

Gravitational Wave Orchestra; Being Sensitive is Crucial!

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GRAVITATIONAL WAVES

The universe is not static! Nor is space time!

GWs are freely propagating oscillations in the geometry of spacetime - ripples in the fabric of spacetime.

accelerating charges

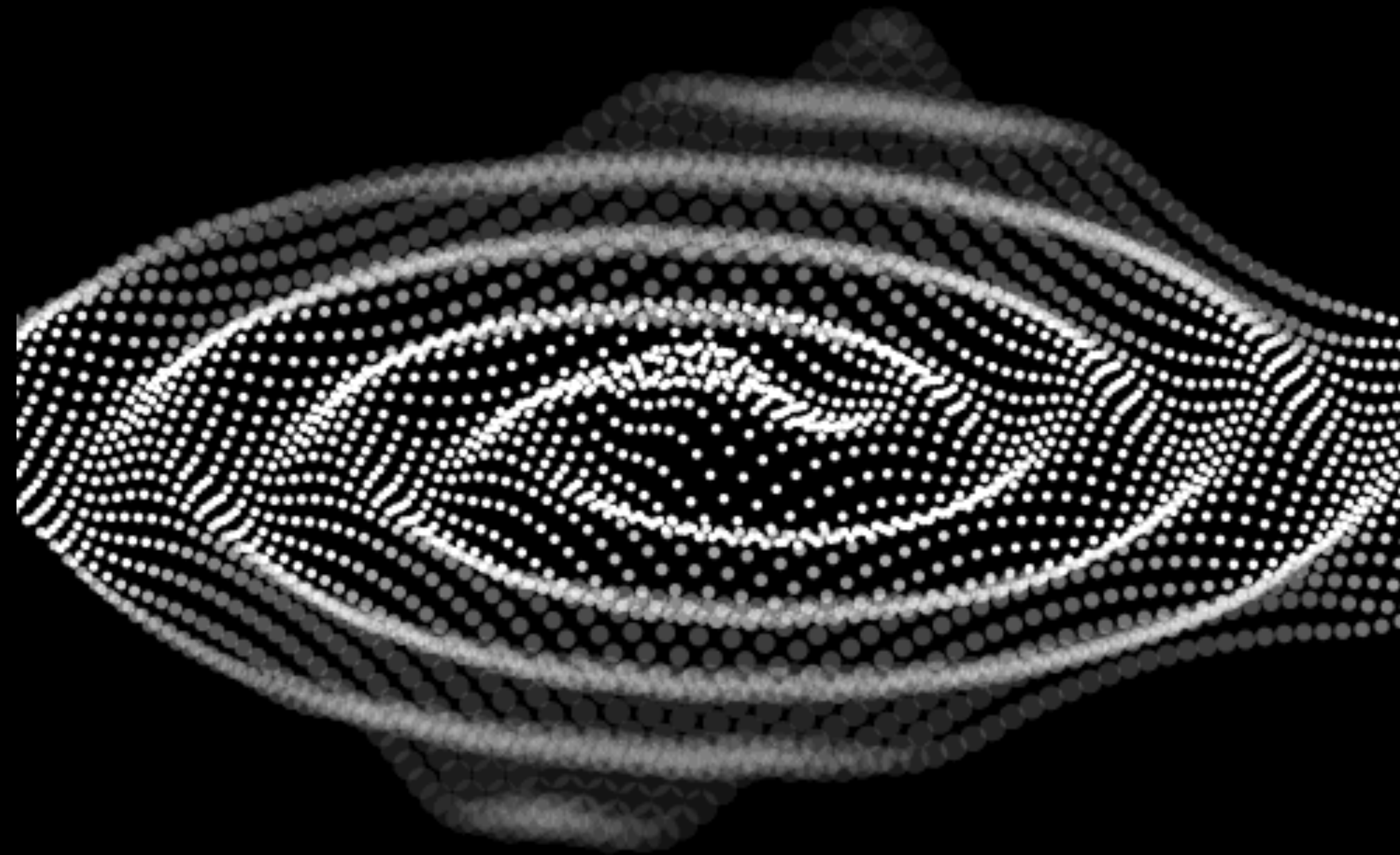


Electromagnetic Waves

accelerating masses

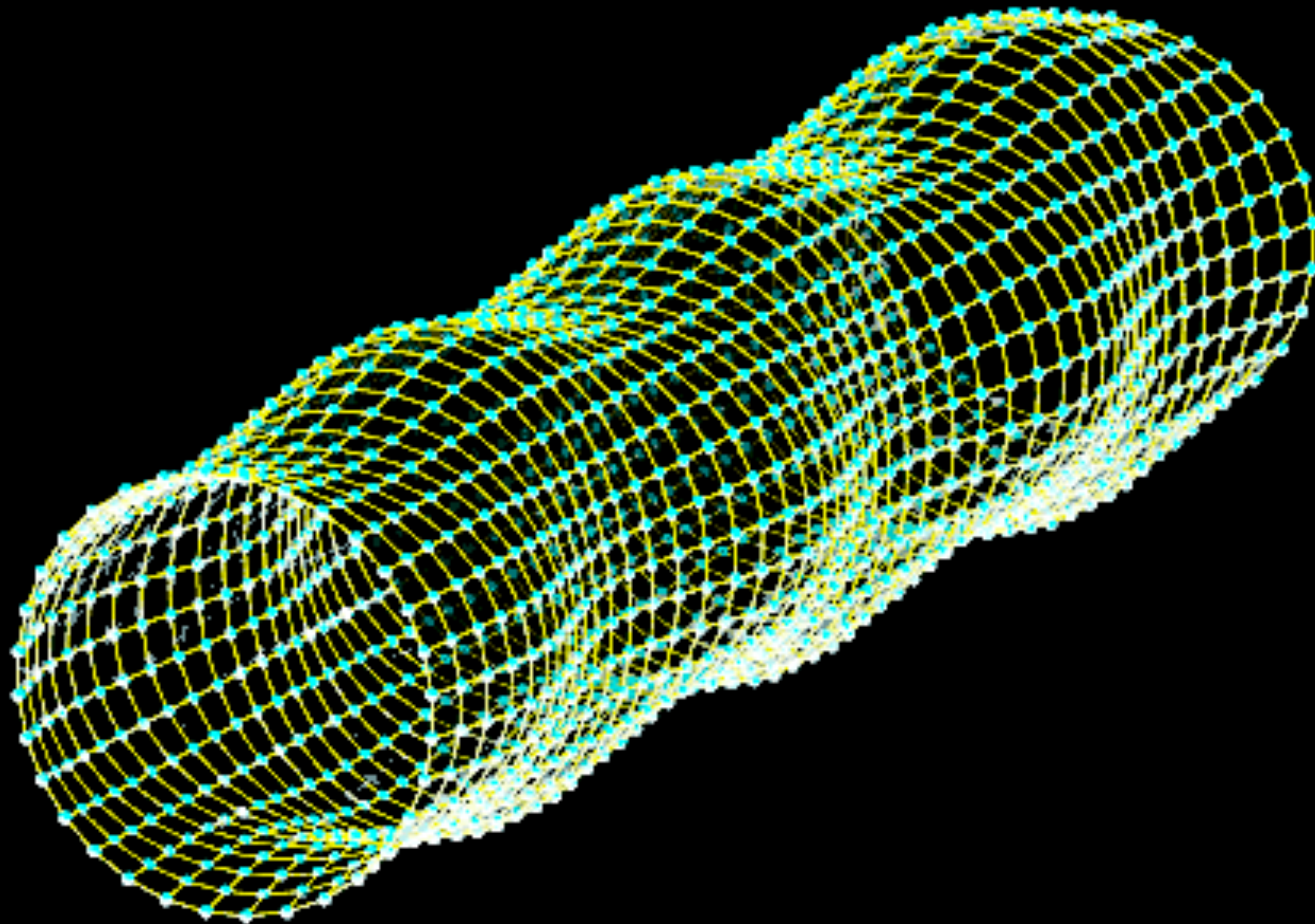


Gravitational Waves



GRAVITATIONAL WAVES

GW produce a time-dependent change in the geometry of the space-time.



credit: Einstein online info

Stretching and squeezing of space

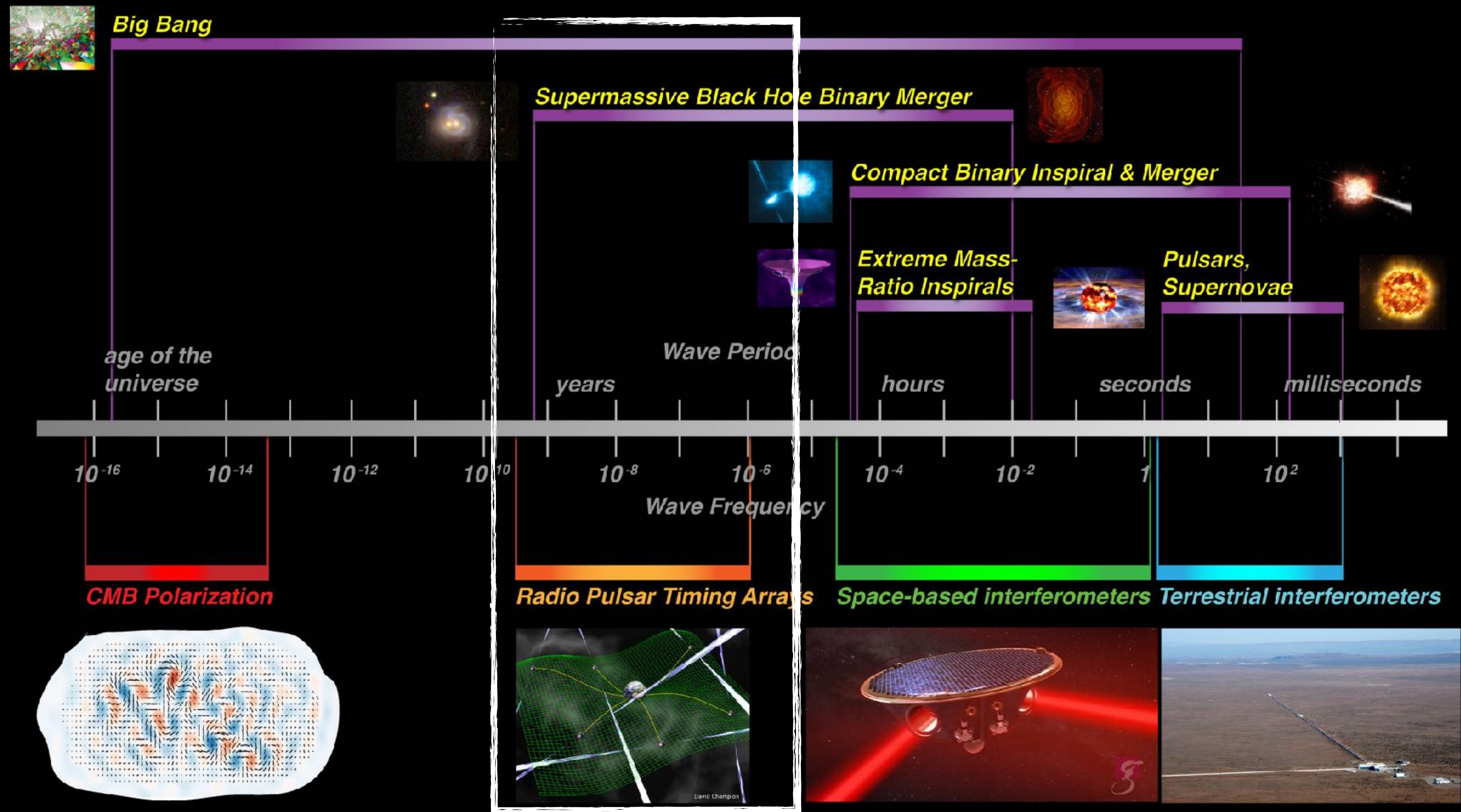
Gravitational waves are 'strains':
Changes in length per unit length,
 $\Delta L/L$

Space-time is very stiff

Strain from astrophysical sources*
on earth is $h \approx 10^{-21}$

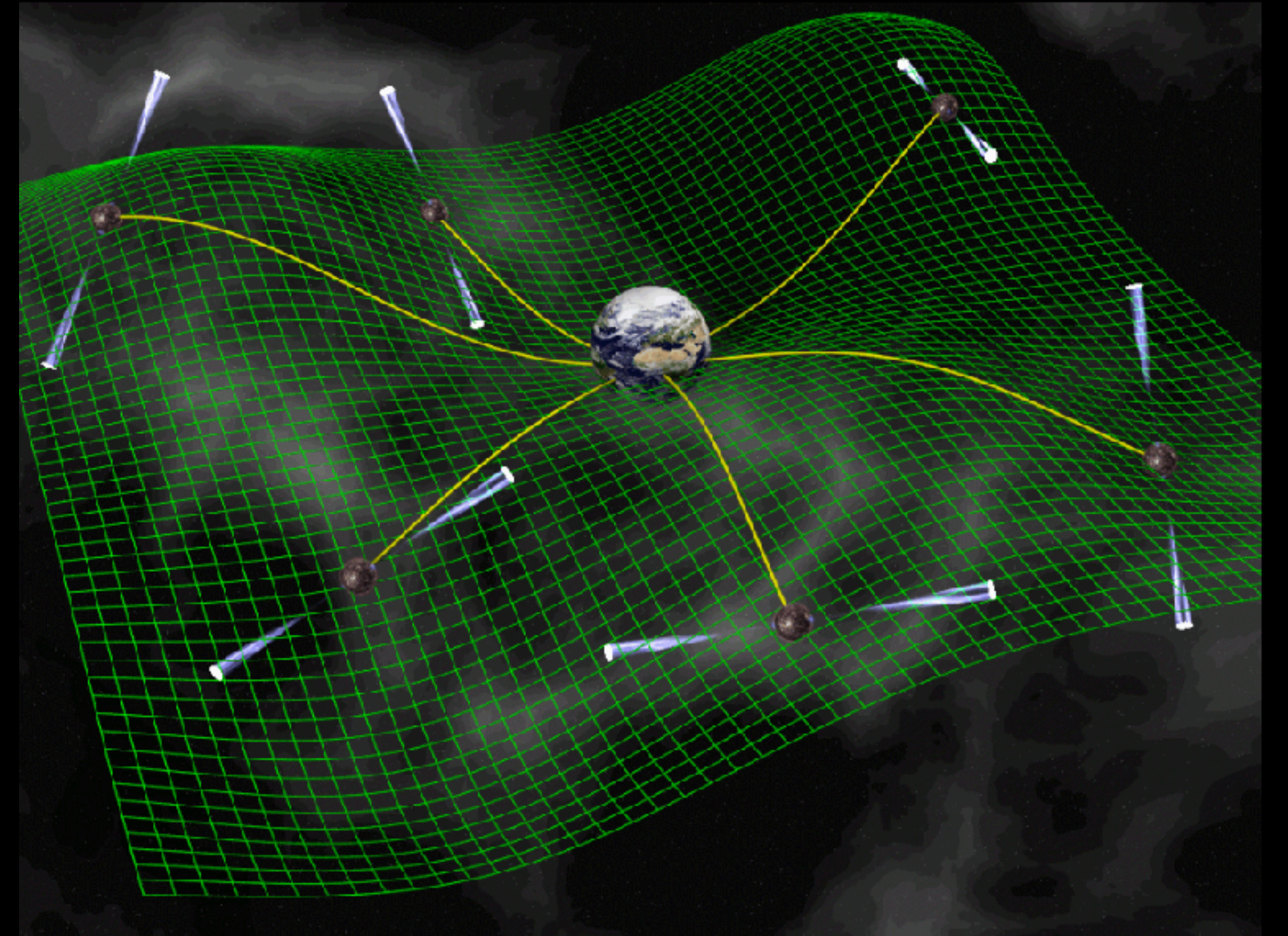
* Two compact object, each 10 times mass of the sun, revolving around each other and merging!

GW ACROSS THE SPECTRUM

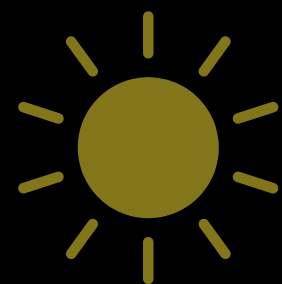


nHz BAND

- Pulsars; rapidly rotating neutron stars
- Measure radio pulse arrivals
- Can predict pulse arrival time to nano-second precision
- GWs cause change in expected arrival time
- That delay is correlated between different pulsars



GWs perturb the arrival times of pulses -> look for the presence of GWs in the timing residuals

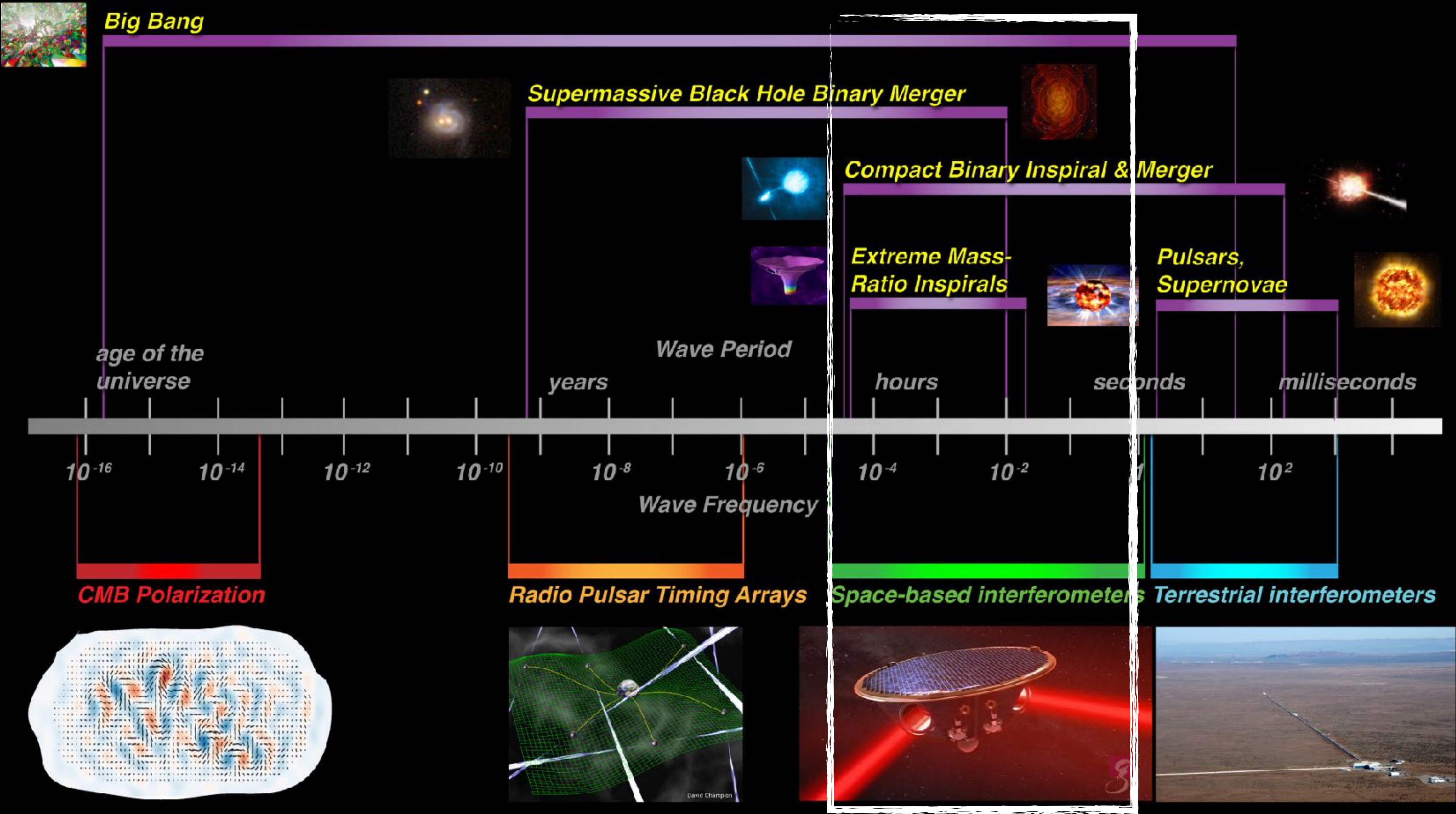


Exciting news from June 2023

Evidence for a GW signal that is correlated among different pulsars

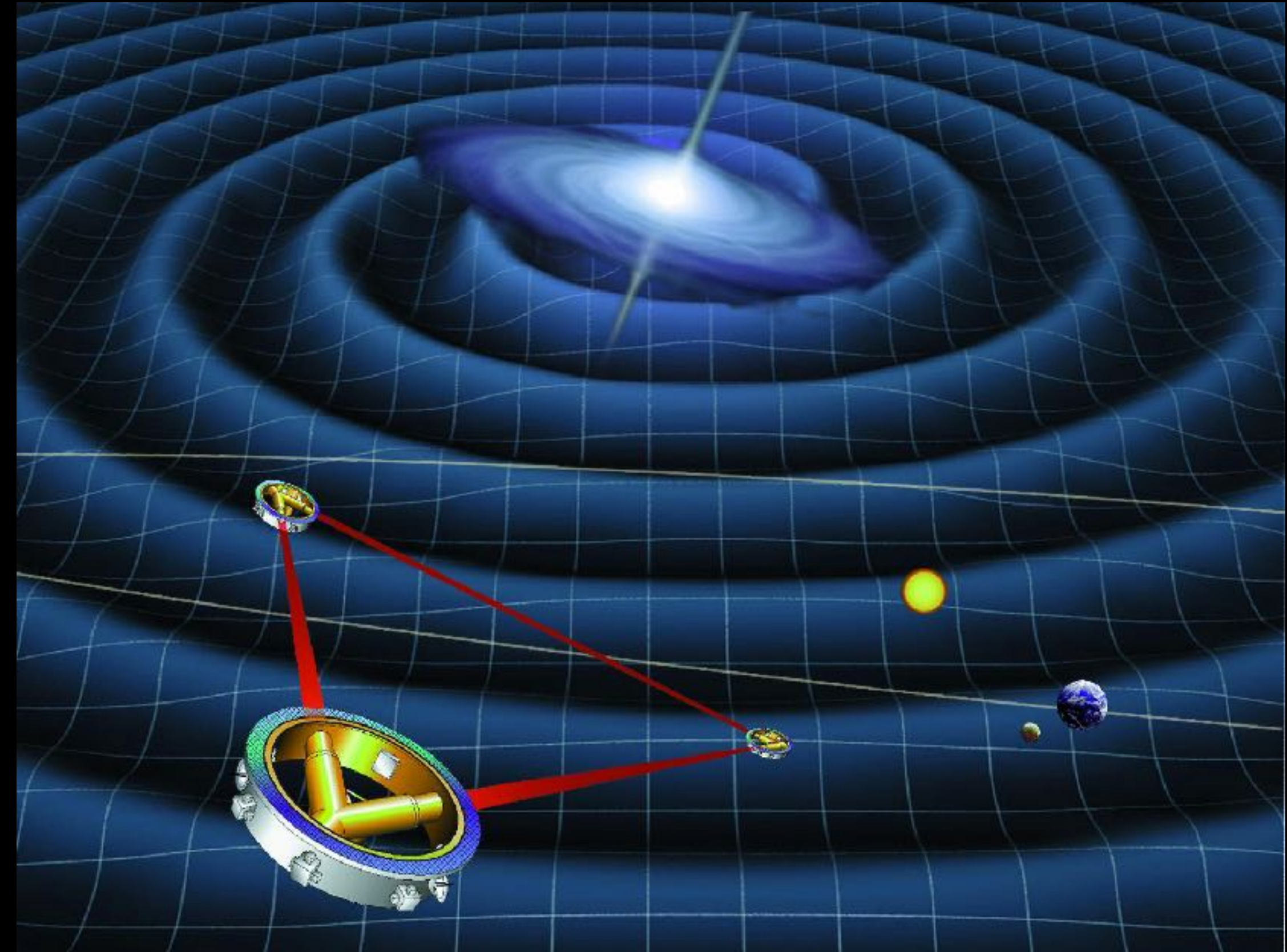
Collaboration	Paper	Link
NANOGrav	GWB Paper	https://arxiv.org/abs/2306.16213
EPTA	GWB Paper	https://arxiv.org/abs/2306.16214
PPTA	GWB Paper	https://arxiv.org/abs/2306.16215
CPTA	GWB Paper	https://arxiv.org/abs/2306.16216
NANOGrav	Data Paper	https://arxiv.org/abs/2306.16217
NANOGrav	Noise Model Paper	https://arxiv.org/abs/2306.16218
NANOGrav	New Physics Paper	https://arxiv.org/abs/2306.16219
NANOGrav	SMBH Binary Paper	https://arxiv.org/abs/2306.16220
NANOGrav	Anisotropy Paper	https://arxiv.org/abs/2306.16221
NANOGrav	Continuous GW Paper	https://arxiv.org/abs/2306.16222
NANOGrav	Code Review Paper	https://arxiv.org/abs/2306.16223
EPTA	Data Paper	https://arxiv.org/abs/2306.16224
EPTA	Noise Model Paper	https://arxiv.org/abs/2306.16225
EPTA	Continuous GW Paper	https://arxiv.org/abs/2306.16226
EPTA	Implications Paper	https://arxiv.org/abs/2306.16227
EPTA	Ultralight Dark Matter	https://arxiv.org/abs/2306.16228
PPTA	Noise Model Paper	https://arxiv.org/abs/2306.16229
PPTA	Data Paper	https://arxiv.org/abs/2306.16230

GW ACROSS THE SPECTRUM

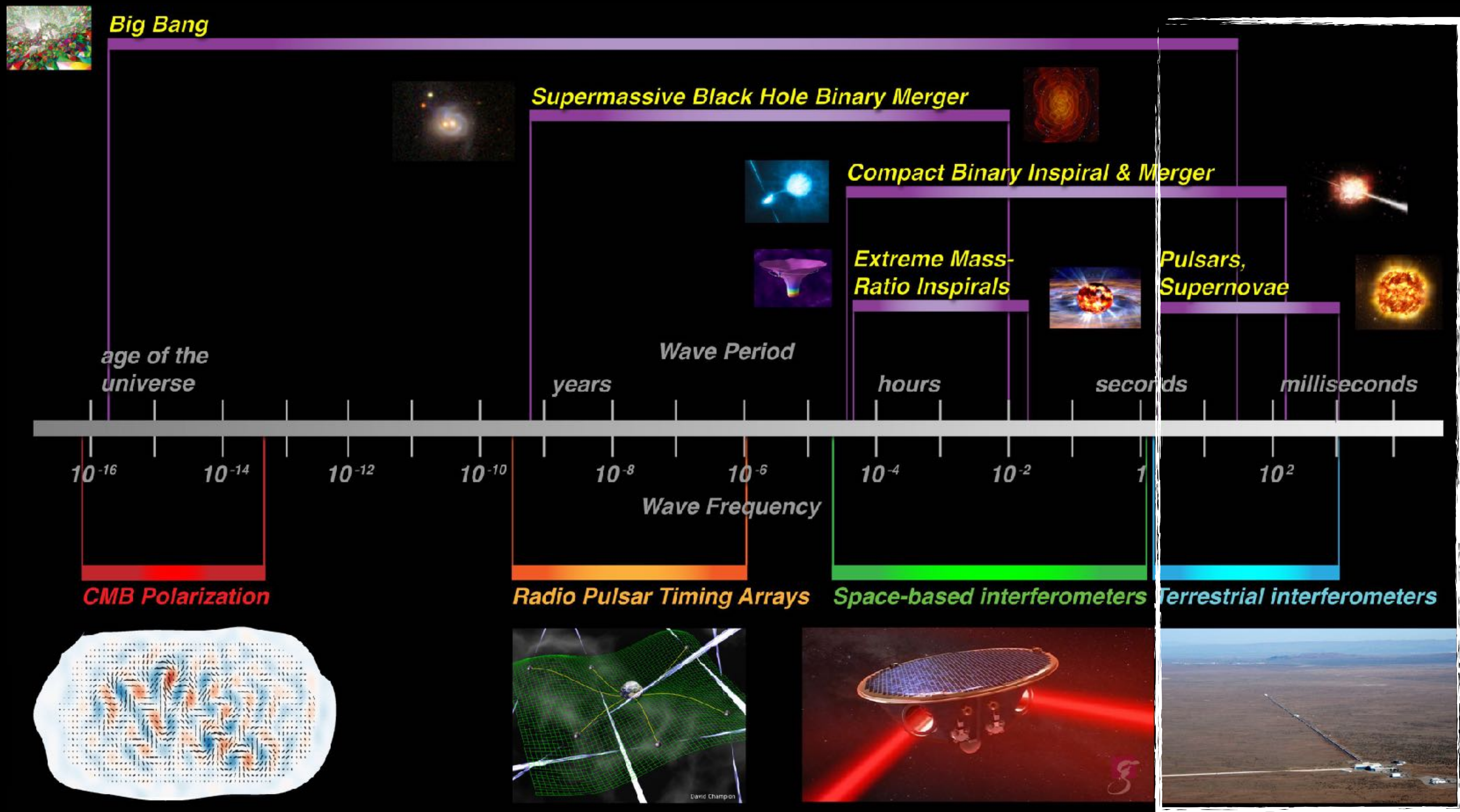


Laser Interferometer Space Antenna (LISA)

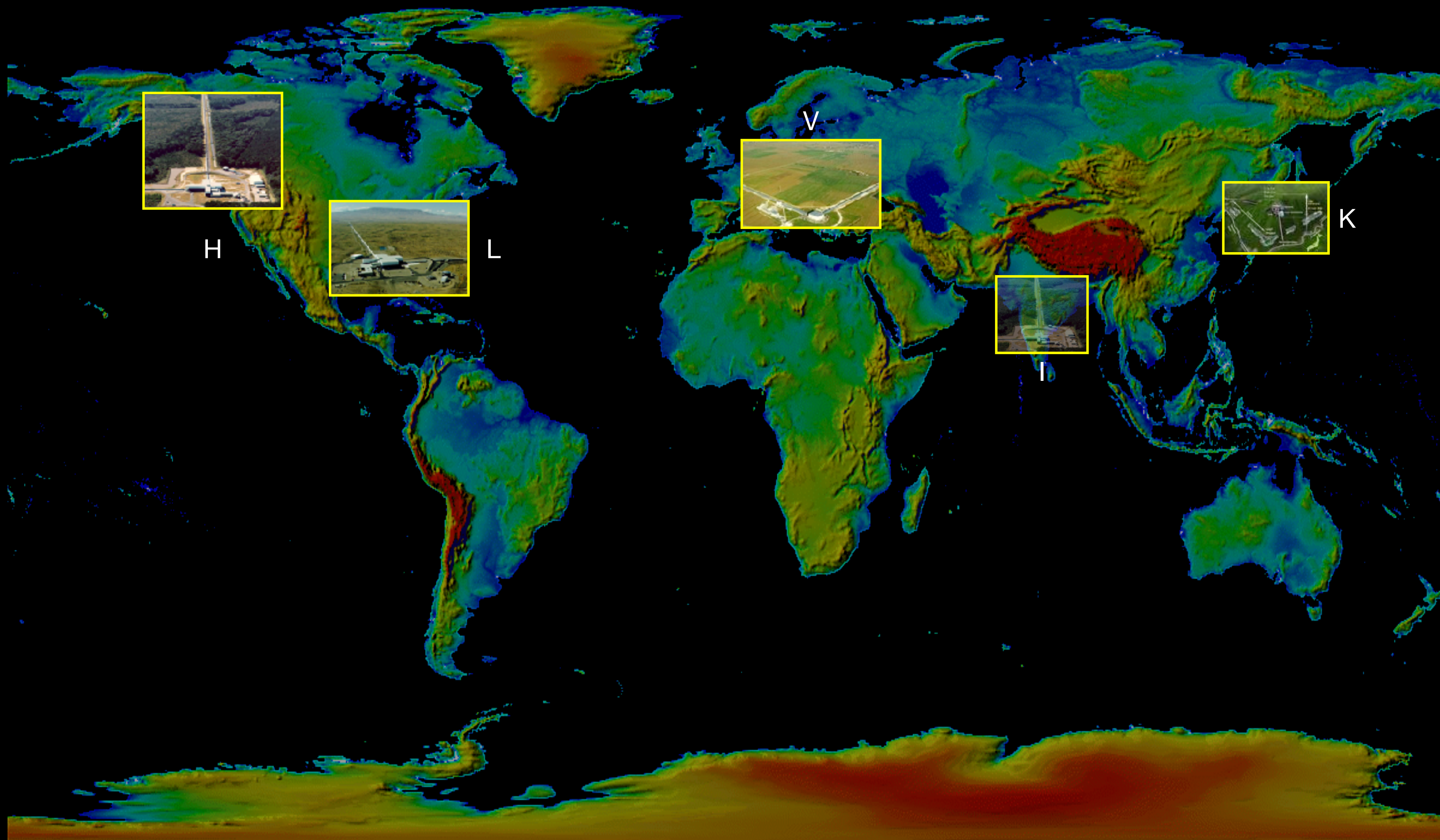
- Constellation of satellites 2.5 million km apart
- uses different type of interferometry
- dominated by white dwarf binaries in the galaxy
- Planned launch date ~2034
- Potential sources to detect: Massive black hole binaries, galactic white dwarf binaries, extreme mass ratio inspirals,



GW ACROSS THE SPECTRUM



GRAVITATIONAL WAVE DETECTOR NETWORK

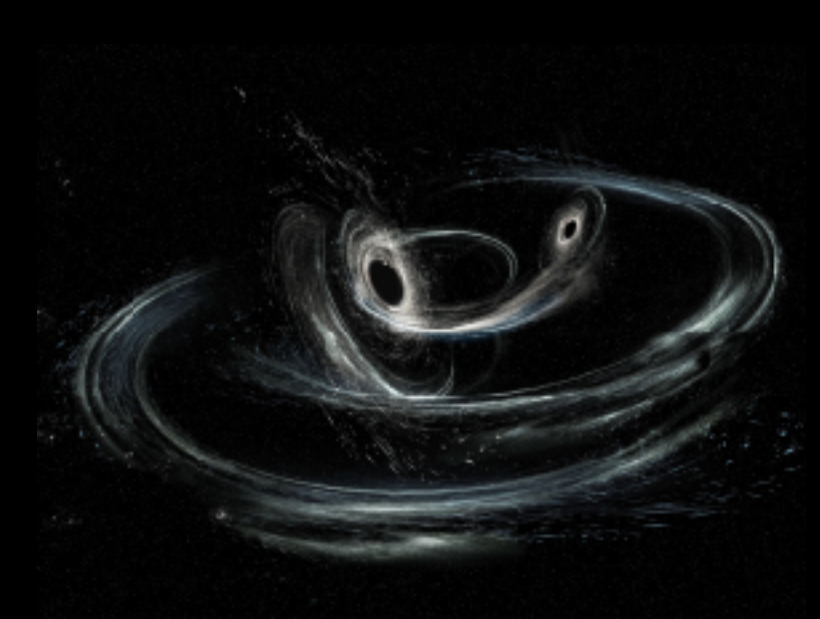


LIGO and Virgo observatories started constructions in '90s, and...

...Observed with the initial detectors 2005-2011,
and saw...

~~nothing~~

We saw no gravitational-wave signals.
But we learned how to build and commission detectors.
We learned how to analyze the data.



1.3 Billion years after the Black Holes merged.. (and multicellular life started on earth...)

100 years after Einstein predicted gravitational waves...

50 years after Rai Weiss invented the detectors...

20 years after LIGO and Virgo were build...

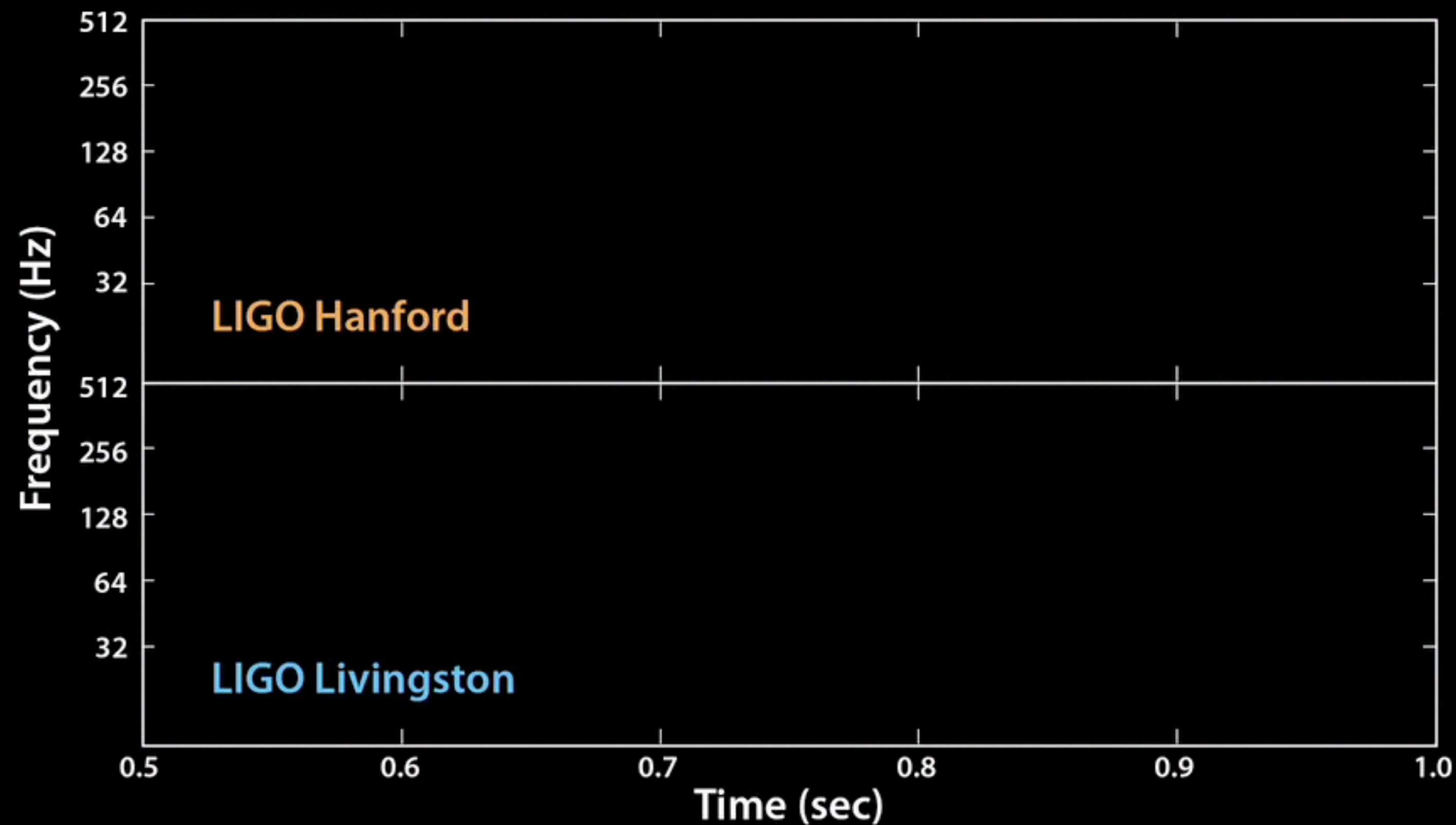
10 years after Advanced LIGO and Virgo got the ok...

6 months after starting detector tuning...

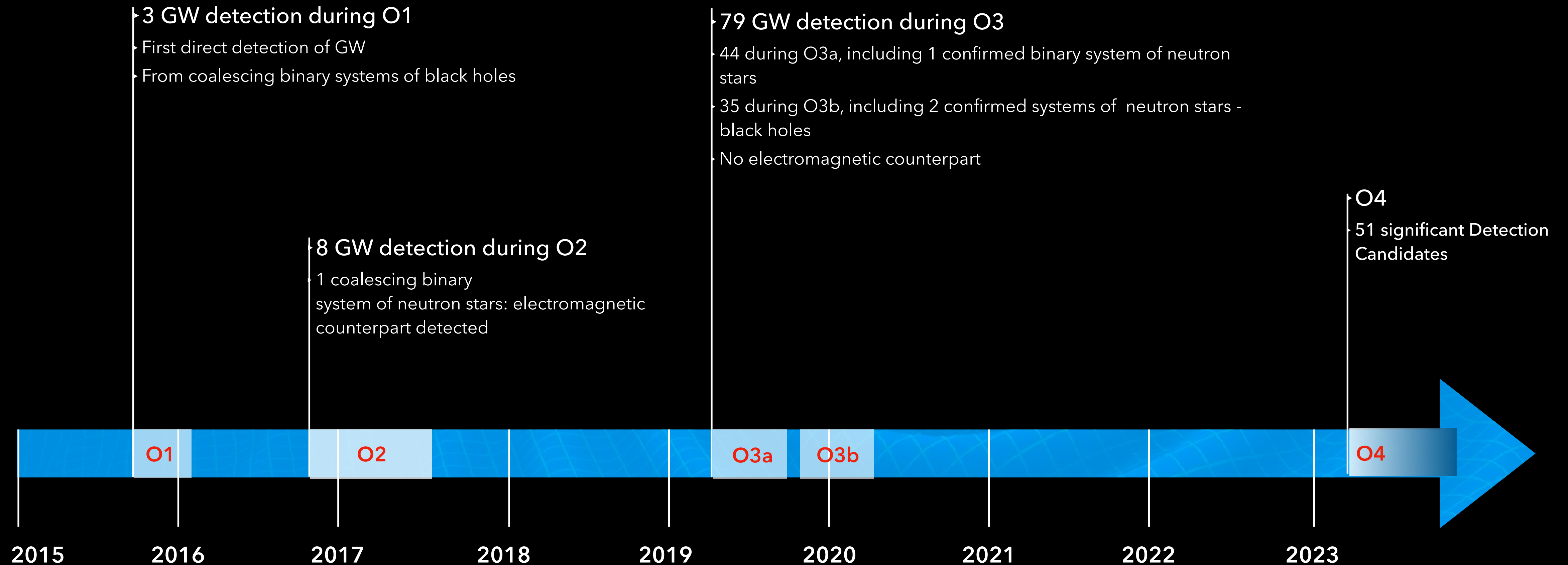
Two days after we started observing...

September 14, 2015 at 09:50 UTC:

Cosmic Rendezvous



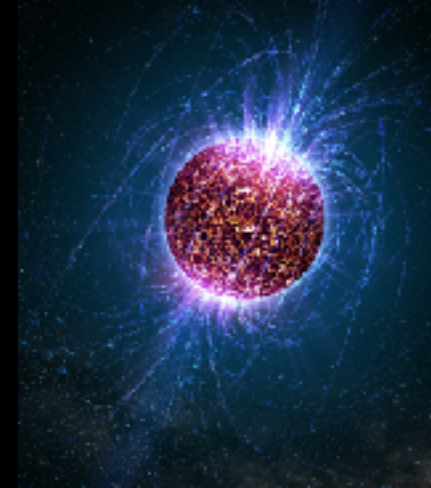
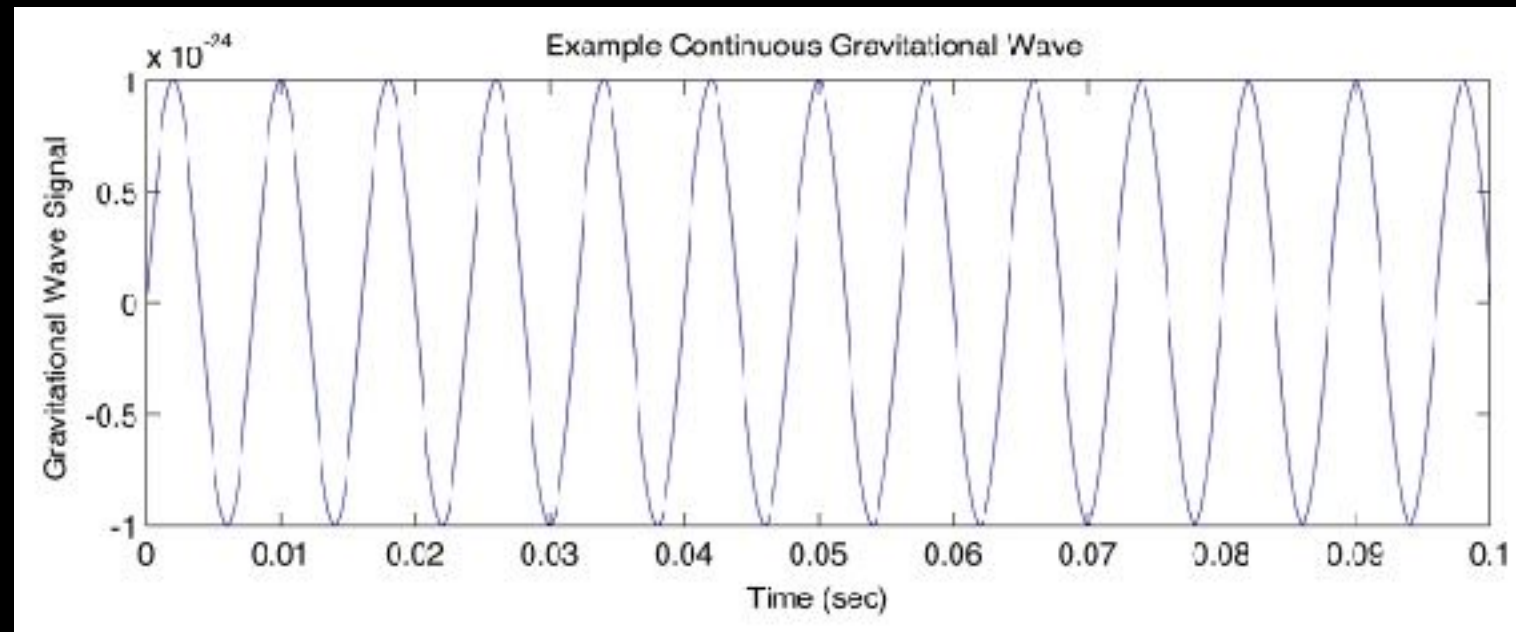
LIGO-Virgo-KAGRA observations so far..



~ 140 GW events/candidates

WHAT'S NEXT

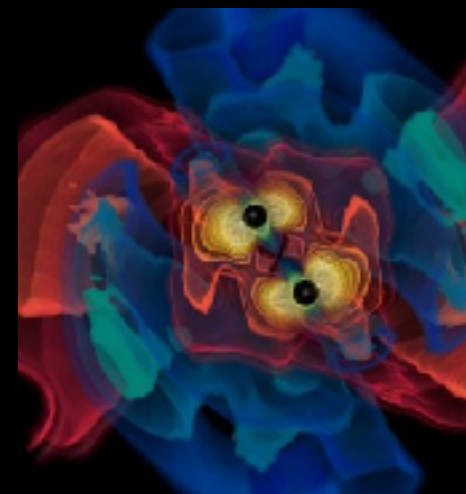
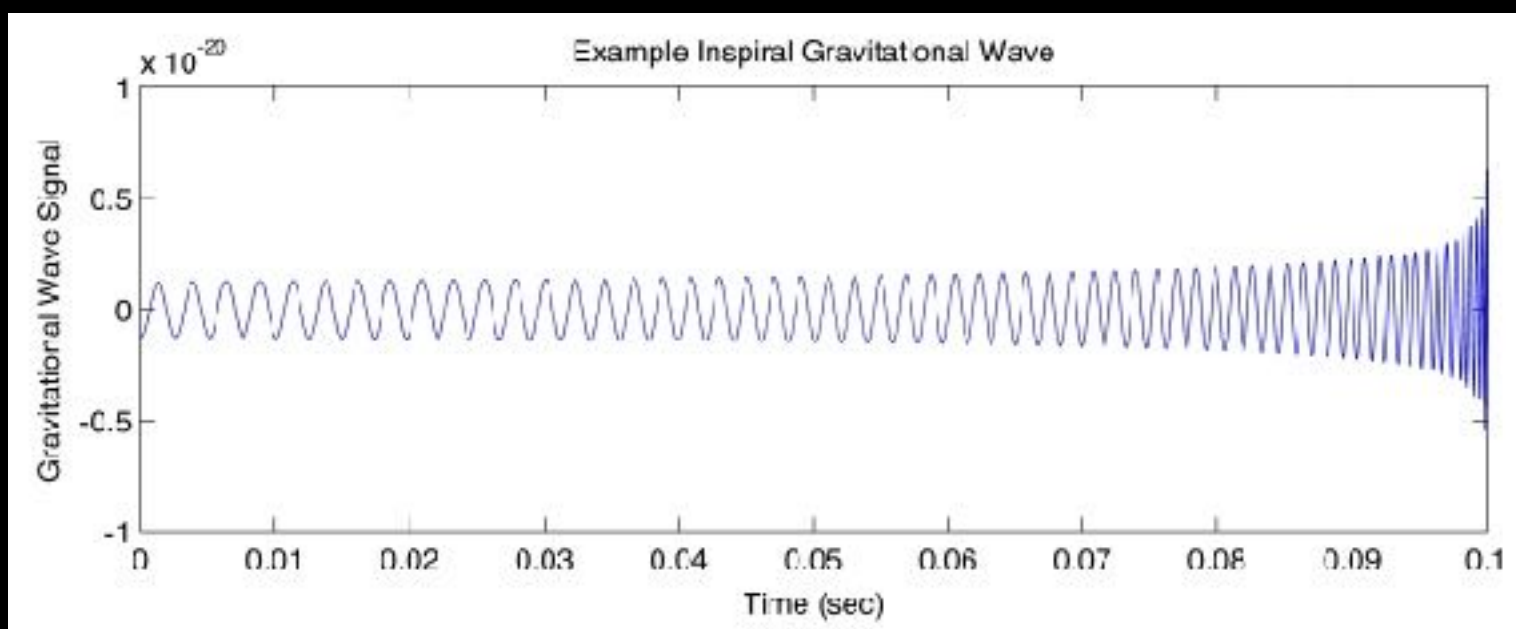
Continuous



A single star swiftly rotating about its axis with a large mountain or other irregularity on it

Expected to produce comparatively weak gravitational waves

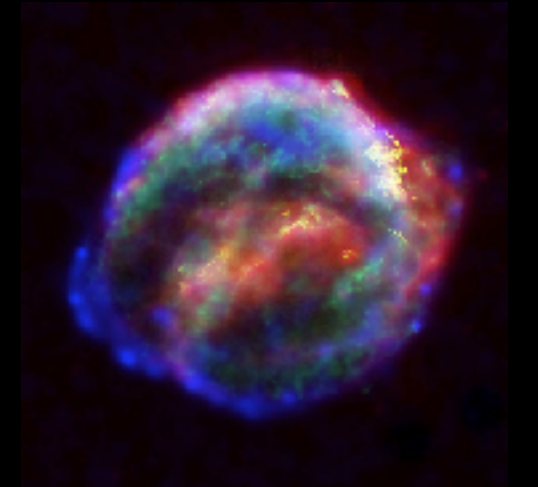
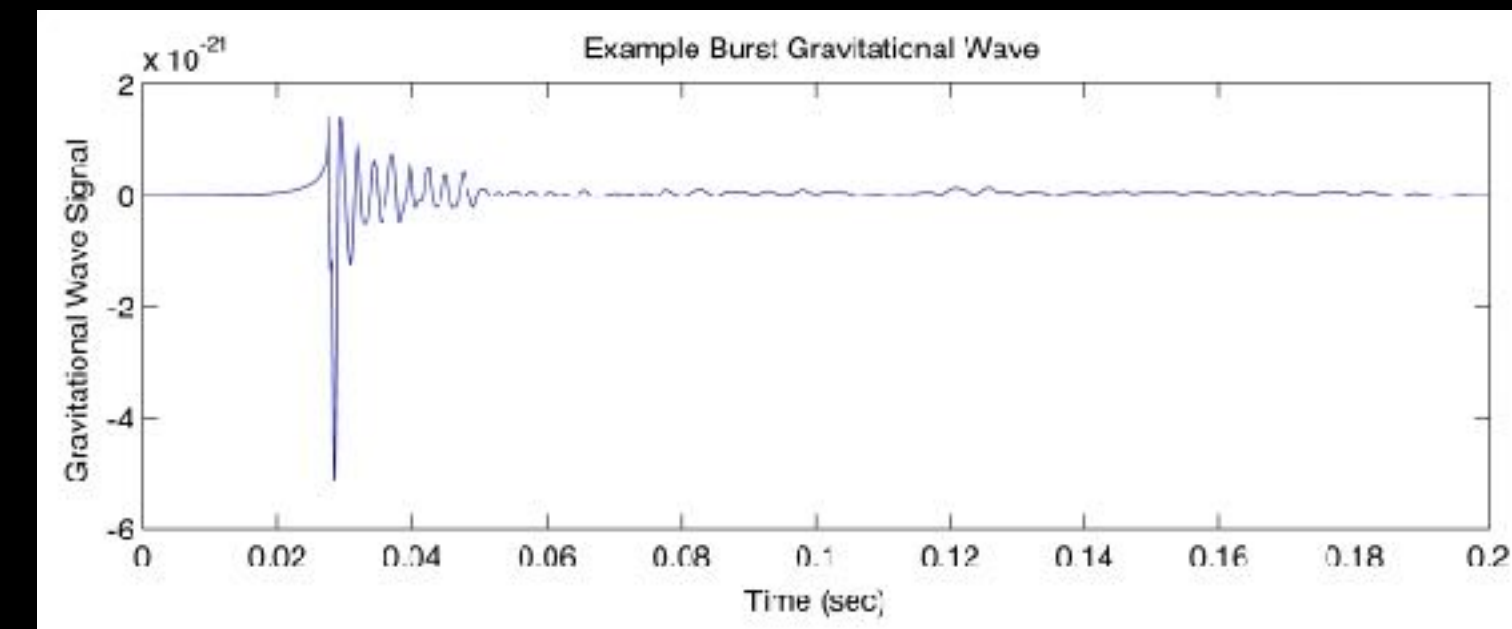
Binary Merger



Generated during the end-of-life stage of binary systems where the two objects merge into one.

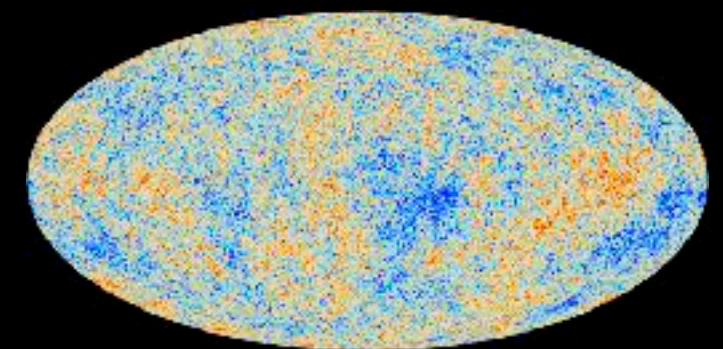
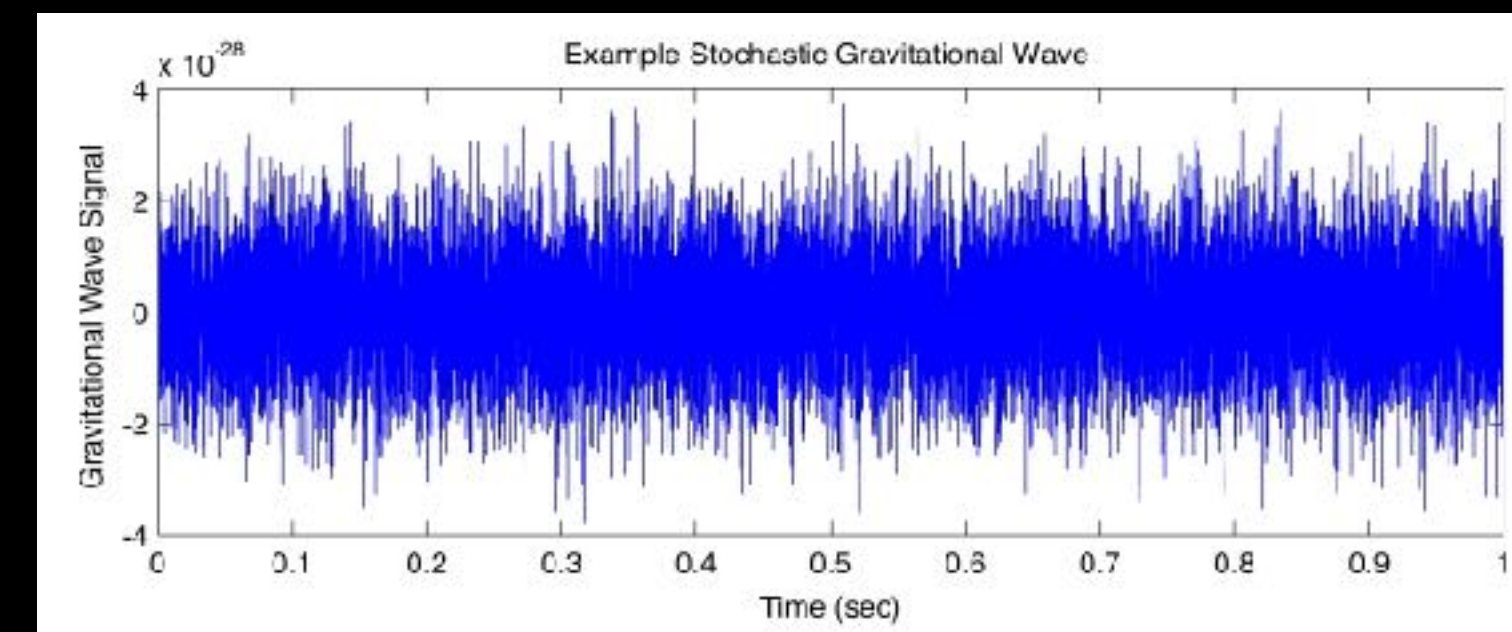
These systems are usually two neutron stars, two black holes, or a neutron star and a black hole

Burst



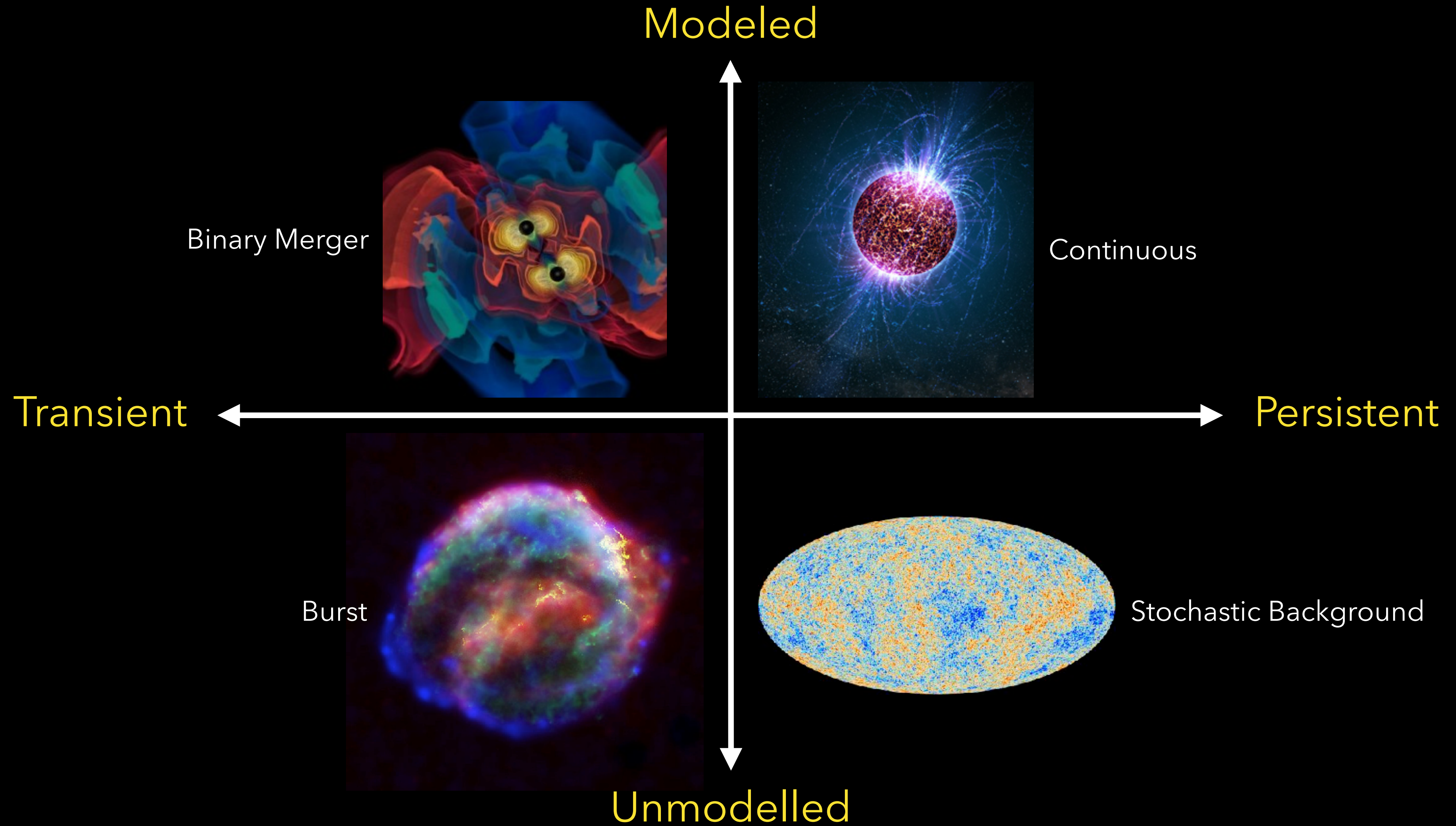
From short-duration unknown or unanticipated sources
There are hypotheses that some systems such as supernovae or gamma ray bursts may produce burst gravitational waves, but too little is known about the details of these systems to anticipate the form these waves will have

Stochastic Background

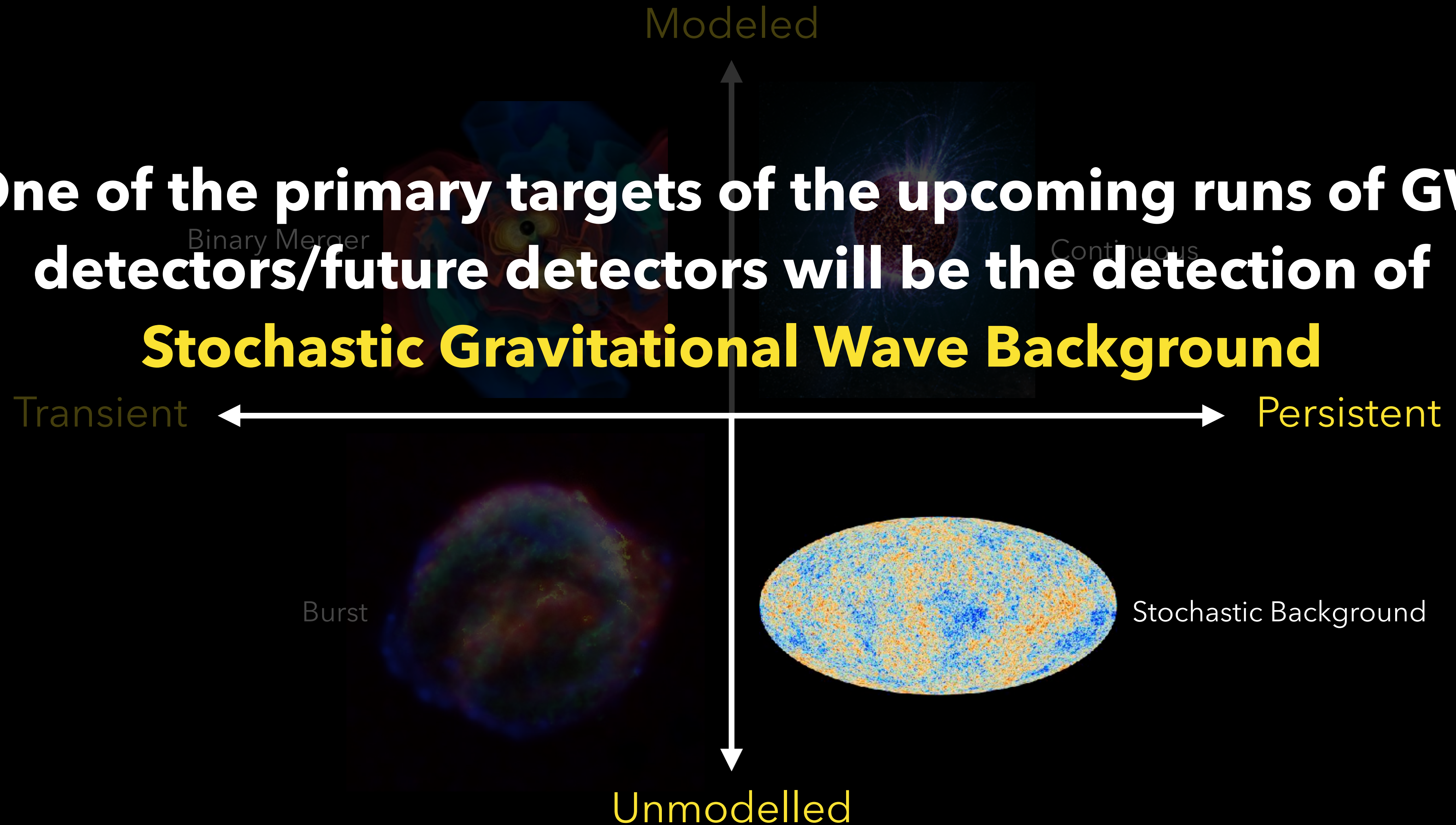


Incoherent superposition of many GW sources. It could be cosmological (for example, vacuum fluctuation from the early universe) and/or astrophysical (for example, adding contribution from all binary black hole coalescence in the universe).

WHAT'S NEXT

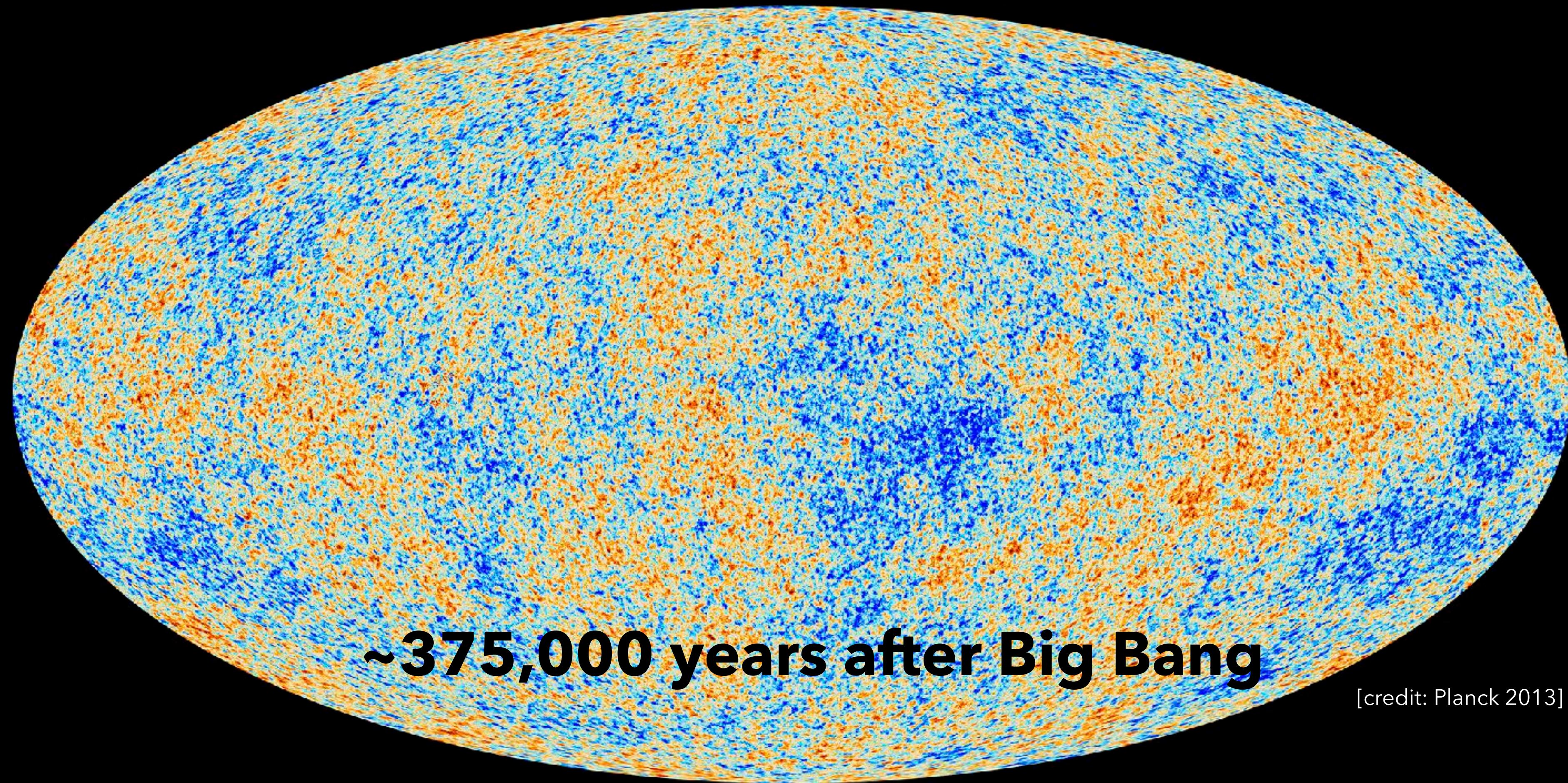


One of the primary targets of the upcoming runs of GW detectors/future detectors will be the detection of **Stochastic Gravitational Wave Background**



WHY SHOULD WE CARE ABOUT SGWB ?

Stochastic background of **EM** radiation



The observation of CMB and its anisotropies has revolutionized our understanding of the universe

WHY SHOULD WE CARE ABOUT SGWB ?

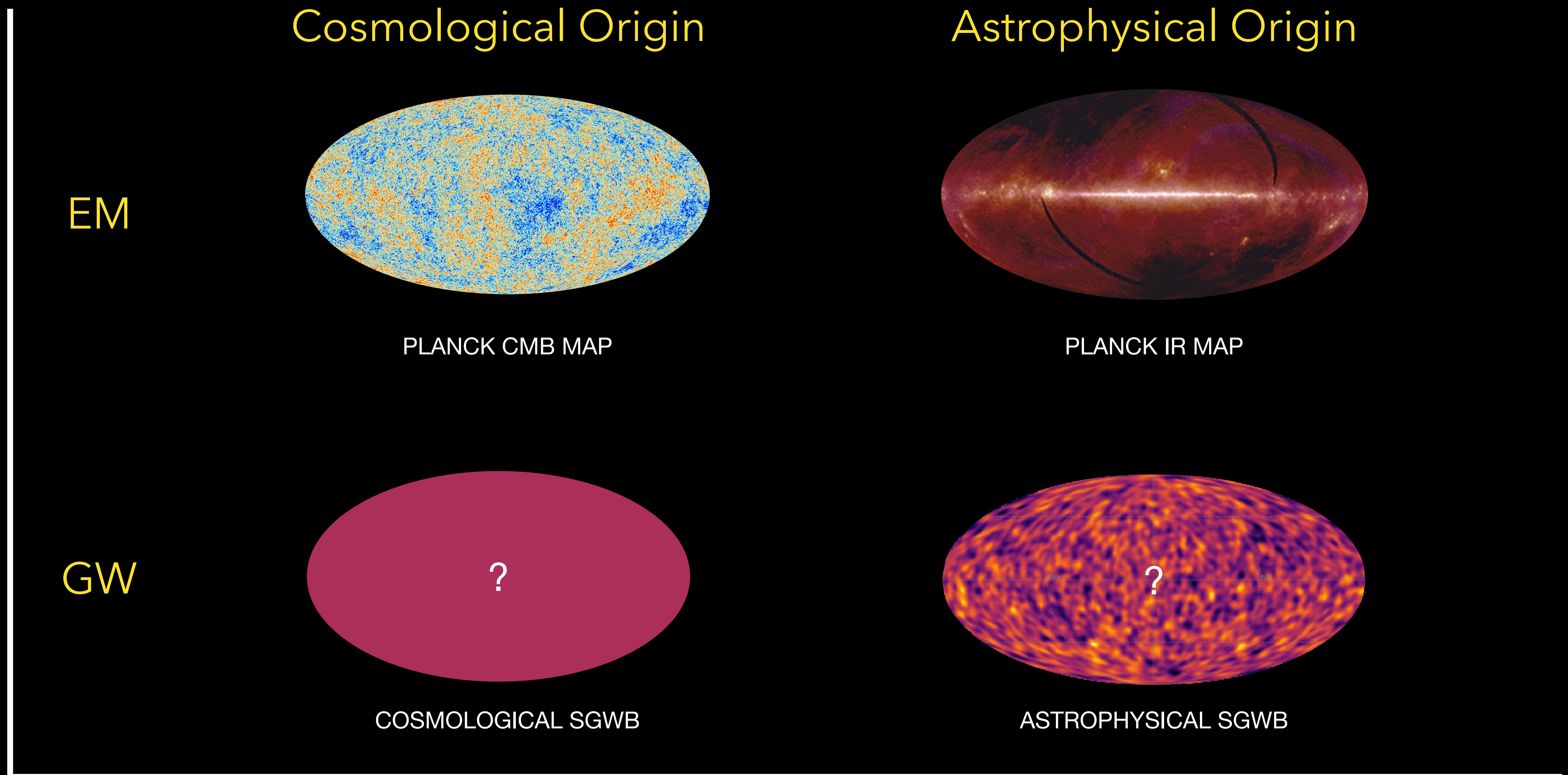
Stochastic background of **GW** radiation

??

$\sim 10^{-32}$ seconds after Big Bang

Weakness of gravity relative to other forces \Rightarrow provide an unprecedented window to the physics of the early universe.

WHICH SGWBs WE ARE SENSITIVE TO?

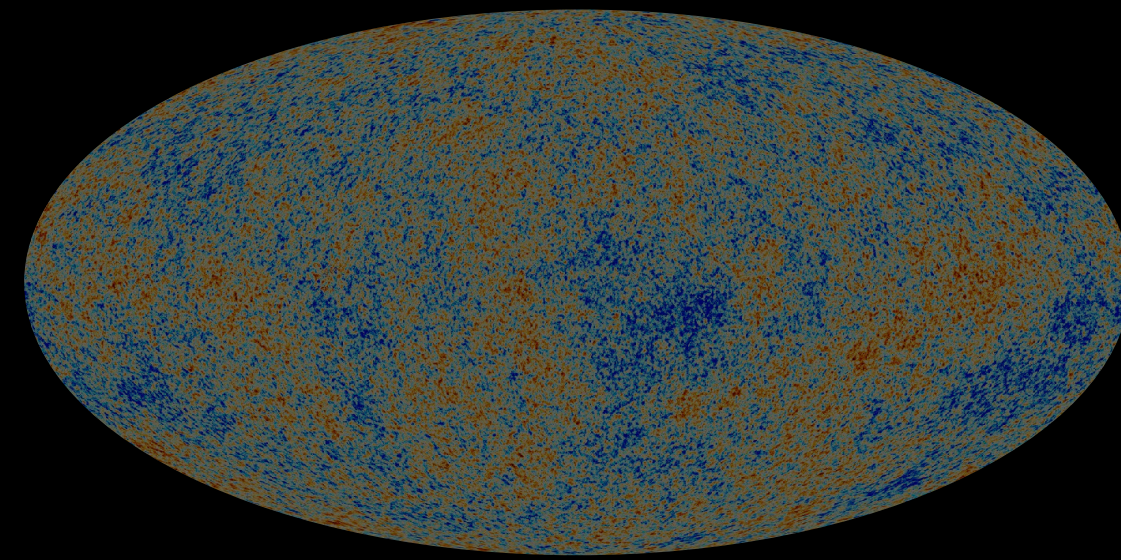


WHICH SGWBs WE ARE SENSITIVE TO?

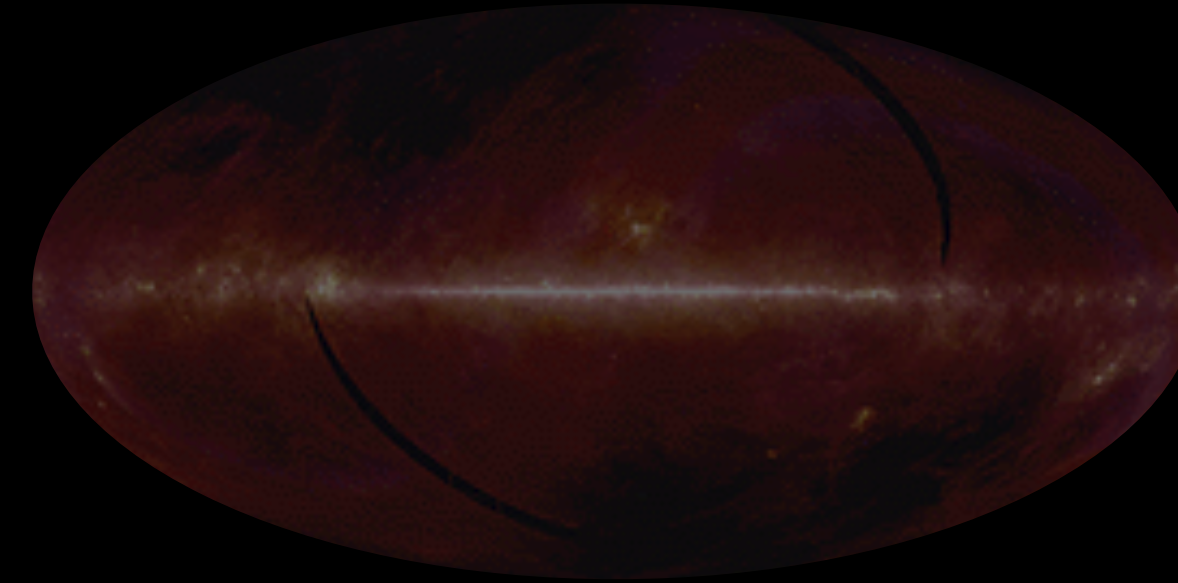
Cosmological Origin

Astrophysical Origin

EM

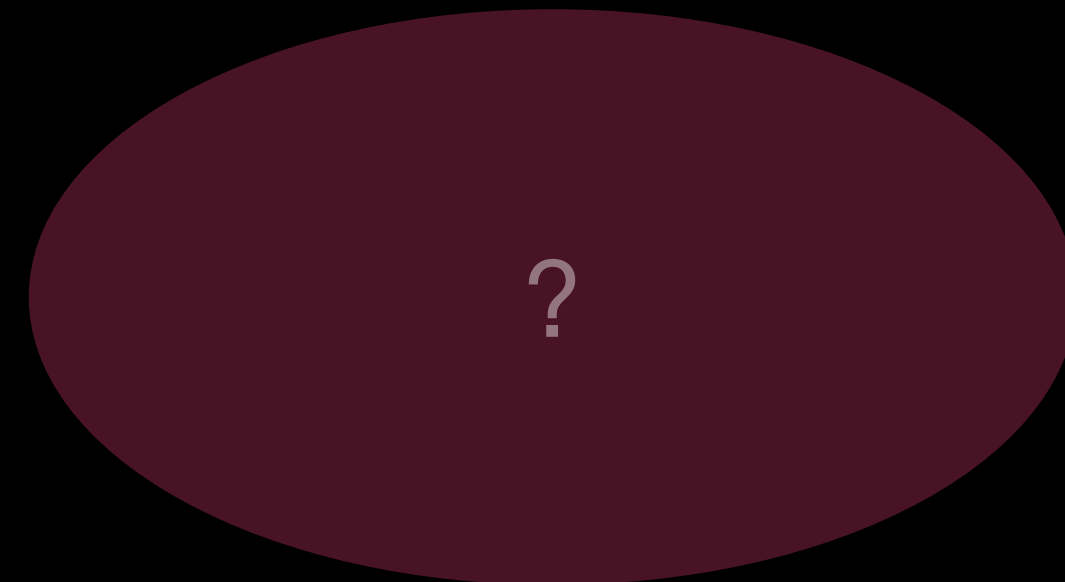


PLANCK CMB MAP

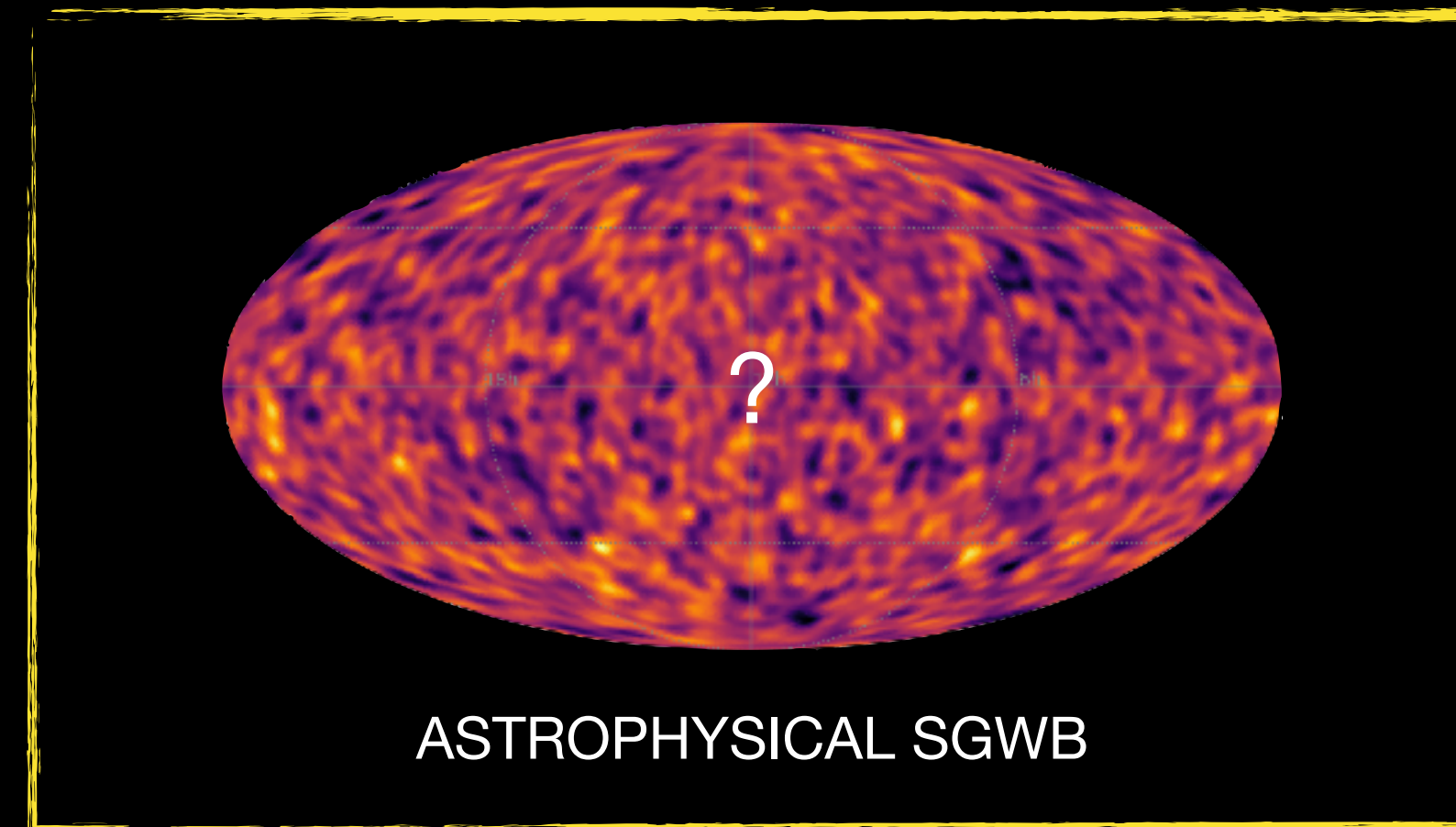


PLANCK IR MAP

GW



COSMOLOGICAL SGWB



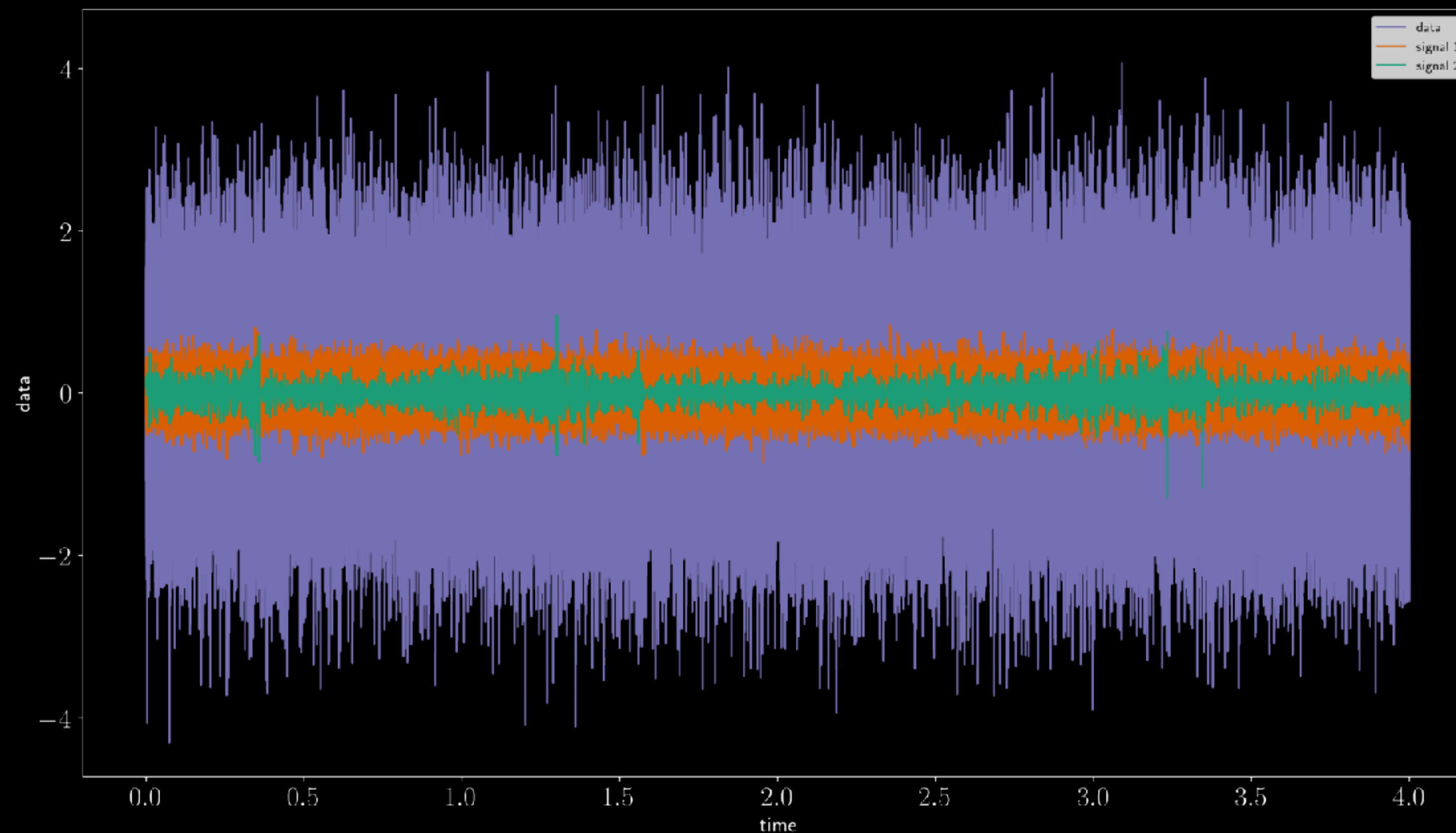
ASTROPHYSICAL SGWB

WHAT ARE THESE SGWBs ?

Superposition of signals **too weak** or **too numerous** to individually detect

Looks **like noise** in a single detector

Characterized **statistically** in terms of moments (ensemble averages) of the metric perturbations



STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

Plane wave expansion of metric perturbations

$$h_{ab}(t, \vec{x}) = \int_{-\infty}^{\infty} df \int d^2\Omega_{\hat{n}} \sum_{A=+, \times} h_A(f, \hat{n}) e_{ab}^A(\hat{n}) e^{i2\pi f(t + \hat{n} \cdot \vec{x}/c)}$$

Strain

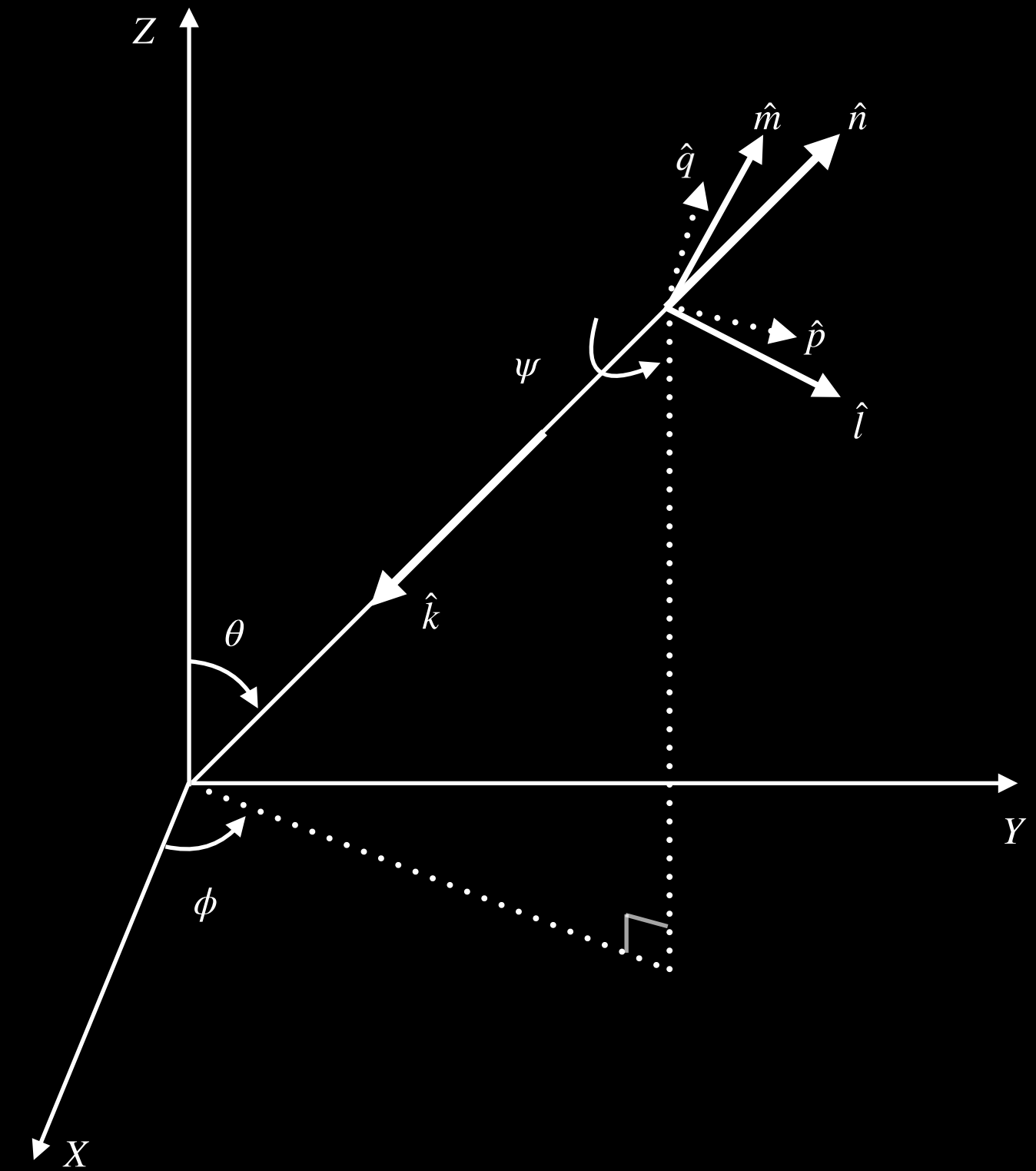
Measured at \vec{x}

Integrate over all sky direction

Integrate over all frequency

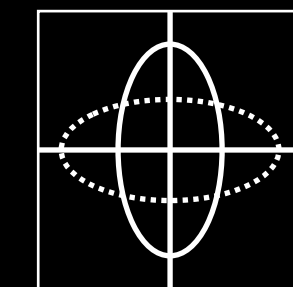
Plane wave coefficients

Plane wave phase



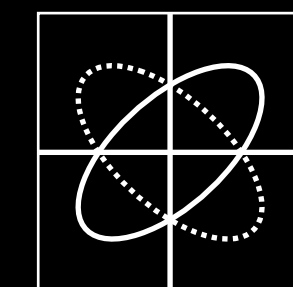
Polarization tensors:

$$e_{ab}^+(\hat{n}) = \hat{l}_a \hat{l}_b - \hat{m}_a \hat{m}_b$$



$A = +$

$$e_{ab}^\times(\hat{n}) = \hat{l}_a \hat{m}_b + \hat{m}_a \hat{l}_b$$

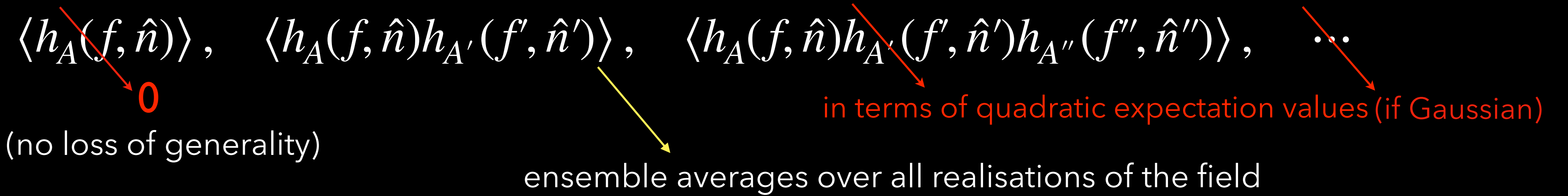


$A = \times$

STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

Statistical properties are encoded in:

$$\langle h_A(f, \hat{n}) \rangle, \quad \langle h_A(f, \hat{n}) h_{A'}(f', \hat{n}') \rangle, \quad \langle h_A(f, \hat{n}) h_{A'}(f', \hat{n}') h_{A''}(f'', \hat{n}'') \rangle, \quad \dots$$



(no loss of generality)

ensemble averages over all realisations of the field

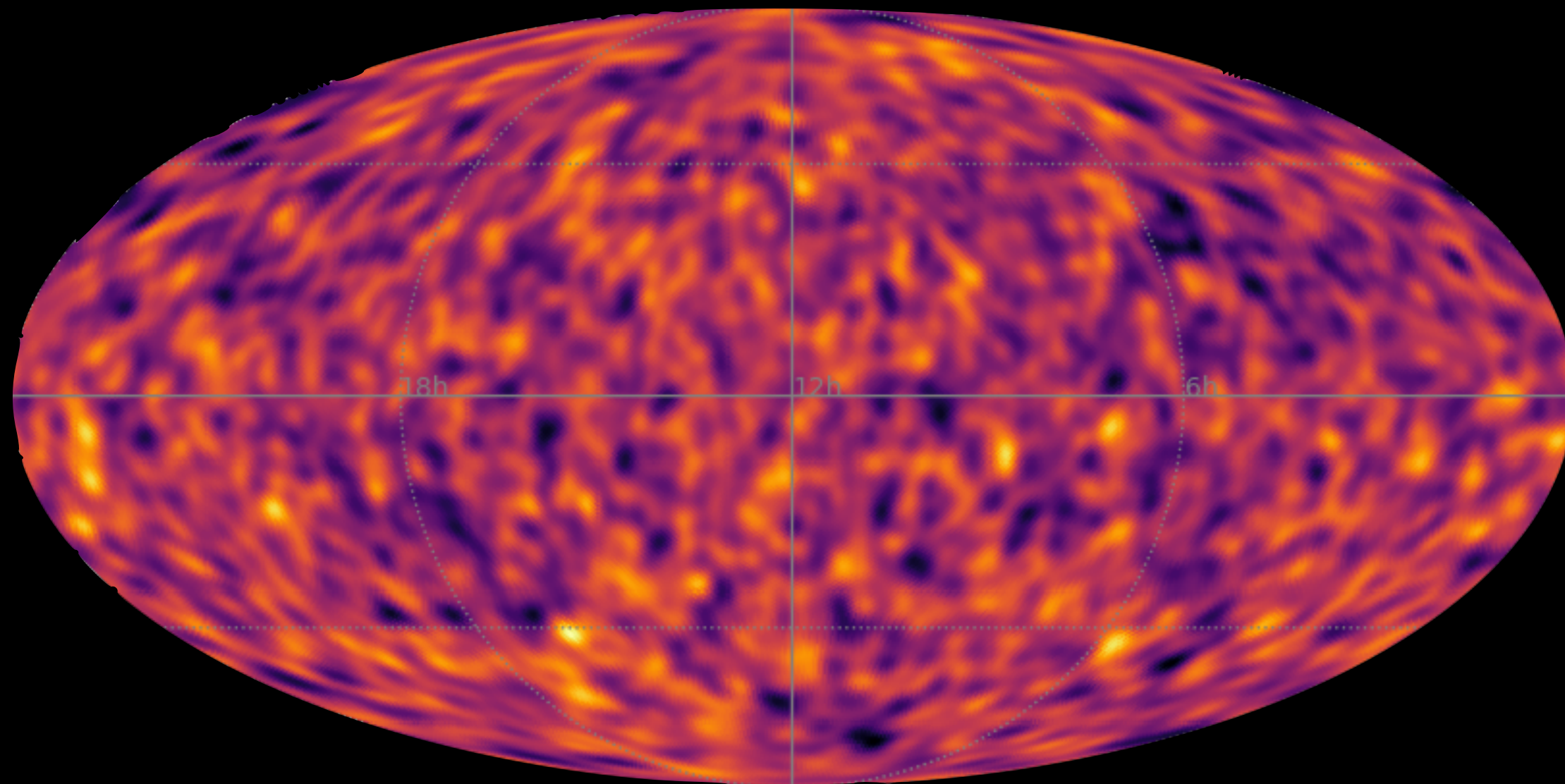
in terms of quadratic expectation values (if Gaussian)

If $h_A(f, \hat{n})$ are mean-zero Gaussian fields, the signal is fully described by its second moment.

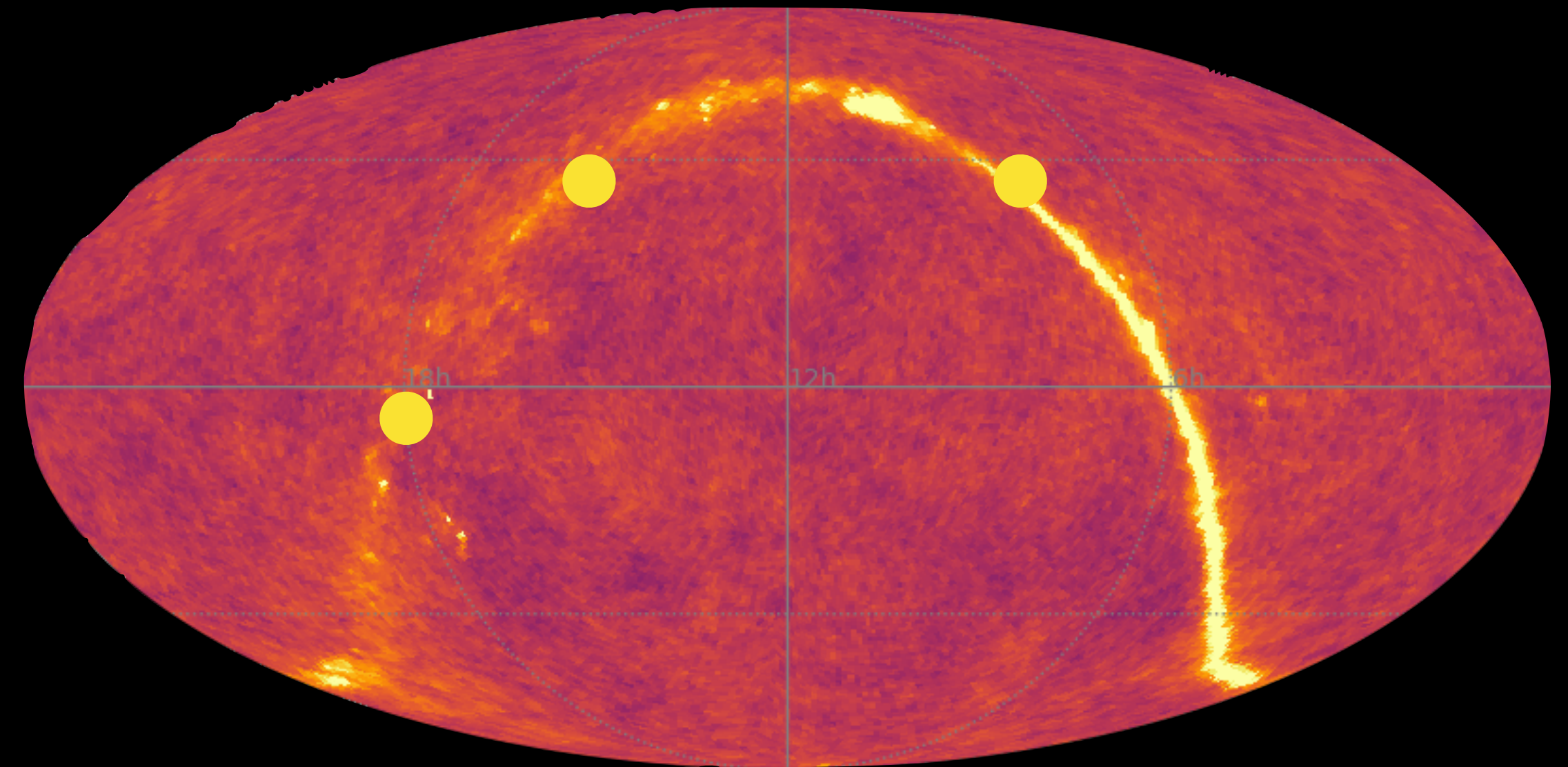
TYPES OF STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

(1) Spatial distribution

isotropic



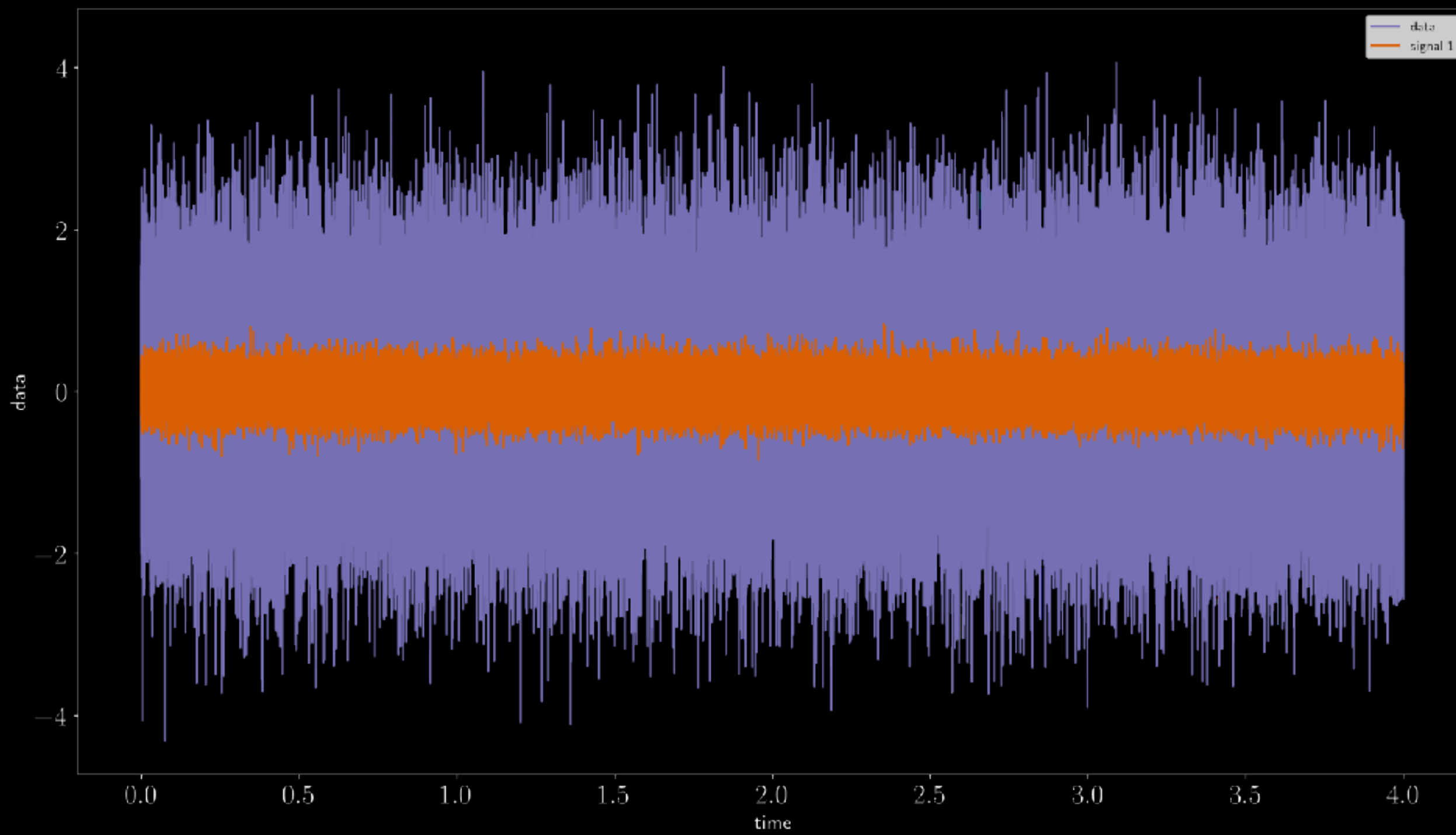
anisotropic



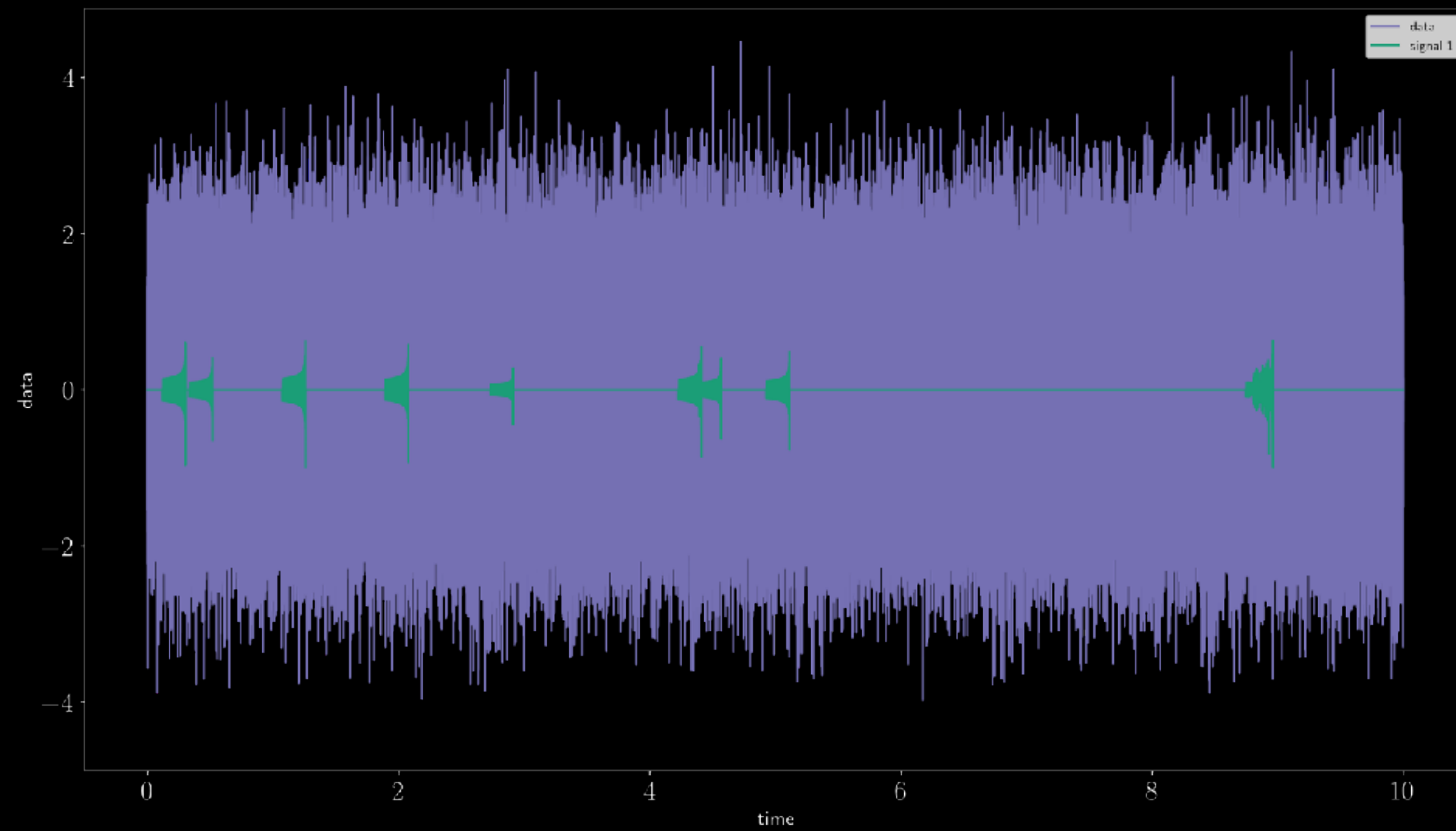
TYPES OF STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

(2) Temporal distribution

Stationary Gaussian

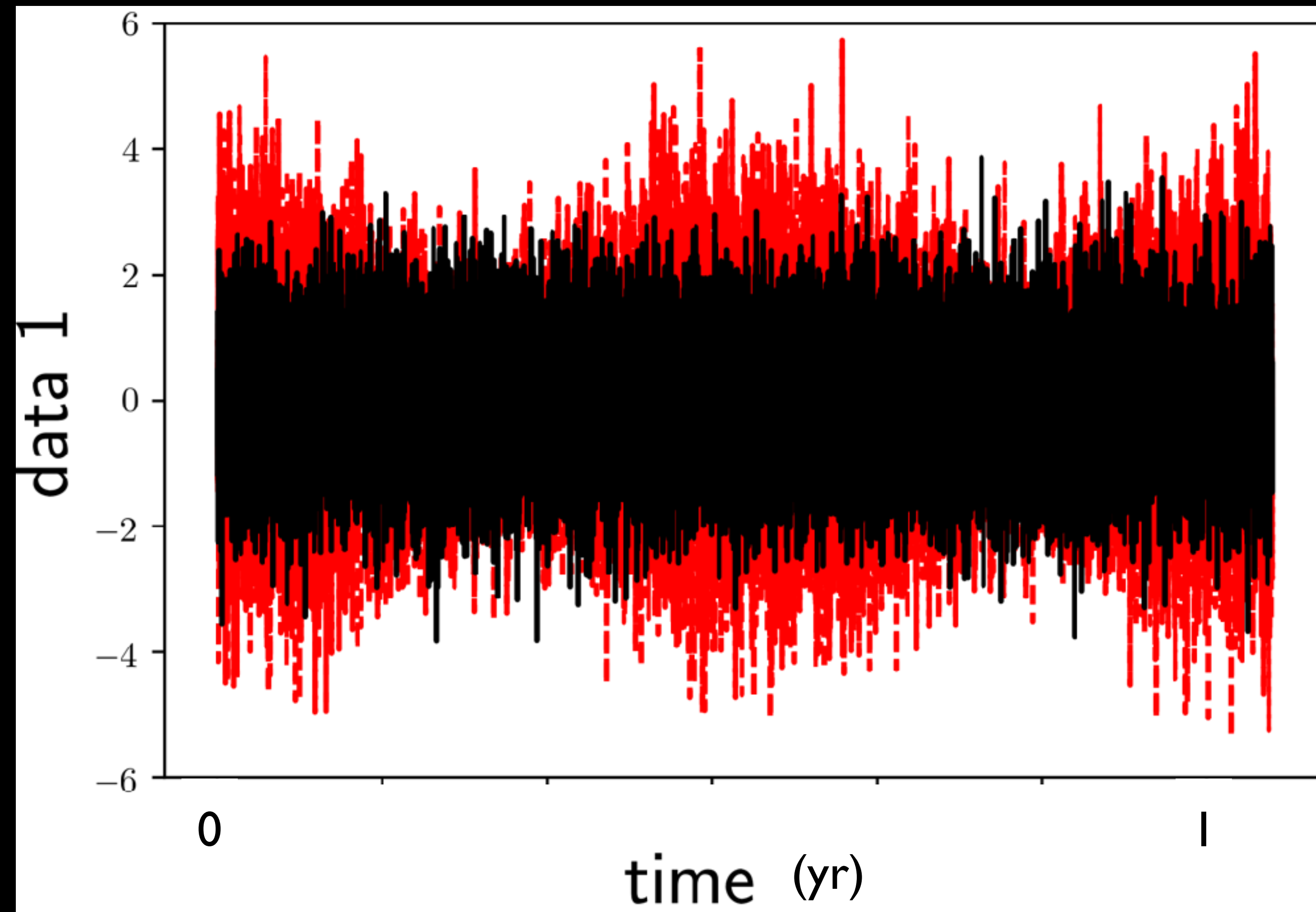


Non-stationary (non-gaussian)



TYPES OF STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

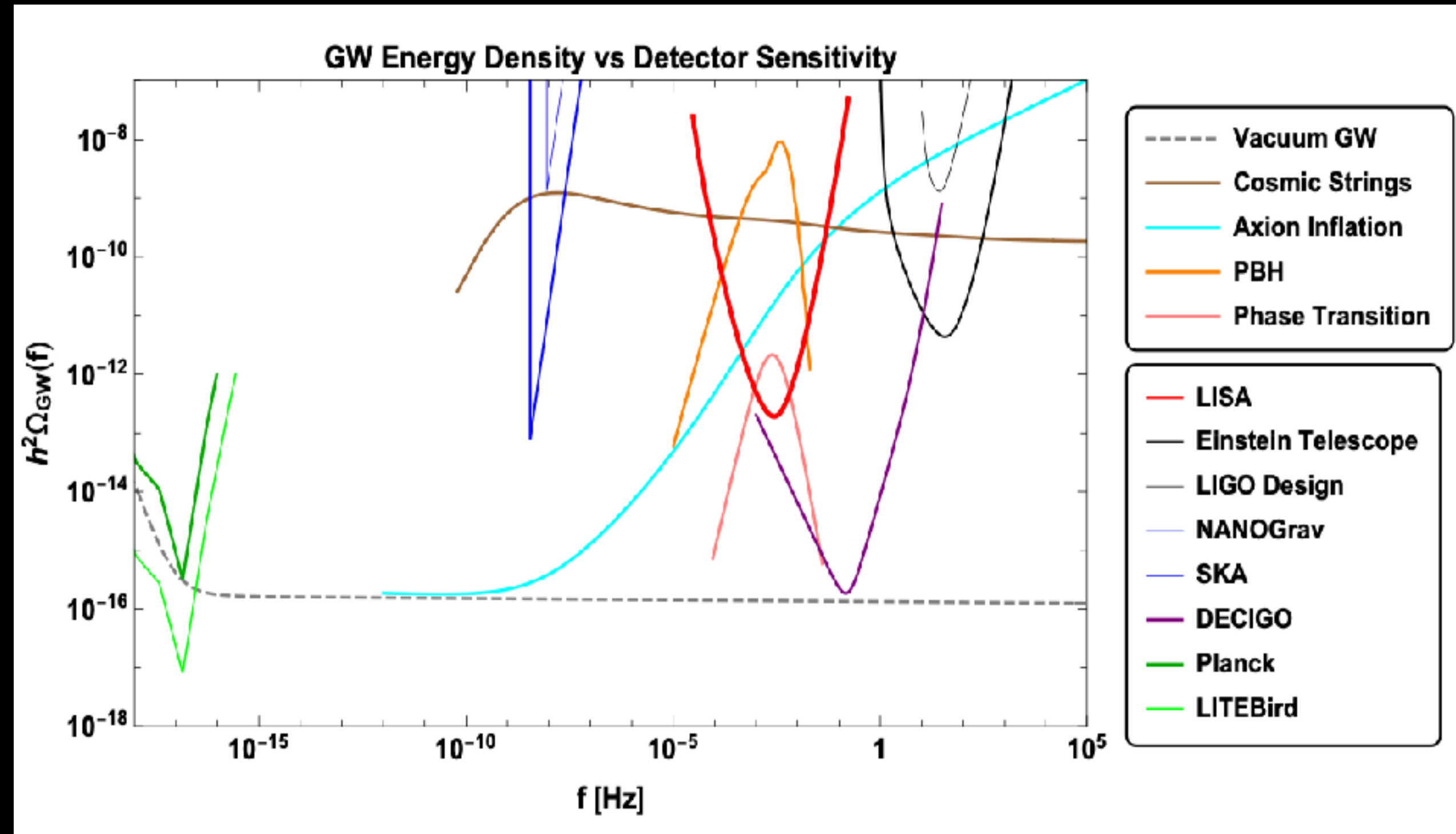
(2) Temporal distribution



(e.g., from galactic white dwarf binaries; modulated by LISA's orbital motion)

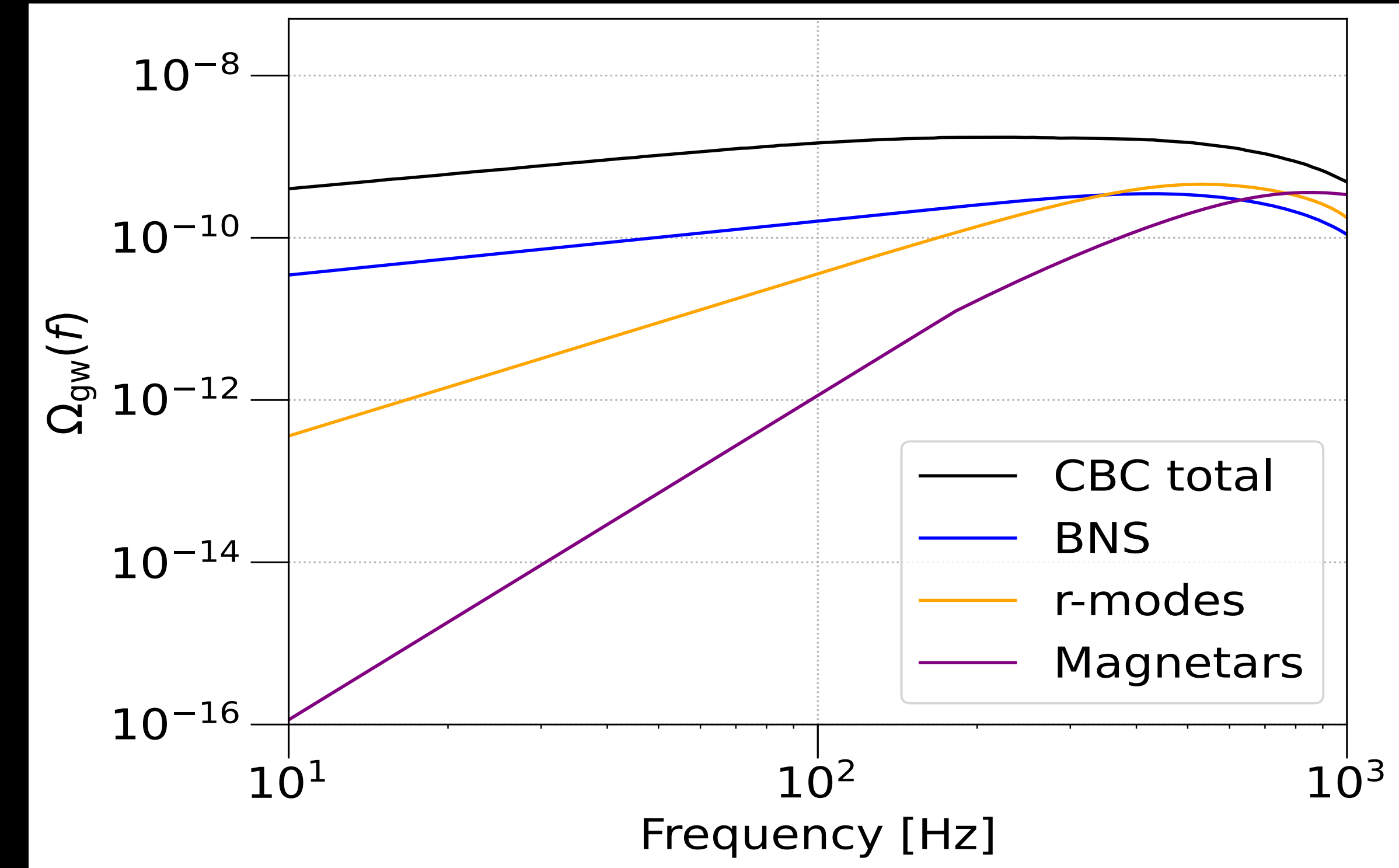
TYPES OF STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

(3) Different Power Spectra



Overview of several proposed model spectra for cosmic SGWB and different experiments' sensitivities.

[arXiv:2204.05434](https://arxiv.org/abs/2204.05434)



Taxonomy of astrophysical SGWB within the frequency range of ground-based detectors.

[F. DeLillo, JS](#) [arXiv:2310.05823](https://arxiv.org/abs/2310.05823)

TYPES OF STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

(4) Different Polarization

So far we have assumed that the gravitational-wave power in the $+$ and \times polarization modes are equal (on average) and are statistically independent of one another.



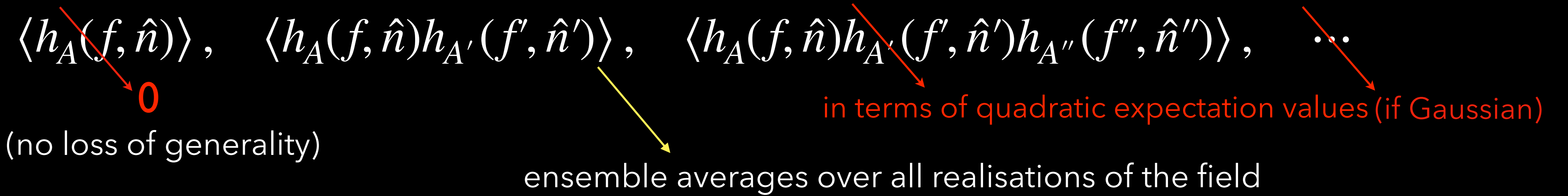
no correlations between the $+$ and \times polarization modes

some processes in the early Universe to give rise to parity violations, which would manifest themselves as an *asymmetry in the amount of right and left circularly polarized gravitational waves*

STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

Statistical properties are encoded in:

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0
(no loss of generality)

ensemble averages over all realisations of the field

in terms of quadratic expectation values (if Gaussian)

If $h_A(f, \hat{n})$ are mean-zero Gaussian fields, the signal is fully described by its second moment.

STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

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$$\langle h_A(f, \hat{n}) \rangle, \quad \langle h_A(f, \hat{n}) h_{A'}(f', \hat{n}') \rangle, \quad \langle h_A(f, \hat{n}) h_{A'}(f', \hat{n}') h_{A''}(f'', \hat{n}'') \rangle, \quad \dots$$

(no loss of generality)
ensemble averages over all realisations of the field
in terms of quadratic expectation values (if Gaussian)

If $h_A(f, \hat{n})$ are mean-zero Gaussian fields, the signal is fully described by its second moment.

Unpolarized, stationary, **isotropic**:

$$\langle h_A(f, \hat{n}) h_{A'}^*(f', \hat{n}') \rangle = \frac{1}{16\pi} S_h(f) \delta(f - f') \delta_{AA'} \delta^2(\hat{n}, \hat{n}')$$

Unpolarized, stationary, **anisotropic**:

$$\langle h_A(f, \hat{n}) h_{A'}^*(f', \hat{n}') \rangle = \frac{1}{4} \mathcal{P}(f, \hat{n}) \delta(f - f') \delta_{AA'} \delta^2(\hat{n}, \hat{n}')$$

where

$$S_h(f) = \frac{3H_0^2}{2\pi^2} \frac{\Omega_{\text{gw}}(f)}{f^3}$$

$$\Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}}{d \ln f} = \frac{f}{\rho_c} \frac{d\rho_{\text{gw}}}{df}$$

energy density spectrum (dimensionless)

ASTROPHYSICAL STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

$$\Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}}{d \ln f} = \frac{f}{\rho_c} \frac{d\rho_{\text{gw}}}{df}$$

$$\rho_{\text{gw}} = \frac{c^2}{32\pi G} \langle \dot{h}_{ab}(t, \vec{x}) \dot{h}^{ab}(t, \vec{x}) \rangle$$

For a collection of sources:

Phinney formula

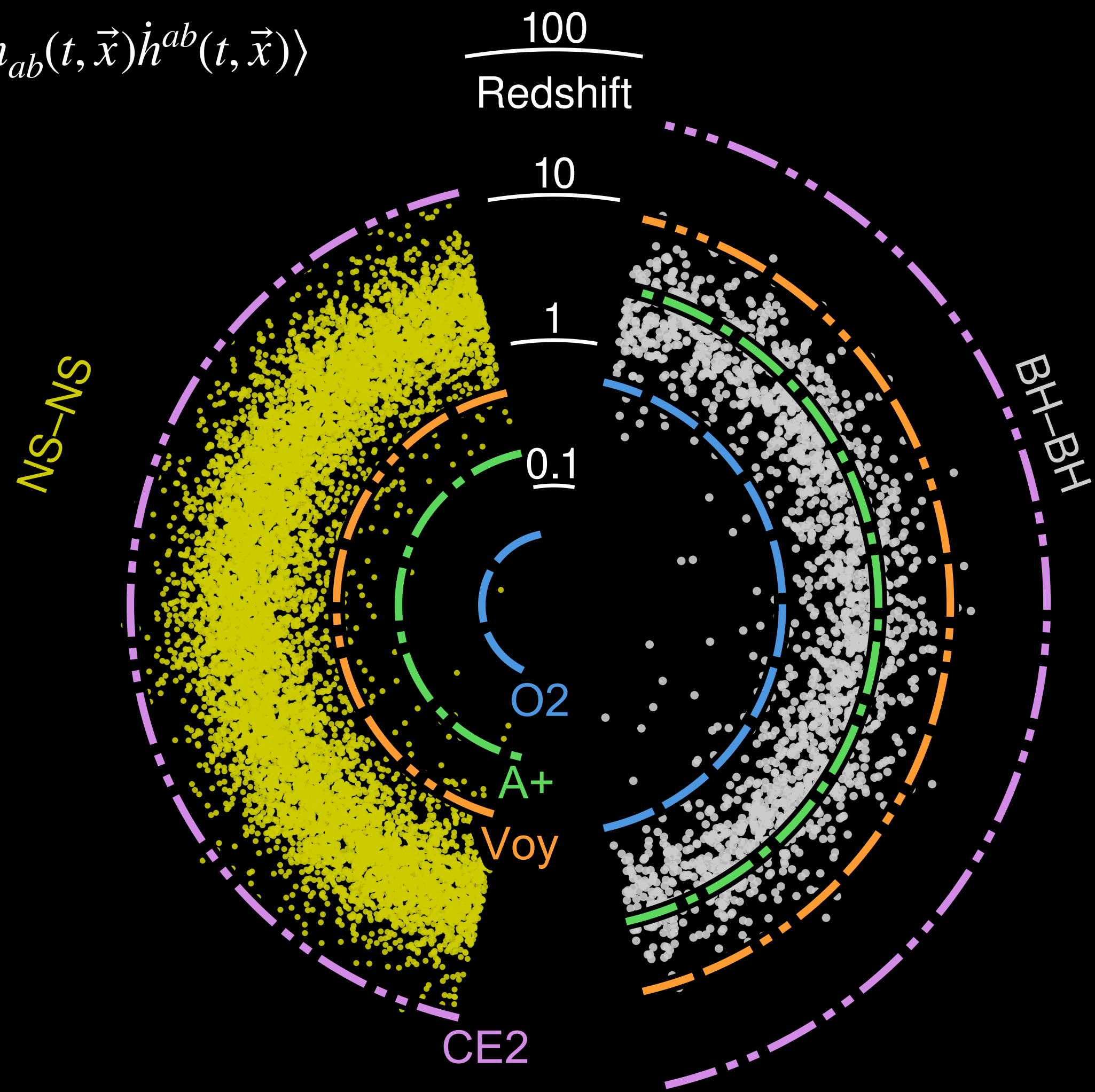
$$\Omega_{\text{gw}}(f) \propto \langle \text{GW energy per source} \rangle \times \langle \text{source rate} \rangle dt$$

$$\Omega_{\text{gw}}(f) = \frac{f}{\rho_c H_0} \int_0^\infty dz R(z) \frac{1}{(1+z)E(z)} \left(\frac{dE_{\text{gw}}}{df_s} \right) \Big|_{f_s=f(1+z)}$$

Event rate

(redshifted) energy radiated per event per source-frame frequency

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \rightarrow \text{cosmology}$$



E. Hall & S. Vitale, LIGO-G1900803

ASTROPHYSICAL STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

$$\Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}}{d \ln f} = \frac{f}{\rho_c} \frac{d\rho_{\text{gw}}}{df}$$

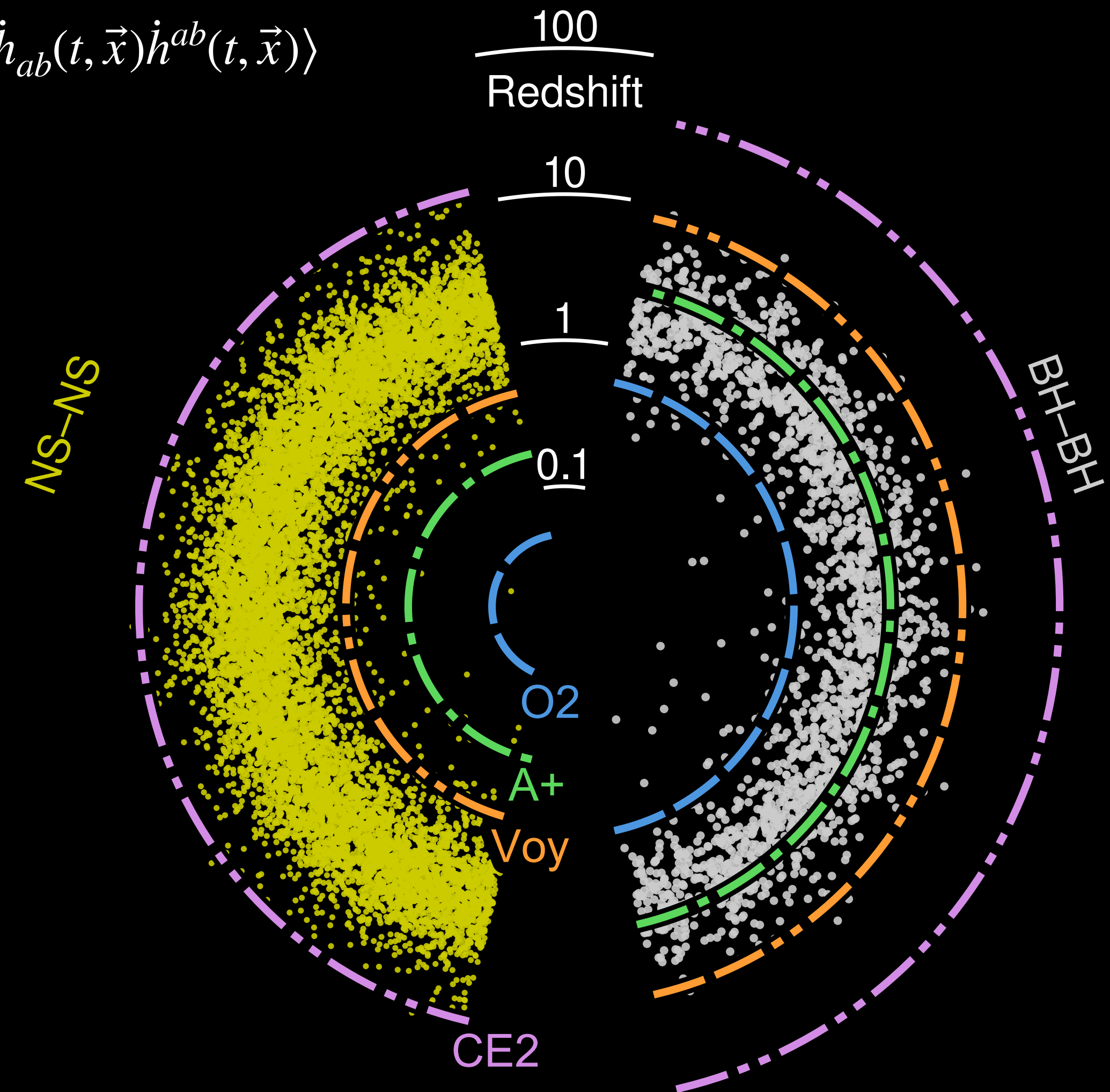
$$\rho_{\text{gw}} = \frac{c^2}{32\pi G} \langle \dot{h}_{ab}(t, \vec{x}) \dot{h}^{ab}(t, \vec{x}) \rangle$$

For a collection of sources:

Phinney formula

Compact Binaries: $\Omega_{\text{gw}}(f) \propto f^{2/3}$

Isolated Neutron star: $\Omega_{\text{gw}}(f) \propto f^4$



E. Hall & S. Vitale, LIGO-G1900803

WHAT DETECTION METHODS CAN WE USE?

The stochastic signal looks more like noise in a single detector.

What can be done:

- Identify features that distinguish between the expected signal and noise.
- Detectors with uncorrelated noise: cross-correlation separates the signal from the noise.

WHAT DETECTION METHODS CAN WE USE?

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Data from two detectors:

$$d_1 = h + n_1 \quad d_2 = h + n_2 \quad h \rightarrow \text{common GW signal component}$$

Cross-correlation:

$$\langle d_1 d_2 \rangle = \langle h^2 \rangle + \langle n_1 n_2 \rangle + \cancel{\langle h n_2 \rangle} + \cancel{\langle n_1 h \rangle} = \langle h^2 \rangle + \langle n_1 n_2 \rangle$$

0 0

Assuming detector noise is uncorrelated*:

$$\langle d_1 d_2 \rangle = \langle h^2 \rangle + \cancel{\langle n_1 n_2 \rangle}$$

0

$$\langle d_1 d_2 \rangle = \langle h^2 \rangle \equiv S_h$$

Cross-correlation separates the signal from the noise

Intensity of the background

DETECTOR RESPONSE

(Slide credit: J. Romano)

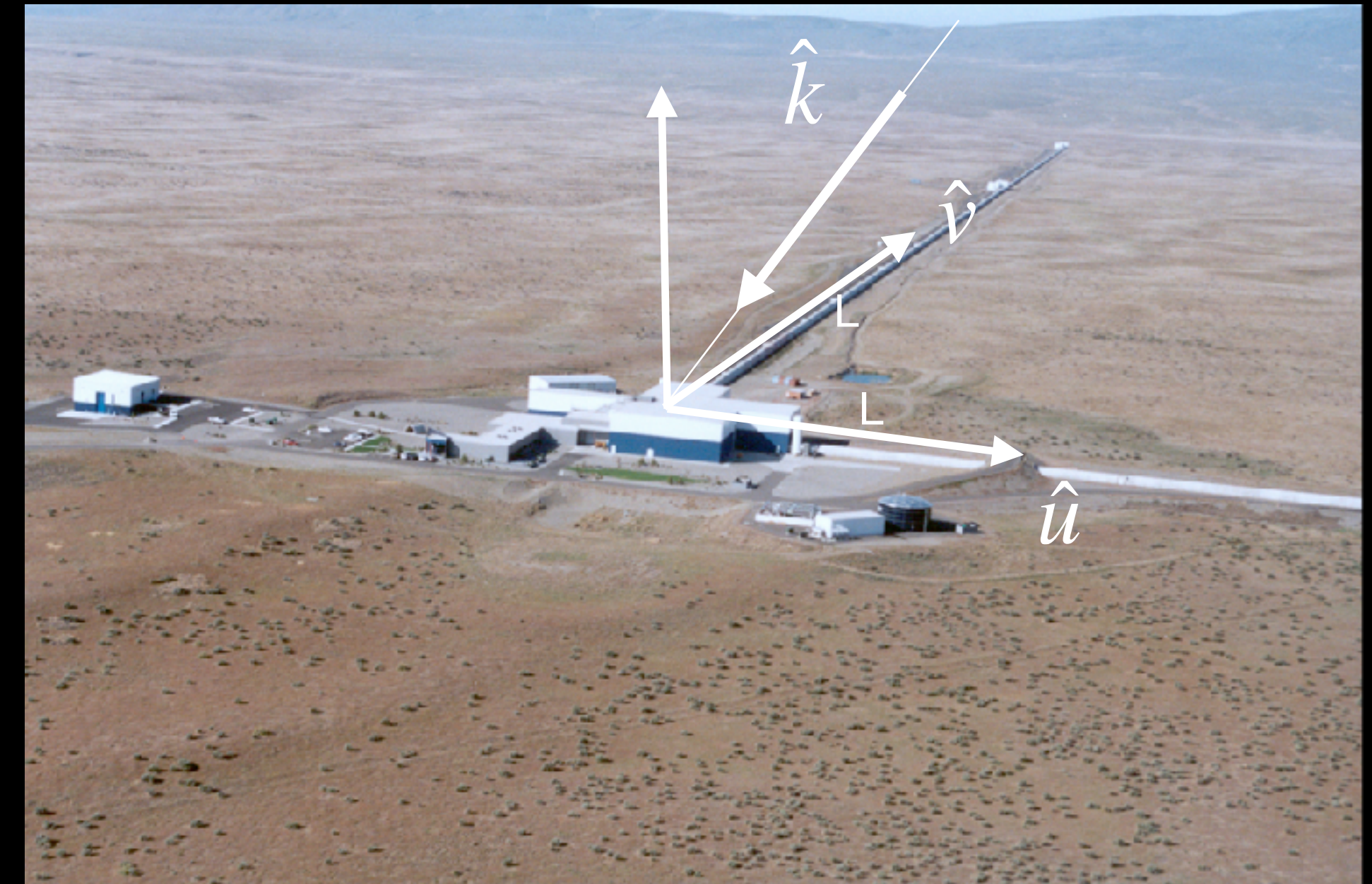
detector acts as a **linear system**, which converts metric perturbations to detector output

$$\begin{array}{c}
 \text{detector} \\
 h_{ab}(t, \vec{x}) \longrightarrow \boxed{R^{ab}(\tau, \vec{y})} \longrightarrow h(t)
 \end{array}$$

detector output

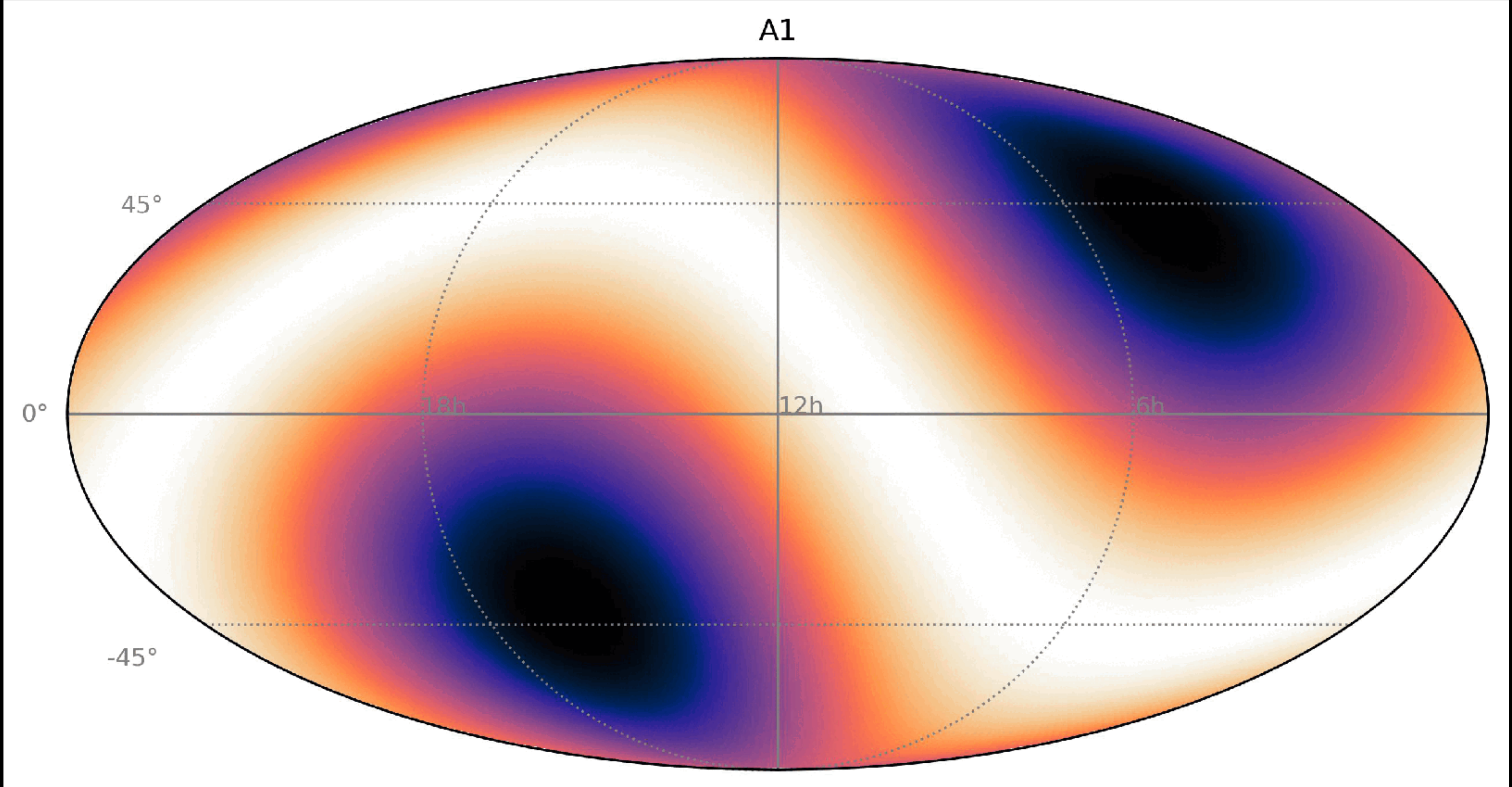
$$\tilde{h}(f) = \int d^2\Omega_{\hat{n}} \sum_A \boxed{R^A(f, \hat{n})} h_A(f, \hat{n})$$

detector response for a plane-wave
with frequency f , direction \hat{n} , polarization A



$$R^A(f, \hat{n}) \simeq \frac{1}{2} (u^a u^b - v^a v^b) e_{ab}^A(\hat{n})$$

BEAM PATTERN FUNCTIONS



OVERLAP REDUCTION FUNCTION

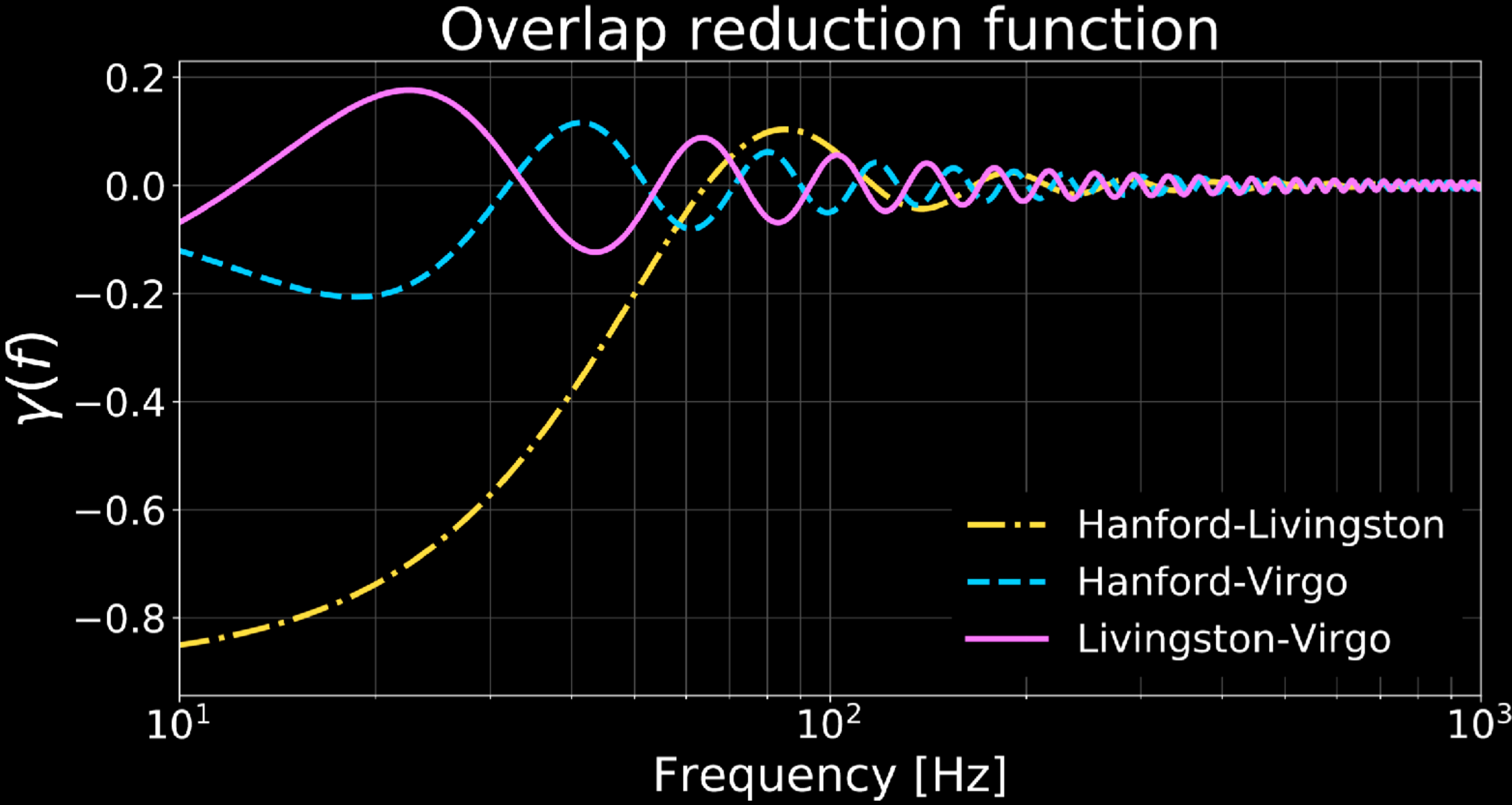
Detectors in **different locations** and with **different orientations** respond differently to a passing GW.

Overlap function encodes reduction in sensitivity of a cross-correlation analysis due to separation and misalignment of the detectors.



$$\gamma_{ft,p}^{\mathcal{J}} = \sum_A F_{\mathcal{J}_1}^A(\hat{\mathbf{n}}_p, t) F_{\mathcal{J}_2}^A(\hat{\mathbf{n}}_p, t) e^{2\pi i f \hat{\mathbf{n}}_p \cdot \Delta \mathbf{x}_{\mathcal{J}}(t)/c}$$

OVERLAP REDUCTION FUNCTION



OPTIMAL FILTERING

What is the optimal way to correlate data from two physically separated and misaligned detectors to search for a SGWB

Cross-correlation estimator $\hat{S}_h \simeq \int_{-\infty}^{\infty} df \int_{-\infty}^{\infty} df' \delta_T(f-f') \tilde{d}_1(f) \tilde{d}_2^*(f') \tilde{Q}^*(f')$

Variance $\sigma^2 \simeq \frac{T}{2} \int_0^{\infty} df P_1(f) P_2(f) |\tilde{Q}(f)|^2$

What we meant by optimal: Choose Q to maximize SNR for fixed spectral shape

$\tilde{Q}(f) \propto \frac{\Gamma_{12}(f) \Omega_t(f)}{P_1(f) P_2(f)}$

Overlap reduction function \rightarrow (points to $\tilde{Q}(f)$)

expected signal spectrum \rightarrow (points to $\Omega_t(f)$)

de-weight correlation when noise is large \rightarrow (points to $P_1(f) P_2(f)$)

OPTIMAL FILTERING

What we meant by optimal: Choose Q to maximize SNR for fixed spectral shape

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Overlap reduction function

expected
signal spectrum

de-weight correlation
when noise is large

We often choose a power-law
functional form for the SGWB template
spectrum

$$\Omega_t(f) = \Omega_{\text{ref}} \left(\frac{f}{f_{\text{ref}}} \right)^\alpha$$

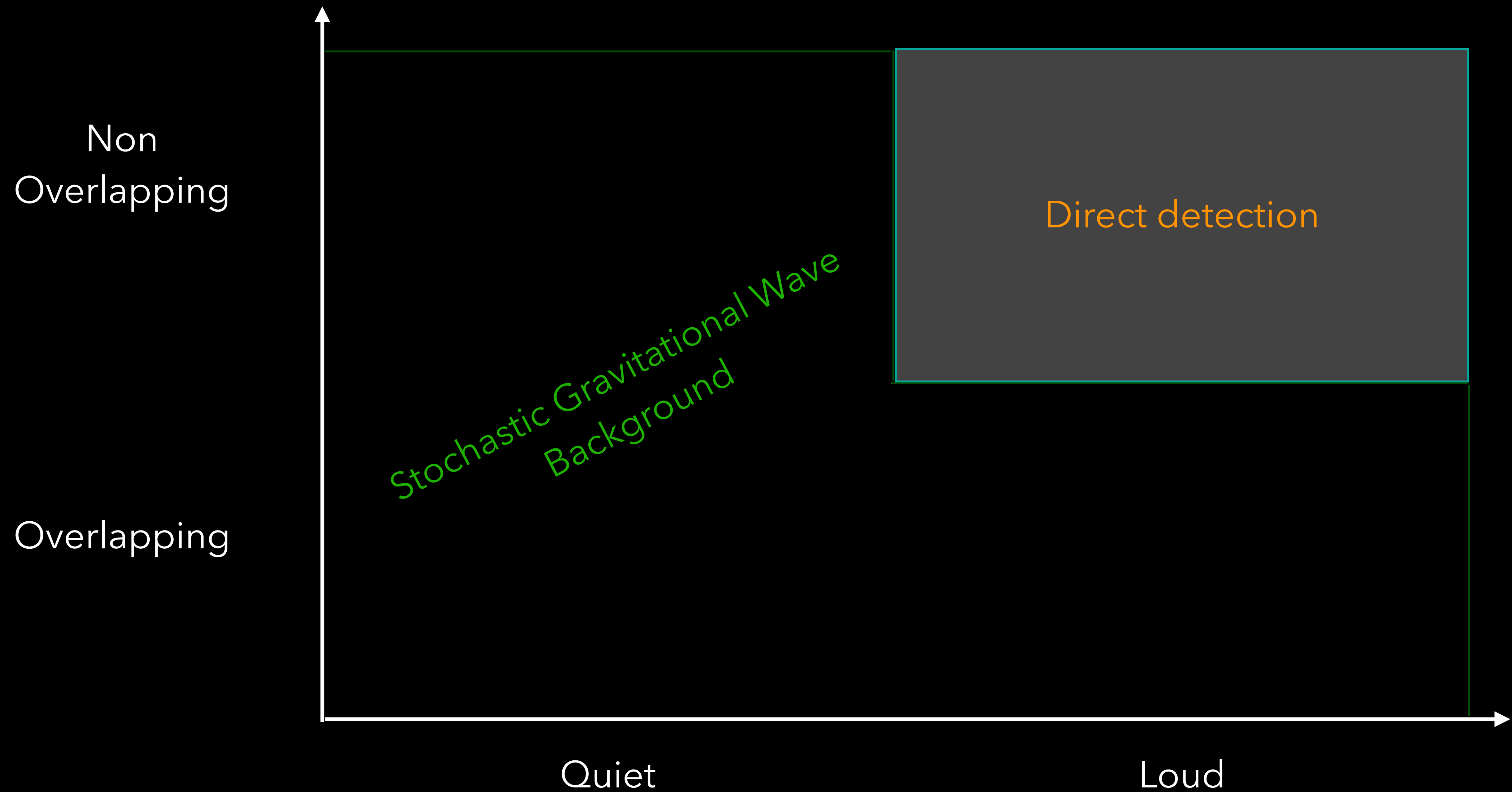
SGWB SEARCHES IN O3

- No detections were made –
 - Used ~200 days of data from 2019-2020
- Set limits on Ω_α for a few values of α .
 - $\alpha = 2/3 \rightarrow$ Unresolved CBCs
 - $\alpha = 3 \rightarrow$ Flat in energy density
 - $\alpha = 0 \rightarrow$ Flat in GW power
- Combine with binary black hole detections to set limits on merger rate vs. redshift
- SGWB searches are useful for constraining high redshifts
- New limits are, in general, 6-12 times better than previous ones.

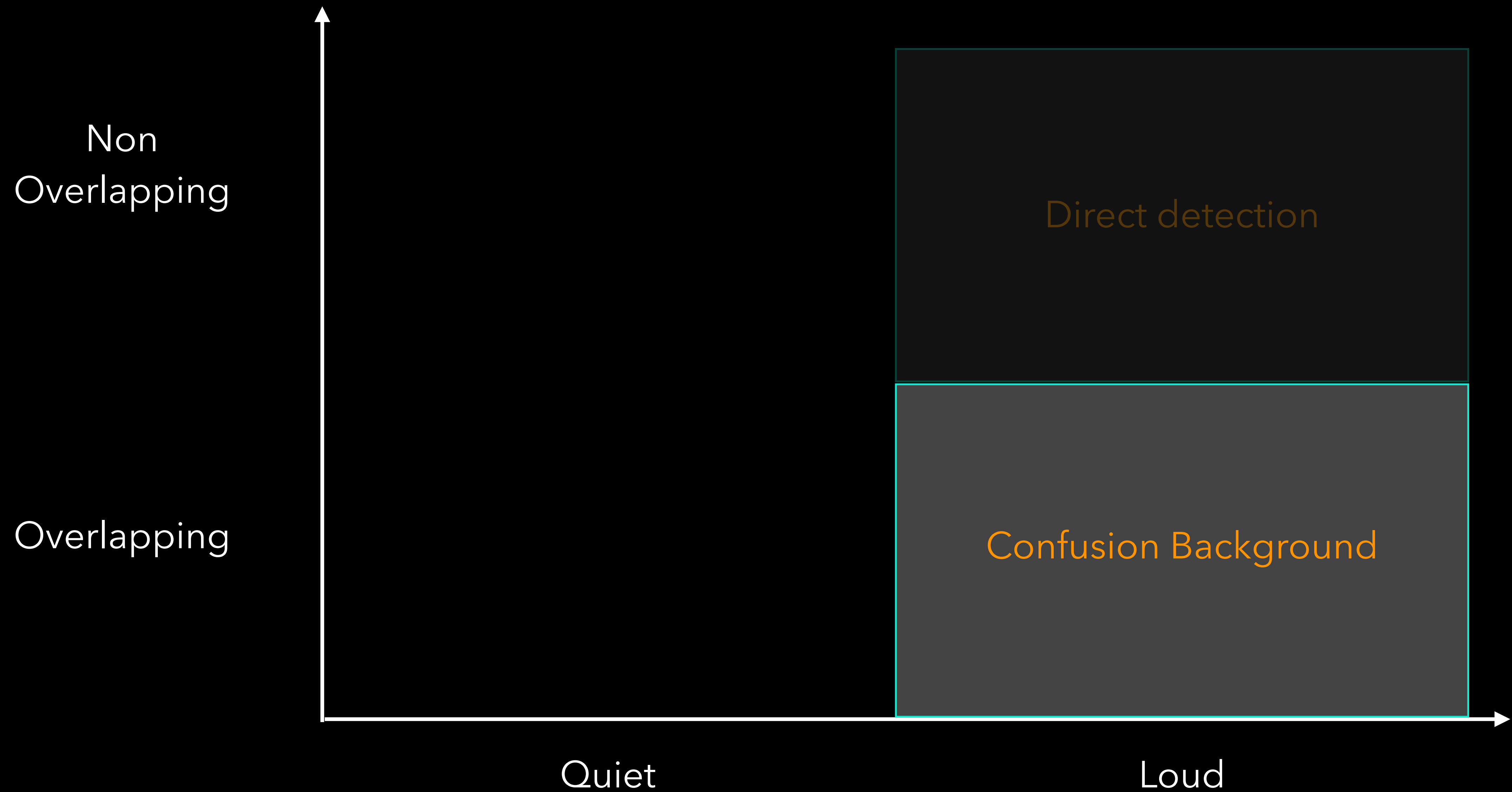
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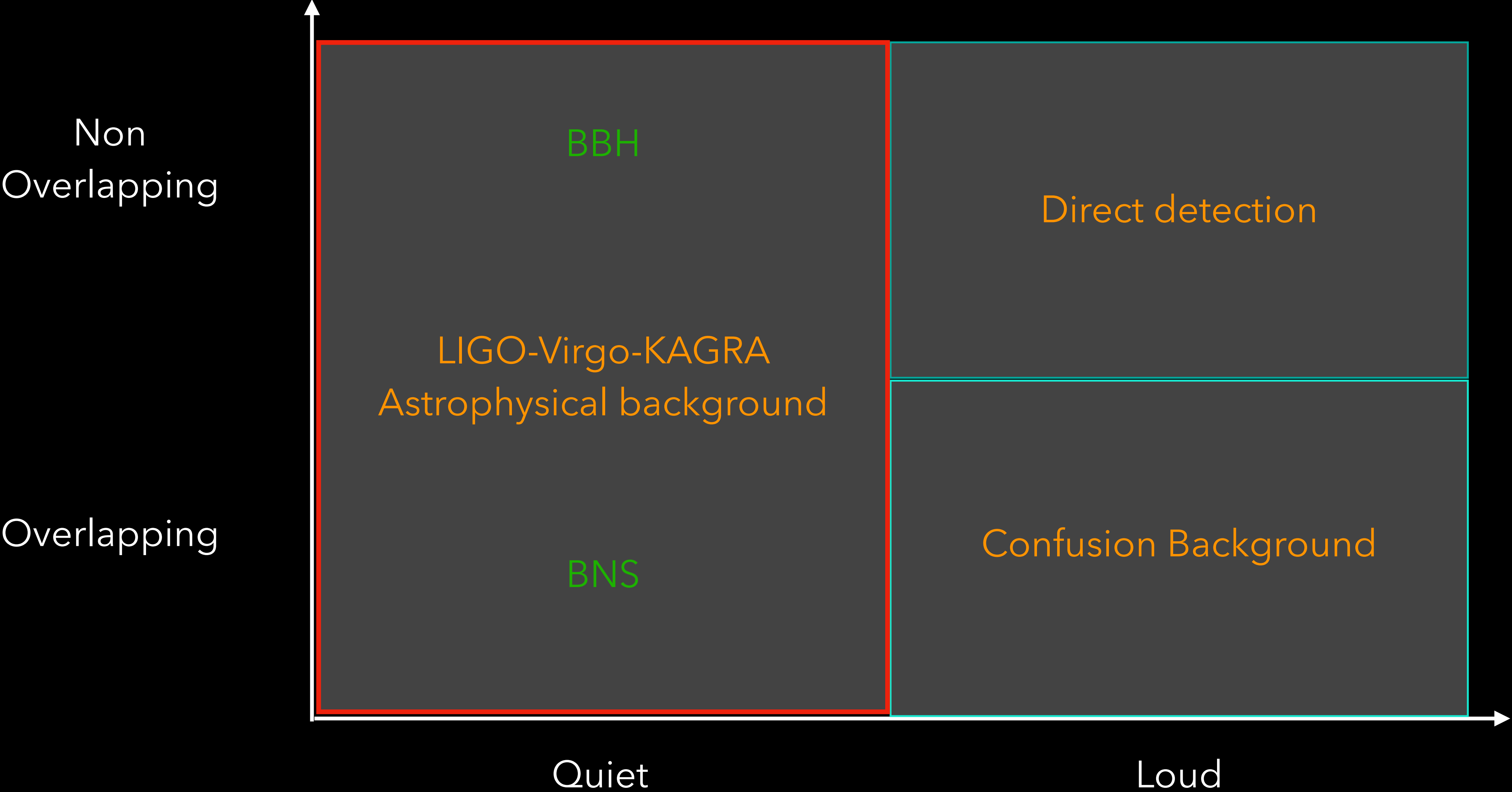
ASTROPHYSICAL STOCHASTIC GRAVITATIONAL WAVE BACKGROUND

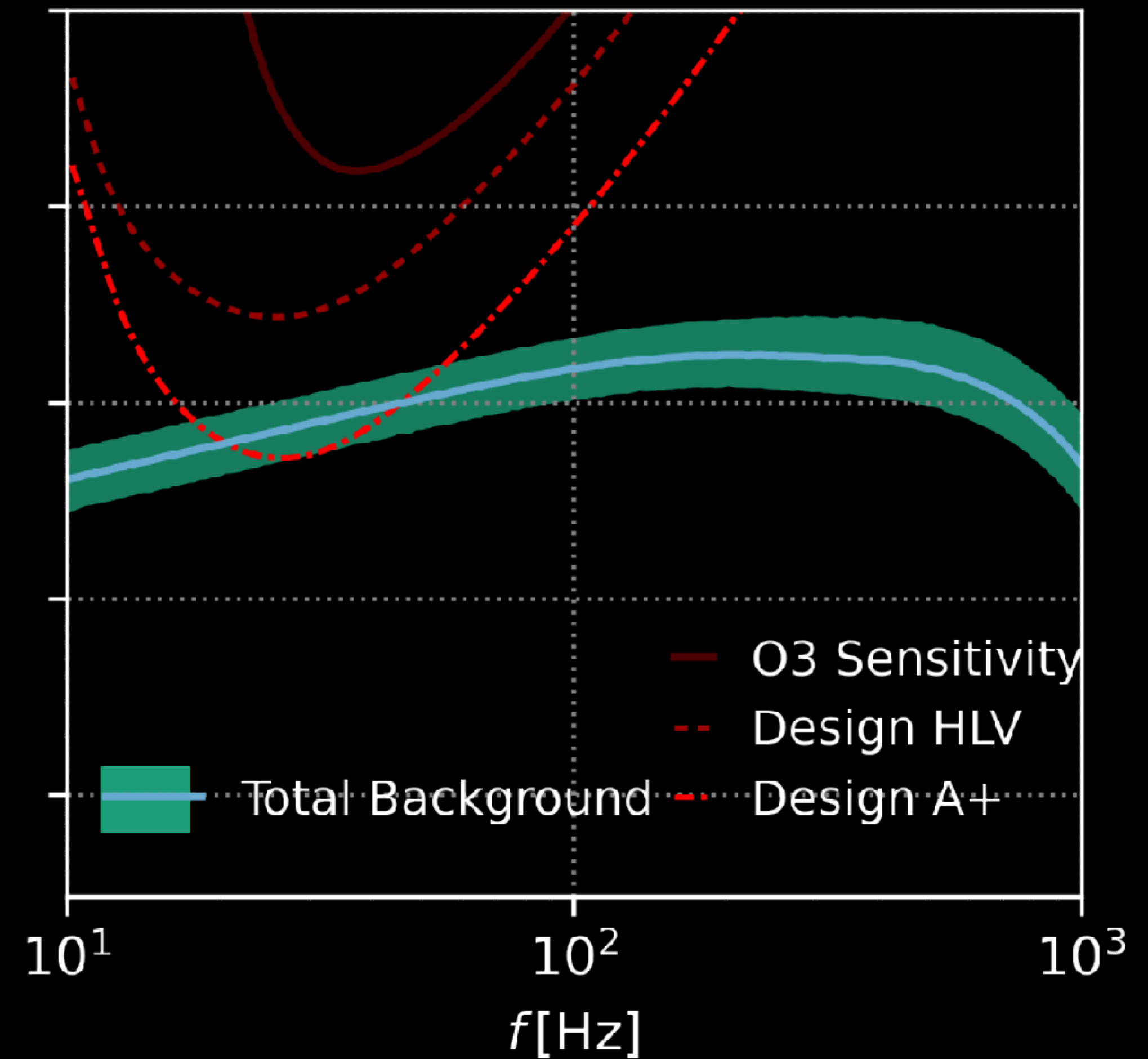
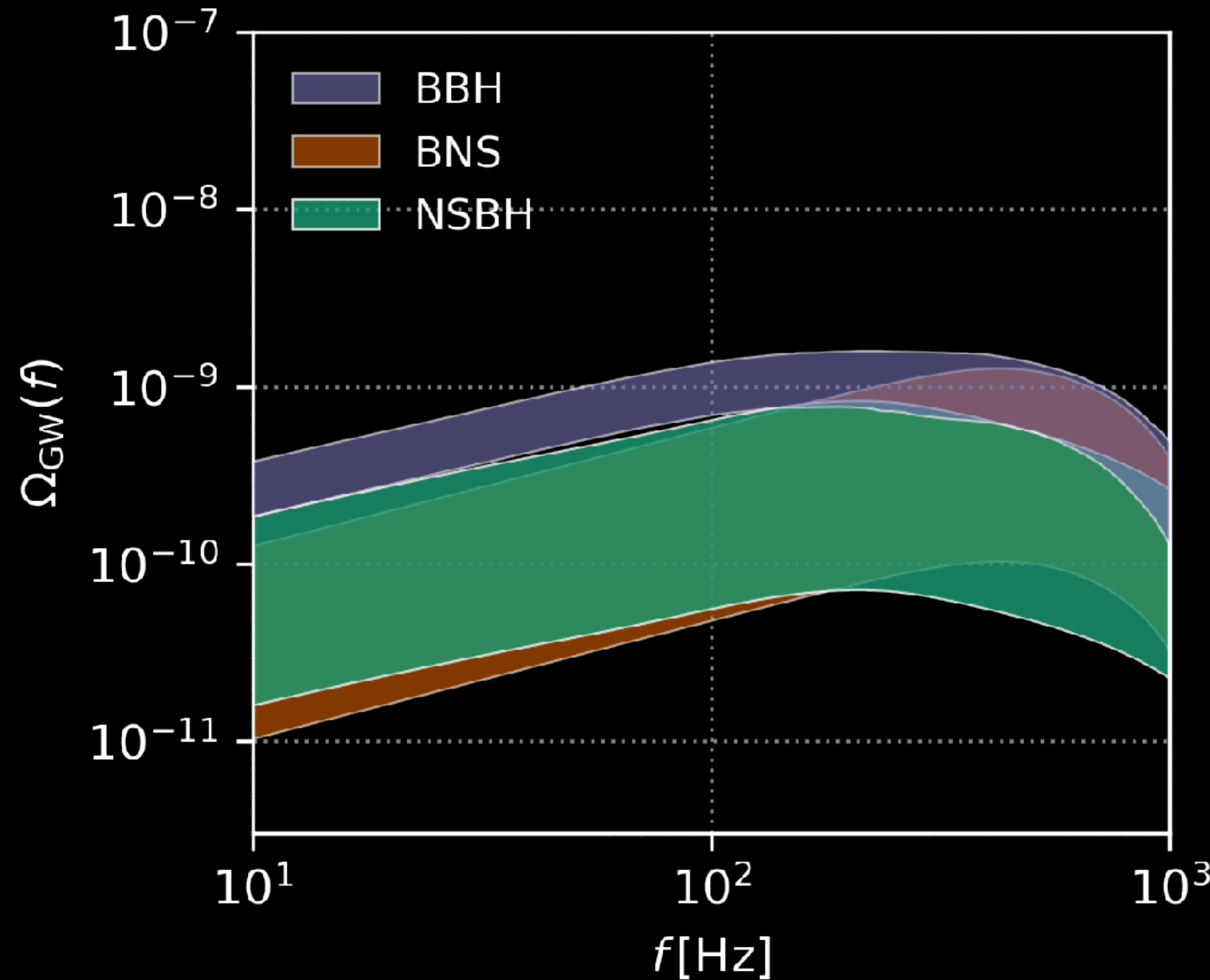


ASTROPHYSICAL STOCHASTIC GRAVITATIONAL WAVE BACKGROUND



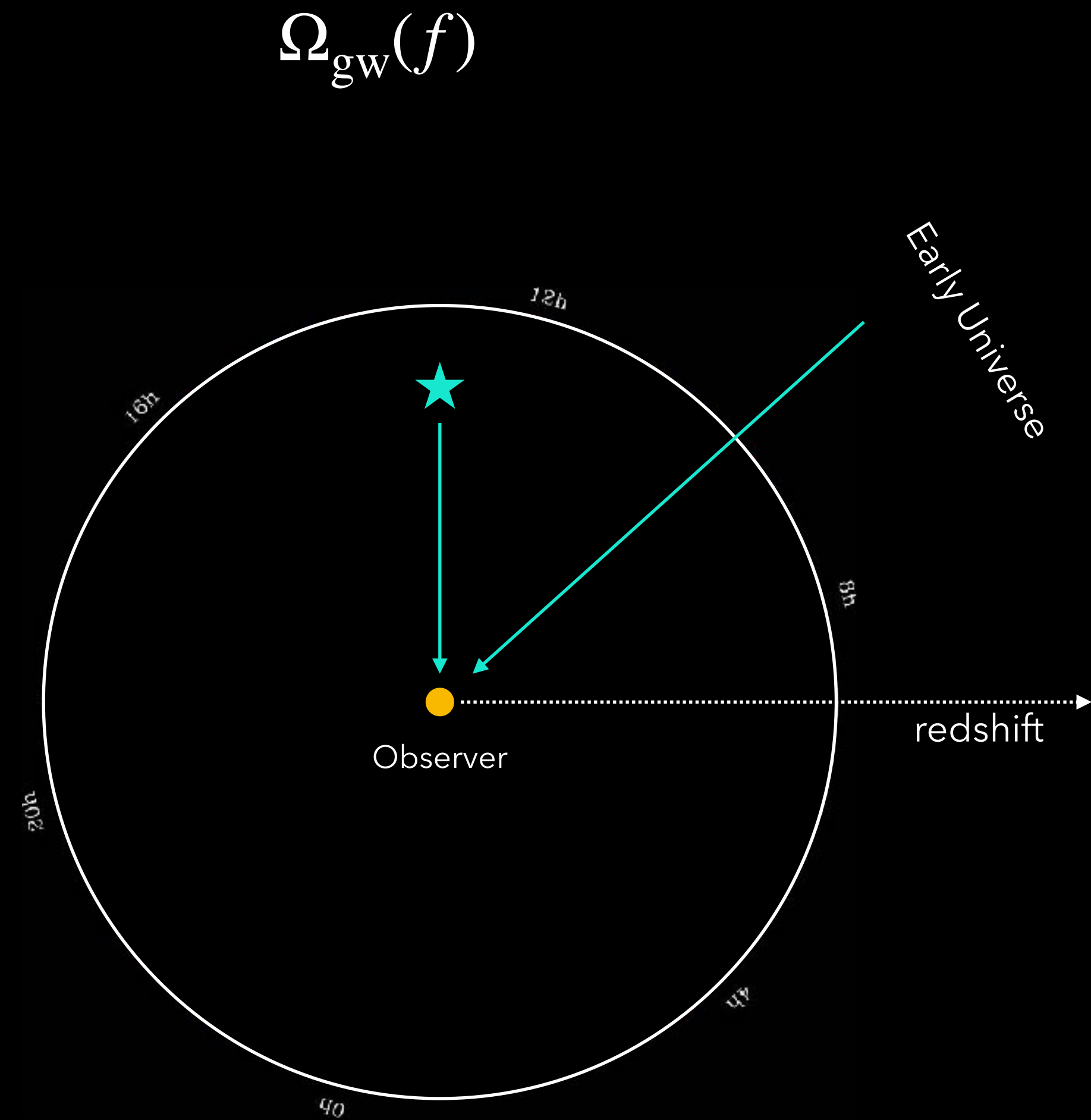
ASTROPHYSICAL STOCHASTIC GRAVITATIONAL WAVE BACKGROUND





The individual contributions expected from the collection of BNS, NSBH, and BBH mergers. While uncertainties on the energy density due to BNS and NSBH are due to Poisson uncertainties in their merger rates, our forecast for the SGWB due to BBHs includes systematic uncertainties associated with their imperfectly known mass distribution. (Right): Estimate of the total gravitational-wave background (green), as well as our current experimental sensitivity (red)

ANISOTROPIC SEARCH



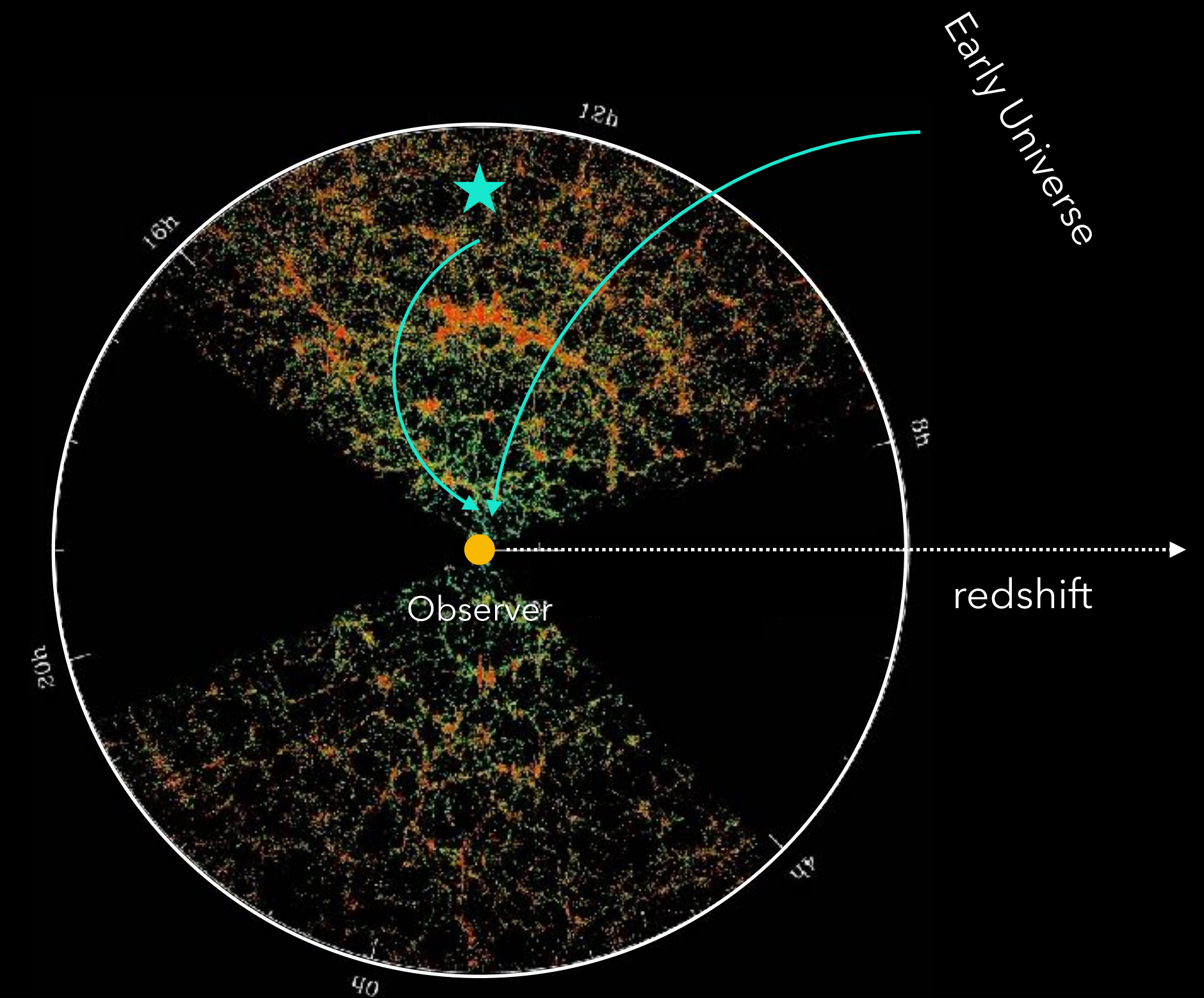
- sources isotropically distributed
- propagation along straight line
- no anisotropy in received flux

ANISOTROPIC SEARCH

A more realistic description

$$\Omega_{\text{gw}}(f, \hat{n})$$

- Anisotropic distribution of the emitting sources.
- Due to propagation: as gravitational-wave propagate, they accumulate line-of-sight effects, crossing different matter density fields which are inhomogeneously distributed in the Universe.



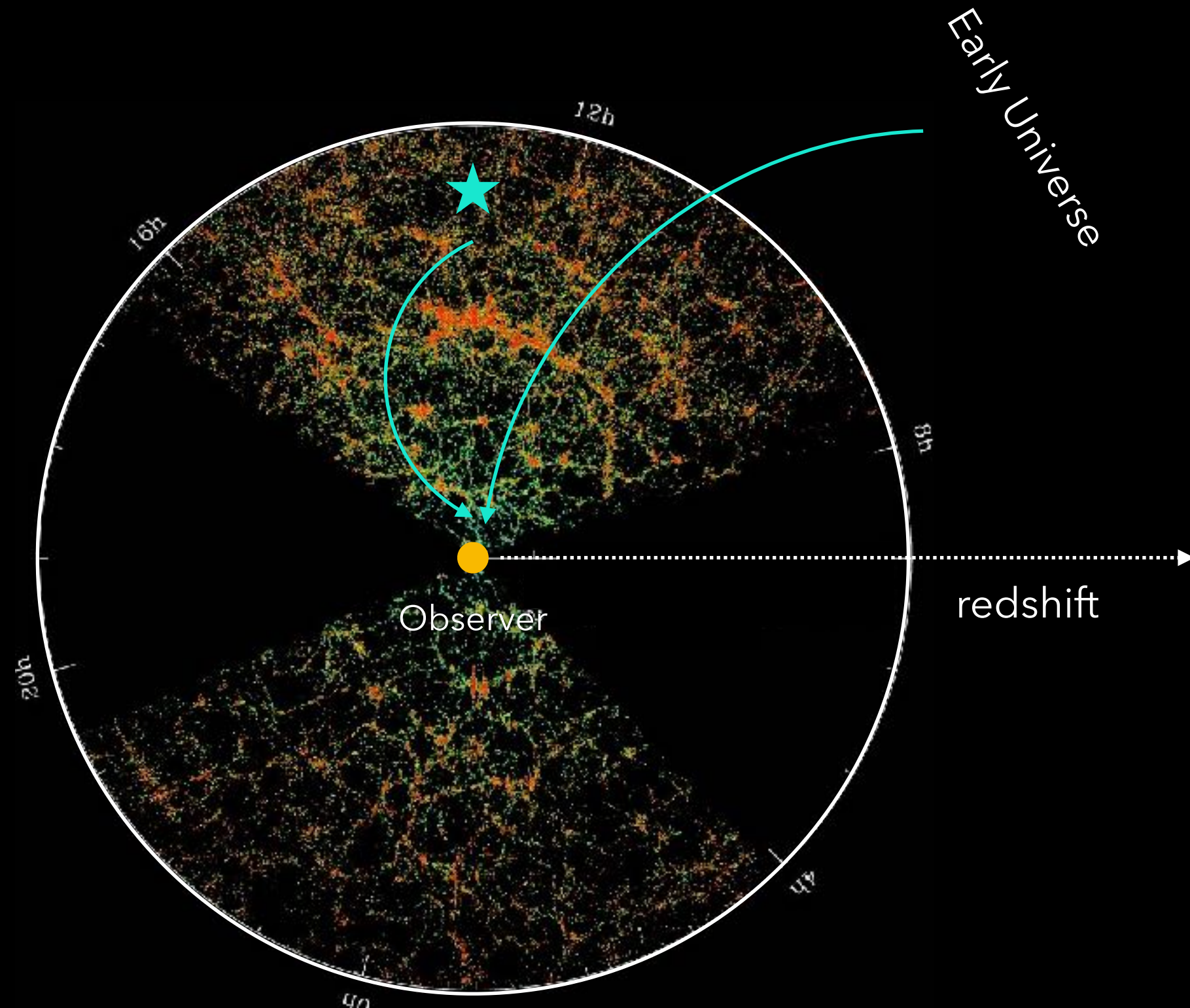
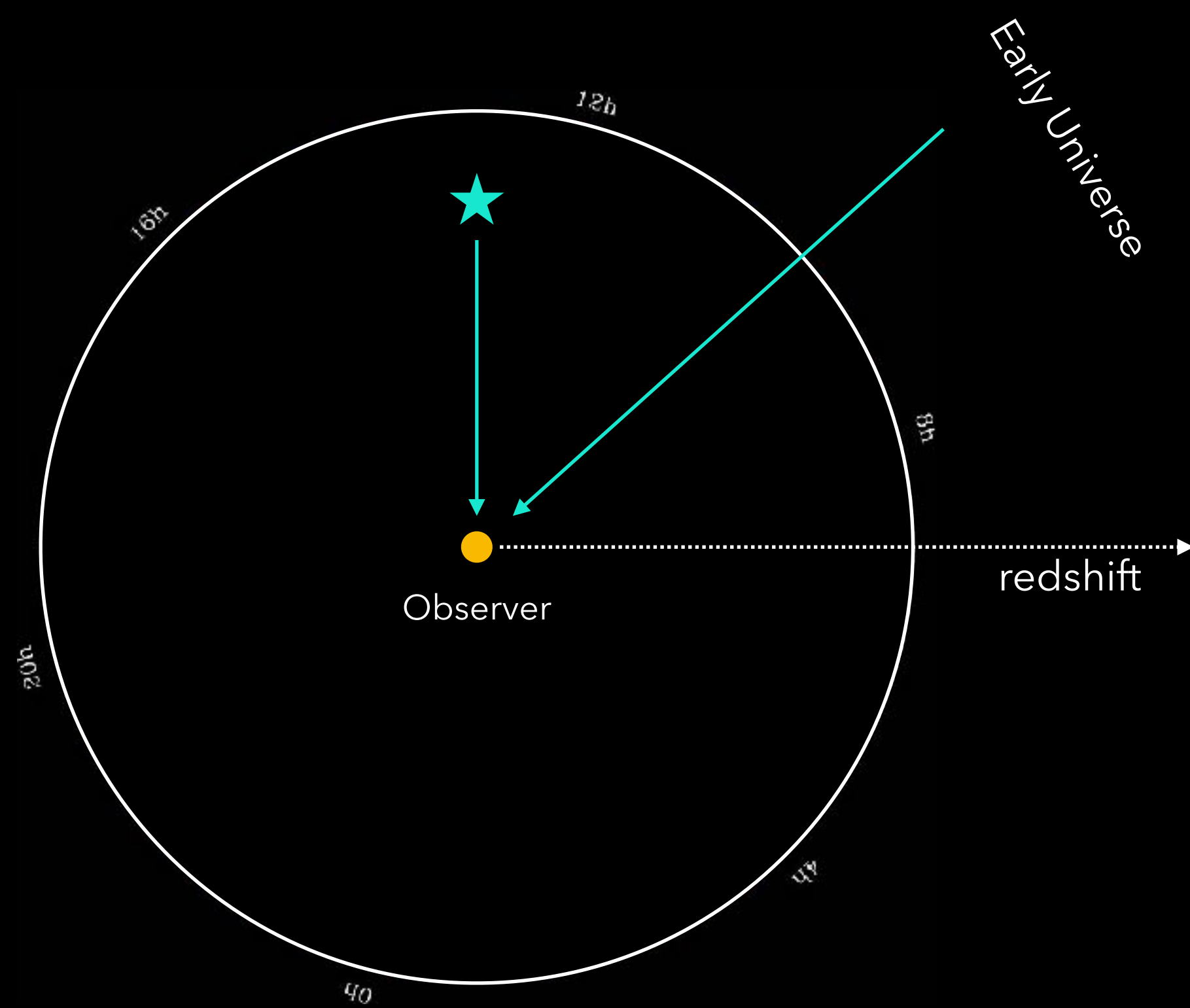
ANISOTROPIC SEARCH

A more realistic description

$$\Omega_{\text{gw}}(f)$$



$$\Omega_{\text{gw}}(f, \hat{n})$$



ANISOTROPIC SEARCH

Anisotropic search tries to measure the direction of the sky from where the signal comes.
In this mapping process, we consider:

- The time delay between two detectors
- Rotation of the earth.

SGWB energy density

$$\Omega_{\text{gw}}(f, \hat{\mathbf{n}}) \equiv \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df} = \frac{2\pi^2}{3H_0^2} f^3 \mathcal{P}(f, \hat{\mathbf{n}})$$

Cross-correlation is essentially a one-dimensional map of the sky.

Anisotropy can be expanded in pixel or spherical harmonic basis

$$\mathcal{P}(f, \hat{\mathbf{n}}) = \sum_p \mathcal{P}_p(f) e_p(\hat{\mathbf{n}})$$

HOW DO WE MAP THE SGWB SKY?

The anisotropy of the SGWB can be characterized using the dimensional energy density parameter

$$\Omega_{\text{gw}}(f, \hat{\mathbf{n}}) \equiv \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df} = \frac{2\pi^2}{3H_0^2} f^3 \mathcal{P}(f, \hat{\mathbf{n}})$$

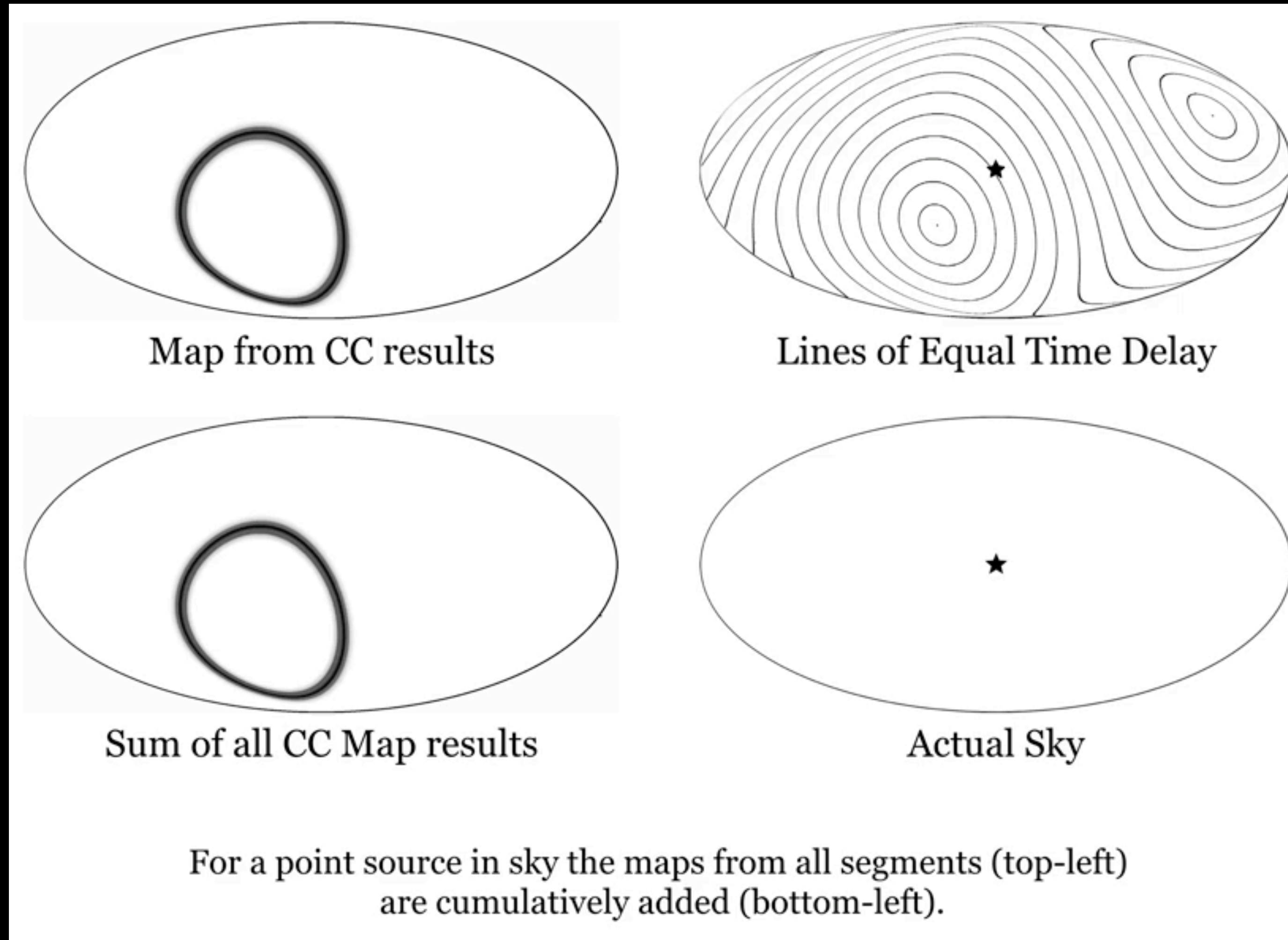
Most of the analysis performed so far assumes that the frequency and direction dependence can be separated: $\mathcal{P}(f, \hat{\mathbf{n}}) = P(\hat{\mathbf{n}}) H(f)$

Where the common choice spectral shape is $H(f) = \left(\frac{f}{f_{\text{ref}}}\right)^\beta$

We will perform a model-independent search

ANISOTROPIC SEARCH

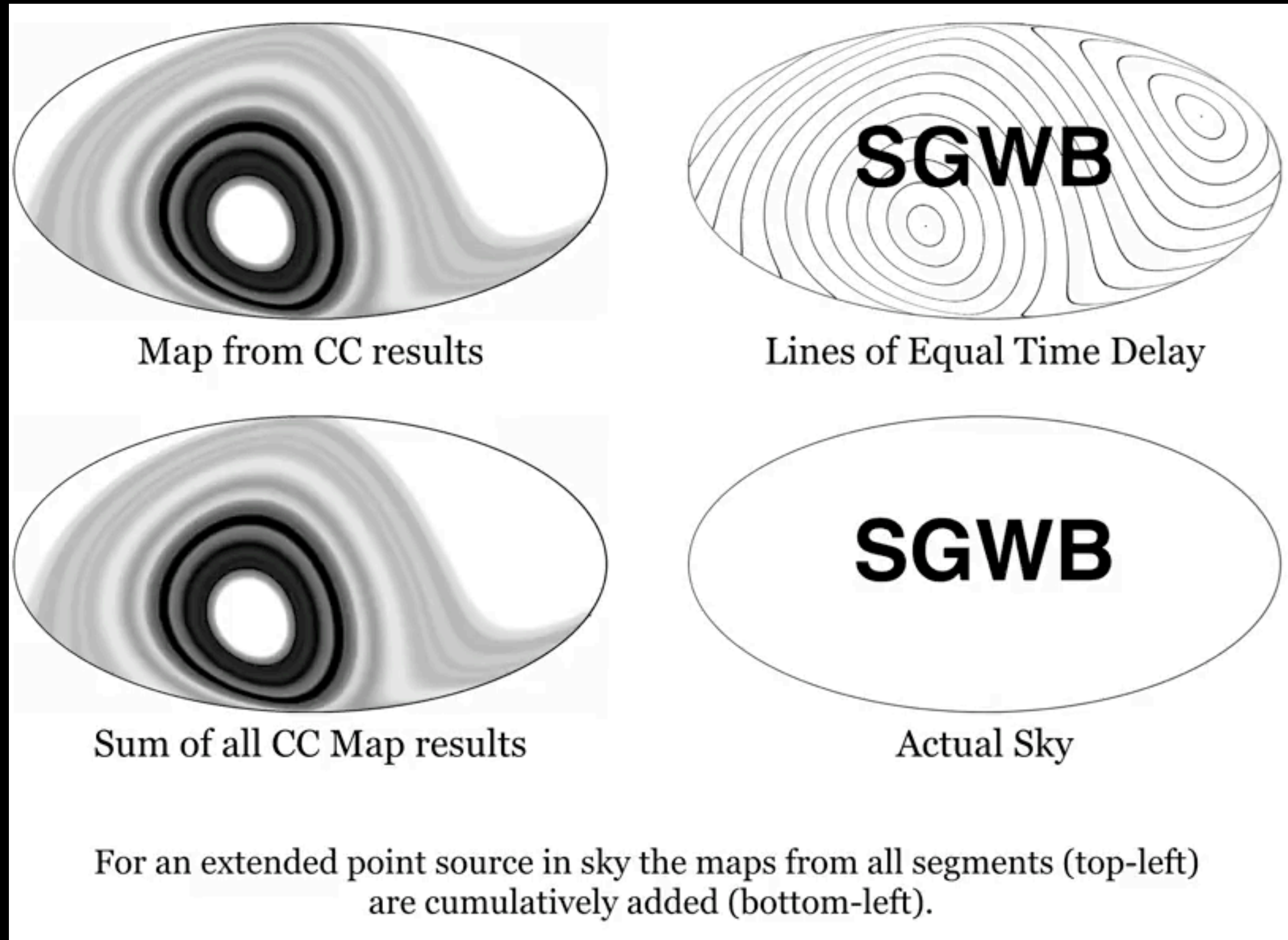
Cross-correlation is essentially a one-dimensional map of the sky.



Animation Credit: A. Ain

ANISOTROPIC SEARCH

Cross-correlation is essentially a one-dimensional map of the sky.



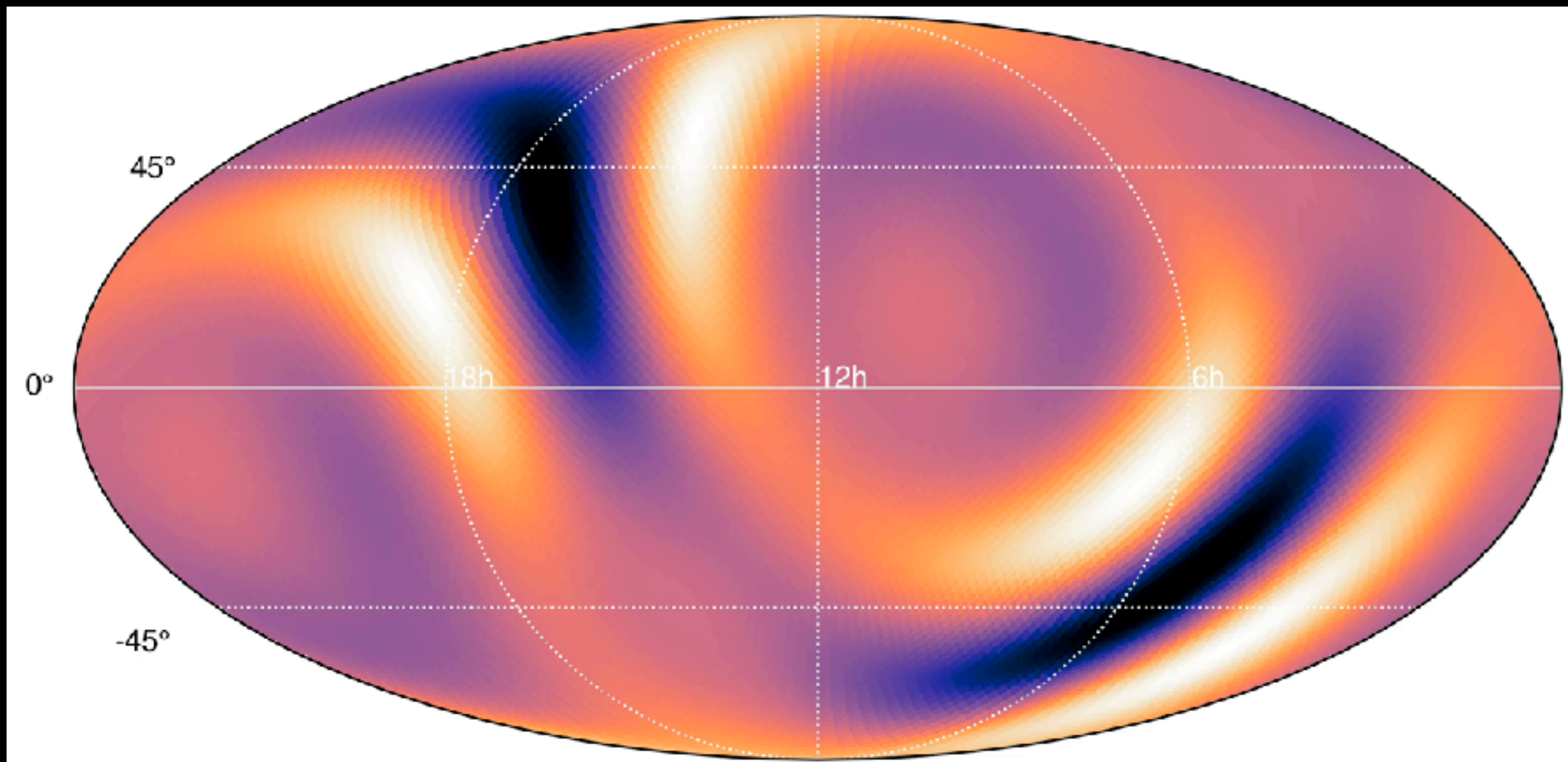
Animation Credit: A. Ain

DIRECTIONAL OVERLAP REDUCTION FUNCTION

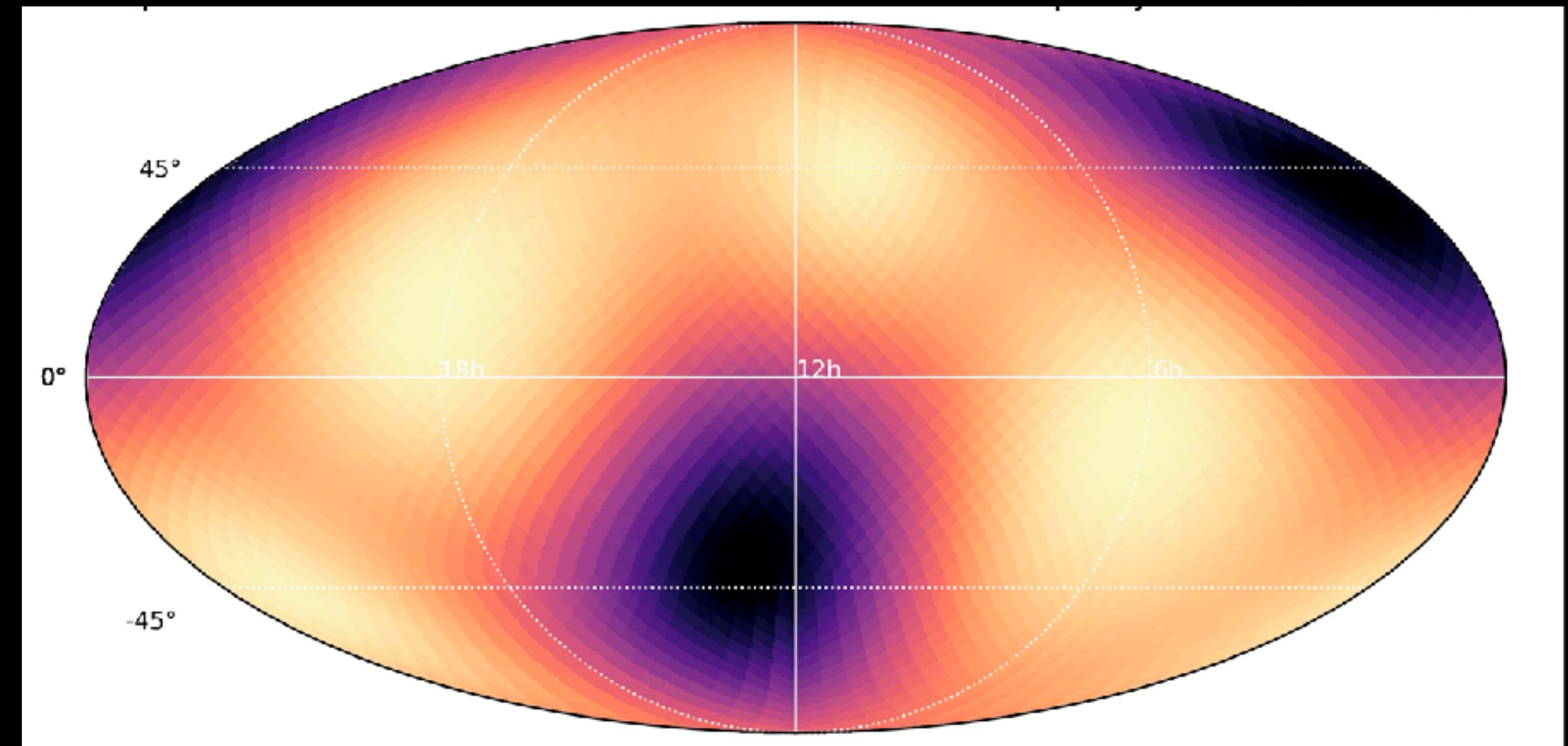
Recall:

The overlap function encodes the reduction in sensitivity of a cross-correlation analysis due to separation and misalignment of the detectors.

$$\gamma_{ft,n} \equiv \sum_A F_1^A(\hat{n}, t) F_2^A(\hat{n}, t) e^{2\pi i f \hat{n} \cdot \Delta x(t) / c}$$



$$\gamma_{t,n} \Big|_{f=100\text{Hz}}$$

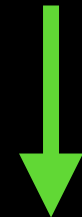


$$\gamma_{f,n} \Big|_{t=1326542418}$$

PyStoch : fast HEALPix based SGWB mapmaking

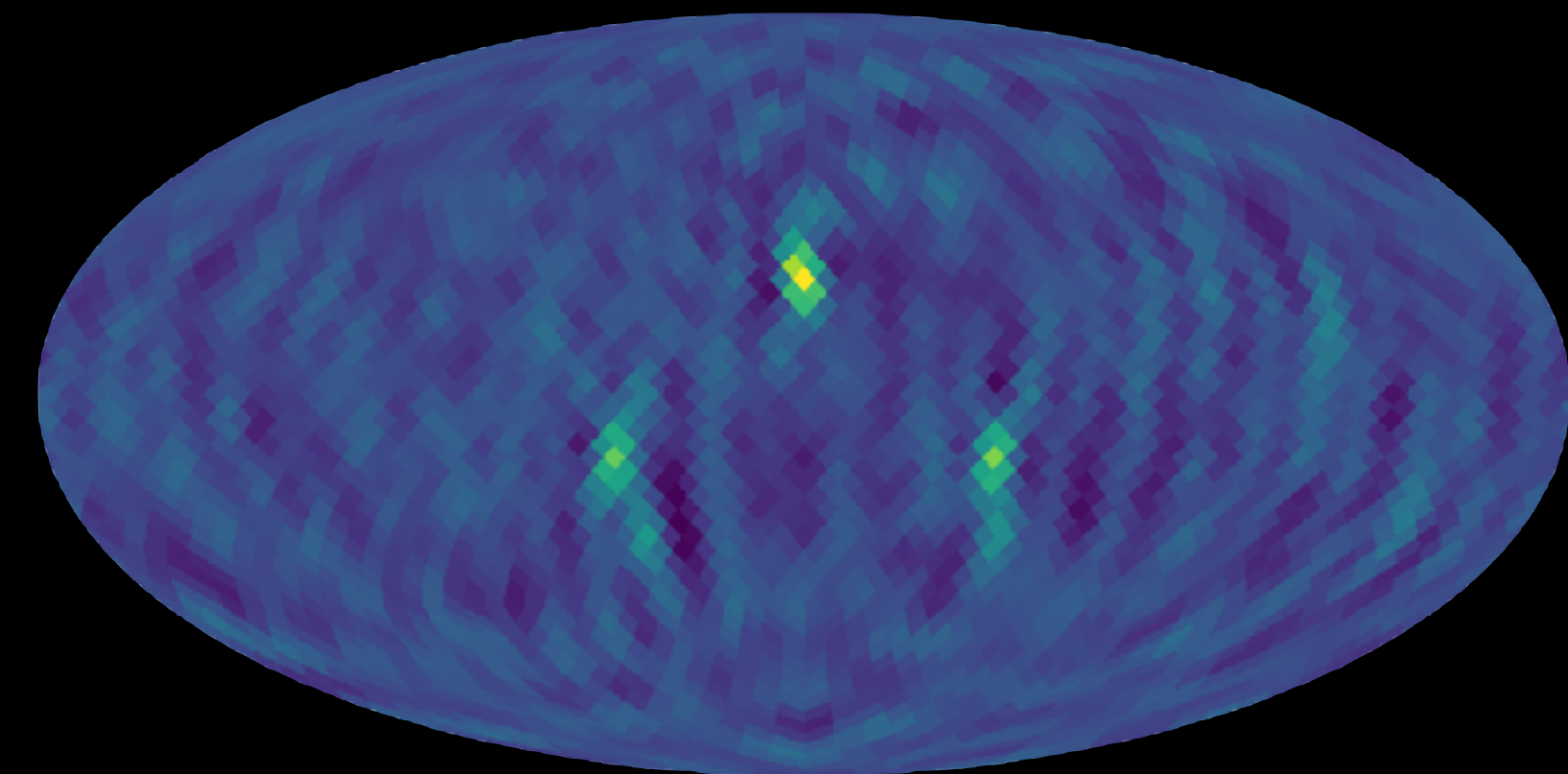
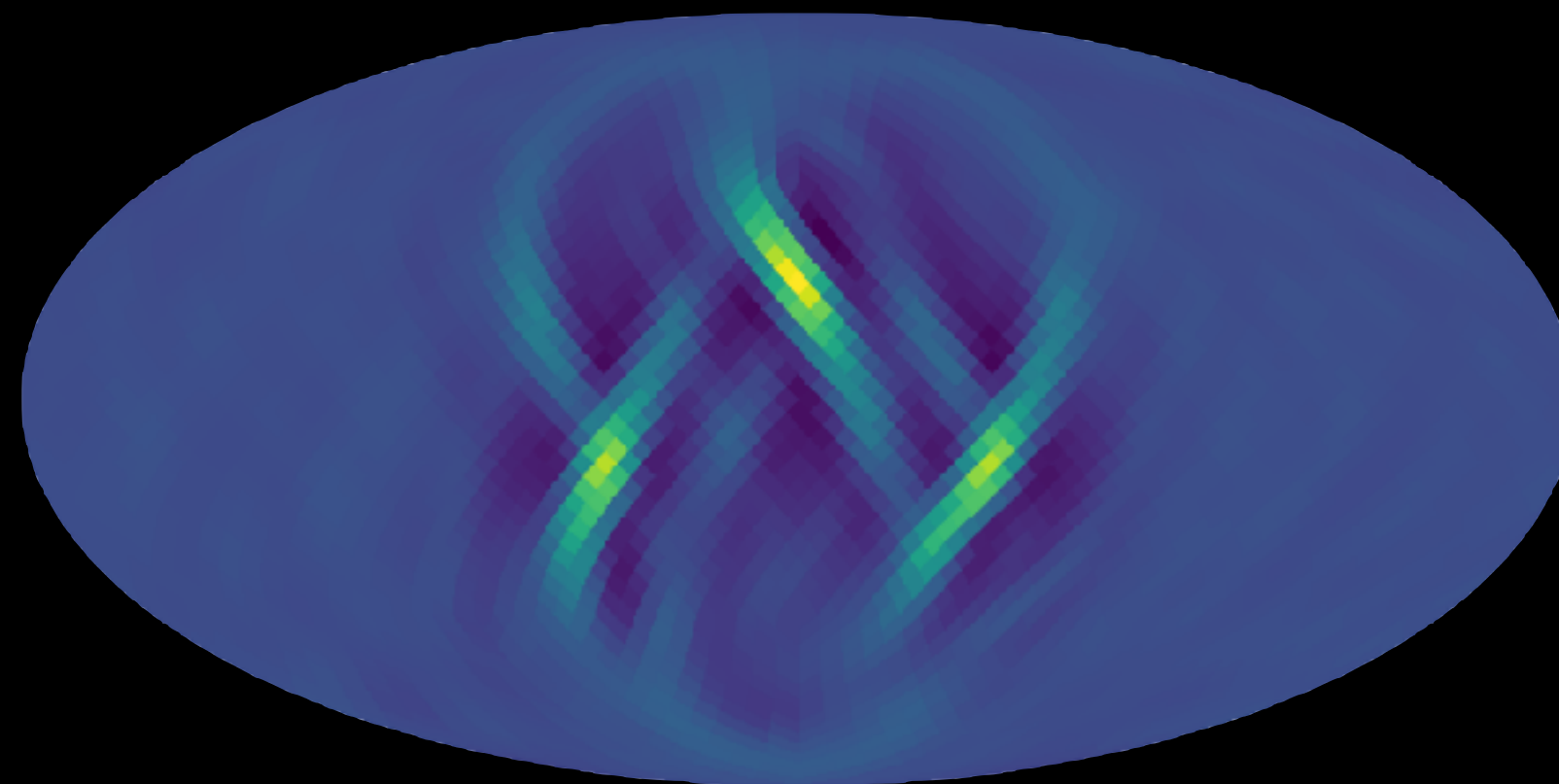
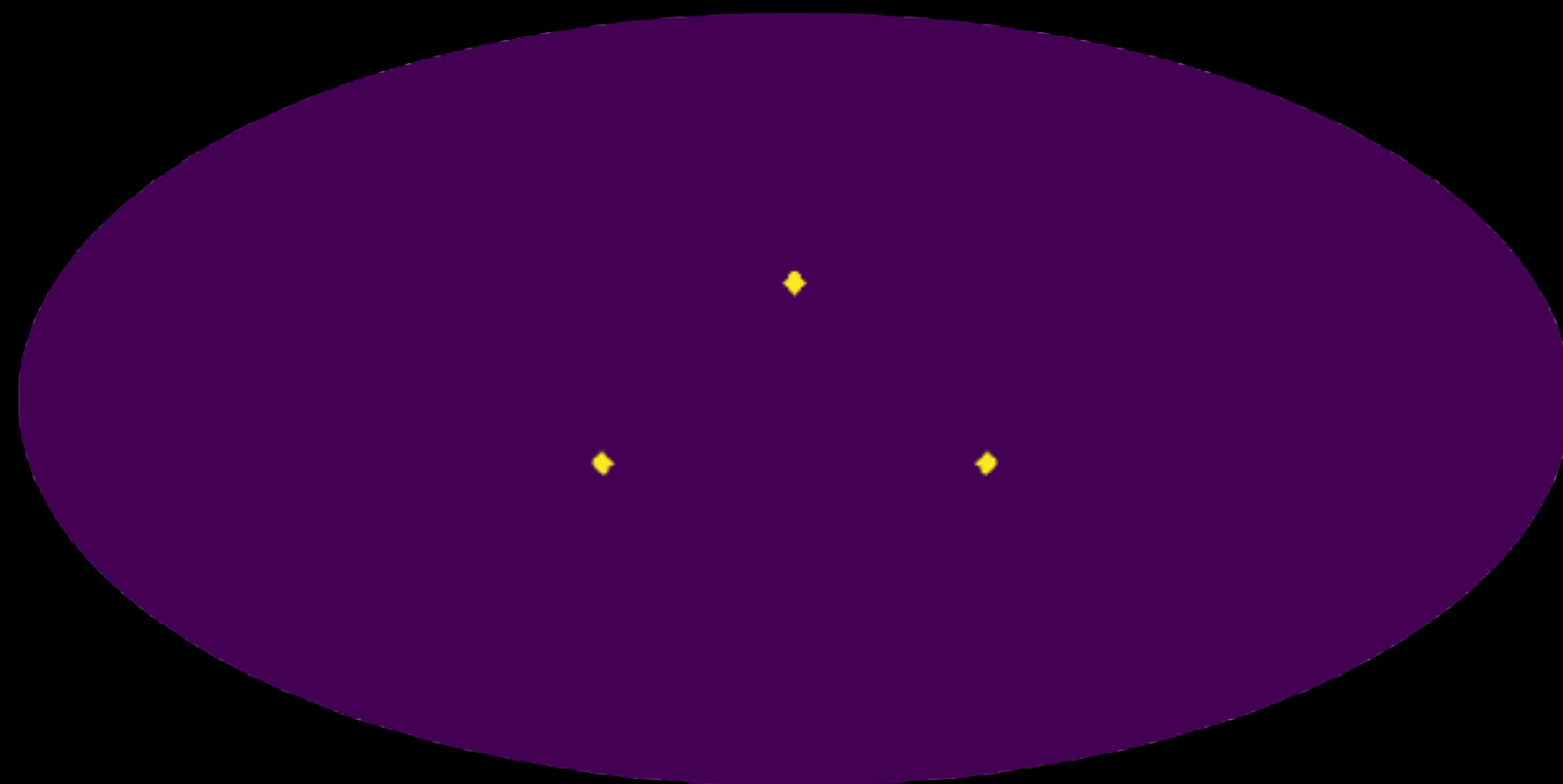


perform the whole analysis on a laptop in a few minutes*



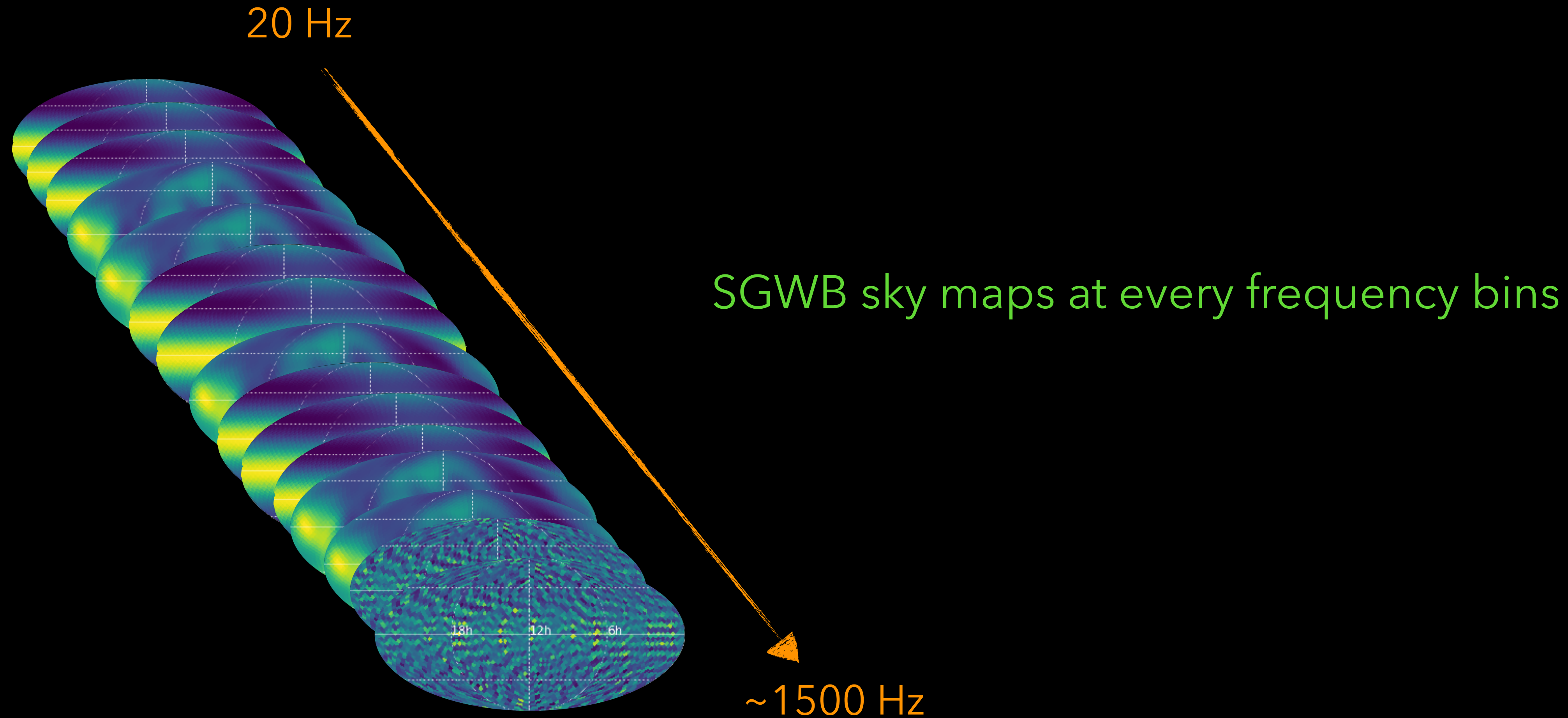
Produces the narrowband maps as an intermediate result

so separate search for different frequency spectra becomes redundant

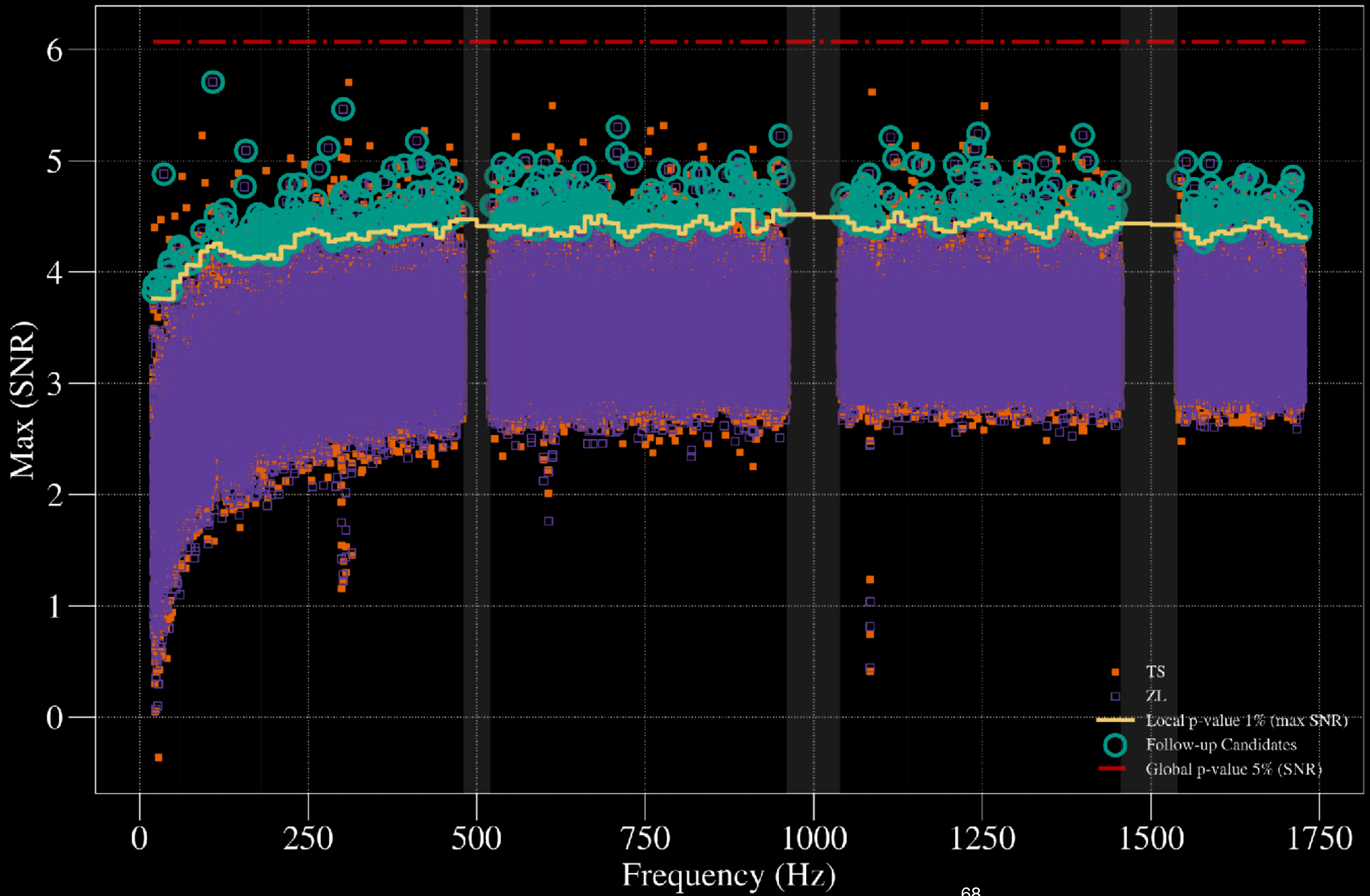


ALL-SKY ALL-FREQUENCY SEARCH

Now we have all the ingredients to perform an all-sky, all-frequency search, which assumes **no** specific power-law model for the SGWB



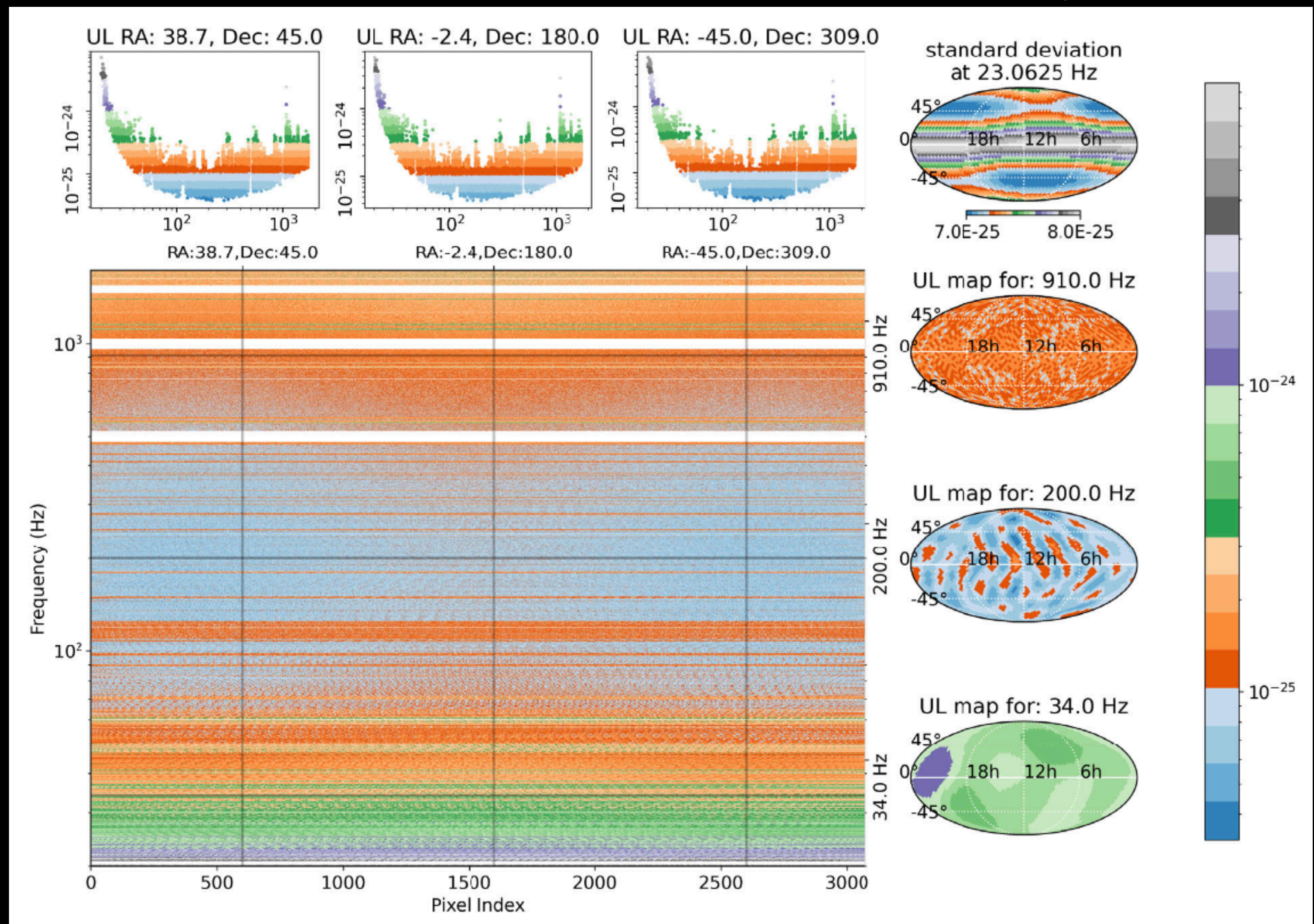
Distribution of maximum SNRs



- The yellow curve shows the 99th percentile of maximum SNR for every 10 Hz frequency bin in TS, smoothed over three neighbouring 10 Hz bins.
- The red line delineates the trials-factor-corrected, one-sided global p-value of 5%
- The points above the yellow curve marked with teal circles are the identified candidates for follow-up studies.

Given no detection, we set the all-sky all-frequency upper limits on the SGWB effective strain*:

$$h(f, \hat{n}) = \sqrt{\mathcal{P}(f, \hat{n}) df}$$



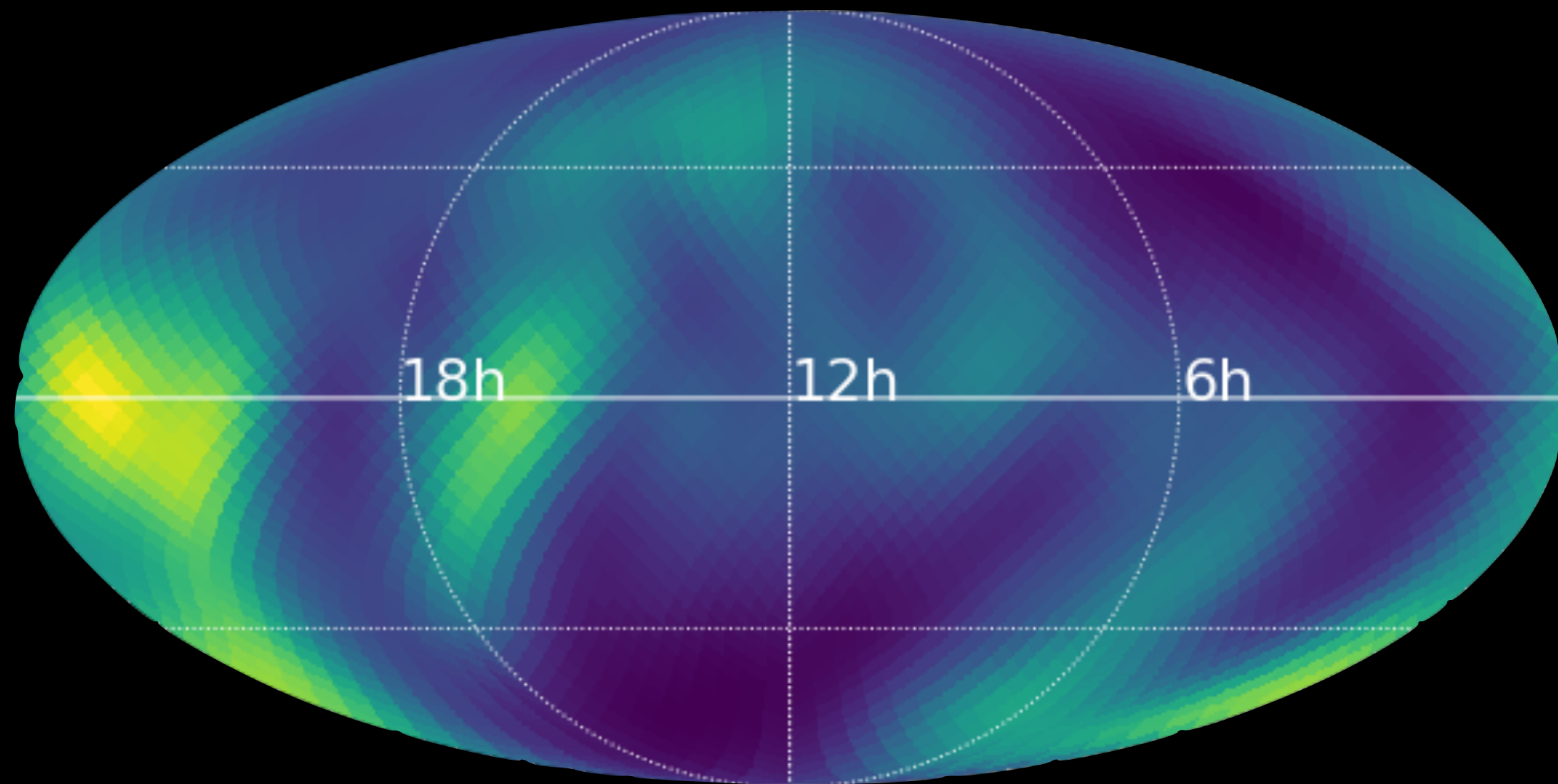
- The colour bar here denotes the range of upper limit variations.
- The vertical cross-section in this diagram shows the frequency-dependent upper limit in a particular direction.
- The Horizontal cross-sections form a map of upper limits in a particular frequency.
- Notched frequencies in a baseline appear as horizontal white bands in the plot.

*circular polarisation without Doppler correction

Assume a power law and combine these narrowband maps to obtain the 'usual' broadband results

$$\hat{\mathcal{P}}(\hat{\mathbf{n}}) = \frac{\sum_f \hat{\mathcal{P}}(f, \hat{\mathbf{n}}) \sigma_{\hat{\mathbf{n}}}^{-2}(f) H(f)}{\sum_f \sigma_{\hat{\mathbf{n}}}^{-2}(f) H^2(f)}$$

$$\sigma_{\hat{\mathbf{n}}} = \left[\sum_f \sigma_{\hat{\mathbf{n}}}^{-2}(f) H^2(f) \right]^{-1/2}$$

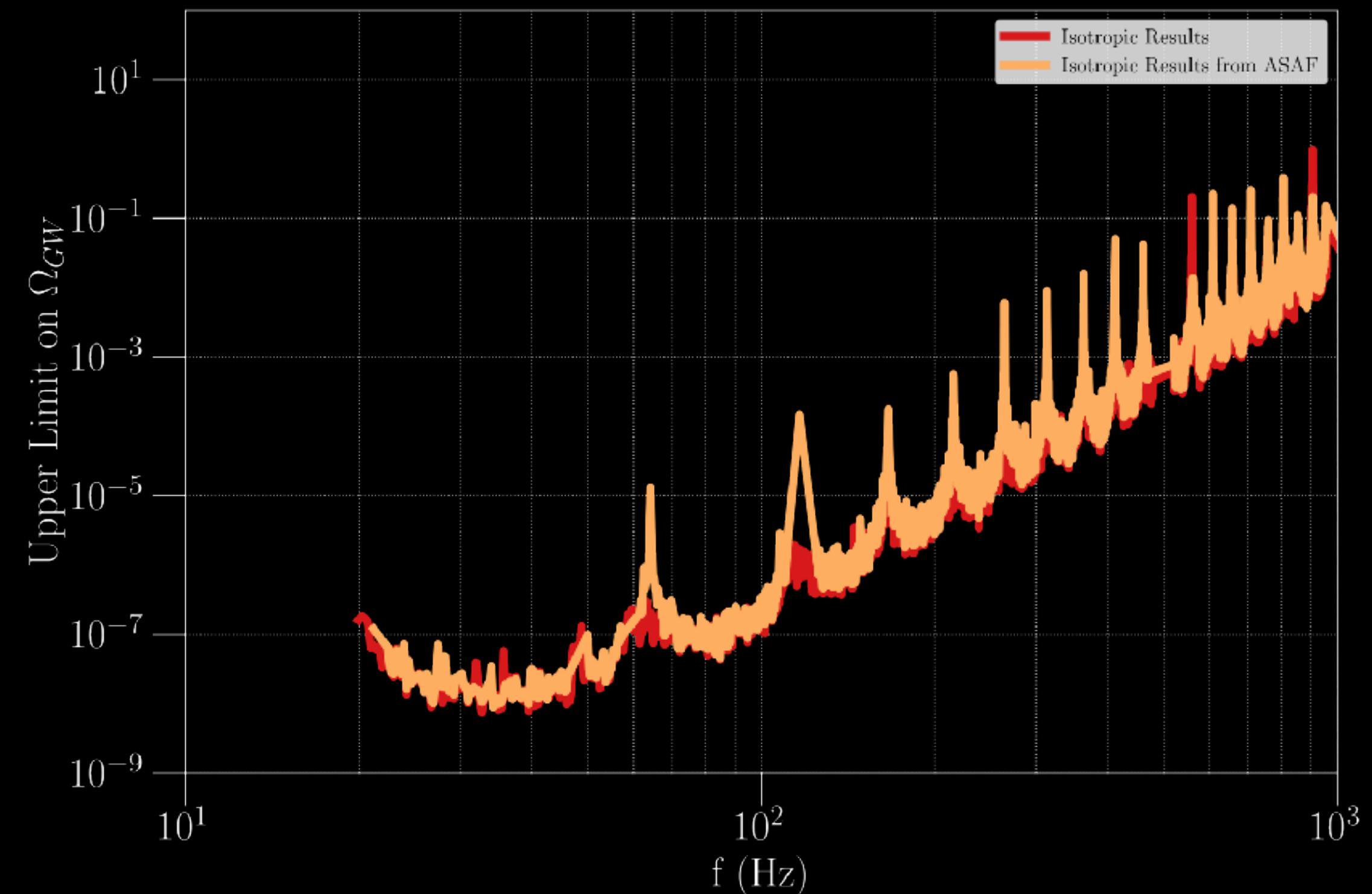


$\alpha = 0$, spectral shape

Assume a power law and sum over all the directions of these narrowband maps to obtain the 'usual' isotropic results

$$\hat{\mathcal{P}}_{\text{iso}}(f) \sigma_{\text{iso}}^{-2}(f) = \frac{5}{4\pi} \int d\hat{\mathbf{n}} \hat{\mathcal{P}}(f, \hat{\mathbf{n}}) \sigma_{\hat{\mathbf{n}}}^{-2}(f)$$

$$\sigma_{\text{iso}}^{-2}(f) = \left(\frac{5}{4\pi} \right)^2 \int d\hat{\mathbf{n}} \int d\hat{\mathbf{n}}' \Gamma_{\hat{\mathbf{n}}, \hat{\mathbf{n}}'}(f)$$



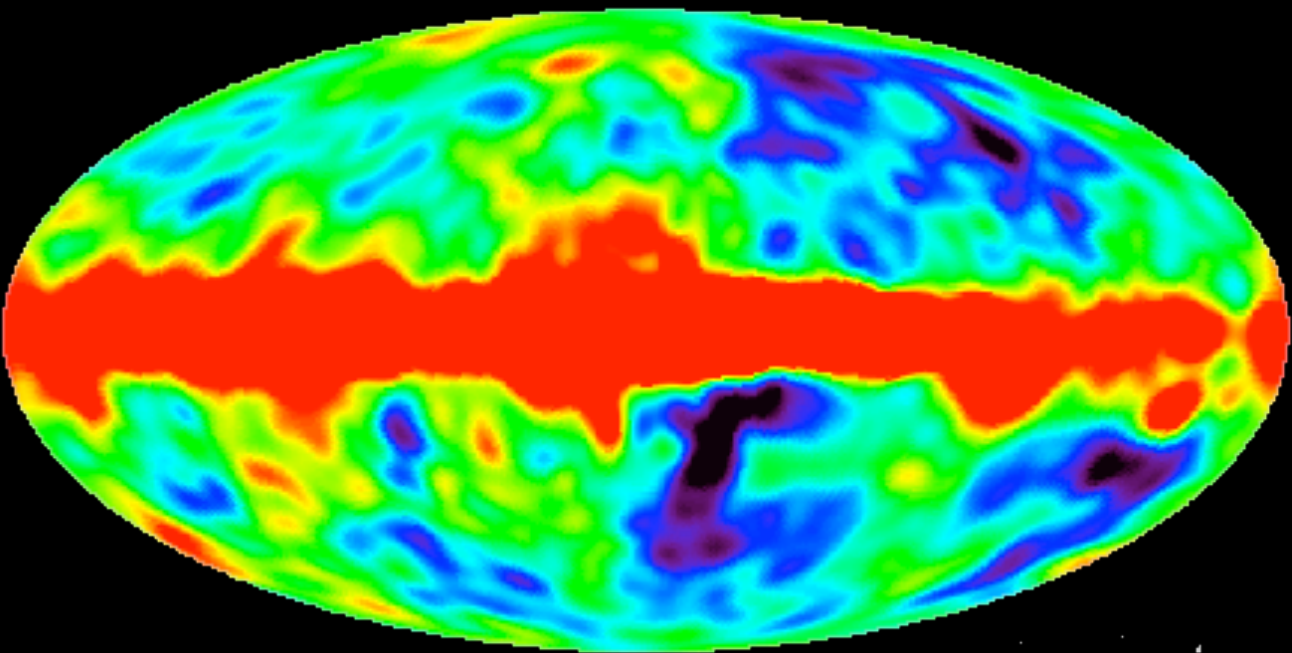
WE ARE NOW AT:

1965: Penzias & Wilson

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE
AT 4080 Mc/s

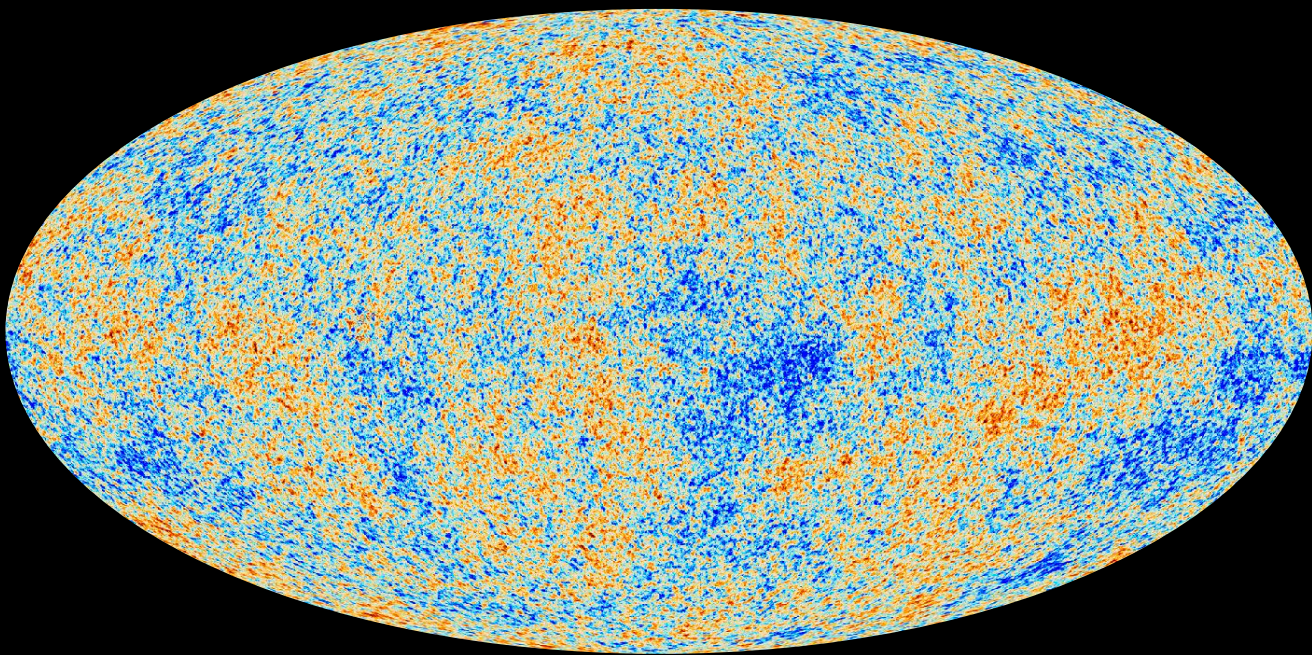
Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964–April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.

1992: COBE



(ang resolution: ~10 degrees)

2013: WMAP, Planck



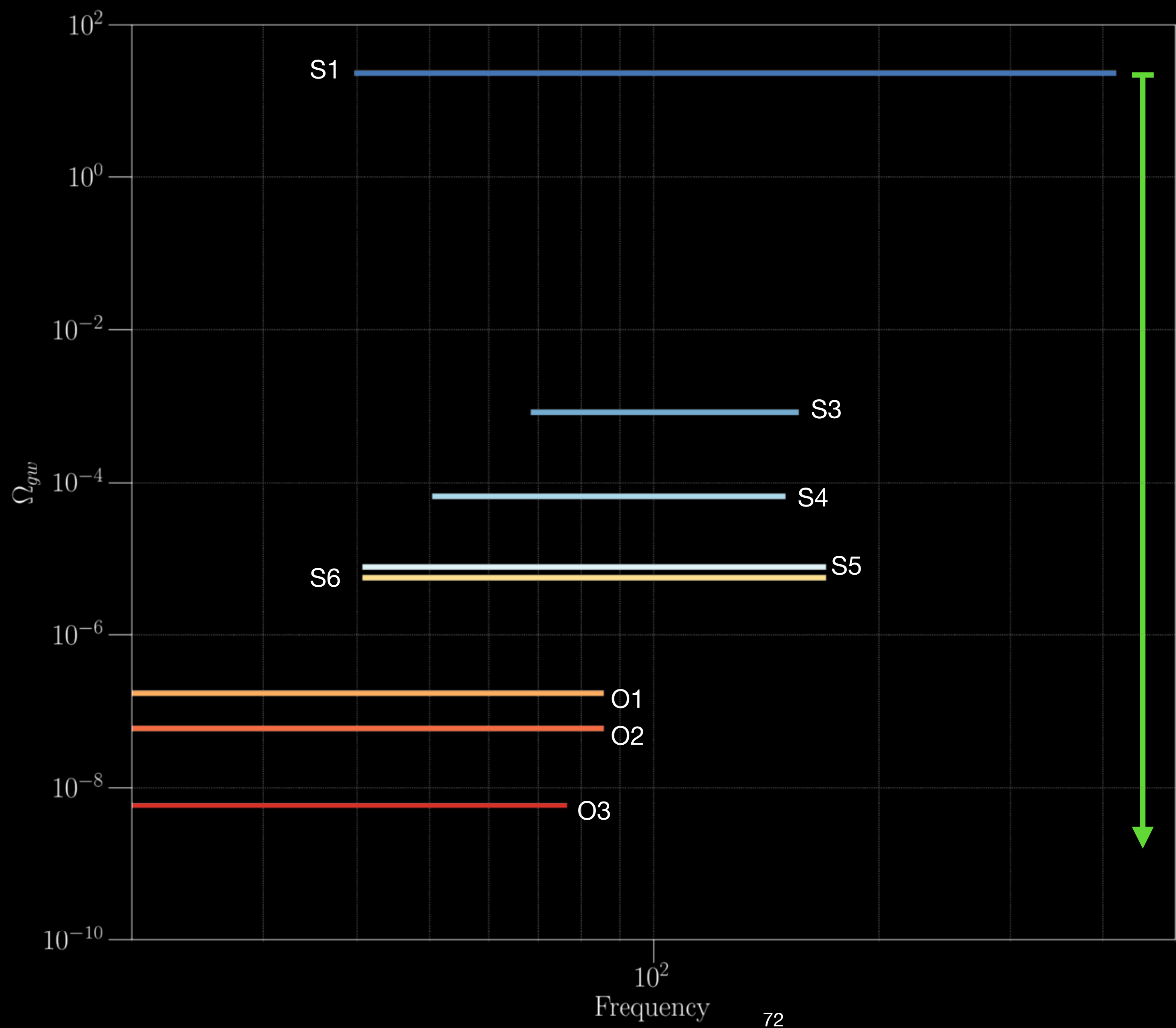
(ang resolution: ~10 arcmin)

2023: We are yet to detect the isotropic SGWB component

Long road ahead.....

WE ARE NOW AT:

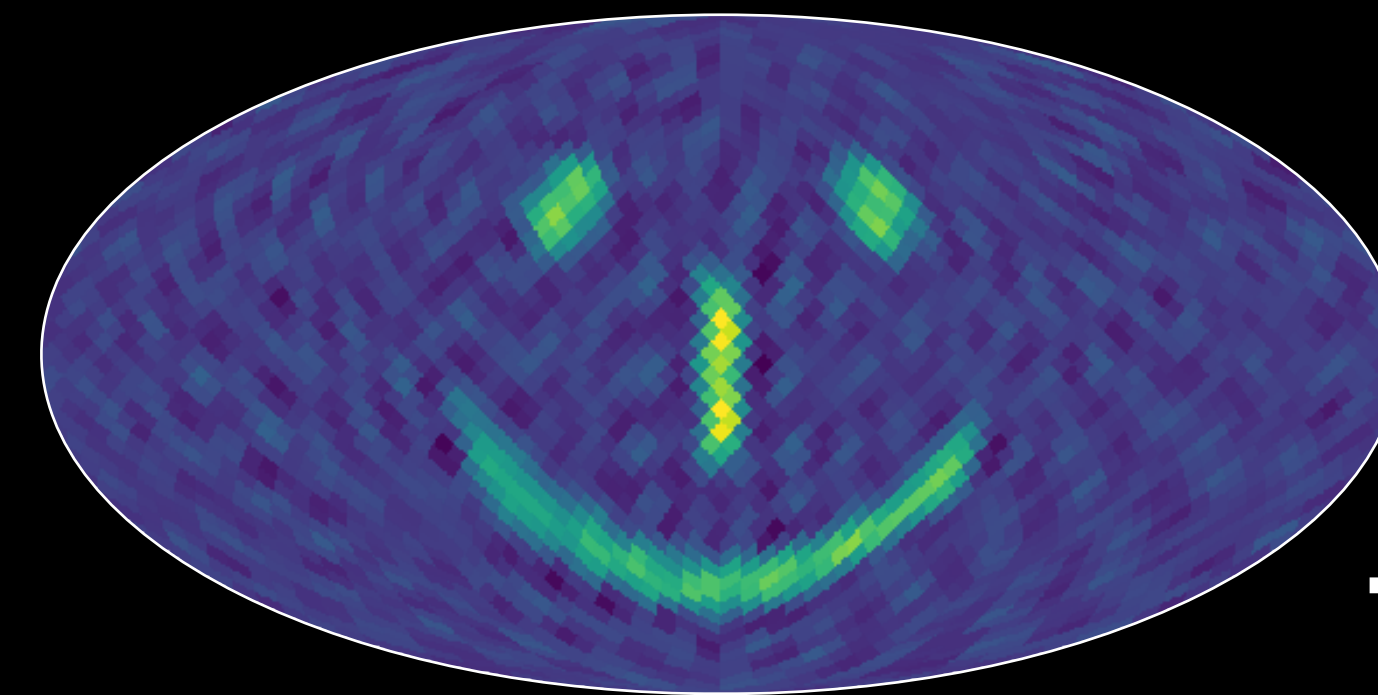
adapted from LIGO S4 paper: APJ 659:918, 2007



We are reaching there...



- New searches and techniques are opening up efficient ways to probe the dark universe.
- Plenty more work to do!
 - More detectors, More signals, More systems, and Dealing with real data.....



Thank you!