f_+(q^2)| \,. \qquad [q^2 = 0 : f_+(0) = f_0(0)]$$

$$\langle \pi(p') | \bar{q} \gamma^\mu Q | D(p) \rangle = f_+(q^2) \left[\Sigma^\mu - \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_D^2 - m_\pi^2}{q^2} q^\mu \,,$$

- "Isospin" form factors, related to $D\pi$, $D\eta$, $D_s \bar{K}$ scattering:

$$\mathcal{F}^{(0,1/2)}(s) \equiv \begin{pmatrix} -\sqrt{\frac{3}{2}} f_0^{D^0 \to \pi^-}(s) \\ -f_0^{D^+ \to \eta}(s) \\ -f_0^{D_s^+ \to K^0}(s) \end{pmatrix} \,, \qquad \mathrm{Im} \mathcal{F}(s) = T^*(s) \Sigma(s) \mathcal{F}(s)$$

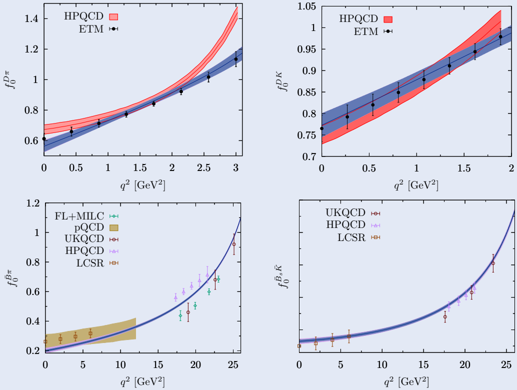
- Write form factors as Omnés matrix times polynomials

$$\mathcal{F}(s) = \Omega(s) \cdot \mathcal{P}(s)$$

- Polynomials fixed so as to reproduce the NLO chiral lagrangian:

$$\mathcal{L}_0 = f_{\mathcal{P}} \left(\mathring{m} P^*_\mu - \partial_\mu P \right) u^\dagger J^\mu \,,$$

$$\mathcal{L}_0 = \beta_1 P \, u \, (\partial_\mu U^\dagger) J^\mu + \beta_2 (\partial_\mu \partial_\nu P) \, u \, (\partial^\nu U^\dagger) \, J^\mu \,.$$



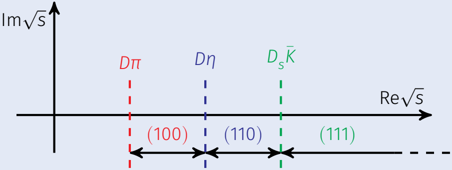
- Points mostly from LQCD
- Also LCSR for $q^2 \to 0$
- Good agreement in general
- CKM matrix can also be calculated
- Definitive results may differ...

	This work	Exp.
$10^3 \lvert V_{ub} \rvert$	4.3(7)	4.49(24) [Incl.] 3.72(19) [Excl.]
$\lvert V_{cd} \rvert$	0.244(22)	0.220(5)
$\lvert V_{cs} \rvert$	0.945(41)	0.995(16)

T-matrix and analytical continuations

- Normalization: $-ip_{ii}(s)T_{ii}(s) = 4\pi\sqrt{s}\left(\eta_i(s)e^{2i\delta_i(s)} - 1\right)$.
- $\mathcal{G}_{ii}(s) = G(s, m_i, M_i)$, regularized with a subtraction constant $a(\mu)$ ($\mu = 1$ GeV).
- Riemann sheets (RS) denoted as $(\xi_1\xi_2\xi_3)$:

$$\mathcal{G}_{ii}(s) \to \mathcal{G}_{ii}(s) + i\frac{p_i(s)}{4\pi\sqrt{s}}\,\xi_i$$



Why is $D_0^*(2300)$ interesting?

- Lightest systems to test ChPT with heavy mesons, besides $D^* \to D\pi$.
- $D\pi$ interactions (where it shows up) are relevant, since $D\pi$ appears as a final state in many reactions that are being considered now (*i.e.*, $Z_c(3900)$ and $\bar{D}^* D\pi$)
- $D_0^*(2300)$ is important in weak interactions and CKM parameters:

  Flynn, Nieves, Phys. Rev. D **76**, 031302 (2007)

  D.-L. Yao, P. Fernández-Soler, MA, F.-K. Guo, J. Nieves, Eur. Phys. J. C **78**, 310 (2018)

  - It determines the shape of the scalar form factor $f_0(q^2)$ in semileptonic $D \to \pi$ decays.
  - Relation to $|V_{cd}|$: $f_+(0) = f_0(0)$ and $d\Gamma \propto |V_{cd} f_+(q^2)|^2$.
  - Even more interesting: the bottom analogue $|V_{ub}|$.

$D\pi$, $D\eta$, $D_s\bar{K}$ energy levels in a finite volume

- Periodic boundary conditions imposes momentum quantization
- Lüscher formalism:

Commun. Math. Phys. **105**, 153 (1986)

Nucl. Phys. B **354**, 531 (1991)

infinite volume	finite volume
$\vec{q} \in \mathbb{R}^3$	$\vec{q} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$
$\int_{\mathbb{R}^3} \frac{d^3q}{(2\pi)^3}$	$\frac{1}{L^3}\sum_{\vec{n}\in\mathbb{Z}^3}$

- In practice, changes in the T-matrix: $T(s) \to \tilde{T}(s,L)$:

Döring *et al.*, Eur. Phys. J. **A** 47, 139 (2011)

$$\mathcal{G}_{ii}(s) \to \widetilde{\mathcal{G}}_{ii}(s,L) = \mathcal{G}_{ii}(s) + \lim_{\Lambda\to\infty}\left(\frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}\,|<\Lambda} I_i(\vec{q}\,) - \int_0^{\Lambda}\frac{q^2 dq}{2\pi^2}\, I_i(\vec{q}\,)\right) ,$$

$$V(s) \to \tilde{V}(s,L) = V(s) ,$$

$$T^{-1}(s) \to \tilde{T}^{-1}(s,L) = V^{-1}(s) - \widetilde{\mathcal{G}}(s,L) ,$$

- Free energy levels: $E^{(i)}_{n,\text{free}}(L) = \omega_{i1}((2\pi n/L)^2) + \omega_{i2}((2\pi n/L)^2)$
- Interacting energy levels $E_n(L)$: $\tilde{T}^{-1}(E_n^2(L), L) = 0$ (poles of the $\tilde{T}$-matrix).

- Other famous two-poles structures rooted in chiral dynamics:

$\Lambda(1405)$ $[\Sigma\pi, N\bar{K}]$

Oller, Meißner, Phys. Lett. B **500**, 263 (2001)
Jido *et al.*, Nucl. Phys. A **725**, 181 (2003)
García-Recio *et al.*, Phys. Lett. B **582**, 49 (2004)
Magas *et al.*, Phys. Rev. Lett. **95**, 052301 (2005)

$K_1(1270)$

Roca *et al.*, Phys. Rev. D **72**, 014002 (2005)
Geng *et al.*, Phys. Rev. D **75**, 014017 (2007)
García-Recio *et al.*, Phys. Rev. D **83**, 016007 (2011)

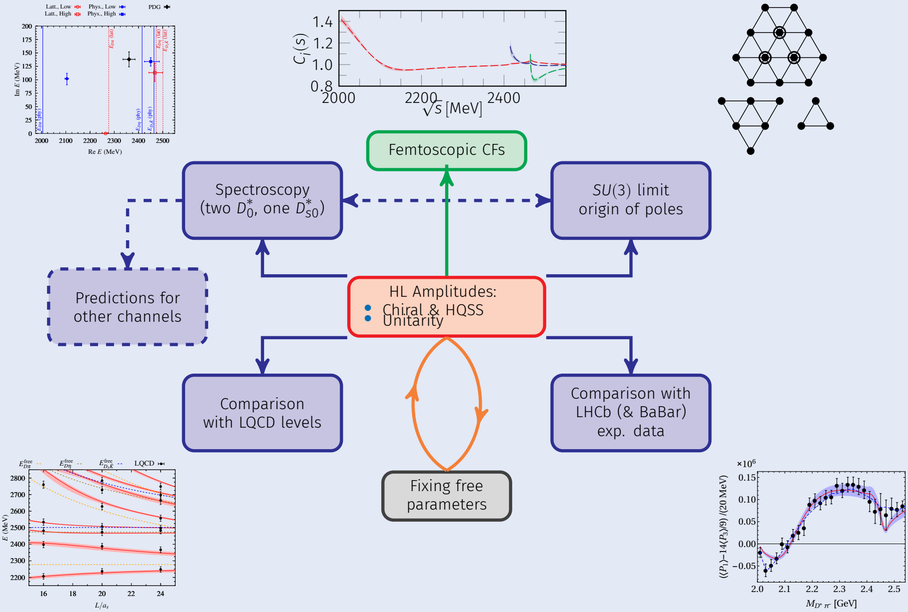
- Recently, Clymton, Kim, 2305.14812 claim a two-pole structure for $b_1(1235)$.

- Chiral dynamics:
  - Incorporates the $SU(3)$ light-flavor structure,
  - Determines the strength of the interaction,
  - Ensures lightness of Goldstone bosons, which in turn separates generating channels from higher hadronic channels.

Summary and conclusions

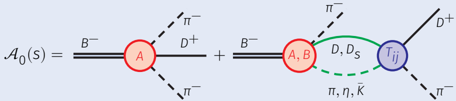
- We have employed unitarized NLO chiral amplitudes to study the open-charm sector.
- The LECs are fixed through LQCD calculations of m_π-dependent scattering lengths.
- The $D_0^*(2300)$ structure is actually produced by two different states (poles), together with complicated interferences with thresholds.
- This two-state structure receives strong support. Without any fit, the amplitudes are compatible with:
  - available LQCD simulations,
  - and experimental data
- This picture, with the lowest D_0^* pole and $D^*(2317)$ being flavour partners nicely solves simultaneously all the puzzles.
- We have used these amplitudes to predict femtoscopy correlation functions in the open-charm sector.
- In particular, we have highlighted the imprints that the two-state structure leaves on $C_{D\pi}$, $C_{D\eta}$, and $C_{D_s\bar{K}}$.
- The measurements of these CFs can shed further light on the (very interesting!) open-charm sector.

Scheme of unitary EFTs relations to LQCD and experiments



Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

- $\mathcal{A}(s,z) = \mathcal{A}_0(s) + \sqrt{3}\mathcal{A}_1(s)P_1(z) + \sqrt{5}\mathcal{A}_2(s)P_2(z) + \dots$
- P-,D-wave as in LHCb paper: D^*, $D^*(2680)$ in P-wave, $D_2^*(2460)$ in D-wave
- S-wave parameterization:



$$\mathcal{A}_0(s) = A\Big\{E_\pi\Big[2 + G_1(s)\left(\frac{5}{3}T_{11}^{1/2}(s) + \frac{1}{3}T^{3/2}(s)\right)\Big] + \frac{1}{3}E_\eta G_2(s)T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}}E_{\bar{K}}G_3(s)T_{31}^{1/2}(s)\Big\} + BE_\eta G_2(s)T_{21}^{1/2}(s),$$

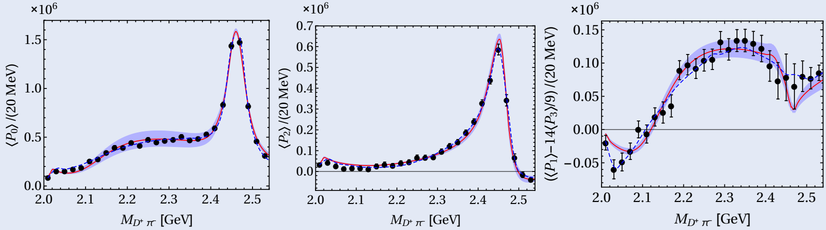
- Angular moments: $\langle P_\ell\rangle(s) = \int dz\,|\mathcal{A}(s,z)|^2\,P_\ell(z)$

$$\langle P_0\rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2\ , \qquad \langle P_2\rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2|\cos(\delta_0-\delta_2)\ ,$$

$$\langle P_{13}\rangle \equiv \langle P_1\rangle - \frac{14}{9}\langle P_3\rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1|\cos(\delta_0-\delta_1)\ .$$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)

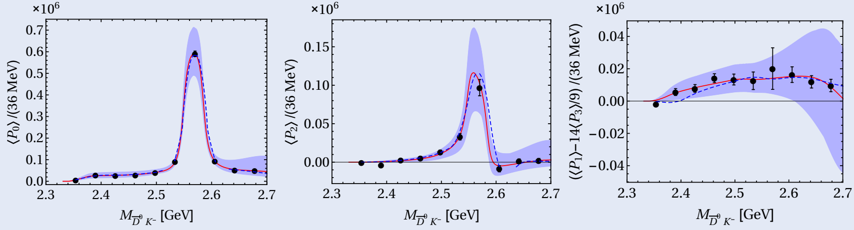
Data: LHCb Collab., PR,D94,072001('16)



- Parameters: $B/A = -3.6 \pm 0.1$, $a_A = 1.0 \pm 0.1$, $\chi^2/\text{d.o.f.} = 1.7$
- — This work. - - - LHCb. Bands: fit uncertainty
- Good agreement with data & with LHCb fit
- Rapid movement in $\langle P_{13}\rangle$ [no $D_2(2460)$] between 2.4 and 2.5 GeV. Related to $D\eta$ and $D_s\bar{K}$ openings.
- Recall: these are the amplitudes with two states in the $D_0^*(2300)$ region, and no fit of the T-matrix parameters is done.

Comparison with experimental data: $B_s^0 \to \bar{D}^0 K^- \pi^+$

Du, MA, Fernández-Soler, Guo, Hanhart, Meißner, Nieves, Yao, PR,D98,094018('18)
Data: LHCb Collab., PR,D90,072003('14)



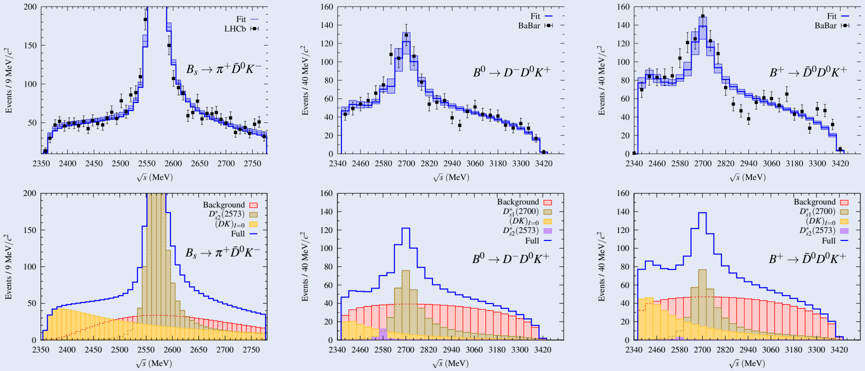
- Exactly the same formalism applied to $B_s^0 \to \bar{D}^0 K^- \pi^+$.

$$\mathcal{A}_0(s) = E_K \left[C + \frac{C+A}{2} G_1(s) T^0_{11}(s) + \frac{C-A}{2} G_1(s) T^1_{11}(s) \right] - \frac{1}{\sqrt{3}} \left(\frac{3}{2} B - C \right) E_\eta G_2(s) T^0_{21}(s)$$

- Take same value of B/A, a_A, as before: $C/A = 4.8^{+3.4}_{-1.7}$, χ^2/d.o.f. $= 1.6$
- — This work. - - - LHCb. Bands: fit uncertainty
- Good agreement with data & with LHCb fit
- See also Du *et al.*, PRL,126,192001('21)

MA, Nieves, Oset, Jido, EPJ,C76,300('16)
Data: LHCb Collab., PR,D90,072003('14)
BaBar Collab., PR,D91,052002('15)

- The LHCb (and BaBar) data had already been analyzed with a similar amplitude



Predictions for other sectors: charm

(S,I)	Channels	$\overline{\mathbf{15}}$	$\mathbf{6}$	$\overline{\mathbf{3}}$	0^+ M	0^+ $\Gamma/2$	1^+ M	1^+ $\Gamma/2$
$(0,\frac{1}{2})$	$D^{(*)}\pi$, $D^{(*)}\eta$, $D_s^{(*)}\bar{K}$	✓	✓	✓	(R) 2105^{+6}_{-8}	102^{+10}_{-12}	(R) 2240^{+5}_{-6}	93^{+9}_{-9}
					(R) 2451^{+36}_{-26}	134^{+7}_{-8}		
$(1,0)$	$D^{(*)}K$, $D_s^{(*)}\eta$	✓	✗	✓	(B) 2315^{+18}_{-28}		(B) 2436^{+16}_{-22}	
$(-1,0)$	$D^{(*)}\bar{K}$	✗	✓	✗	(V) 2342^{+13}_{-41}		–	
$(1,1)$	$D_s^{(*)}\pi$, $D^{(*)}K$	✓	✓	✗	–		–	

- HQSS relates 0^+ ($D_{(s)}P$) and 1^+ ($D^*_{(s)}P$) sectors: similar resonance pattern.
- Two pole structure: higher D_1 pole probably affected by ρ channels.
- $D\bar{K}$ $[0^+, (-1,0)]$: this virtual state (from $\mathbf{6}$) has a large impact on the scattering length, $a^{D\bar{K}}_{(-1,0)} \simeq 0.8$ fm. (Rest of scattering lengths are $|a| \simeq 0.1$ fm.)

Predictions for other sectors: bottom

(S,I)	Channels	$\overline{\mathbf{15}}$	$\mathbf{6}$	$\overline{\mathbf{3}}$	0^+ M	0^+ $\Gamma/2$	1^+ M	1^+ $\Gamma/2$
$\left(0,\frac{1}{2}\right)$	$\bar{B}^{(*)}\pi$, $\bar{B}^{(*)}\eta$, $\bar{B}_s^{(*)}\bar{K}$	✓	✓	✓	(R) 5537^{+9}_{-11} (R) 5840^{+12}_{-13}	116^{+14}_{-15} 25^{+6}_{-5}	(R) 5581^{+9}_{-11}	115^{+13}_{-15}
$(1,0)$	$\bar{B}^{(*)}K$, $\bar{B}_s^{(*)}\eta$	✓	✗	✓	(B) 5724^{+17}_{-24}		(B) 5768^{+17}_{-23}	
$(-1,0)$	$\bar{B}^{(*)}\bar{K}$	✗	✓	✗	(V–B) thr.		(V–B) thr.	
$(1,1)$	$\bar{B}_s^{(*)}\pi$, $\bar{B}^{(*)}K$	✓	✓	✗	–		–	

- Heavy flavour symmetry relates charm (D) and bottom ($\bar{B}$) sectors.
- $(0,\frac{1}{2})$: B_0^*, two-pole pattern also observed.
- $(-1,0)$: $[B^{(*)}\bar{K}]$: very close to threshold. Relevant prediction.
  Can be either bound or virtual $(\mathbf{6})$ within our errors.
- $(1,1)$: $[\bar{B}_s\pi, \bar{B}K, 0^+]$, $X(5568)$ channel. No state is found: $\overline{\mathbf{15}}$ and $\mathbf{6}$. If it exists, it is not generated with these $B_s\pi$, $B\bar{K}$ interactions.
  M. A. *et al.*, Phys. Lett. B **757**, 515 (2016); Guo *et al.*, Commun. Theor. Phys. **65**, 593 (2016)
- $(1,0)$: Our results for B_{s0}^* and B_{s1} agree with other results from LQCD:
  Lang *et al.*, Phys. Lett. B **750**, 17 (2015); M. A. *et al.* Eur. Phys. J. C**77**, 170 (2017)
- Comparison of 0^+, 1^+ beauty states by Colangelo *et al.*, Phys. Rev. D **86** 054024 (2012): agreement in $(1,0)$ $[b\bar{s}]$, but not in $(0,1/2)$ $[b\bar{q}]$.

Accesing isospin-definite $C(k)$ from physical channels

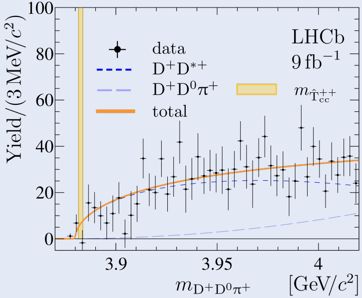
MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)

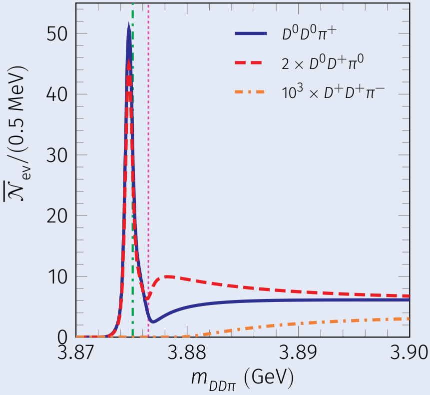
- We are considering $I_z = +\frac{1}{2}$ channels: $D^+\pi^0/D^0\pi^+$, $D^+\eta$, $D_s^+\overline{K}^0$.
- We avoid Coulomb interaction, which shows up in $I_z = -\frac{1}{2}$, through $D^+\pi^-$ (not in $D^0\pi^0$) and $D_s^+K^-$.
- Q: Does it make sense to use isospin channels, when physical ones are measured?
  - Physical channels will have combination of $I = \frac{1}{2}$ and $I = \frac{3}{2}$.
  - Potentially, you can also have Coulomb interaction.

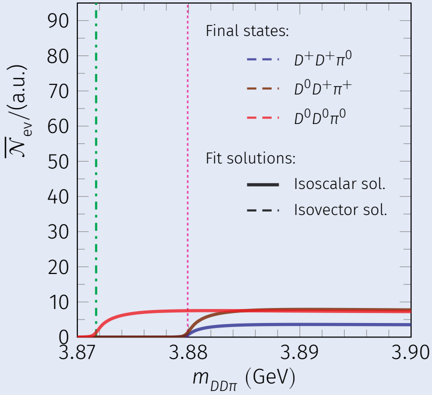
  A: Let us see...
- Inserting isospin decompositions in the amplitudes, we get interesting relations:
  - $I = \dfrac{1}{2}$: $\quad C_{D\pi}^{(1/2)} = 2C_{D^0\pi^+} - C_{D^+\pi^0} = \dfrac{3C_{D^0\pi^+} - C_{D^0\pi^-}}{2}$
  - $I = \dfrac{3}{2}$: $\quad C_{D\pi}^{(3/2)} = 2C_{D^+\pi^0} - C_{D^0\pi^+} = C_{D^0\pi^-}$

Isospin $I = 0$ or $I = 1$, and fit degeneracy

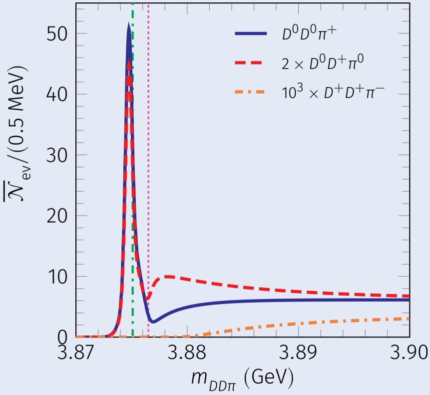
- **Q**: Can I know the T_{cc}^{+} isospin from this analysis? **A**: No
- There's a degeneracy in the solutions: the model is "invariant" under a simultaneous exchange: $C_0 \leftrightarrow C_1$ and $\beta \leftrightarrow -\beta$
- Physically, this is due to the fact that $D^0 D^0 \pi^+$ has $I_z = 0$, so you cannot know whether T_{cc}^{+} is $|I\, I_z\rangle = |1\,0\rangle$ or $|0\,0\rangle$.
- We will keep both solutions $I = 0$ or $I = 1$ for T_{cc}^{+} and discuss their differences (whenever they exist!)
- LHCb [2109.01056] has shown an additional spectrum, $D^+ D^0 \pi^+$ ($I_z = 1$), in which no sign of a T_{cc}^{++} is observed, but with much less statistics:

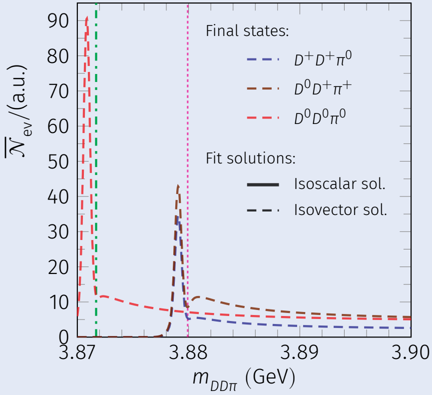



$D^0D^0\pi^+$
$2 \times D^0D^+\pi^0$
$10^3 \times D^+D^+\pi^-$
$\overline{\mathcal{N}}_{ev}/(0.5\ \text{MeV})$
$m_{DD\pi}$ (GeV)
0
10
20
30
40
50
3.87
3.88
3.89
3.90

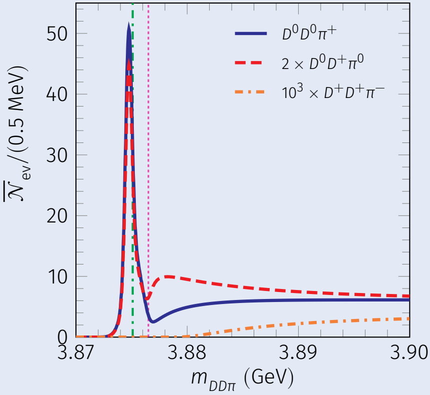

Final states:
$D^+D^+\pi^0$
$D^0D^+\pi^+$
$D^0D^0\pi^0$
Fit solutions:
Isoscalar sol.
Isovector sol.
$\overline{\mathcal{N}}_{ev}/(\text{a.u.})$
$m_{DD\pi}$ (GeV)
0
10
20
30
40
50
60
70
80
90
3.87
3.88
3.89
3.90

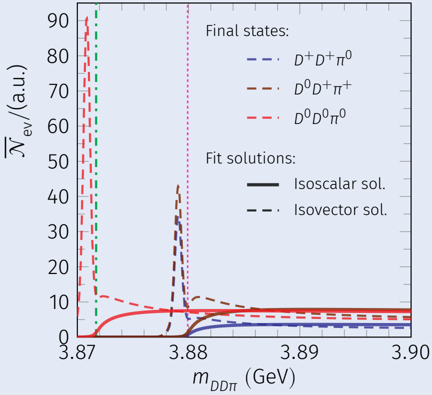
Other distributions





Other distributions



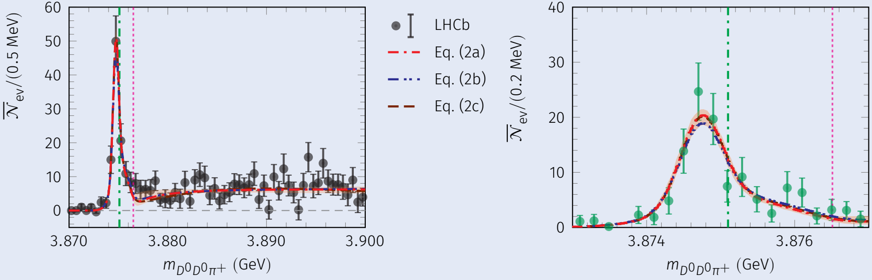


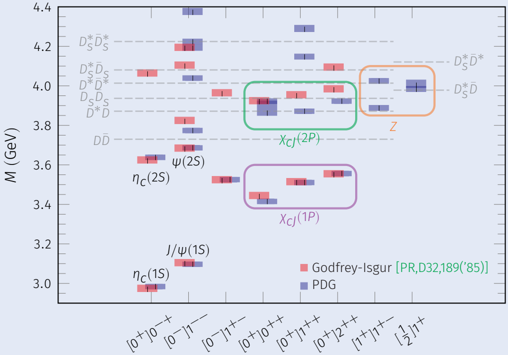
- Other paramterizations could lead to different line shapes and/or properties: pole position, scattering length, molecular probability,...
- No large variations are observed when the most inmediate generalizations are employed
- In particular, always large molecular probability

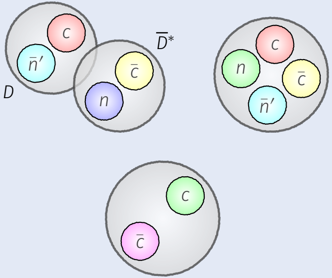
$$V_0(s) = C_0(\Lambda) \quad \longrightarrow \quad V_0(s) = C_0(\Lambda) + b_0(\Lambda)k^2 \ , \tag{2a}$$

$$V_0(s) = C_0(\Lambda) \quad \longrightarrow \quad V_0(s) = \frac{d_0(\Lambda)}{E - M_0(\Lambda)} \ , \tag{2b}$$

$$V_1(s) = C_1(\Lambda) \quad \longrightarrow \quad V_1(s) = C_1(\Lambda) + b_1(\Lambda)k^2 \ . \tag{2c}$$



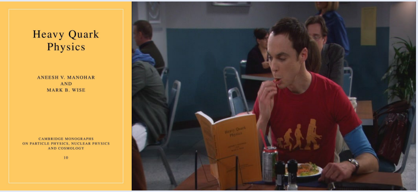




Appropriate tool: Weinberg's compositeness
[Weinberg PR,137,B672('65)]
[MA, Nieves, EPJ,C82,724('22)]

- $\chi_{cJ}(1P)$ well established, "very CQM model" state.
- $X(3872)$ discovered by Belle [PRL,91,262001('03)] (also 2003!)
- $J^{PC} = 1^{++}$ and $\Gamma \simeq 1\,\text{MeV}$ established by LHCb (*e.g.* [JHEP,08(2020),123])
- $\chi_{cJ}(2P)$ Not established. Influence of open thresholds? $X(3872)$ a molecular state, $4q$,...?
- Z_c/Z_{cs} states have $I = 1$ or $1/2$, clearly "tetraquarks" ($c\bar{c}u\bar{d}$, ...)
- Many theoretical and lattice and experimental works: can't cite them properly here! (many references in [PR,D106,094002('22)])

- HQSS: $H_a^{(Q)} = \frac{1+\not{v}}{2}\left(P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right)$ (with $v\cdot P_a^{*(Q)}=0$) [Grinstein *et al.*, NP,B380('92); Alfiky *et al.*, PL,B640('06), ...]
- $SU(3)$ light flavour symmetry: $H_a^{(Q)} \sim \left(Q\bar{u}, Q\bar{d}, Q\bar{s}\right) \sim \left(D^0, D^+, D_s^+\right)$



- $H\bar{H} \to H\bar{H}$ LO lagrangian (S-wave contact interactions):

$$\begin{aligned}\mathcal{L}_{4H} &= \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\right]\left(F_A\,\delta_a^b\delta_c^d + F_A^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right)\\ &+ \frac{1}{4}\,\mathrm{Tr}\left[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\mu\gamma_5\right]\mathrm{Tr}\left[H^{(\bar{Q})c}\bar{H}_d^{(\bar{Q})}\gamma^\mu\gamma_5\right]\left(F_B\,\delta_a^b\delta_c^d + F_B^\lambda\,\vec{\lambda}_a^b\cdot\vec{\lambda}_c^d\right),\end{aligned}$$

- Only 4 constants, any linear combination can be used

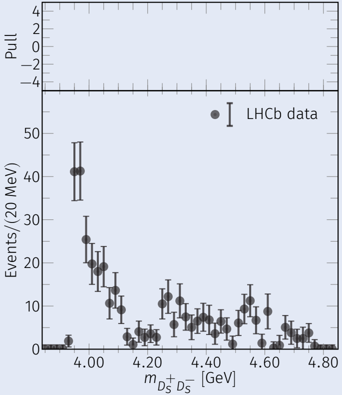
$$\begin{aligned}\mathcal{C}_{0a} &= F_A + \frac{10F_A^\lambda}{3}, \qquad \mathcal{C}_{1a} = F_A - \frac{2}{3}F_A^\lambda,\\ \mathcal{C}_{0b} &= F_B + \frac{10F_B^\lambda}{3}, \qquad \mathcal{C}_{1b} = F_B - \frac{2}{3}F_B^\lambda.\end{aligned}$$

$D_s^+ D_s^-$ interaction and $B^+ \to D_s^+ D_s^- K^+$ decay

Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

- Scattering amplitude: $T^{-1}(E) = V^{-1} - G(E)$
  - $V = 4m_{D_s}^2 \dfrac{C_{0a} + C_{1a}}{2}$
  - $G(E)$: loop functions, once-subtracted DR, $G(E_{th}) = G_\Lambda(E_{th})$
- Simple production model:
  $$T_B(E) = P + P\,G(E)T(E) = P\frac{1}{1 - V\,G(E)}$$
- $$\frac{d\Gamma}{dE} = \frac{1}{(2\pi)^3}\frac{k\,p}{4m_B^2}\left|T_B(E)\right|^2$$

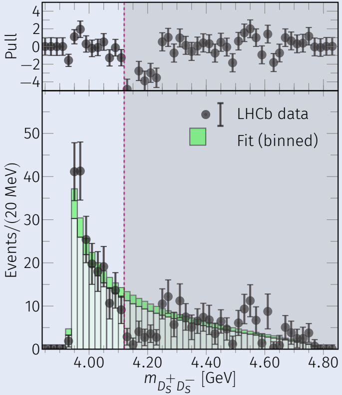
[LHCb: 2210.15153, 2211.05034]
[LHC Seminar (July 5th, 2022)]



Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

- Scattering amplitude: $T^{-1}(E) = V^{-1} - G(E)$
  - $V = 4m_{D_s}^2 \dfrac{C_{0a} + C_{1a}}{2}$
  - $G(E)$: loop functions, once-subtracted DR, $G(E_{\rm th}) = G_\Lambda(E_{\rm th})$
- Simple production model:
  $T_B(E) = P + P\,G(E)T(E) = P\dfrac{1}{1 - V\,G(E)}$
- $\dfrac{d\Gamma}{dE} = \dfrac{1}{(2\pi)^3}\dfrac{k\,p}{4m_B^2}\left|T_B(E)\right|^2$
- Fit: two solutions (virtual or bound), in both:
  $M_{X(3960)} = 3928(3)$ MeV
  $2M_{D_s} - M_{X(3960)} = 8(3)$ MeV
- LHCb: $M = 3956(5)(11)$ MeV, $\Gamma = 43(13)(7)$ MeV
- [Prelovsek *et al.*, JHEP 06,035('20)]: Bound state $B = 6.2^{+2.0}_{-3.8}$ MeV (*cf.* also [Bayar, Feijoo, Oset, 2207.08490])

[LHCb: 2210.15153, 2211.05034]
[LHC Seminar (July 5th, 2022)]



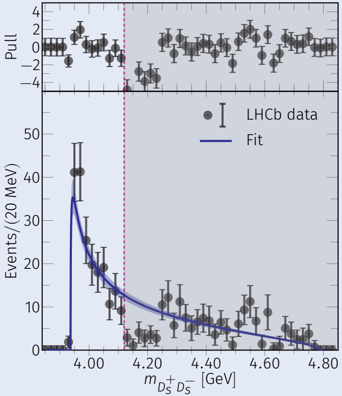
	Vir. (S-I)		Bou. (S-II)	
	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV
$C_{D_s\bar{D}_s}$ (fm^2)	$-0.74^{+0.04}_{-0.04}$	$-0.46^{+0.02}_{-0.02}$	$-3.36^{+0.56}_{-1.02}$	$-0.91^{+0.05}_{-0.06}$

$D_s^+ D_s^-$ interaction and $B^+ \to D_s^+ D_s^- K^+$ decay

Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

- Scattering amplitude: $T^{-1}(E) = V^{-1} - G(E)$
  - $V = 4m_{D_s}^2 \dfrac{C_{0a} + C_{1a}}{2}$
  - $G(E)$: loop functions, once-subtracted DR, $G(E_{\rm th}) = G_\Lambda(E_{\rm th})$
- Simple production model:
  $T_B(E) = P + P\,G(E)T(E) = P\dfrac{1}{1 - V\,G(E)}$
- $\dfrac{d\Gamma}{dE} = \dfrac{1}{(2\pi)^3}\dfrac{k\,p}{4m_B^2}\left|T_B(E)\right|^2$
- Fit: two solutions (virtual or bound), in both:
  $M_{X(3960)} = 3928(3)$ MeV
  $2M_{D_s} - M_{X(3960)} = 8(3)$ MeV
- LHCb: $M = 3956(5)(11)$ MeV, $\Gamma = 43(13)(7)$ MeV
- [Prelovsek *et al.*, JHEP 06,035('20)]: Bound state $B = 6.2^{+2.0}_{-3.8}$ MeV (*cf.* also [Bayar, Feijoo, Oset, 2207.08490])

[LHCb: 2210.15153, 2211.05034]
[LHC Seminar (July 5th, 2022)]



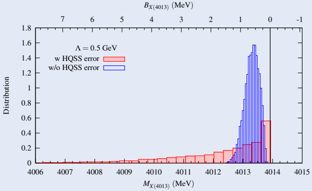
	Vir. (S-I)		Bou. (S-II)	
	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV	$\Lambda = 1.0$ GeV
$C_{D_s\bar{D}_s}$ (fm^2)	$-0.74^{+0.04}_{-0.04}$	$-0.46^{+0.02}_{-0.02}$	$-3.36^{+0.56}_{-1.02}$	$-0.91^{+0.05}_{-0.06}$

- Lagrangian $\mathcal{L}$ with HQSS and light-flavour $SU(3)$ symmetry has 4 constants (C_{0a}, C_{0b}, C_{1a}, C_{1b})
- Some relations can be independently useful. Some examples:

1. $X(3872)$ and X_{2++}

[MA, Guo, Hidalgo-Duque, Nieves, Pavón, EPJ,C75,547('15)]

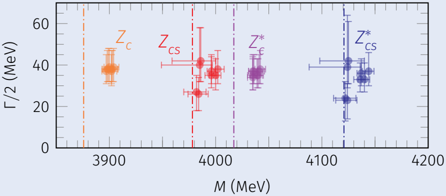
$$\left.\begin{array}{l}\left\langle D\bar{D}^*;\, 0(1^{++})\middle|\hat{T}\middle\rangle D\bar{D}^*;\, 0(1^{++})\right) \\ \left\langle D^*\bar{D}^*;\, 0(2^{++})\middle|\hat{T}\middle\rangle D^*\bar{D}^*;\, 0(2^{++})\right)\end{array}\right\} = C_{0a} + C_{0b}$$



2. Z_c, Z_c', Z_{cs}, Z_{cs}'

[Du, MA, Guo, Nieves, PR,D105,074018('22); see below]

$$\left.\begin{array}{l}\left\langle D\bar{D}^*;\, 1(1^{+-})\middle|\hat{T}\middle\rangle D\bar{D}^*;\, 1(1^{+-})\right) \\ \left\langle D_s\bar{D}^*;\, \tfrac{1}{2}(1^{+})\middle|\hat{T}\middle\rangle D_s\bar{D}^*;\, \tfrac{1}{2}(1^{+})\right) \\ \left\langle D^*\bar{D}^*;\, 1(1^{+-})\middle|\hat{T}\middle\rangle D^*\bar{D}^*;\, 1(1^{+-})\right) \\ \left\langle D_s^*\bar{D}^*;\, \tfrac{1}{2}(1^{+})\middle|\hat{T}\middle\rangle D_s^*\bar{D}^*;\, \tfrac{1}{2}(1^{+})\right)\end{array}\right\} = C_{1a} - C_{1b}$$



Fixing all constants

Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

- $X(3960)$: fixes $C_{D_s\bar{D}_s} = \left(C_{0a} + C_{1a}\right)/2$ (as previously seen)
- $Z_c(3900)$: fixes $C_{1Z} = C_{1a} - C_{1b}$
  - Assume virtual state $M = 3813^{+28}_{-21}$ MeV ([2201.08253; 1512.03638] from a fit to BESIII data)
- $X(3872)$: fixes $\begin{cases} C_{0X} = \left(C_{0a} + C_{0b}\right)/2 \\ C_{1X} = \left(C_{1a} + C_{1b}\right)/2 \end{cases}$
  - Experimental information:

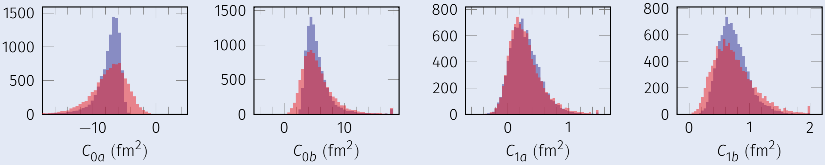
$$\begin{cases} \text{[LHCb, 2204.12597]} & R^{\rm exp}_{X(3872)} = 0.29(4) \\ \text{[LHCb, PR,D102,092005('20)]} & B^{\rm exp}_{X(3872)} = [-150, 0]\ \text{keV} \longleftarrow M^{\rm exp}_{X(3872)} = 3871.69^{+0.00\,+0.05}_{-0.04\,-0.13}\ \text{MeV} \end{cases}$$

  - Theoretically: [0911.4407; 1210.5431; 1504.00861]

$$V = \frac{1}{2}\begin{pmatrix} C_{0X} + C_{1X} & C_{0X} - C_{1X} \\ C_{0X} - C_{1X} & C_{0X} + C_{1X} \end{pmatrix}, \qquad T = (\mathbb{I} - VG)^{-1}V\,.$$

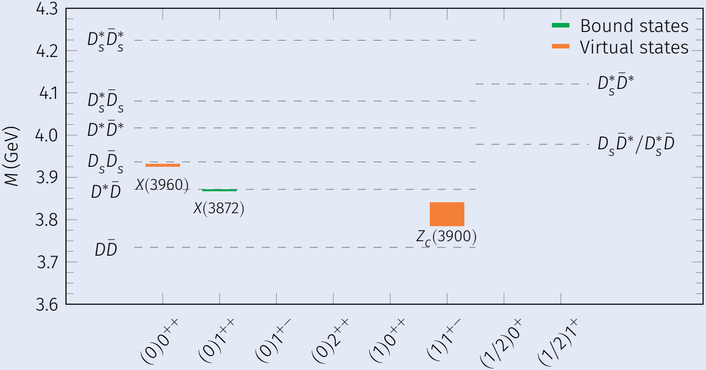
$$R_{X(3872)} = \frac{\hat\Psi_n - \hat\Psi_c}{\hat\Psi_n + \hat\Psi_c}, \quad \frac{\hat\Psi_n}{\hat\Psi_c} = \frac{1 - (2m_{D^+}m_{D^{*-}})\,G_2\left(C_{0X} + C_{1X}\right)}{(2m_{D^+}m_{D^{*-}})\,G_2\left(C_{0X} - C_{1X}\right)} = \frac{(2m_{D^0}m_{D^{*0}})\,G_1\left(C_{0X} - C_{1X}\right)}{1 - (2m_{D^0}m_{D^{*0}})\,G_1\left(C_{0X} + C_{1X}\right)},$$

- $SU(3)$ and HQSS breaking corrections (30%) are taken into account for the LECs



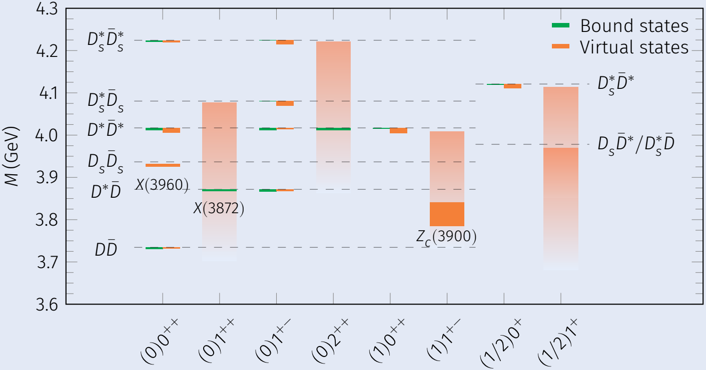
Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

With the four LECs (C_{0a}, C_{0b}, C_{1a}, C_{1b}) fixed, poles can be looked for in every sector.



Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

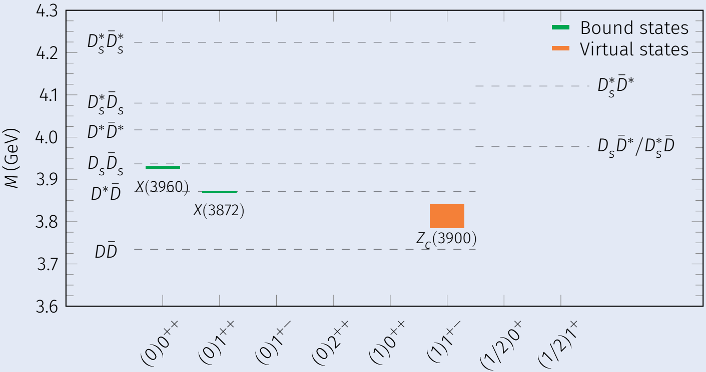
With the four LECs (C_{0a}, C_{0b}, C_{1a}, C_{1b}) fixed, poles can be looked for in every sector.



Predictions (complete multiplet): $X(3960)$ as bound

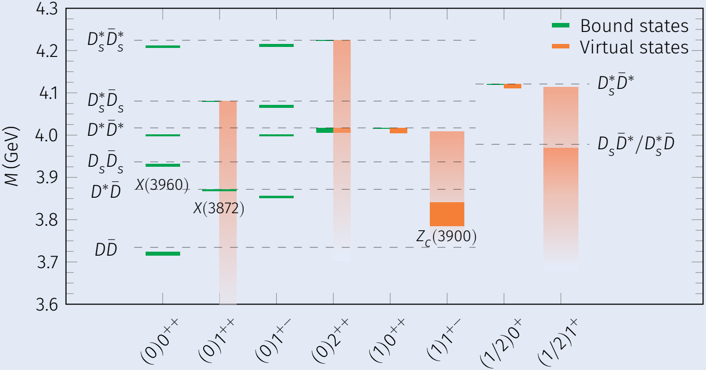
Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

With the four LECs (C_{0a}, C_{0b}, C_{1a}, C_{1b}) fixed, poles can be looked for in every sector.



Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)

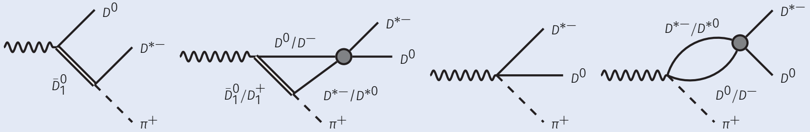
With the four LECs (C_{0a}, C_{0b}, C_{1a}, C_{1b}) fixed, poles can be looked for in every sector.



- $\left\langle D\bar{D}^*;\, 1(1^{+-}) \middle| \hat{T} \middle| D\bar{D}^*;\, 1(1^{+-}) \right\rangle = \left\langle D_s\bar{D}^*;\, \frac{1}{2}(1^{+}) \middle| \hat{T} \middle| D_s\bar{D}^*;\, \frac{1}{2}(1^{+}) \right\rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant V and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach in two directions [MA, Guo, Hidalgo-Duque, Nieves, PL,B755,337('16)]:

  1. Coupled channels $\begin{cases} I = 1 & \left(\frac{1}{\sqrt{2}} \left[D\bar{D}^* - D^*\bar{D} \right],\, J/\psi\,\pi \right) \\ I = \frac{1}{2} & \left(\frac{1}{\sqrt{2}} \left[D_s\bar{D}^* - D_s^*\bar{D} \right],\, J/\psi\, K \right) \end{cases}$ (also necessary for $e^+e^- \to J/\psi\pi\pi$ data)
  2. Energy dependence $C_{1Z} \to C_{1Z} + b\,\dfrac{s - E_{\rm th}^2}{2E_{\rm th}}$

- Production mechanism (includes triangle singularity!) for $Y \to D^0\, D^{*-}\, \pi^+$



$$\overline{|\mathcal{A}_2(s,t)|^2} = \left| \frac{1}{t - m_{D_1}^2} + I(s)T_{22}(s) \right|^2 q_\pi^4(s) + \frac{1}{2} \left| E_\pi(s)\,\frac{h_S}{h_D} \left[\frac{1}{t - m_{D_1}^2} + I(s)T_{22}(s) \right] + \beta \left[1 + G_2(s)T_{22}(s) \right] \right|^2$$

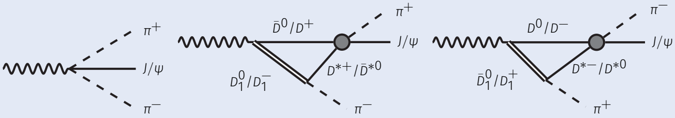
A closer look into $Z_c(3900)$ and $Z_{cs}(3985)$

Du, MA, Guo, Nieves, PR,D105,074018('22)

- $\left\langle D\bar{D}^*;\, 1(1^{+-})\middle|\hat{T}\middle|D\bar{D}^*;\, 1(1^{+-})\right\rangle = \left\langle D_s\bar{D}^*;\, \tfrac{1}{2}(1^+)\middle|\hat{T}\middle|D_s\bar{D}^*;\, \tfrac{1}{2}(1^+)\right\rangle = C_{1a} - C_{1b} \equiv C_{1Z}$
- Constant V and single channel: only virtual or bound states (pole condition $V^{-1} = G$)
- Extension of the previous approach in two directions [MA, Guo, Hidalgo-Duque, Nieves, PL,B755,337('16)]:

  1. Coupled channels $\begin{cases} I = 1 & \left(\frac{1}{\sqrt{2}}\left[D\bar{D}^* - D^*\bar{D}\right],\, J/\psi\,\pi\right) \\ I = \frac{1}{2} & \left(\frac{1}{\sqrt{2}}\left[D_s\bar{D}^* - D_s^*\bar{D}\right],\, J/\psi\, K\right) \end{cases}$ (also necessary for $e^+e^- \to J/\psi\pi\pi$ data)
  2. Energy dependence $C_{1Z} \to C_{1Z} + b\,\dfrac{s - E_{\rm th}^2}{2E_{\rm th}}$

- Production mechanism (includes triangle singularity!) for $Y \to J/\psi\,\pi^+\,\pi^-$



$$\overline{|\mathcal{A}_1(s,t)|^2} = |\tau(s)|^2 q_\pi^4(s) + |\tau(t)|^2 q_\pi^4(t) + \frac{3\cos^2\theta - 1}{2}\left[\tau(s)\tau(t)^* + \tau(s)^*\tau(t)\right] q_\pi^2(s) q_\pi^2(t)$$
$$+ \frac{1}{2}\left\{|\tau'(s)|^2 E_\pi^2(s) + |\tau'(t)|^2 E_\pi^2(t) + \left[\tau'(s)^*\tau'(t) + \tau'(s)\tau'(t)^*\right] E_\pi(s) E_\pi(t)\right\}$$
$$\tau(s) = \sqrt{2} I(s) T_{12}(s) + \alpha \qquad \tau'(s) = \sqrt{2} I(s) T_{12}(s) \times (h_S/h_D)$$

Fit to data

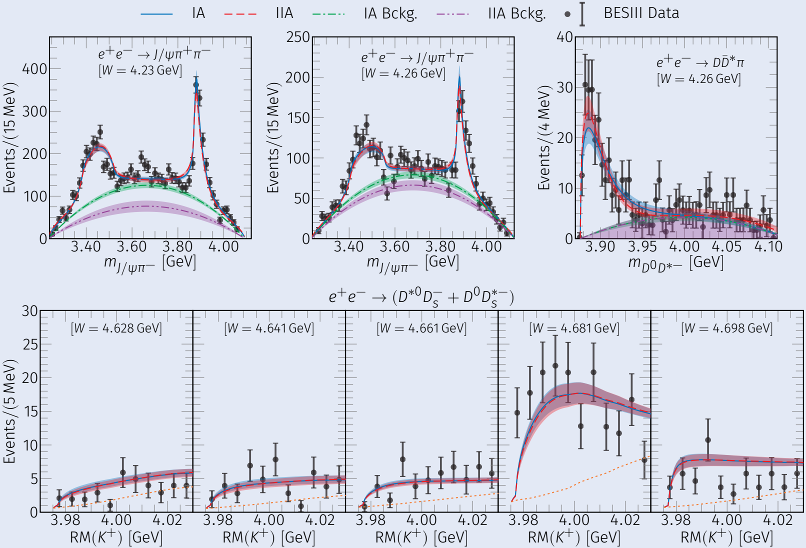
Du, MA, Guo, Nieves, PR,D105,074018('22)

- Fitted data:
  - $J/\psi\pi^-$ distribution in $e^+e^- \to J/\psi\pi^+\pi^-$ [BESIII, PRL,119('17)]
  - $D^0 D^{*-}$ distribution in $e^+e^- \to D^0 D^{*-}\pi^+$ [BESIII,PR, D92('15)]
  - $e^+e^- \to \left(D^{*0}D_s^- + D^0 D_s^{*-}\right) K^+$ [BESIII, PRL,126('21)]
- Some production/background/normalization constants are also fitted (not shown here)
- Four schemes:
  - A or B: $b = 0$ or free
  - I or II: $h_s = 0$ or not (D- or S- and D-waves)

Scheme	$D_1 D^*\pi$	$a_2(\mu)$	χ^2/dof	C_{12} [fm^2]	C_Z [fm^2]	b [fm^3]
IA	D	-2.5	1.62	0.005(1)	$-0.226(10)$	0^*
		-3.0	1.62	0.005(1)	$-0.177(6)$	0^*
IIA	$S+D$	-2.5	1.83	0.006(1)	$-0.217(10)$	0^*
		-3.0	1.83	0.006(1)	$-0.171(6)$	0^*
IB	D	-2.5	1.24	0.007(4)	$-0.222(6)$	$-0.447(44)$
		-3.0	1.21	0.008(1)	$-0.177(4)$	$-0.255(30)$
IIB	$S+D$	-2.5	1.37	0.005(1)	$-0.203(7)$	$-0.473(45)$
		-3.0	1.27	0.005(1)	$-0.171(5)$	$-0.270(30)$

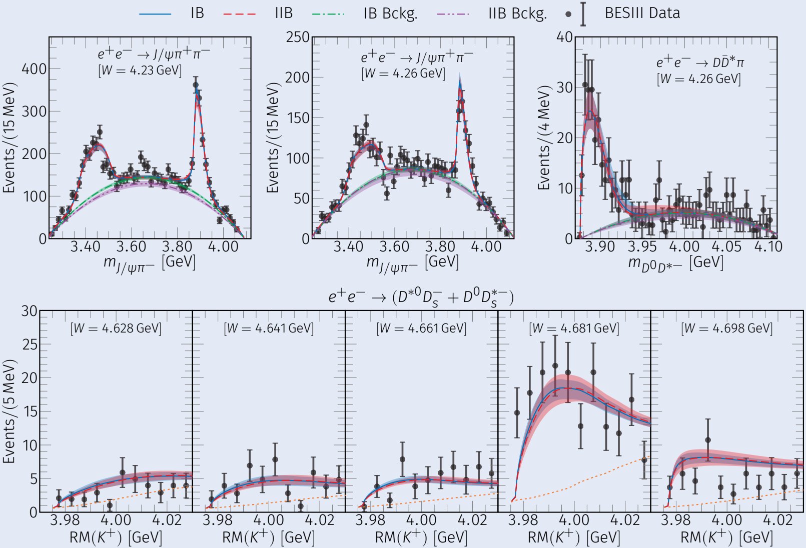
Fit to data (scheme A)

Du, MA, Guo, Nieves, PR,D105,074018('22)



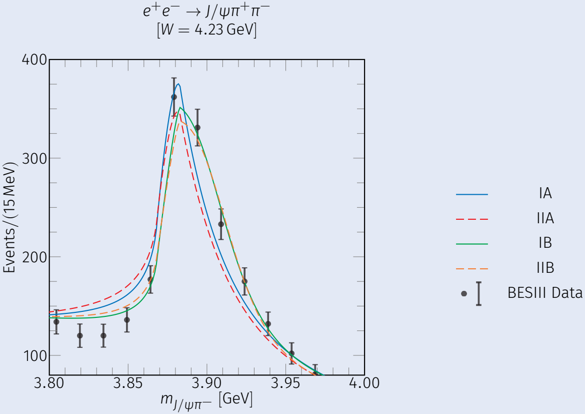
Fit to data (scheme B)

Du, MA, Guo, Nieves, PR,D105,074018('22)



Not possible to distinguish different scenarios in $J/\psi\pi$ spectrum

Du, MA, Guo, Nieves, PR,D105,074018('22)



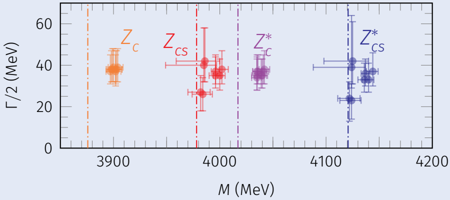
- The effect/peak produced by a virtual pole is always at threshold
- The peak of a resonance can be shifted from $\text{Re}\sqrt{s}_{\text{pole}}$
- And, in this case, the effect is very close to threshold
- We had observed this already [MA *et al.*, Phys. Lett. , **B755**, 337 (2016)]

$Z_{cs}^{(*)}$ and $Z_c^{(*)}$ poles

Du, MA, Guo, Nieves, PR,D105,074018('22)

	$D_1D^*\pi$	$a_2(\mu)$	Z_c [MeV] Mass	$\Gamma/2$	Z_{cs} [MeV] Mass	$\Gamma/2$	Z_c^* [MeV] Mass	$\Gamma/2$	Z_{cs}^* [MeV] Mass	$\Gamma/2$
IA	D	-2.5	3813^{+21}_{-28}	vir.	3920^{+18}_{-26}	vir.	3962^{+19}_{-25}	vir.	4069^{+12}_{-16}	vir.
		-3.0	3812^{+22}_{-26}	vir.	3924^{+19}_{-23}	vir.	3967^{+19}_{-22}	vir.	4078^{+17}_{-13}	vir.
IIA	$S+D$	-2.5	3799^{+24}_{-33}	vir.	3907^{+22}_{-31}	vir.	3949^{+22}_{-30}	vir.	4057^{+20}_{-28}	vir.
		-3.0	3798^{+25}_{-31}	vir.	3911^{+17}_{-27}	vir.	3955^{+22}_{-27}	vir.	4067^{+19}_{-25}	vir.
IB	D	-2.5	3897^{+4}_{-4}	37^{+8}_{-6}	3996^{+4}_{-4}	37^{+8}_{-6}	4035^{+4}_{-4}	37^{+8}_{-6}	4137^{+4}_{-4}	36^{+7}_{-6}
		-3.0	3898^{+5}_{-5}	38^{+10}_{-7}	3996^{+5}_{-6}	35^{+9}_{-6}	4035^{+4}_{-5}	34^{+9}_{-6}	4136^{+5}_{-6}	33^{+8}_{-6}
IIB	$S+D$	-2.5	3902^{+6}_{-6}	38^{+9}_{-6}	4002^{+6}_{-6}	38^{+9}_{-7}	4042^{+5}_{-5}	38^{+9}_{-7}	4144^{+5}_{-6}	37^{+9}_{-7}
		-3.0	3902^{+5}_{-5}	37^{+9}_{-6}	4000^{+5}_{-6}	35^{+8}_{-7}	4039^{+5}_{-6}	35^{+8}_{-6}	4140^{+5}_{-6}	33^{+8}_{-6}

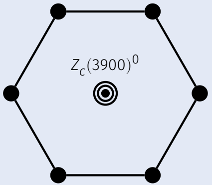
- Z_c exp.: $M = 3881.2(4.2)(52.7)$ MeV, $\Gamma/2 = 25.9(2.3)(18.0)$ MeV
- Z_{cs} exp.: $M = 3982.5(2.6)(2.1)$ MeV, $\Gamma/2 = 6.4(2.6)(1.5)$ MeV
- [Yang *et al.*, PR,D103('21)] (Z_{cs} and distribution)
- [Ikeno, Molina, Oset, PL,B814('21)] (Z_{cs} threshold effect)
- [JPAC, PL,B772('17)] Several possibilities for Z_c



- Both schemes (IB and IIB)
- Including $SU(3)$ breaking effects

Z_c and Z_{cs} form a $J^{PC} = 1^{+-}$ octet

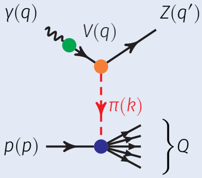
Ji, Dong, MA, Du, Guo, Nieves, PR,D106,094002('22)



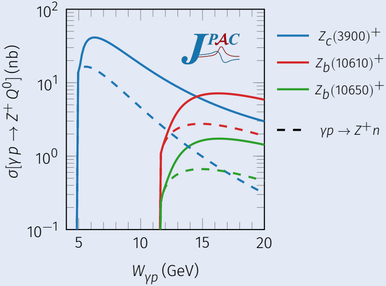
- $3 \otimes \bar{3} = 8 \oplus 1$
- A $\bar{Z}^0_{cs}$ has also been found by BESIII
  [PRL,129,112003('22)]
- In the case of $I = 0$ one cannot make direct identification (mixing *vs.* coupled channels)

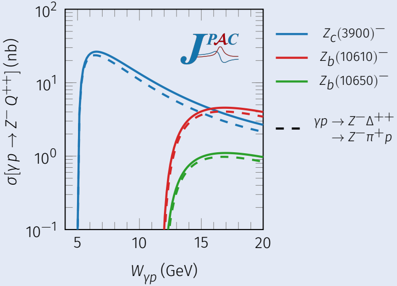
Photoproduction of Z states would be beneficial...

JPAC, PR,D106,094009('22)

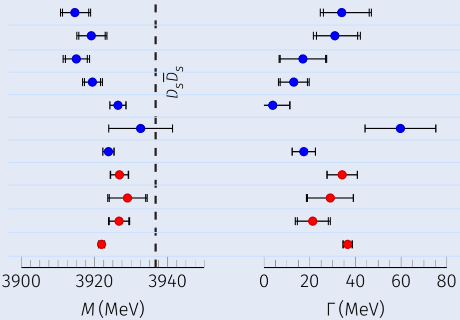


- A new method to confirm or discard these new XYZ states
- In principle, photoproduction is free of triangle-singularities that can give rise to resonance-like effects
- Different background, easier to pin down the scattering amplitude part





$\chi_{c0}(3915)$ and $\chi_{c2}(3930)$?



Reaction	Collab.	Ref.
$B^{(+,0)} \to J/\psi\omega K^{(+,0)}$	BaBar	BaBar:2007vxr
$B^{(+,0)} \to J/\psi\omega K^{(+,0)}$	BaBar	BaBar:2010wfc
$\gamma\gamma \to J/\psi\omega$	Belle	Belle:2009and
$\gamma\gamma \to J/\psi\omega$	BaBar	BaBar:2012nxg
$e^+e^- \to \gamma J/\psi\omega$	BESIII	BESIII:2019qvy
$e^+e^- \to \gamma J/\psi\omega$	BESIII	BESIII:2019qvy
$B^+ \to D^+D^-K^+$	LHCb	LHCb:2020pxc
$B^+ \to D^+D^-K^+$	LHCb	LHCb:2020pxc
$\gamma\gamma \to D\bar{D}$	Belle	Belle:2005rte
$\gamma\gamma \to D\bar{D}$	BaBar	BaBar:2010jfn
$pp \to D\bar{D}$ + anything	LHCb	LHCb:2019lnr

Weinberg's compositeness

[Weinberg, PR,137,B672('65)]

- Identity resolution:

$$\mathbb{1} = \sum_n |n\rangle\langle n| + \int d\alpha \, |\alpha\rangle\langle\alpha|$$

- $\hat{H} = \hat{H}_0 + \hat{V}$
- Eigenstates of $\hat{H}_0$:
  - continuum $|\alpha\rangle$, $\hat{H}_0 |\alpha\rangle = E(\alpha) |\alpha\rangle$
  - bare elementary $|n\rangle$, $\hat{H}_0 |n\rangle = E_n |n\rangle$
- Eigenstates of $\hat{H}$:
  - $|d\rangle$ ("deuteron") $\hat{H} |d\rangle = E_B |d\rangle$
- Normalized:

$$\langle\alpha|\beta\rangle = \delta(\beta - \alpha) \qquad \langle\alpha|n\rangle = 0$$
$$\langle m|n\rangle = \delta_{mn} \qquad \langle d|d\rangle = 1$$

- Identity resolution:

$$\mathbb{1} = \sum_n |n\rangle\langle n| + \int d\alpha\, |\alpha\rangle\langle\alpha|$$

- "Sandwich" $\langle d|\mathbb{1}|d\rangle = 1$

$$1 = Z + \int d\alpha \frac{|\langle\alpha|V|d\rangle|^2}{(E(\alpha) - E_B)^2}$$

- $\hat{H} = \hat{H}_0 + \hat{V}$
- Eigenstates of $\hat{H}_0$:
  - continuum $|\alpha\rangle$, $\hat{H}_0\, |\alpha\rangle = E(\alpha)\, |\alpha\rangle$
  - bare elementary $|n\rangle$, $\hat{H}_0\, |n\rangle = E_n\, |n\rangle$
- Eigenstates of $\hat{H}$:
  - $|d\rangle$ ("deuteron") $\hat{H}\, |d\rangle = E_B\, |d\rangle$
- Normalized:

$$\langle\alpha|\beta\rangle = \delta(\beta - \alpha) \qquad \langle\alpha|n\rangle = 0$$
$$\langle m|n\rangle = \delta_{mn} \qquad \langle d|d\rangle = 1$$

Fundamental quantity $Z = \sum_n |\langle n|d\rangle|^2$

«(...) Z is the probability of finding the deuteron in a bare elementary-particle state.» [Weinberg, PR,137,B672]

Weinberg's compositeness

[Weinberg, PR,137,B672('65)]

- Identity resolution:

$$\mathbb{1} = \sum_n |n\rangle\langle n| + \int d\alpha \, |\alpha\rangle\langle\alpha|$$

- "Sandwich" $\langle d|\mathbb{1}|d\rangle = 1$

$$1 = Z + \int d\alpha \frac{|\langle\alpha|V|d\rangle|^2}{(E(\alpha) - E_B)^2}$$

- 1st crucial approximation: $\langle\alpha|V|d\rangle \simeq g/(2\pi)^{3/2}$

$$1 - Z = g^2 \frac{\mu}{2\pi^2} \int_{E_{\text{th}}}^{\infty} dW \frac{k(W)}{(W - E_B)^2} = \frac{\mu^2 g^2}{2\pi\gamma_B}$$

- $\hat{H} = \hat{H}_0 + \hat{V}$
- Eigenstates of $\hat{H}_0$:
  - continuum $|\alpha\rangle$, $\hat{H}_0 |\alpha\rangle = E(\alpha) |\alpha\rangle$
  - bare elementary $|n\rangle$, $\hat{H}_0 |n\rangle = E_n |n\rangle$
- Eigenstates of $\hat{H}$:
  - $|d\rangle$ ("deuteron") $\hat{H} |d\rangle = E_B |d\rangle$
- Normalized:

$$\langle\alpha|\beta\rangle = \delta(\beta - \alpha) \qquad \langle\alpha|n\rangle = 0$$
$$\langle m|n\rangle = \delta_{mn} \qquad \langle d|d\rangle = 1$$

Fundamental quantity $Z = \sum_n |\langle n|d\rangle|^2$

«(...) Z is the probability of finding the deuteron in a bare elementary-particle state.» [Weinberg, PR,137,B672]

- Measure: $d\alpha = d^3\vec{k} = 4\pi\,\mu\,k(W)dW$

- $1-Z$ written in terms of g^2: $\quad 1-Z = \frac{\mu^2 g^2}{2\pi\gamma_B}$

  Next step, move to *low energy n-p scattering*, and relate g^2 to a, r
- Start from *a version* of the Low equation:

$$T(E) = \frac{g^2}{E-E_B} + \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\sqrt{\vec{k}^2}}{E-E_{\text{th}}-\vec{k}^2/(2\mu)} \left|T(E(\vec{k}^2))\right|^2$$

- *A solution* proposed by Weinberg reads
  [2nd crucial assumption]:

$$T^{-1}(E) = \frac{E-E_B}{g^2} - \frac{\mu^2}{2\pi\gamma_B}(E+E_B-2E_{\text{th}}) + i\frac{\mu k(E)}{2\pi}$$

- This solution for the amplitude exactly satisfies the Effective Range Expansion (ERE):

$$-\frac{2\pi}{\mu}T^{-1}(E) = k\cot\delta - ik = \frac{1}{a} + \frac{1}{2}r\,k^2 + \cdots - ik$$

Weinberg's compositeness (II)

[Weinberg, PR,137,B672('65)]

- $1-Z$ written in terms of g^2: $\quad 1-Z = \dfrac{\mu^2 g^2}{2\pi\gamma_B}$

  Next step, move to *low energy n-p scattering*, and relate g^2 to a, r
- Start from *a version* of the Low equation:

$$T(E) = \frac{g^2}{E-E_B} + \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\sqrt{\vec{k}^2}}{E-E_{th}-\vec{k}^2/(2\mu)} \left|T(E(\vec{k}^2))\right|^2$$

- *A solution* proposed by Weinberg reads [2nd crucial assumption]:

$$T^{-1}(E) = \frac{E-E_B}{g^2} - \frac{\mu^2}{2\pi\gamma_B}(E+E_B-2E_{th}) + i\frac{\mu k(E)}{2\pi}$$

- This solution for the amplitude exactly satisfies the Effective Range Expansion (ERE):

$$-\frac{2\pi}{\mu}T^{-1}(E) = k\cot\delta - ik = \frac{1}{a} + \frac{1}{2}r k^2 + \cdots - ik$$

- Relation of g^2 and γ_B to a and r:

$$a\gamma_B = -2\left(1+\frac{2\pi\gamma_B}{\mu^2 g^2}\right)^{-1}$$

$$r\gamma_B = \left(1-\frac{2\pi\gamma_B}{\mu^2 g^2}\right)$$

- $1-Z$ written in terms of g^2: $\quad 1-Z=\dfrac{\mu^2 g^2}{2\pi\gamma_B}$

  Next step, move to *low energy n-p scattering*, and relate g^2 to a, r
- Start from *a version* of the Low equation:

$$T(E)=\frac{g^2}{E-E_B}+\int\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\frac{\sqrt{\vec{k}^2}}{E-E_{\rm th}-\vec{k}^2/(2\mu)}\left|T(E(\vec{k}^2))\right|^2$$

- *A solution* proposed by Weinberg reads
  [2nd crucial assumption]:

$$T^{-1}(E)=\frac{E-E_B}{g^2}-\frac{\mu^2}{2\pi\gamma_B}(E+E_B-2E_{\rm th})+\ i\frac{\mu k(E)}{2\pi}$$

- This solution for the amplitude exactly satisfies the Effective Range Expansion (ERE):

$$-\frac{2\pi}{\mu}T^{-1}(E)=k\cot\delta-ik=\frac{1}{a}+\frac{1}{2}r\,k^2+\cdots-\ ik$$

- Relation of g^2 and γ_B to a and r:

$$a\gamma_B=-2\left(1+\frac{2\pi\gamma_B}{\mu^2 g^2}\right)^{-1}$$

$$r\gamma_B=\left(1-\frac{2\pi\gamma_B}{\mu^2 g^2}\right)$$

- $1-Z$ written in terms of g^2: $1-Z = \frac{\mu^2 g^2}{2\pi\gamma_B}$

  Next step, move to *low energy n-p scattering*, and relate g^2 to a, r
- Start from *a version* of the Low equation:

$$T(E) = \frac{g^2}{E-E_B} + \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{\sqrt{\vec{k}^2}}{E-E_{th}-\vec{k}^2/(2\mu)} \left|T(E(\vec{k}^2))\right|^2$$

- *A solution* proposed by Weinberg reads
  [2nd crucial assumption]:

$$T^{-1}(E) = \frac{E-E_B}{g^2} - \frac{\mu^2}{2\pi\gamma_B}(E+E_B-2E_{th}) + i\frac{\mu k(E)}{2\pi}$$

- This solution for the amplitude exactly satisfies the Effective Range Expansion (ERE):

$$-\frac{2\pi}{\mu}T^{-1}(E) = k\cot\delta - ik = \frac{1}{a} + \frac{1}{2}r k^2 + \cdots - ik$$

- Relation of g^2 and γ_B to a and r:

$$a\gamma_B = -2\left(1+\frac{2\pi\gamma_B}{\mu^2 g^2}\right)^{-1}$$

$$r\gamma_B = \left(1-\frac{2\pi\gamma_B}{\mu^2 g^2}\right)$$

Weinberg's compositeness condition(s)

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} \qquad r = -\frac{1}{\gamma_B}\frac{Z}{1-Z}$$

- r, a are expansions in $\frac{1}{\beta}\left(\frac{\gamma_B}{\beta}\right)^{n-1}$, with β^{-1} an interaction range [in np, $\beta \sim m_\pi$]

$$r = \overbrace{-\frac{1}{\gamma_B}\frac{Z}{1-Z}}^{r_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1})$$

$$a = \underbrace{-\frac{2}{\gamma_B}\frac{1-Z}{2-Z}}_{a_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1})$$

Ⓐ r, a are expansions in $\frac{1}{\beta}\left(\frac{\gamma_B}{\beta}\right)^{n-1}$, with β^{-1} an interaction range [in np, $\beta \sim m_\pi$]

$$r = \overbrace{-\frac{1}{\gamma_B}\frac{Z}{1-Z}}^{r_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad\Longrightarrow\quad r = \overbrace{r_{\rm LO}(Z) + \delta r}^{r_{\rm NLO}(Z,\delta r)} + \mathcal{O}(\gamma_B/\beta^2)$$

$$a = \underbrace{-\frac{2}{\gamma_B}\frac{1-Z}{2-Z}}_{a_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad\Longrightarrow\quad a = \underbrace{a_{\rm LO}(Z) + \delta a}_{a_{\rm NLO}(Z,\delta a)} + \mathcal{O}(\gamma_B/\beta^2)$$

Extension of Weinberg's compositeness

- A r, a are expansions in $\frac{1}{\beta}\left(\frac{\gamma_B}{\beta}\right)^{n-1}$, with β^{-1} an interaction range [in np, $\beta \sim m_\pi$]

$$r = \overbrace{-\frac{1}{\gamma_B}\frac{Z}{1-Z}}^{r_{\text{LO}}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad \Longrightarrow \quad r = \overbrace{r_{\text{LO}}(Z) + \delta r}^{r_{\text{NLO}}(Z,\delta r)} + \mathcal{O}(\gamma_B/\beta^2)$$

$$a = \underbrace{-\frac{2}{\gamma_B}\frac{1-Z}{2-Z}}_{a_{\text{LO}}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad \Longrightarrow \quad a = \underbrace{a_{\text{LO}}(Z) + \delta a}_{a_{\text{NLO}}(Z,\delta a)} + \mathcal{O}(\gamma_B/\beta^2)$$

- B From ERE one gets a relation that works very well: $\gamma_B \simeq -\frac{1}{a} + \frac{1}{2}r\gamma_B^2$

Extension of Weinberg's compositeness

[MA, J. Nieves, EPJ,C82,8('22)]

(A) r, a are expansions in $\frac{1}{\beta}\left(\frac{\gamma_B}{\beta}\right)^{n-1}$, with β^{-1} an interaction range [in np, $\beta \sim m_\pi$]

$$r = \overbrace{-\frac{1}{\gamma_B}\frac{Z}{1-Z}}^{r_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad \Longrightarrow \quad r = \overbrace{r_{\rm LO}(Z) + \delta r}^{r_{\rm NLO}(Z,\delta r)} + \mathcal{O}(\gamma_B/\beta^2)$$

$$a = \underbrace{-\frac{2}{\gamma_B}\frac{1-Z}{2-Z}}_{a_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad \Longrightarrow \quad a = \underbrace{a_{\rm LO}(Z) + \delta a}_{a_{\rm NLO}(Z,\delta a)} + \mathcal{O}(\gamma_B/\beta^2)$$

(B) From ERE one gets a relation that works very well: $\gamma_B \simeq -\frac{1}{a} + \frac{1}{2} r \gamma_B^2$

- *The trick* is to combine A & B to correlate δr and δa:
  1. Introduce $a_{\rm NLO}$ and $r_{\rm NLO}$ above,
  2. Expand in powers of γ_B,
  3. Solve δa in terms of δr such that the difference is $\mathcal{O}(\gamma_B^3/\beta^2)$.

(A) r, a are expansions in $\frac{1}{\beta}\left(\frac{\gamma_B}{\beta}\right)^{n-1}$, with β^{-1} an interaction range [in np, $\beta \sim m_\pi$]

$$r = \overbrace{-\frac{1}{\gamma_B}\frac{Z}{1-Z}}^{r_{LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad \Longrightarrow \quad r = \overbrace{r_{LO}(Z)+\delta r}^{r_{NLO}(Z,\delta r)} + \mathcal{O}(\gamma_B/\beta^2)$$

$$a = \underbrace{-\frac{2}{\gamma_B}\frac{1-Z}{2-Z}}_{a_{LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1}) \quad \Longrightarrow \quad a = \underbrace{a_{LO}(Z)+\delta a}_{a_{NLO}(Z,\delta a)} + \mathcal{O}(\gamma_B/\beta^2)$$

(B) From ERE one gets a relation that works very well: $\gamma_B \simeq -\frac{1}{a} + \frac{1}{2} r\gamma_B^2$

- *The trick* is to combine A & B to correlate δr and δa:
  1. Introduce a_{NLO} and r_{NLO} above,
  2. Expand in powers of γ_B,
  3. Solve δa in terms of δr such that the difference is $\mathcal{O}(\gamma_B^3/\beta^2)$.
- Final equations, main result of [MA, J. Nieves, EPJ,C82,8('22)]:

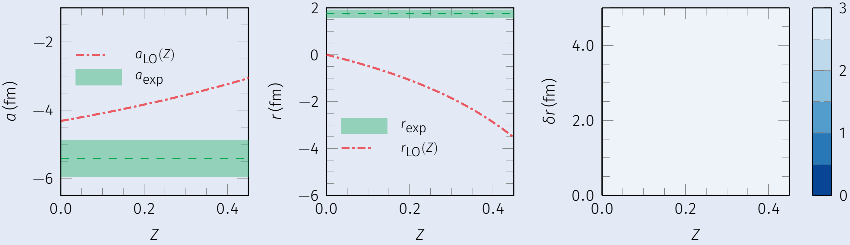
$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \delta r + \mathcal{O}(\gamma_B/\beta^2)$$

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} - 2\delta r\left(\frac{1-Z}{2-Z}\right)^2 + \mathcal{O}(\gamma_B/\beta^2)$$

Canonical example: the deuteron

- Exp. data: $\left\{ \begin{array}{ll} a_{\rm exp} & = -5.42(1)\,\text{fm} \\ r_{\rm exp} & = +1.75(1)\,\text{fm} \\ \gamma_{B_{\rm exp}} & = 45.7\,\text{MeV} \end{array} \right\}$

If Z is naively evaluated from these data, one gets $P = 1 - Z = 1.68$ [which makes no sense!]



Canonical example: the deuteron

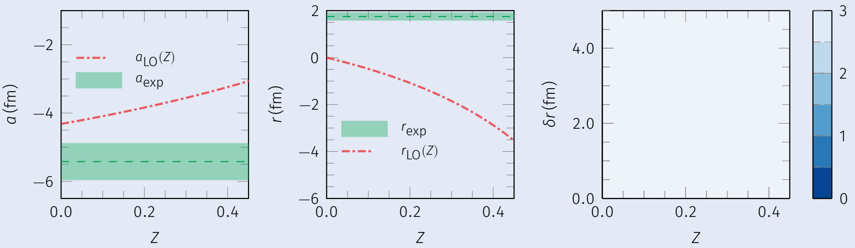
- Exp. data: $\left\{ \begin{array}{ll} a_{\text{exp}} & = -5.42(1)\,\text{fm} \\ r_{\text{exp}} & = +1.75(1)\,\text{fm} \\ \gamma_{B_{\text{exp}}} & = 45.7\,\text{MeV} \end{array} \right\}$

If Z is naively evaluated from these data, one gets $P = 1 - Z = 1.68$ [which makes no sense!]

- Additional tool, likelihood estimator $\mathcal{L}(Z, \delta r)$:

$$\mathcal{L}(Z,\delta r) = \frac{1}{3}\left[\left(\frac{a_{\text{exp}} - a_{\text{NLO}}}{\Delta a_{\text{exp}}}\right)^2 + \left(\frac{r_{\text{exp}} - r_{\text{NLO}}}{\Delta r_{\text{exp}}}\right)^2 + \left(\frac{\gamma_b^{\text{exp}} - \gamma_b^{\text{NLO}}}{\Delta \gamma_b^{\text{exp}}}\right)^2\right]$$

- Δa_{exp} (*et al.*): relative error taken as $(\gamma_B/m_\pi)^2 \simeq 0.1$ because exp. error is smaller



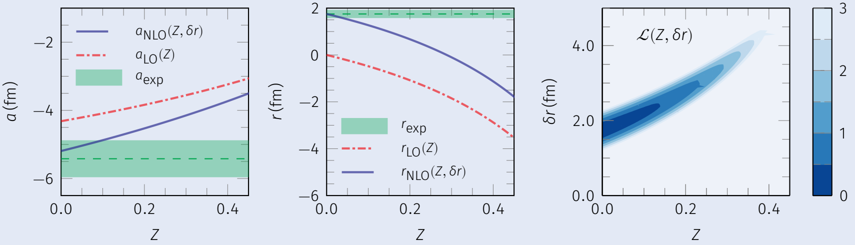
- Exp. data: $\left\{ \begin{array}{ll} a_{\rm exp} & = -5.42(1)\,\text{fm} \\ r_{\rm exp} & = +1.75(1)\,\text{fm} \\ \gamma_{B_{\rm exp}} & = 45.7\,\text{MeV} \end{array} \right\}$

If Z is naively evaluated from these data, one gets $P = 1 - Z = 1.68$ [which makes no sense!]

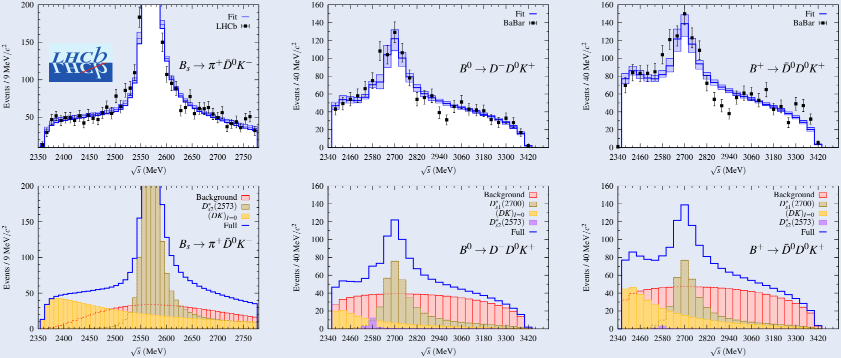
- Additional tool, likelihood estimator $\mathcal{L}(Z, \delta r)$:

$$\mathcal{L}(Z,\delta r) = \frac{1}{3}\left[\left(\frac{a_{\rm exp} - a_{\rm NLO}}{\Delta a_{\rm exp}}\right)^2 + \left(\frac{r_{\rm exp} - r_{\rm NLO}}{\Delta r_{\rm exp}}\right)^2 + \left(\frac{\gamma_b^{\rm exp} - \gamma_b^{\rm NLO}}{\Delta \gamma_b^{\rm exp}}\right)^2\right]$$

- $\Delta a_{\rm exp}$ (*et al.*): relative error taken as $(\gamma_B/m_\pi)^2 \simeq 0.1$ because exp. error is smaller
- The NLO expressions improve the agreement with data for $Z \simeq 0$ (molecular case), as expected
- The minimum is found for $\delta r \simeq r_{\rm exp} = 1.75\,\text{fm}$ ($\simeq m_\pi^{-1}$, as expected)

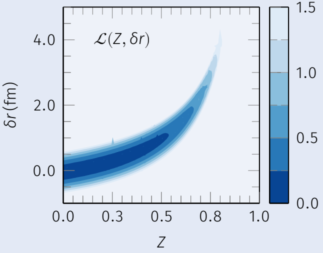


- DK $(I = 0)$ interaction taken from Heavy Meson ChPT



Fit	$M_{D^*_{s0}}$ (MeV)	P_{DK}(%)	a_0 (fm)
LHCb	$2326^{+16}_{-16}{}^{+1}_{-5}$	$74^{+7}_{-6}{}^{+9}_{-1}$	$-1.10^{+0.19}_{-0.39}{}^{+0.01}_{-0.02}$
BaBar	$2306^{+14}_{-23}{}^{+16}_{-9}$	$67^{+5}_{-7}{}^{+6}_{-10}$	$-0.87^{+0.15}_{-0.15}{}^{+0.11}_{-0.18}$
Combined	$2315^{+12}_{-17}{}^{+10}_{-5}$	$70^{+4}_{-6}{}^{+4}_{-8}$	$-0.95^{+0.15}_{-0.15}{}^{+0.08}_{-0.13}$

[MA, J. Nieves, EPJ,C82,8('22)]



- a and r: [Martínez-Torres *et al.*, JHEP,05,153('15)] [Mohler *et al.*, PRL,111,222001('13); PR,D90,034510('14)]

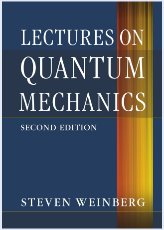
$$a = -1.3(5)\text{ fm}$$
$$r = -0.1(3)\text{ fm}$$

[Values compatible with those in the previous slide]

- $E_B = -45(4)$ MeV [from PDG compilation]

- Molecular probabilities $P = 1 - Z \gtrsim 0.5$
- We are not as specific as in other cases:
  - Larger uncertainties on the input (a, r, E_B)
  - Formalism pushed to (or beyond?) the limits: $\gamma_B/\beta \sim 0.6$ $[(\gamma_B/\beta)^2 = 36\%$

A curiosity...



Thus, expanding in bare two-particle states, Eq. (8.8.17) gives

$$1 = |g|^2 \int_0^\infty \frac{p(E)\, dE}{(E+B)^2}$$

and so[14]

$$|g|^2 = \frac{1}{\pi}\sqrt{\frac{2B}{\mu}}. \tag{8.8.19}$$

Using this in the solution (8.8.16) of the Low equation, we have

$$t(E) = \frac{1}{\pi\sqrt{2\mu}}\left[\sqrt{B} + i\sqrt{E}\right]^{-1}. \tag{8.8.20}$$

[14] More generally, if in addition to the continuum the eigenstates of H_0 include an elementary particle state with the same quantum numbers as the bound state, $|g|$ is less than the value given in Eq. (8.8.19) by a factor $1 - Z$, where Z is the probability that an examination of the bound state will find it in the elementary particle state rather than the two-particle state. The case $Z \neq 0$ is studied in detail by S. Weinberg, *Phys. Rev.* **B137**, 672 (1965).

«Mostly composite»

PHYSICAL REVIEW VOLUME 137, NUMBER 3B 8 FEBRUARY 1965

Evidence That the Deuteron Is Not an Elementary Particle*

STEVEN WEINBERG†
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 30 September 1964)

$$a_s = +5.41\ \mathrm{F}; \quad r_e = +1.75\ \mathrm{F}. \tag{5}$$

In contrast, if the deuteron had an appreciable probability Z of being found in an elementary bare-particle state then a_s would be less than R, and more striking, *r_e would be large and negative*. This is clearly contradicted by the experimental values (5), so we may conclude that Z is small (say <0.2), and therefore the deuteron is at least mostly composite.[3]