



UNIVERSITY OF
Southampton
School of Physics
and Astronomy

HIDDe

Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Neutrinos, Martin and Me



Hirschfest
Celebrating the death of countless models



23-24 January 2024
IFIC (CSIC/UV) Valencia

Neutrino Physics in 2001 (when Martin was in Southampton)

Atmospheric ν_μ disappear, large θ_{23} (1998) SK

Solar ν_e disappear, large θ_{12} (2002) SK, SNO

Solar ν_e are converted to $\nu_\mu + \nu_\tau$ (2002) SNO

Reactor anti- ν_e disappear/reappear (2004) Kamland

Accelerator ν_μ disappear (2006) MINOS

Accelerator ν_μ converted to ν_τ (2010) OPERA

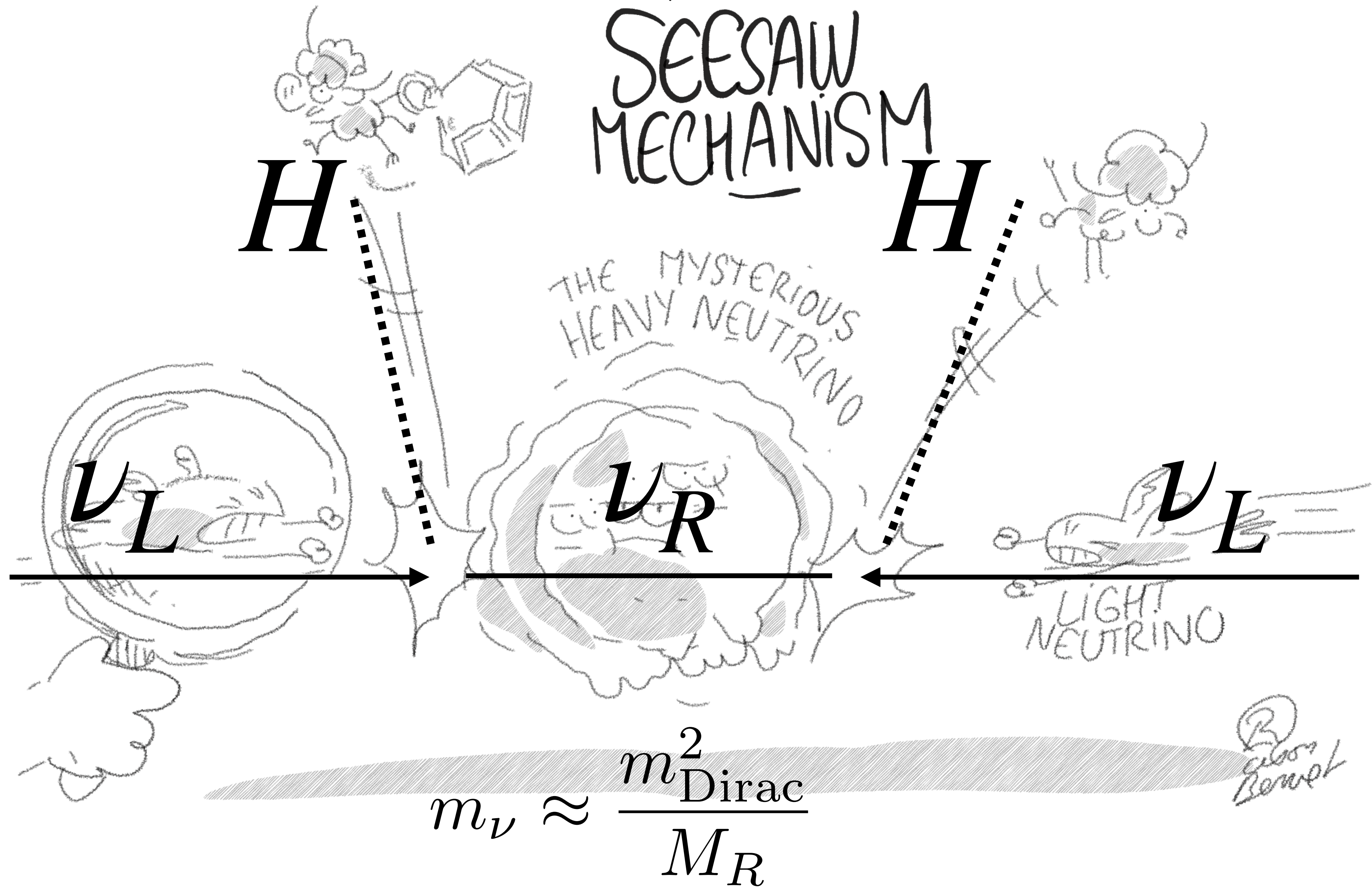
Accelerator ν_μ converted to ν_e , θ_{13} hint (2011) T2K

Reactor anti- ν_e disapp θ_{13} meas. (2012) DB, Reno, DC

Before Martin (BM)

Back in the day I assumed the...

Type Ia
SEESAW
MECHANISM



Single RHN model (1998)

Just add a single RHN to the SM

$$(H_u/v_u)(d\bar{L}_e + e\bar{L}_\mu + f\bar{L}_\tau)\nu_R^{\text{atm}} + M_{\text{atm}}\overline{\nu_R^{\text{atm}}}(\nu_R^{\text{atm}})^c$$

To explain atmospheric neutrino oscillations assume

$$d \ll e \sim f$$

Assume charged lepton mass matrix is approximately diagonal (like the quarks)

So that

$$\tan \theta_{23} \sim e/f \sim 1$$

Maximal atmospheric
mixing

$$\tan \theta_{13} \sim d/\sqrt{e^2 + f^2} \ll 1$$

Small reactor mixing

Two RHN Model (1999)

hep-ph/9904210

hep-ph/9912492

Add a second RHN to the SM to account for solar neutrino oscillations as well

Solar
$$\frac{(H_u/v_u)(a\bar{L}_e + b\bar{L}_\mu + c\bar{L}_\tau)\nu_R^{\text{sol}}}{+ M_{\text{sol}}\overline{\nu_R^{\text{sol}}}(\nu_R^{\text{sol}})^c} + \frac{(H_u/v_u)(d\bar{L}_e + e\bar{L}_\mu + f\bar{L}_\tau)\nu_R^{\text{atm}}}{+ M_{\text{atm}}\overline{\nu_R^{\text{atm}}}(\nu_R^{\text{atm}})^c}$$
 Atmospheric

Simpler matrix notation

$$m^D = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix} \quad M_R = \begin{pmatrix} M_{\text{sol}} & 0 \\ 0 & M_{\text{atm}} \end{pmatrix}$$

Assume diagonal M_R

Assume charged lepton mass matrix is approximately diagonal (like the quarks)

Seesaw matrix

$$m^\nu = m^D M_R^{-1} (m^D)^T = \begin{pmatrix} \frac{a^2}{M_{\text{sol}}} + \frac{d^2}{M_{\text{atm}}} & \frac{ab}{M_{\text{sol}}} + \frac{de}{M_{\text{atm}}} & \frac{ac}{M_{\text{sol}}} + \frac{df}{M_{\text{atm}}} \\ \frac{ab}{M_{\text{sol}}} + \frac{de}{M_{\text{atm}}} & \frac{b^2}{M_{\text{sol}}} + \frac{e^2}{M_{\text{atm}}} & \frac{bc}{M_{\text{sol}}} + \frac{ef}{M_{\text{atm}}} \\ \frac{ac}{M_{\text{sol}}} + \frac{df}{M_{\text{atm}}} & \frac{bc}{M_{\text{sol}}} + \frac{ef}{M_{\text{atm}}} & \frac{c^2}{M_{\text{sol}}} + \frac{f^2}{M_{\text{atm}}} \end{pmatrix}$$

Single RHN Dominance

$$\boxed{\frac{(e, f)^2}{M_{\text{atm}}}} \gg \boxed{\frac{(a, b, c)^2}{M_{\text{sol}}}}$$

$$d = 0$$

Atmospheric mixing from dominant RHN

$$\tan \theta_{23} \sim \frac{e}{f},$$

$$\tan \theta_{12} \sim \frac{\sqrt{2}a}{b - c}$$

Solar mixing from subdominant RHN

Reactor angle
 $\theta_{13} \lesssim m_2/m_3$

Martin Era (ME)

Leptogenesis with single right-handed neutrino dominance

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(Received 4 July 2001; published 5 November 2001)

We make an analytic and numerical study of leptogenesis in the framework of the (supersymmetric) standard model plus the seesaw mechanism with a $U(1)$ family symmetry and single right-handed neutrino dominance. In presenting our analytic and numerical results we make a clear distinction between the theoretically clean asymmetry parameter ϵ_1 and the baryon asymmetry Y_B . In calculating Y_B we propose and use a fit to the solutions to the Boltzmann equations which gives substantially more reliable results than parametrizations previously used in the literature. Our results show that there is a decoupling between the low energy neutrino observables and the leptogenesis predictions, but that nevertheless leptogenesis is capable of resolving ambiguities within classes of models which would otherwise lead to similar neutrino observables. For example we show that models where the dominant right-handed neutrino is the heaviest are preferred to models where it is the lightest and study an explicit example of a unified model of this type.

Discriminating neutrino see-saw models

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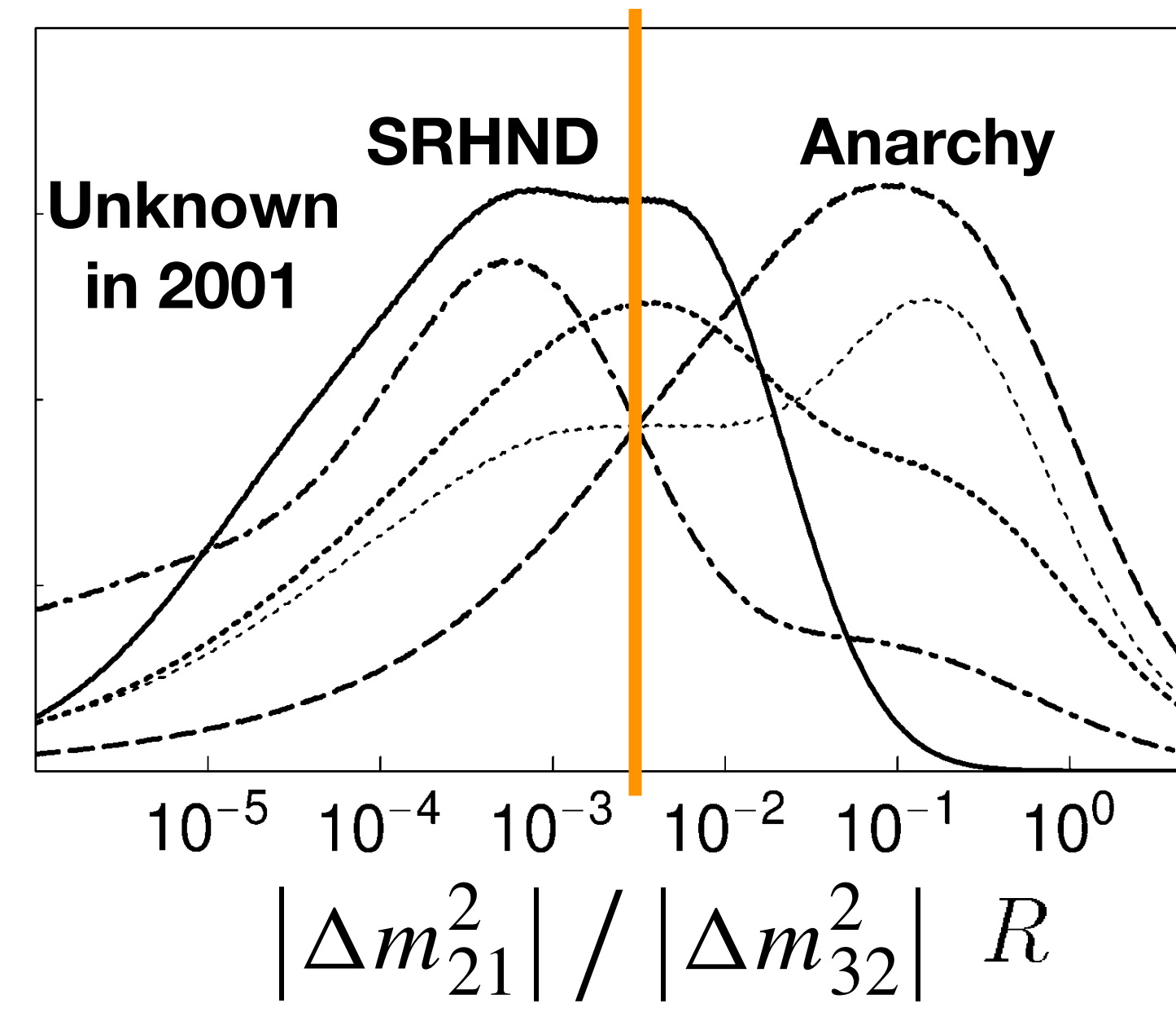
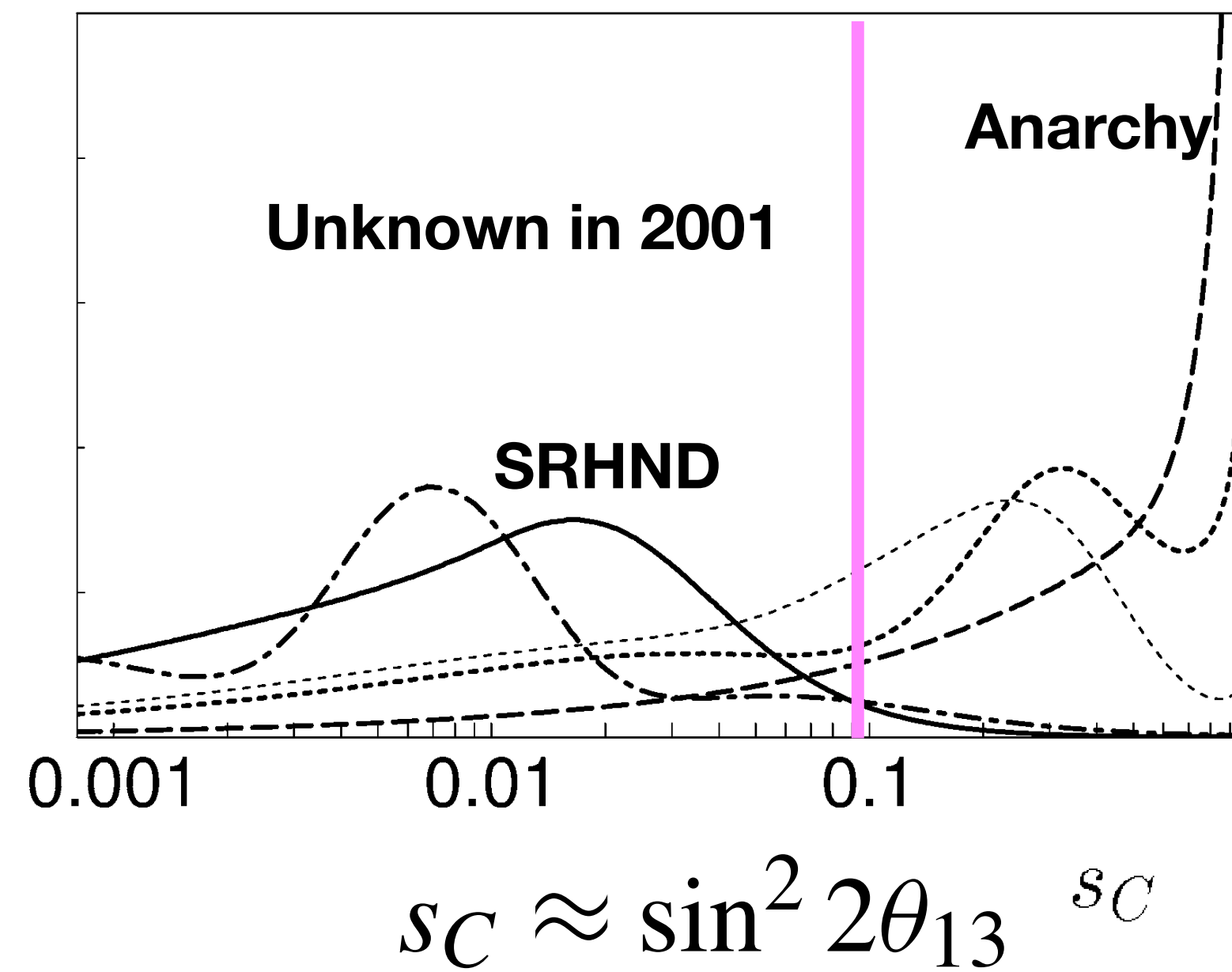
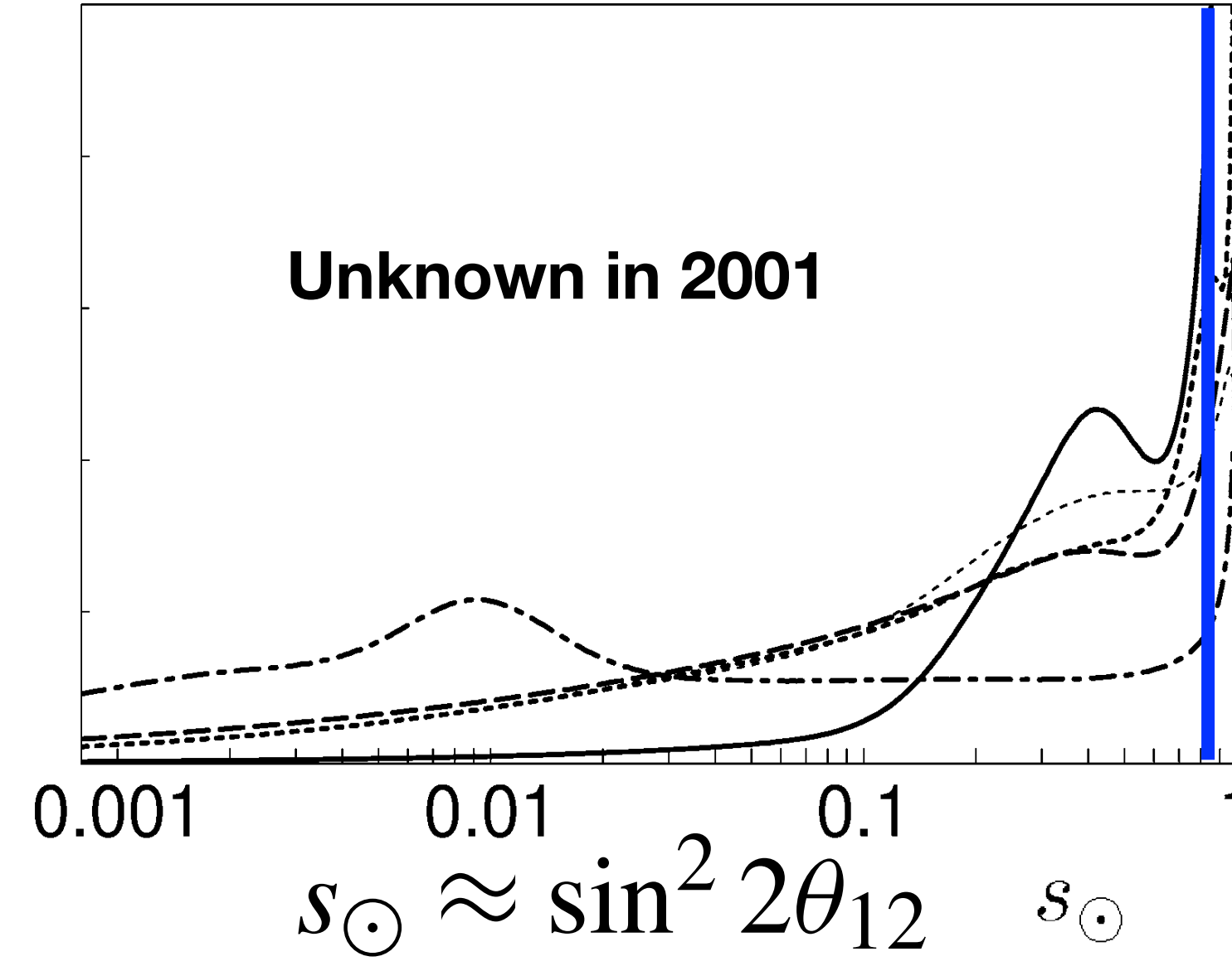
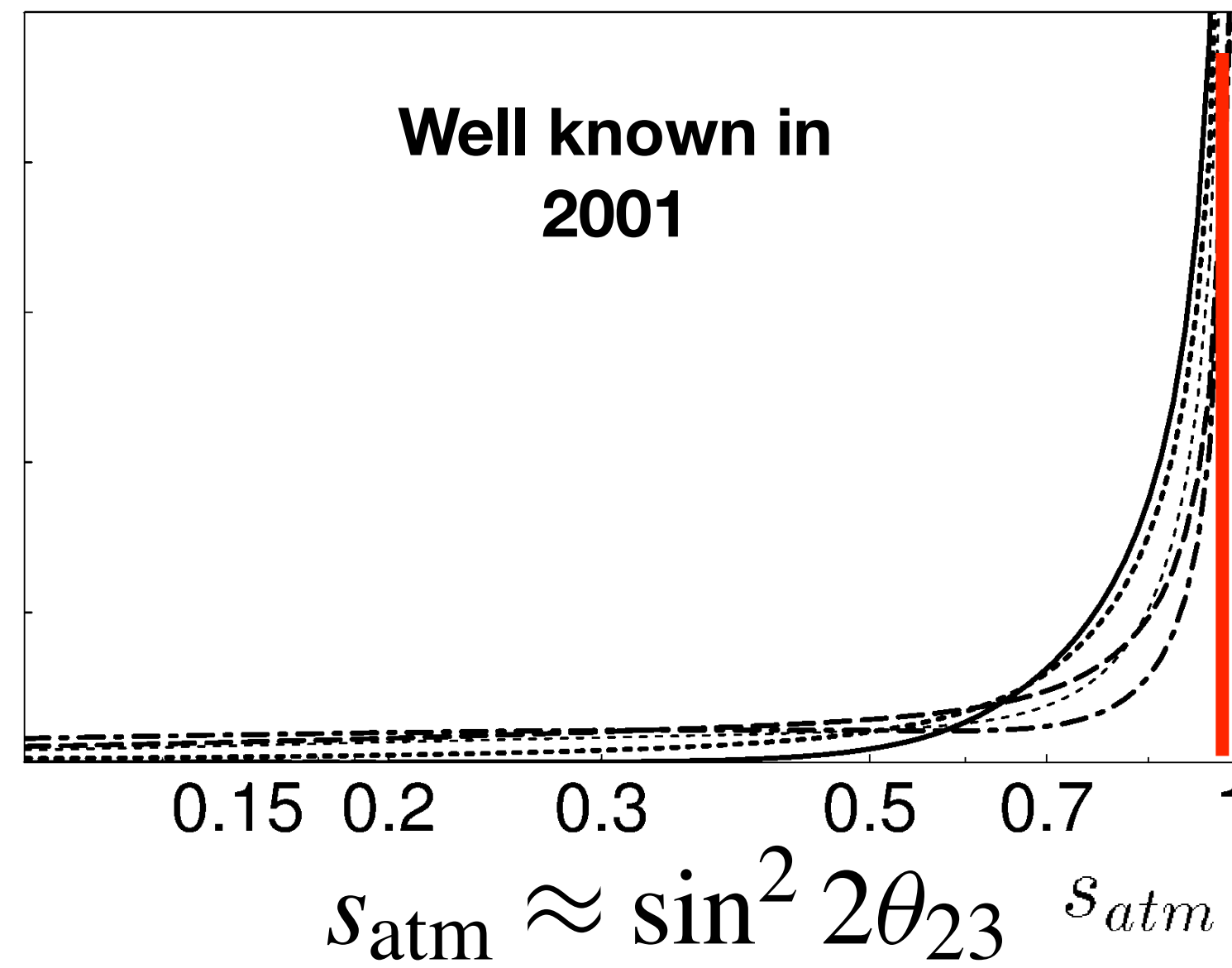
Received 6 June 2001; received in revised form 16 July 2001; accepted 17 July 2001

Editor: P.V. Landshoff

Abstract

We consider how well current theories can predict neutrino mass and mixing parameters, and construct a statistical discriminator which allows us to compare different models to each other. As an example we consider see-saw models based on family symmetry, and single right-handed neutrino dominance, and compare them to each other and to the case of neutrino anarchy with random entries in the neutrino Yukawa and Majorana mass matrices. The predictions depend crucially on the range of the undetermined coefficients over which we scan, and we speculate on how future theories might lead to more precise predictions for the coefficients and hence for neutrino observables. Our results indicate how accurately neutrino masses and mixing angles need to be measured by future experiments in order to discriminate between current models. © 2001 Elsevier Science B.V. All rights reserved.

Arguably
SRHND has
fared better
than Anarchy



After Martin (AM)

Constrained Sequential Dominance (2005)

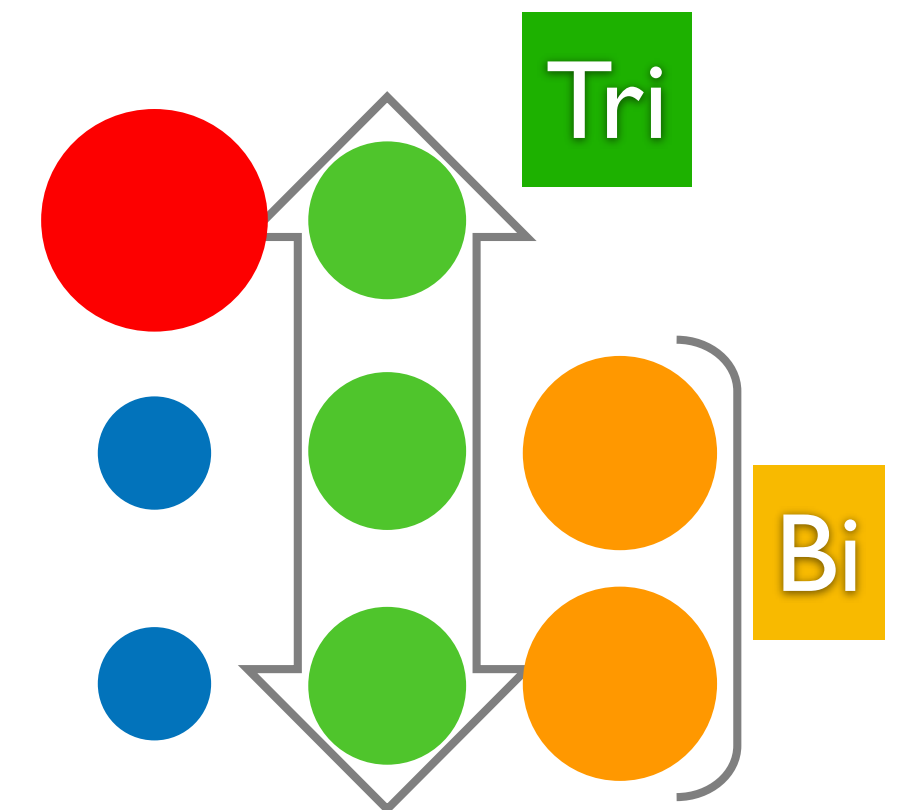
Recall $m^D = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$


Assume charged lepton mass matrix is exactly diagonal


We now add further constraints to enhance predictivity

$$d = 0 \quad e = f \quad \tan \theta_{23} \sim e/f \sim 1$$

$$a = b = -c \quad \tan \theta_{12} \sim \sqrt{2}a/(b - c) \sim 1/\sqrt{2}$$



It turns out that this gives exact tri-bimaximal mixing with $\theta_{13} = 0$  Accidentally occurs due to orthogonality of two columns

More general examples called CSD(n) give approximate TBM with $\theta_{13} \neq 0$ 

CSD(n)

(n=real number)

More generally assume the two columns of the Dirac matrix are proportional to

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix} \quad \text{(can be enforced by symmetry)}$$

$$\tan \theta_{23} \sim e/f \sim 1$$

$$\tan \theta_{12} \sim \sqrt{2}a/(b-c) \sim 1/\sqrt{2}$$

Approximate TBM
independently of n
(which cancels) but
depends on phases

The case $n = 1$ corresponds to the exact TBM case previously

The $n \neq 1$ results depend on relative phase of columns, find... $\theta_{13} \neq 0$ 😊

Results for CSD(n)

Assume charged lepton mass matrix is exactly diagonal

$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix}$ gives seesaw mass matrix in terms of three effective input parameters (for given n)

$$m_{\nu(n)}^{\nu} = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & n & n-2 \\ n & n^2 & n(n-2) \\ n-2 & n(n-2) & (n-2)^2 \end{pmatrix}$$

$m_a \sim \frac{(e, f)^2}{M_{\text{atm}}} \gg \frac{(a, b, c)^2}{M_{\text{sol}}} \sim m_b$

Bjorkeroth et al 1412.6996

n	m_a (meV)	m_b (meV)	η (rad)	θ_{12} ($^{\circ}$)	θ_{13} ($^{\circ}$)	θ_{23} ($^{\circ}$)	$ \delta_{\text{CP}} $ ($^{\circ}$)	m_2 (meV)	m_3 (meV)	χ^2	
1	24.8	2.89	3.14	35.3	0	45.0	0	8.66	49.6	485	CSD(1)=TBM
2	19.7	3.66	0	34.5	7.65	56.0	0	8.85	48.8	95.1	CSD(2) Antusch et al 1108.4278
3	27.3	2.62	2.17	34.4	8.39	44.5	92.2	8.69	49.5	3.98	CSD(3) 1304.6264
4	36.6	1.95	2.63	34.3	8.72	38.4	120	8.61	49.8	8.82	CSD(4) 1305.4846
5	45.9	1.55	2.88	34.2	9.03	34.4	142	8.53	50.0	33.8	

$m_1 = 0$

$$\theta_{13} \sim (n-1) \frac{\sqrt{2}}{3} \frac{m_2}{m_3}$$

Find best fit for n~3

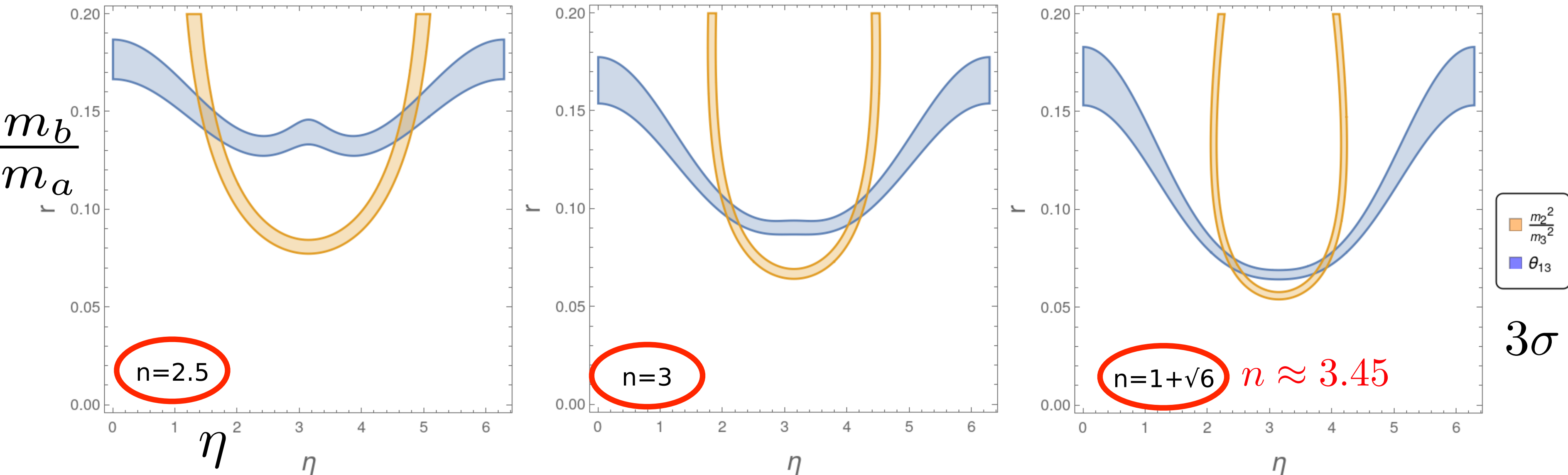
Highly predictive - 3 inputs for 9 observables (6 so far measured)

CSD(~ 3) = “Littlest Seesaw”

1512.07531

F.Costa et al 2307.13895

2 inputs fix all the neutrino observables (up to overall mass scale)



Modular symmetry fixes

$$n = 1 + \sqrt{6}$$
$$n \approx 3.45$$

Predictions for 6 observables

Modular Littlest seesaw			Flipped modular Littlest seesaw		
$n = 1 + \sqrt{6}$	bf	allowed ranges	$n = 1 + \sqrt{6}$	bf	allowed ranges
η/π	1.240	[1.197, 1.276]	η/π	0.742	[0.725, 0.806]
$r \frac{m_b}{m_a}$	0.0734	[0.0684, 0.0786]	r	0.0758	[0.0683, 0.0786]
$\sin^2 \theta_{13}$	0.0223	[0.0205, 0.0240]	$\sin^2 \theta_{13}$	0.0231	[0.0205, 0.0240]
$\sin^2 \theta_{12}$	0.318	[0.317, 0.319]	$\sin^2 \theta_{12}$	0.318	[0.317, 0.319]
$\sin^2 \theta_{23}$	0.447	[0.408, 0.483]	$\sin^2 \theta_{23}$	0.535	[0.517, 0.595]
δ_{CP}/π	-0.575	[-0.640, -0.522]	δ_{CP}/π	-0.452	[-0.478, -0.354]
β/π	0.474	[0.408, 0.555]	β/π	-0.441	[-0.562, -0.409]
m_2^2/m_3^2	0.0297	[0.0270, 0.0321]	m_2^2/m_3^2	0.0283	[0.0270, 0.0321]

Littlest Modular Seesaw

Ding et al, 1910.03460, 2311.09282

de Medeiros Varzielas, M.Levy et al 2211.00654, 2309.15901

de Anda et al 2304.05958, 2312.09010

Ioannina, 28 May 2015 at 20:54





Mercado Colon, 19 June 2019 at 23:30



Happy 60th Birthday Martin!

Valencia, 22 January 2024 at 22:00

Backup slides

Flipped CSD(n)

N.B. Both cases predict normal hierarchy $m_{\text{lightest}}=0$

Non-flipped

Flipped

(n= real number)

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ n-2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n-2 \\ n \end{pmatrix}$$

The two predictions only differ in atmospheric angle and CP phase (solar angle, reactor angle and neutrino mass unchanged)

Octant flipped

$$\tan \theta_{23} \rightarrow \cot \theta_{23} \quad \delta \rightarrow \delta + \pi$$

Alternatively we could use the following (only differs by unphysical phases):

$$\begin{pmatrix} d \\ e \\ f \end{pmatrix} \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ n \\ 2-n \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} \propto \begin{pmatrix} 1 \\ 2-n \\ n \end{pmatrix}$$

Littlest Modular Seesaw

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$	Loc
L	1	1	3	0	0	0	\mathbb{T}_C^2
e^c	1	1	1	0	0	-6	\mathbb{T}_C^2
μ^c	1	1	1	0	0	-4	\mathbb{T}_C^2
τ^c	1	1	1	0	0	-2	\mathbb{T}_C^2
N_a^c	1	1	1	0	-4	0	\mathbb{T}_B^2
N_s^c	1	1	1	-2	0	0	\mathbb{T}_A^2
Φ_{BC}	1	3	3	0	0	0	Bulk
Φ_{AC}	3	1	3	0	0	0	Bulk

Yuk/Mass	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
$Y_e(\tau_3)$	1	1	3	0	0	6
$Y_\mu(\tau_3)$	1	1	3	0	0	4
$Y_\tau(\tau_3)$	1	1	3	0	0	2
$Y_a(\tau_2)$	1	3	1	0	4	0
$Y_s(\tau_1)$	3	1	1	2	0	0
$M_a(\tau_2)$	1	1	1	0	8	0
$M_s(\tau_1)$	1	1	1	4	0	0

Yukawa couplings
are modular forms
evaluated at the
fixed points of the
moduli fields (the
lattice vectors)

	τ	$Y_{\mathbf{3}}^{(2)}(\tau), Y_{\mathbf{3},\mathbf{I}}^{(6)}(\tau)$		$Y_{\mathbf{3}}^{(4)}(\tau), Y_{\mathbf{3}'}^{(6)}(\tau)$
τ_1	i	$(1, 1 + \sqrt{6}, 1 - \sqrt{6})$		$(1, -\frac{1}{2}, -\frac{1}{2})$
	$i + 1$	$(1, -\frac{\omega}{3}(1 + i\sqrt{2}), -\frac{\omega^2}{3}(1 + i\sqrt{2}))$		$(0, 1, -\omega)$
τ_2	$i + 2$	$(1, \frac{1}{3}(-1 + i\sqrt{2}), \frac{1}{3}(-1 + i\sqrt{2}))$		$(0, 1, -1)$
	$i + 3$	$(1, \omega(1 + \sqrt{6}), \omega(1 - \sqrt{6}))$		$(1, -\frac{\omega}{2}, -\frac{\omega^2}{2})$
	τ	$Y_{\mathbf{3}}^{(2)}(\tau)$	$Y_{\mathbf{3}}^{(4)}(\tau), Y_{\mathbf{3}'}^{(4)}(\tau)$	$Y_{\mathbf{3},\mathbf{II}}^{(6)}(\tau), Y_{\mathbf{3}'}^{(6)}(\tau)$
τ_3	ω	$(0, 1, 0)$	$(0, 0, 1)$	$(1, 0, 0)$
	$\omega + 1$	$(1, 1, -\frac{1}{2})$	$(1, -\frac{1}{2}, 1)$	$(1, -2, -2)$
	$\omega + 2$	$(1, -\frac{\omega^2}{2}, \omega)$	$(1, \omega^2, -\frac{\omega}{2})$	$(1, -2\omega^2, -2\omega)$
	$\omega + 3$	$(1, \omega, -\frac{\omega^2}{2})$	$(1, -\frac{\omega}{2}, \omega^2)$	$(1, -2\omega, -2\omega^2)$

$\omega = e^{\frac{2\pi i}{3}}$

$$\frac{1}{\Lambda} \left[L\Phi_{BC}Y_aN_a^c + L\Phi_{AC}Y_sN_s^c \right] H_u + \left[LY_e e^c + LY_\mu \mu^c + LY_\tau \tau^c \right] H_d + \frac{1}{2}M_aN_a^cN_a^c + \frac{1}{2}M_sN_s^cN_s^c.$$

$$\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

Charged leptons

$$\begin{pmatrix} 0 & b \\ a & b(1 - \sqrt{6}) \\ -a & b(1 + \sqrt{6}) \end{pmatrix}$$

Dirac neutrinos