Lattice Form-Factors for Exclusive b->s modes

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- Outline

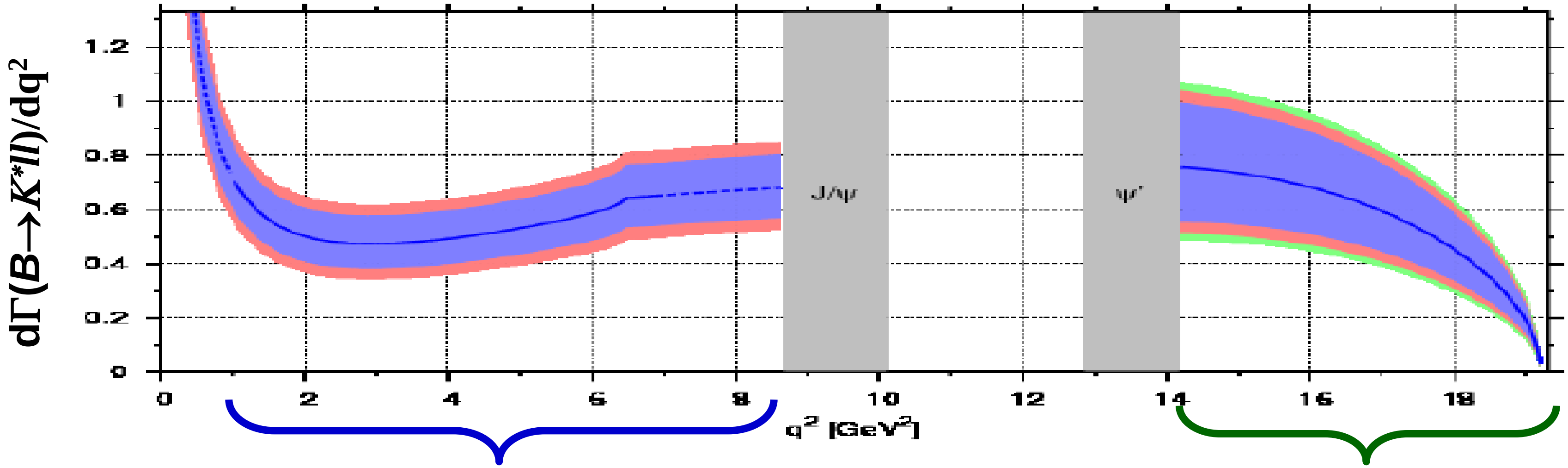
➊ b->s FCNC transitions:

➲ $B\to K^*\gamma$, $B\to Kll$, $B\to K^*ll$,

➋ Status of form factor calculations on the lattice

3rd Flavour Workshop, Valencia, February 4-5, 2013

*Example of observable in B→K*ll process*



large recoil region: $1\text{GeV}^2 < q^2 < 6\ \text{GeV}^2$ **low recoil region:** $q^2 > 14.2\ \text{GeV}^2$

7 form factors in QCD: $V(q^2)$, $A_{0,1,2}(q^2)$, $T_{1,2,3}(q^2)$

❑ ***$m_b \to \infty$, $E_{K^*} \to \infty$***: low $q^2 \sim 0$

✓LEET + QCDF expansion:

$$\xi_\perp(E_{K^*}) \quad \xi_{||}(E_{K^*})$$

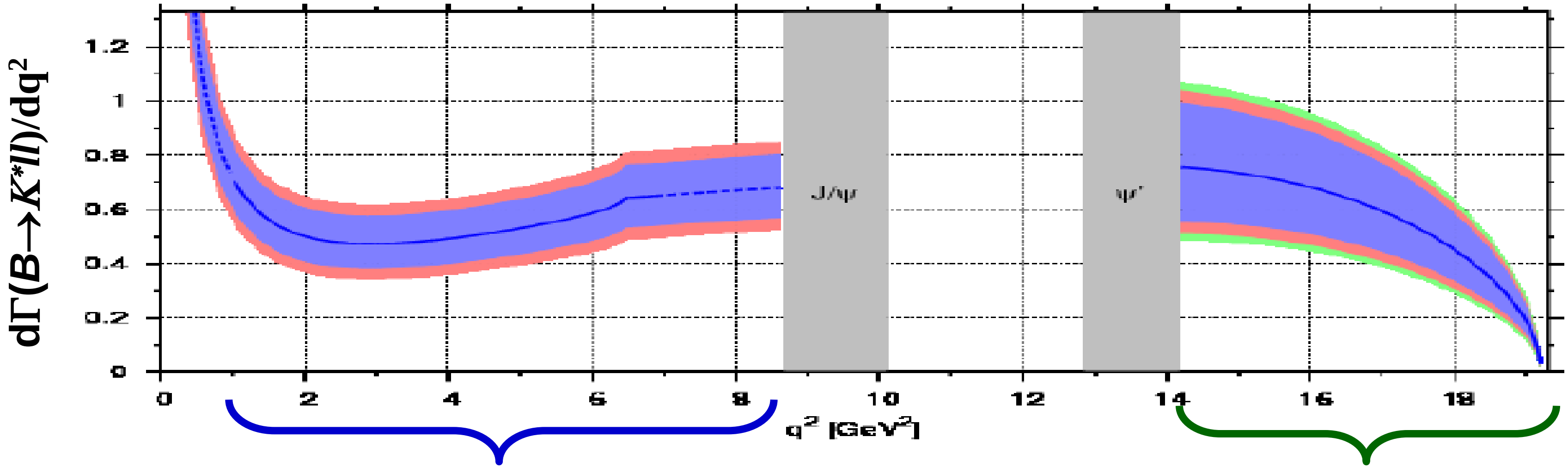
➲ ***2 independent ffs: $V(q^2)$, $A_2(q^2)$***

☺ ***ffs by LCSR → ☺ at low q^2*** **Khodjamirian *et al.*'10**
P. Ball, Zwicky '05

☹ final uncertainties of $O(\Lambda/m_b)$ from QCDF & LEET relations

❑ **Satisfactory scenario at large recoil;** ☹ tough to improve ff!

Example of observable in $B \to K^* ll$ process



large recoil region: 1GeV^2 <q^2<6 GeV^2

low recoil region: q^2>14.2 GeV^2

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- ❑ **Satisfactory scenario at large recoil:** ☹ tough to improve!

- ❑ $m_b \to \infty$, $E_{K^*} \to 0$: large $q^2 \sim m_b$

✓HQET + OPE→ ☺ *$O(\Lambda^2/m_b^2)$ uncertainties* ☺

✓Isgur-Wise relations→ 😐 *$O(\Lambda/m_b)$ uncertainties* 😐

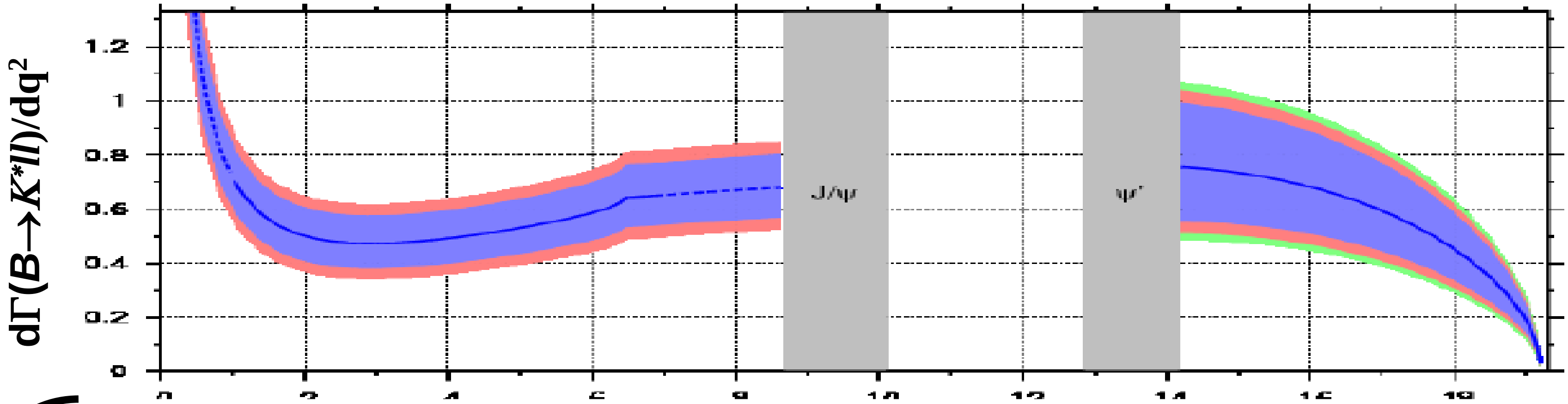
➲ ***3 independent ffs: $V(q^2)$, $A_{1,2}(q^2)$***

☹ ***ffs by LCSR extrapolated ☹ at large q^2***

☹ **Unsatisfactory scenario at low recoil**

☺ **But room to improve -> LATTICE QCD**

Example of observable in $B \to K^* ll$ process



MAIN STRATEGY -> use clean observables at large and low recoil

see Virto's talk

$$O = \left\{P_1, P_2, P_3, P_4', P_5', P_6', M_1, M_2, S_1, S_2\right\}$$ at 1GeV^2 <q^2<8.68 GeV^2

F.Kruger, J.Matias '05; J.Matias, F.M., M.Ramon, J.Virto '12;
D.Becirevic, E. Schneider '12; S. Descotes-Genon, J. Matias, M.Ramon, J.Virto '12

$$O = \left\{H_T^{(2,3,4,5)}\right\}$$ at q^2>14.2 GeV^2 **C.Bobeth, G. Hiller & D. Van Dvk '10,'12**

However, it is still worth to improve form factor calculations:

1) to better assess (residual) hadronic uncertainties in clean obs:
2) to exploit all info available from exp. data:
Moreover, thanks to Lattice QCD concrete and doable possibility at low recoil:
at low $p_{B,K}$ momenta: small discretization errors and better signal for the ground state!

Decays

$$B \to K^* \gamma$$
$$B_s \to \phi \gamma$$
$$B \to (\rho/\omega)\gamma$$

$$B \to K^{(*)} \ell^+ \ell^-$$
$$B_s \to \phi \ell^+ \ell^-$$

$$\Lambda_b \to \Lambda \gamma$$
$$\Lambda_b \to \Lambda \ell^+ \ell^-$$

SM operators

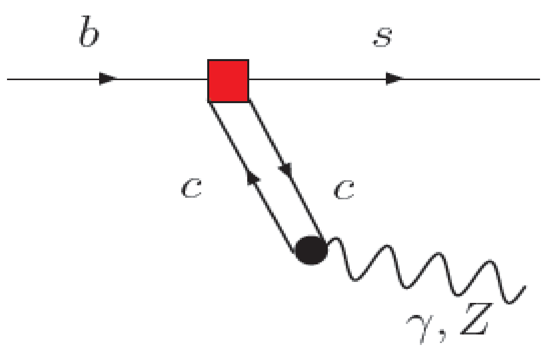
$$O_7 = \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu}$$
$$O_9 = \left(\bar{s}\gamma_L^\mu b\right)\bar{\ell}\gamma^\mu \ell$$
$$O_{10} = \left(\bar{b}\gamma_L^\mu s\right)\bar{\ell}\gamma^\mu \gamma_5 \ell$$

$$O_2 = \left(\bar{s}\gamma_L^\mu c\right)\left(\bar{c}\gamma_L^\mu b\right)$$

BSM operators

$$O_{S(P)} = \left(\bar{s}_L b_R\right)\bar{\ell}\ell_{S(P)}, O_T = \left(\bar{s}_L \sigma^{\mu\nu} b_R\right)\bar{\ell}\sigma^{\mu\nu}\ell$$

Charm Loops



Under control (to some extent)
at low and large *q*²*, out of resonance region*

Jaeger & J. Martin Camalich
talks

GOAL: calculate Matrix elements of 2-quark operators between 2 hadrons (Form factors)

SM operators

$$O_7 = \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu}$$

$$O_9 = \left(\bar{s}\gamma_L^{\mu} b\right)\bar{\ell}\gamma^{\mu}\ell$$

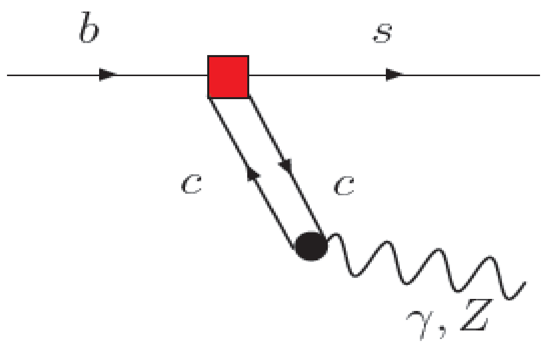
$$O_{10} = \left(\bar{b}\gamma_L^{\mu} s\right)\bar{\ell}\gamma^{\mu}\gamma_5\ell$$

$$O_2 = \left(\bar{s}\gamma_L^{\mu} c\right)\left(\bar{c}\gamma_L^{\mu} b\right)$$

BSM operators

$$O_{S(P)} = \left(\bar{s}_L b_R\right)\bar{\ell}\ell_{S(P)}, \quad O_T = \left(\bar{s}_L \sigma^{\mu\nu} b_R\right)\bar{\ell}\sigma^{\mu\nu}\ell$$

Charm Loops



Under control (to some extent)
at low and large q^2, *out of resonance region*

Jaeger & J. Martin Camalich
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Form Factor Definition for $B\to K^*\gamma$, $B\to K^*ll$

$$\langle V(p',\varepsilon)|\bar q\hat\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p'_\rho p_\sigma$$

$$\begin{aligned}\langle V(p',\varepsilon)|\bar q\hat\gamma^\mu\hat\gamma^5 b|B(p)\rangle &= 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu \\ &+(m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right) \\ &-A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+p')^\mu-\frac{m_B^2-m_V^2}{q^2}q^\mu\right)\end{aligned}$$

$$q^\nu\langle V(p',\varepsilon)|\bar q\hat\sigma_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau p'^\sigma$$

$\longrightarrow$ $Br(B\to K^*\gamma)$ one ff. at $q^2=0$

$$\begin{aligned}q^\nu\langle V(p',\varepsilon)|\bar q\hat\sigma_{\mu\nu}\hat\gamma^5 b|B(p)\rangle &= iT_2(q^2)[\varepsilon^*_\mu(m_B^2-m_V^2)-(\varepsilon^*\cdot q)(p+p')_\mu] \\ &+iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu-\frac{q^2}{m_B^2-m_V^2}(p+p')_\mu\right]\end{aligned}$$

$Br(B\to K^*ll)$: 7 form factors in QCD

*! LATTICE QCD: only to compute the full ff basis at **large** q^2:*
☺ no $O(\Lambda/m_b)$ uncertainty from Isgur-Wise relation at LO!

Form Factor Definition for $B \to K ll$

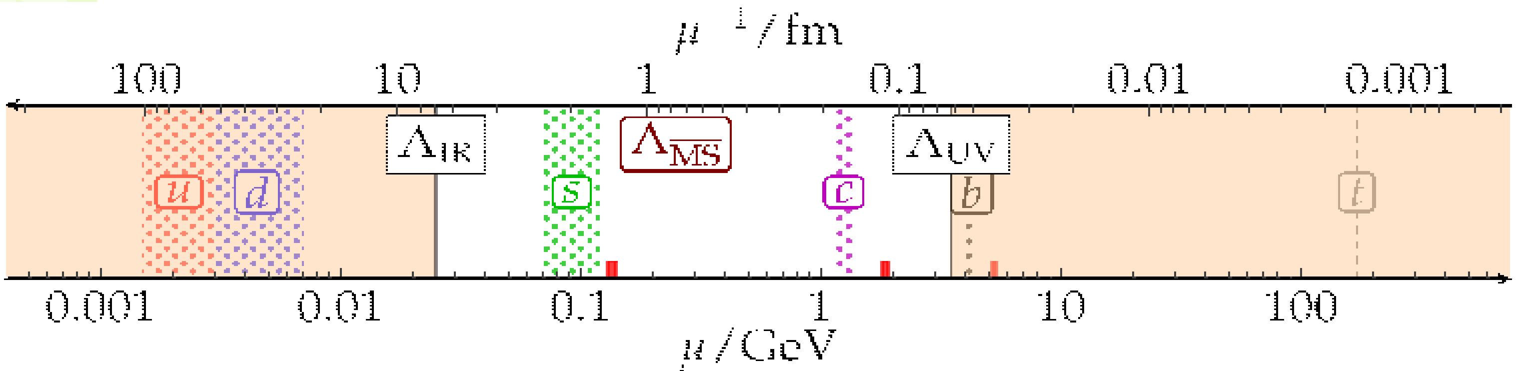
$$\langle B(p)|\bar{b}\gamma^{\mu}s|K(k)\rangle = \left(p^{\mu} + k^{\mu} - \frac{m_B^2 - m_K^2}{q^2}q^{\mu}\right)f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2}q^{\mu}f_0(q^2)$$

$$\langle B(p)|\bar{b}\sigma^{\mu\nu}s|K(k)\rangle = \frac{i f_T}{m_B + m_K}\left[(p^{\mu} + k^{\mu})q^{\nu} - (p^{\nu} + k^{\nu})q^{\mu}\right]$$

Br($B \to K ll$): 3 form factors in QCD

*! LATTICE QCD: only to compute the full ff basis at **large** q^2:*
☺ no $O(\Lambda/m_b)$ uncertainty from Isgur-Wise relation at LO!

Challenge of B-physics: the multi scale-problem of QCD



hierarchy of disparate physical scales to be covered:

$$\Lambda_{IR} = L^{-1} \ll m_\pi, \ldots, m_D, m_B \ll a^{-1} = \Lambda_{UV}$$

$$\left\{ O(e^{-L m_\pi}) \Rightarrow \boxed{L \gtrsim \frac{4}{m_\pi} \sim 6\,\mathrm{fm}} \right\} \curvearrowright \; L/a \gtrsim 120 \; \curvearrowleft \left\{ a m_D \lesssim \frac{1}{2} \Rightarrow \boxed{a \approx 0.05\,\mathrm{fm}} \right\}$$

Currently a^{-1}<4 GeV, *b* quarks cannot be directly simulate at their physical mass due to large discretization errors ($a\, m_b$« 1)

Michele's talk

- *effective theories: like NRQCD action*
- *simulate heavy quark in the charm region and extrapolate to the B + HQET.*

Studies of form-factor calculations on the Lattice:

$B \to K ll$

N_F=0: Quenched lattice QCD: **relativistic fermions**

❖ D. Becirevic, N. Kosnik, F. M., E. Schneider, 2012

N_F=2+1 staggered fermions: NRQCD

❖ FNAL/MILC, 2012

❖ HPQCD, 2012

❖ Cambridge, 2012

→ $f_+(q^2)$, $f_+(q^2)$, $f_T(q^2)$

$B \to K^* ll$

N_F=0: Quenched lattice QCD: **relativistic fermions**

❖ D. Becirevic, V. Lubicz & F. M. 2007 → $T_{123}(q^2)$

N_F=2+1 staggered fermions: **NRQCD**

❖ Cambridge, 2012 → $T_{123}(q^2)$, $V(q^2)$, $A_{012}(q^2)$

☹ *very preliminary unquenched activities:*

overall agreement between Quenched and LCSR

☹ *q^2 dependence: further complication with respect to f_B or B_B*

$B \to K^* ll$ - STRATEGY 1: QCD + extrapolating from charm region:

D.Becirevic, F.M.,V. Lubicz 2007

Lattice setup:

QCD action: 1.2 GeV<m_H<3 GeV, a^{-1}~4 GeV

$$\langle V|\bar{q}\sigma_{\mu\nu}Q|H\rangle = e^{\alpha}\,\varepsilon_{\mu\nu\alpha\beta}P^{\beta}\,T_1(q^2) + e^{\alpha}\,\varepsilon_{\mu\nu\alpha\beta}q^{\beta}\,T_2(q^2) + e^{\alpha}\,p_{\alpha}/(M_H - M_V)^2\,\varepsilon_{\mu\nu\beta\gamma}q^{\gamma}p^{\beta}\,T_3(q^2)$$

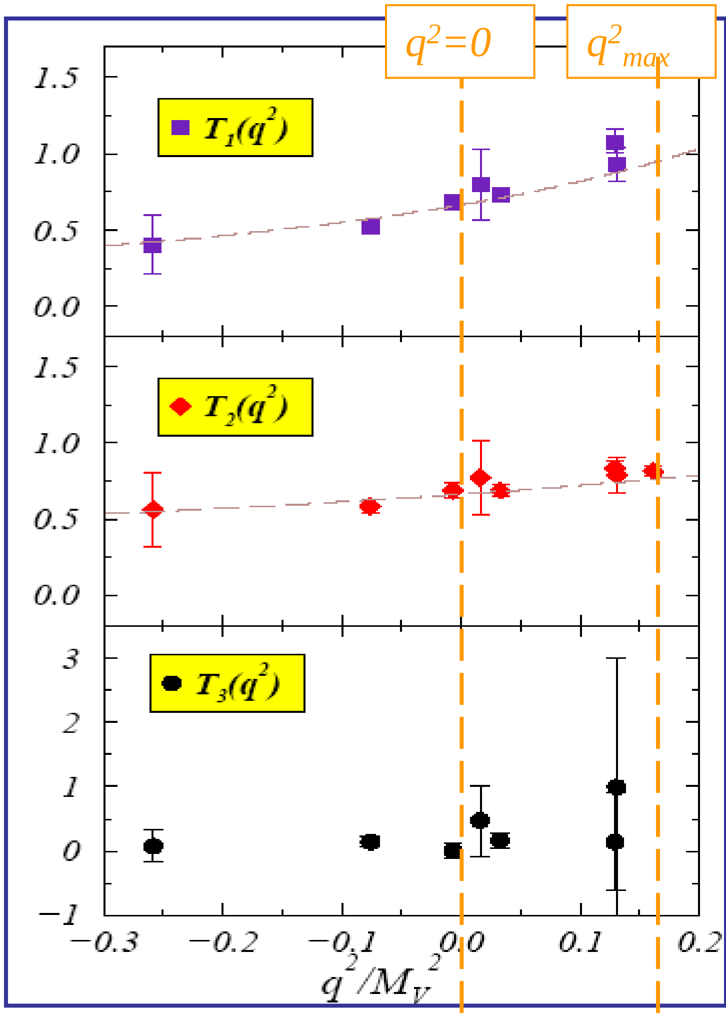
$$T_i(q^2) = T_i^{\vec{p}\vec{q}}(q^2, M_H^2, M_V^2)$$

$$q^2 = m_H^2 + m_V^2 - 2M_H v\cdot p'$$

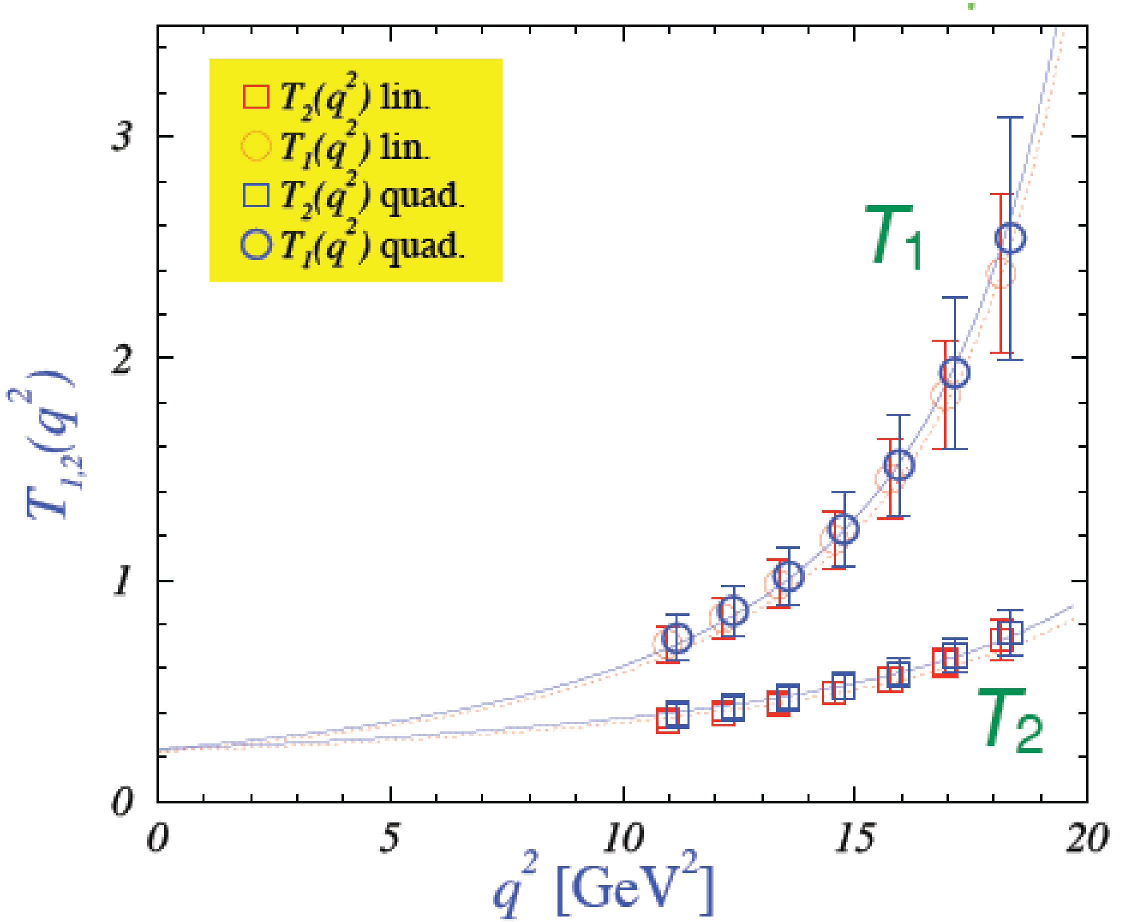
$\Rightarrow$ Extrapolation **to m_B** from **HQET scaling laws**

$$\frac{T_1(v\cdot p')}{\sqrt{M_H}} = d_0(v\cdot p') + \frac{d_1(v\cdot p')}{M_H} + \frac{d_2(v\cdot p')}{M_H^2}$$

$$vp' = (M_H^2 + m_V^2 - q^2)/2M_H$$



B→K*ll - *STRATEGY 1: QCD + extrapolating from charm region:*

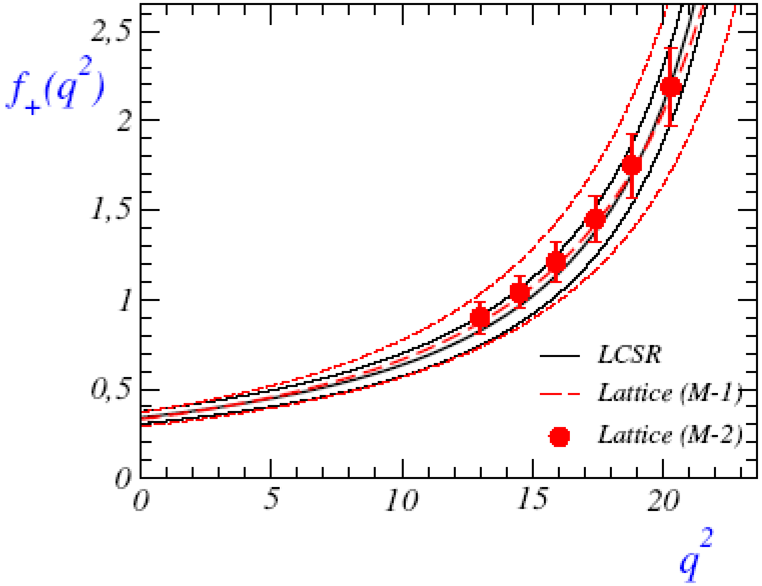


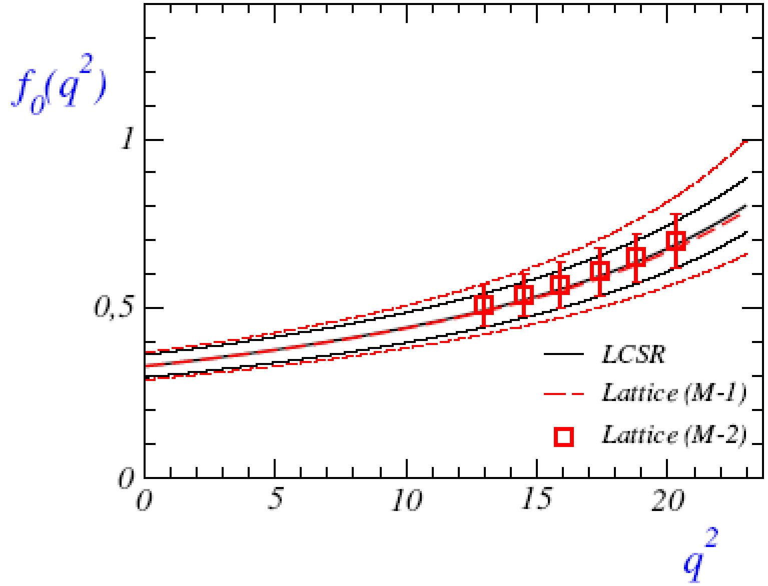
1) relative th. error at 15%. Quenched result!
2) **Lattice points at large q^2**

$$q^2 = m_H^2 + m_V^2 - 2M_H v \cdot p'$$

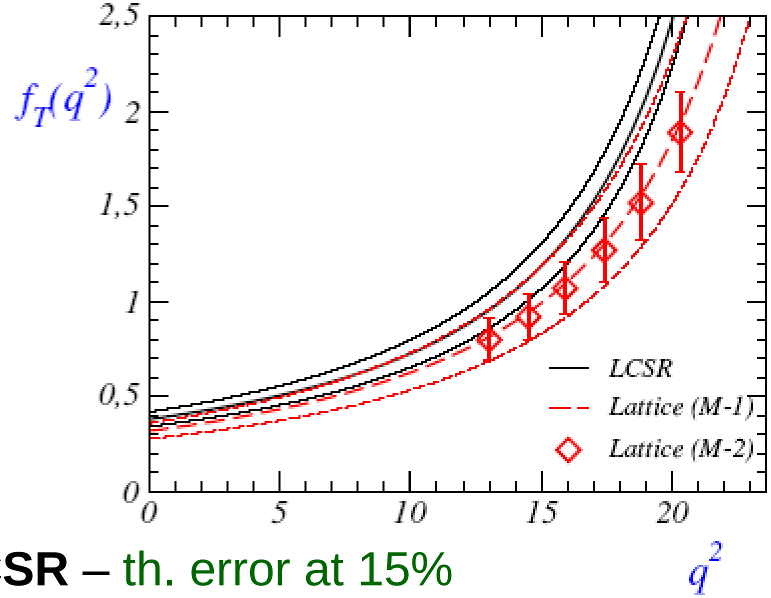
☺ **helpful the HQET ffs at q^2_{max}**

$B \to K ll$ - *STRATEGY 1: QCD + extrapolating from charm region:*





Lattice QCD Quenched:
f_+, f_0 -> D. Becirevic et al. 2012
f_T -> D. Becirevic et al. 2007



Light cone QCD sum rules [Ball'05, Khodjamirian'07, '10]

1) **Lattice QCD and LCSR** – th. error at 15%
2) **Lattice points at large q^2**
3) **Agreement with LQSR**

☺ **helpful the HQET ffs at q^2_{max}**

Theory: Hadronic Uncertainties $B \to K ll$

$B \to K ll$

Dominant uncertainties come from the form 3 factors: $f_+(q^2)$, $f_0(q^2)$, $f_T(q^2)$

$$\langle K | \bar{b}\gamma^\mu\gamma_5 s | B\rangle \Leftrightarrow f_{+,0}(q^2) \qquad \langle K | \bar{b}\sigma^{\mu\nu} s | B\rangle \Leftrightarrow f_T(q^2)$$

- $C_{9,10}^{(\prime)} \to f_+(q^2), f_0(q^2), C_{S,P}^{(\prime)} \to f_0/m_b \quad C_7^{(\prime)} \to f_T,$
- Wide range of $q^2=[0,(m_B-m_K)^2]$ -> Opportunities for different nonperturbative techniques: Lattice QCD and LCSR – relative error 30%

$$\mathrm{Br}\left(B \to K\ell^+\ell^-\right)_{\mathrm{SM}} = \begin{cases} (7.5 \pm 1.4) \times 10^{-7} & \mathrm{LQCD}\,, \\ (6.8 \pm 1.6) \times 10^{-7} & \mathrm{LCSR}\,. \end{cases},$$

Our average

$$\mathrm{Br}\left(B \to K\ell^+\ell^-\right)_{\mathrm{SM}} = (7.0 \pm 1.8) \times 10^{-7}$$

still th. error large 30%

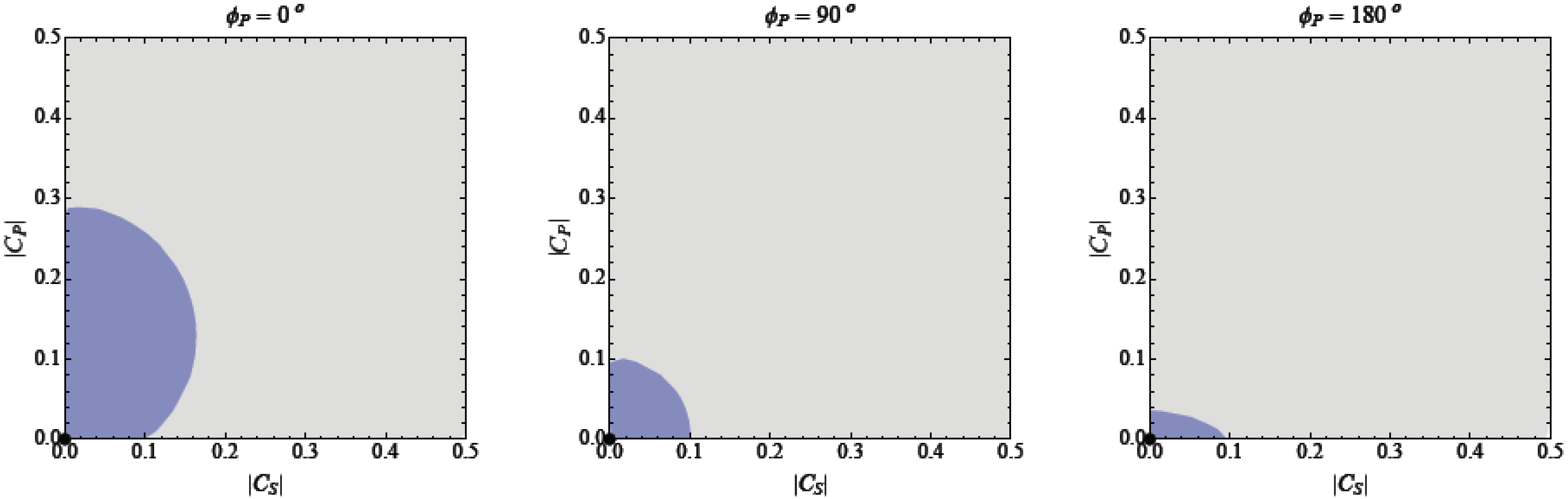
BaBar'12

Br($B \to K\, ll$) = (4.7±0.6) × 10^{-7}

LHCb'12

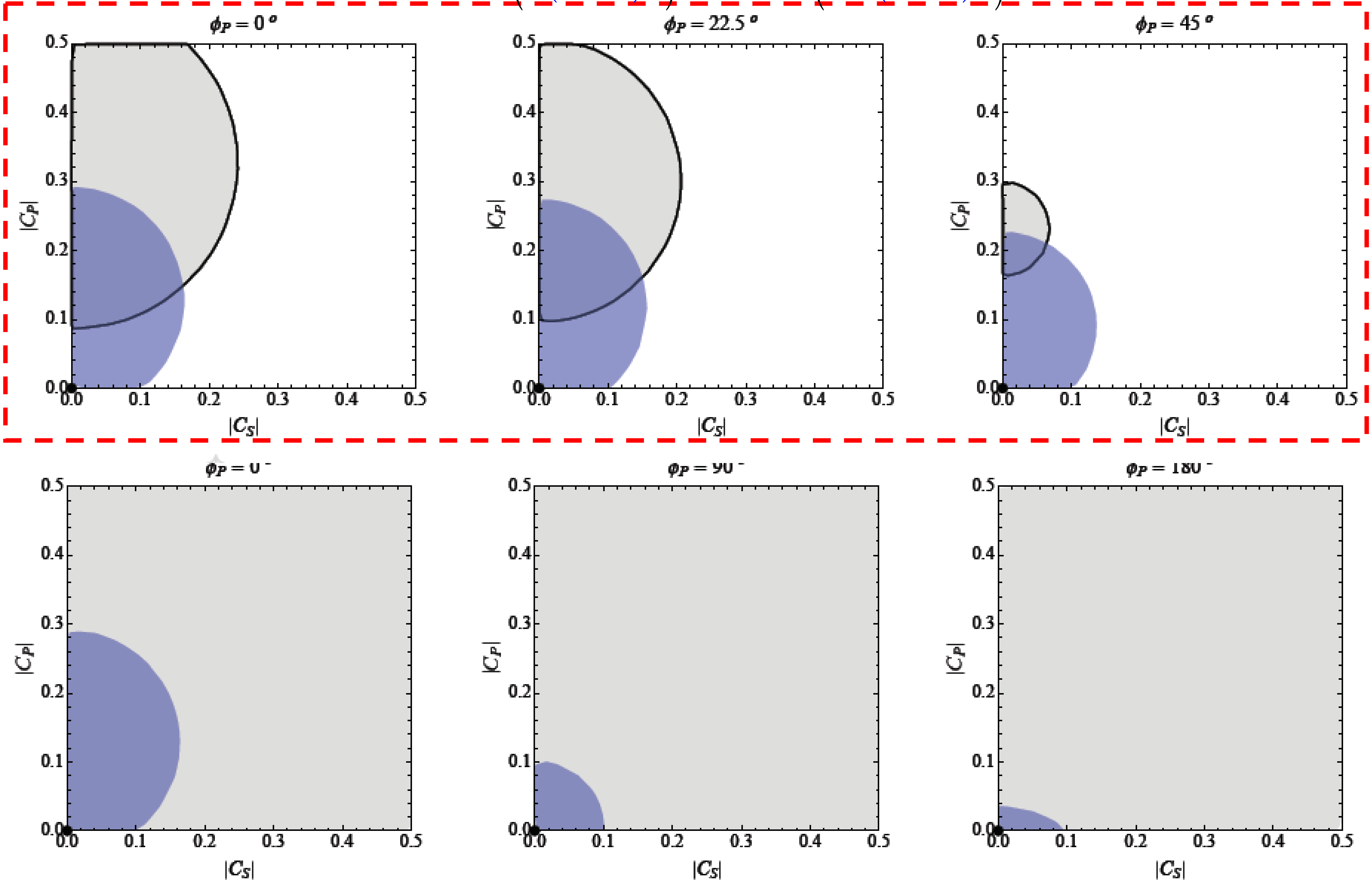
Br($B^+ \to K^+ \mu\mu$) = (3.1±0.7) × 10^{-7}

New Physics: SM $+ C_S\left(\bar{b}(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell + C_P\left(\bar{b}\gamma_5(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell$



Lowering th. error on *B->Kll: 20% smaller than now*

New Physics: SM + $C_S\left(\bar{b}(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell + C_P\left(\bar{b}\gamma_5(1-\gamma_5)s\right)\bar{\ell}\gamma_5\ell$



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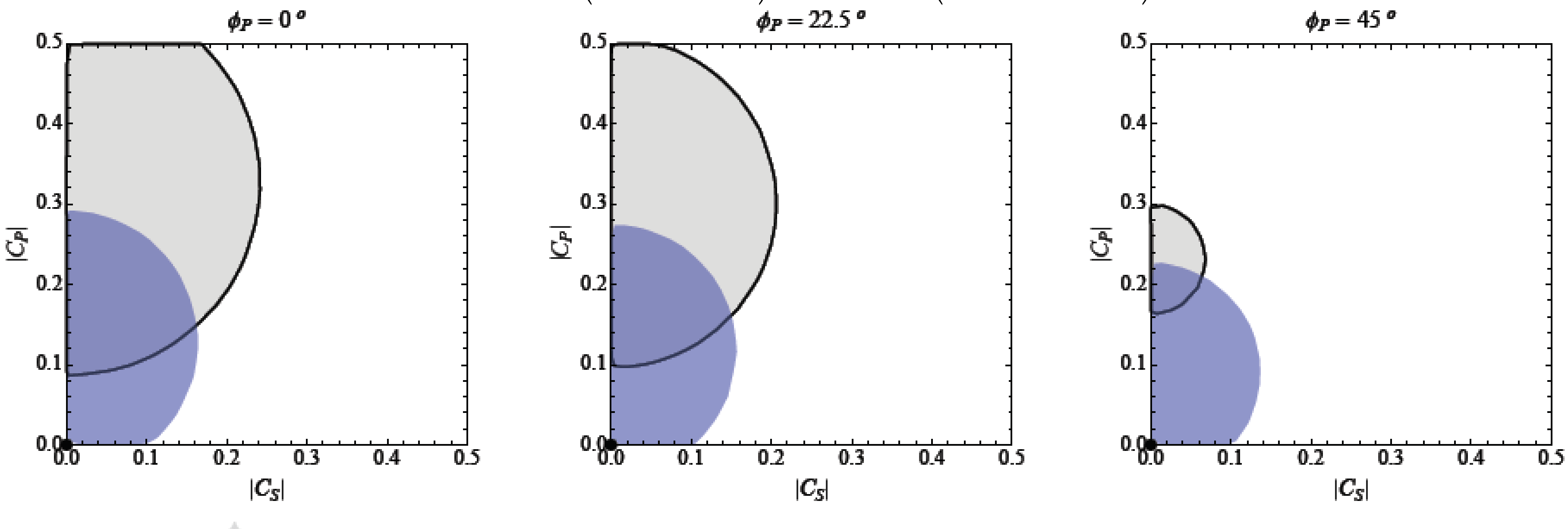


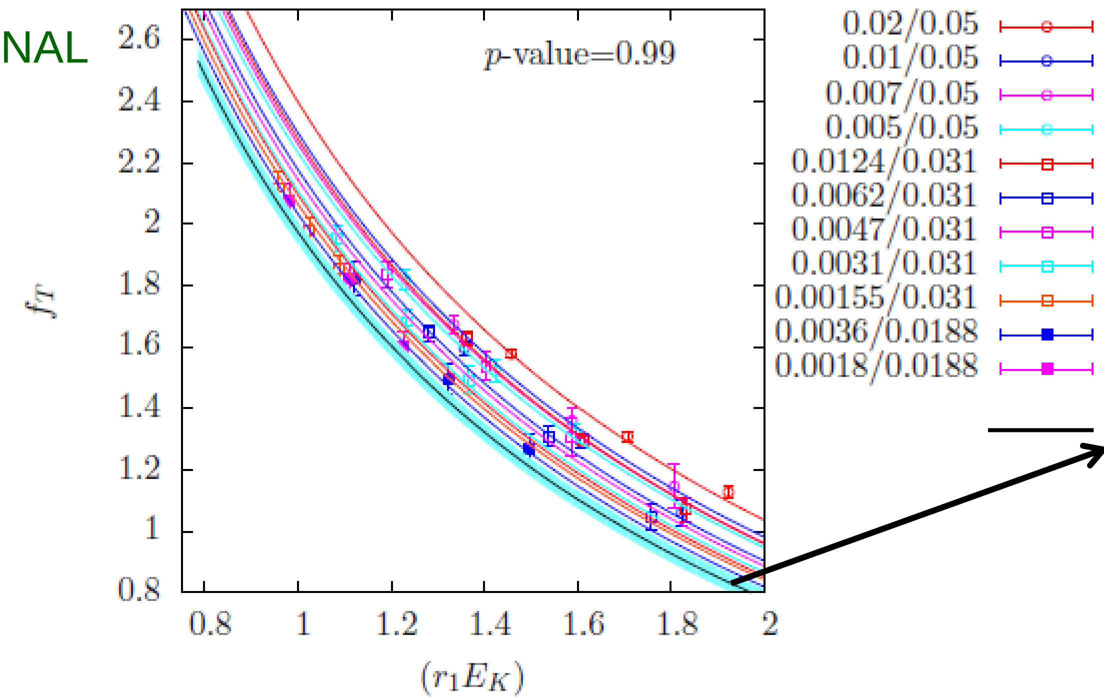
This "toy-scenario" would prefer nonzero C_P.

Lowering th. error on *B->Kll: 20% smaller than now*

$B \to Kll$ - *STRATEGY 2: (NRQCD) effective action for the b quark, v<<c*

❖ sea quarks

❖ MILC-FNAL



❖1/a=1.7 GeV

❖1/a=2.2 GeV

❖1/a=3.3 GeV

continuum

m_π (MeV)
~300
~460
~320

❖ **fit in q^2, m_{sea}, and lattice spacing (a)**

$$f_{\parallel} = \frac{C_{\parallel}^{(0)}}{f_\pi}\left[1 + \text{logs} + C_{\parallel}^{(1)} m_l + C_{\parallel}^{(2)}(2m_l + m_s) + C_{\parallel}^{(3)} E_K + C_{\parallel}^{(4)} E_K^2 + C_{\parallel}^{(5)} a^2\right],$$

❖ **<u>preliminary</u> unquenched results (NF=2+1)**

-> stats + (systs) errors at 5%

3 coarse lattice spacings -. Continuum limit

😐 Comments:

- ***Discretized NRQCD action***

  ➢*Quite Sophisticated procedure!*

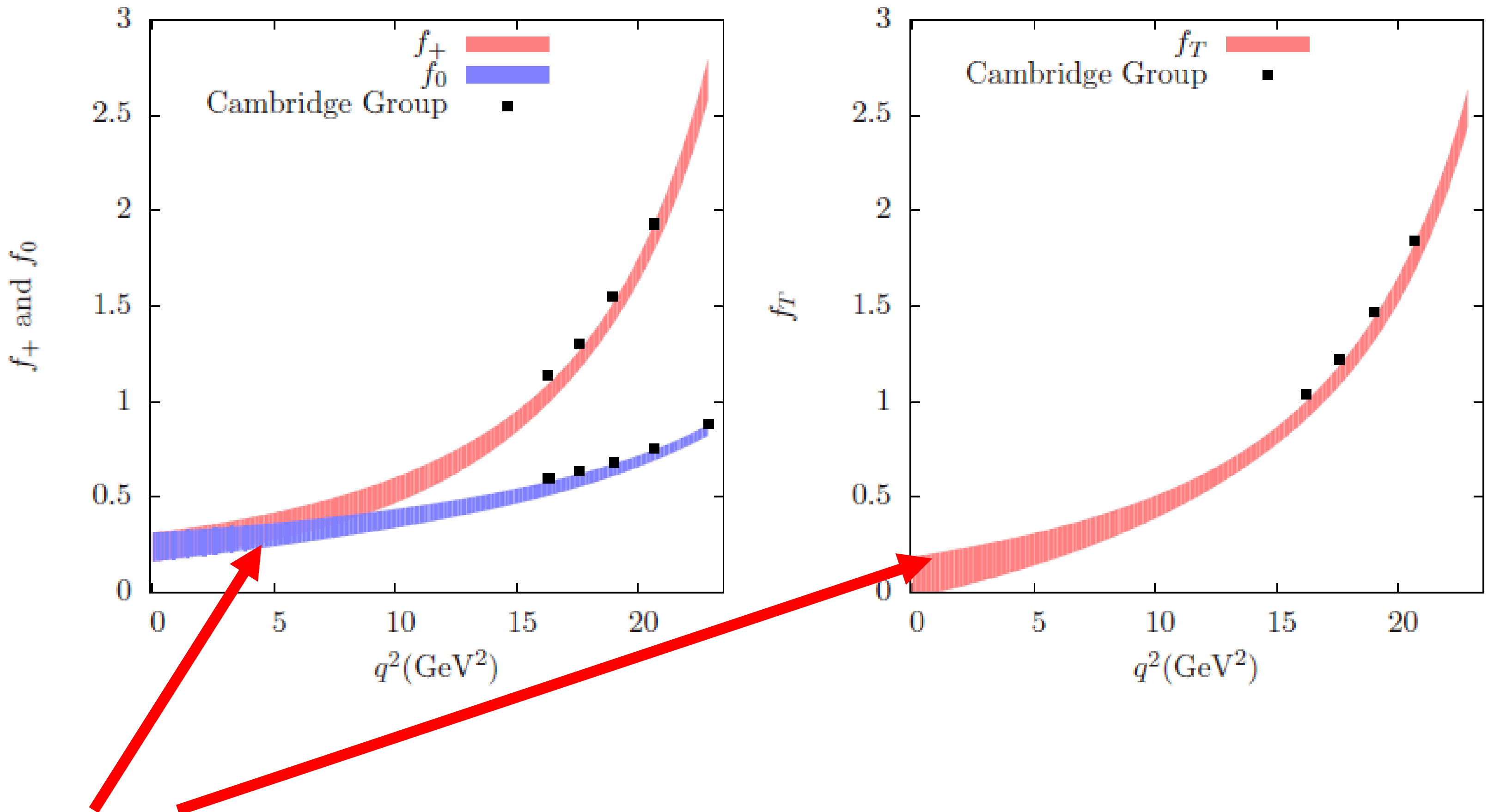
  ➲ *larger set of $1/m_Q$ corrections on the lattice w.r.t the continuum*

  ➢$O[\alpha_s^n/(am_Q)]$ divergences to be subtracted to get the continuum limit

➢*On the other hand, large experience from MILC/FNAL/HPQCD*

➢☺ ***Succeful strategy for f_B when comparing with unquenched results from strategy 1***

B→Kll - *STRATEGY 2: (NRQCD) effective action for the b quark, v<<c*

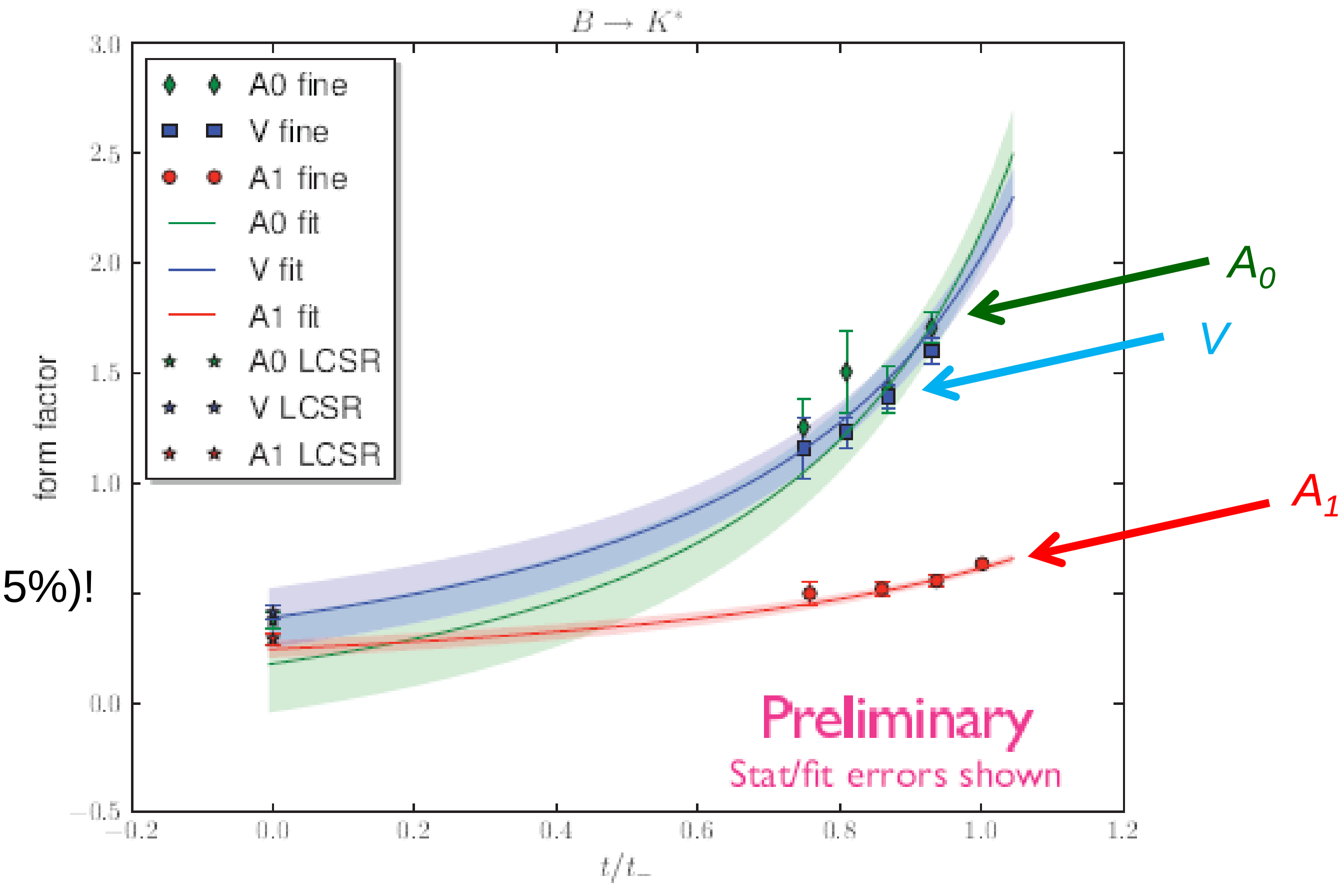


❖ MILC-FNAL vs **Cambridge group**

❖Overall agreement among unquenched results.

❖All use staggered fermions but different sets, masses and fit procedure

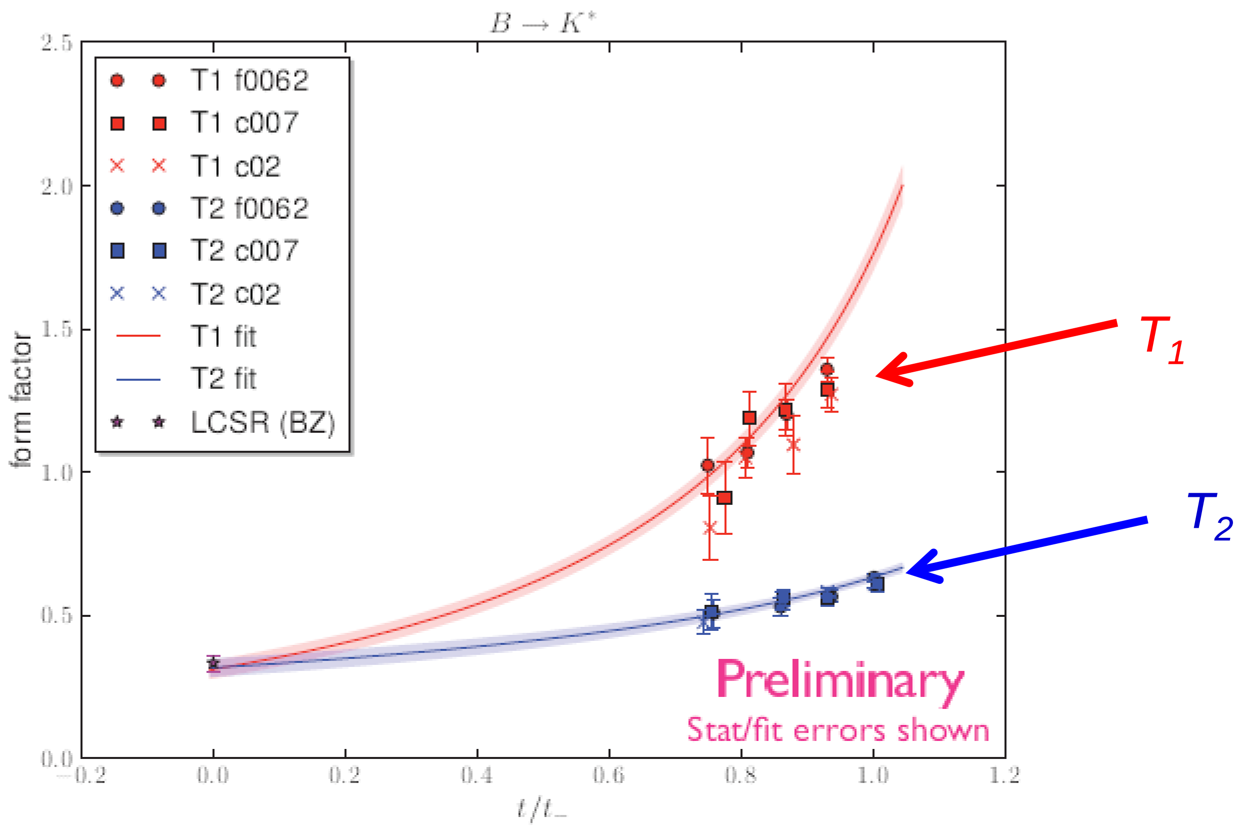
$B \to K^* ll$ form factors from Cambridge/W&M/Edinburgh.



- Only stats errors (at 5%)!
- Promising study

Preliminary results on $B \to K^* ll$ V, A_0, and A_1 vs. $q^2/q^2_{\rm max}$. (by M. Wingate at lattice 2012)

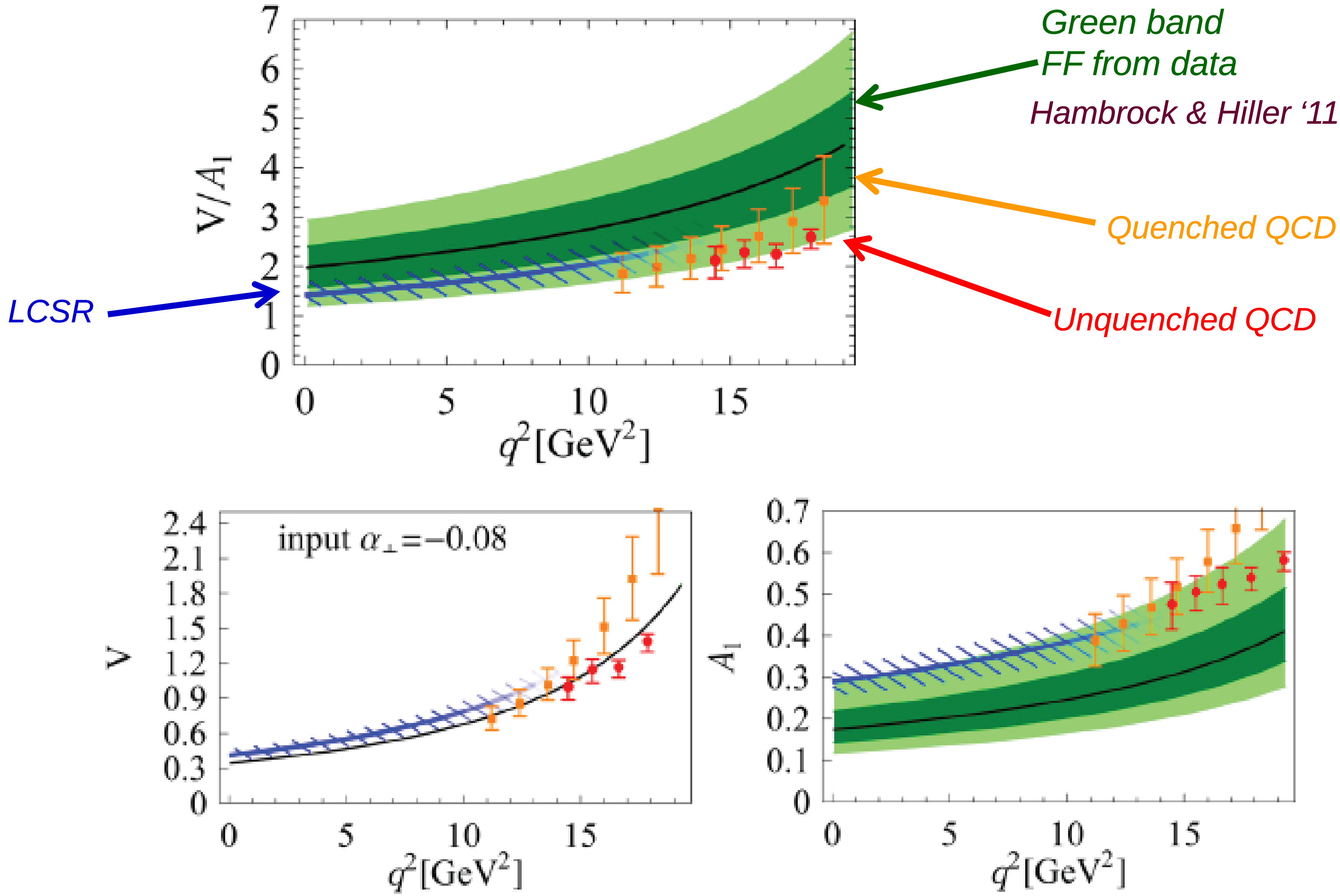
$B \to K^* ll$ form factors from Cambridge/W&M/Edinburgh.



Preliminary results on $B \to K^* ll$ T_1 and T_2 vs. $q^2/q^2_{\max}$ (by M. Wingate at lattice 2012)

- **Only stats errors at 5%!**
- **Promising study**

Comparison of B→K*ll form factor calculations



Conclusions:

LATTICE QCD -> touchable progress in recent years:

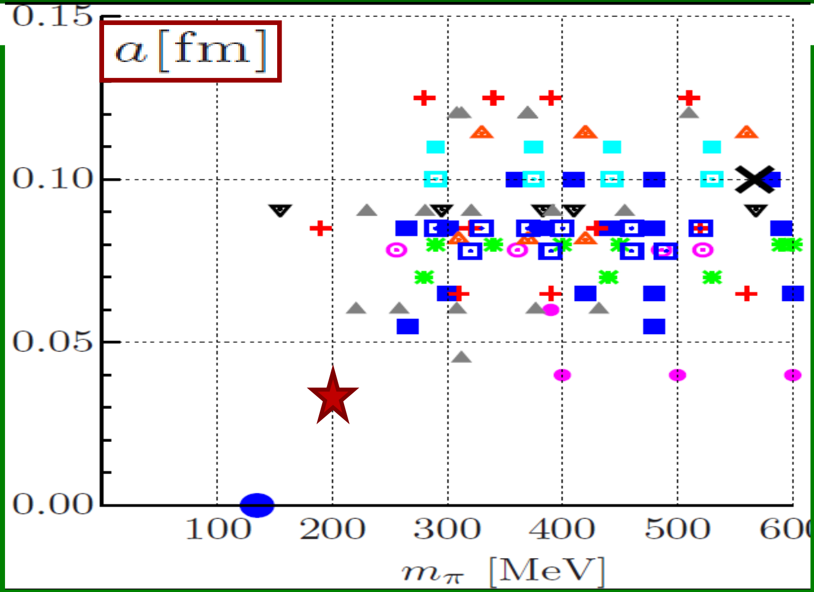
- reliable *unquenched simulations with pions close to the physical point* => m_{π}=156 MeV (PACS-CS), **m_{π}**=190 MeV (BMW)
- f_K/f_{π} & f_B *paradigma of present lattice progress!*
- *promising studies at percent level on the way for B Physics ffs*

Still a long work to assess 1%-precision needed for B physics

1. ***discretization errors:*** $a*m_B \ll 1$

   => $a \sim$ **0.033 fm** (6 GeV): ($a \geq$ 0.07 fm)
2. ***finite volume effects:*** $L*m_{\pi} \gg 1$

   => $L \geq$ **4.5 fm** ($L \leq$ 3 fm)
3. ***chiral regime***: **200 ≤ m_{π} ≤ 300 MeV**



courtesy of G. Herdoiza